



Dark Matter And Sterile Neutrino

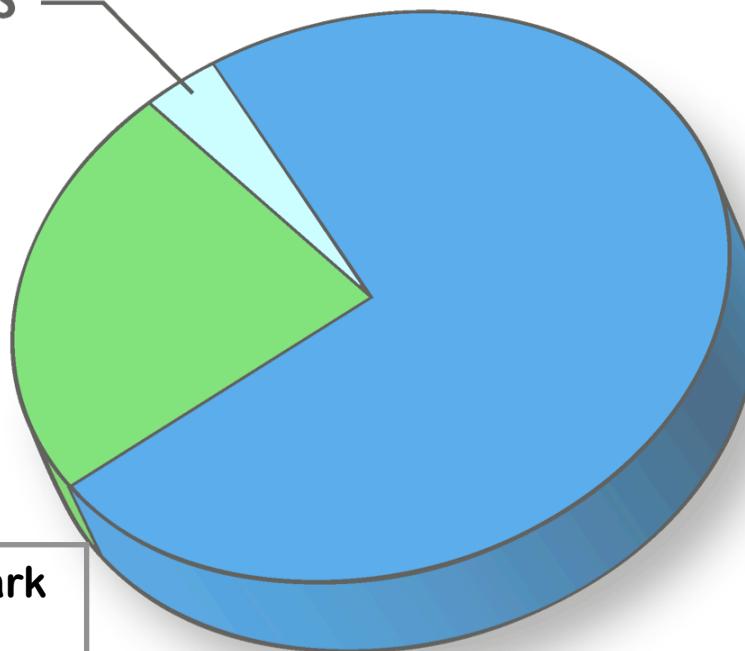
Thierry Lasserre
Meudon
27/11/2014

Our Understanding of the Universe

- Stars & galaxies: only ~0.5%

Atoms
4.6%

Dark Matter
23%



- Need of non baryonic Dark Matter (WIMP, Axion, ...)
- Neutrinos only < few %

- Almost no anti-Matter...
(baryo- lepto- genesis)

Dark Energy
72%

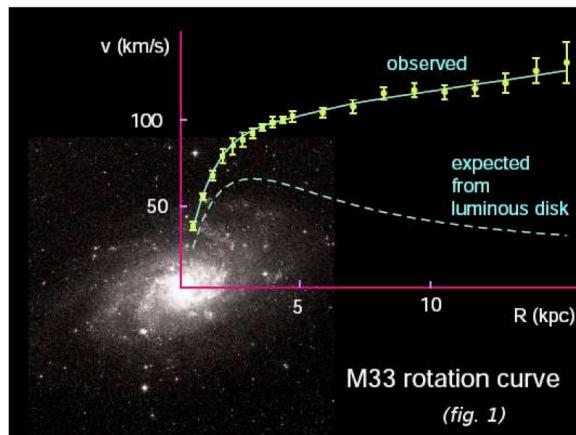
- $\rho_{\text{vacuum}} / \rho_{\Lambda} \sim 10^{120}$
- $\rho_{\Lambda} \sim m_{\nu}$

Dark Matter & Dark Energy

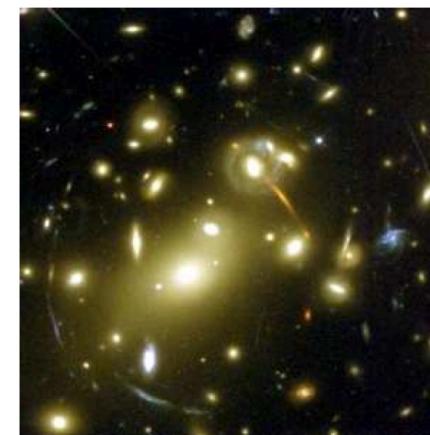
- 95% of our Universe
- not yet understood ...

Dark Matter (23%)

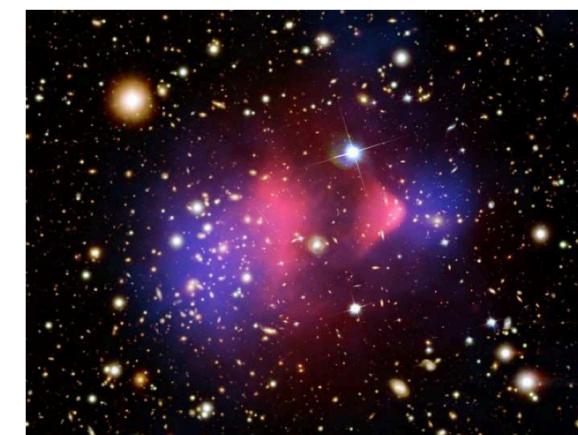
- The dominant gravitating component of the Universe cannot be the ordinary matter



Rotation Curves
of Galaxies



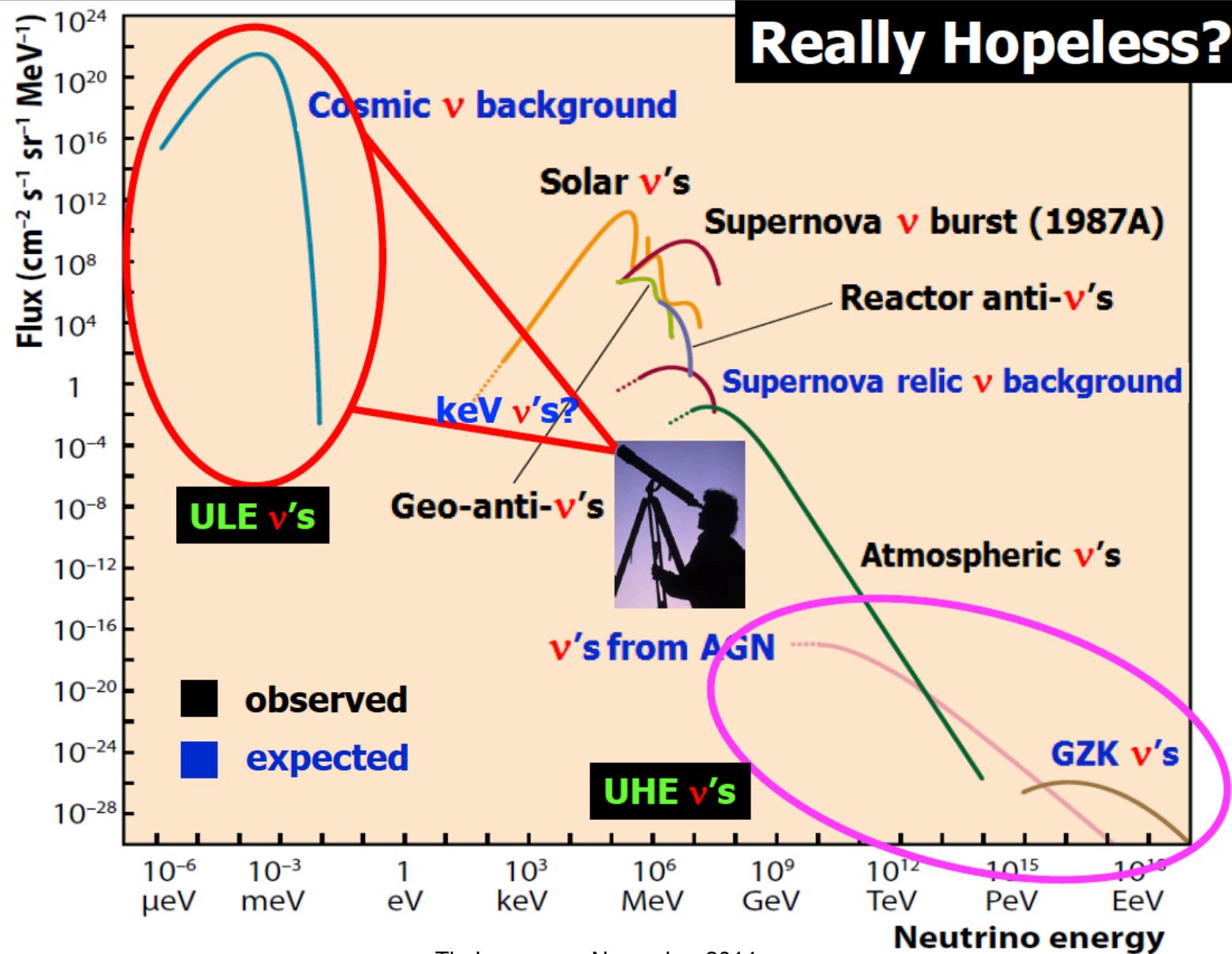
Gravitational
Lensing



Bullet Cluster & other
similar clusters

- Any DM candidate must be:
 - Very weakly interacting with EM radiation (“dark”)
 - Cosmologically long-lived or stable
 - Produced in the early Universe with the right abundance

Neutrinos in the Universe



C_vB is guaranteed but not significant

- Thermal relic background of active neutrinos in the Universe: $n_\nu = 112 \text{ cm}^{-3}$, for each specie
- Neutrinos should not overclose the Universe
 - $\rho_\nu = \sum m_{\nu_i} n_\nu < \rho_c \Omega_{\text{DM}}$ & $\rho_c = 3 H^2 m_{\text{Pl}} / 8 \pi \approx 10.5 h^2 \text{ KeV cm}^{-3}$
 $\rightarrow \sum m_{\nu_i} < 94 \Omega_{\text{DM}} h^2 \text{ eV} \approx 13 \text{ eV} \rightarrow \text{Hot Dark Matter (HDM)}$
- CvB : issues for Hot Dark Matter
 - Structure form in a top-down scenario with galaxies & clusters forming too late via fragmentation
 - Too many large galaxy clusters
- Experiments show that neutrinos have a mass
 - Terrestrial: $0.01 < m < 2 \text{ eV}$
 - LSS & CMS: $\sum m_{\nu_i} < 0.5\text{-}1 \text{ eV (95\% CL)}$ $\rightarrow \Omega_\nu < \text{few \% } \Omega_{\text{DM}}$

Particle DM Candidates

- DM particle must be beyond the Standard Model
- WIMPS (CDM)
 - Particles with masses $\sim 10 \text{ GeV} - 10^4 \text{ GeV}$
- Axions (CDM)
 - Light pseudo-scalars, mass $\sim 10^{-5} \text{ eV}$
- ...
- Neutrinos
 - Sterile – eV scale (but HDM ...)
 - Sterile – keV scale (WDM)
- Ockham's razor principle: one dominant DM particle?
Caveat: CDM and WDM may co-exist !

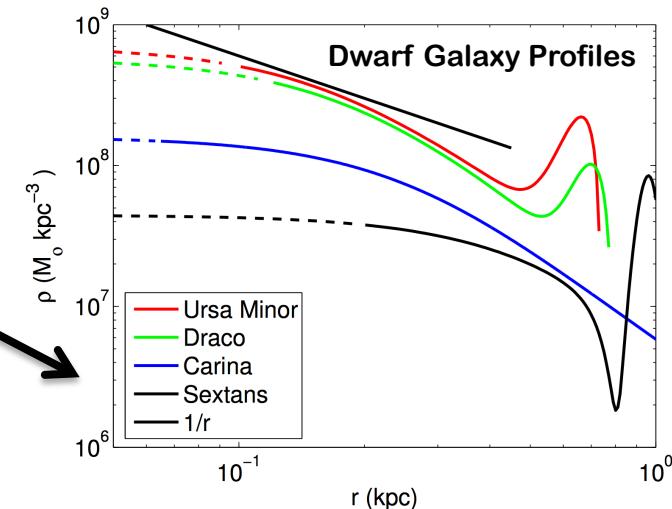
eV-Sterile Neutrinos & Dark Matter

- **Particle Physics:**
 - Reactor anomaly, LSND, MiniBooNE, Gallex/SAGE...
 - Light sterile ν 's? but eV scale and sizable mixings
 - New and better data are needed to clarify the situation
- **CMB:**
 - extra eV-ish neutrinos not excluded
- **BBN:**
 - extra ν 's possible
- **Astrophysics:**
 - Effects of keV-ish sterile ν 's on pulsar kicks?
- They would be non-relativistic now. Density per specie expected may differ to that of active ν 's

Modern Paradigm: Λ CDM

- DM is “cold” (CDM).
 - Structure formation is *bottom-up*
 - Smaller objects formed first: stars → galaxies → clusters

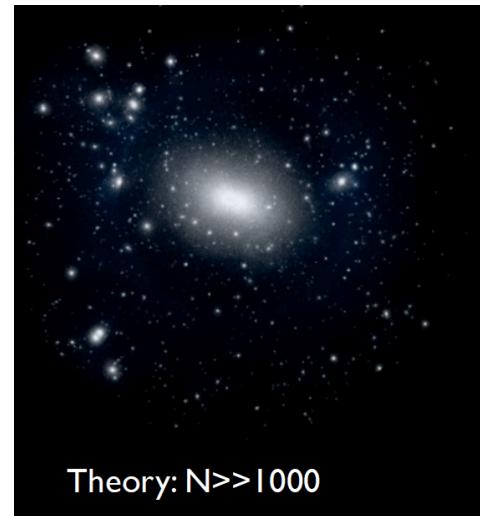
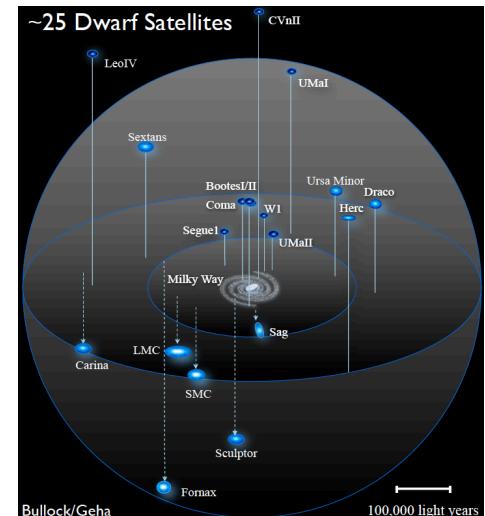
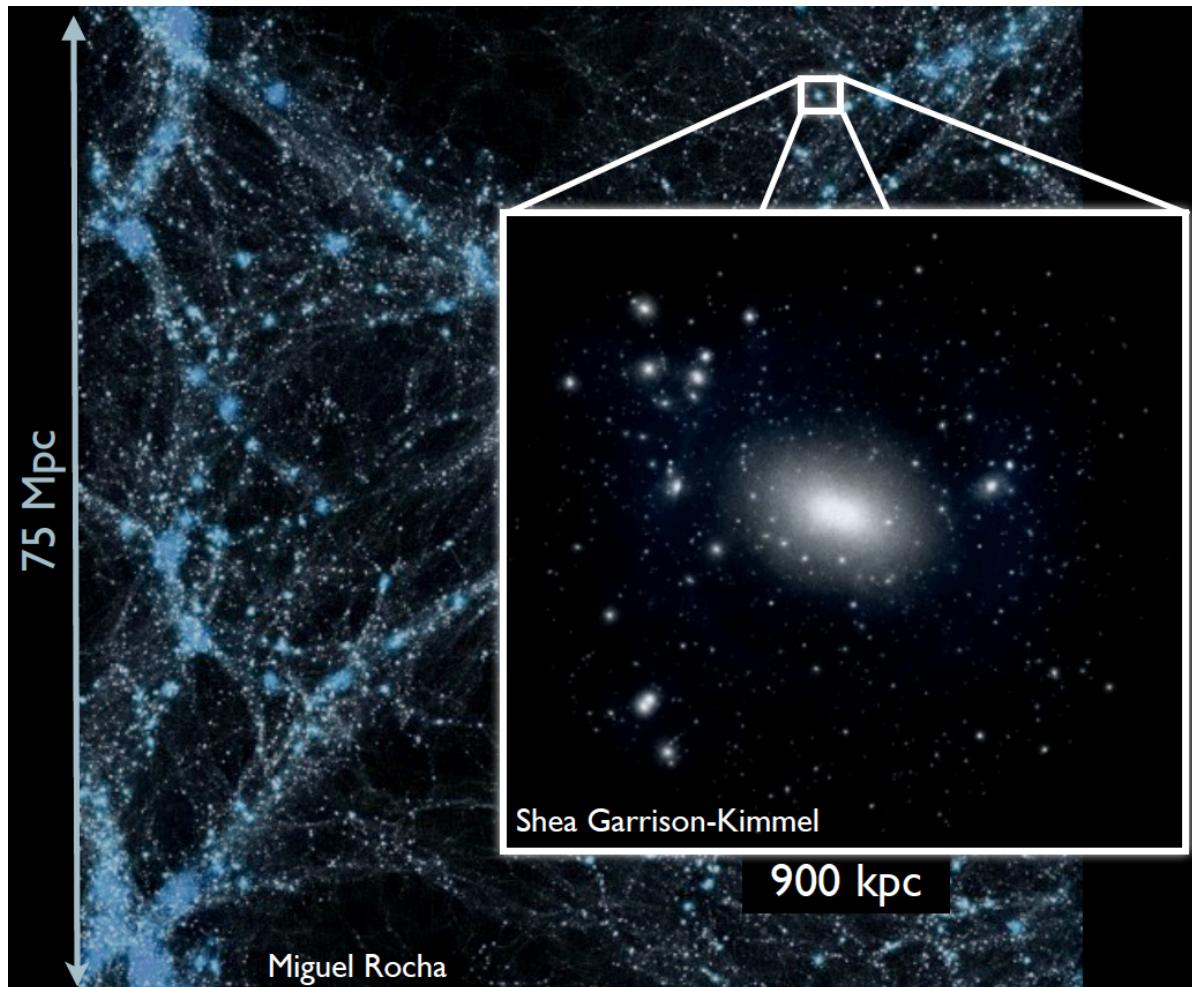
- CDM simulation issues...
 - Matching small scale data?
 - **Cuspy halo profiles predicted**
Galaxies → Clusters
 - **Too many satellites predicted**
Could be an observational bias?



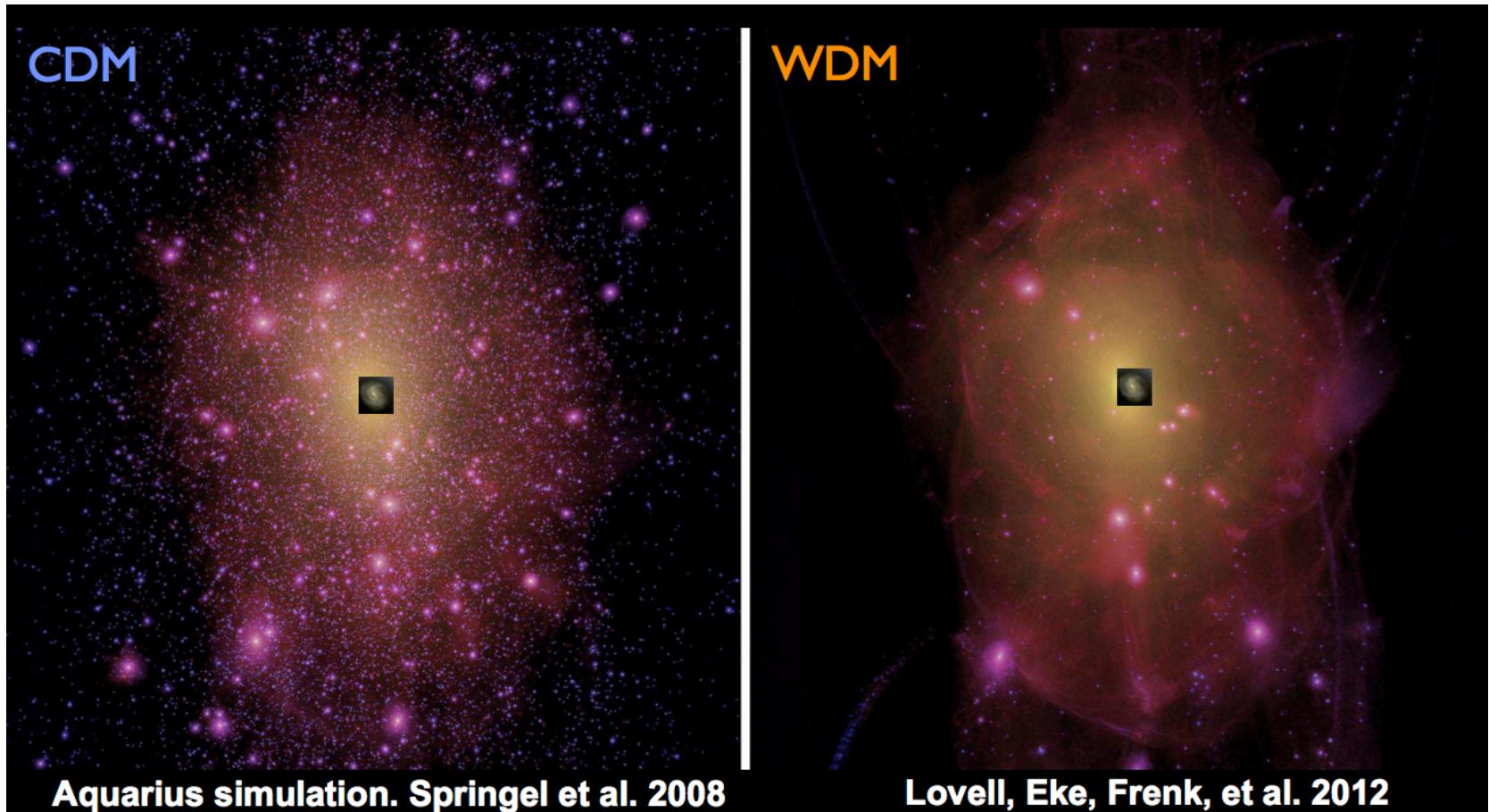


CDM: Missing Satellites Issue?

- CDM N-Body Simulations: too many small substructures (dwarf galaxies DM dominated) predicted
- Failure of CDM model? Observational bias?



WDM: No Missing Satellites Issue?

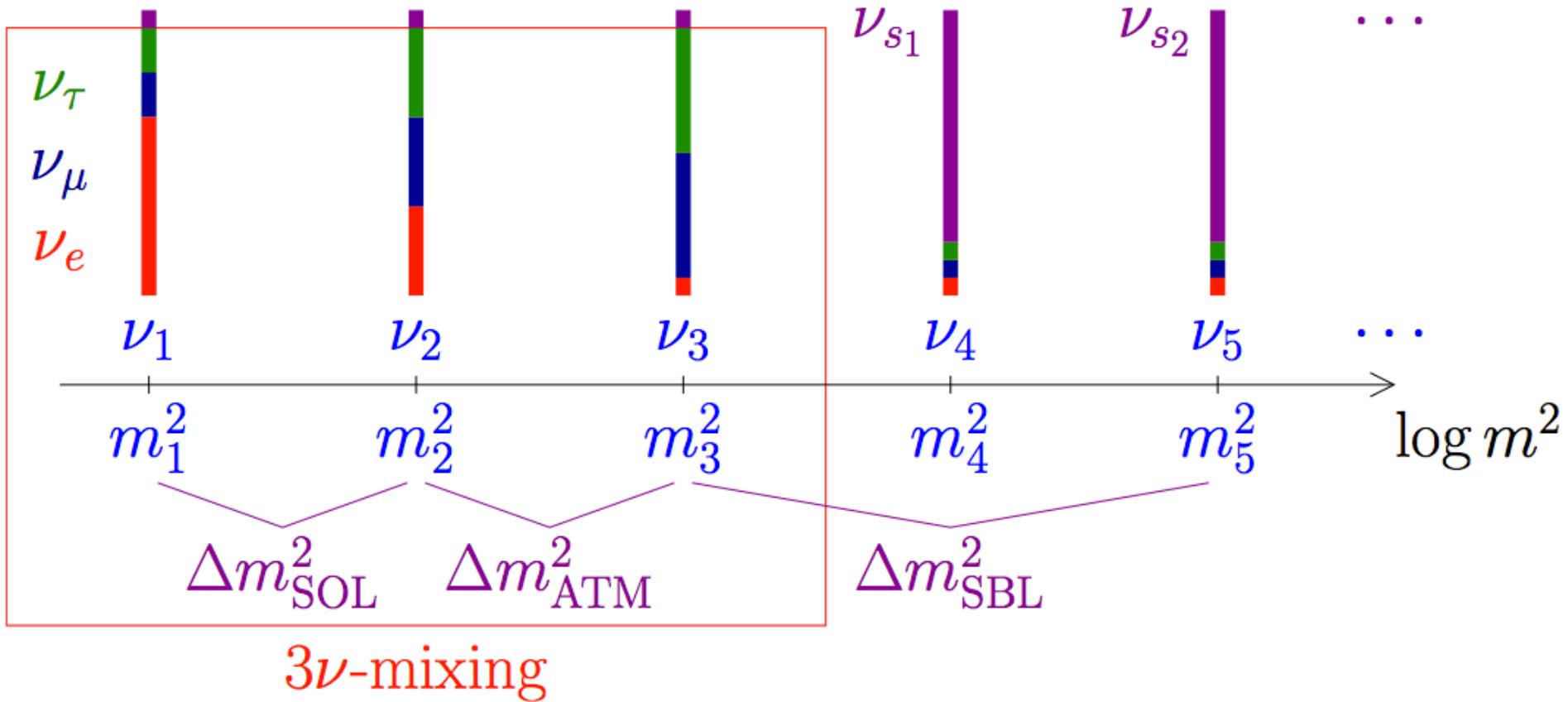


But WDM may be in conflict with other observations?

Sterile neutrino hypothesis

- Generic extension of SM model
- Add a SM singlet fermion
- Mixing with active ν 's

No or tiny SM model interaction

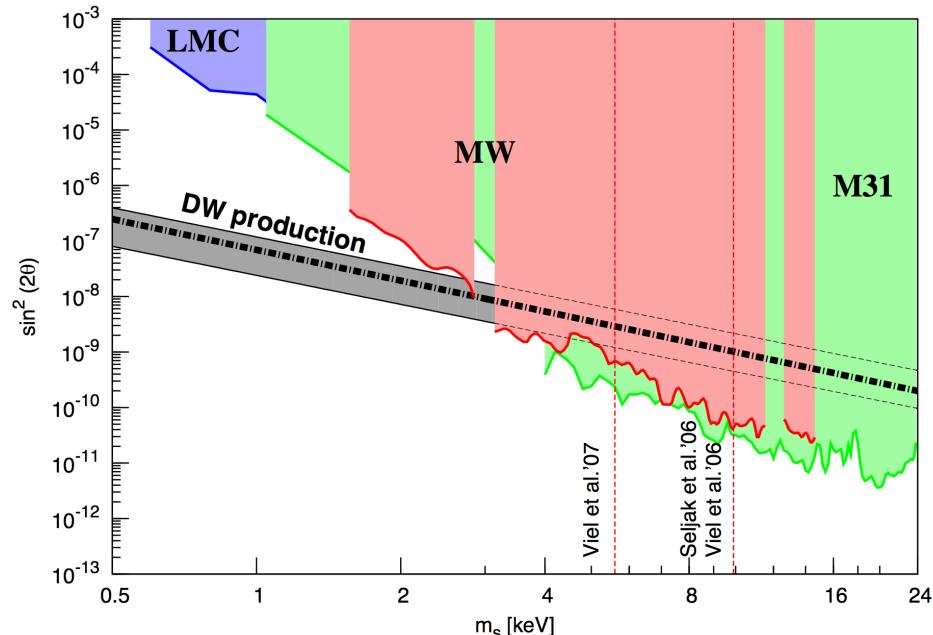
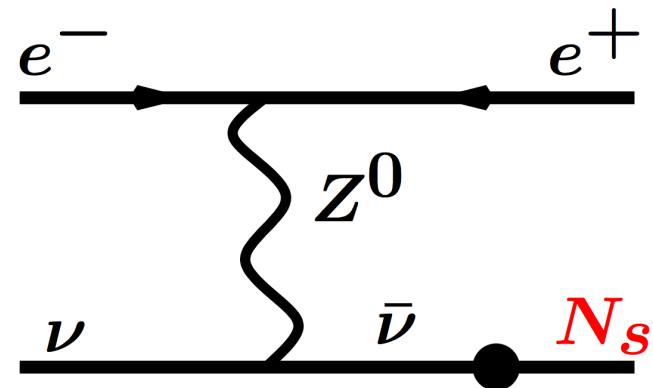


Sterile neutrinos – viable WDM candidate

- WDM in the form of keV sterile ν 's can suppress the formation of dwarf galaxies and other small-scale structures.
- Experiments on neutrino oscillations provides compelling evidence of physics beyond the SM
- Adding right-handed neutrinos to the SM:
 - The simplest and natural extension of the SM → new eigenstates
 - Mix (oscillate) with active neutrinos
 - Could break CP and allow for lepto- baryo- genesis
- Sterile neutrino are suitable WDM candidates:
 - Can be produced in the Early Universe (Warm or Cold), but abundance depend on production history
 - Can have cosmological life-time
 - Wide range of masses...
 - requires non-thermal production from other particles
 - But not any strong prior theoretical motivating keV sterile ν 's

ν_s : Dodelson-Widrow (94)

- The theory is SM + 1 ν_s
- Cosmological production is due to active – sterile neutrino oscillations in the early Universe, including matter effects
- Produced at $T_{\max} \sim 130 (m_s/\text{keV})^{1/3} \text{ MeV}$
- But model is in tension with Lyman- α data



ν_s Production: Shi-Fuller (98)

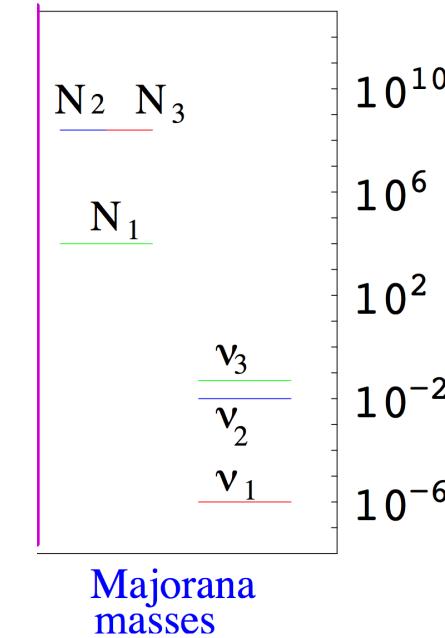
- **Resonant MSW-like oscillation production**
 - Smaller θ are enough for efficient production
- **Spectra of sterile neutrino DM is very non-thermal and colder than Dodelson-Widrow**
- **Necessitate Lepton Asymmetry**
 - Being converted into the sterile neutrino
- **This pushes Lyman- α bounds down and the X-ray bounds up, opening the allowed region of parameters**

ν MSM: a comprehensive model

Shaposhnikov, PLB 620, 17 (2005)

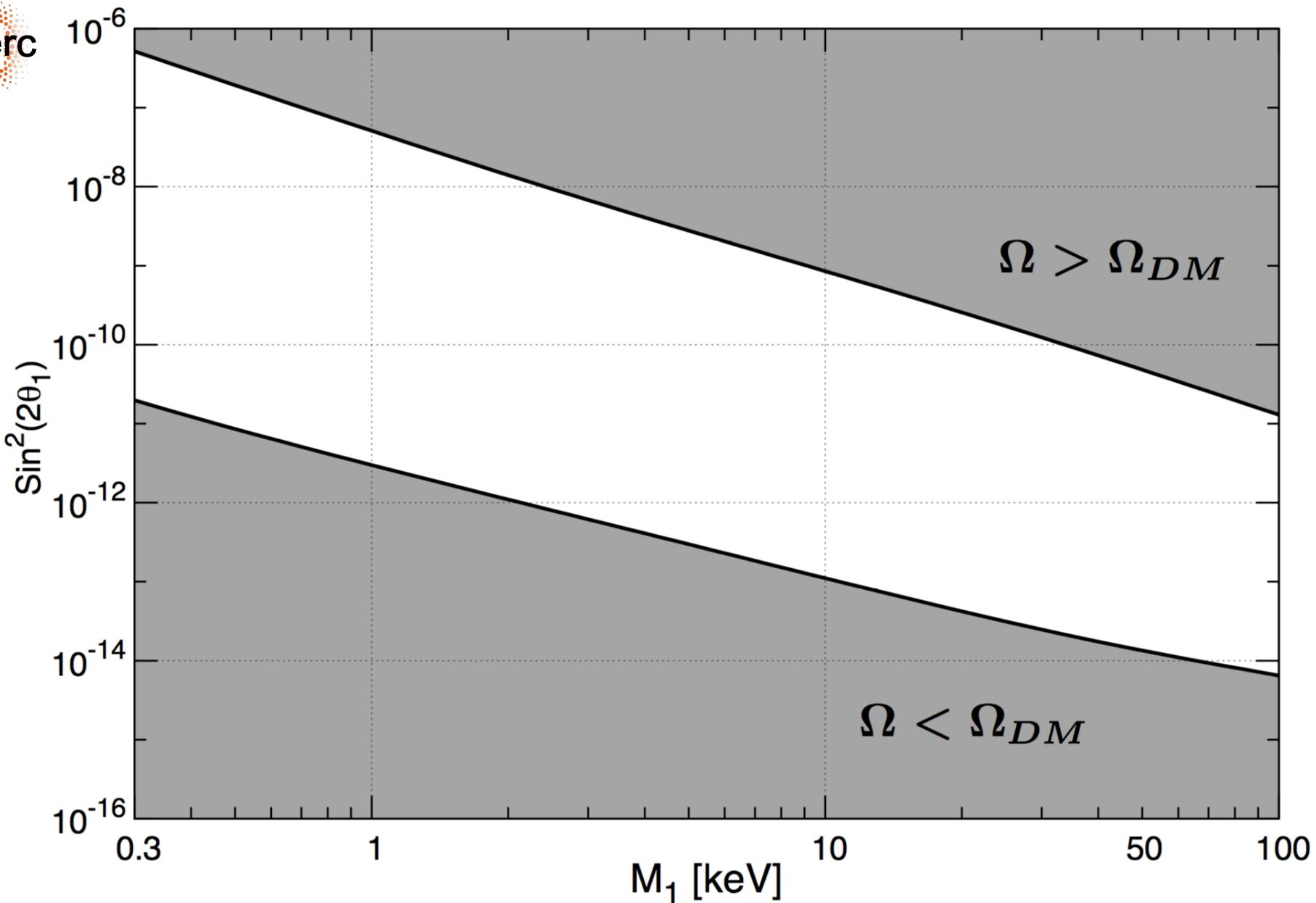
| Three Generations of Matter (Fermions) spin $\frac{1}{2}$ | | | | | | | | |
|---|---------------------------|--------------------------|--------------------------|------------------------|------------------------|------------------------|--|--|
| | I | II | III | | | | | |
| mass \rightarrow | 2.4 MeV | 1.27 GeV | 171.2 GeV | | | | | |
| charge \rightarrow | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | | | | | |
| name \rightarrow | Left u up Right | Left c charm Right | Left t top Right | | | | | |
| Quarks | d down | s strange | b bottom | | | | | |
| Leptons | ν_e electron neutrino | ν_μ muon neutrino | ν_τ tau neutrino | N_1 sterile neutrino | N_2 sterile neutrino | N_3 sterile neutrino | | |
| | Left -1 e electron Right | Left -1 μ muon Right | Left -1 τ tau Right | | | | | |

| Bosons (Forces) spin 1 | | |
|------------------------|----------------------|--------------------|
| 0 g gluon | 0 γ photon | 0 Z^0 weak force |
| 0 H^0 Higgs boson | 0 W^\pm weak force | spin 0 |



- SM + 3 MJ masses, 3 Dirac masses, 6 mixing angles and 6 CP-phases
- Role of N_1 with mass in keV region: **Dark Matter**
- Role of N_2, N_3 with mass in 100 MeV – GeV region: “give” masses to ν 's and produce **baryon asymmetry** of the Universe
- The Higgs and $N_{1,2,3}$ and provides dynamical **dark energy**

ν MSM: keV Neutrino Abundance



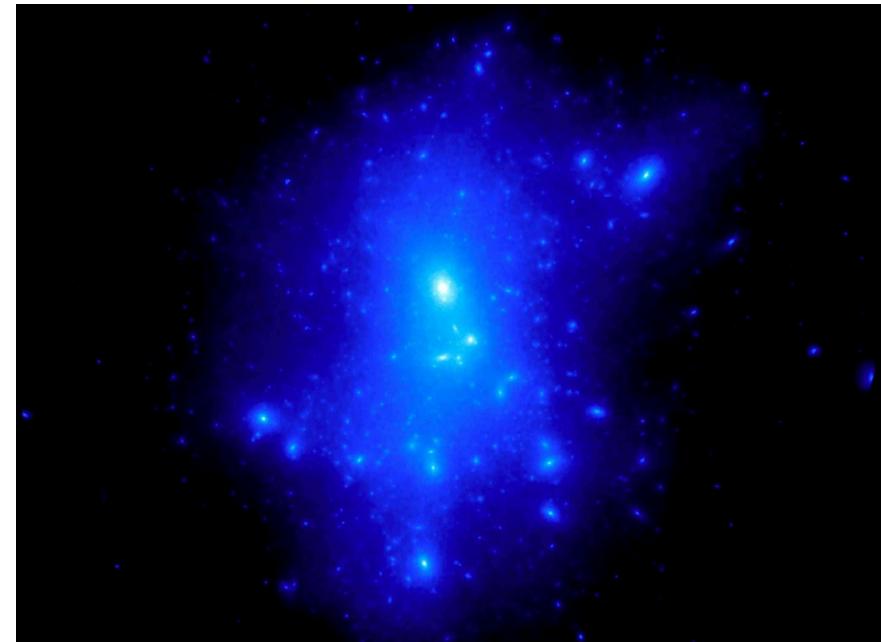
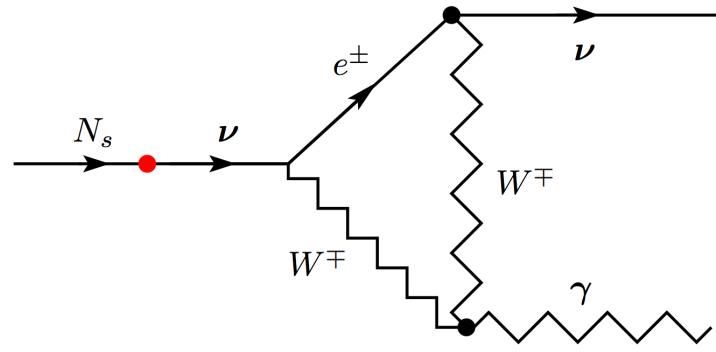
Major Constraints on ν WDM models

- Production of DM density in the right range
- Phase-space density & structure formation
 - $m > 1 \text{ keV}$
- Satellite X-ray observation on decay $\nu_s \rightarrow \nu + \gamma$
 - Non-observations bring down the upper limit on DM sterile neutrino mass
- Lyman- α forest
 - Non-observations restricts the mass of the sterile neutrino DM from below
 - Bounds are strongly model dependent
 - $m >$ few keV for resonant production
 - $m >$ 30 keV for Dodelson-Widrow model

keV Neutrino DM: not fully Dark...



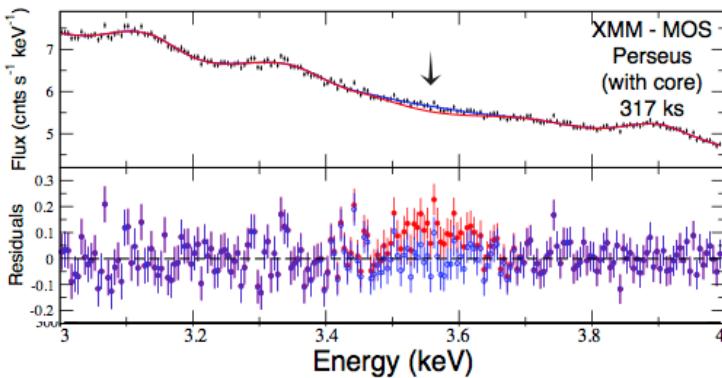
- Radiative decay channel:
 - $\nu_s \rightarrow \nu \gamma$
- Emitted Photon energy:
 - $E_\gamma = m_s / 2$
- Detection: X-ray telescope in Space observing galaxy clusters
- (faint) hint of signal
 - arxiv:1402.2301
 - $m = 7.1 \text{ keV}$
 - $\sin^2(\Theta) \approx 10^{-10} \dots$



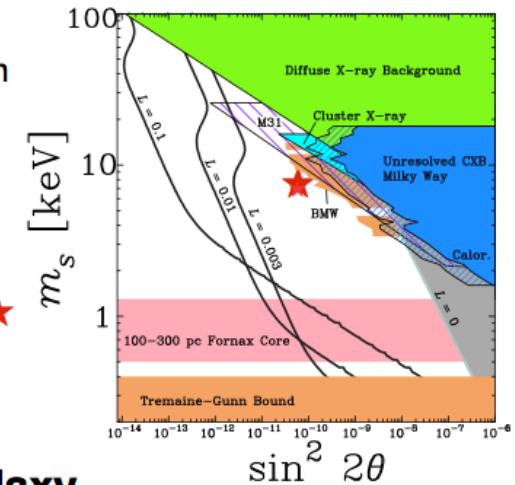
New Evidence for 7 keV Neutrino

DETECTION OF AN UNIDENTIFIED EMISSION LINE IN THE STACKED X-RAY SPECTRUM OF GALAXY CLUSTERS arXiv:1402.2301

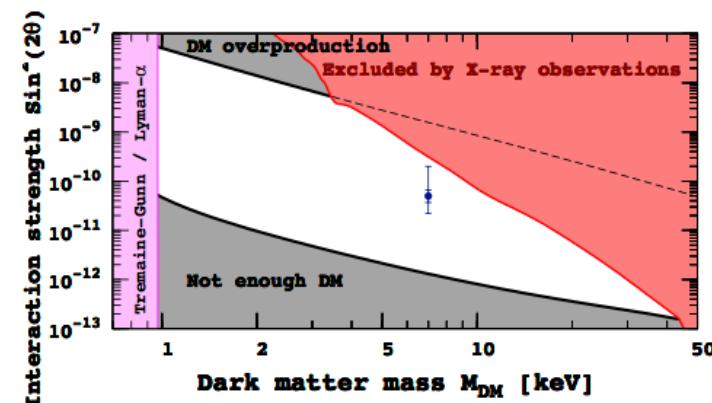
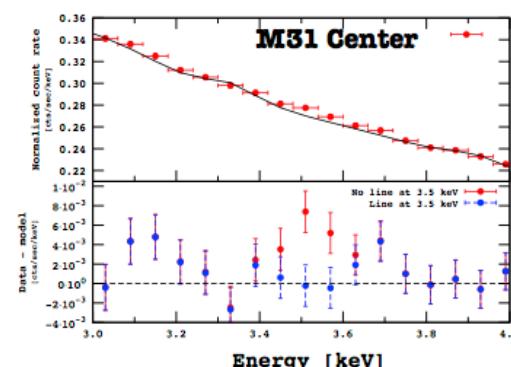
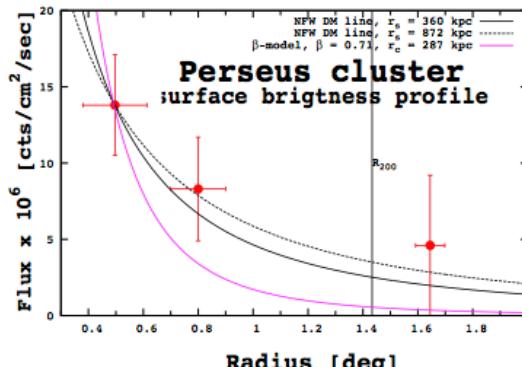
Esra Bulbul, Maxim Markevitch, Adam Foster, Randall K. Smith, Michael Loewenstein, and Scott W. Randall



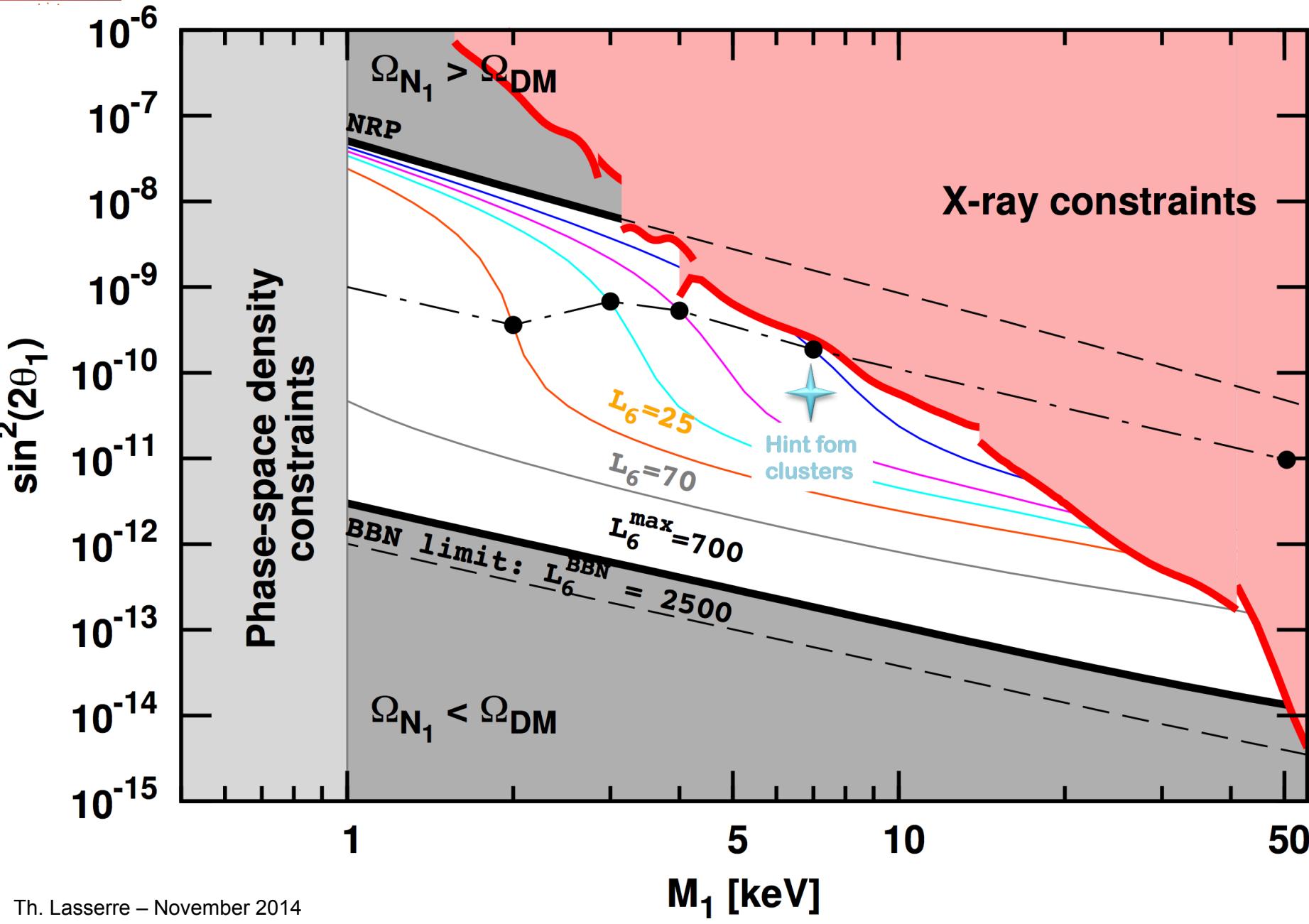
Recent constraints on sterile neutrino dark matter production models (Abazajian+07). Lines in black show theoretical predictions assuming sterile neutrinos are the dark matter with lepton number $L = 0, L = 0.003, L = 0.01, L = 0.1$. The \star is consistent with upper limits.



An unidentified line in X-ray spectra of the Andromeda galaxy and Perseus cluster arXiv:1402.4119 A. Boyarsky, O. Ruchayskiy, D. Iakubovskyi and J. Franse

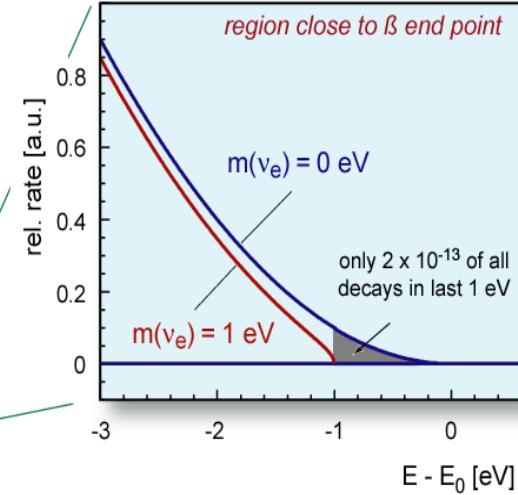
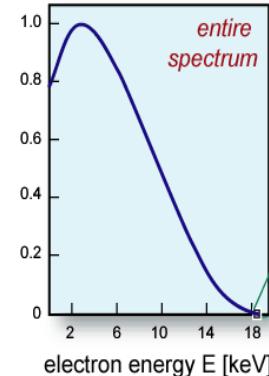
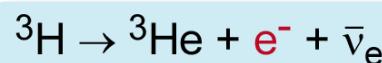
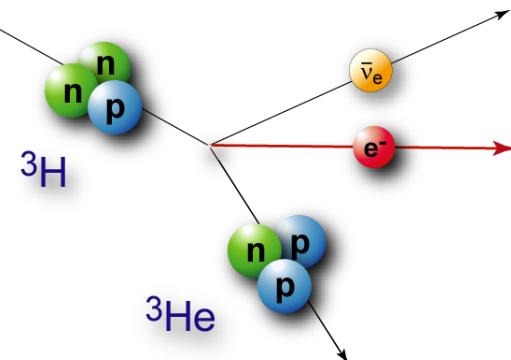
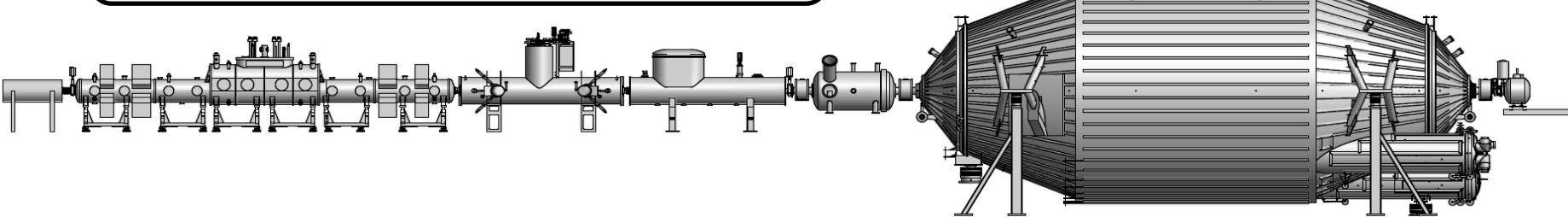


ν MSM: Observational Constraints

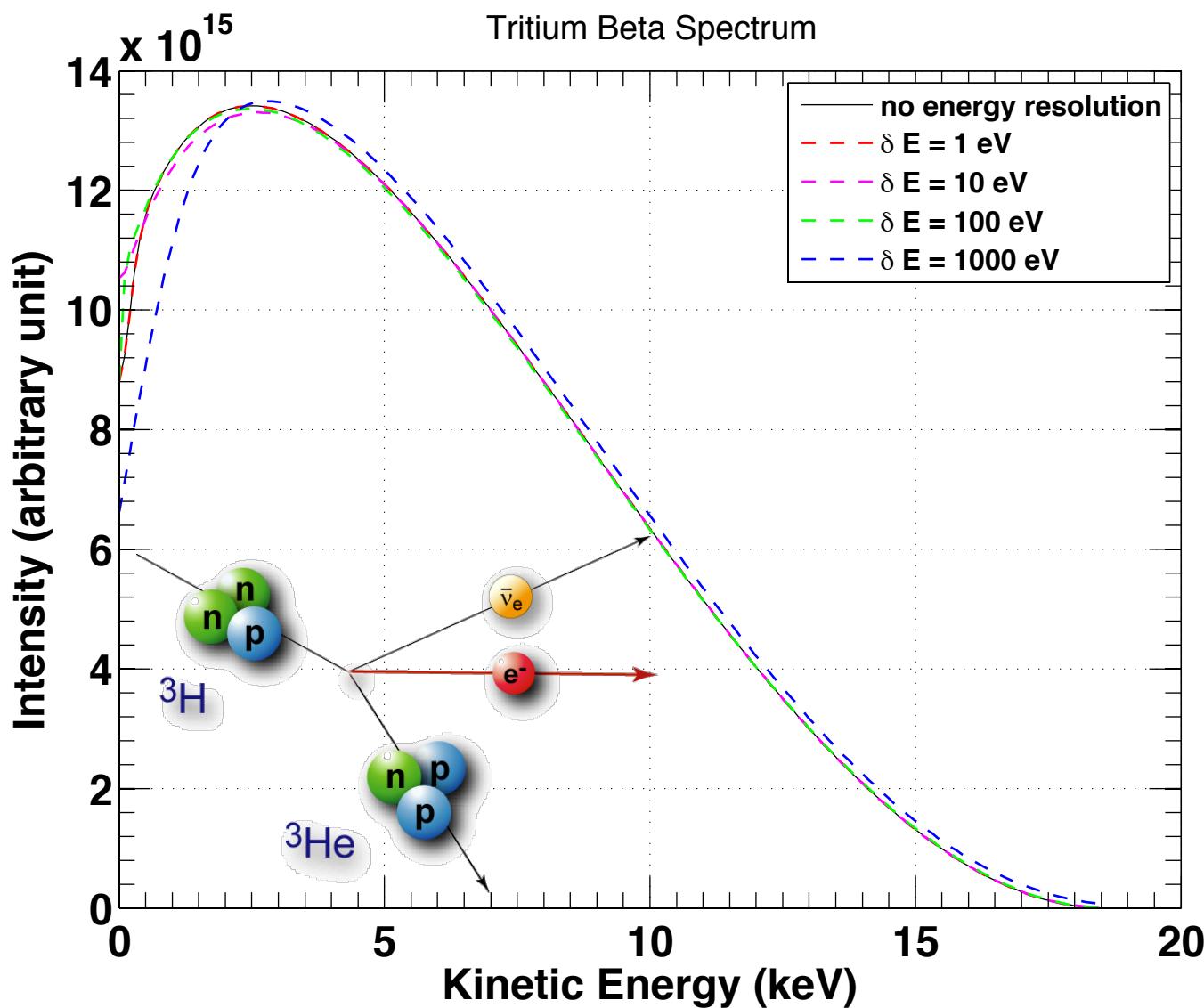


The KATRIN experiment

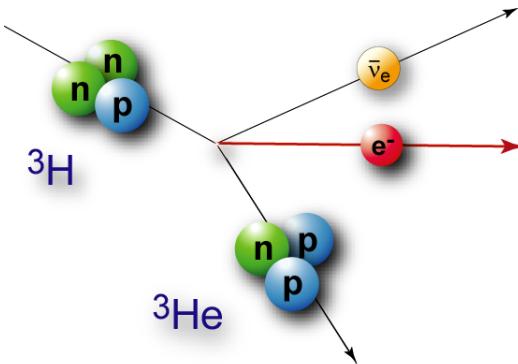
Goal: Direct neutrino mass measurement
Sensitivity = 200 meV [90% C.L.]



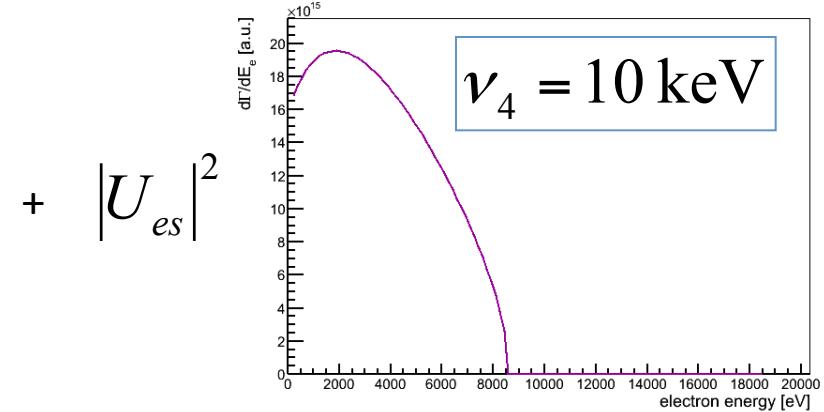
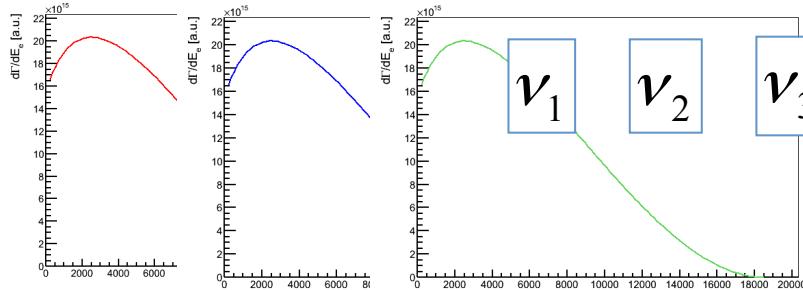
Tritium Beta Decay



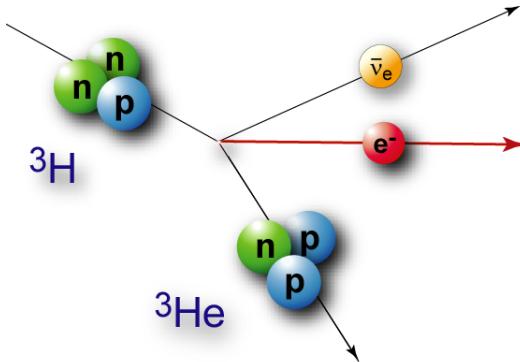
Imprint of keV neutrino



$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & 0 \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & 0 \\ 0 & 0 & 0 & U_{s4} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix}$$

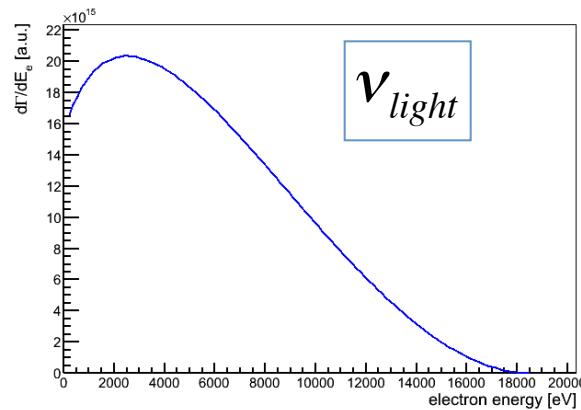


Imprint of keV neutrino



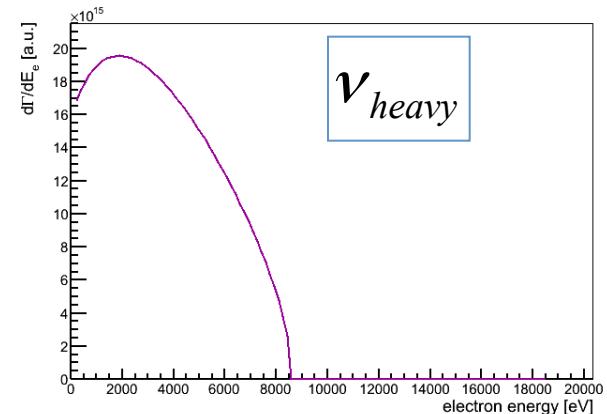
$$\begin{pmatrix} \nu_e \\ \nu_s \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_{light} \\ \nu_{heavy} \end{pmatrix}$$

$$\cos^2(\theta)$$



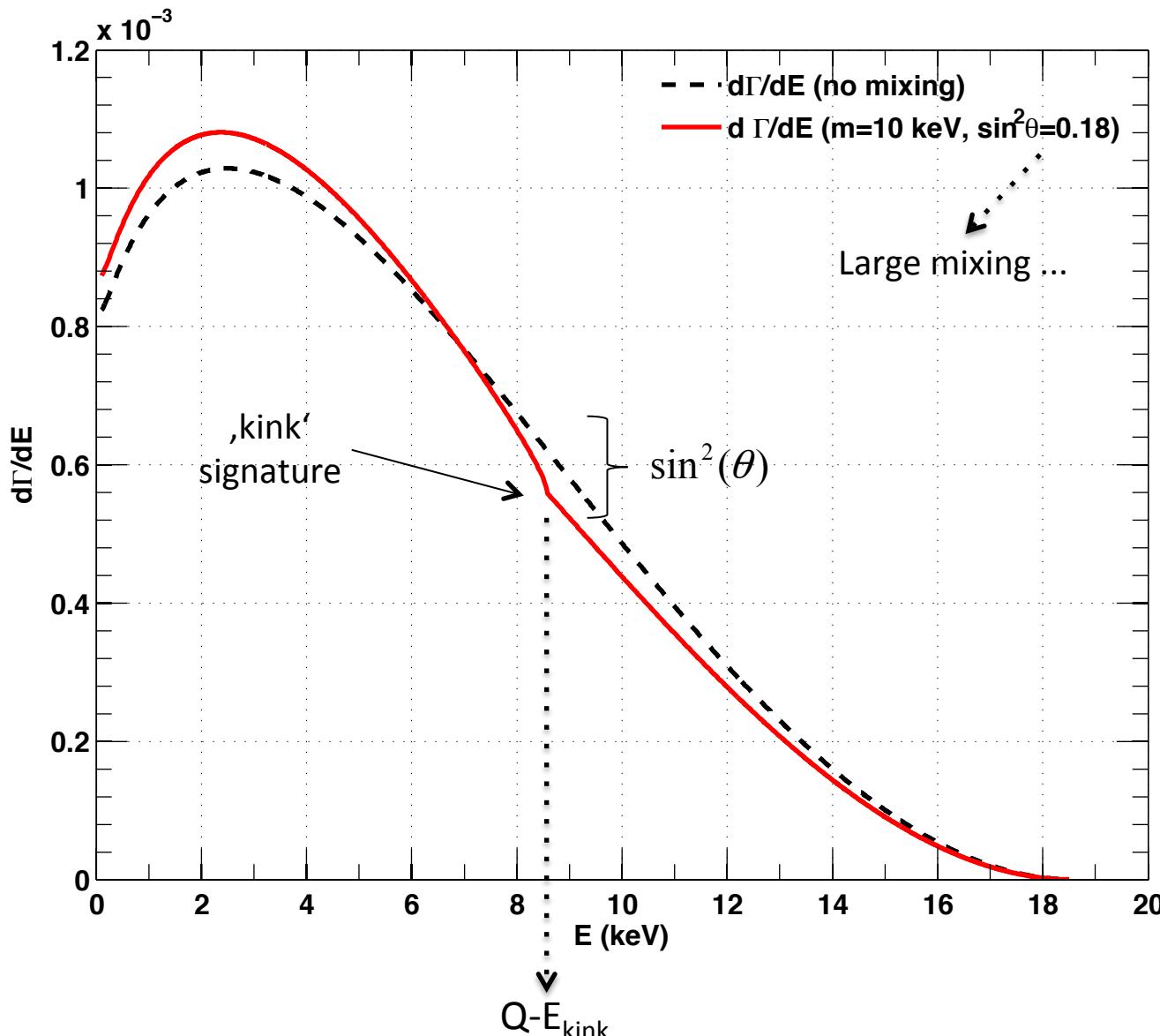
ν_{light}

$$+ \sin^2(\theta)$$

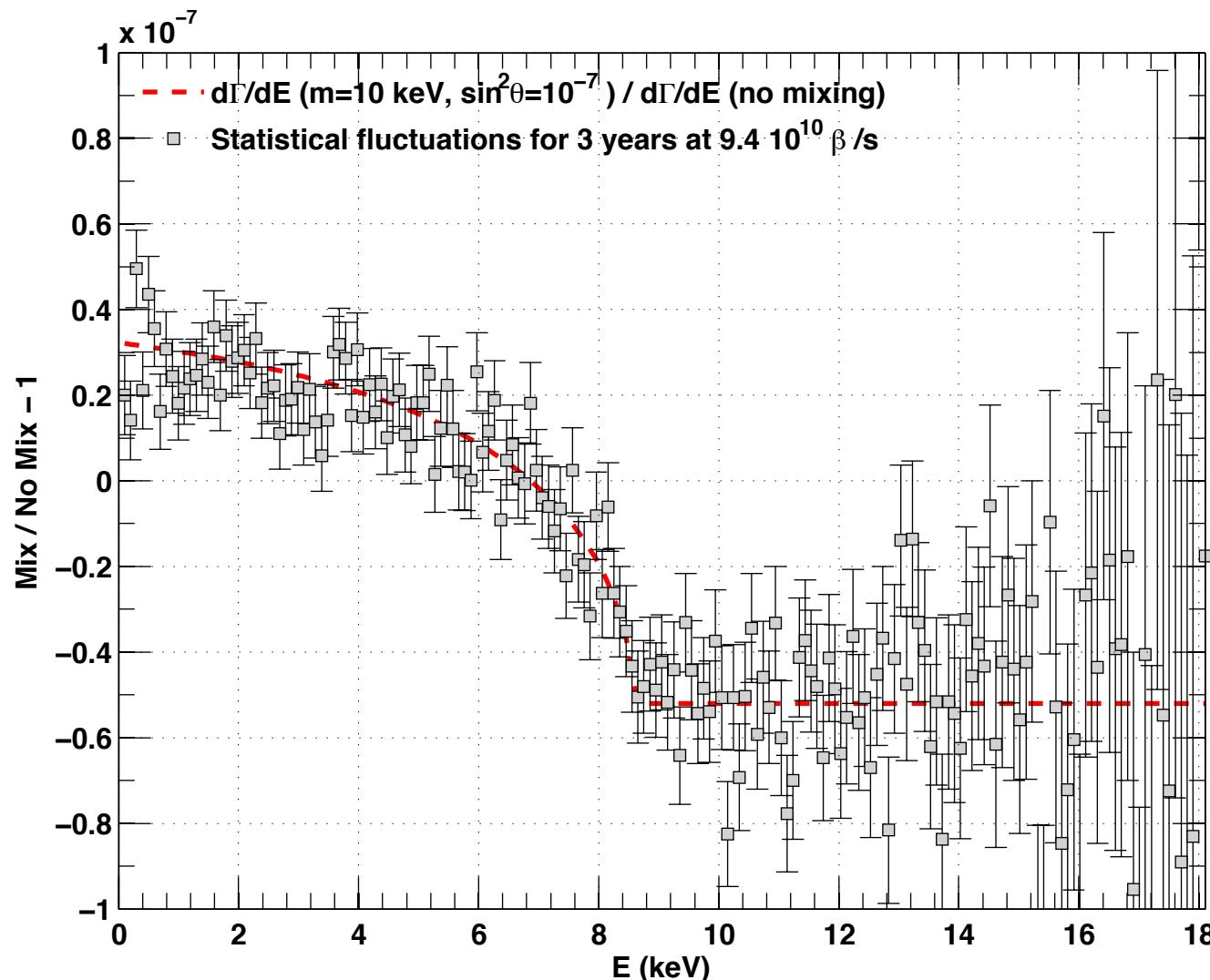


ν_{heavy}

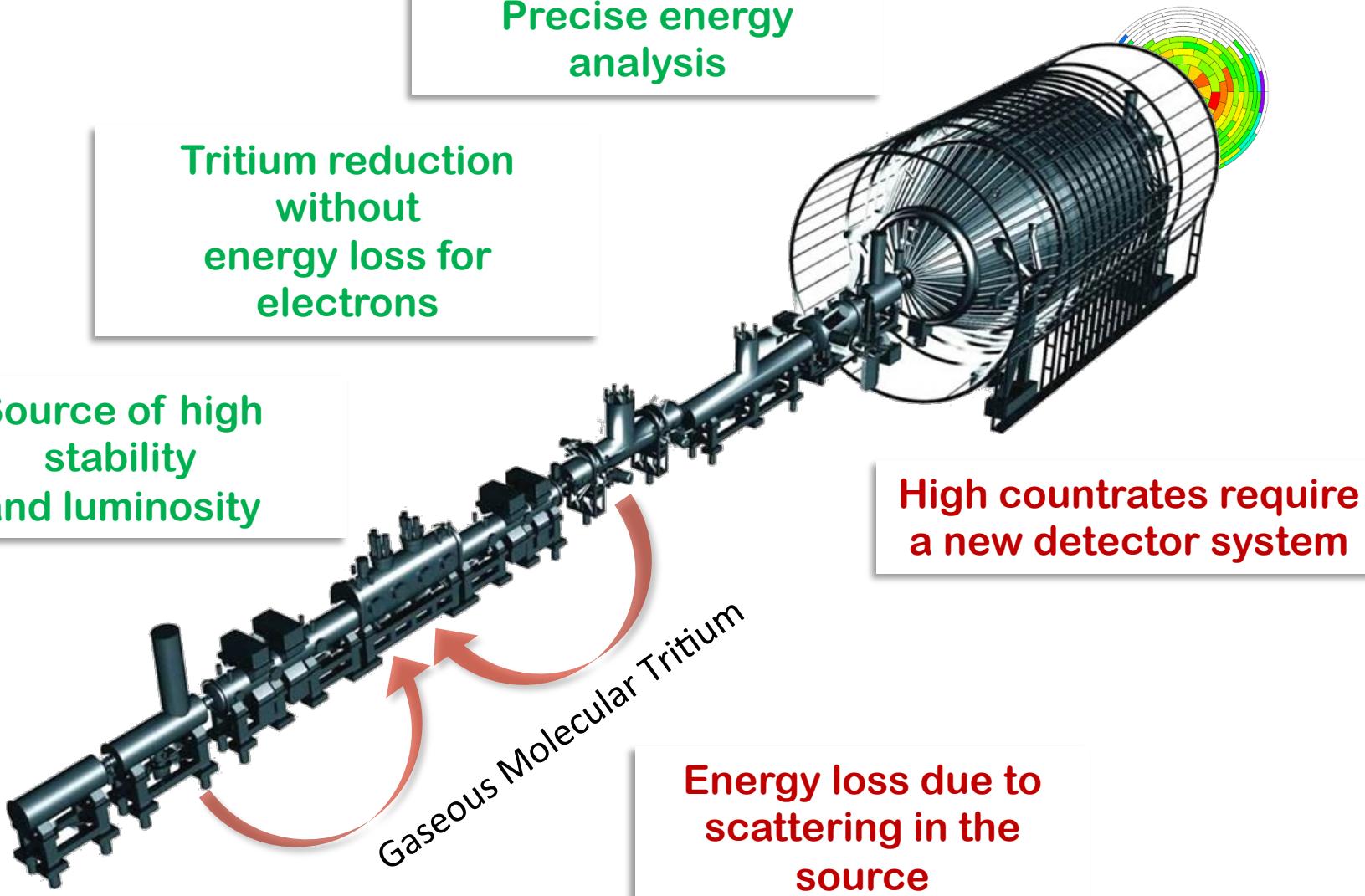
Imprint of keV neutrino



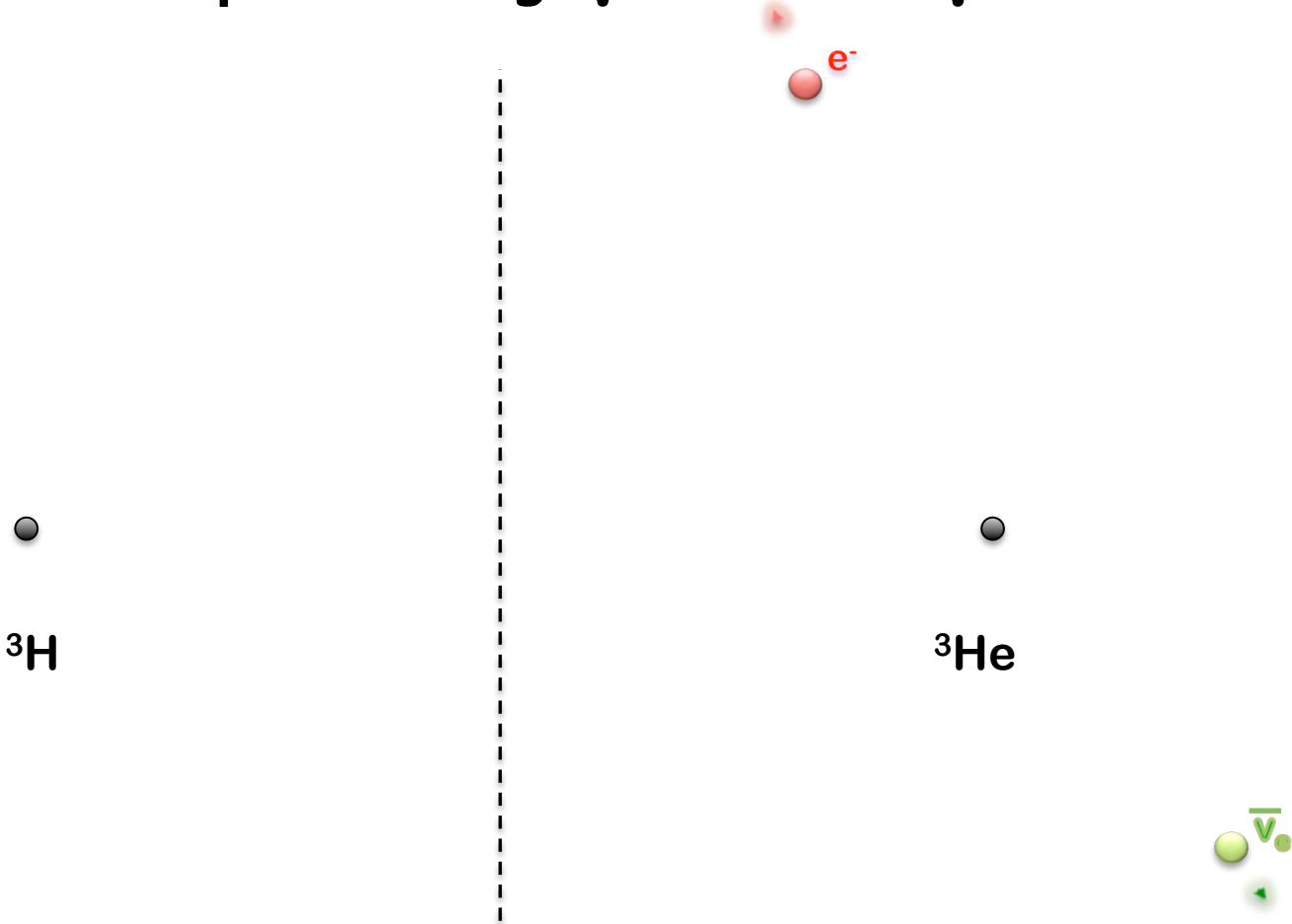
Mixing/No Mixing Ratio with tiny mixing relevant for WDM



A KATRIN-like experiment for keV ν



Tritium β decay phase space



$$\frac{d\Gamma}{dE_e}(m_{\nu_i}) = C \cdot p_e E_e \cdot \sqrt{(E_e - E_0)^2 - m_{\nu_i}^2} \cdot (E_e - E_0)$$

β -decay spectrum modelization



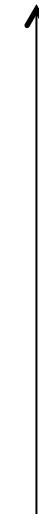
$$\frac{d\Gamma}{dE_e}(m_{\nu_i}) = C \cdot p_e E_e \cdot \sqrt{(E_e - E_0)^2 - m_{\nu_i}^2} \cdot (E_e - E_0) \cdot F(E_e, Z)$$



Normalization constant



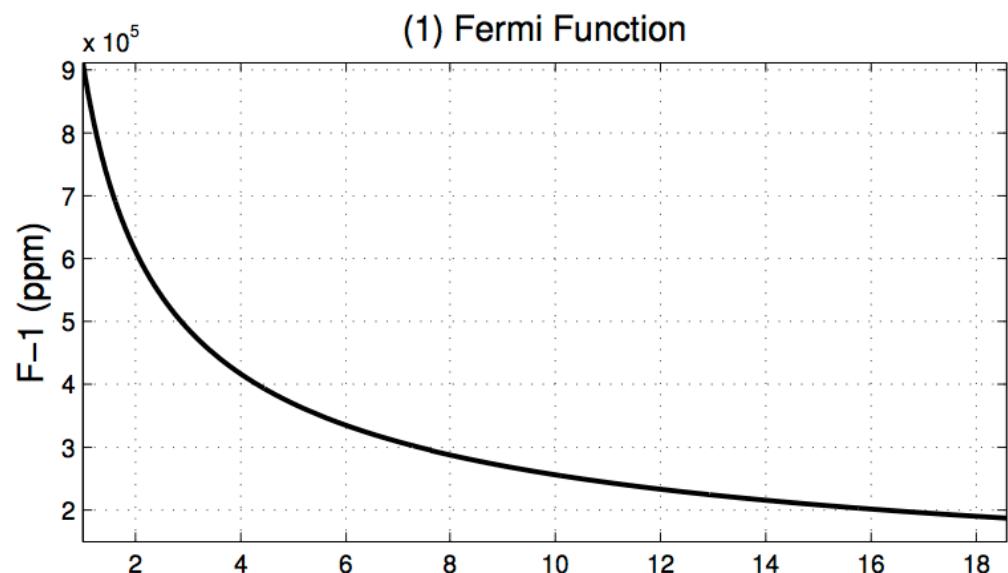
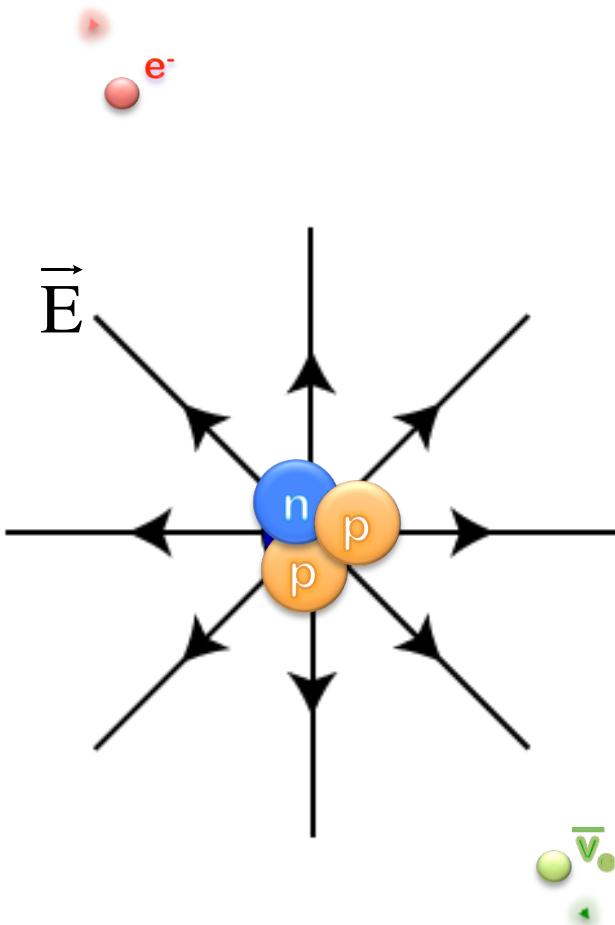
Phase space factor



Relativistic Fermi function

→ Extreme precision needed → must add theoretical corrections

Coulomb field of the daughter He nucleus



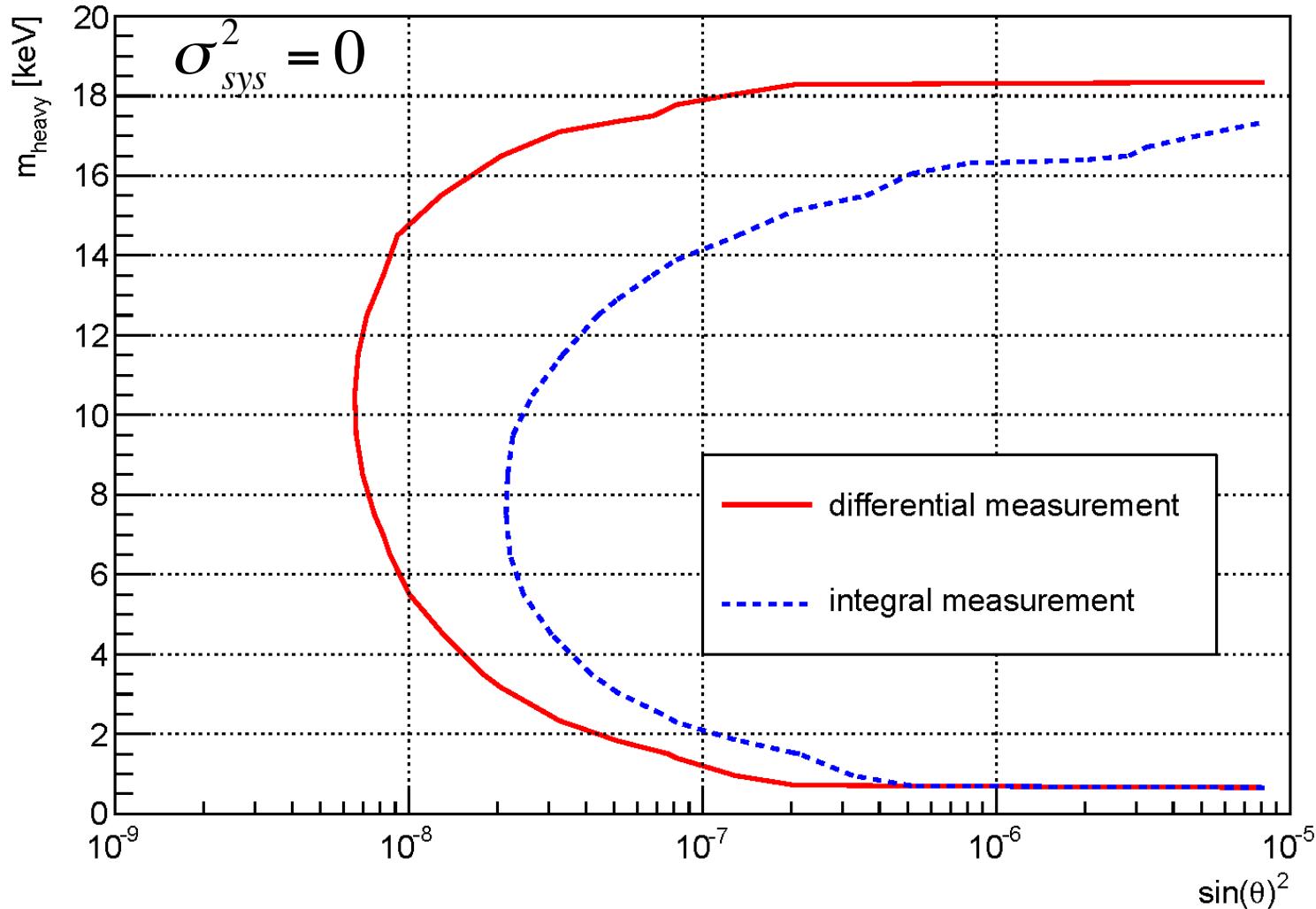
The Spectral Fit Approach

- Simulation of a KATRIN-like experiment
- 3 years of data – 100% efficiency – 10^{18} counts!
- Null hypothesis: no keV neutrino (no kink)
- Least squares estimator, non correlated uncertainties

$$\chi^2 = \sum_{i=1}^N \left| \frac{y_i - f(x_i | \alpha)}{\sigma_i} \right|^2 \quad \sigma^2 \Rightarrow \sigma_{stat}^2 + \sigma_{sys}^2$$

Statistical Sensitivity

3 years of data – 100% efficiency – 10^{18} counts



β -decay spectrum modelization

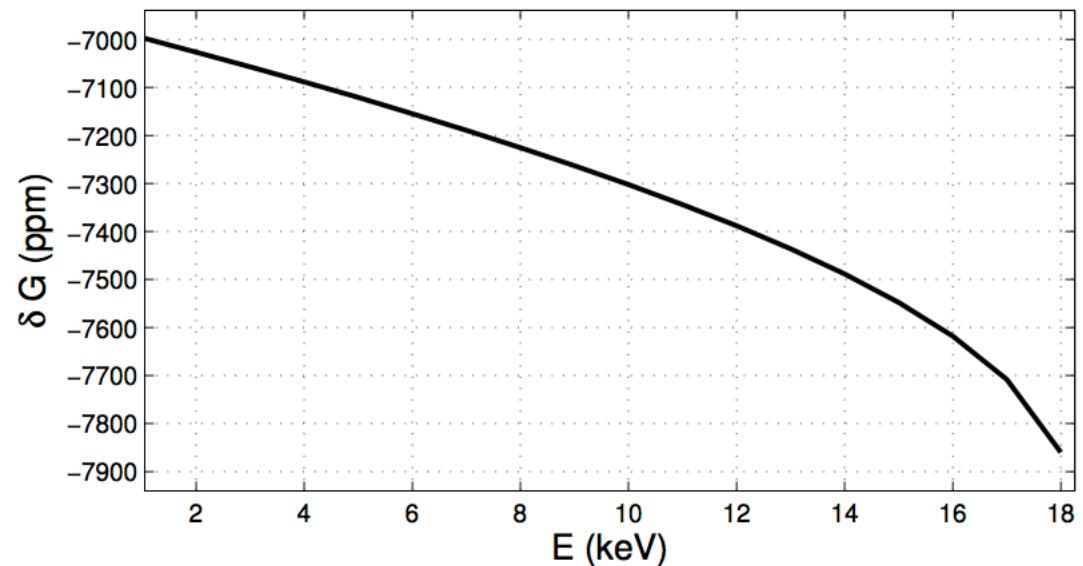
$$\left(\frac{d\Gamma}{dE_e} \right)^{corr} = \frac{d\Gamma}{dE_e} \cdot \left[\prod_{\Psi=L_0, S, E, Q, R, G} \Psi(E_e, Z) \right]$$

- **Screening Correction (S)**
- **Beta electron to orbital electron exchange (E)**
- **He recoil corrections (R):**
 - 3 body decay
 - weak magnetism
 - V-A correction
- **Recoiling Coulomb field (Q)**
- **Finite extension of the nucleus**
 - Coulomb field
 - weak interaction (L_0 , C)
- **Radiative corrections (G)**

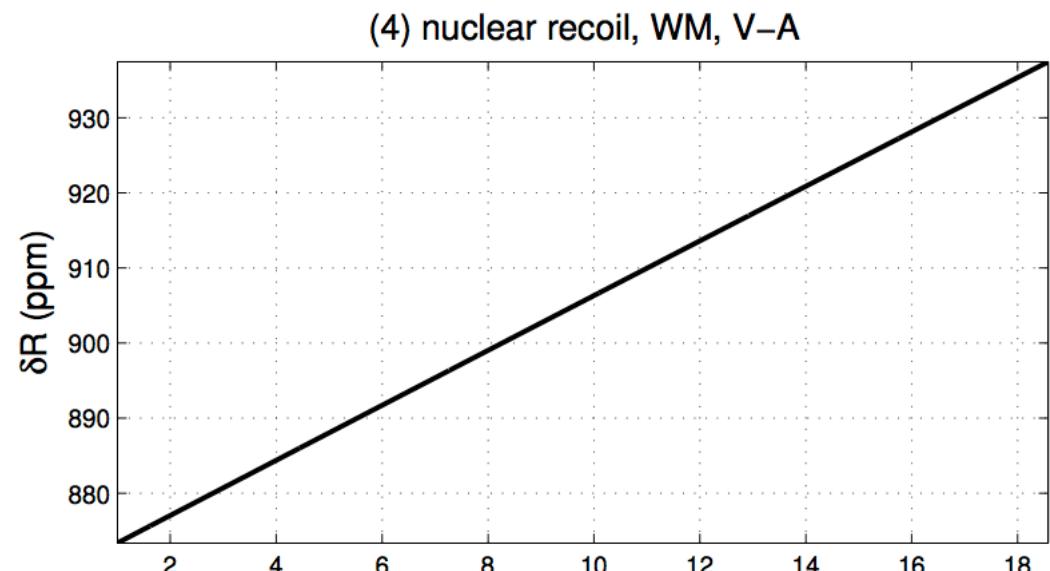
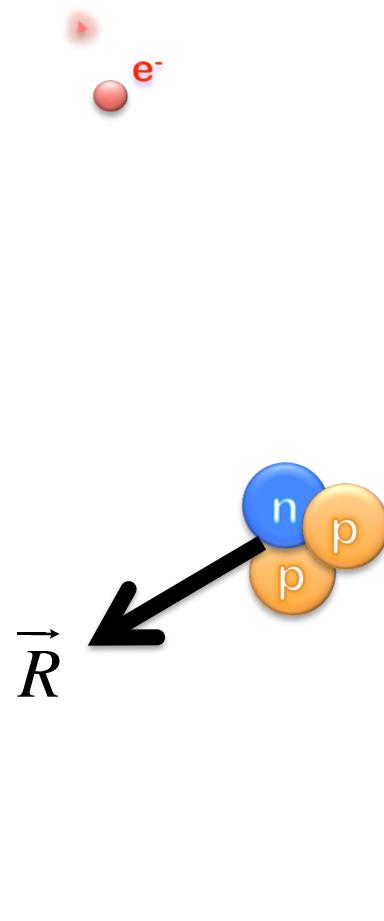
Radiative Correction



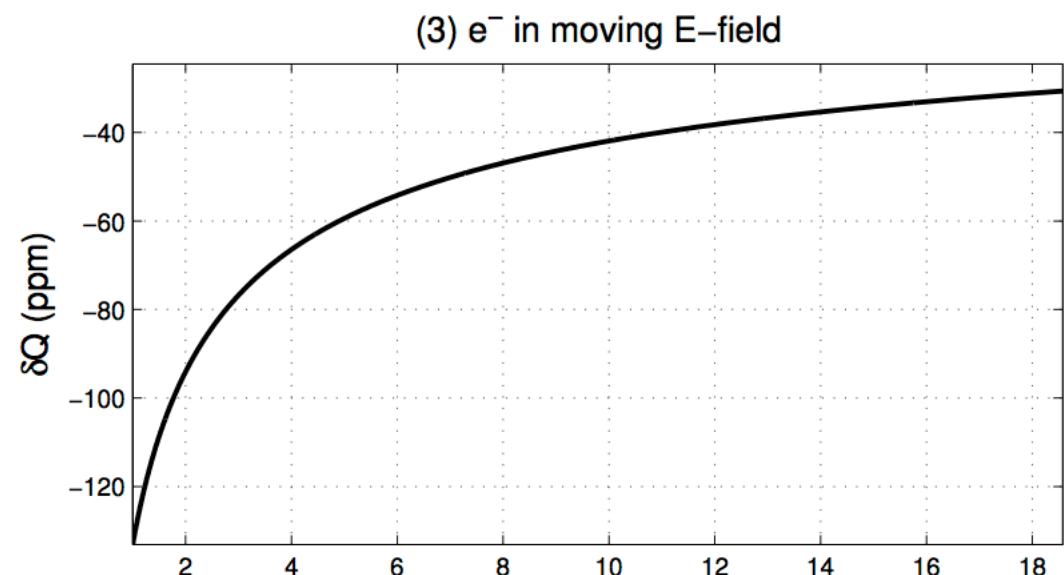
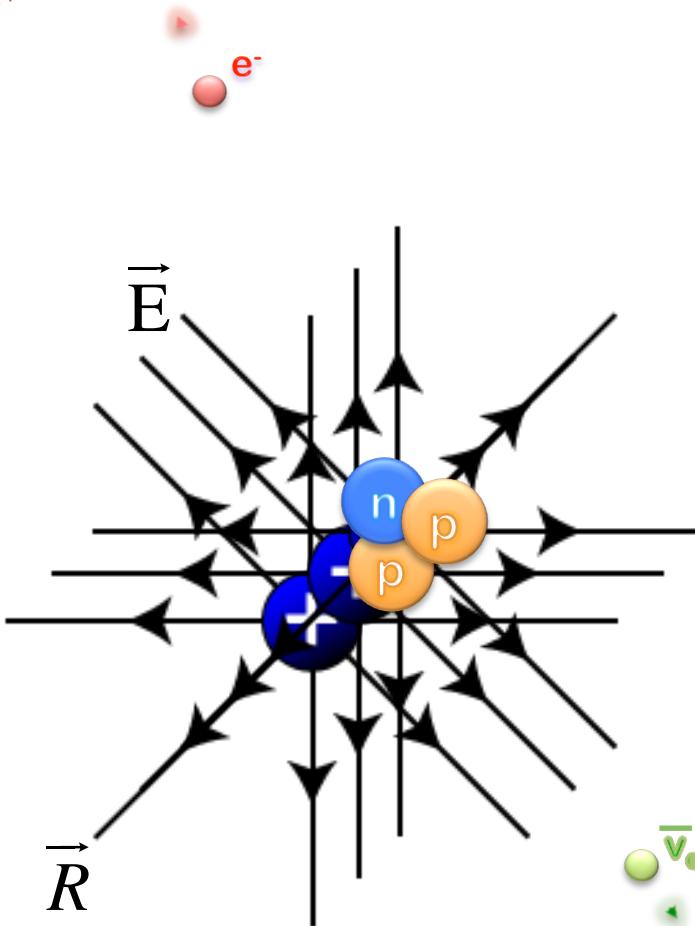
(8) radiative correction



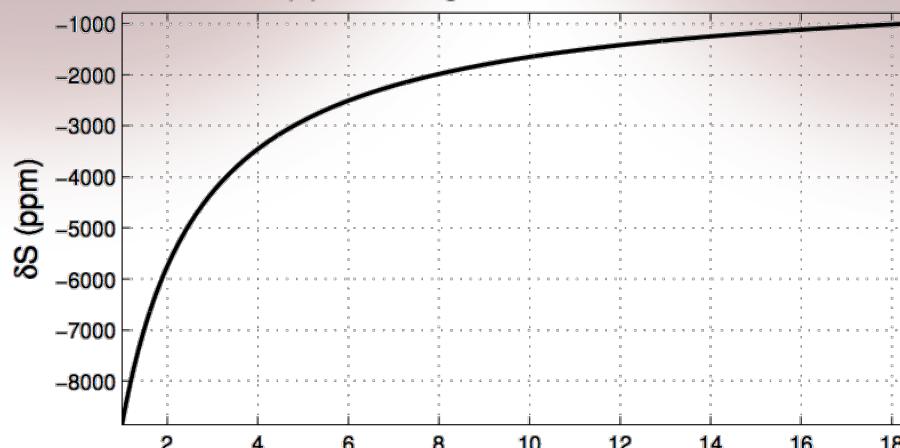
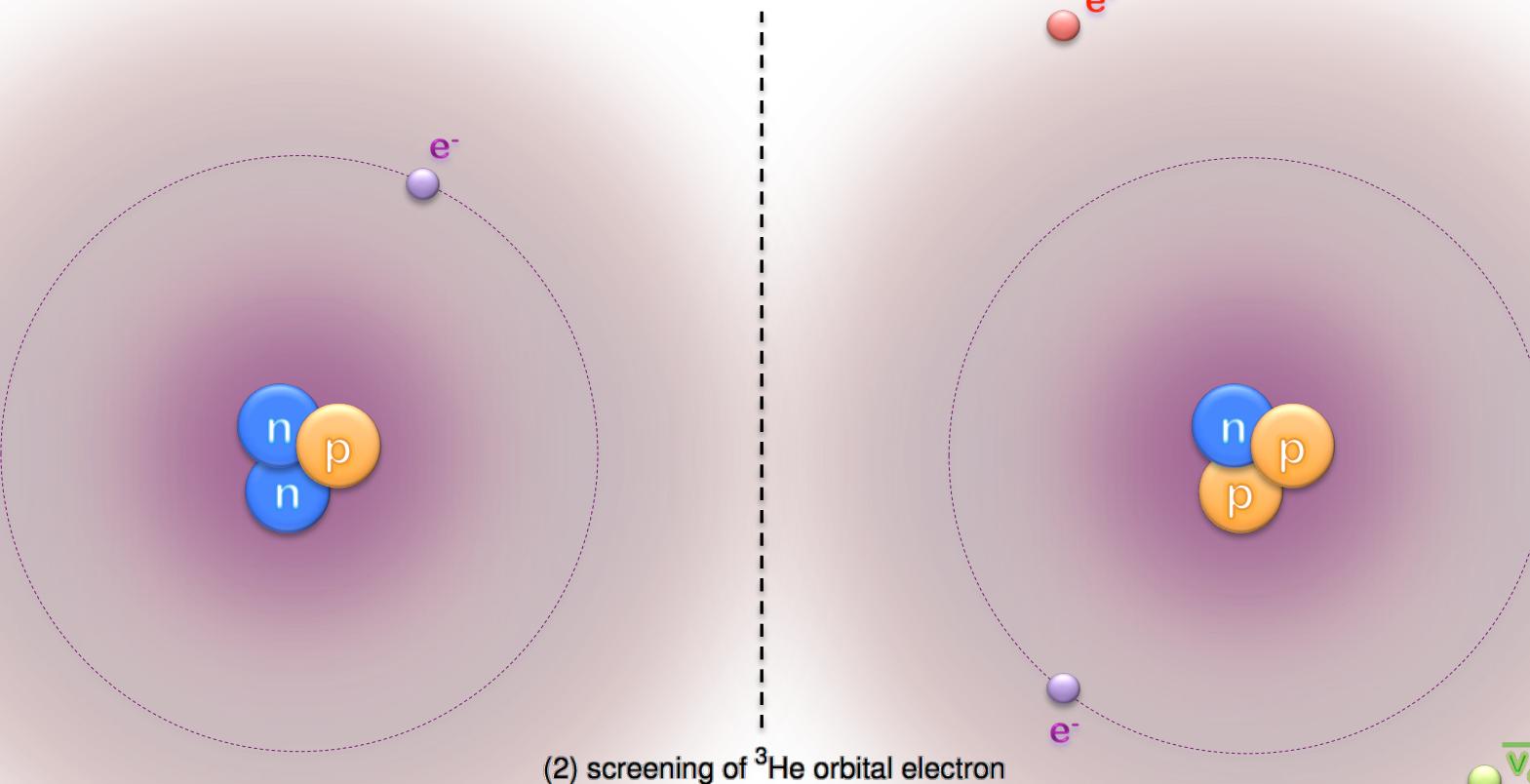
Recoil of the daughter nucleus (+ WM + V-A Corrections)



Recoiling Coulomb field of ${}^3\text{He}$

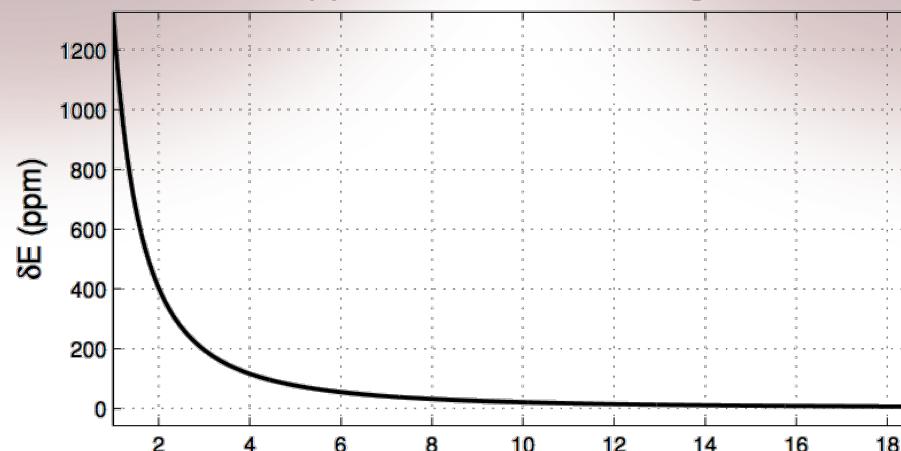
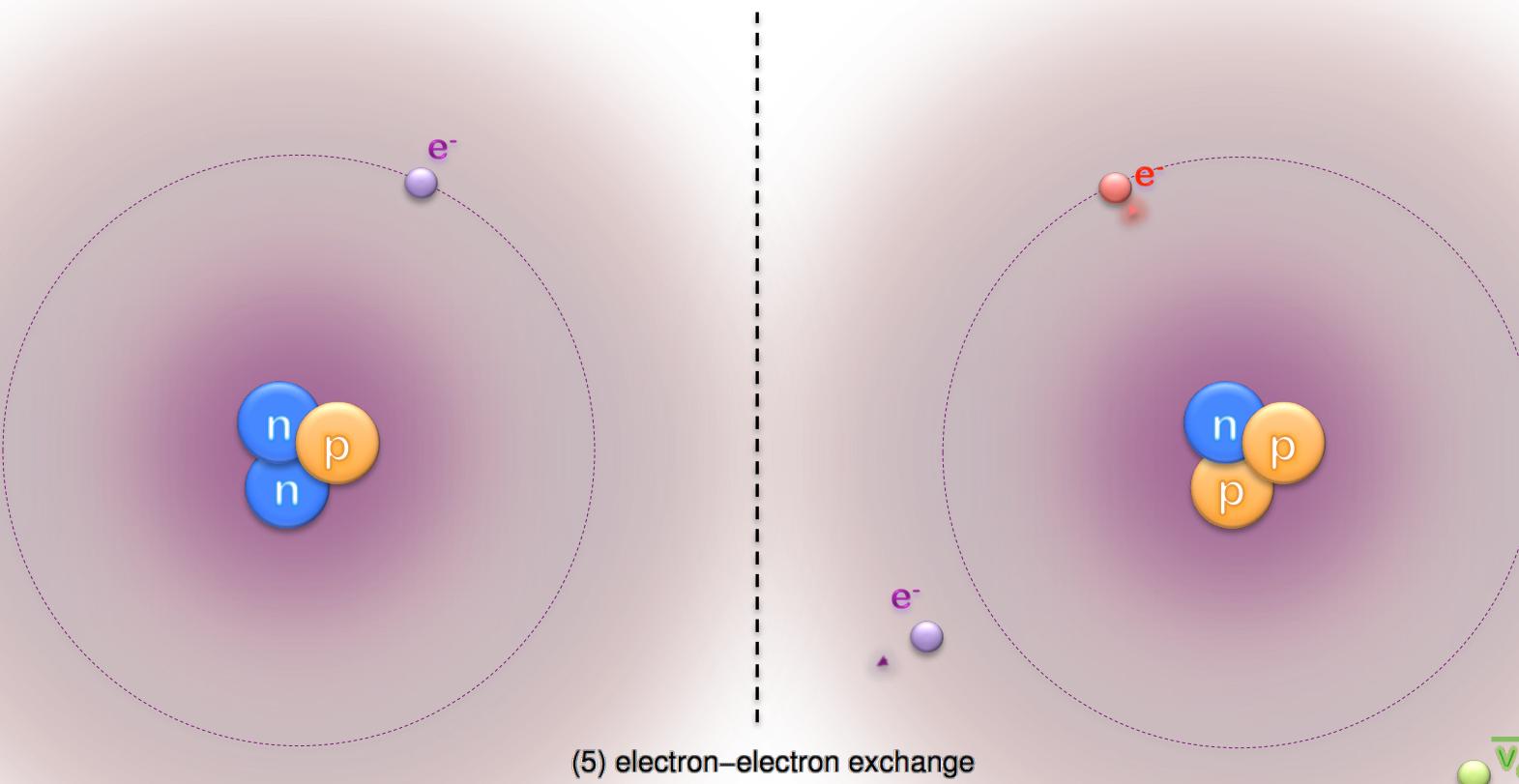


Screening of the orbital electron

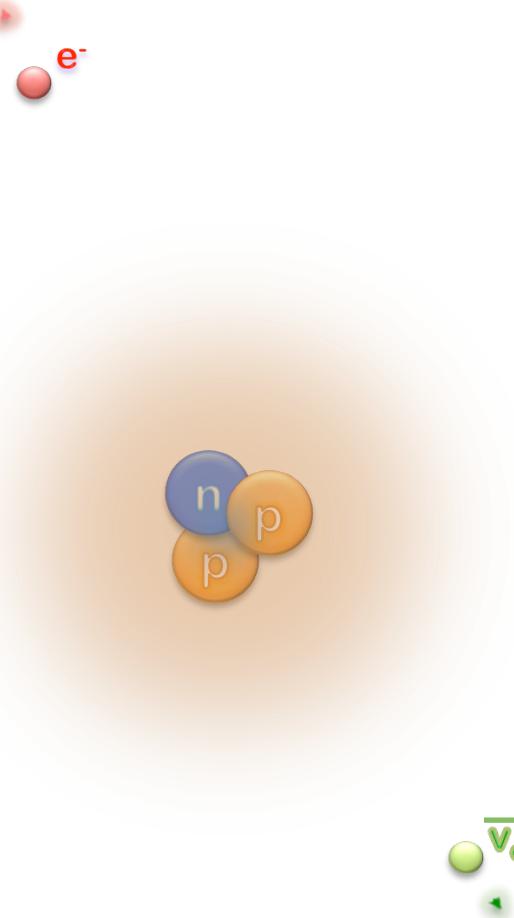


β electron - orbital electron exchange

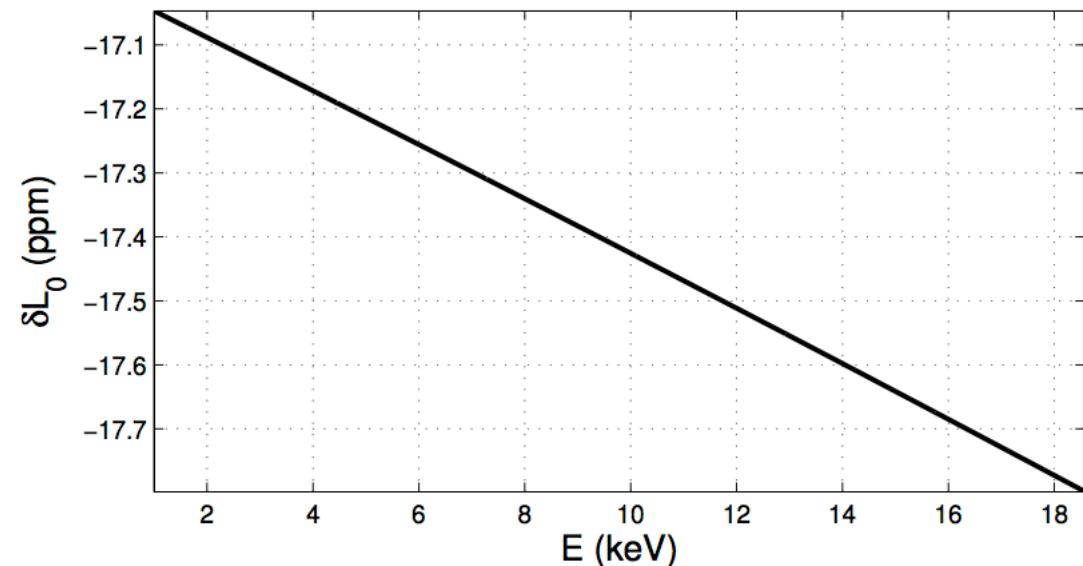
erc



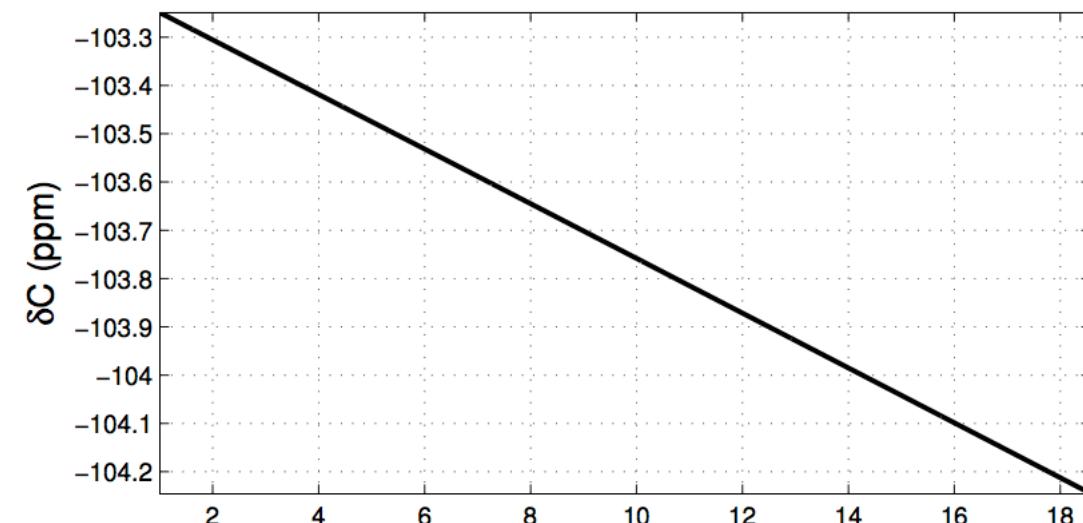
Finite Extension of the nucleus



(7) extension of nucleus charge



(6) weak interaction finite size



Taken the unknown into account...

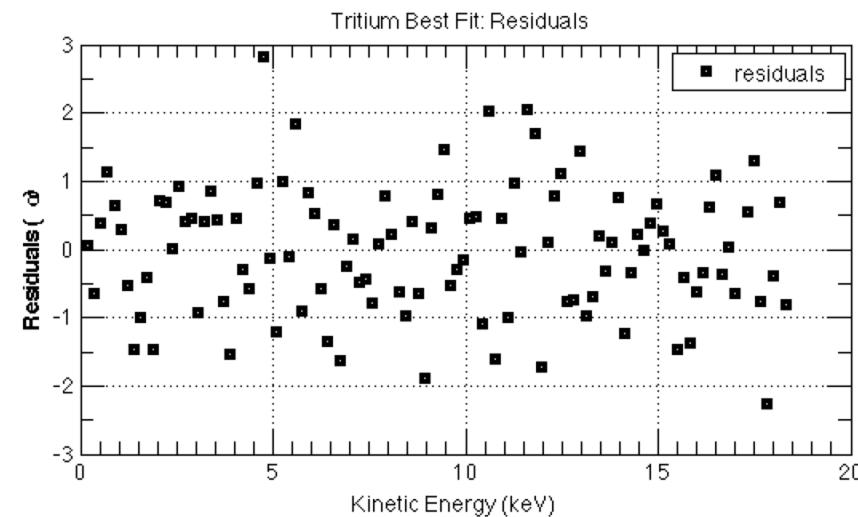
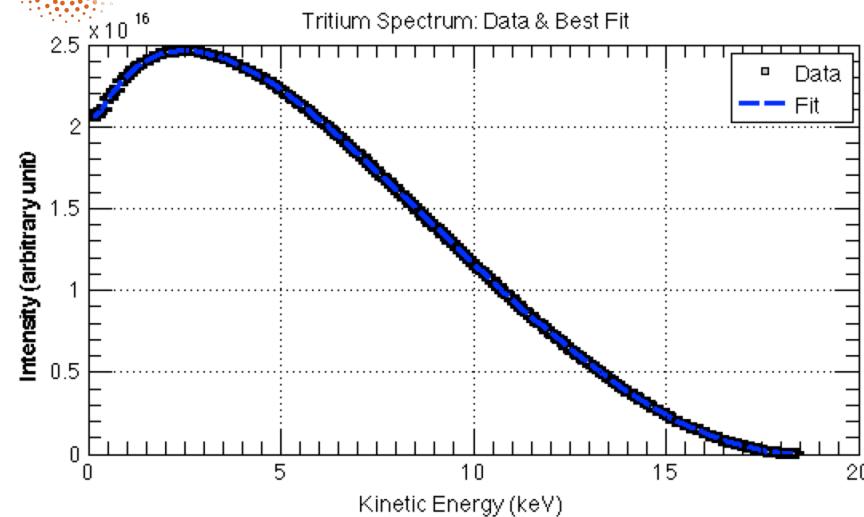
$$\left(\frac{d\Gamma}{dE_e} \right)^{corr} = \frac{d\Gamma}{dE_e} \cdot \left[\prod_{\Psi=L_0, S, E, Q, R, G} \Psi(E_e, Z) \right] \cdot SF(E_e)$$

$SF = \text{polynomial shape factor } (\beta, \gamma, \delta)$

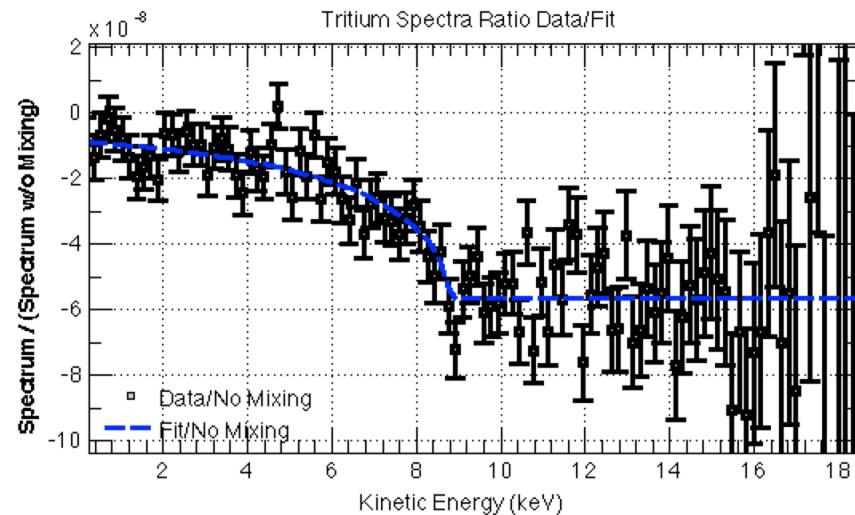
$$\frac{d\Gamma}{dE_e}(m_{\nu_i}) = C \cdot p_e E_e \cdot \sqrt{(E_e - E_0)^2 - m_{\nu_i}^2} \cdot (E_e - E_0) \cdot F(E_e, Z)$$

Fit of Spectrum with Corrections

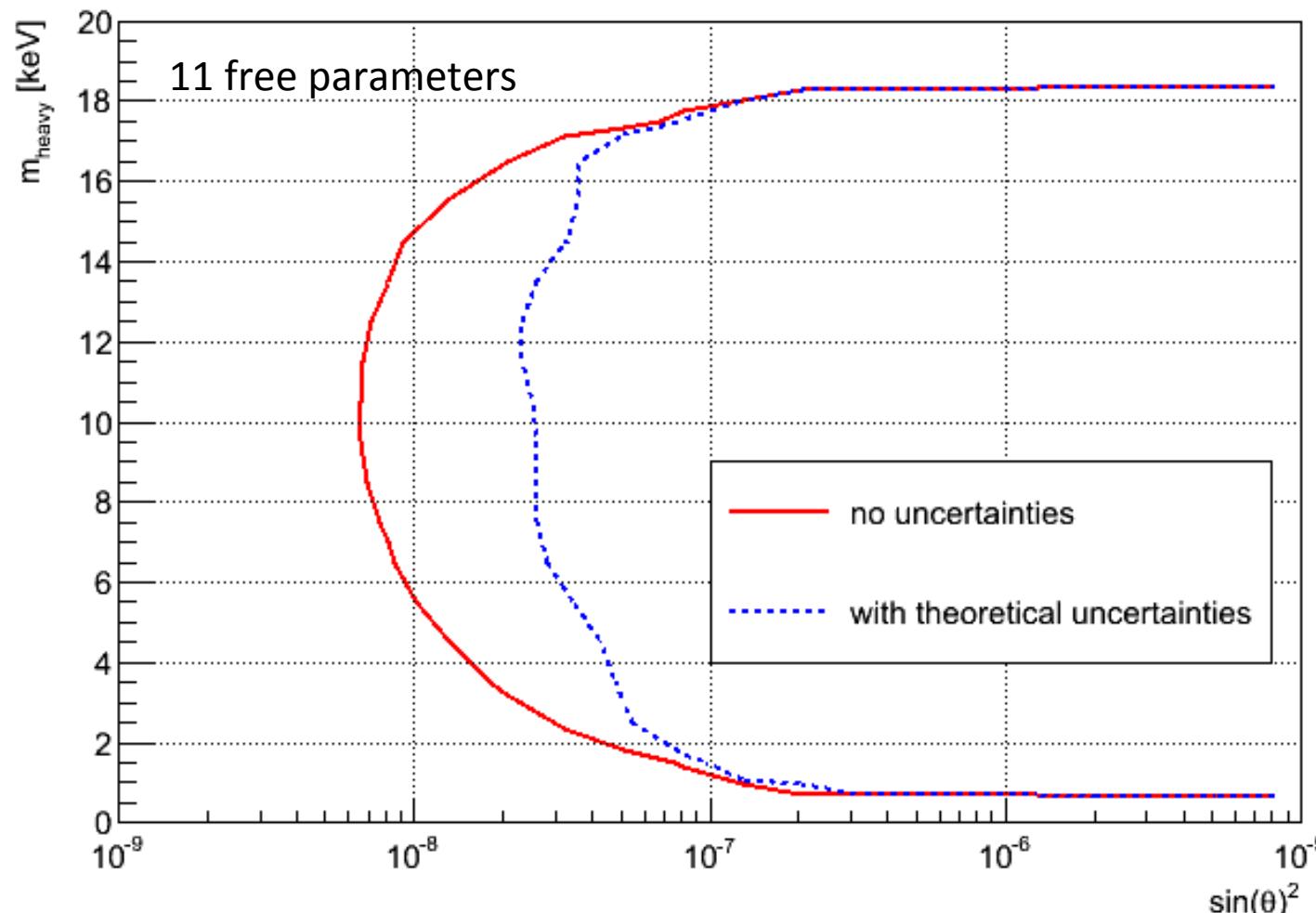
Blind test with an unknown small non-polynomial correction



| parameter | initialization | pull uncertainty | truth | best fit | fit uncertainty |
|---------------------|----------------|------------------|----------|-----------|-----------------|
| 1: norm | 1.00e-99 | | | 7.21e-03 | 1.00e+00 |
| 2: ν -mass | 9.80e+00 | | | 9.80e+00 | |
| 3: $\sin(\theta)^2$ | 5.00e-08 | | | 5.64e-08 | 6.79e-07 |
| 4: SF lin | | 1.00e-01 | | -7.91e-05 | 1.00e+00 |
| 5: SF quad | | 1.00e-01 | | -7.25e-07 | 1.00e+00 |
| 6: SF cub | | 1.00e-02 | | -1.40e-08 | 1.00e+00 |
| 7: SF 1/lin | 1.00e-99 | 1.00e-07 | | 1.00e-99 | |
| 8: G | 1.00e-99 | 1.00e-01 | | 1.00e-99 | |
| 9: R | 1.00e-99 | 1.00e-02 | | 1.00e-99 | |
| 10: E | 1.00e-99 | 1.00e-02 | | 5.51e-07 | 1.00e+00 |
| 11: C | 1.00e-99 | 1.00e-02 | | -1.64e-03 | 1.00e+00 |
| 12: L0 | 1.00e-99 | 1.00e-02 | | 7.11e-05 | 1.00e+00 |
| 13: Q | 1.00e-99 | 1.00e-02 | | -1.56e-05 | 1.00e+00 |
| 14: S | 1.00e-99 | 1.00e-02 | | 1.00e-99 | |
| 15: GS | 1.00e-99 | 2.00e-02 | 1.00e-99 | 1.00e-99 | |
| 16: ES | 1.00e-99 | 2.00e-02 | 1.00e-99 | 1.00e-99 | |
| χ^2 | 99.9 | 93 (dof) | | | |

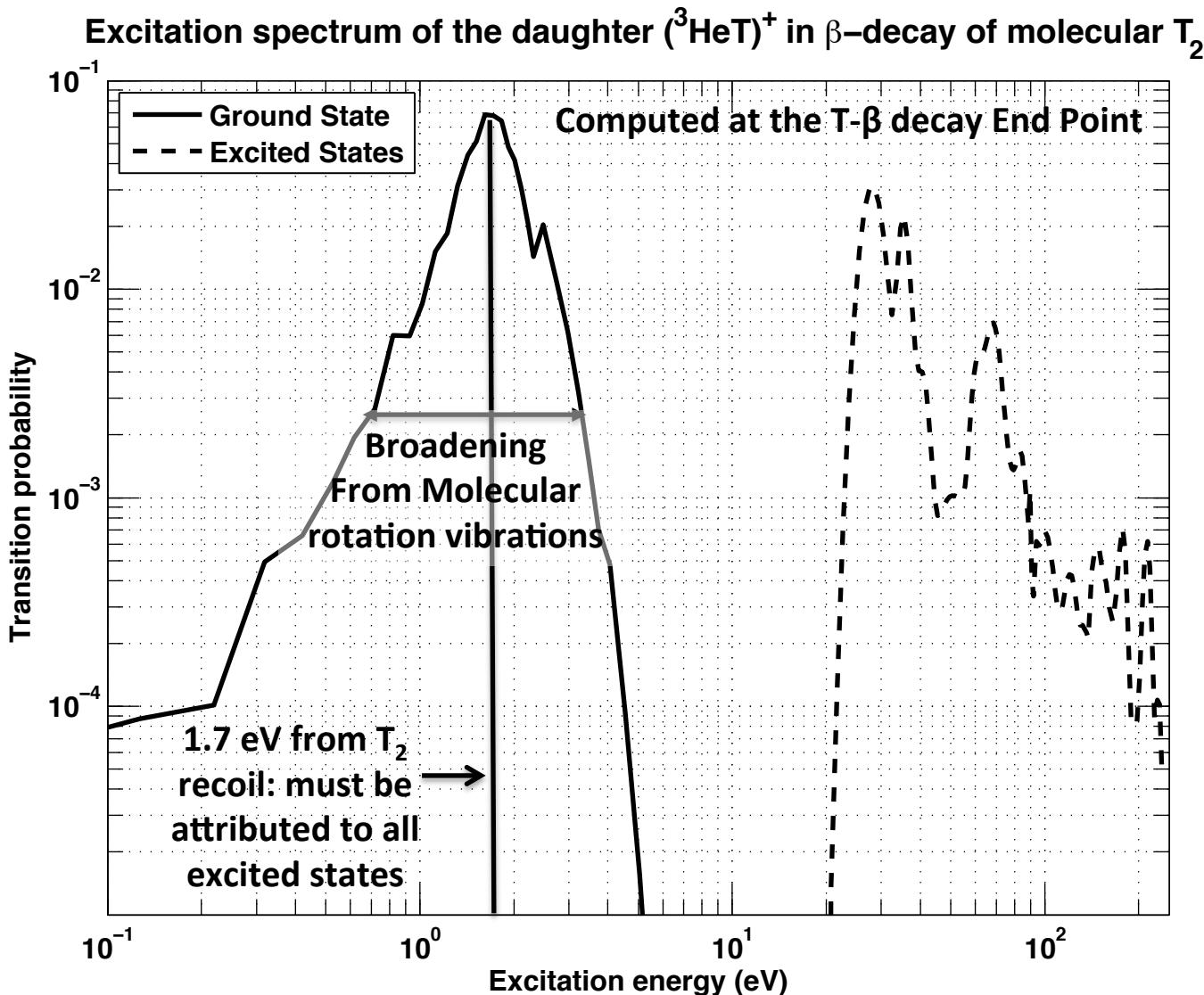


Impact of Theoretical Corrections



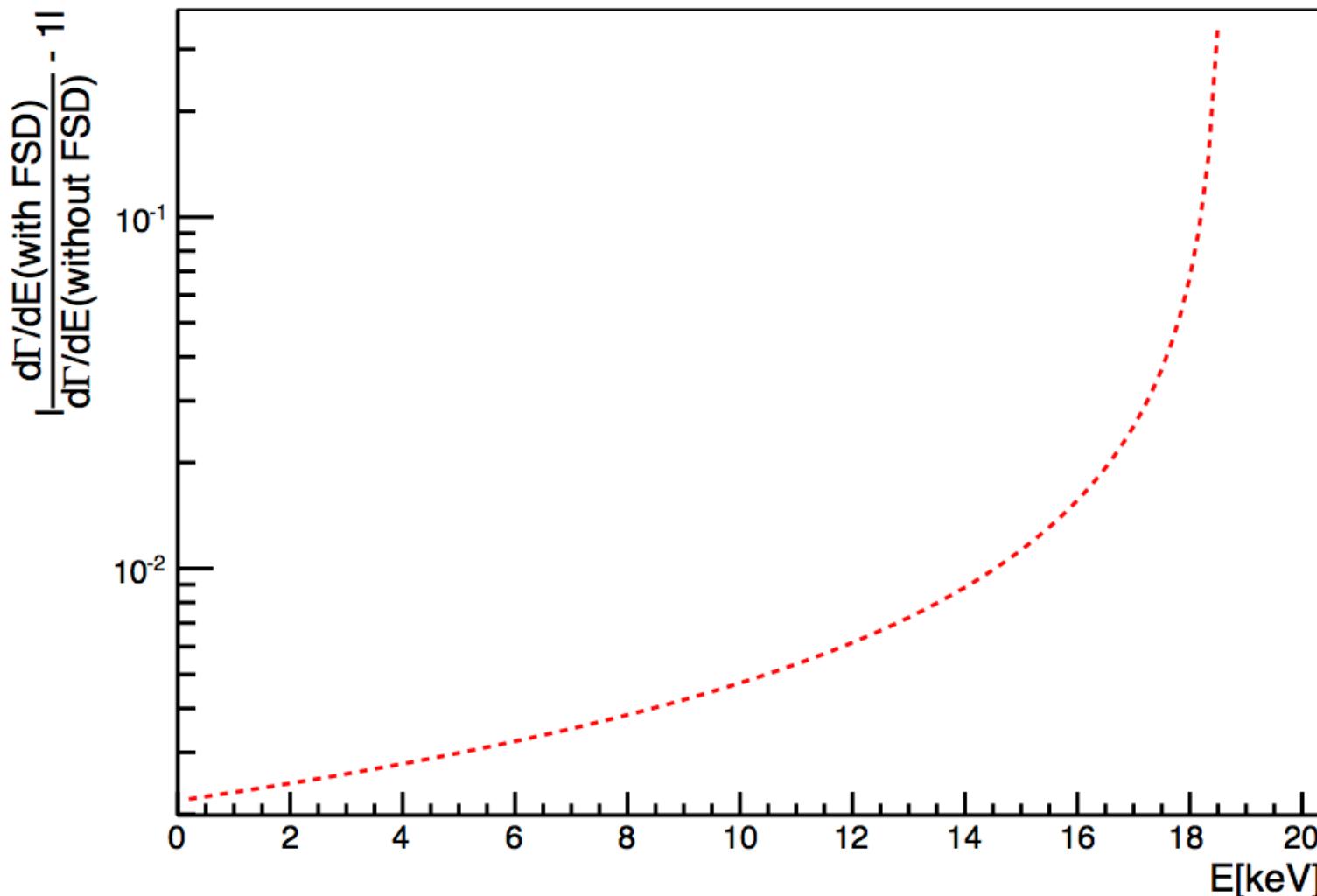
- Smooth corrections do not fake a kink signal $\sin^2(\theta) > 10^{-7}$
- Accurate Parameterization is necessary in order to perform the fit
- (Modelization harder at high/low energy due to steeply varying corrections)

Systematics: Decay to Excited States

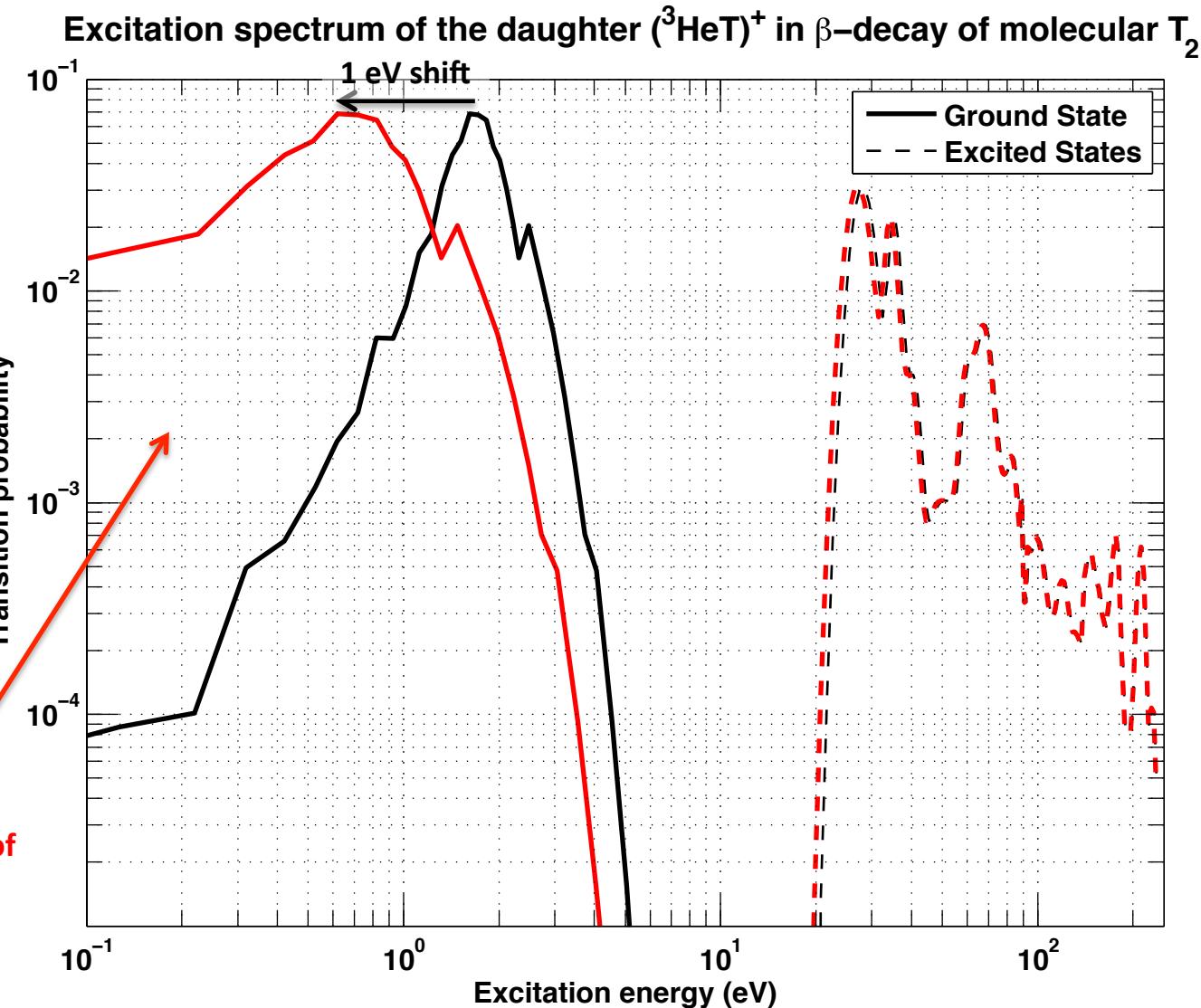


Systematics: Decay to Excited States

(1-Ratio) of tritium beta-decay spectrum with excited states / no excited states: **a large effect**



Systematics: Decay to Excited States

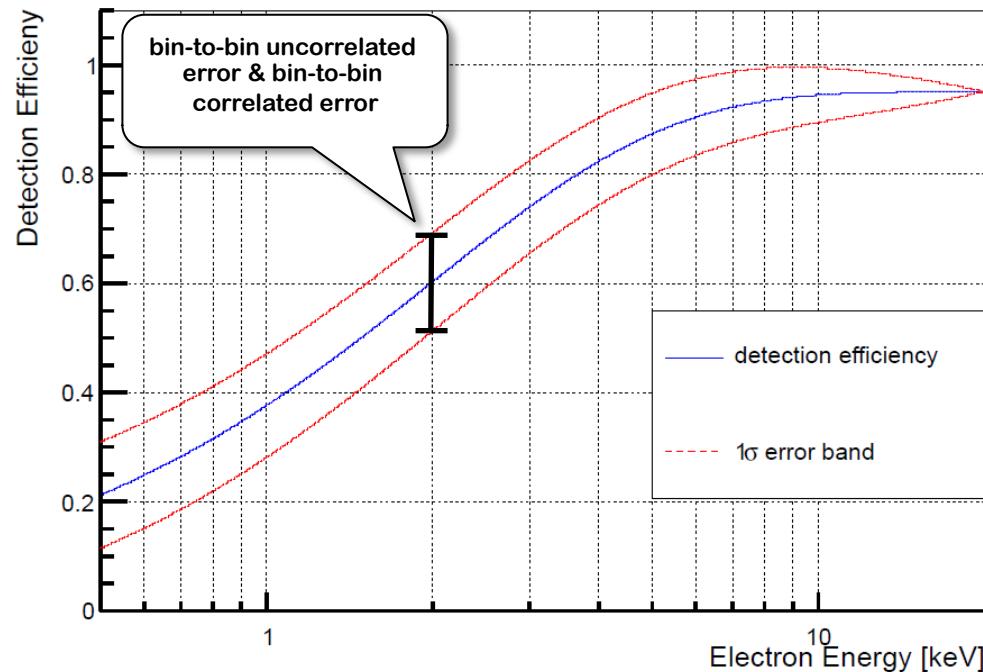


As a calculation of the FSD as a function of β -electron energy is not available but it is expected to be feasible

Systematics: Detector Efficiency

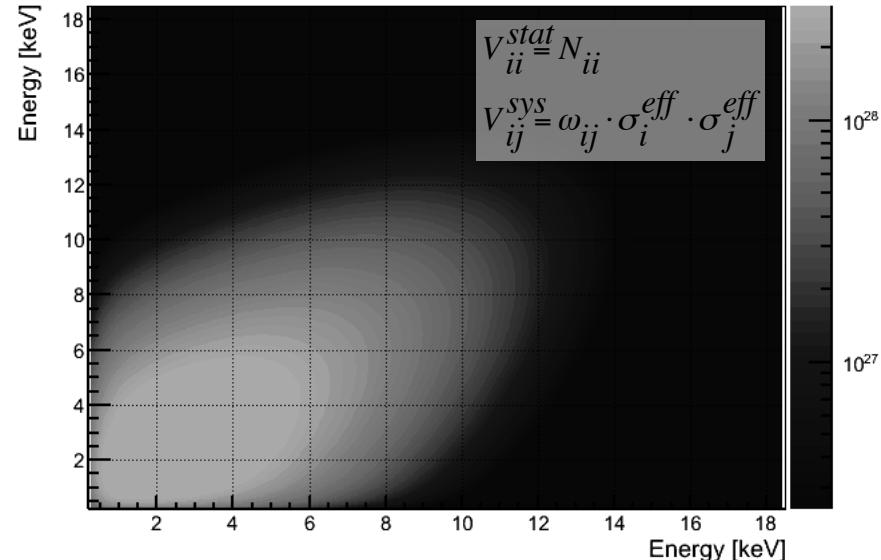
$$\Upsilon(E) = \epsilon_{\max} \times (1 - e^{-x \cdot \frac{E}{E_0}})$$

$$\delta\Upsilon(E) = \rho \cdot \left(1 - \frac{E}{E_0}\right)$$

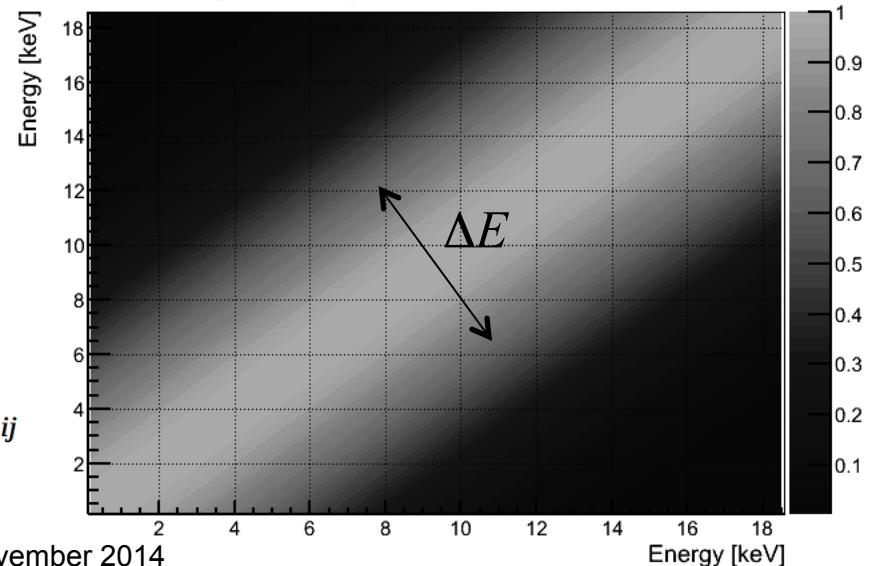


$$\chi^2 = \sum_{i=1}^N \sum_{j=1}^N (y_i - f(x_i|\alpha))(y_j - f(x_j|\alpha))(V^{-1})_{ij}$$

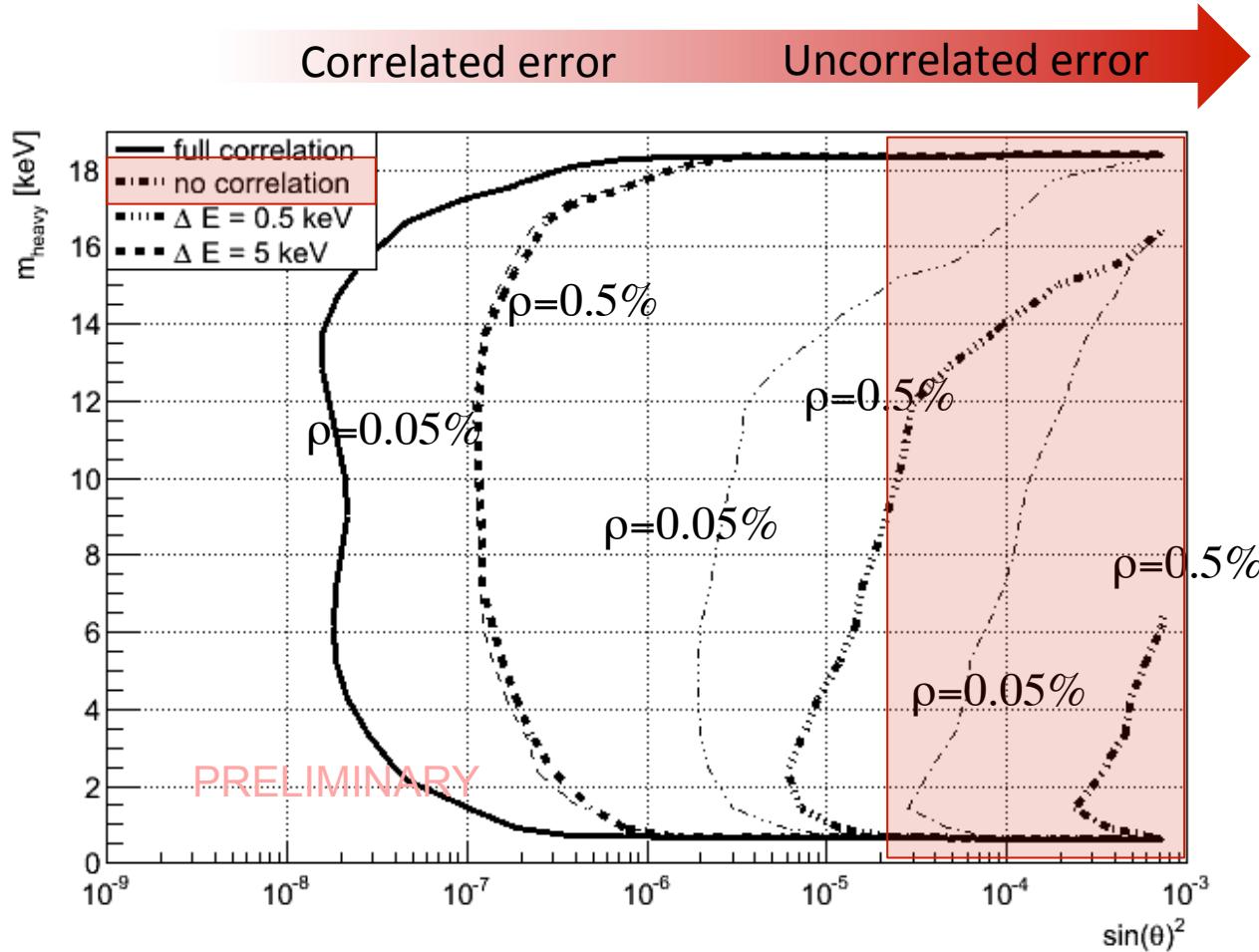
Covariance matrix



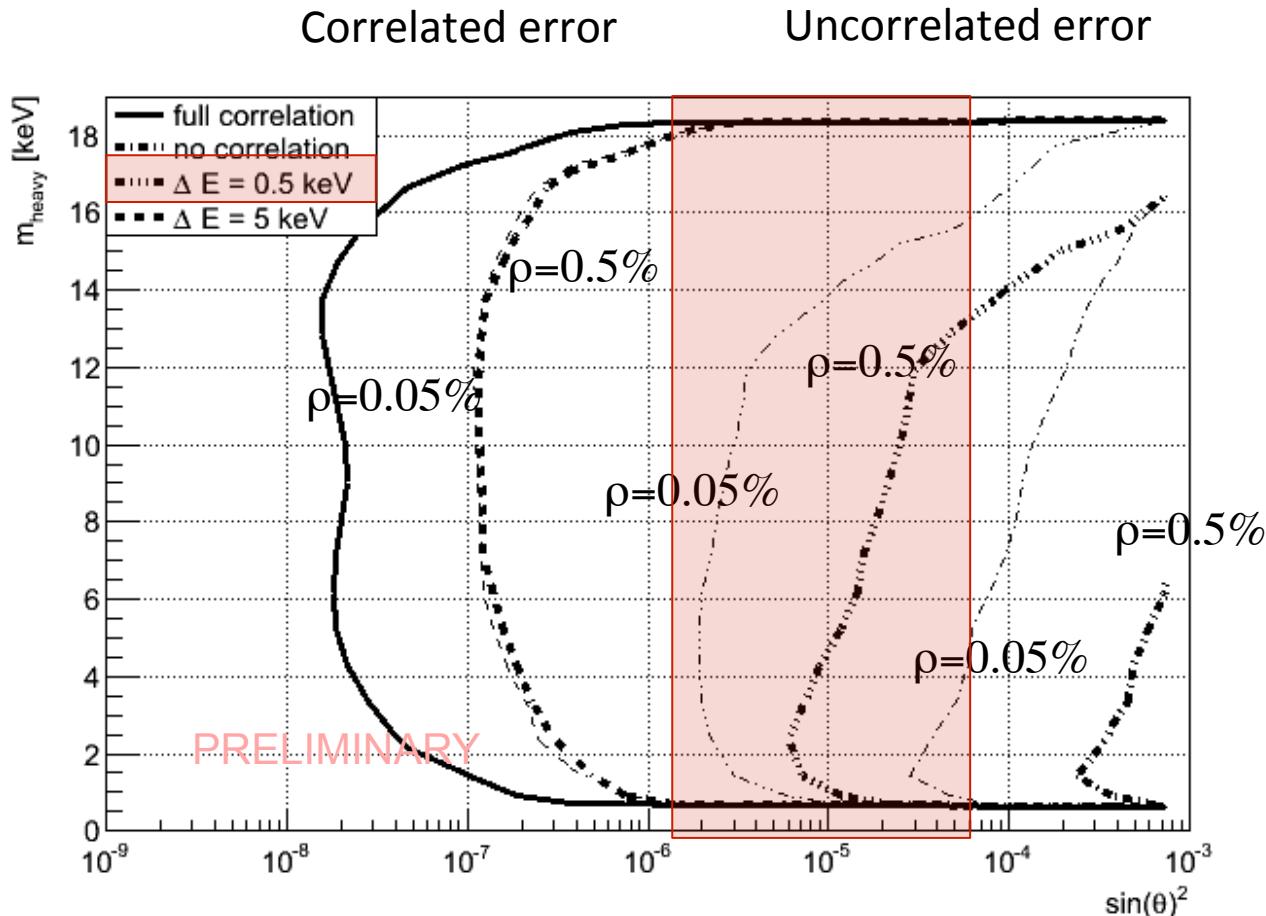
Weighting factors



Systematics: Detector Efficiency

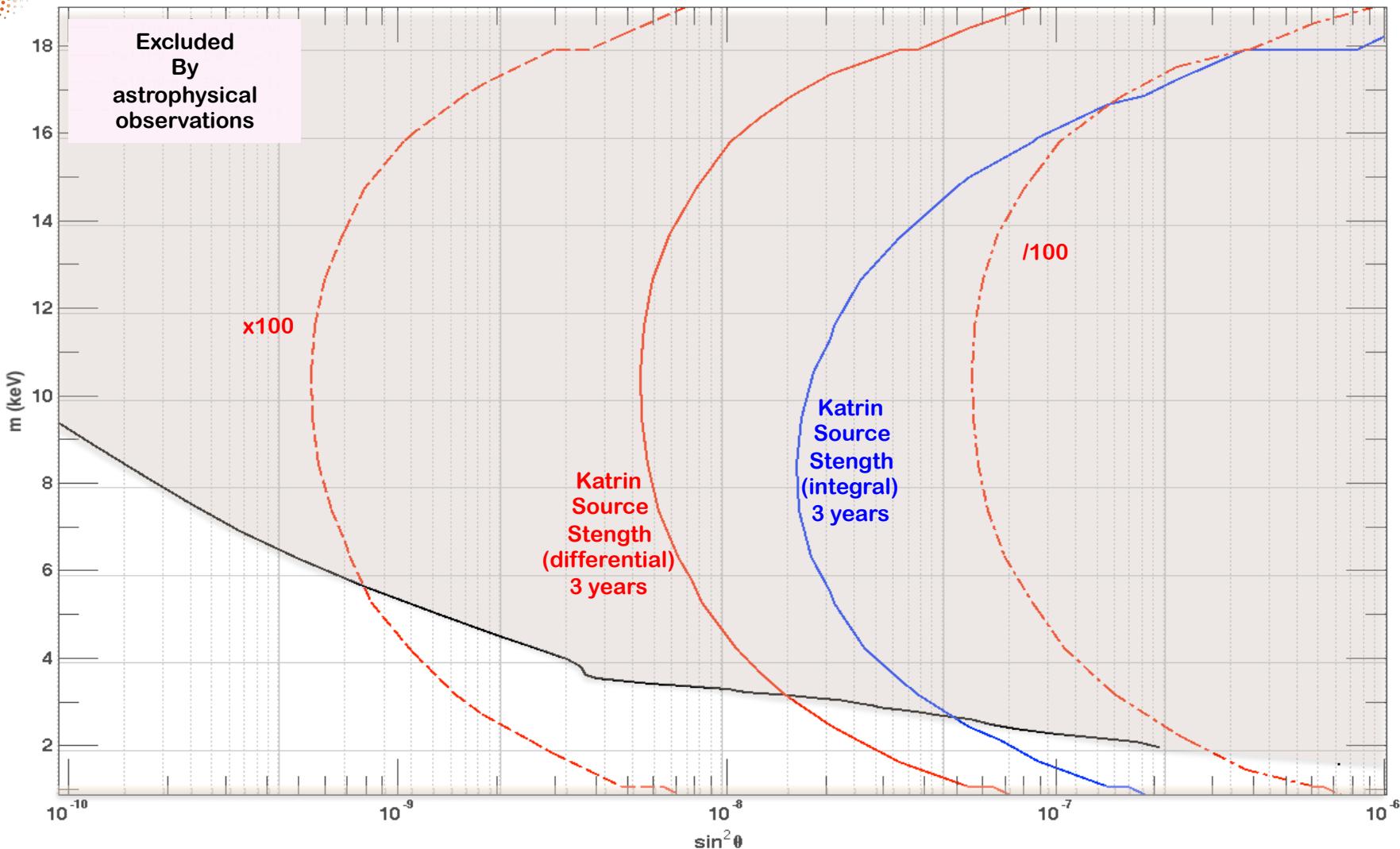


Systematics: Detector Efficiency





keV neutrinos search with Tritium



Summary and Outlook

- **A keV-ish neutrino Suitable Warm Dark Matter candidate**
 - Warm dark matter: suppress the small-scale structures.
 - Simplest realization: ν MSM (non-thermal production)
- **Search in the lab: KATRIN-like experiment (upgrade)**
 - KATRIN source strength + a new detector system
 - Statistically: $\sin^2(\theta) > 10^{-8} \rightarrow$ into cosmological favored region
- **Theoretical corrections:**
 - Smooth corrections can not fake a keV neutrino signature with $\sin^2(\theta) > 10^{-7}$ but full understanding of spectrum is needed
- **Generic Experimental effect:**
 - Uncorrelated uncertainties reduce sensitivity.
 - Need of detector development
- Article: arXiv:1409.0920
- Neutrino & Dark Matter White Paper in Preparation
http://irfu.cea.fr/en/Phocea/Vie_des_labos/Ast/ast_visu.php?id_ast=3446

Dark Matter & Neutrinos: White Paper



Outline and Section Editors

Editorial Committee: Marco Drewes, Thierry Lasserre, Alexander Merle, Susanne Mertens

I - Neutrinos in the Standard Model of Particle Physics and Beyond

(*Section Editors: Carlo Giunti and André de Gouvea*)

1. Current status of Neutrino Masses and Oscillations
2. Open questions in Neutrino Physics
 - 2.1 Neutrino Masses
 - 2.2 Neutrino Nature, Dirac or Majorana
 - 2.3 Neutrino Mass Hierarchy
 - 2.4 Neutrino CP violation
 - 2.5 Additional neutrino states
3. Sterile Neutrinos
 - 3.1 eV-scale
 - 3.2 keV-scale
 - 3.3 GeV, TeV, and >>TeV scales

II - Neutrinos in The Standard Model of Cosmology and Beyond

(*Section Editors: Julien Lesgourgues and Alessandro Mirizzi*)

1. Cosmological Concordance Model (J. Hamann, G. Mangano)
2. Active neutrinos in Cosmology (J. Lesgourgues, S. Pastor)
3. Sterile neutrinos in Cosmology
 - 3.1 eV-scale (M. Archidiacono, N. Saviano)
 - 3.2 KeV-scale (A. Boiarskyi, O. Ruchayskiy)
 - 3.3 Mev-scale (S. Pascoli)
 - 3.4 GeV-TeV (A. Ibarra)
 - 3.5 Leptogenesis (P. Di Bari)

Dark Matter & Neutrinos: White Paper



III - Dark Matter at Galactic Scales: Observational Constraints & Simulations (Astro)

(*Section Editors: Aurel Schneider, Francesco Shankar and Oleg Ruchayskiy*)

1. The Matter Power Spectrum & Status of Galactic-scale Structure Simulations
2. The too-big-to-fail issue
3. The Galactic Halo Cusps issue
4. The Missing Satellite issue
5. Proposed solution: CDM / WDM / Mixed-DM

IV - Observables Related to keV Neutrino Dark Matter

(*Section Editor: George Fuller, to be written in close collaboration with Section V*)

1. Pauli blocking (D. Gorbunov)
2. Lyman alpha (M. Vie)
3. X-rays - Hint for keV neutrino X-ray signal (O. Ruchaiskiy and A. Neronov)
4. Laboratory constraints on keV neutrinos (S. Mertens)

V - Constraining keV Neutrino Production Mechanisms

(*Section Editors: Marco Drewes, Fedor Bezrukov and G. Fuller*)

1. Dodelson-Widrow (G. Fuller, M. Drewes)
2. Shi-Fuller (G. Fuller, M. Drewes)
3. Scalar Decay (I. Tkachev)
4. Entropy Dilution
5. Discussion

Dark Matter & Neutrinos: White Paper



VI - keV Neutrino Theory and Model Building (Particle Physics)

(*Section Editors: Alexander Merle and Viviana Niro*)

1. General Principles of keV Neutrino Model Building (Alexander Merle and Viviana Niro)
2. Models based on Suppression Mechanisms:
 - 2.1 Split seesaw + extensions (A. Kusenko, R. Takahashi)
 - 2.2 Froggatt-Nielsen + variants (A. Merle, V. Niro)
 - 2.3 Minimal radiative inverse seesaw (A. Pilaftsis, B. Dev)
 - 2.4 Further models based on loop suppressions (D. Borah, R. Adhikari)
3. Models based on Symmetry Breaking:
 - 3.1 $L_e-L_\mu-L_\tau$ (A. Merle, V. Niro)
 - 3.2 A_4 (A. Merle)
 - 3.3 Q_6 (T. Araki)
4. Models based on other principles:
 - 4.1 Extended seesaw (J. Heeck)
 - 4.2 Dynamical mass generation/composite neutrinos (D. Robinson, Y. Tsai)
 - 4.3 331-models (A. Gomes Dias, N. Anh Ky)
 - 4.4 Geometric torsions/string theory (N. Mavromatos, A. Pilaftsis)

VII - Current & Future keV Neutrino Search with Astrophysical Experiments

(*Section Editors: Steen Hansen and Alexei Boyarsky*)

1. X-ray telescopes
2. Lyman alpha
3. Pulsar kick
4. Supernovae

Dark Matter & Neutrinos: White Paper



VIII - Current & Future keV Neutrino Search with Laboratory Experiments

(*Section Editors: Susanne Mertens and Loredana Gastaldo*)

1. Effect of keV sterile neutrinos on beta decay spectra

1.1 The case of H-3

- Troitsk
- KATRIN (S. Mertens, T. Lasserre)
- Project8 (B. Monreal)
- Ptolemy (C. Tully)
- Full kinematic reconstruction

1.2 The case of Re-187

- MANU (F. Gatti, M. Galeazzi)
- MIBETA (A. Nucciotti)
- MARE (F. Gatti, A. Nucciotti)

2. Effect of keV sterile neutrinos on electron capture spectra

2.1 The case of Ho-163

- ECHo (L. Gastaldo, Amand Faessler, T. Lasserre)
- HOLMES (F. Gatti, A. Nuciotti)
- NuMECS (M. Rabin)

2.2 Other EC isotopes (L. Gastaldo, Y. Novikov)

IX - Discussion - Pro and Cons for keV Neutrino as Dark Matter and Perspectives

Summary of Theoretical Corrections

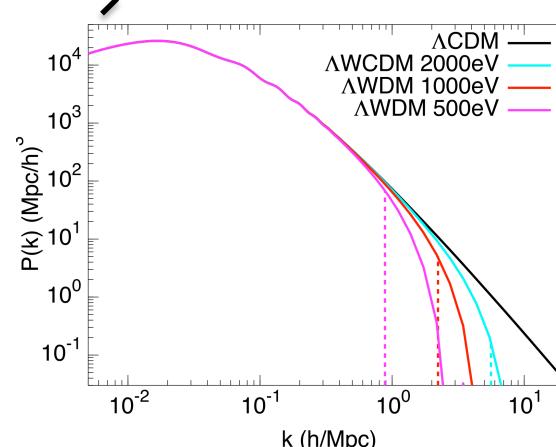
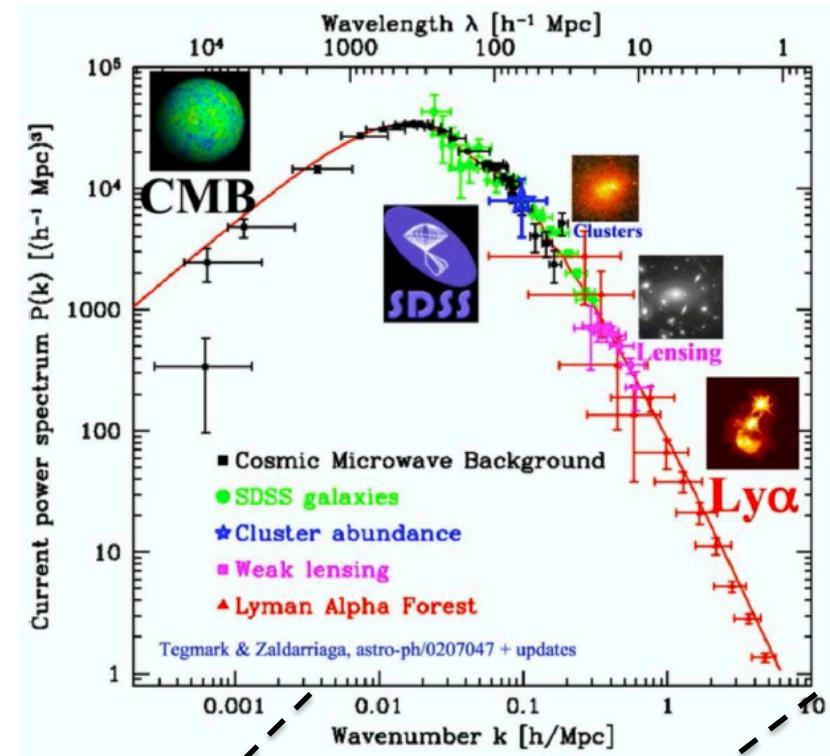
Table 1. For each multiplicative function Ψ_i ($\Psi_i = S, E, G, Q, R, C, L_0$) and the final state distribution (FSD) correction we define $\delta\Psi_i$ as $\frac{(d\Gamma/dE)^{\text{corr}}}{(d\Gamma/dE)^{\text{uncorr}}} - 1$, featuring the magnitude of each physical effect, computed at $E = 1, 9$, and 18 keV, in parts per million (ppm). The variation over the whole energy spectrum is defined by $\Delta A = |\delta\Psi_i(1 \text{ keV}) - \delta\Psi_i(18 \text{ keV})|$. The energy dependence of the FSD, the atomic corrections (S,E), and the radiative corrections (G) largely dominate. σ_{Ψ_i} provides a rough estimate of the uncertainty of each physical effects, obtained by varying key parameters or comparing different calculation methods as described in the 6th column. Additionally, the 6th column contains comments about the current status of the computation.

| Correction | $E=1$ keV [ppm] | $E=9$ keV [ppm] | $E=18$ keV [ppm] | ΔA [ppm] | Comment/Error estimation method | Ref. |
|-----------------|--------------------|--------------------|---------------------|---------------------|---|------|
| $\delta FSD(E)$ | 1400 | -635 | -351175 | 352575 | Computed only for the endpoint | [44] |
| σ_{FSD} | — | — | — | — | | |
| $\delta S(E)$ | -8850 | -1765 | -995 | 7860 | V_0 computed only for ${}^3\text{He}^+$ ion | [48] |
| σ_S | 1780 | 360 | 200 | | V_0 varied by $\pm 10\%$ | |
| $\delta E(E)$ | 2470 | 45 | 10 | 1320 | Excitations computed only for ${}^3\text{He}^+$ ion | [50] |
| σ_E | 1145 | 20 | 5 | | Diff. between [49] & this work | |
| $\delta G(E)$ | -6995 | -7270 | -8110 | 1115 | Only first order considered | [54] |
| σ_G | 25 | 260 | 830 | | Diff. between [54] & [54] approx. | |
| $\delta Q(E)$ | -135 | -45 | -30 | 105 | — | [52] |
| σ_Q | <1 | <1 | <1 | | λ_t varied by $\pm 1\%$ (3σ) | |
| $\delta R(E)$ | 875 | 905 | 935 | 60 | — | [51] |
| σ_R | 5 | 5 | 5 | | λ_t varied by $\pm 1\%$ (3σ) | |
| $\delta C(E)$ | -105 | -105 | -105 | 1 | — | [49] |
| σ_C | 3 | 3 | 3 | | R varied by $\pm 5\%$ | |
| $\delta L_0(E)$ | -20 | -20 | -20 | 1 | — | [49] |
| σ_L | 6 | 6 | 6 | | R varied by $\pm 5\%$ | |

Dark Matters: Cold or Warm?

- **Large scales:**
 - CMB, LSS survey's
 - HDM (active ν 's) ruled out
 - CDM or WDM fit the observed structures equally well

- **Small scales:**
 - Lyman- α , lensing...
 - CDM & WDM differ
 - WDM erases the ‘smallest’ structures



WDM Constraint: Lyman- α Forest

erc

- Observe redshifted H absorption lines in quasar spectrum
- Provide constraints on structure-size at scales of $0.3 \text{ h/Mpc} < k < 3 \text{ h/Mpc}$
- Constraint depends on model of ν_s dark matter
- Uncertainties
 - analysis of quasar spectra
 - modelling of hydrogen clouds
 - simulation of DM clustering
 - Data & fit to astrophysical parameters - Systematics

