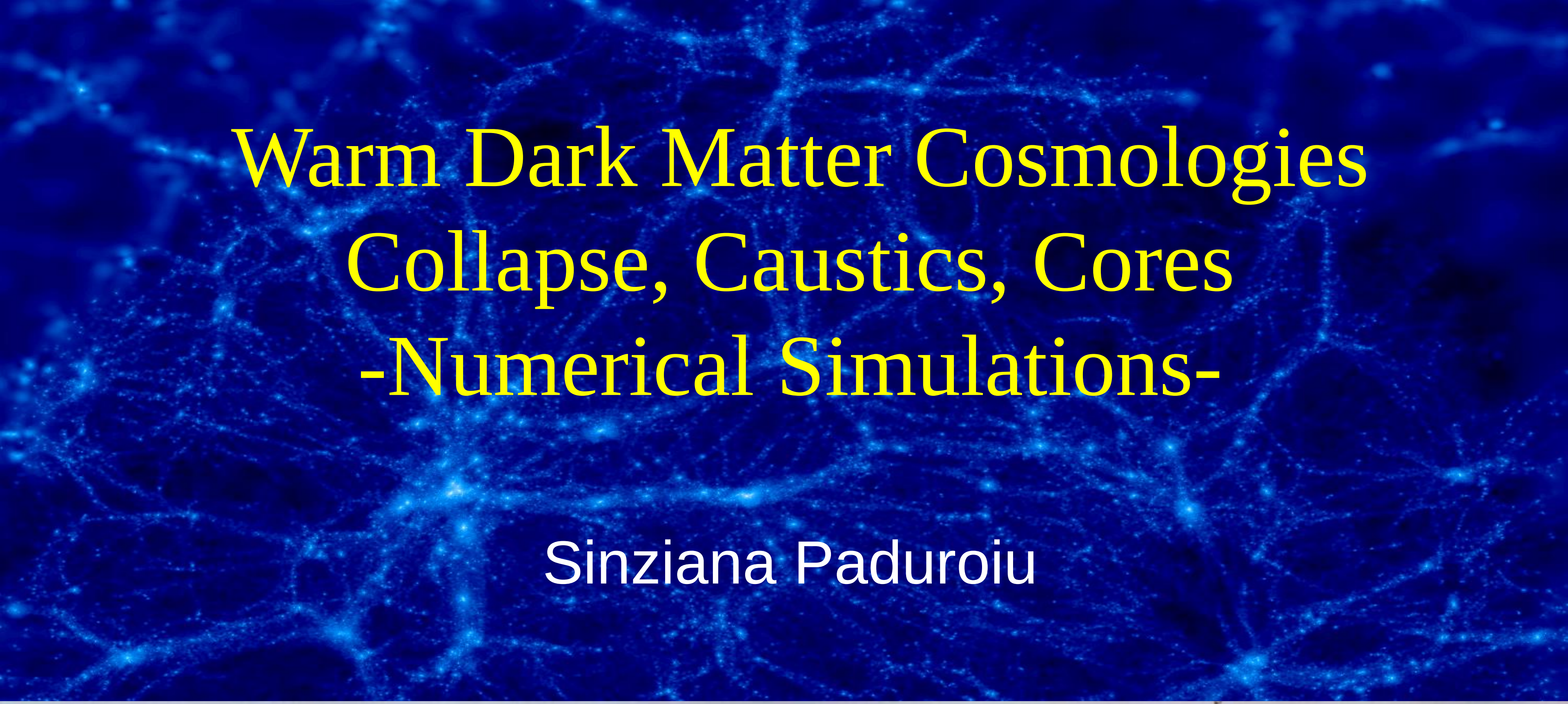
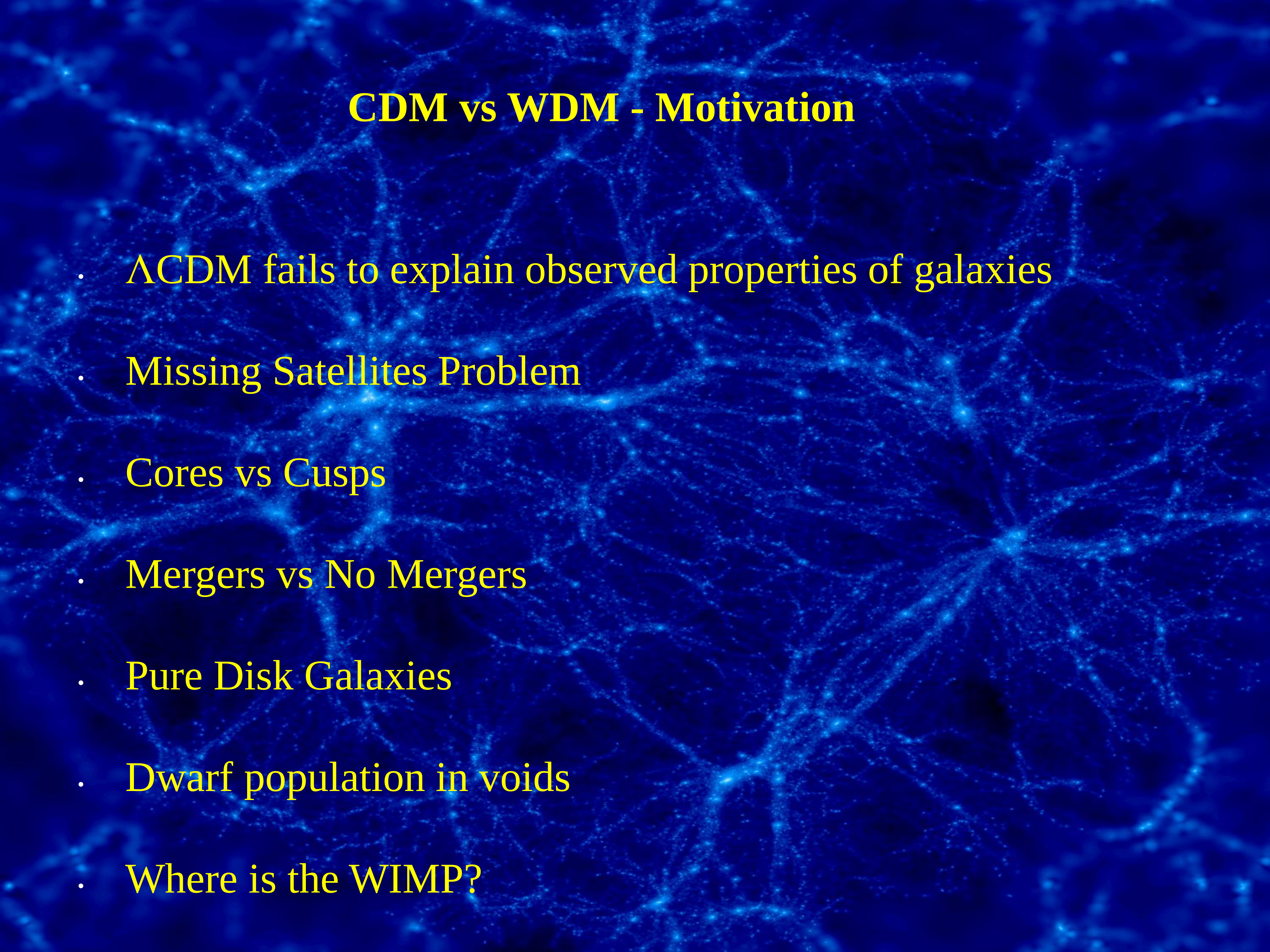


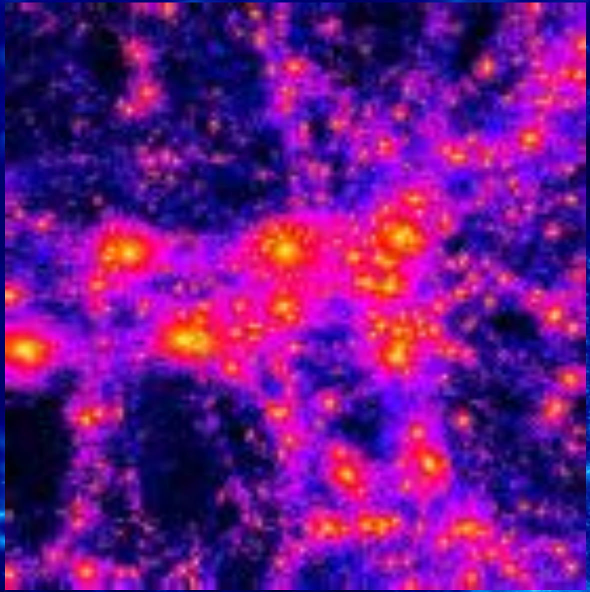
Warm Dark Matter Cosmologies
Collapse, Caustics, Cores
-Numerical Simulations-

Sinziana Paduroiu

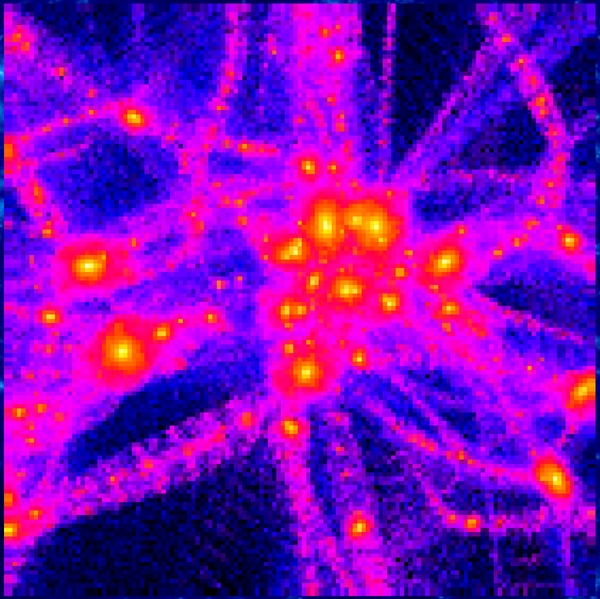


CDM vs WDM - Motivation

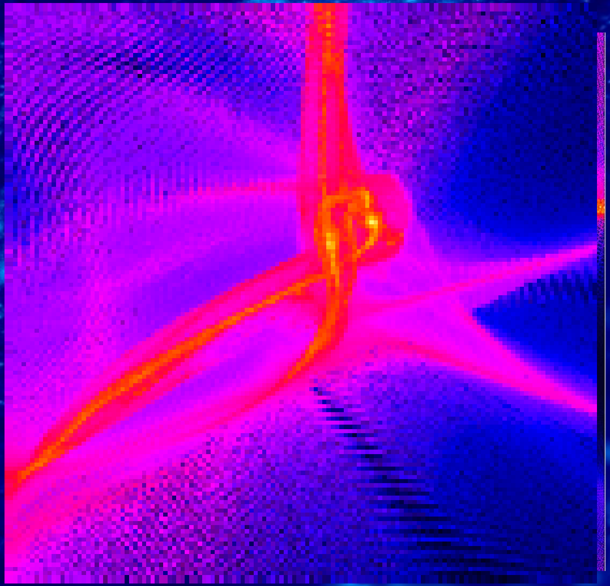
- ΛCDM fails to explain observed properties of galaxies
- Missing Satellites Problem
- Cores vs Cusps
- Mergers vs No Mergers
- Pure Disk Galaxies
- Dwarf population in voids
- Where is the WIMP?



CDM



WDM – 200 eV



HDM – 50 eV

Simulating the Warm Dark Matter - The Challenges

- How to treat the particles?
- How to cut the power spectrum?
- What about velocities ? The impact of velocity disperson.
- How to compare WDM sims with CDM sims and observations
- Resolution and softening
- Fragmentation
- Structure formation and its hidden treasures
- Halo internal structure
- The trustworthy factor and the catch 22

Simulating the WDM…

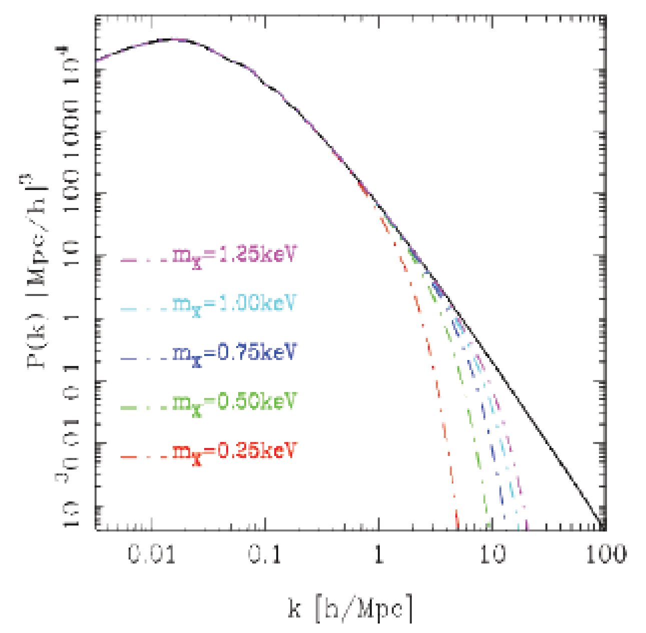
$$\frac{v_0(z)}{1+z} = .012 \left(\frac{\Omega_X}{0.3}\right)^{\frac{1}{3}} \left(\frac{h}{0.65}\right)^{\frac{2}{3}} \left(\frac{1.5}{g_X}\right)^{\frac{1}{3}} \left(\frac{keV}{m_X}\right)^{\frac{4}{3}} \mathrm{km\,s^{-1}}$$

Bode, Turok, and Ostriker 2001

$$T^2(k) = \frac{P^{WDM}}{P^{CDM}} = [1 + (\alpha k)^{2\nu}]^{-10/\nu}$$

$$\alpha = 0.049 \cdot \left(\frac{m_x}{1keV}\right)^{-1.11} \cdot \left(\frac{\Omega_\nu}{0.25}\right)^{0.11} \cdot \left(\frac{h}{0.7}\right)^{1.22} h^{-1}\mathrm{Mpc}.$$

Viel et al. 2005



$$k_s \approx \left(\frac{0.3}{\Omega_X}\right)^{0.15} \left(\frac{m_X}{keV}\right)^{1.15} Mpc^{-1}$$

Smith & Markovic 2011

Speed Matters

Bode, Ostriker & Turok 2001

Assumptions: entropy production & negligible chemical potential

$$\frac{n_X}{n_\gamma} = \left(\frac{43/4}{g_{\rm dec}}\right)\left(\frac{4}{11}\right)\frac{g_X}{2}$$

$$\Omega_X h^2 \approx \frac{115}{g_{\rm dec}}\frac{g_X}{1.5}\frac{m_X}{\rm keV} \; .$$

$$\frac{v_0(z)}{1+z} = .012\left(\frac{\Omega_X}{0.3}\right)^{\frac{1}{3}}\left(\frac{h}{0.65}\right)^{\frac{2}{3}}\left(\frac{1.5}{g_X}\right)^{\frac{1}{3}}\left(\frac{keV}{m_X}\right)^{\frac{4}{3}} \,{\rm km\,s^{-1}}$$

$$(\exp(v/v_0)+1)^{-1}$$

$$\frac{v_0(z)}{1+z} \approx 0.12\left(\frac{1}{g_{\rm dec}}\right)^{1/3}\frac{\rm keV}{m_X}\,{\rm km\,s^{-1}}$$

$$g_{\rm dec} = 1000\,(g_X/1.5)^{1/3}$$

Different Assumptions:

Paduroiu et al. 2015

- number conservation
- non-entropy production
- quantum pressure
- thermalization caused by an exchange potential

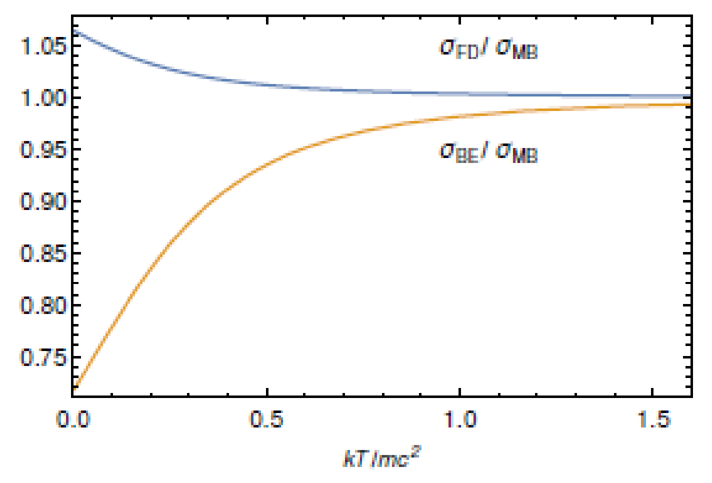
$$f(\mathrm{p}) = \frac{g}{(2\pi\hbar)^3}\frac{1}{\exp((\epsilon-\mu)/kT)\pm 1}$$

$$n(T,\mu) = \frac{4\pi g}{h^3}\int_0^\infty \frac{p^2}{\exp((\epsilon-\mu)/kT)\pm 1}\,dp$$

$$\epsilon(T,\mu) = \frac{4\pi g}{h^3}\int_0^\infty \frac{p^2\epsilon}{\exp((\epsilon-\mu)/kT)\pm 1}\,dp$$

$$P(T,\mu) = \frac{4\pi g}{h^3}\int_0^\infty \frac{p^2}{\exp((\epsilon-\mu)/kT)\pm 1}\,\frac{1}{3}\,\frac{c^2p^2}{\epsilon+mc^2}\,dp$$

$$v \;=\; 0.2226\left(\frac{n}{115\,\mathrm{cm}^{-3}}\frac{1}{g}\right)^{1/3}\left(\frac{mc^2}{\mathrm{keV}}\right)^{-1}\mathrm{km\,s}^{-1}$$



CAVEAT EMPTOR

$$\frac{v_0(z)}{1+z} = .012\left(\frac{\Omega_X}{0.3}\right)^{\frac{1}{3}}\left(\frac{h}{0.65}\right)^{\frac{2}{3}}\left(\frac{1.5}{g_X}\right)^{\frac{1}{3}}\left(\frac{keV}{m_X}\right)^{\frac{4}{3}}\,\mathrm{km\,s^{-1}}$$

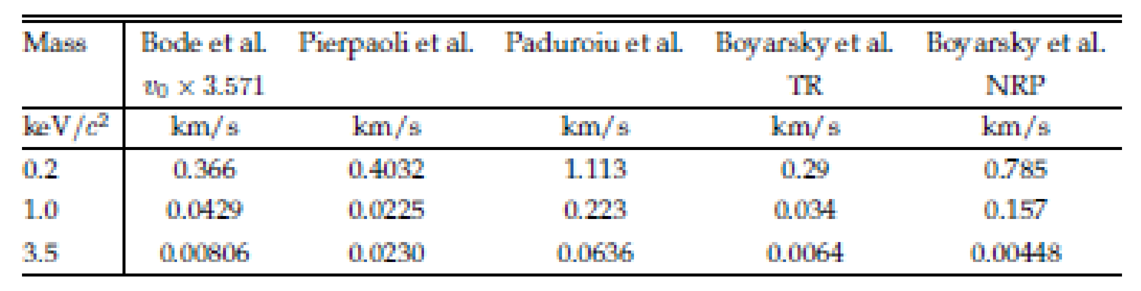
Bode et al. 2001

$$\Omega_X h^2 \approx \frac{115}{g_{\rm dec}}\,\frac{g_X}{1.5}\,\frac{m_X}{\rm keV}\ .$$

'valid' only for the case in which the full dark matter content is made up by only one type of particles

$$v = 0.2226\left(\frac{n}{115\,\mathrm{cm^{-3}}}\frac{1}{g}\right)^{1/3}\left(\frac{mc^2}{\rm keV}\right)^{-1}\,\mathrm{km\,s^{-1}}$$

Paduroiu et al. 2015



Mass	Bode et al. $v_0 \times 3.571$	Pierpaoli et al.	Paduroiu et al.	Boyarsky et al. TR	Boyarsky et al. NRP
keV/c^2	km/s	km/s	km/s	km/s	km/s
0.2	0.366	0.4032	1.113	0.29	0.785
1.0	0.0429	0.0225	0.223	0.034	0.157
3.5	0.00806	0.0230	0.0636	0.0064	0.00448

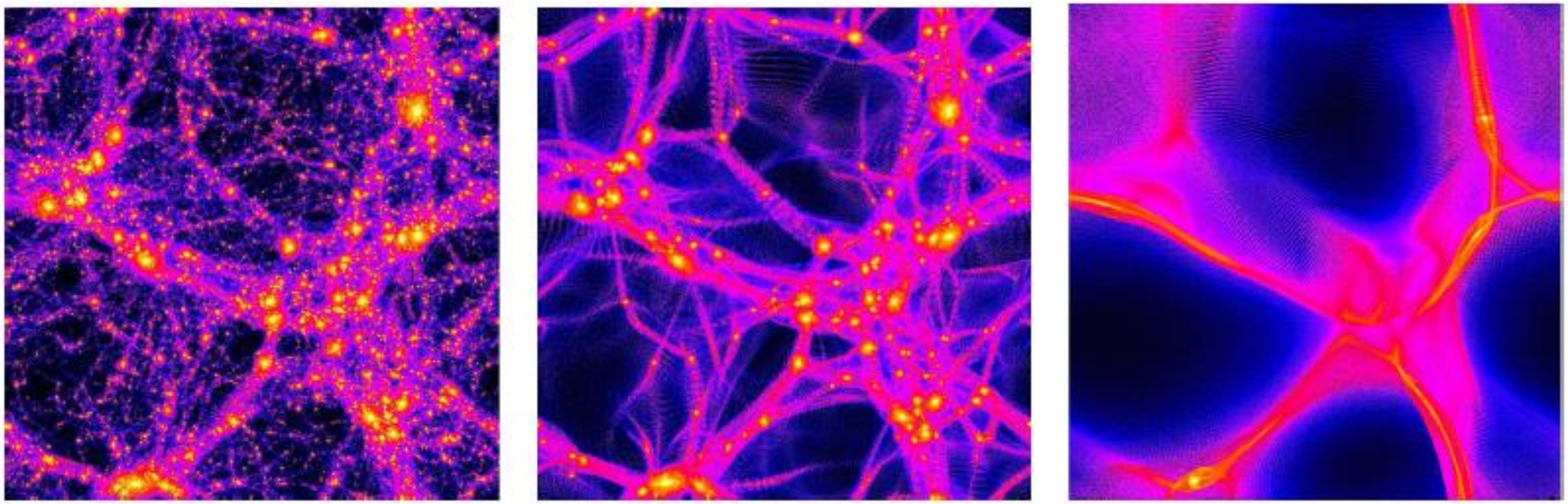


Figure 1. Three snapshots of different simulations at redshift $z = 0$. CDM, WDM3 and WDM4 are shown from left to right.

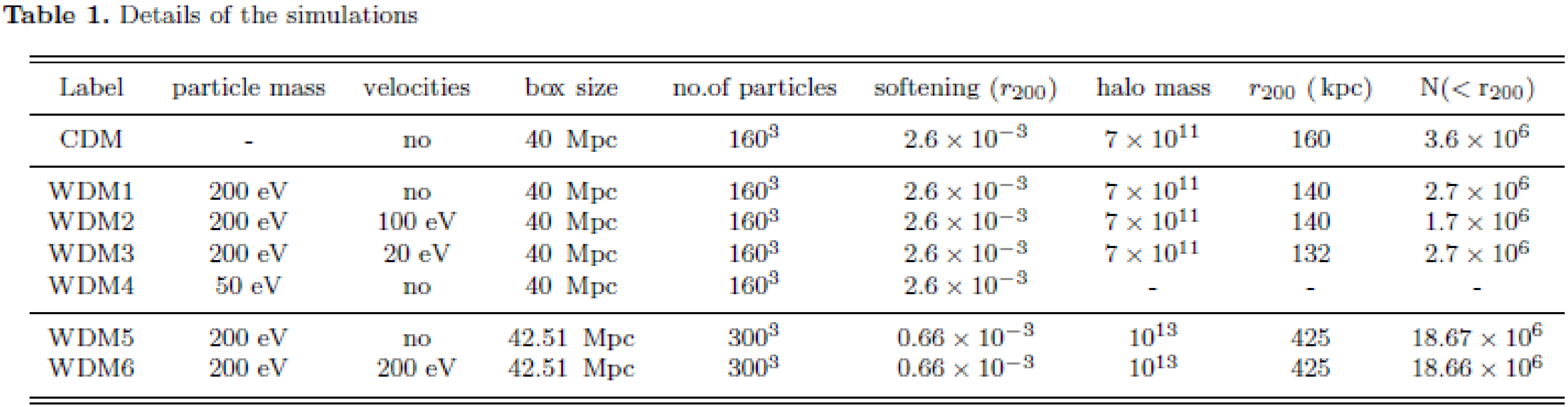
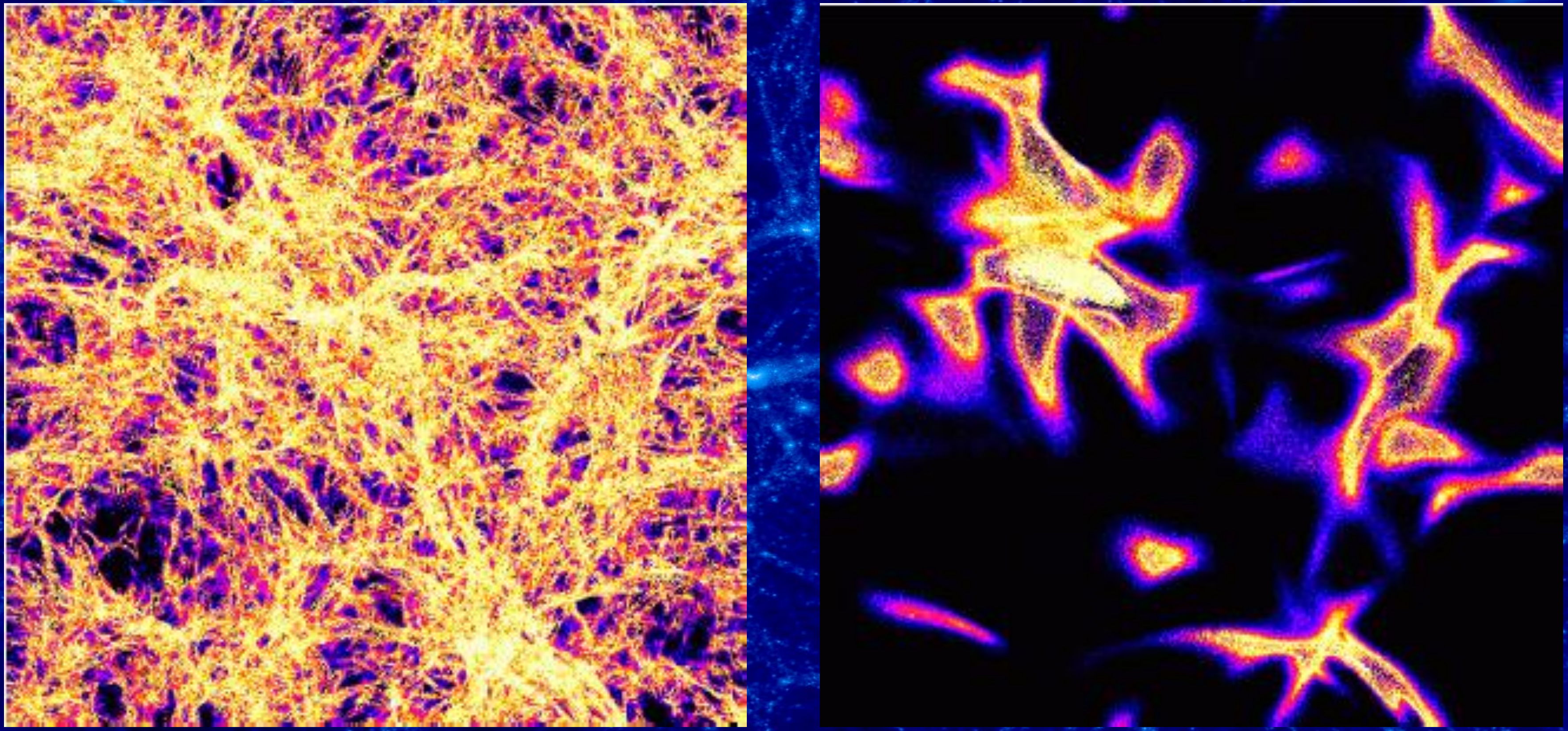


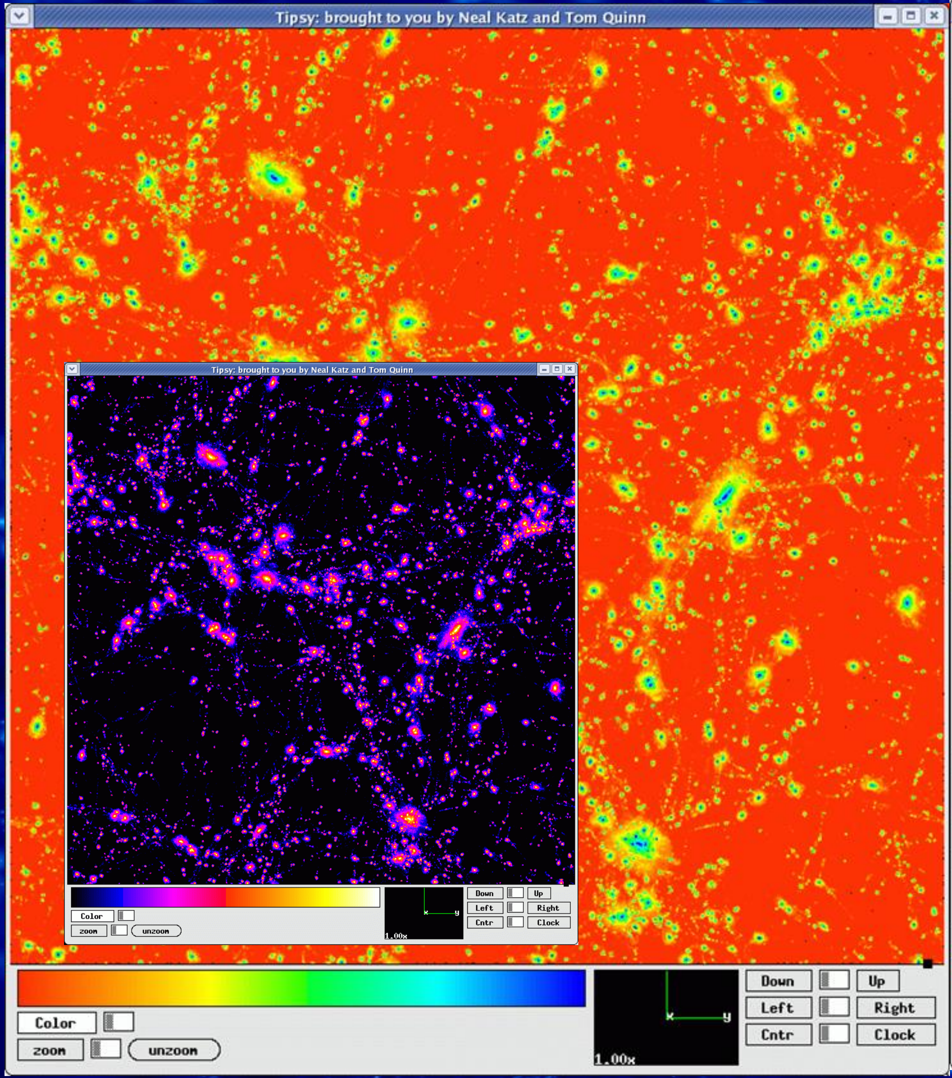
Table 1. Details of the simulations

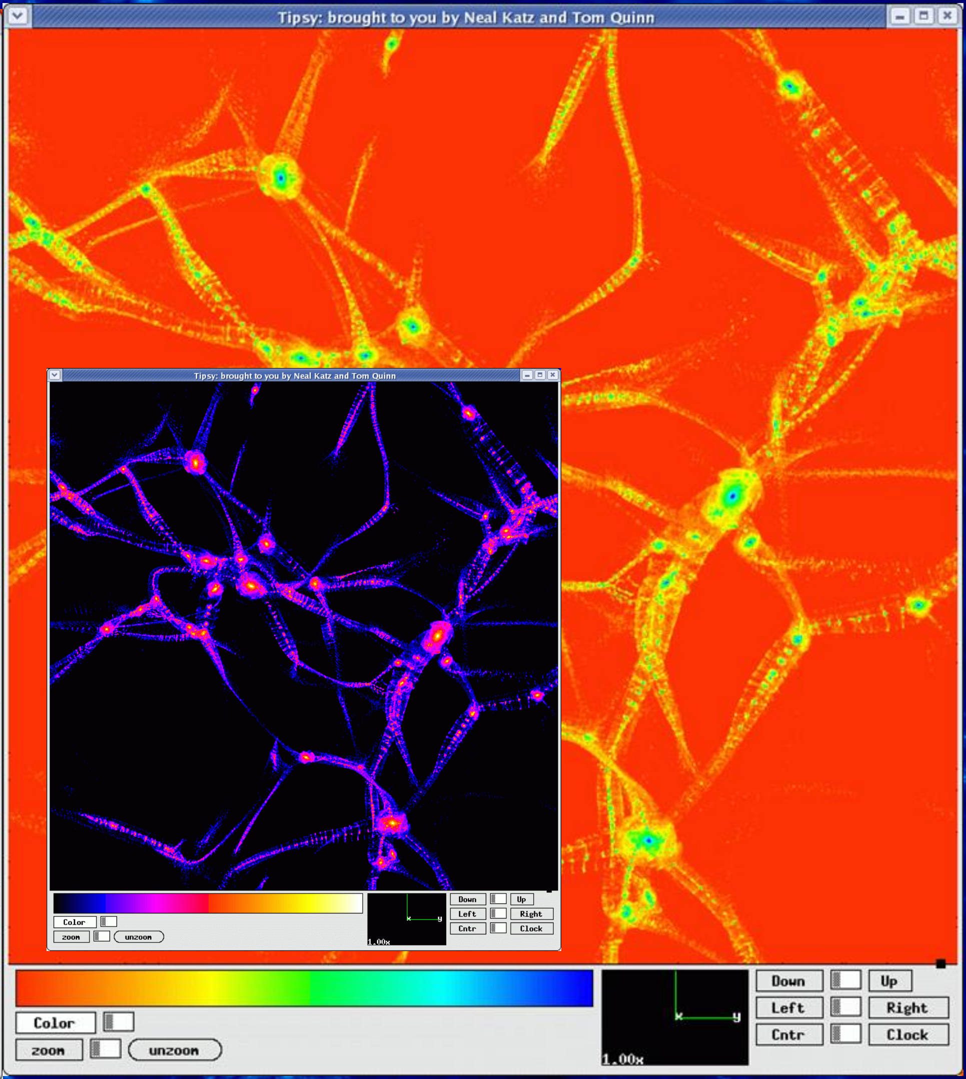
Label	particle mass	velocities	box size	no.of particles	softening (r_{200})	halo mass	r_{200} (kpc)	N($< r_{200}$)
CDM	-	no	40 Mpc	160^3	2.6×10^{-3}	7×10^{11}	160	3.6×10^6
WDM1	200 eV	no	40 Mpc	160^3	2.6×10^{-3}	7×10^{11}	140	2.7×10^6
WDM2	200 eV	100 eV	40 Mpc	160^3	2.6×10^{-3}	7×10^{11}	140	1.7×10^6
WDM3	200 eV	20 eV	40 Mpc	160^3	2.6×10^{-3}	7×10^{11}	132	2.7×10^6
WDM4	50 eV	no	40 Mpc	160^3	2.6×10^{-3}	-	-	-
WDM5	200 eV	no	42.51 Mpc	300^3	0.66×10^{-3}	10^{13}	425	18.67×10^6
WDM6	200 eV	200 eV	42.51 Mpc	300^3	0.66×10^{-3}	10^{13}	425	18.66×10^6



Mildly non-linear regions at z=3 in CDM and WDM

i.e. overdensities between 1 and 5 w.r.t. mean



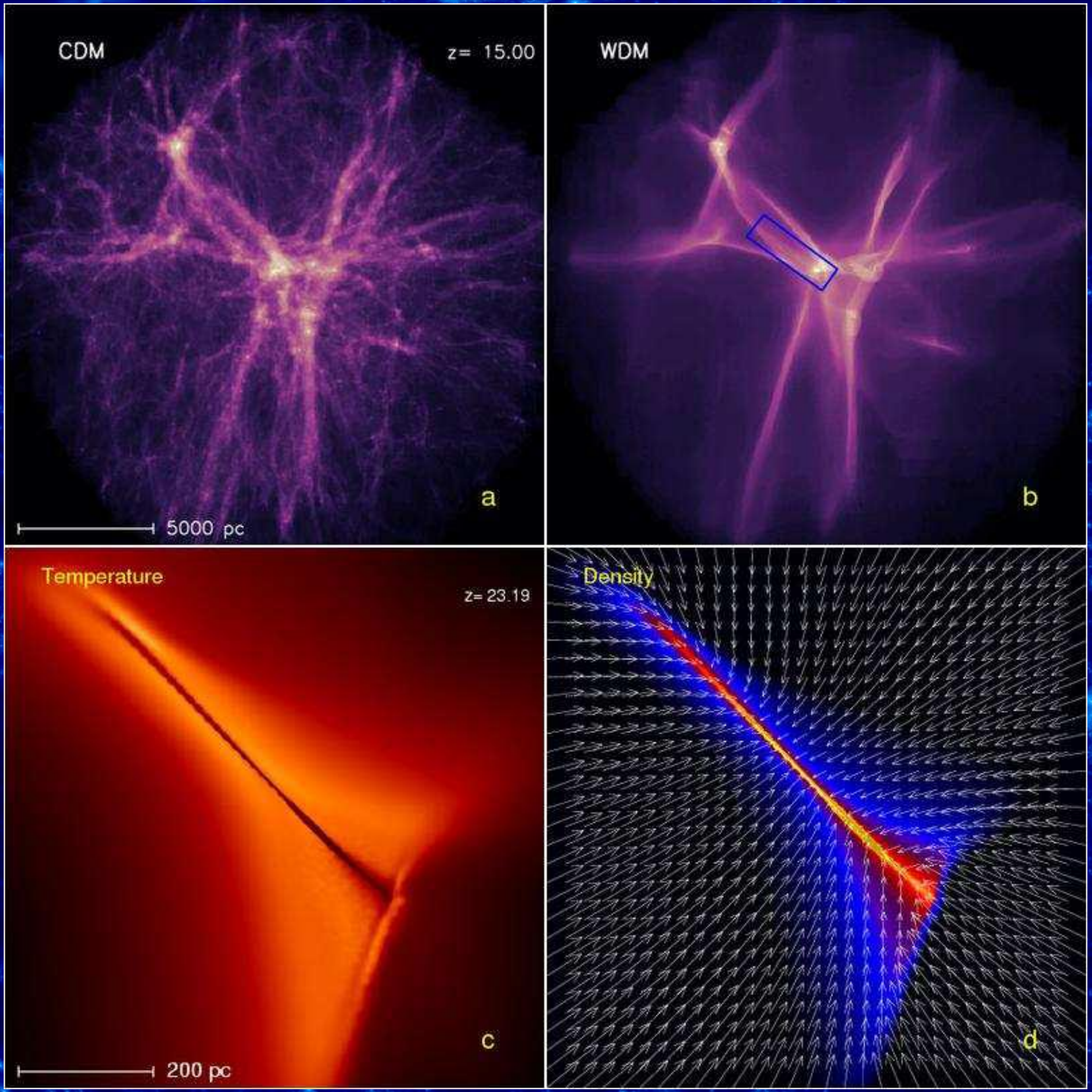


Virialised regions at z=3 in CDM and WDM

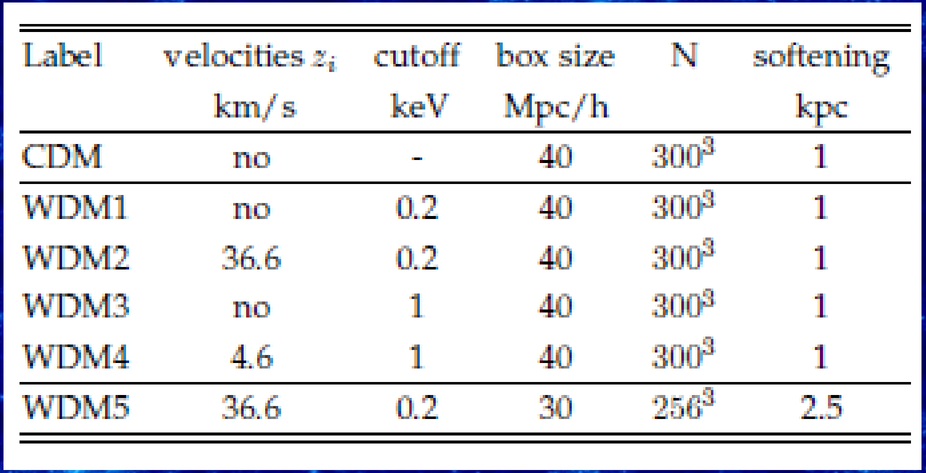
i.e. overdensities higher than 100 w.r.t. mean

Thermal Velocities

No Thermal Velocities

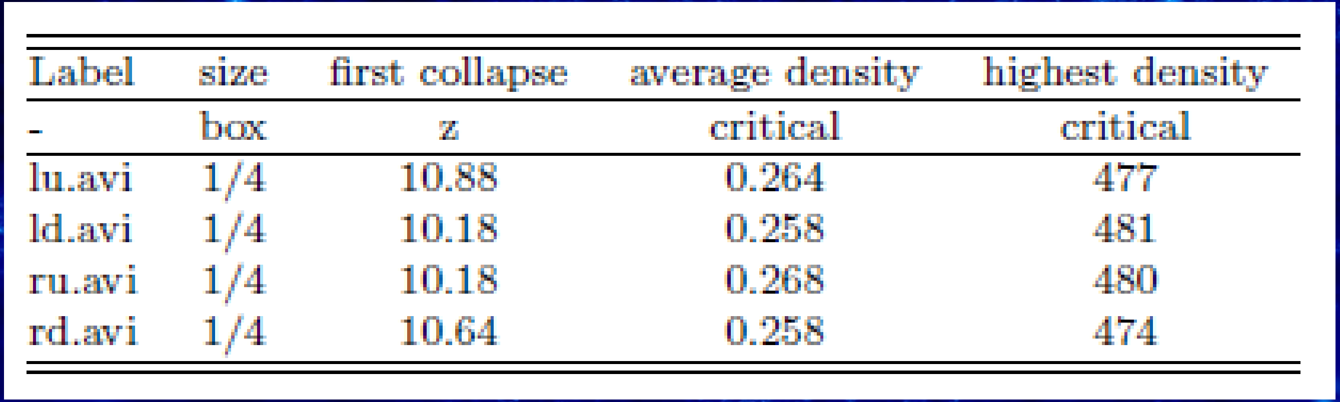


Liang Gao & Tom Theuns, Science 2007



Label	velocities z_i km/s	cutoff keV	box size Mpc/h	N	softening kpc
CDM	no	-	40	300^3	1
WDM1	no	0.2	40	300^3	1
WDM2	36.6	0.2	40	300^3	1
WDM3	no	1	40	300^3	1
WDM4	4.6	1	40	300^3	1
WDM5	36.6	0.2	30	256^3	2.5

Simulations details: 2.72x10^5 M$_o$ / particle
355 pc spline gravitational softening
WMAP7 cosmological parameters
z=100 initial redshift



Label	size	first collapse	average density	highest density
-	box	z	critical	critical
lu.avi	1/4	10.88	0.264	477
ld.avi	1/4	10.18	0.258	481
ru.avi	1/4	10.18	0.268	480
rd.avi	1/4	10.64	0.258	474

Movie Time…

https://www.youtube.com/playlist?list=PLnGS4wkStJ1aqi3M9hTDaUzuZ-vs-Qg6i

or on demand …

Hybrid mechanism of structure formation

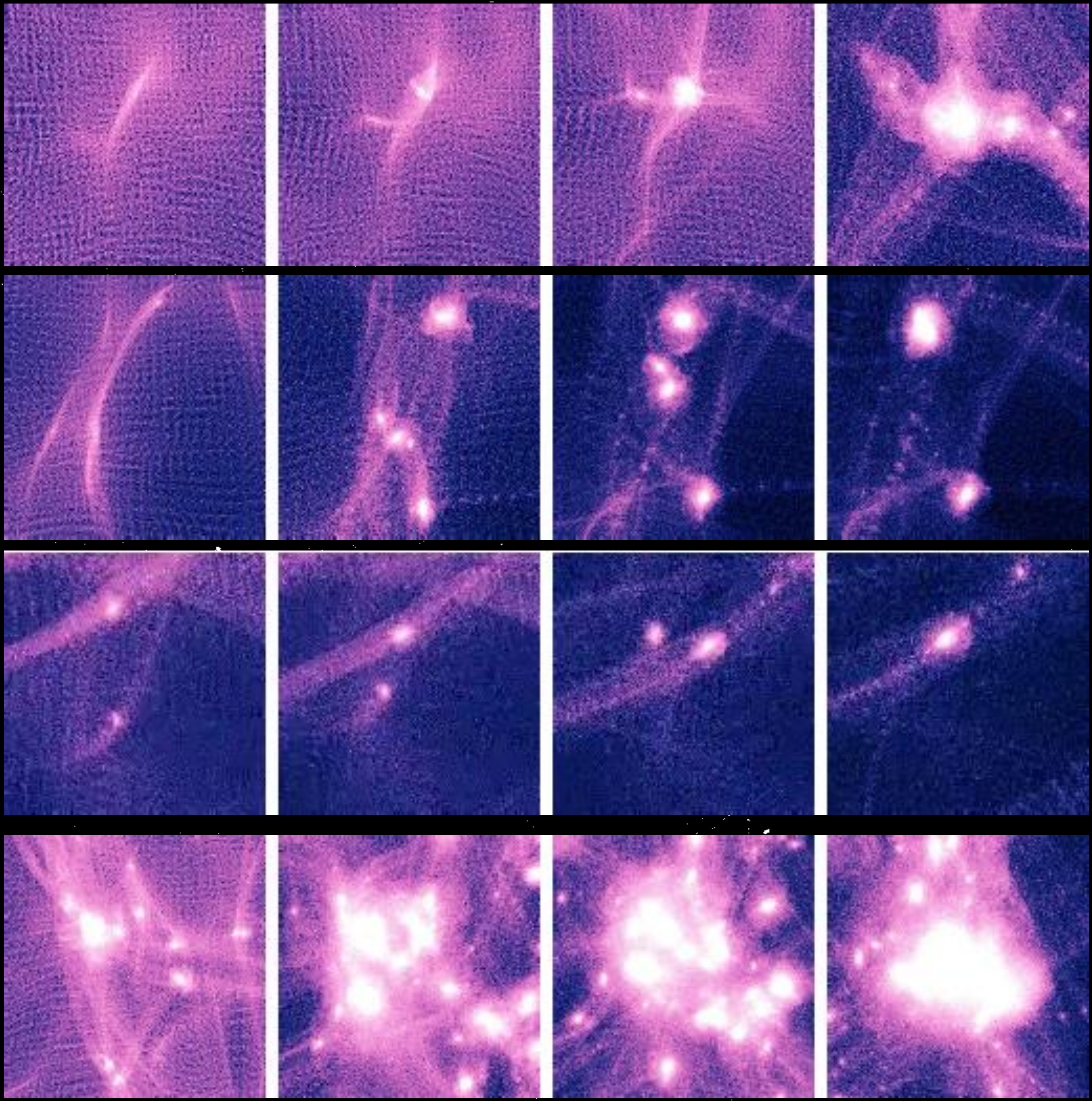
- During the early stages one sees the formation of well contoured filaments.

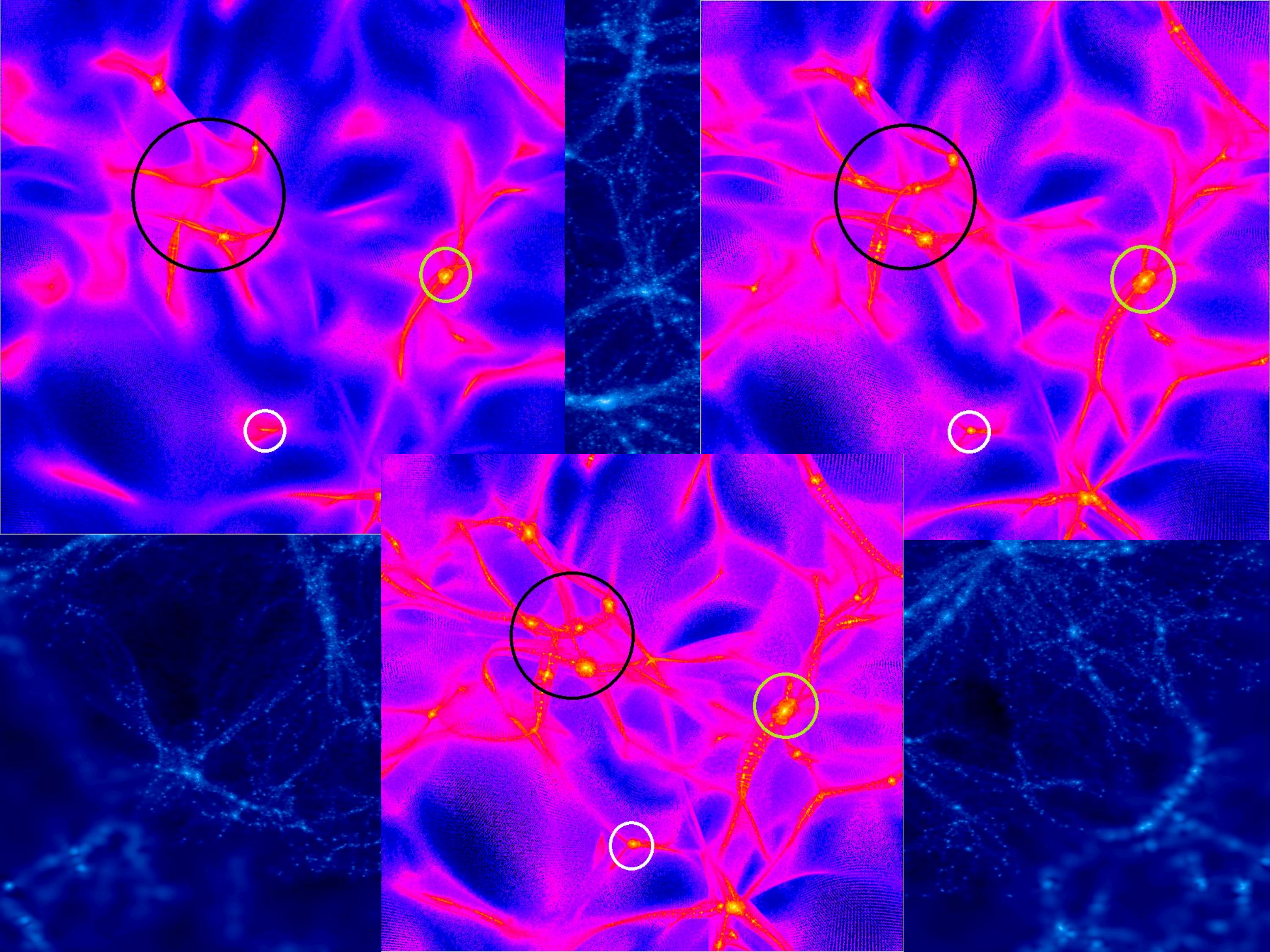
- In the higher density regions, usually situated at the intersection of such filaments, the first halos are formed through gravitational collapse. These halos continue growing into larger ones by accreting particles from the disrupted filaments.

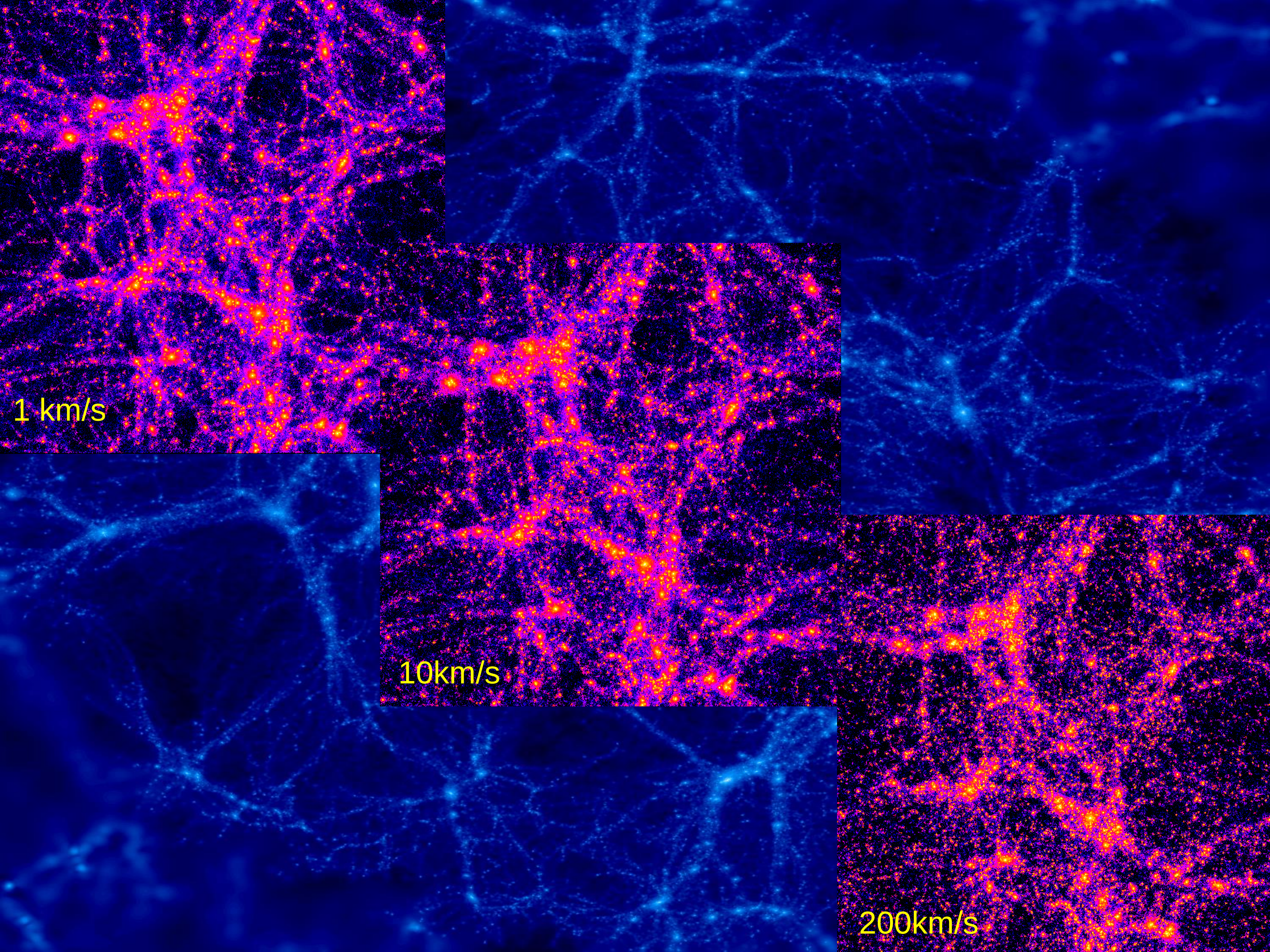
- In medium density regions, haloes show a hierarchical formation trend. Small haloes collapse first and then merge into bigger haloes.

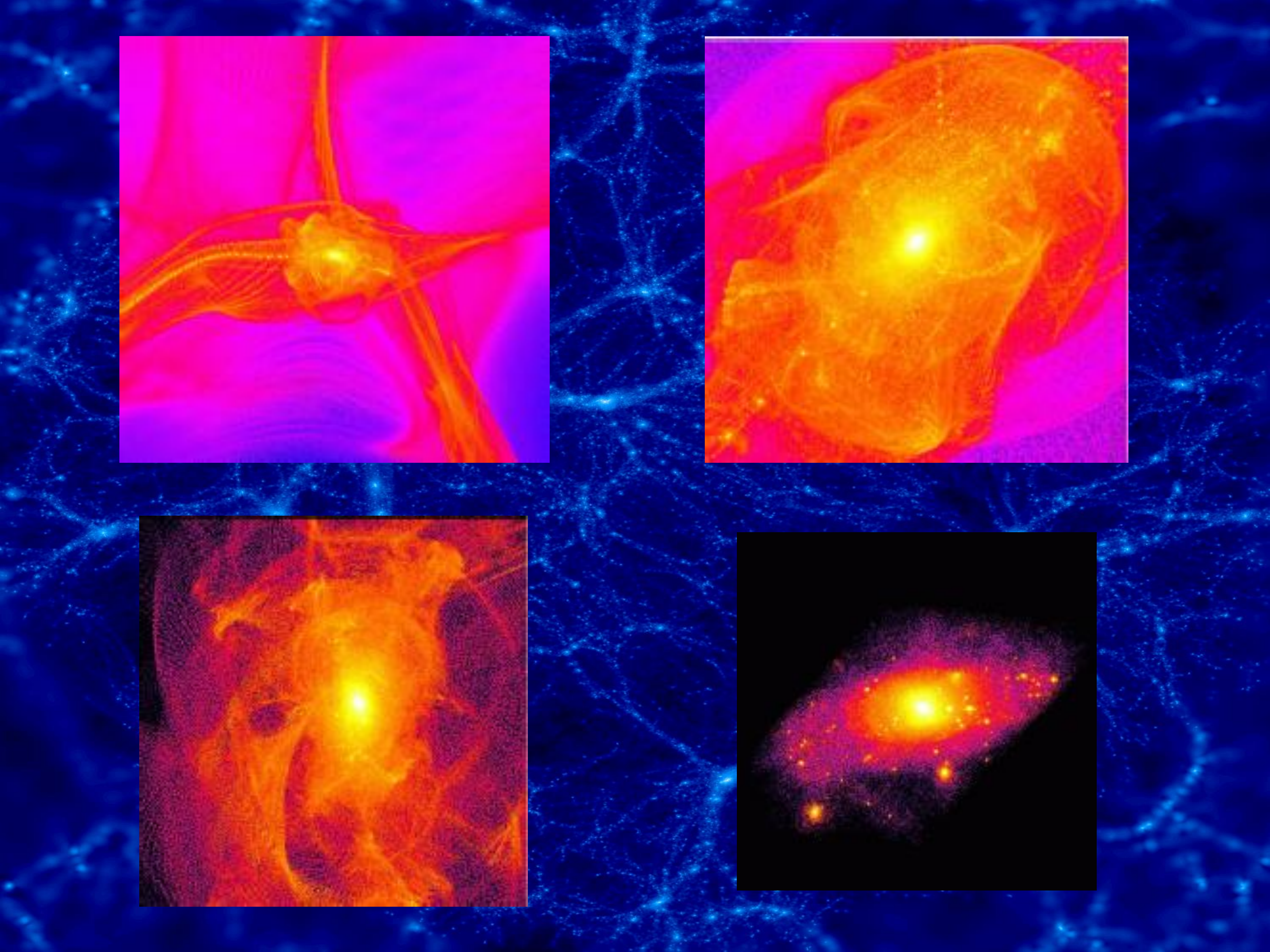
- In less dense regions, the ones isolated by voids and have a very slow evolution, we have observed filaments that collapse very late. The top down formed halo survives without any mergers until redshift zero.

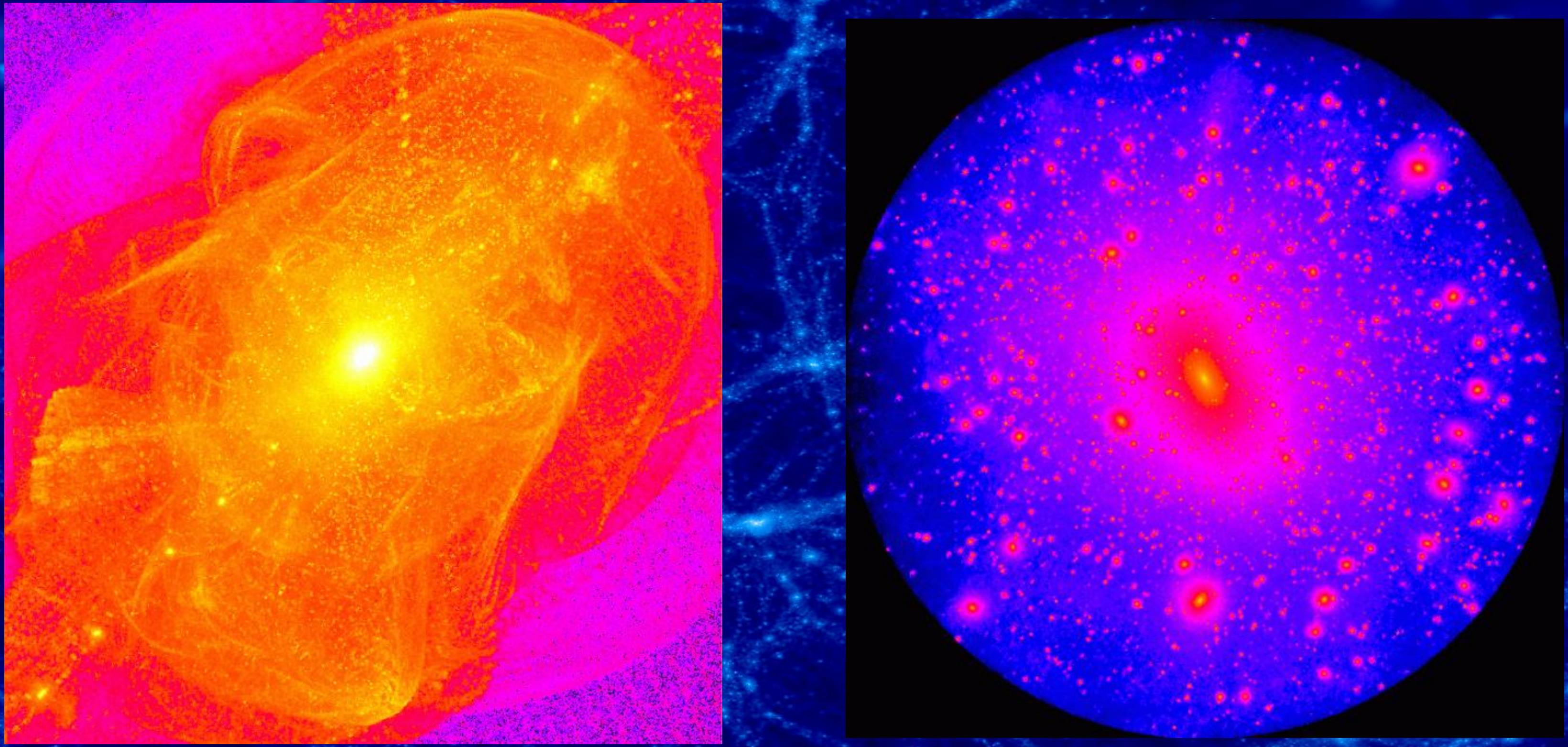
- Finally there is the more complex scenario in which we observe large haloes formed earlier which merge into clusters.






1 km/s
10km/s
200km/s

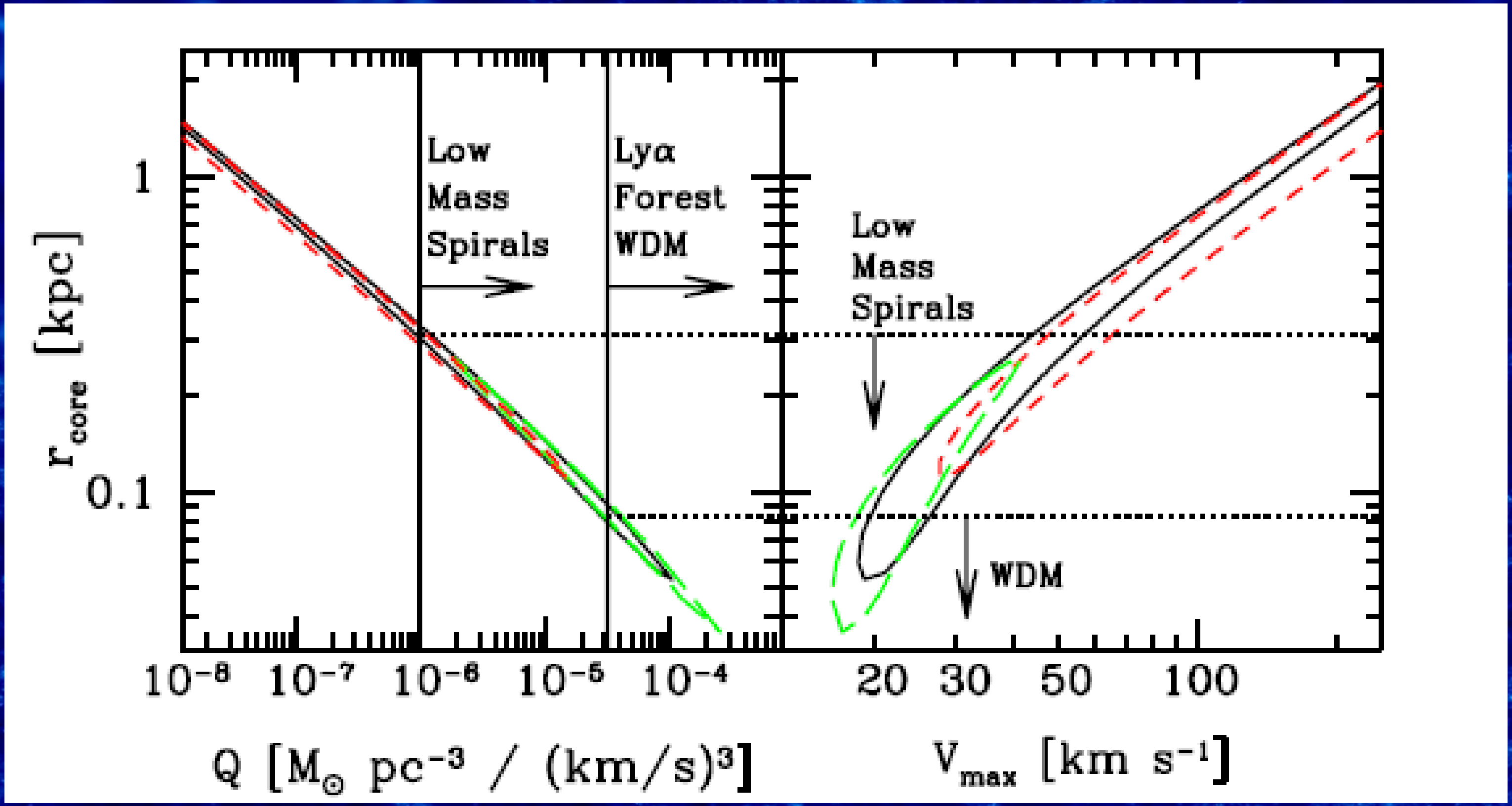




	Number of part.	r_{vir} [kpc]	mass p. part. $[M_\odot]$	boxsize [Mpc]
m<200eV	18 mil.	630	10^5	42.5
m>2keV	50 mil.	200	10^7	40

Assumptions in determining the core radius:

- Isothermal spheres
- Liouville – Phase space density (PSD) is conserved
- Pauli exclusion principle
- PSD constant as mixing occurs
- Velocity dispersion in central halo = constant
- Density profile in central halo = constant

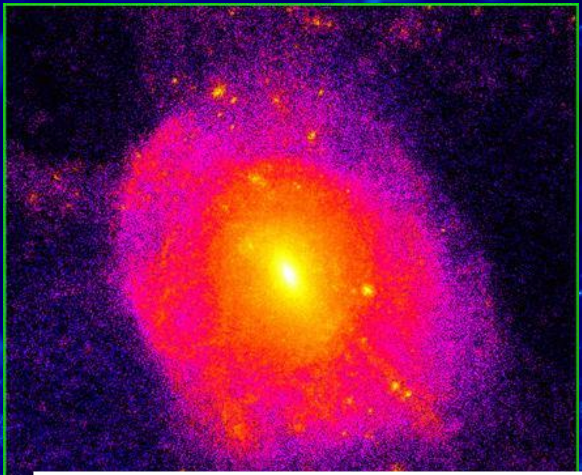


Constraints on the core radius of Fornax as a function of the central phase-space density and maximum circular velocity derived from the velocity dispersion profile

Strigari et al 2006

$$\rho(r) = \frac{\rho_0}{\left[1 + (r/r_0)^{\alpha}\right]^{3/\alpha}}$$

CATCH 22



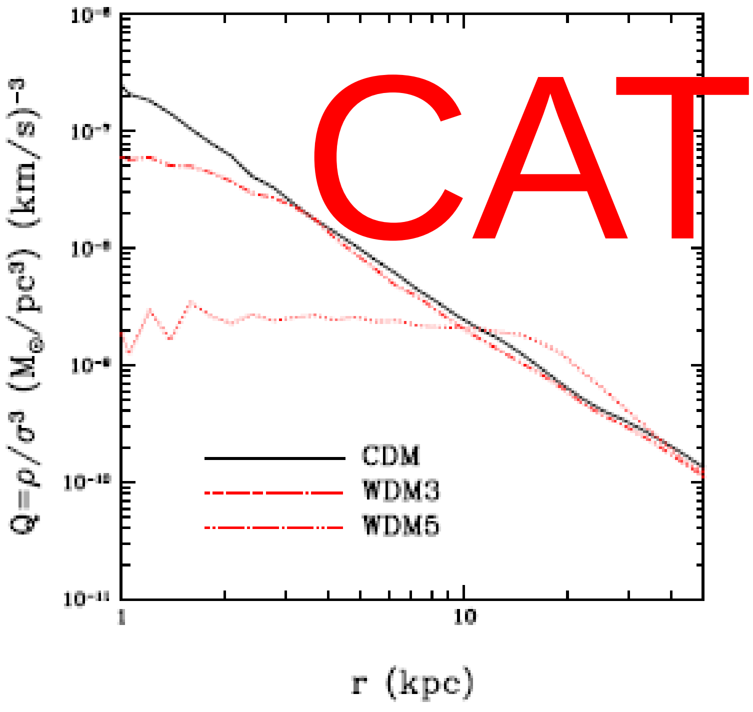


Figure 4. Phase space density profile for the CDM, WDM3 and WDM5 models at $z = 0$.

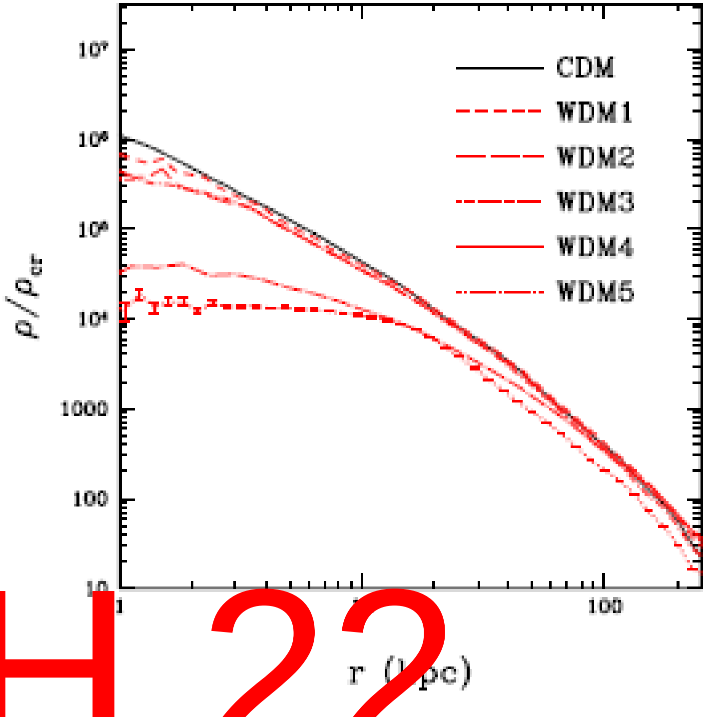
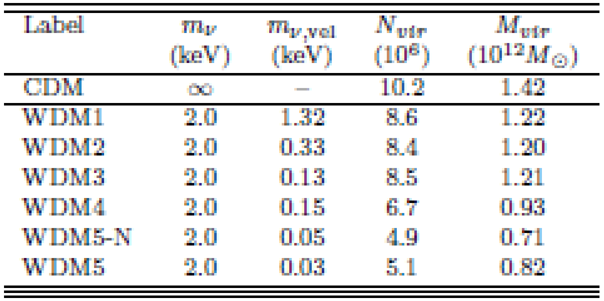


Figure [illegible]. The spherically averaged density profiles for CDM, WDM1-5 haloes.



Label	m_ν (keV)	$m_{\nu,vel}$ (keV)	N_{vir} (10^6)	M_{vir} ($10^{12} M_\odot$)
CDM	∞	–	10.2	1.42
WDM1	2.0	1.32	8.6	1.22
WDM2	2.0	0.33	8.4	1.20
WDM3	2.0	0.13	8.5	1.21
WDM4	2.0	0.15	6.7	0.93
WDM5-N	2.0	0.05	4.9	0.71
WDM5	2.0	0.03	5.1	0.82

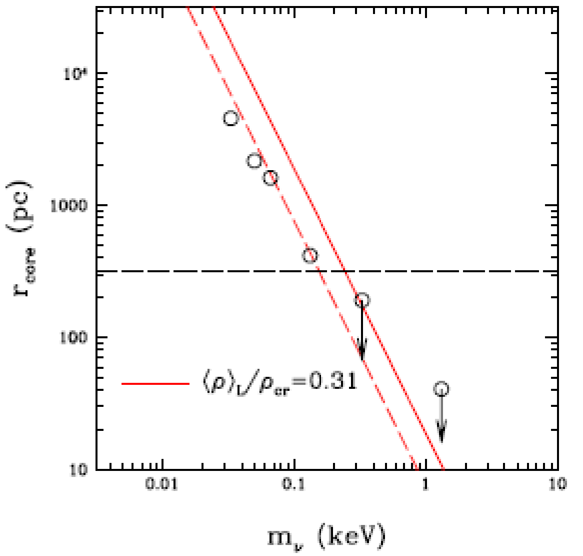


Figure 1. Comparison between core size in simulations (open symbols) and the theoretical expectation for a $M = 10^{12} M_\odot$ halo (solid line). The dashed horizontal line is the gravitational softening of our simulations. All points below this line should be considered as upper limits on the core size. The red dashed line is a linear fit to the simulation results.

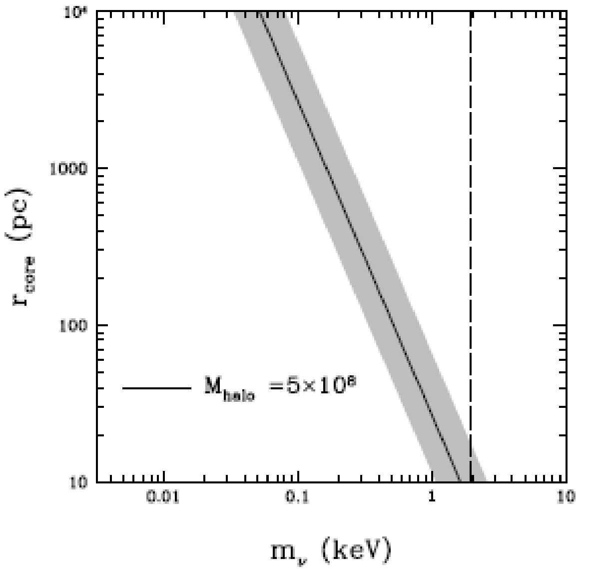
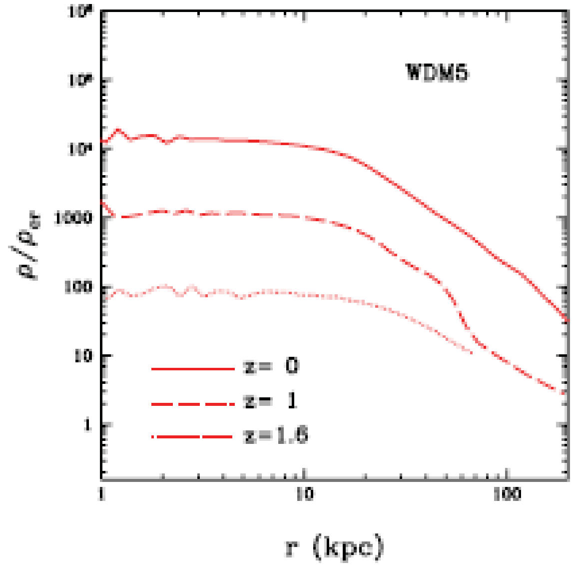
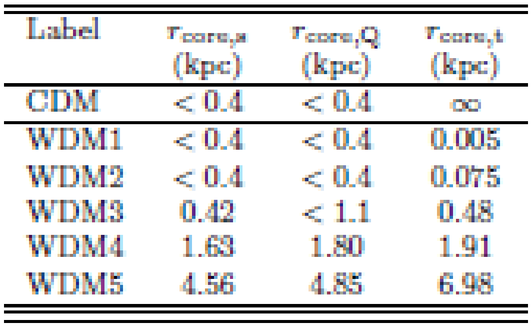


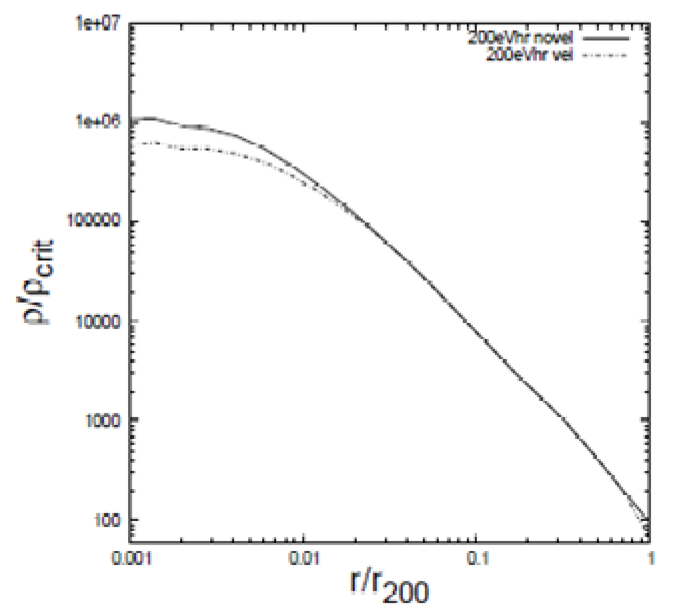
Figure 2. Expected core size for the typical dark matter mass of Milky Way satellites as a function of the WDM mass m_ν. The shaded area takes into account possible different values of the local density parameter $0.15 < \Omega_m < 0.6$. The vertical dashed line shows the current limits on the WDM mass from large scale structure observations.



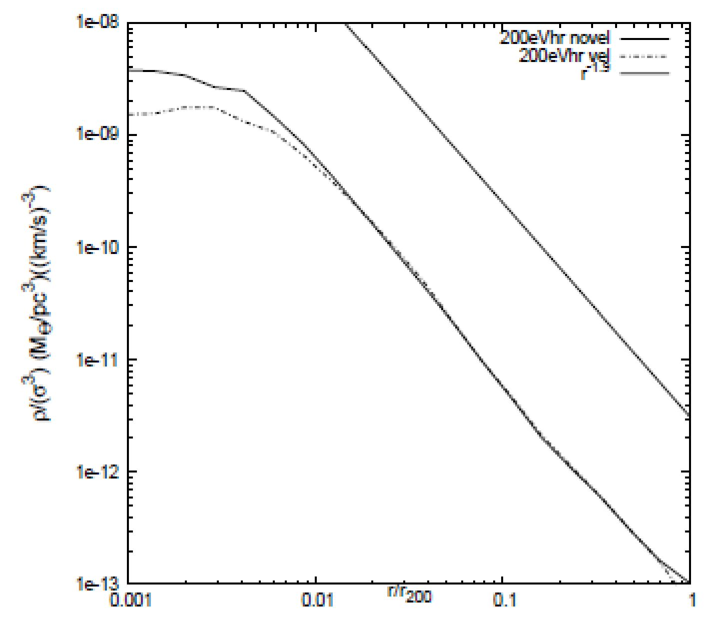


Label	$r_{\rm core,s}$ (kpc)	$r_{\rm core,Q}$ (kpc)	$r_{\rm core,t}$ (kpc)
CDM	< 0.4	< 0.4	∞
WDM1	< 0.4	< 0.4	0.005
WDM2	< 0.4	< 0.4	0.075
WDM3	0.42	< 1.1	0.48
WDM4	1.63	1.80	1.91
WDM5	4.56	4.85	6.98

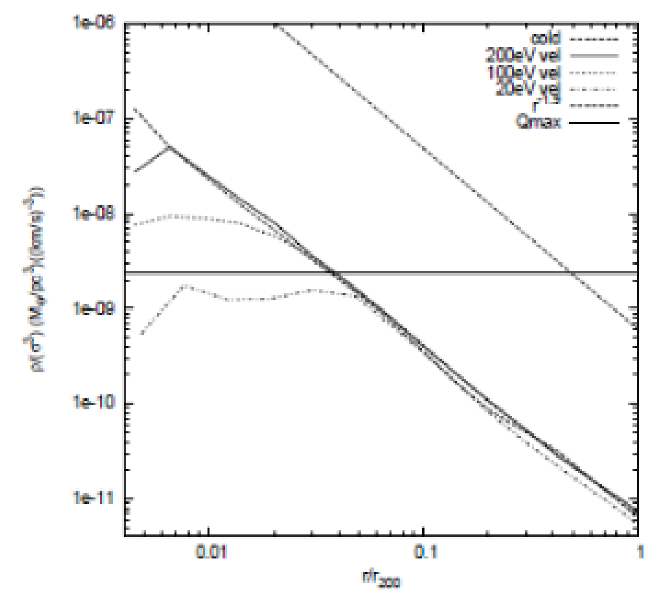
Maccio et al. 2012



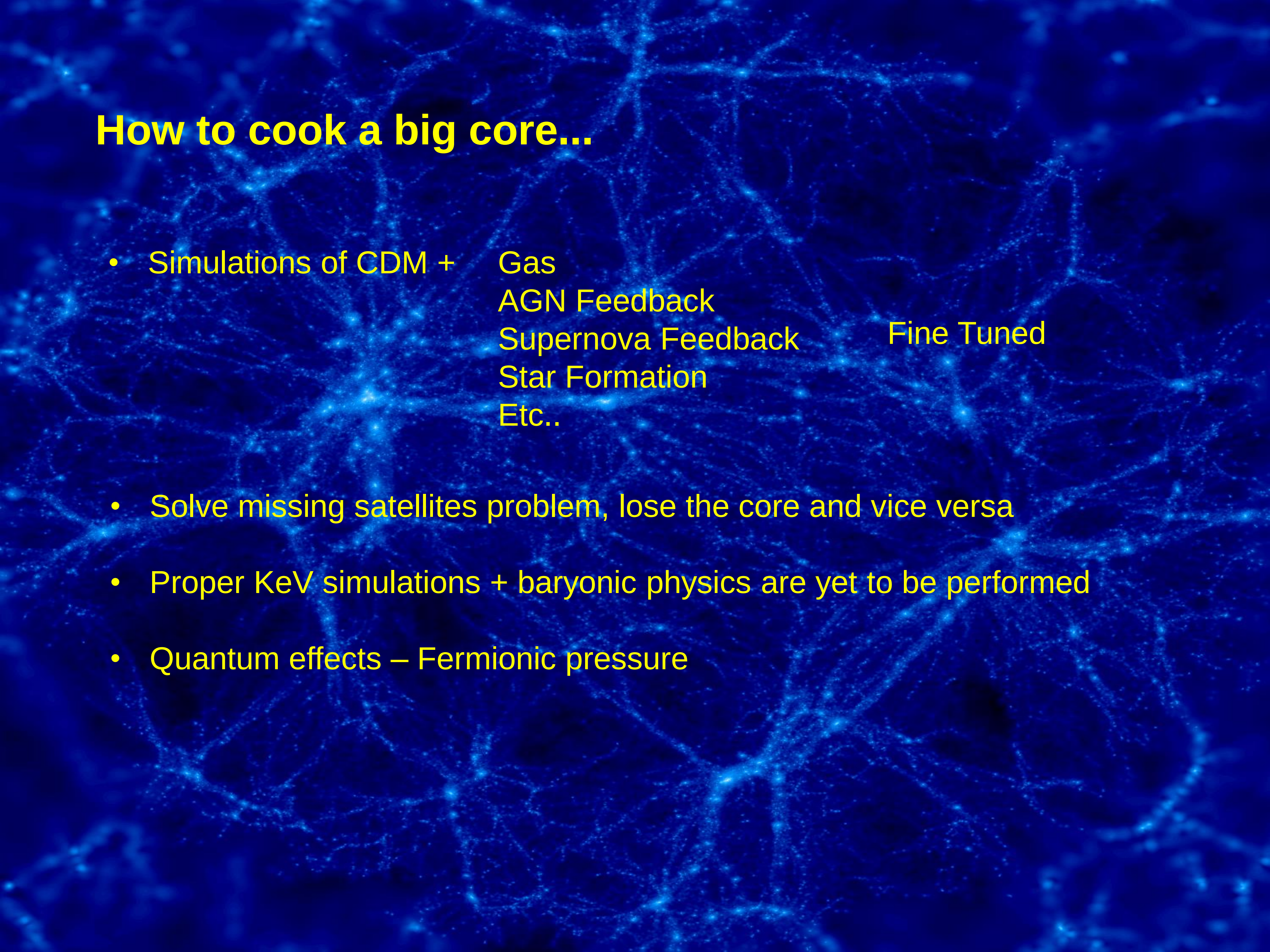
ρ/ρ_{crit}
1e+07
1e+06
100000
10000
1000
100
0.001
0.01
0.1
1
r/r_{200}
200eVhr novel
200eVhr vel



$\rho/(\sigma^3)$ $(M_\odot/pc^3)((km/s)^{-3})$
1e-08
1e-09
1e-10
1e-11
1e-12
1e-13
0.001
0.01
0.1
1
r/r_{200}
200eVhr novel
200eVhr vel
$r^{-1.9}$



$\rho/(\sigma^3)$ $(M_\odot/pc^3)((km/s)^{-3})$
1e-06
1e-07
1e-08
1e-09
1e-10
1e-11
0.01
0.1
1
r/r_{200}
cold
200eV vel
100eV vel
20eV vel
$r^{-1.9}$
Qmax

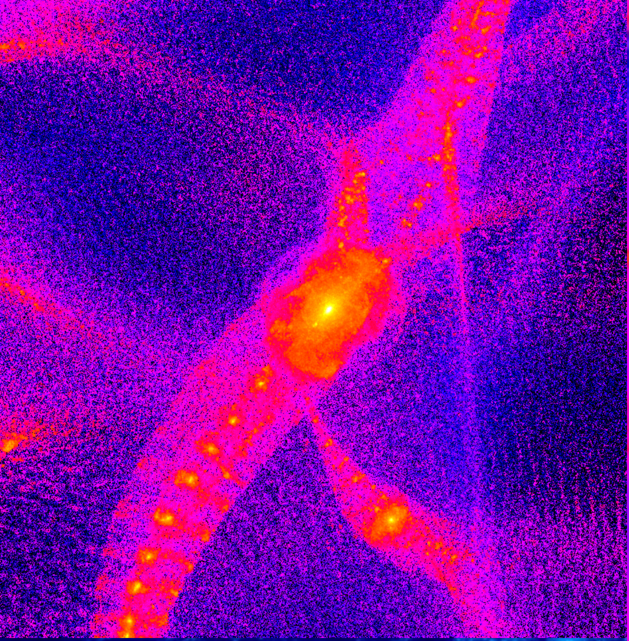


How to cook a big core...

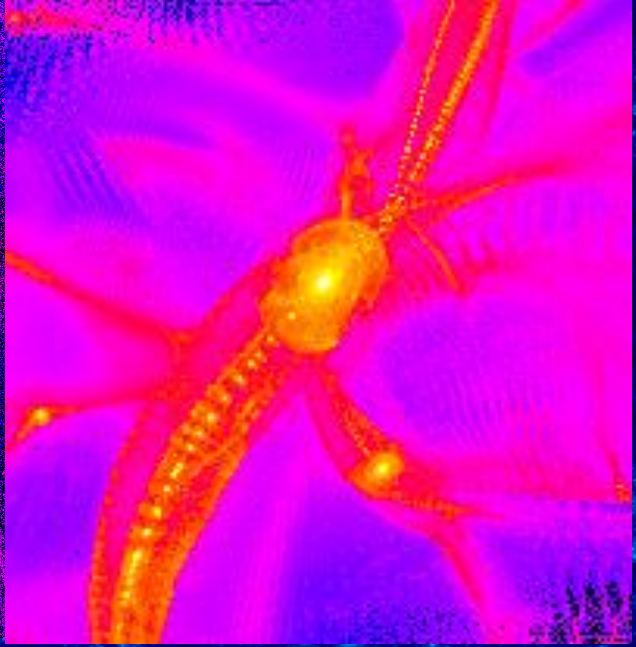
- Simulations of CDM + Gas
  AGN Feedback
  Supernova Feedback
  Star Formation
  Etc..

Fine Tuned

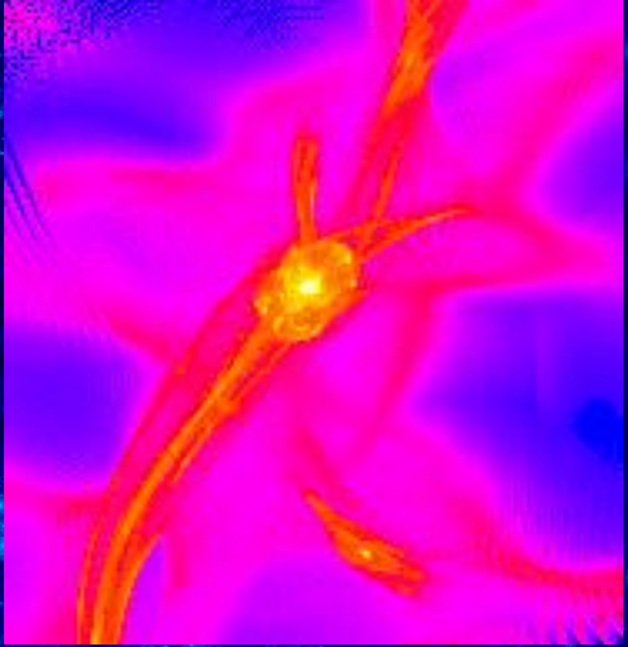
- Solve missing satellites problem, lose the core and vice versa
- Proper KeV simulations + baryonic physics are yet to be performed
- Quantum effects – Fermionic pressure



“Low” resolution

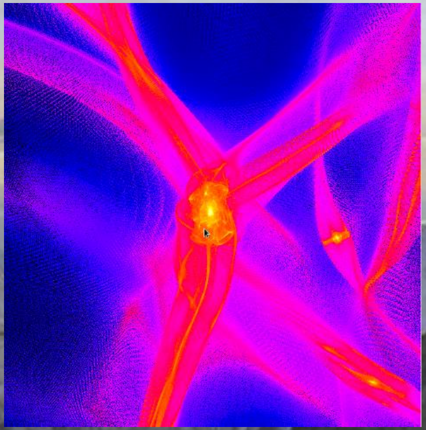


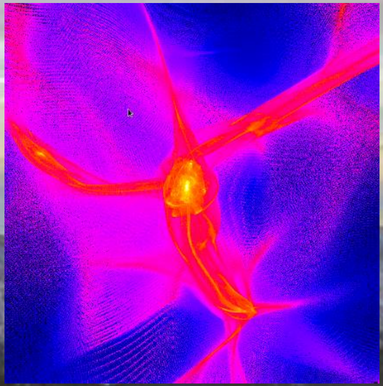
“High” resolution

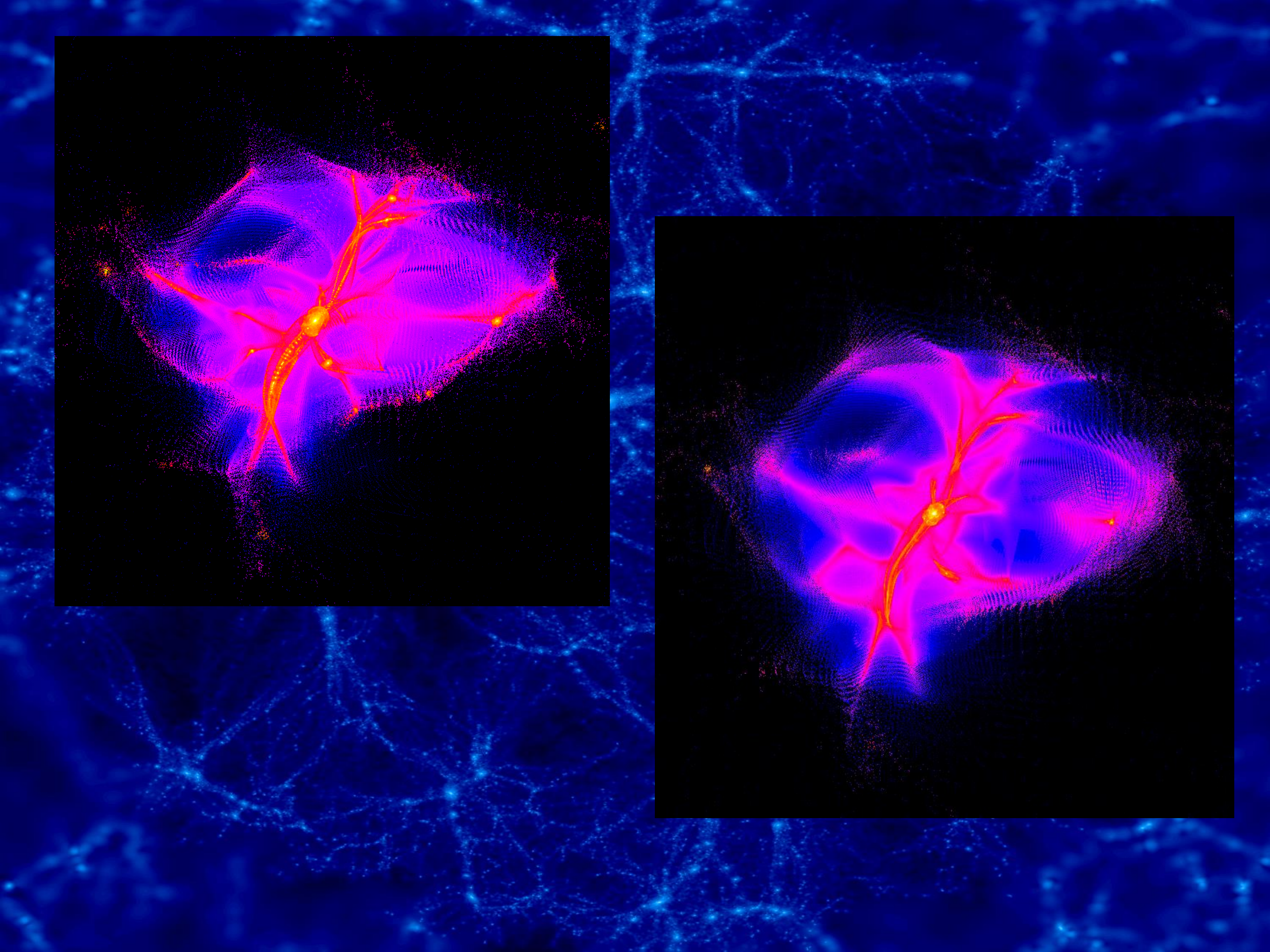


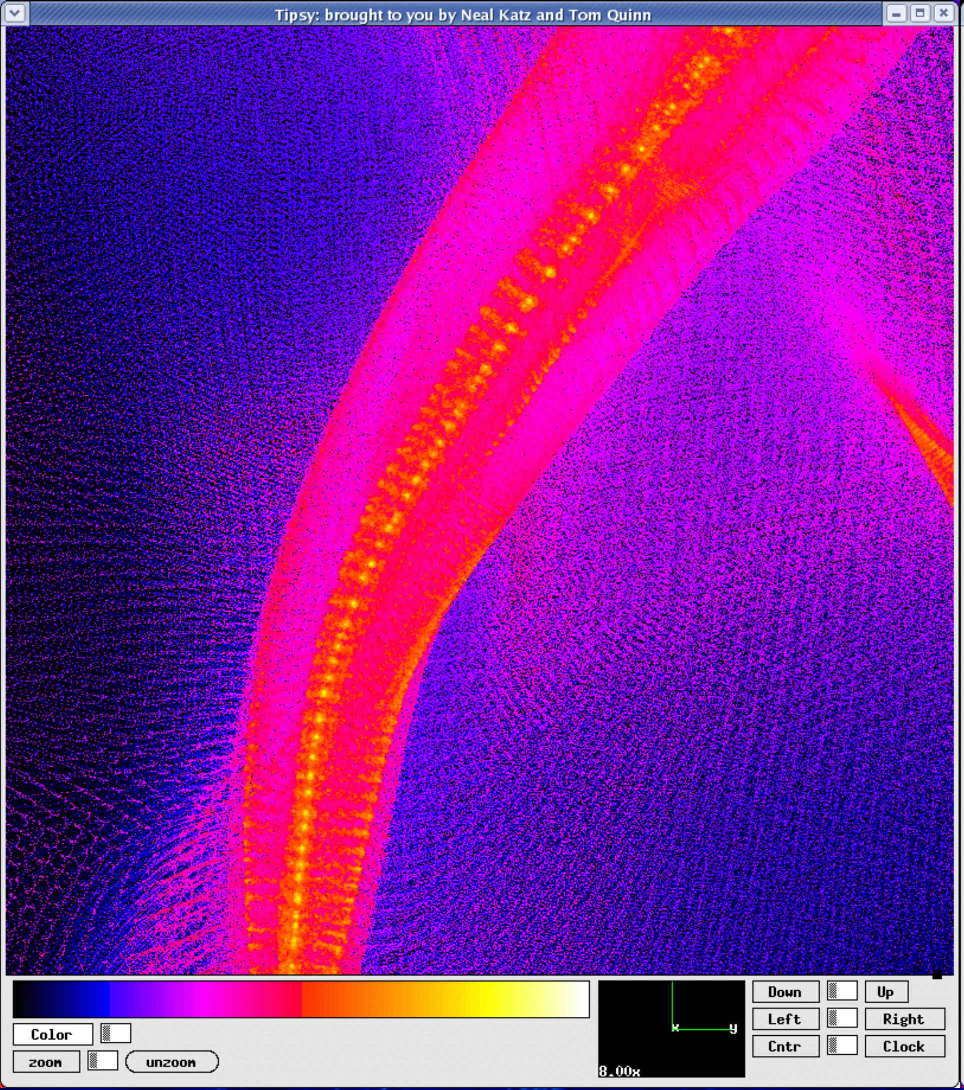
“High” resolution

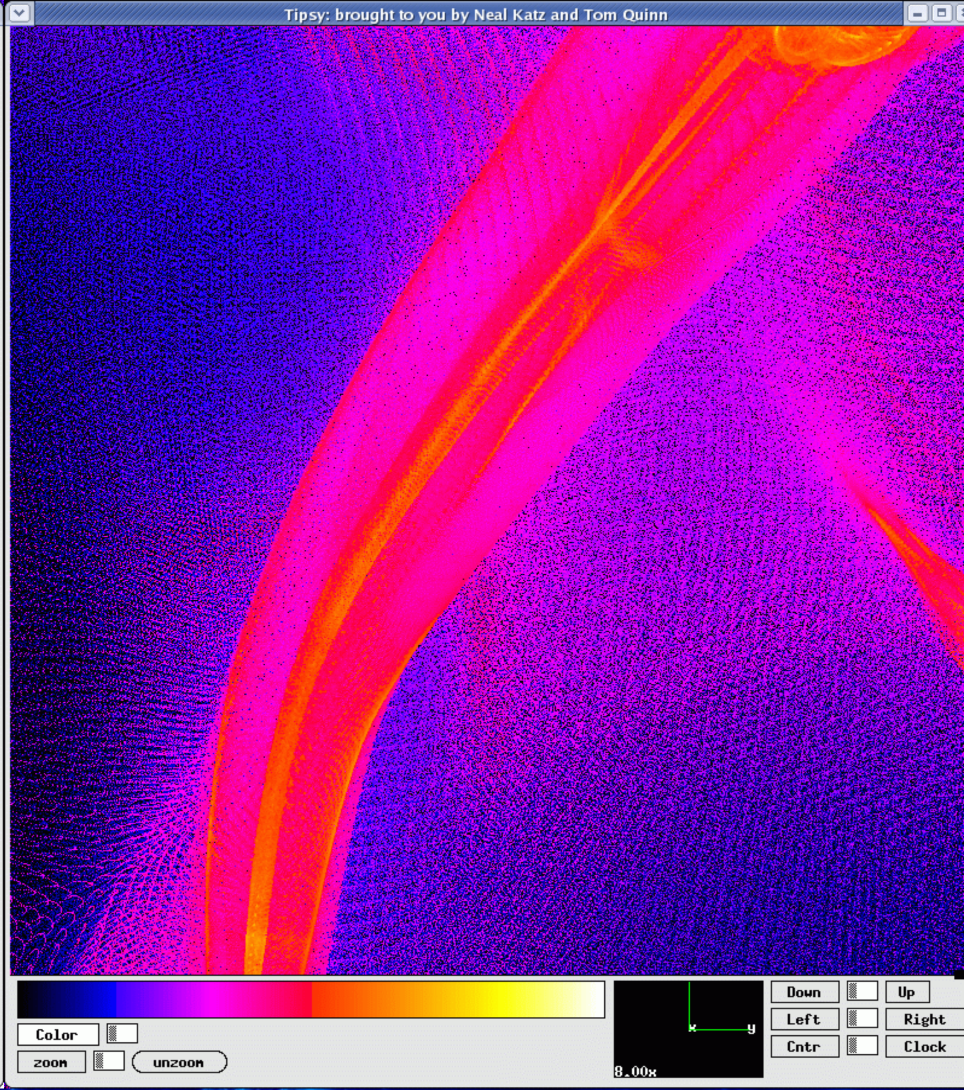
Large Softening

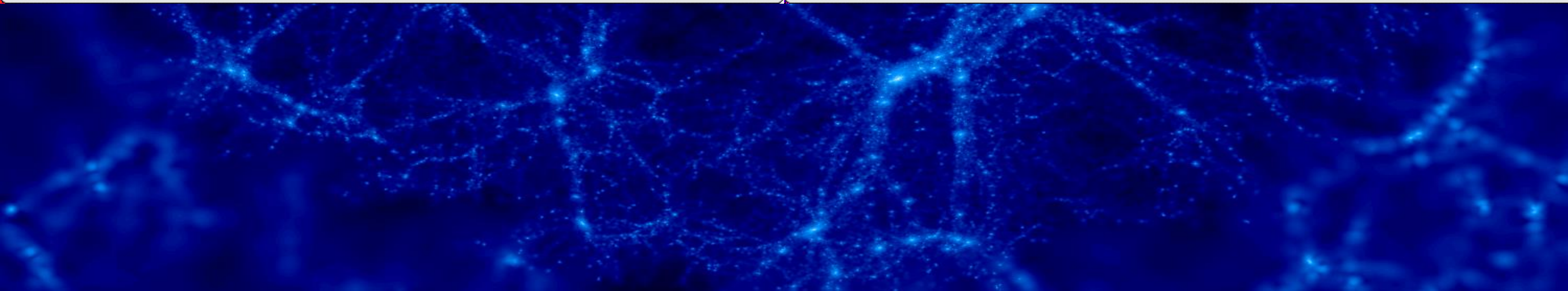


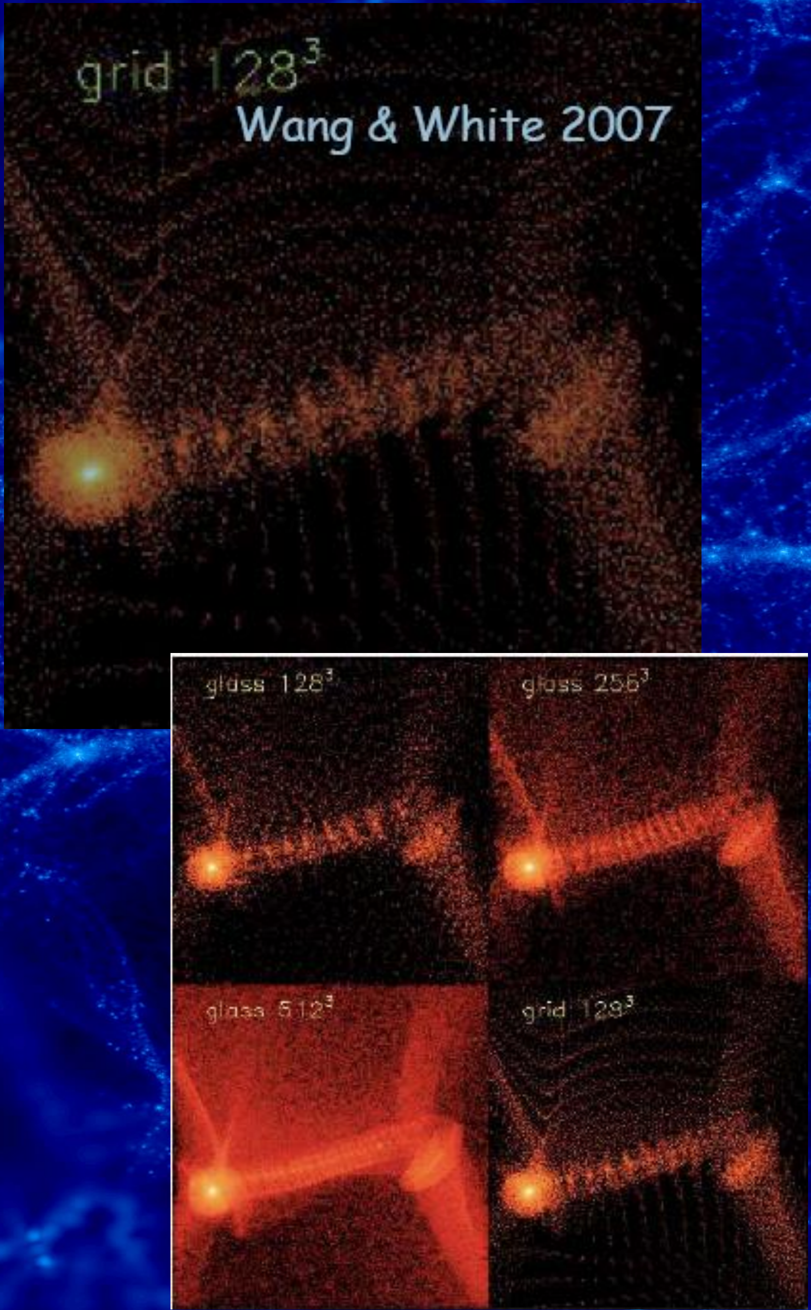


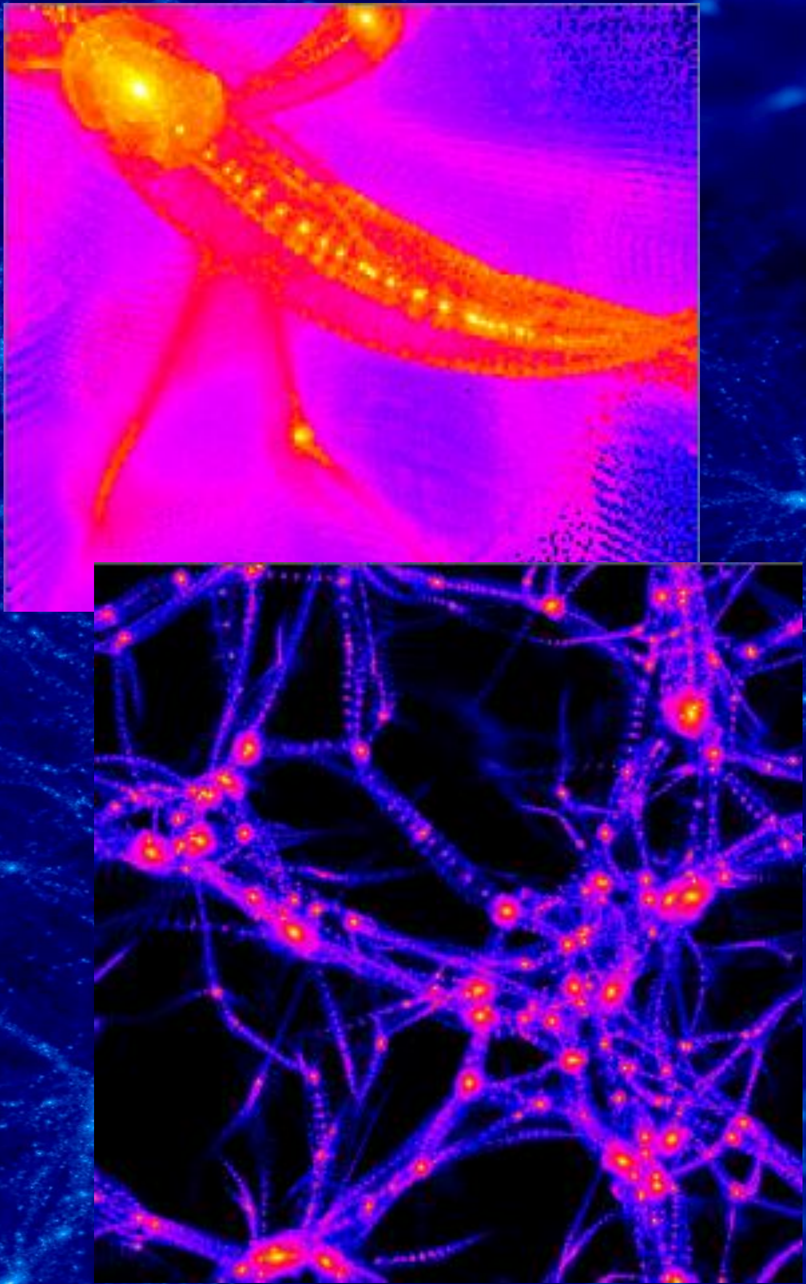



Tipsy: brought to you by Neal Katz and Tom Quinn
Color
zoom
unzoom
x
y
8.00x
Down
Up
Left
Right
Cntr
Clock

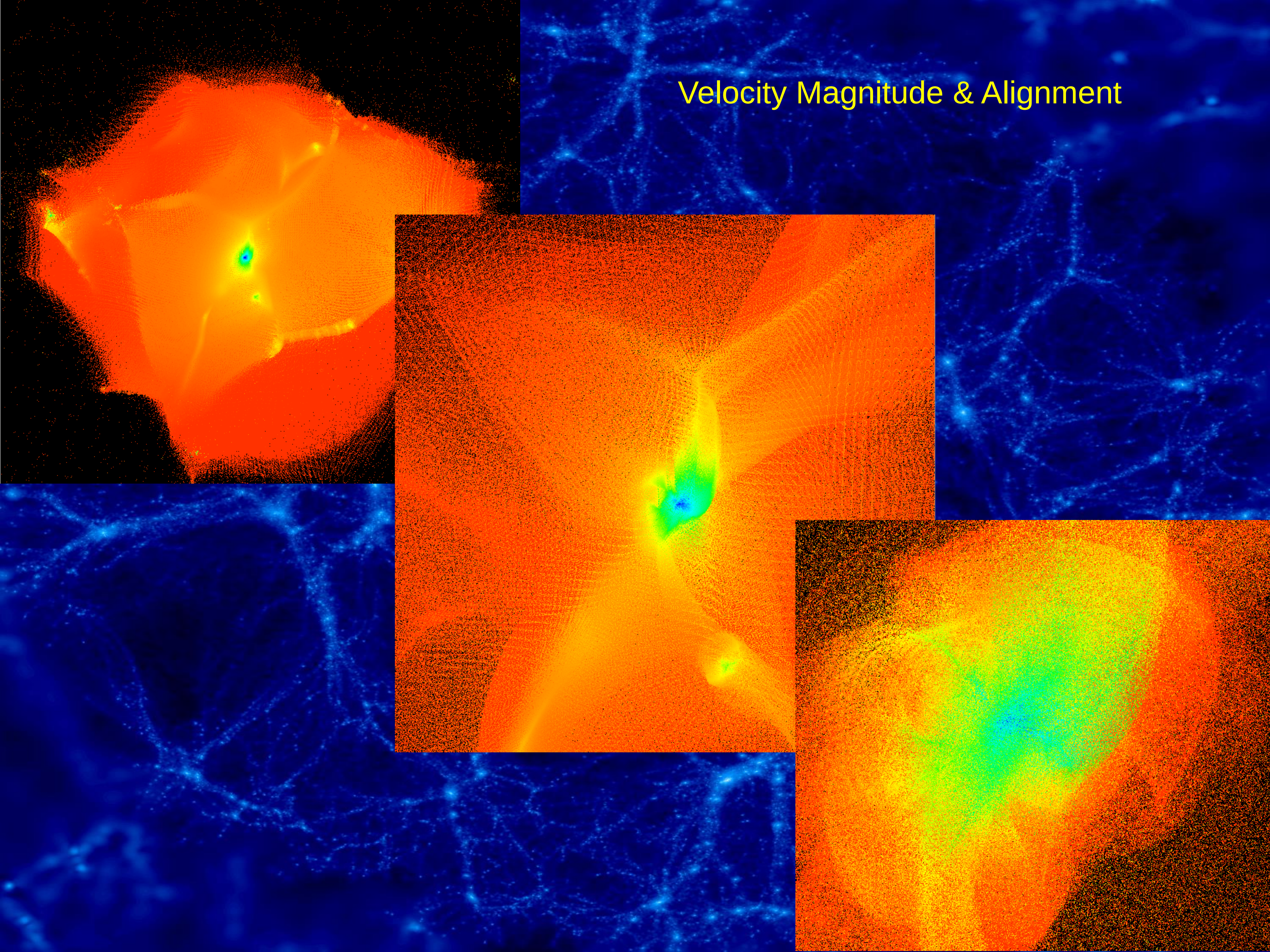

Tipsy: brought to you by Neal Katz and Tom Quinn
Color
zoom
unzoom
x
y
8.00x
Down
Up
Left
Right
Cntr
Clock




Wang & White 2007



Velocity Magnitude & Alignment



CONCLUSIONS and COMPLICATIONS

- Formation of haloes in WDM models differs from CDM.
  Hybrid mechanism - Top-Down & Hierarchical; long distance & nearest neighbours
  Looking at high redshift galaxies for T-D memory.

- The exact recipe for structure formation seems to depend only on the morphology and architecture of the environment
- ☼ Quantum Pressure; Baryons and their physics

- Warm dark matter haloes contain visible caustics and shells.

- Galaxies that do not suffer mergers

- The turn over in PSD results in constant density core with characteristic size.

- Spurious fragmentation below the free streaming scale hard to overcome
  $$ Adaptive softening?

- The velocity dispersion is crucial in describing warm dark matter particles!
  There is no universal one to one correspondence between mass and thermal velocity
  Mass constraints on the particle’s mass, are not accurate.

Outlook and Perspectives

- Phase space density studies for dwarfs
- High resolution halos without mergers
- High resolution simulations of WDM + baryons
- Shells, caustics and 3.5 keV line
- Simulations for different WDM particles (power spectra)
- Softening and resolution scaling
- Arcs in the sky and early supermassive black holes