Neutrino physics using quantum coherence

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Outline of this talk

Introduction:
remaining important questions in neutrino physics
detection principle of relic 1.9 K neutrino

(Quantum coherence: an example of adiabatic Raman excitation) covered by Sasao

Coherent quantum synchrotron

Present status of neutrino physics

- Oscillation experiments
  - Finite mass
  - Flavor mixing
  - Only mass-squared difference can be measured.

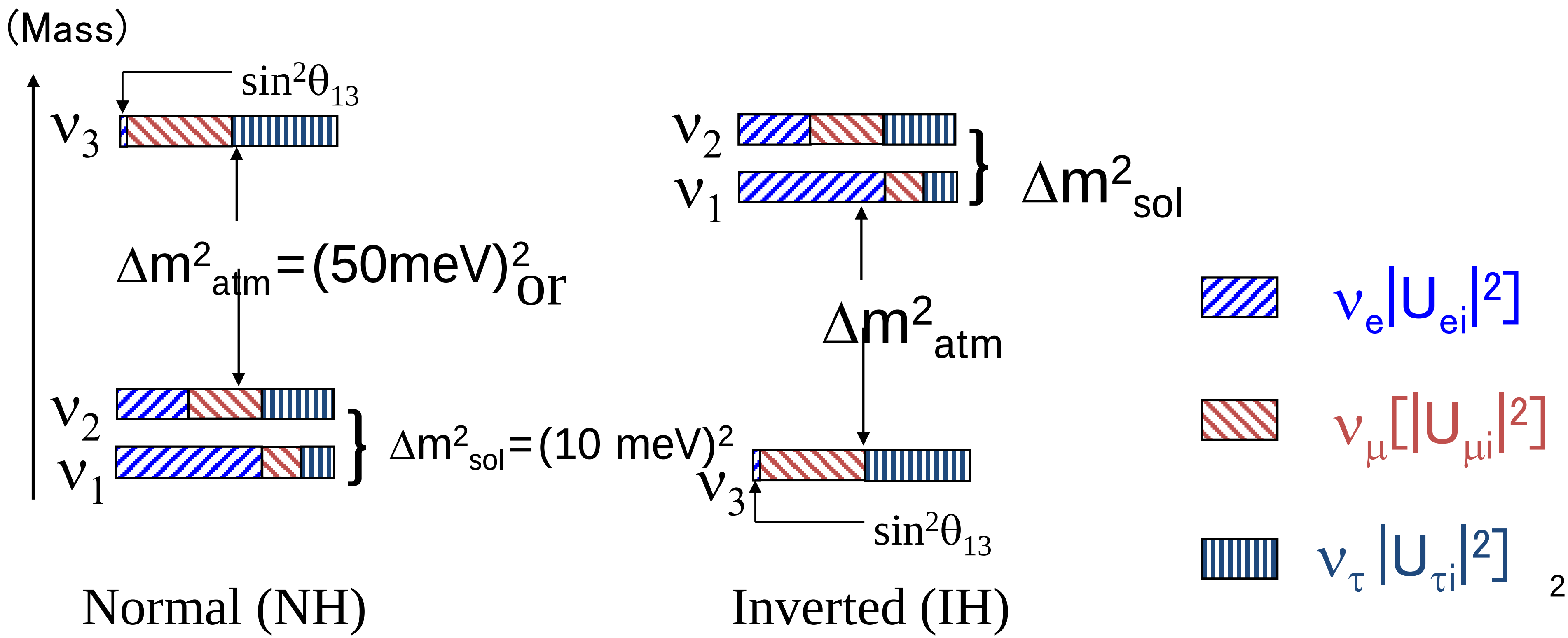
$$U = VP, \tag{A8}$$

where

$$V = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}, \tag{A9}$$

with $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. The diagonal unitary matrix P may be expressed by

$$P = \mathrm{diag.}(1, e^{i\alpha}, e^{i\beta}), \tag{A10}$$



Important questions left in neutrino physics

- Absolute mass scale and the smallest mass (oscillation experiments are sensitive to mass squared differences alone)
- Majorana vs Dirac distinction
- CPV phase （Majorana case has 2 extra phases） $\alpha\,,\ \beta\,,\ \delta\ (\mathrm{KM}-\mathrm{type})$
- Detection of relic 1.9K neutrino

These are relevant to explanation of matter-antimatter imbalance of universe and physics beyond the standard theory.

CP asymmetry in leptogeneis

$$\approx \frac{3y_1^2}{4\pi}\left(-2(\frac{m_3}{m_2})^3 s_{13}^2 \sin 2(\delta+\alpha-\beta) + \frac{m_1}{m_2}\sin(2\alpha)\right)$$

+ (high energy phases inaccessible in low energy experiments)

Majorana vs Dirac equations chirally projected solutions

Dirac eq.: degenerate 2 Majorana

$$(i\partial_t - i\vec{\sigma}\cdot\vec{\nabla})\varphi = m\chi\,, \quad (i\partial_t + i\vec{\sigma}\cdot\vec{\nabla})\chi = m\varphi$$

$$\psi_D = (1-\gamma_5)\psi/2$$

$$\psi_D = b(\vec{p}\,,h)e^{-ip\cdot x}u(\vec{p}\,,h) + d^\dagger(\vec{p}\,,h)e^{ip\cdot x}\sqrt{\frac{E_p+hp}{E_p-hp}}(-i\sigma_2)u^*(\vec{p}\,,h)$$

Particle annihilation Anti-particle creation

2-component in weak process involved

Majorana eq. : particle=antiparticle

$$(i\partial_t - i\vec{\sigma}\cdot\vec{\nabla})\varphi = im\sigma_2\varphi^*$$

$$\varphi_{\vec{p},h}(x) = c(\vec{p}\,,h)e^{-ip\cdot x}u(\vec{p}\,,h) + c^\dagger(\vec{p}\,,h)e^{ip\cdot x}\sqrt{\frac{E_p+hp}{E_p-hp}}(-i\sigma_2)u^*(\vec{p}\,,h)\,,$$

$$u(\vec{p}\,,h) = \frac{1}{2}\sqrt{\frac{E_p-hp}{pE_p(p+hp_3)}}\begin{pmatrix} p+hp_3 \\ h(p_1+ip_2)\end{pmatrix}.$$

2 neutrino wave functions are anti-symmetrized

Significance of Majorana neutrinos

- Theoretical prejudice: Neutral leptons consist of 4 components like all other quarks and leptons, the ordinary massless neutrino and the other 2-component partner having a much larger mass of Majorana-type than the Fermi scale
- -> Seesaw mechanism with a Dirac-type coupling via Higges $\frac{m^2}{M}$
- Plausible scenario of lepto-genesis

Heavy Majorana decay responsible for generation of lepton asymmetry, being converted to baryon asymmetry via strong electroweak B, L violation keeping B-L conserved.

Prerequisite: ordinary neutrinos are massive、but very light Majorana.

New CPV sources related to heavy partners of mass >> Fermi scale

Detection of relic neutrinos of 1.9 K

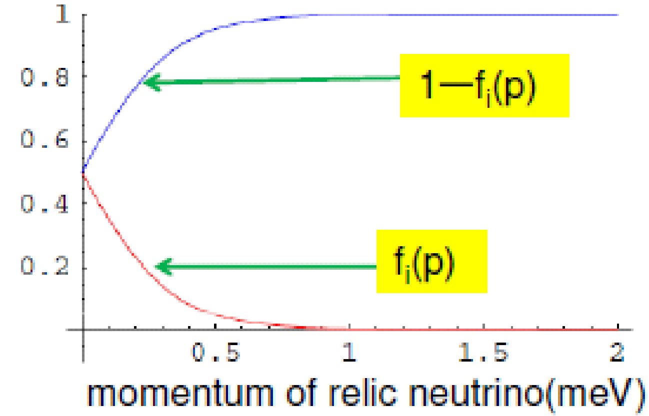
- Direct remnant at a few seconds after the big bang
- Prove that neutrinos were in thermal equilibrium, giving the important basis of light element synthesis such as 4He
- T differs from 2.7K of microwave, because electron-positron annihilation occurred after the neutrino decoupling at a few MeV, heating up matter in equilibrium
- Prediction is firm: $(4/11)^{(1/3)}$ 2.7 K = 1.9 K, 110cm^-3

- Spectrum distortion by the Pauli blocking caused by ambient relic neutrinos

Neutrino distribution function

$$f(p) = \frac{1}{\zeta e^{\sqrt{p^2+m^2/(z_d+1)^2}/T}+1} \approx \frac{1}{\zeta e^{p/T}+1}$$

$$\zeta = e^{-\mu_d/T_d}\,, \qquad z_d = O(10^{10})$$



Blocking given by 1-f(p)

$$F^A_{ij}(\omega;T_\nu) = \frac{1}{8\pi\omega}\int_{E_-}^{E_+} dE_1\, g^A_{ij}(E_1)\cdot\left(1 - f(\sqrt{E_1^2-m_i^2})\right)\left(1-\bar{f}(\sqrt{(\epsilon_{eg}-\omega-E_1)^2-m_j^2})\right)\,,$$

$$g^M_{ii}(E) = -E^2 + (\epsilon_{eg}-\omega)E + \frac{1}{2}m_i^2 - \frac{1}{4}\epsilon_{eg}(\epsilon_{eg}-2\omega) + \delta_M\frac{m_i^2}{2}\,,$$

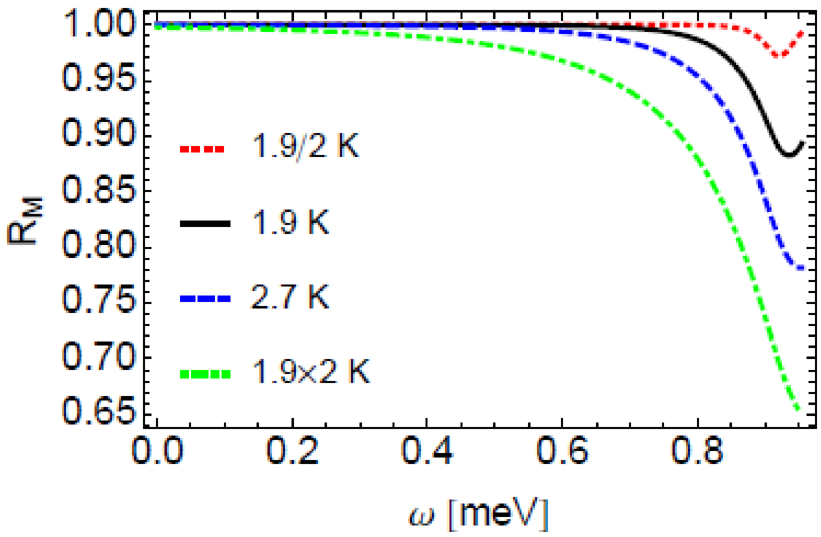
$$g^S_{ij}(E) = -\frac{1}{3}E^2 + \frac{1}{3}(\epsilon_{eg}-\omega)E + \frac{1}{12}\epsilon_{eg}(\epsilon_{eg}-2\omega) - \frac{1}{12}(m_i^2+m_j^2) - \delta_M\frac{m_i m_j}{2}\,,$$

$$E_\pm = \frac{1}{2}\left((\epsilon_{eg}-\omega)(1+\frac{m_i^2-m_j^2}{\epsilon_{eg}(\epsilon_{eg}-2\omega)}) \pm \omega\Delta_{ij}(\omega)\right)\,, \quad \Delta_{ij}(\omega) = \left\{\left(1-\frac{(m_i+m_j)^2}{\epsilon_{eg}(\epsilon_{eg}-2\omega)}\right)\left(1-\frac{(m_i-m_j)^2}{\epsilon_{eg}(\epsilon_{eg}-2\omega)}\right)\right\}^{1/2}.$$

Temperature measurement possible for RENP ?

Ratio of rates: with to without Pauli blocking

with/without Pauli blocking



Level spacing 11 meV
Smallest mass 5meV

Difference of distortions for 1.9 and 2.7 K

10% level

For small level spacing, temperature measurement possible.

Coherent quantum beam

Neutrino pair beam and fundamental oscillation experiments

By N. Sasao and M. Yoshimura
Okayama University

References

Neutrino pair and gamma beams from circulating excited ions

arXiv: 1505.07572v2 [hep-ph]

Determination of CP violation parameter using neutrino pair beam

arXiv: 1506.08003v1 [hep-ph]

Conventional neutrino sources: pi-, mu-, beta-decay

We shall use de-excitation of circulating excited heavy ions, producing pairs of neutrino and anti-neutrino.

Two useful processes: pair emission and RENP (radiative neutrino pair emission) from circulating excited ions

Without and with a photon emission

$$|e^{\pm}\rangle \to |g^{\pm}\rangle + \nu_i \bar{\nu}_j$$
$$|e^{\mp}\rangle \to |g^{\pm}\rangle + \gamma + \nu_i \bar{\nu}_j$$

1st giving useful and unusual neutrino pair beam for oscillation experiments and 2nd giving opportunities to resolve the Majorana/Dirac distinction and determining the smallest neutrino mass.

Neutrino pair emission occurs similarly to synchrotron radiation,

But, in the keV energy region with extremely small rates, hence completely negligible for electron synchrotron and heavy ion circulation in the ground state

New feature for excited ions

Input of excitation energy in the crucial time integral of phase factor, leading to a kind of non-linear resonance given by stationary points （positive and negative phases cancellation)

How to calculate neutrino pair emission rate

- Semi-classical approximation: classical ion CM motion and quantum internal state
- Spin current dominance from valence electron transition

Source current
$$J^{\alpha}_{eg}(x) = S^{\alpha}\frac{1}{\sqrt{\gamma}}\int dt\rho_{eg}(t)\,\delta^{(4)}(x - x_A(t)\,)\,,$$

hamiltonian
$$H_w = \int d^3x\frac{G_F}{\sqrt{2}}J^{\alpha}_{eg}(x)\cdot\sum_{i=e,\mu,\tau}\nu_i^{\dagger}(x)\sigma_{\alpha}\nu_i(x)\,,$$

coherence
$$\rho_{eg}(t) = \rho_{eg}(0)\exp[-(i\epsilon_{eg} + \frac{1}{T_2})\frac{t}{\gamma}]\,,$$

CP-even

Mixture of well-defined phase

Spin factor
$$(S_{\alpha}) = (\gamma\vec{\beta}\cdot\vec{S}_e\,,\vec{S}_e + \frac{\gamma^2}{\gamma+1}(\vec{\beta}\cdot\vec{S}_e)\vec{\beta}\,) \sim \gamma(\vec{\beta}\cdot\vec{S}_e\,,(\vec{\beta}\cdot\vec{S}_e)\vec{\beta}\,)\,,$$

$$-> \qquad \gamma^2\frac{S_e^2}{3}\left(1 + \frac{1}{3}\frac{\vec{p}_1\cdot\vec{p}_2}{E_1E_2} - \frac{m_1m_2}{2E_1E_2}\delta_M\right)$$

Ion trajectory
$$\vec{r}_A(t) = \rho\left(\sin\frac{vt}{\rho}\,,1 - \cos\frac{vt}{\rho}\,,0\right)$$

Resonance behavior of phase integral: Airy-type

$$\int_0^\infty dt \cos\Phi(t)\,, \quad \Phi(t) = \frac{E_1+E_2}{2\rho\gamma}\sqrt{D}(t-\frac{\rho}{\gamma}D)^2$$

$$D = 1 - \frac{2\epsilon_{eg}\gamma + \gamma^2(m_1^2/E_1 + m^2/E_2)}{E_1+E_2} - (\text{quadratic function of angles})$$

$$\text{resonance in time domain}:\quad t_r = \frac{\rho}{\gamma}D \approx 10\text{ps}\,,$$

$$\text{width;}\quad \Delta t_r = \rho\sqrt{\frac{2}{(E_1+E_2)t_r}} \gg t_r$$

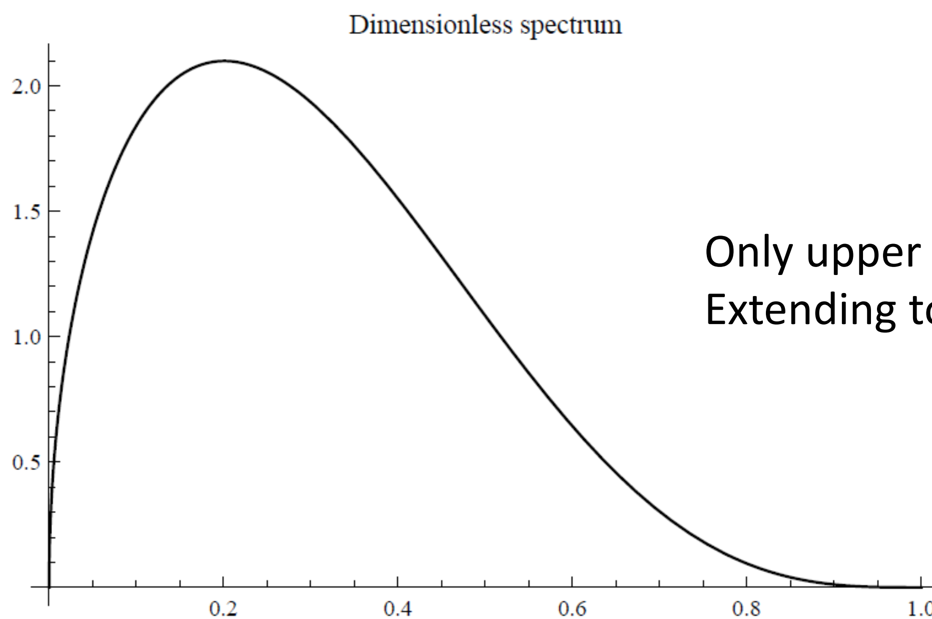
$$\int_0^\infty dt\cos\Phi(t) \sim \int_0^\infty dt\cos\frac{(t-t_r)^2}{(\Delta t_r)^2} \sim \sqrt{\frac{2\pi}{3}}\Delta t_r = \sqrt{\frac{\pi}{3}}(\frac{\rho\gamma}{E_1+E_2})^{1/2}D^{-1/4}$$

for GeV neutrinos

Differential and total production rates

$$\Gamma = \sum_i \Gamma_i \sim 3.1 \times 10^{21}\mathrm{Hz}(\frac{\rho}{4\mathrm{km}})^{1/2}\frac{S_e^2 N|\rho_{eg}(0)|^2}{10^8}(\frac{\gamma}{10^4})^4(\frac{\epsilon_{eg}}{50\mathrm{keV}})^{11/2},$$

$$\text{with } E_m = 2\epsilon_{eg}\gamma = 1\mathrm{GeV}\frac{\epsilon_{eg}}{50\mathrm{keV}}\frac{\gamma}{10^4}.$$



Only upper bound of total energy
Extending to GeV region

Intuitive understanding

- A kind of non-linear resonance: orbital energy balanced against internal ion energy, giving non-linear resonance oscillation. Its width around the stationary point gives a broad resonance-like behavior in time domain.

- Key concept for its success: quantum coherence typically realized by ionic system under laser irradiation, but may persist without phase relaxation.

Simple example of quantum coherence: adiabatic Raman process

Difference from usual synchrotron radiation

Ground state ion

X = rescaled time

$$\int_0^\infty dx h(x) \cos \xi (\frac{1}{2}x^3 + \frac{3}{2}x) \rightarrow \sqrt{\frac{\pi}{6}} e^{-\xi} \frac{h(0)}{\sqrt{\xi}} .$$

$$\xi = \rho(E_1 + E_2) \times \text{ a function of } \left(\frac{E_1}{E_2}, \frac{\epsilon_{eg}}{E_1 + E_2}, \gamma, \text{angles} \right)$$

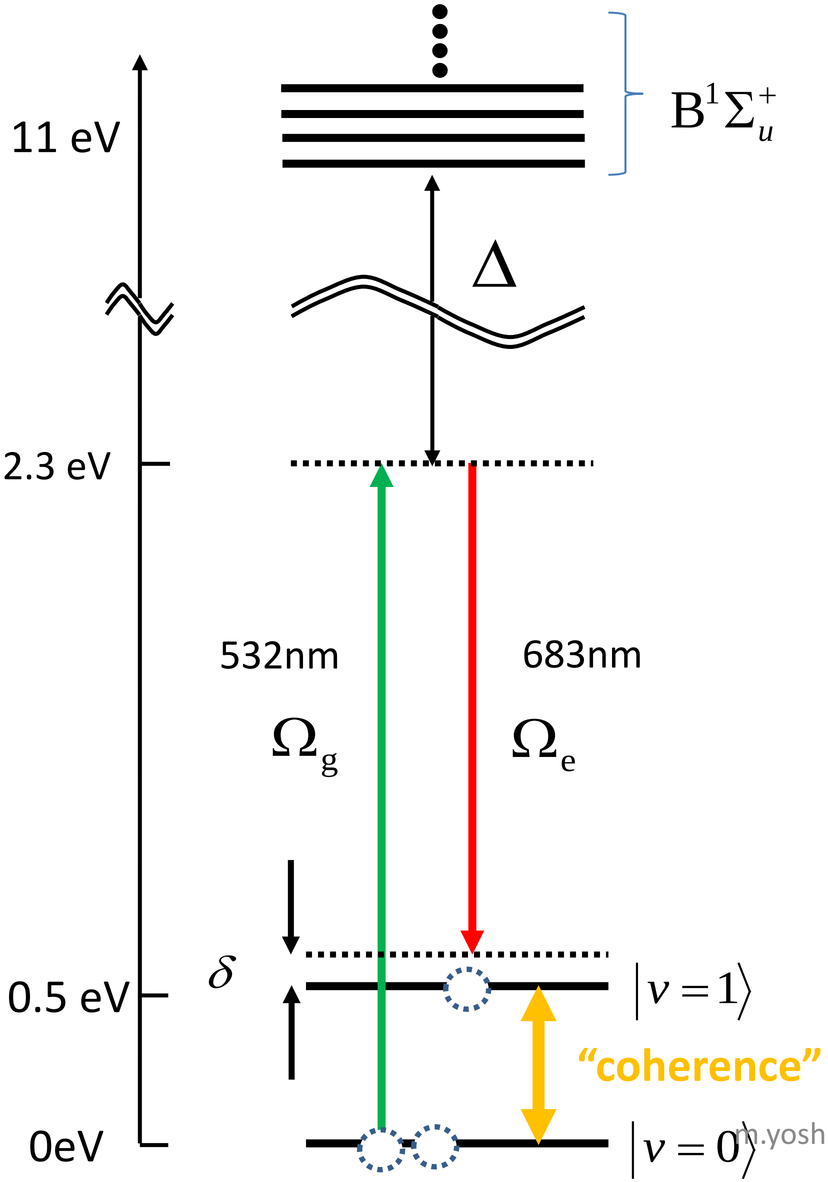
- Always the same sign phase added, leading to exponential dampling

Excited ion with coherence

$$\int_0^\infty dx h(x) \cos \xi (\frac{1}{2}x^3 - \frac{3}{2}x) \rightarrow \sqrt{\frac{2\pi}{3}} \cos(\xi - \frac{\pi}{4}) \frac{h(1)}{\sqrt{\xi}}$$

Cancellation of positive and negative phases
Energy input leads to resonance-like behavior

Preparation of initial coherence – Adiabatic Raman -



Two laser fields irradiates p-H2

Two photon Rabi frequency $\Omega_{ge} \cong \dfrac{\Omega_g \Omega_e}{\Delta}$

→ |g> and |e> are mixed with an angle

$$\tan\theta \cong \frac{\Omega_{ge}}{\delta}$$

Non-degenerate Superposition States:

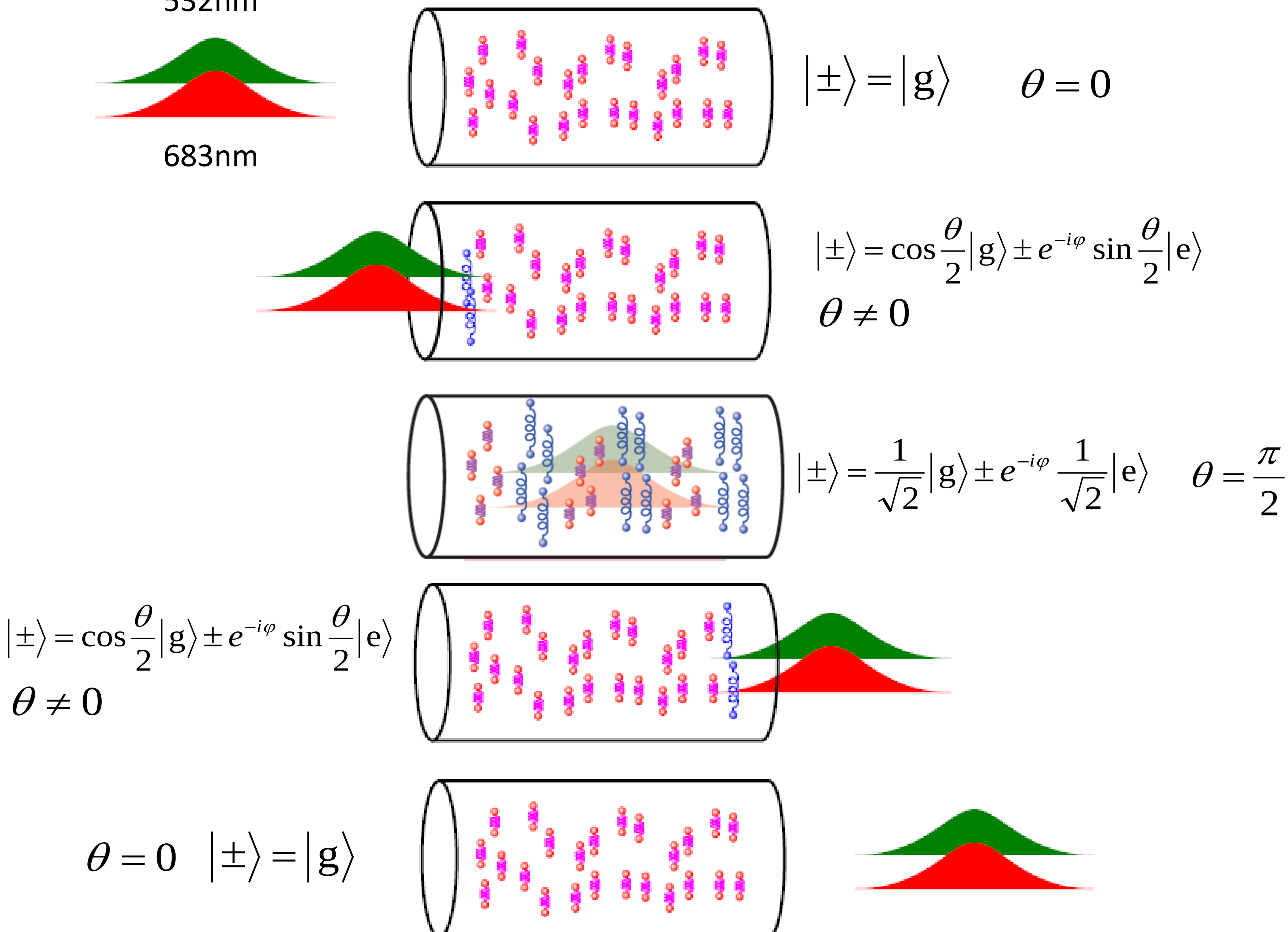
$$|+\rangle = \cos\frac{\theta}{2}|g\rangle + e^{-i\varphi}\sin\frac{\theta}{2}|e\rangle$$

$$|-\rangle = \cos\frac{\theta}{2}|g\rangle - e^{-i\varphi}\sin\frac{\theta}{2}|e\rangle$$

Coherence between |e> and |g>

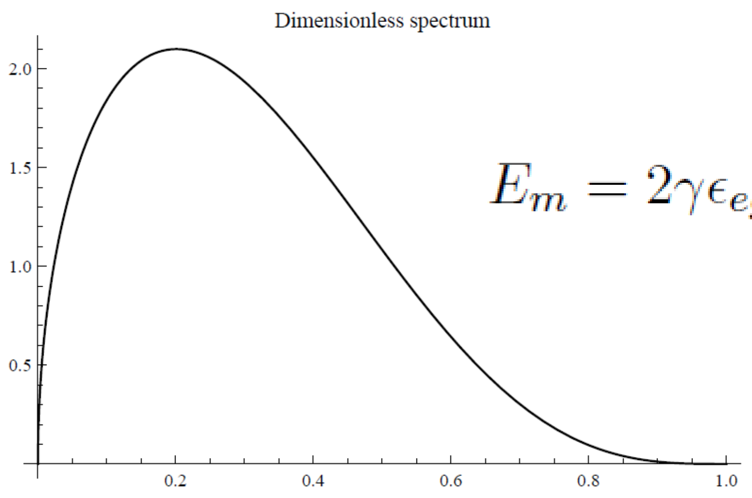
$$\boxed{|\rho_{eg}| = \frac{1}{2}\sin\theta}$$

p-H2 gas

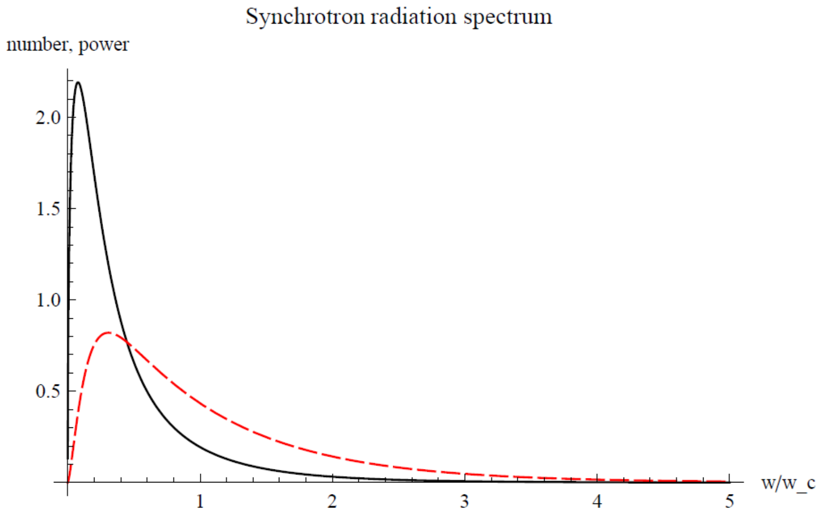


532nm
683nm
$|\pm\rangle = |g\rangle \qquad \theta = 0$
$|\pm\rangle = \cos\frac{\theta}{2}|g\rangle \pm e^{-i\varphi}\sin\frac{\theta}{2}|e\rangle$
$\theta \neq 0$
$|\pm\rangle = \frac{1}{\sqrt{2}}|g\rangle \pm e^{-i\varphi}\frac{1}{\sqrt{2}}|e\rangle \qquad \theta = \frac{\pi}{2}$
$|\pm\rangle = \cos\frac{\theta}{2}|g\rangle \pm e^{-i\varphi}\sin\frac{\theta}{2}|e\rangle$
$\theta \neq 0$
$\theta = 0 \quad |\pm\rangle = |g\rangle$

Energy spectrum



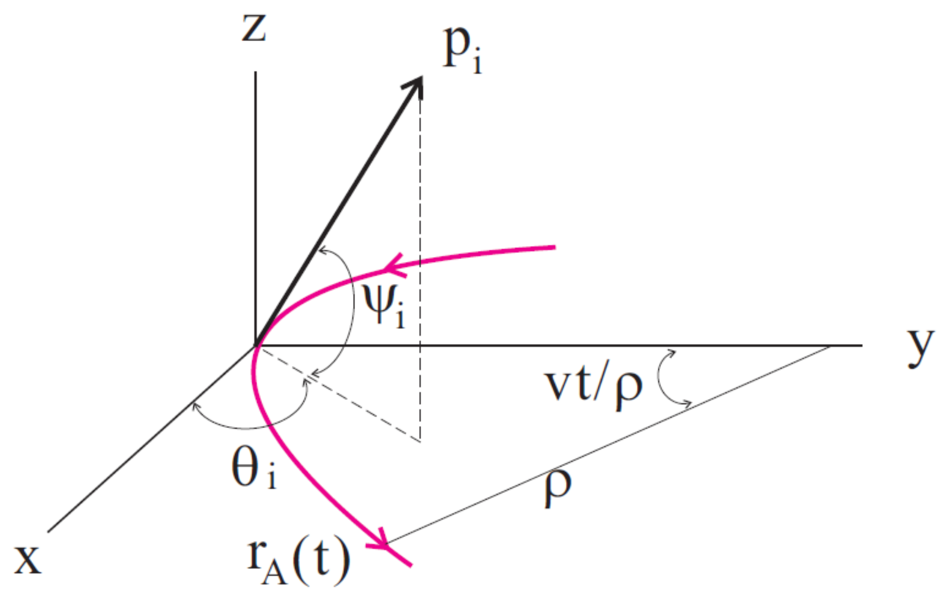
$$E_m = 2\gamma\epsilon_{eg}$$



Our gamma beam

Synchrotron radaiton

$$\text{exponential cutoff } eB\gamma^2/m_e = \gamma^3/\rho \sim 2\gamma^3\text{neV}(100\text{m}/\rho)$$



$$\frac{d^4\Gamma_{ij}}{dE_1 dE_2 d\Omega_1 d\Omega_2} = \frac{4G_F^2}{2^{7/4}\cdot 3\sqrt{3\pi}(2\pi)^6}|C_{ij}|^2 S_e^2 N|\rho_{eg}(0)|^2\gamma\sqrt{\rho}E_1^2E_2^2F^{-1/4}\,,$$

$$F = (E_1+E_2)(\frac{\epsilon_{eg}}{\gamma} - \frac{E_1+E_2}{2\gamma^2}) - \frac{1}{2}(E_1^2\psi_1^2 + E_2^2\psi_2^2) - \frac{E_1E_2}{2}(\theta_1-\theta_2)^2 - \frac{\epsilon_{eg}}{2\gamma}(E_1\theta_1^2 + E_2\theta_2^2)\,.$$

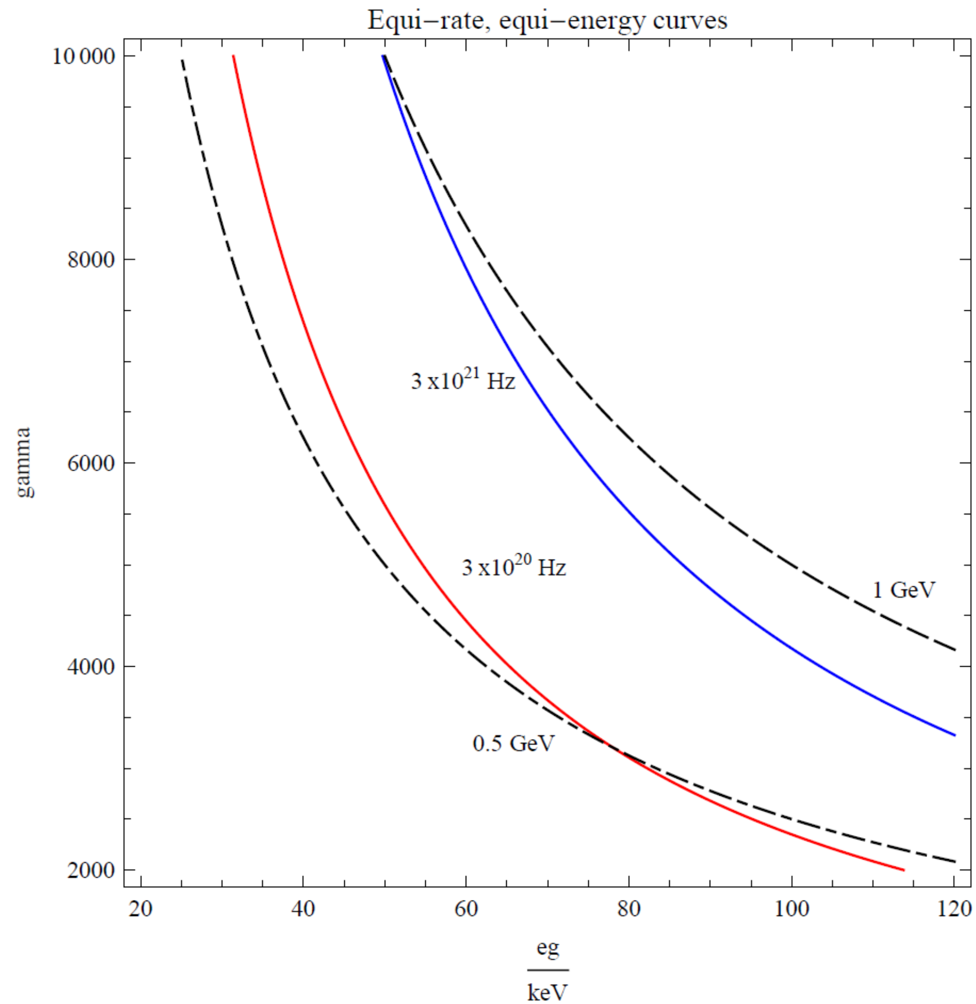
$$\Delta\psi = O\left(\frac{1}{\gamma}\sqrt{\frac{2(E_m-2E)}{E}}\right)\,,\quad \Delta\theta = O\left(\sqrt{\frac{E_m-2E}{E_m}}\right)\,,\quad E_m = 2\gamma\epsilon_{eg}\,.$$

$$\Delta|\theta_1-\theta_2| < O\left(\frac{1}{\gamma}\sqrt{\frac{(E_1+E_2)(E_m-E_1-E_2)}{E_1E_2}}\right)$$

$100\,\mu\text{radian}\ 10^4/\gamma$

Scaling law

$$\Gamma \propto \frac{1}{\gamma} \cdot N|\rho_{eg}(0)|^2 \cdot G_F^2 E_m^5 \cdot \sqrt{\rho\epsilon_{eg}} \propto \gamma^4 \epsilon_{eg}^{11/2}$$



Neutrino oscillation

Detection of neutrino pair away from synchrotron

$$\sum_b (\frac{G_F}{\sqrt{2}})^2 \bar{\nu}_a \gamma_\alpha (1-\gamma_5) l_a J^\alpha \bar{l}_c \gamma_\beta (1-\gamma_5)\nu_c (J^\beta)^\dagger \langle \bar{a}|e^{-iHL}|\bar{b}\rangle \langle c|e^{-iHL}|b\rangle \mathcal{P}_{\bar{b}b}(1,2)$$

$$H = U \begin{pmatrix} \frac{m_1^2}{2E} & 0 & 0 \\ 0 & \frac{m_2^2}{2E} & 0 \\ 0 & 0 & \frac{m_3^2}{2E} \end{pmatrix} U^\dagger \mp \sqrt{2} G_F n_e \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- Single neutrino detection eliminates oscillation pattern

$$\langle c|e^{-iHL}|b\rangle = \sum_i V_{ci}^* V_{bi} e^{-i\lambda_i L}\,, \qquad \langle \bar{a}|e^{-iHL}|\bar{b}\rangle = \sum_i \bar{V}_{ai}^* \bar{V}_{bi} e^{-i\bar{\lambda}_i L}\,,$$

$$\sum_b \langle \bar{a}|e^{-iHL}|\bar{b}\rangle \langle c|e^{-iHL}|b\rangle c_b = \sum_{ij} V_{ci}^* \bar{V}_{aj}^* \xi_{ij} e(\lambda_j, \lambda_i)\,, \qquad (c_b) = \frac{1}{2}(1,-1,-1)\,,$$

$$\sum_c |\sum_{ij} V_{ci}^* \bar{V}_{\mu j}^* \xi_{ij} e(\lambda_j, \lambda_i)|^2 = \sum_{ijkl} \sum_c V_{ci}^* V_{ck} \bar{V}_{\mu j} \bar{V}_{\mu l}^* \xi_{ij} \xi_{kl}^* e(\lambda_j, \lambda_i) e^*(\lambda_l, \lambda_k)$$

$$= \sum_{jl} \bar{V}_{\mu j} \bar{V}_{\mu l}^* e(\lambda_j, \lambda_i) e^*(\lambda_l, \lambda_i) \sum_i \xi_{ij} \xi_{il}^* = \frac{1}{4} \sum_{jl} \bar{V}_{\mu j} \bar{V}_{\mu l}^* e(m_j, m_i) e^*(m_l, m_i) \delta_{jl} = \frac{1}{4}\,,$$

Unusual feature of neutrino pair beam

$$\text{measured in oscillation experiments ;} \quad \sum_{a=e,\mu,\tau} |\langle b, c| e^{-iHL} |a, a\rangle|^2$$

$$\text{unlike flavor} - \text{tagged case} \;\; ; \quad |\langle b, c| e^{-iHL} |a, a\rangle|^2$$

Short baseline experiments

$$\sqrt{2}G_F n_e L \sim 1 \times \frac{n_e}{1.4\times 6\times 10^{23}\mathrm{cm}^{-3}}\frac{L}{1860\mathrm{km}}$$

$$|\sum_{ij} V_{ci}^* \bar{V}_{\mu j}^* \xi_{ij} e(\bar{\lambda}_j, \lambda_i)|^2 \frac{d^4\Gamma}{dE_1 dE_2 d\Omega_1 d\Omega_2}\frac{d^2\sigma}{dE_+ d\sin\psi_+}\frac{d^2\sigma}{dE_- d\sin\psi_-}$$

Plotted quantities

$$P_{\bar{a}c} = |\sum_{ij} U_{ci}^* U_{aj} \xi_{ij} e^{-iL(m_j^2/E_1 + m_i^2/E_2)}|^2$$

$$A(\delta) = \frac{d\Gamma(\delta : G_F) - d\Gamma(-\delta : -G_F)}{d\Gamma(\delta : G_F) + d\Gamma(-\delta : -G_F)}\,.$$

Double rates $10 \sim 100$ mHz for a 100 kt class

$\sigma n_N l \sim 10^{-11} \sim 10^{-10}$ for a single detection

Oscillation and CPV asymmetry

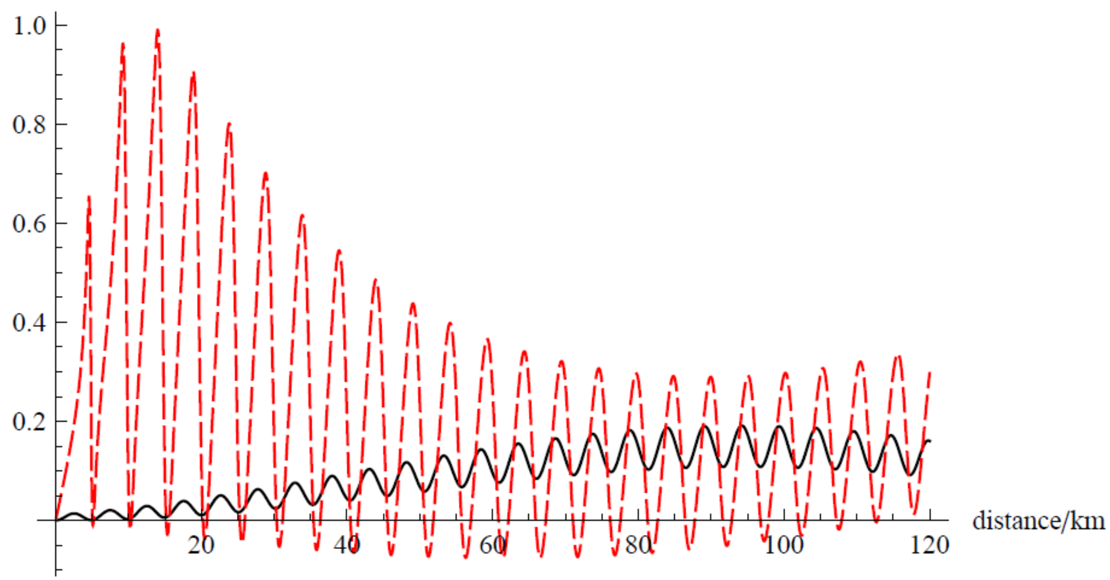


Figure 1: Oscillation pattern given by $P_{\bar{\mu}e}$ of eq.(13) (in solid black) and asymmetry (in dashed red) at various distances for $\bar{\nu}_\mu \nu_e$ CC double events. $\delta = \pi/4, E_{\bar{\nu}_\mu} = 500\text{MeV}, E_{\nu_e} = 5\text{MeV}$.

How short baseline exp. became effective

- Two factors of L/E, one 10km/10MeV instead of 500km/500 MeV giving the same oscillation pattern

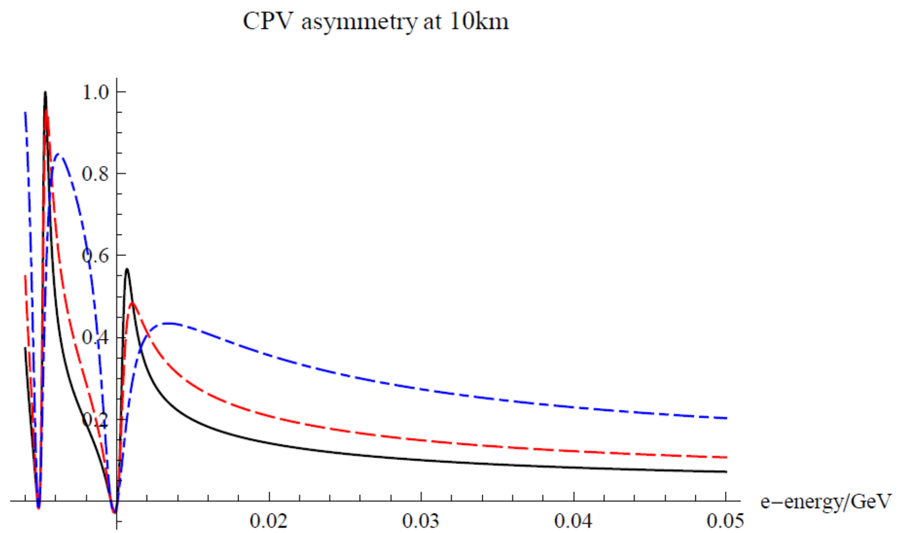


Figure 3: Asymmetry vs electron neutrino energy for $\bar{\nu}_\mu \nu_e$ CC double events. $E_{\bar{\nu}_\mu} = 500\mathrm{MeV}$ and $\delta = \pi/6$ in solid black, $\pi/4$ in dashed red, and $\pi/2$ in dash-dotted blue. NH of smallest mass zero is assumed.

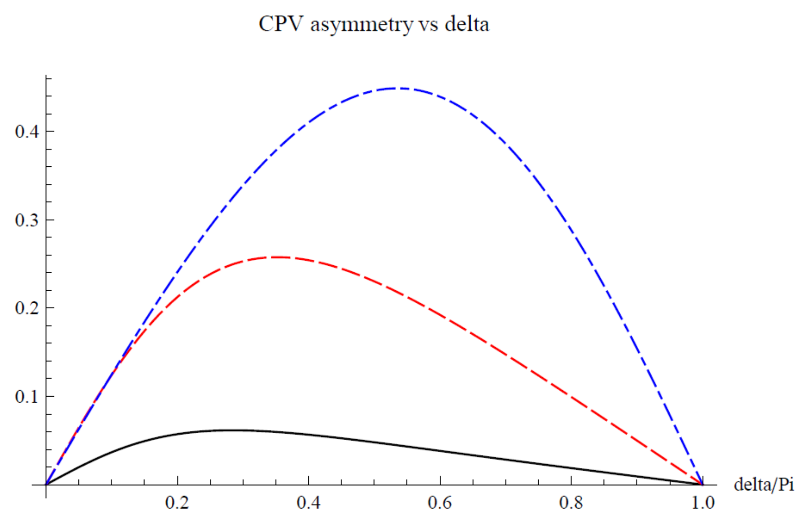


Figure 4: Asymmetry vs CPV δ for $\bar{\nu}_\mu \nu_e$ CC double events. $E_{\bar{\nu}_\mu} = 500\mathrm{MeV}, E_{\nu_e} = 5\mathrm{MeV}$ at 10 km away in solid black, 50km in dashed red, and 100km in dash-dotted blue. NH of smallest mass zero is assumed.

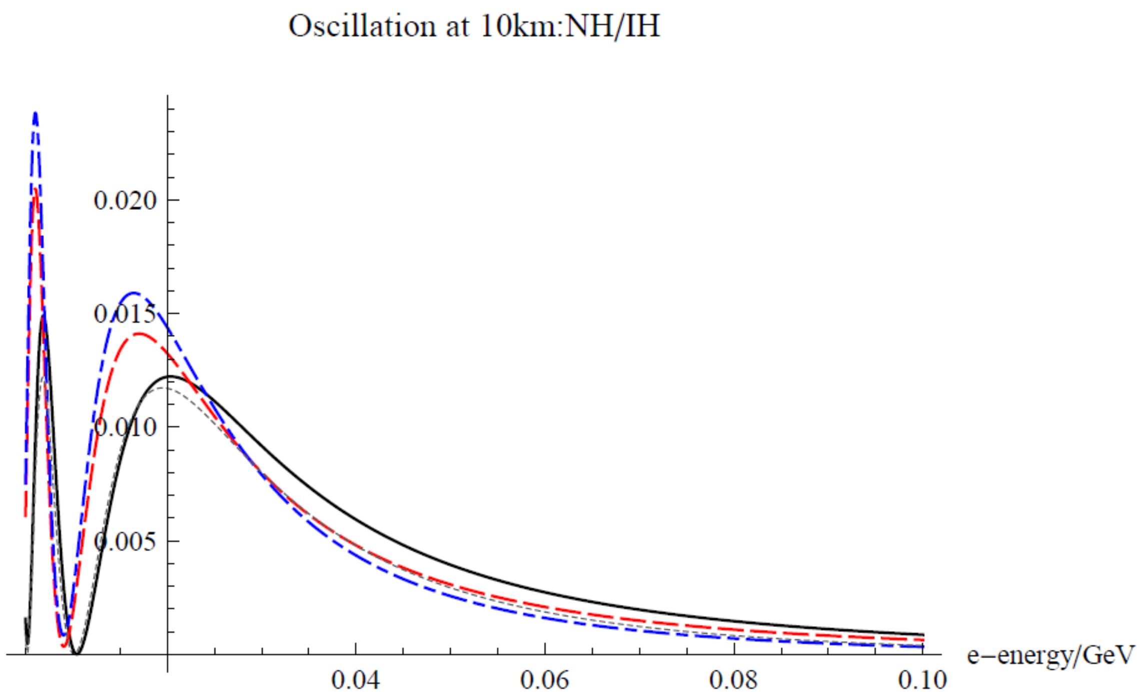


Figure 5: NH vs IH distinction at 10 km away from the synchrotron, given by asymmetric energy combinations: $P_{\bar{\mu}e}$ is plotted for $E_{\bar{\nu}_\mu} = 500, 200\text{MeV}$, fixed and variable E_{ν_e}. NH in blacks, 500 MeV in solid and 200 MeV in dotted lines, and IH in colored, 500MeV, in dashed red and 200 MeV in dash-dotted blue. $\delta = 0$.

Features of pair beam

- Double detection required for oscillation experiments
- Short baseline experiments recommended to avoid the earth matter effect
- Excellent opportunity for ¥delta and NH/IH

RENP using pair beam

- New work in progress
- Absolute mass determination and MD distinction expected
- Kinematics different from SPAN: energy and momentum conservation not obeyed, and only the sum of photon and two neutrinos limited
- Rate scales with gamma^6

Detection principles

- Majorana/Dirac distinction: identical fermion effects, different effects from SPAN because energy-momentum conservation do not hold and mass threshold regions exist in all photon energy regions

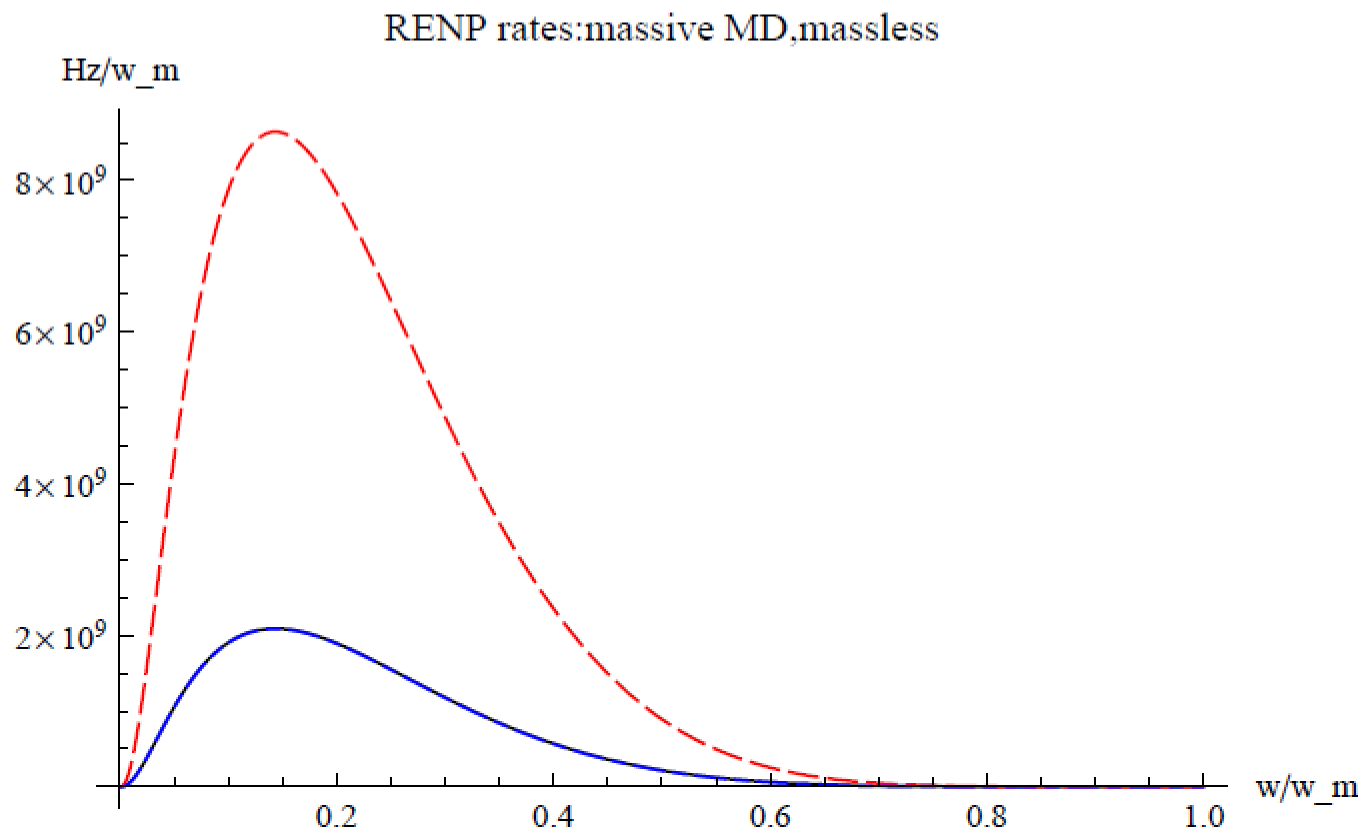


Figure 3: Single photon spectral rates: the massive (NH of zero smallest mass) Majorana case in solid black, the Dirac (NH of zero smallest mass) case dashed-red, and the massless case in dash-dotted blue. $N|\rho_{eg}(0)|^2 = 10^8, \gamma = 5000, \gamma_{pe} = 100\text{MHz}, \rho = 4\text{km}$ and $\delta = 0, \epsilon_{eg} = 1, \epsilon_{pe} = 0.1\text{keV}$ are assumed. The maximum photon energy $\omega_m = 5\text{MeV}$.

$$\omega_m \frac{d\Gamma}{d\omega} = RF(\frac{\omega}{\omega_m}) \,, \qquad \omega_m = 2\epsilon_{eg}\gamma \,,$$

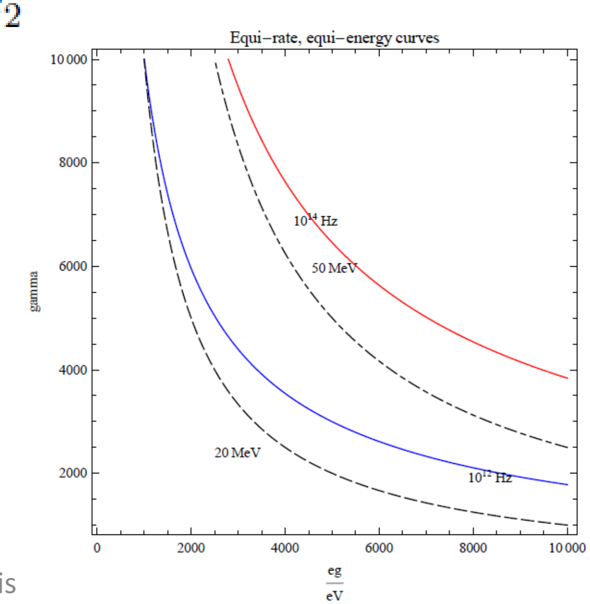
$$R = \frac{\sqrt{\pi}}{2\sqrt{3}(2\pi)^8} v_5 G_F^2 e^2 \vec{r}_{ep}^{\,2} \gamma^6 N |\rho_{eg}(0)|^2 \sqrt{\rho}\,\epsilon_{eg}^{19/2} \frac{1}{\epsilon_{pe}^2}$$

$$\sim 0.99 \times 10^{12}\mathrm{Hz} \frac{N|\rho_{eg}(0)|^2}{10^8} \frac{\gamma_{pe}}{100\mathrm{MHz}} \sqrt{\frac{\rho}{4\mathrm{km}}} (\frac{\gamma}{10^4})^6 (\frac{\epsilon_{eg}}{1\mathrm{keV}})^{15/2} \,,$$

$$v_5 = \int dV_5 (1 - r^2)^{-1/4} \sim 9.1 \times 10^{-6} \,, \qquad F(y) = \int_{m_i/\epsilon_{eg}}^1 dx_1 dx_2 H(y, x_1, x_2) \,,$$

$$H(y, x_1, x_2) = y^{5/2} (1 + \frac{2\epsilon_{eg}}{\epsilon_{pe}} y)^{-2} x_1 x_2 (x_1 + x_2 + y)^{1/4} G(x_1, x_2, y)^{9/4} \Theta(G(x_1, x_2, y)) \,,$$

$$G(x_1, x_2, y) = 1 - x_1 - x_2 - y - \frac{1}{4\epsilon_{eg}^2} (\frac{m_1^2}{x_1} + \frac{m_2^2}{x_2}) \,.$$



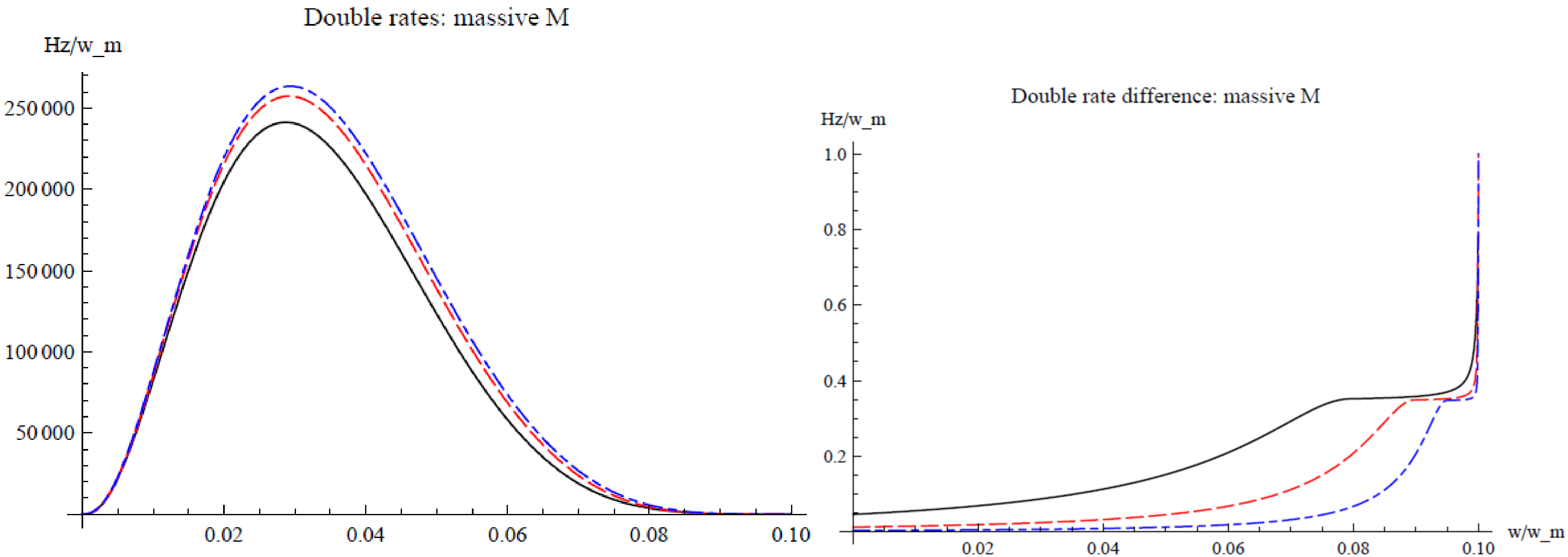


Figure 5: Double spectral rates of photon with a detected neutrino of fixed energy $0.9\omega_m$: the smallest neutrino mass 20 meV in solid black, 10 meV in dashed red, and 0 meV in dashed-dotted blue, all for the Majorana NH cases. $N|\rho_{eg}(0)|^2 = 10^8, \gamma_{pe} = 100\text{MHz}, \rho = 4\text{km}$ and $\gamma = 5 \times 10^3, \epsilon_{eg} = 1\text{keV}, \epsilon_{pe} = 0.1\text{keV}$ are assumed.

Summary of this talk

- We should maximally exploit quantum coherence towards the ultimate clarification of mysteries of neutrino.
- Coherent quantum heavy ion synchrotron is excellent for CPV parameter, NH/IH hierarchy measurements (pair beam) and MD distinction and the smallest mass determination (RENP).
- Accelerator R & D works crucial to obtain a high coherence beam.