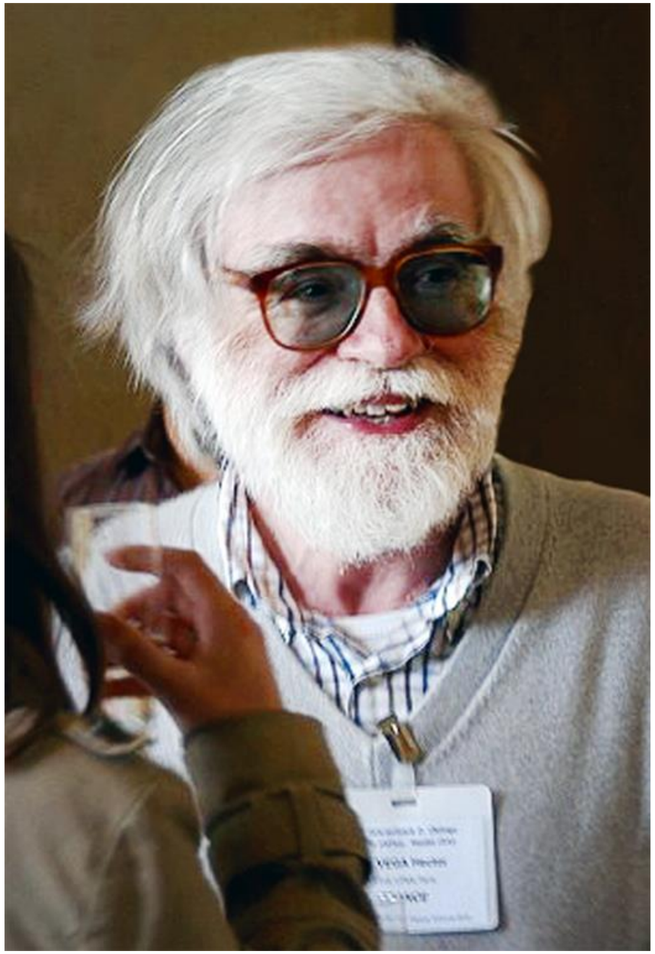
Work with Héctor J. DE VEGA
on *Warm Dark Matter Cosmology &*
the Thomas-Fermi galaxy structure theory



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Chalonge de Vega School

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UPDATE and CLARIFICATIONS

ΛCDM agrees with CMB + LSS **BUT**

ΛCDM **DOES NOT agree with SSS (GALAXIES)**

ΛWDM agrees with CMB + LSS + SSS (GALAXIES)

The Standard Model of the Universe is LWDM =

{GR, Newtonian Gravity, Field Theory, QFT}

Sentences like « CMB confirms the ΛCDM model é »

Must be completed by adding: « in the large scalesò »

and must be updated with the sentence:

CMB confirms the ΛWDM model in large scales

NEW: Gravity and Quantum Mechanics in Galaxies. Newton, Fermi and Dirac meet together in Galaxies because of keV WDM

Dark Matter in the Universe

81 % of the matter of the universe is DARK (DM).
DM is the dominant component of galaxies.

DM interacts through gravity.

Further DM interactions unobserved so far. Such couplings must be very weak: much weaker than weak interactions.

DM is outside the standard model of particle physics.

Proposed candidates:

- Cold Dark Matter: CDM, WIMPS, $m \sim 1 - 1000$ GeV.
  IN BIG TROUBLE.
- Warm Dark Matter: WDM, sterile neutrinos $m \sim$ keV.
  THE ANSWER !

DM particles decouple due to the universe expansion, their distribution function freezes out at decoupling.
Early decoupling: $T_d \sim 100$ GeV

Structure Formation in the Universe

Structures in the Universe as galaxies and cluster of galaxies form out of the small primordial quantum fluctuations originated by inflation just after the big-bang.

These small linear primordial fluctuations grow due to gravitational unstabilities (Jeans) and then classicalize.

Structures form through non-linear gravitational evolution.

Hierarchical formation starts from small scales first.

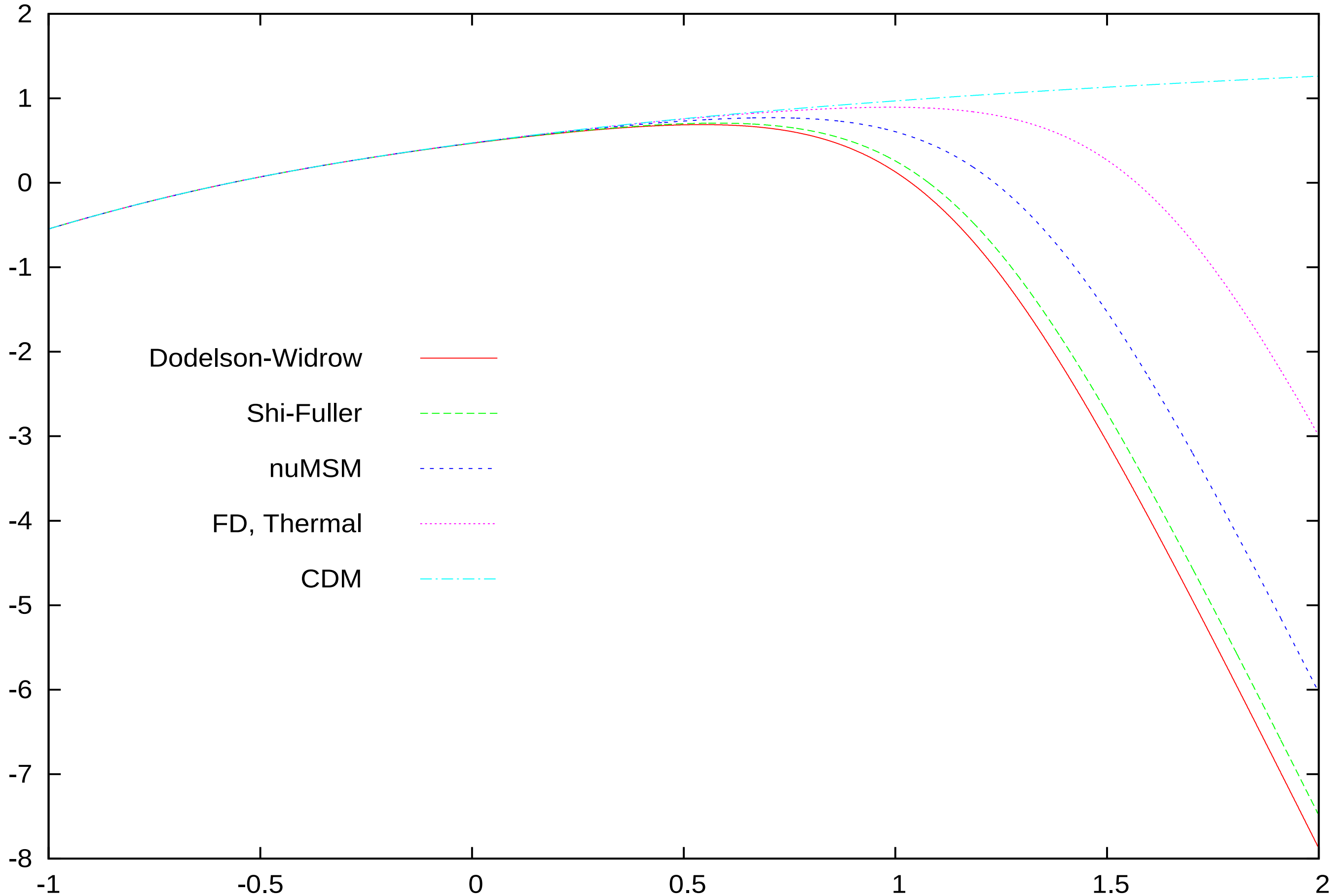
N-body CDM simulations fail to produce the observed structures for small scales less than some kpc.

Both N-body WDM and CDM simulations yield identical and correct structures for scales larger than some kpc.

WDM predicts correct structures for small scales (below kpc) when its quantum nature is taken into account.

Primordial power $\Delta^2(k)$: first ingredient in galaxy formation.

Linear primordial power spectrum $\Delta^2(k)$ vs. k Mpc $/h$



$\log_{10} \Delta^2(k)$ vs. $\log_{10}[k \text{ Mpc}/h]$ for a physical mass of 2.5 keV in four different WDM models and in CDM. WDM cuts $\Delta^2(k)$ on small scales. $r \lesssim 73\ (\text{keV}/m)^{1.45}$ kpc/h.
CDM and WDM are identical for CMB.

WDM Primordial Power Spectrum

The WDM Primordial Power Spectrum is obtained solving the linear Boltzmann-Vlasov equations.

We define the transfer function ratio $T^2(k) \equiv \frac{\Delta^2_{wdm}(k)}{\Delta^2_{cdm}(k)}$

Reproduced by the analytic formula $T^2(k) = \left[1 + \left(\frac{k}{\kappa}\right)^a\right]^{-b}$

a and b are independent of the WDM particle mass m.

κ scales with m. In our best fit:

$$a = 2.304 \; , \quad b = 4.478 \; , \quad \kappa = 14.6 \; (m_{FD}/\mathrm{keV})^{1.12} \; h/\mathrm{Mpc}$$

At the wavenumber $k_{1/2} : \; T^2(k_{1/2}) = 1/2$ and

$$k_{1/2} = 6.72 \; (m_{FD}/\mathrm{keV})^{1.12} \;\; h/\mathrm{Mpc}$$

The characteristic length scale is

$$l_{1/2} = 1/k_{1/2} = 207 \; \mathrm{kpc} \; (\mathrm{keV}/m_{FD})^{1.12}$$

This scale reproduces the sizes of the DM galaxy cores when the WDM mass is in the keV scale !!

TRANSFER *FUNCTION ratio T(k)*

$$T^2(k) \equiv \frac{\Delta^2_{wdm}(k)}{\Delta^2_{cdm}(k)}$$

$T^2(k)$ tends to 1 for large scales $k \ll 1/l_{fs}$.

$T^2(k)$ vanishes for small scales $k \gg 1/l_{fs}$

de Vega, Sanchez PRD 2012, Destri, de Vega, Sanchez , PRD 2013

$$T^2(k) = \frac{1}{[1 + (k/\kappa)^a]^b}$$

a and *b* are independent of the WDM particle mass *m*, while the coefficient κ scales with *m*.

$a = 2.304$, $b = 4.478$, $\kappa = 14.6 \ (m_{FD} / \text{keV})^{1.12}$ h/ Mpc

$ab = 10.3$

In the usual literature:

fit $T^2(k)$ with only two free parameters: κ and α

$$T^2(k) = [\, 1 + (\alpha\, k)^{2\nu} \,]^{-10/\nu}, \quad \nu = 1.11$$

which corresponds to the choice: *ab*=20.

While with the precise values of a, b we have: *ab* = 10.3

Our $T^2(k)$ gives a χ^2 3 times smaller than fitting the same CAMB results with the usual $T^2(k)$ with *ab*=20.

Our formula provides a better fit than from Refs in the usual literature, independently of the WDM particle mass.

TABLE I

WDM particle masses providing the same WDM power spectrum

and therefore the same differential mass functions

in different WDM particle models. DdVS PRD 2013

Fermi Dirac (thermal keV)	**Dodelson Widrow (Kev)**	dDVS PRD 2013 **Shi Fueller (keV)**	***3* MSM (keV)**
2.5	**9.67**	**6.38**	**4.75**
0.91	**2.5**	**2.31**	**1.72**
0.98	**2.78**	**2.5**	**1.86**
1.32	**4.11**	**3.36**	**2.5**

WDM mass particle CONVERSION FACTORS

WDM particles in the different WDM particle models behave just as if their masses are different . The masses of WDM particles in different models with the same power spectrum are related by: de Vega & Sanchez, PRD 2012

$$m_{DW} \simeq 2.85 \ \mathrm{keV}(m_{FD} / \mathrm{keV})^{4/3} \qquad m_{SF} \simeq 2.55 m_{FD} \qquad m_{\nu MSM} \simeq 1.9 m_{FD}$$

FD : WDM fermions decoupling in thermal equilibrium (TE) , Fermi-Dirac.
DW: WDM sterile ns out of TE in Dodelson-Widrow.
SF : WDM sterile ns out of TE in the Shi-Fuller model
*ν*MSM : WDM sterile ns out TE in the *ν*MSM model

These relations ensure identical density and anisotropic stress fluctuations of WDM and neutrinos in the coupled evolution Volterra equations derived in dVS PRD 2012
Therefore, the WDM spectrum is the same for thermal fermions and out of equilibrium sterile neutrinos when these relations hold.
The same power spectrum implies an identical differential mass function $S(M,z)$.
Whether the fermions are Dirac or Majorana, the WDM power spectrum is slightly different.
Identical power for Dirac and Majorana fermions with masses related as:
m_{Maj}=(21/4) m_{Dirac} in FD, SF and *ν*MSM models; m_{Maj} = (21/3) m_{Dirac} in DW model.

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The characteristic length scale is

$$l_{1/2} = 1/k_{1/2} = 207 \; \mathrm{kpc} \; (\mathrm{keV}/m_{FD})^{1.12}$$

This scale reproduces the sizes of the DM galaxy cores when the WDM mass is in the keV scale !!

WDM free streaming scale

The scale $l_{1/2}$ is where the WDM power spectrum is one-half of the CDM power spectrum:

$$l_{1/2} = 1/k_{1/2} = 207\,\mathrm{kpc}\ \left(\mathrm{keV}/m_{FD}\right)^{1.12}$$

This scale reproduces the sizes of the observed DM galaxy cores when the WDM mass is in the keV scale !!

$l_{1/2}$ is similar but more precise than the free streaming scale (or Jeans' scale):

$$r_{Jeans} = 210\,\mathrm{kpc}\,\frac{\mathrm{keV}}{m_{FD}}\ \left(\frac{100}{g_d}\right)^{\frac{1}{3}}\ ,$$

$g_d =$ number of UR degrees of freedom at decoupling.

Small structure formation in WDM

DM particles can freely propagate over distances of the order of the free streaming scale.

Therefore, structures at scales smaller or of the order of $l_{1/2}$ are erased which agrees with the observed structures in galaxies !!

WDM sterile neutrinos in different particle models behave primordially just as if their masses were different (FD = thermal fermions):

$$\frac{m_{DW}}{\mathrm{keV}} \simeq 2.85 \left(\frac{m_{FD}}{\mathrm{keV}}\right)^{\frac{4}{3}}, \; m_{SF} \simeq 2.55 \, m_{FD}, \; m_{\nu\mathrm{MSM}} \simeq 1.9 \, m_{FD}.$$

DW: Dodelson-Widrow model, SF: Shi-Fuller model

H J de Vega, N Sanchez, Warm Dark Matter cosmological fluctuations, Phys. Rev. D85, 043516 and 043517 (2012).

CDM free streaming scale

For CDM particles with $m \sim 100$ GeV $\Rightarrow\ r_{Jeans} \sim 0.1$ pc. Hence CDM structures keep forming till scales as small as the solar system.

This is a robust result of N-body CDM simulations but never observed in the sky. Including baryons do not cure this serious problem. There is over abundance of small structures in CDM ('satellite problem') which are too dense.

CDM has many further serious conflicts with observations:

CDM needs ad-hoc merging and environment to grow gal. Observations show that galaxy mergers are rare ($< 10\%$). Pure-disk galaxies (bulgeless) are observed whose formation through CDM is unexplained.

CDM predicts cusped density profiles: $\rho(r) \sim 1/r$ for small r. Observations show cored profiles: $\rho(r)$ bounded for small r. Adding by hand strong enough feedback from baryons does not eliminate cusps (F. Marinacci et al., MNRAS 437, 1750 (2014)).

Summary Warm Dark Matter, WDM: $m \sim$ keV

- Large Scales, structures beyond ~ 100 kpc: WDM and CDM yield identical results which agree with observations
- Intermediate Scales: WDM simulations give the correct abundance of substructures.
- Inside galaxy cores, below ~ 100 pc: N-body classical physics simulations are incorrect for WDM because of important quantum effects.
- Quantum calculations (Thomas-Fermi) give galaxy cores, galaxy masses, velocity dispersions and densities in agreement with the observations.
- Direct Detection of the main WDM candidate: the sterile neutrino. Beta decay and electron capture. ^{3}H, Re, Ho.

So far, not a single valid objection arose against WDM.
Baryons (<16%DM) expected to give a correction to WDM

Summary: keV scale DM particles

- The phase-space density evolution since DM decoupling till today (observed in galaxies) implies keV scale DM particles (de Vega, Sanchez, MNRAS 2010).
- The Thomas-Fermi approach gives physical galaxy magnitudes: mass, halo radius, phase-space density and velocity dispersion fully compatible with observations from the largest spiral galaxies till the ultracompact dwarf galaxies for a WDM particle mass around 2 - 3 keV. Compact dwarf galaxies are close to a degenerate WDM Fermi gas while large galaxies are classical WDM Boltzmann gases.
- The galaxy surface density $\Sigma_0 \equiv \rho_0 \, r_0$ value $\Sigma_0 \simeq 120 \; M_\odot/pc^2 \simeq (18 \text{ MeV})^3$ is reproduced by WDM (de Vega, Salucci, Sanchez, New Astronomy, 2012). CDM simulations give 1000 times the observed value of μ_0 (Hoffman et al. ApJ 2007).

Quantum physics in Galaxies

de Broglie wavelength of DM particles $\lambda_{dB} = \frac{\hbar}{m\, v}$

$d =$ mean distance between particles, $v =$ mean velocity

$d = \left(\frac{m}{\rho}\right)^{\frac{1}{3}} \quad , \quad Q = \rho/v^3 \quad , \quad Q =$ phase space density.

ratio: $\qquad \mathcal{R} = \frac{\lambda_{dB}}{d} = \hbar \, \left(\frac{Q}{m^4}\right)^{\frac{1}{3}}$

Observed values: $2 \times 10^{-3} \, \left(\frac{\mathrm{keV}}{m}\right)^{\frac{4}{3}} < \mathcal{R} < 1.4 \, \left(\frac{\mathrm{keV}}{m}\right)^{\frac{4}{3}}$

The larger $\mathcal{R}$ is for ultracompact dwarfs.
The smaller $\mathcal{R}$ is for big spirals.

$\mathcal{R}$ near unity (or above) means a QUANTUM OBJECT.

Observations alone show that compact dwarf galaxies are quantum objects (for WDM).
No quantum effects in CDM: $m \gtrsim$ GeV $\Rightarrow \mathcal{R} \lesssim 10^{-8}$

Quantum pressure vs. gravitational pressure

quantum pressure: $P_q =$ flux of momentum $= n\ v\ p$ repulsive

$v =$ mean velocity, momentum $= p \sim \hbar/\Delta x \sim \hbar\ n^{\frac{1}{3}}$,

particle number density $= n = \frac{M_q}{\frac{4}{3}\ \pi\ R_q^3\ m}$

galaxy mass $= M_q$, galaxy halo radius $= R_q$

gravitational pressure (attractive): $P_G = \frac{G\ M_q^2}{R_q^2} \times \frac{1}{4\,\pi\ R_q^2}$

Equilibrium: $P_q = P_G \Longrightarrow$

$$R_q = \frac{3^{\frac{5}{3}}}{(4\ \pi)^{\frac{2}{3}}}\ \frac{\hbar^2}{G\ m^{\frac{8}{3}}\ M_q^{\frac{1}{3}}} = 10.6\ \text{pc} \left(\frac{10^6\ M_\odot}{M_q}\right)^{\frac{1}{3}}\ \left(\frac{\text{keV}}{m}\right)^{\frac{8}{3}}$$

$$v = \left(\frac{4\,\pi}{81}\right)^{\frac{1}{3}}\ \frac{G}{\hbar}\ m^{\frac{4}{3}}\ M_q^{\frac{2}{3}} = 11.6\ \frac{\text{km}}{\text{s}}\ \left(\frac{\text{keV}}{m}\right)^{\frac{4}{3}}\ \left(\frac{M_q}{10^6\ M_\odot}\right)^{\frac{2}{3}}$$

for WDM the values of $M_q,\ R_q$ and v are consistent with the dwarf galaxy observations !! .

Dwarf galaxies can be supported by the fermionic quantum pressure of WDM. Analogous to neutron stars and white dwarfs.

Self-gravitating Fermions in the Thomas-Fermi approach

WDM is non-relativistic in the MD era. A single DM halo in late stages of formation relaxes to a time-independent form especially in the interior.

Chemical potential: $\mu(r) = \mu_0 - m\,\phi(r)$, $\phi(r) =$ grav. pot.

Poisson's equation: $\frac{d^2\mu}{dr^2} + \frac{2}{r}\,\frac{d\mu}{dr} = -4\,\pi\,G\,m\,\rho(r)$

$\rho(0) =$ finite for fermions $\Longrightarrow \frac{d\mu}{dr}(0) = 0$.

Density $\rho(r)$ and pressure $P(r)$ in terms of the distribution function $f(E)$:

$$\rho(r) = \frac{m}{\pi^2\,\hbar^3}\int_0^\infty p^2\;dp\;f[\frac{p^2}{2\,m} - \mu(r)]$$

$$P(r) = \frac{1}{3\,\pi^2\;m\,\hbar^3}\int_0^\infty p^4\;dp\;f[\frac{p^2}{2\,m} - \mu(r)]$$

These are self-consistent non-linear Thomas-Fermi equations that determine $\mu(r)$.

Galaxy surface density

The surface density: $\Sigma_0 \equiv r_h \, \rho_0 \simeq 120 \; M_\odot/\mathrm{pc}^2$,

takes nearly the same value for galactic systems (spirals, dwarf irregular and spheroidals, elliptics) spanning over 14 magnitudes in luminosity and over different Hubble types.

We take Σ_0 as physical scale to express the galaxy magnitudes in the Thomas-Fermi approach.

Dimensionless variables: $\xi, \;\; \nu(\xi)$.

$$r = l_0 \, \xi \quad , \quad \mu(r) = T_0 \, \nu(\xi) \quad , \quad \rho_0 \equiv \rho(0).$$

$T_0 =$ effective galaxy temperature, l_0 characteristic length.

From the Thomas-Fermi equations:

$$l_0 \equiv \left(\frac{9\pi}{2^9}\right)^{\frac{1}{5}} \left(\frac{\hbar^6}{G^3 m^8}\right)^{\frac{1}{5}} \left[\frac{\xi_h \, I_2(\nu_0)}{\Sigma_0}\right]^{\frac{1}{5}} =$$

$$4.2557 \, [\xi_h \, I_2(\nu_0)]^{\frac{1}{5}} \left(\frac{2 \, \mathrm{keV}}{m}\right)^{\frac{8}{5}} \left(\frac{120 \, M_\odot}{\Sigma_0 \mathrm{pc}^2}\right)^{\frac{1}{5}} \mathrm{pc}$$

$$I_n(\nu) \equiv (n+1) \; \int_0^\infty y^n \, dy \; f(y^2 - \nu) \quad , \quad \nu_0 \equiv \nu(0)$$

WDM Thomas-Fermi equations

Self-consistent dimensionless Thomas-Fermi equation:

$$\frac{d^2\nu}{d\xi^2} + \frac{2}{\xi}\,\frac{d\nu}{d\xi} + I_2(\nu) = 0 \quad , \quad \nu'(0) = 0$$

Core size r_h of the halo defined as for Burkert profile:

$$\frac{\rho(r_h)}{\rho_0} = \frac{1}{4} \quad , \quad r_h = l_0\,\xi_h$$

Fermi-Dirac Phase-Space distribution function $f(E/T_0)$:

Contrasting the theoretical Thomas-Fermi solution with galaxy data, T_0 turns to be $10^{-3}\ {}^o$K $< T_0 <$ 20 ^{o}K

colder = ultracompact, warmer = large spirals.

$T_0 \sim m\ < v^2 >_{\rm observed}$ for $m \sim 2$ keV.

All results are independent of any WDM particle physics model, they only follow from the gravitational interaction of the WDM particles and their fermionic nature.

Lower bound on the particle mass m

In the degenerate quantum limit $\nu_0 \to +\infty, \; T_0 \to 0$ the galaxy mass and halo radius take their minimum values

$$r_h^{min} = 11.3794 \; \left(\frac{2\,\mathrm{keV}}{m}\right)^{\frac{8}{5}} \; \left(\frac{120 \; M_\odot}{\Sigma_0 \; \mathrm{pc}^2}\right)^{\frac{1}{5}} \; \mathrm{pc}$$

$$M_h^{min} = 30998.7 \; \left(\frac{2\,\mathrm{keV}}{m}\right)^{\frac{16}{5}} \; \left(\frac{\Sigma_0 \; \mathrm{pc}^2}{120 \; M_\odot}\right)^{\frac{3}{5}} \; M_\odot$$

Observed halo masses must be larger or equal than M_h^{min}

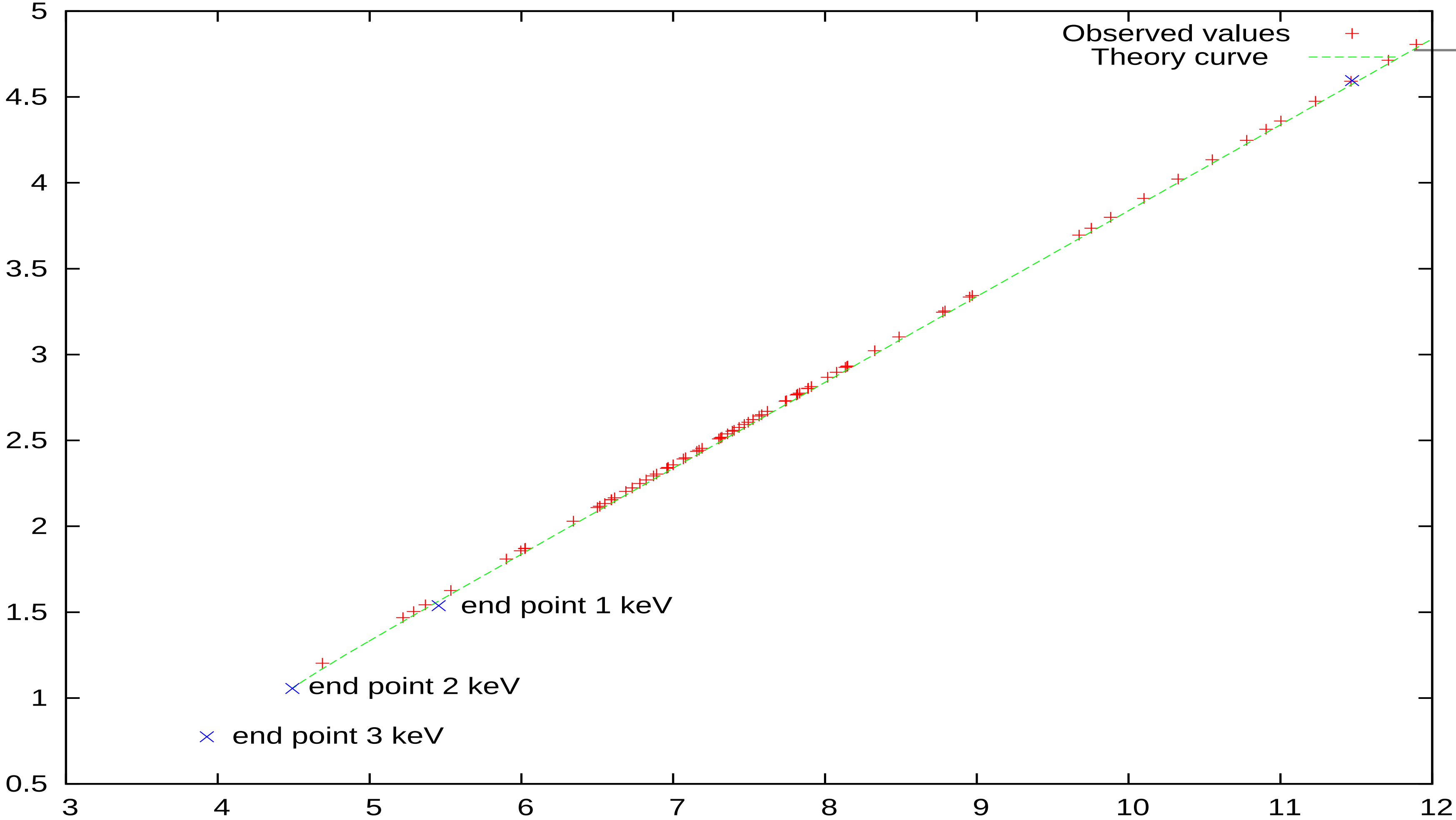
From the minimum observed value of the halo mass M_h^{min} a lower bound for the WDM particle mass m follows

$$m \geq m_{min} \equiv 1.387 \; \mathrm{keV} \; \left(\frac{10^5 \; M_\odot}{M_h^{min}}\right)^{\frac{5}{16}} \; \left(\frac{\Sigma_0 \; \mathrm{pc}^2}{120 \; M_\odot}\right)^{\frac{3}{16}}$$

The minimal known halo mass is for Willman I:

$M_{Willman\,I} = 3.9 \; 10^4 \; M_\odot$ which implies $m \geq 1.86 \; \mathrm{keV}$

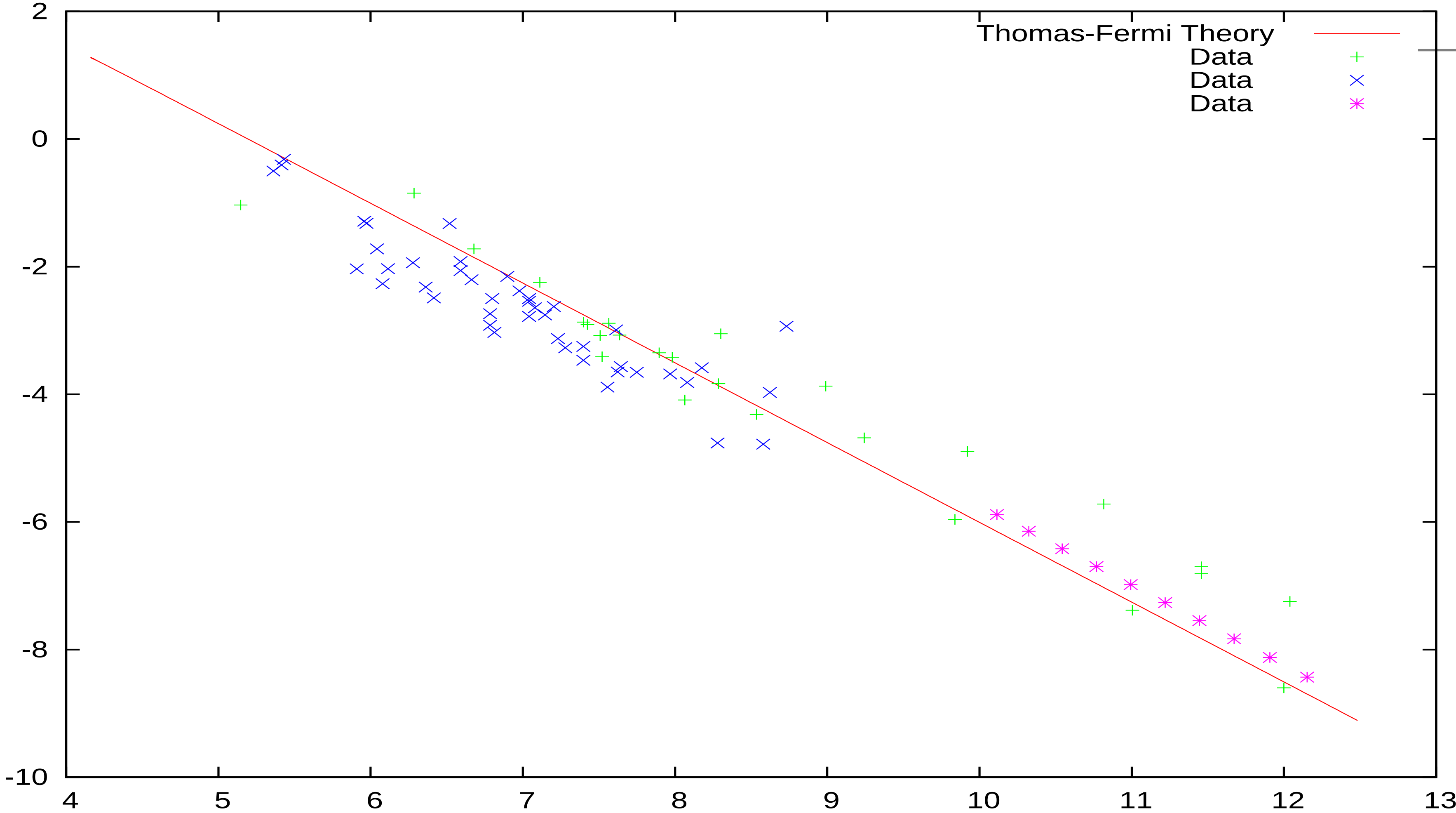
Galaxy halo radius vs. Galaxy halo Mass



$\hat{r}_h = r_h \left(\Sigma_0 \text{ pc}^2/[120\ M_\odot]\right)^{\frac{1}{5}}$ vs. $\hat{M}_h = M_h \left(120\ M_\odot/[\Sigma_0 \text{ pc}^2]\right)^{\frac{3}{5}}$.

r_h follows with precision the square-root of M_h and the amplitude factor as predicted theoretically.

Galaxy Phase-space density Q vs. Galaxy halo Mass



$\log_{10} Q$ vs. $\log_{10} \hat{M}_h$ theory and data.

$Q \equiv \rho(0)/\sigma^3(0)$. Theoretical curve Q obtained from the Thomas-Fermi expression.

Classical and Quantum regimes of WDM Galaxies

I. Diluted and classical regime:

$\hat{M}_h \gtrsim 10^6 \ M_\odot \ , \quad \nu_0 \lesssim -5 \ , \quad T_0 \gtrsim 0.017$ K.

The density and the velocity profiles are universal.

Exact scaling laws for $r_h, \ M_h$ and $Q(0)$.

II. Quantum compact regime:

$10^6 \ M_\odot \gtrsim \hat{M}_h \gtrsim \hat{M}_{h,min} = 3.1 \ 10^4 M_\odot \ ,$

$\nu_0 \gtrsim -5 \ , \quad 0 \leq T_0 \lesssim 0.017$ K.

The density and the velocity profiles are non-universal: the profiles depend on the galaxy mass M_h.

Small deviations from the scaling laws for $r_h, \ M_h$ and $Q(0)$.

III. Degenerate limit

$\hat{M}_h = \hat{M}_{h,min} = 3.1 \ 10^4 \ M_\odot \ , \quad \nu_0 = +\infty \ , \quad T_0 = 0$

Diluted regime of Galaxies

In the diluted regime of Galaxies

$M_h \gtrsim 10^6 \; M_\odot \; , \quad \nu_0 \lesssim -5 \; , \quad T_0 \gtrsim 0.017$ K = 17 mK.

$r_h , \; M_h$ and $Q(0)$ scale as functions of each other.

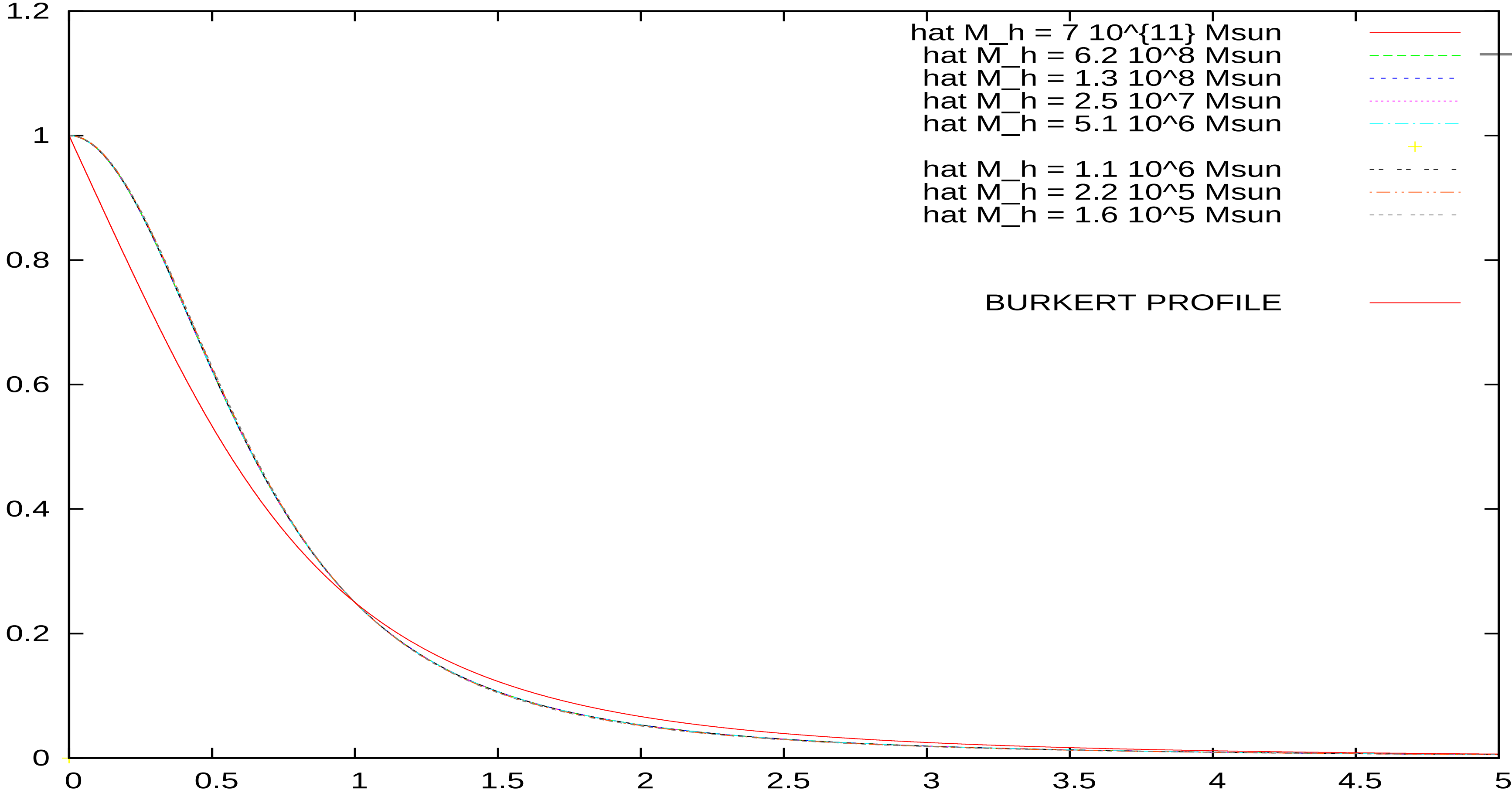
$$M_h = 1.75572 \; \Sigma_0 \; r_h^2 \quad , \quad r_h = 68.894 \sqrt{\frac{M_h}{10^6 \; M_\odot}} \sqrt{\frac{120 \; M_\odot}{\Sigma_0 \; \mathrm{pc}^2}} \;\; \mathrm{pc}$$

$$Q(0) = 1.2319 \; \left(\frac{10^5 \; M_\odot}{M_h}\right)^{\frac{5}{4}} \; \left(\frac{\Sigma_0 \; \mathrm{pc}^2}{120 \; M_\odot}\right)^{\frac{3}{4}} \; \mathrm{keV}^4$$

These scaling behaviours are very accurate except near the degenerate limit.

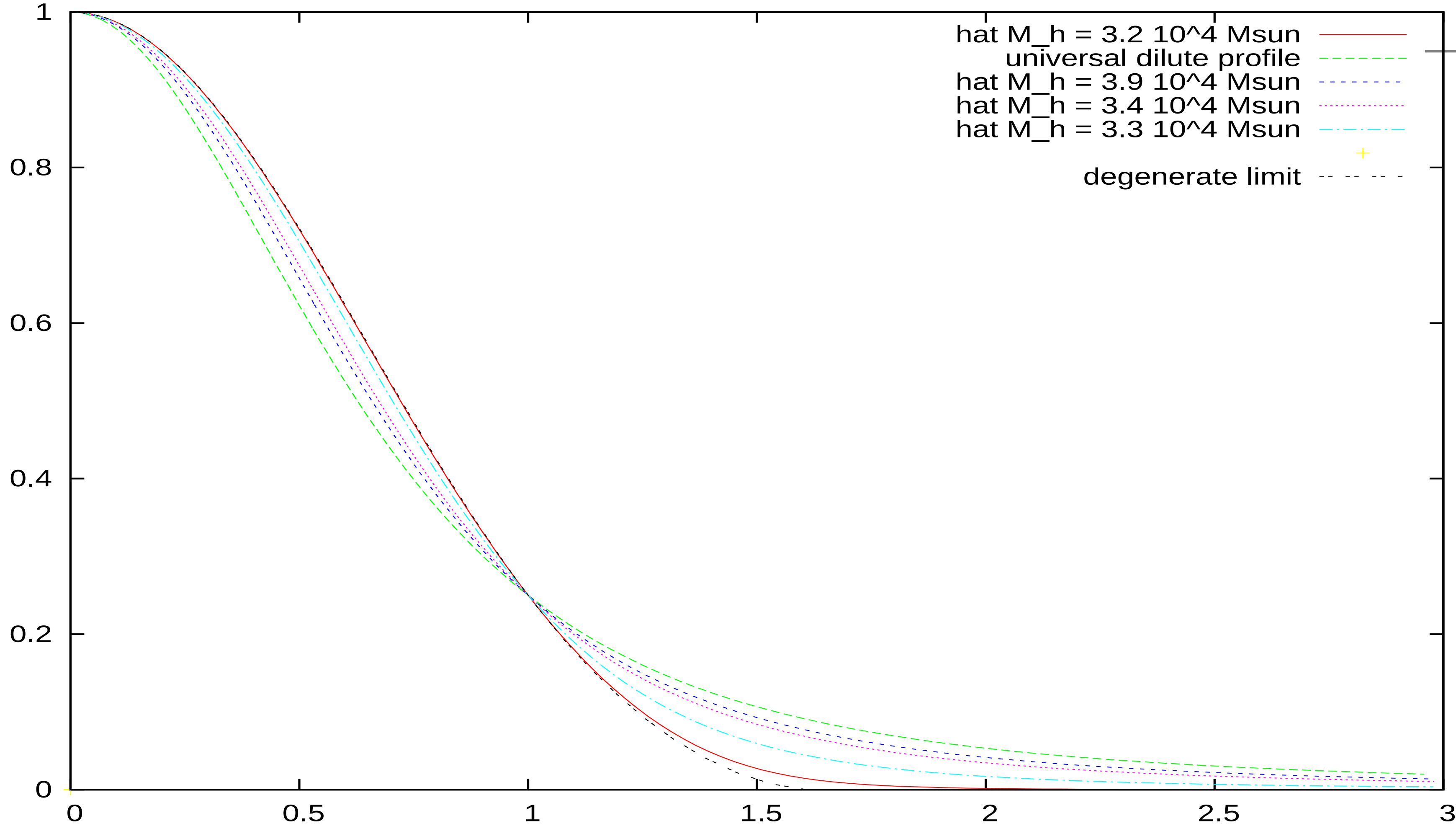
C. Destri, H. J. de Vega, N. G. Sanchez, New Astronomy **22**, 39 (2013) and Astroparticle Physics, **46**, 14 (2013).
H. J. de Vega, P. Salucci, N. G. Sanchez, arXiv:1309.2290, to appear in MNRAS.
H. J. de Vega, N. G. Sanchez, arXiv:1310.6355.

Theoretical vs. observational density profiles



$\rho(r)/\rho(0)$ as functions of r/r_h. ALL the theoretical profiles in the diluted regime: $1.4\ 10^5\ M_\odot < \hat{M}_h < 7.5\ 10^{11}\ M_\odot$ fall into the same and universal density profile in very good agreement with the empirical Burkert profile.

Density profiles in the Quantum regime



$\rho(r)/\rho(0)$ as functions of r/r_h: Non-Universal.

Galaxy halo masses $M_h^{min} = 3.1\ 10^4\ M_\odot \leq \hat{M}_h < 3.9\ 10^4\ M_\odot$ in the quantum regime exhibit shrinking density profiles for $r > r_h$.

Circular Velocities and Density Profiles

The circular velocity $v_c(r)$ follows from the virial theorem

$$v_c(r) = \sqrt{\frac{G\ M(r)}{r}} = \sqrt{-\frac{r}{m}\ \frac{d\mu}{dr}}$$

The circular velocity normalized at the core radius r_h

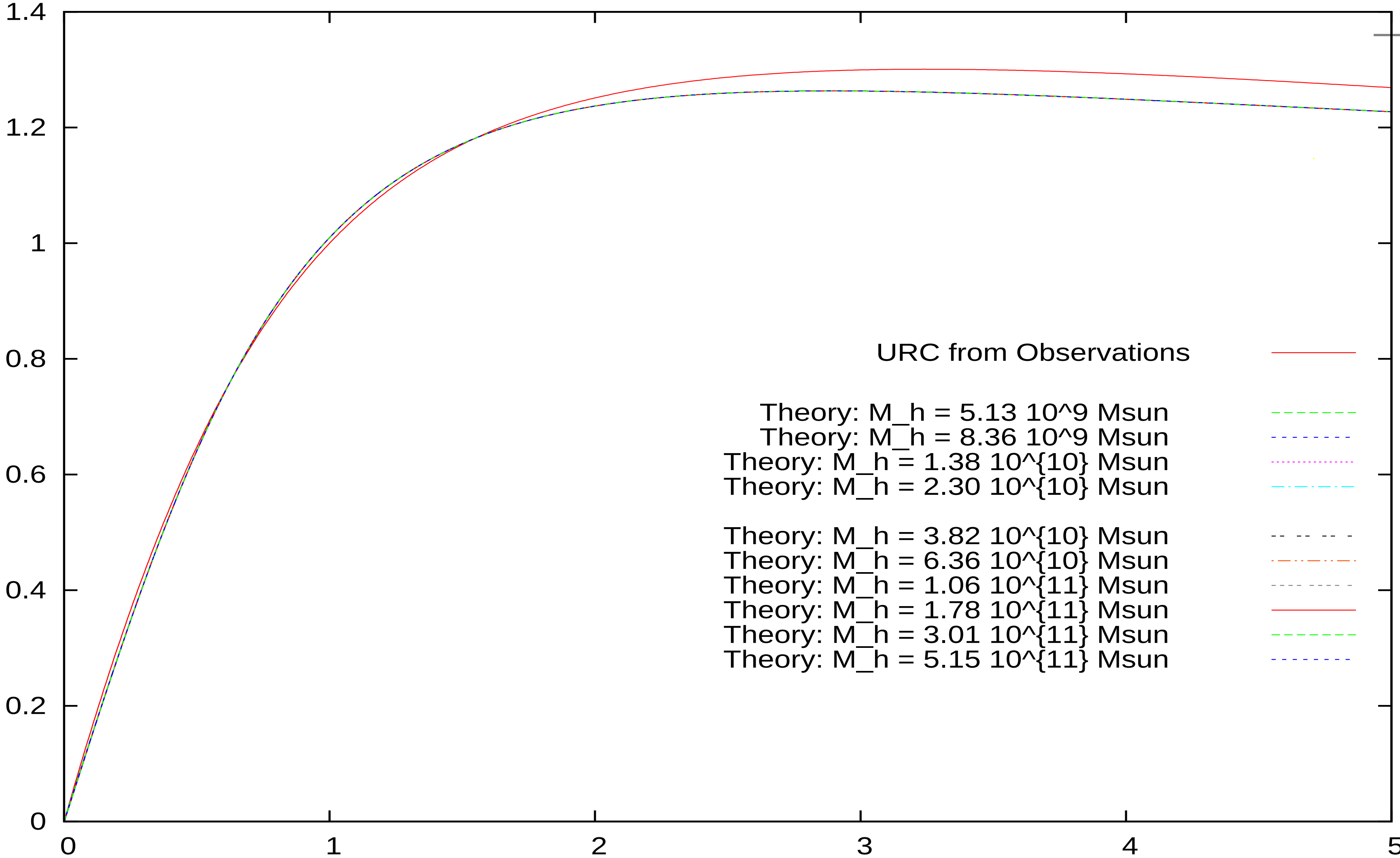
$$U(x) \equiv \frac{v_c(r)}{v_c(r_h)} \quad , \quad x = \frac{r}{r_h}$$

Solving the Thomas-Fermi equations we find:

- $U(x) = v_c(r)/v_c(r_h)$ is only function of $x = r/r_h$.
- $U(x)$ takes the same values for all galaxy halo masses in the range $5.1\ 10^9\ M_\odot$ till $5.1\ 10^{11}\ M_\odot$.
- $U(x)$ turns to be an universal function.
- The observational universal curves and the theoretical Thomas-Fermi curves coincide for $r \lesssim 2\ r_h,\ x \lesssim 2$.

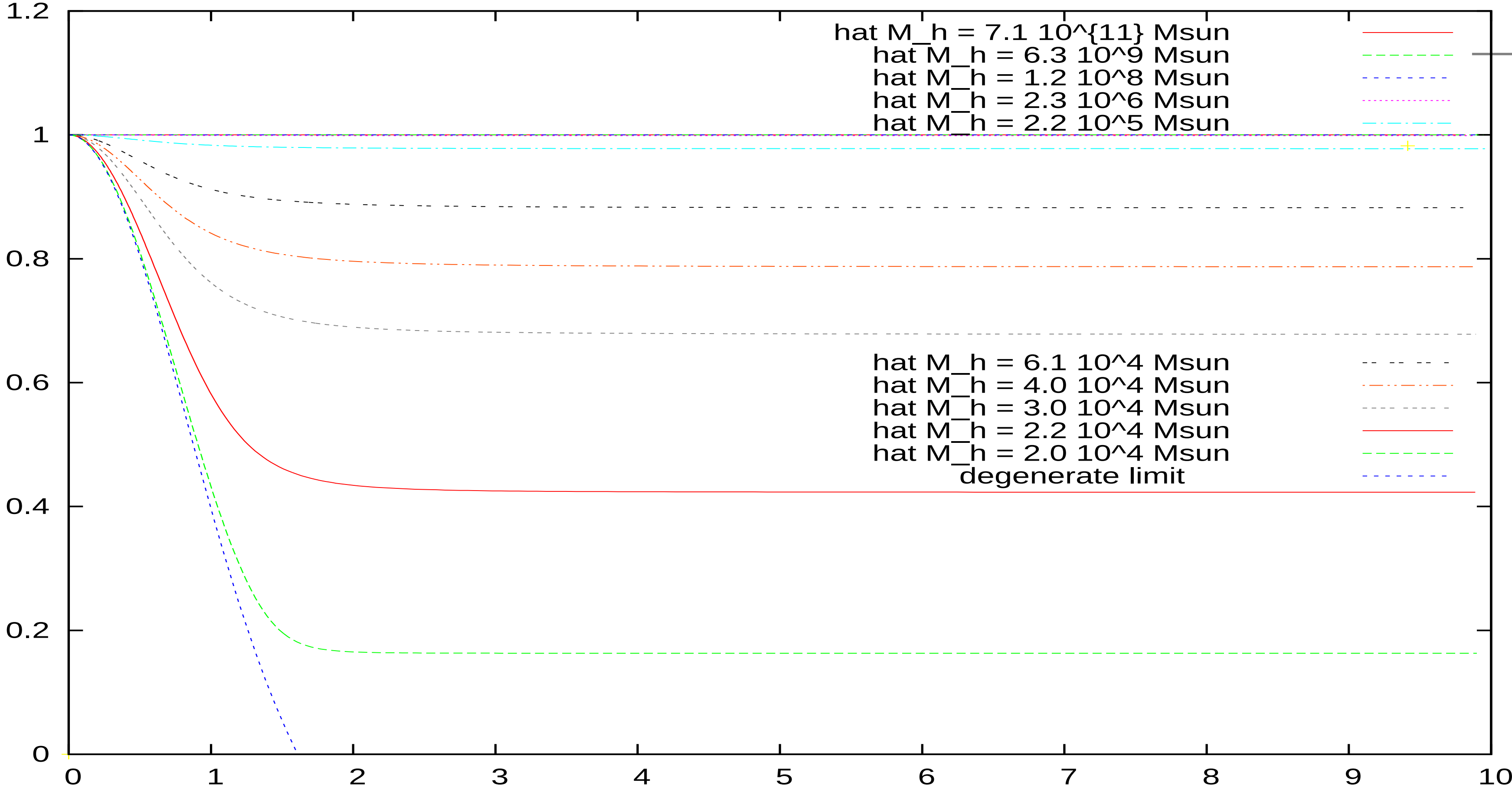
These are remarkable results !!

Normalized circular velocities



$U(x) = v_c(r)/v_c(r_h)$ **vs.** $x = r/r_h$.

Velocity dispersion profiles $\sigma^2(r)/\sigma^2(0)$ vs. $x = r/r_h$



ALL velocity profiles in the classical diluted regime $\hat{M}_h > 2.3\ 10^6\ M_\odot$ fall into a constant universal value.

In the quantum regime: $1.6\ 10^6 M_\odot > \hat{M}_h > \hat{M}_{h,min}$ the profiles are not universal and do depend on $\hat{M}_h$ and x.

The local equation of state of WDM Galaxies

The pressure $P(r)$ as a function of the density $\rho(r)$

$$\rho = \frac{m^{\frac{5}{2}}}{3\,\pi^2\,\hbar^3}\;(2\,T_0)^{\frac{3}{2}}\;I_2(\nu) \quad , \quad P = \frac{m^{\frac{3}{2}}}{15\,\pi^2\,\hbar^3}\;(2\,T_0)^{\frac{5}{2}}\;I_4(\nu).$$

through the potential ν from the Thomas-Fermi equation.

$P = \frac{T_0}{m}\,\rho \quad , \quad \nu \ll -1$, WDM diluted galaxies.

$P = \frac{\hbar^2}{5}\;\left(\frac{3\,\pi^2}{m^4}\right)^{\frac{2}{3}}\;\rho^{\frac{5}{3}}\;,\;\nu \gg 1$, WDM degenerate quantum limit.

Simple formula accurately representing the exact equation of state obtained by solving the Thomas-Fermi equation:

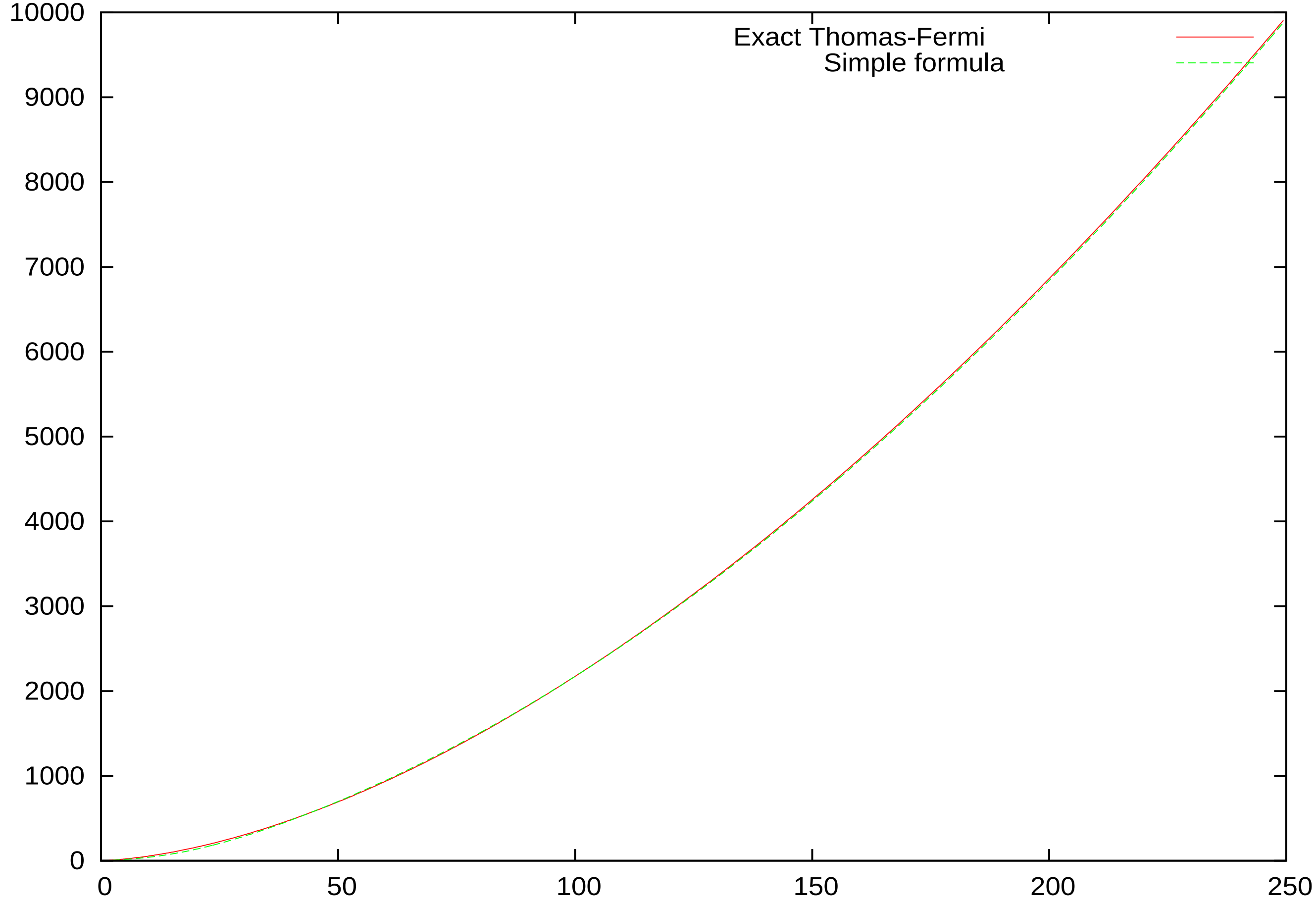
$$P = \frac{m^{\frac{3}{2}}\;(2\,T_0)^{\frac{5}{2}}}{15\,\pi^2\,\hbar^3}\;\left(1 + \frac{3}{2}\,e^{-\beta_1\,\tilde{\rho}}\right)\;\tilde{\rho}^{\dfrac{1}{3}\left(5 - 2\,e^{-\beta_2\,\tilde{\rho}}\right)},$$

$$\tilde{\rho} \equiv \frac{3\,\pi^2\,\hbar^3}{m^{\frac{5}{2}}\;(2\,T_0)^{\frac{3}{2}}}\;\rho = I_2(\nu),$$

best fit to the Thomas-Fermi equation of state for:

$$\beta_1 = 0.047098 \quad , \quad \beta_2 = 0.064492$$

he equation of state of Galaxies: exact T-F and simple formu



The equation of state $\tilde{P}$ vs. $\tilde{\rho}$ obtained by solving the Thomas-Fermi equation and the simple formula.

$$\tilde{P} = \frac{15\,\pi^2\,\hbar^3}{m^{\frac{3}{2}}\,(2\,T_0)^{\frac{5}{2}}}\,P = I_4(\nu)\;,\;\tilde{\rho} \equiv \frac{3\,\pi^2\,\hbar^3}{m^{\frac{5}{2}}\,(2\,T_0)^{\frac{3}{2}}}\,\rho = I_2(\nu)$$

The Eddington equation for Dark Matter in Galaxies

$f(E)$ DM distribution function, $E = p^2/(2m) - \mu$,
m DM particle mass, μ the chemical potential.

Equilibrium condition: $\mu(r) = \mu_0 - m\,\phi(r)$,

$\phi(r) =$ gravitational potential.

The Poisson equation takes the self-consistent form:

$$\frac{d^2\mu}{dr^2} + \frac{2}{r}\,\frac{d\mu}{dr} = -4\pi G m\,\rho(r) = -\frac{4\,G\,m^2}{\pi\,\hbar^3}\int_0^\infty dp\,p^2 f\left[\frac{p^2}{2m} - \mu(r)\right]$$

Dimensionless variables: $q,\ \nu(q)$:

$$r = r_h\,q \quad , \quad \mu(r) = T_0\,\nu(q) \quad , \quad f(E) = \Psi(E/T_0)$$

T_0 plays the role of the temperature and depends on the galaxy mass. The density profile is known from the observations:

$$\rho(r) = \rho_0\,F\left(\frac{r}{r_h}\right) = \rho_0\,F(q)\ ,\ \rho_0 \equiv \rho(0)\ ,\ F(1) = 1/4.$$

To be determined: the DM distribution function $\Psi(E/T_0)$.

Abel's equation and its solution

Dimensionless Poisson's equation:

$$\frac{d^2\nu}{dq^2} + \frac{2}{q}\,\frac{d\nu}{dq} = -b_0\; F(q)\;,\; b_0 \equiv 4\,\pi\; G\;\rho_0\; r_h^2\;\frac{m}{T_0}$$

$$\nu(q) = \nu(0) + b_0\;\varepsilon(q)\;,\; \varepsilon(q) - \int_0^q \left(1 - \frac{q'}{q}\right) q'\; F(q')\; dq'$$

Self-consistent Poisson equation in dimensionless variables:

$$\rho(r) = \frac{\sqrt{2}}{\pi^2}\; m^{\frac{5}{2}}\; T_0^{\frac{3}{2}} \int_{\nu(\infty)}^{\nu} d\nu' \; \sqrt{\nu - \nu'}\;\; \Psi(-\nu')\;,\; \nu' \equiv \nu - \frac{p^2}{2\,m\,T_0}.$$

and in terms of the density profile $F(q)$

$$F(\nu) = \frac{\sqrt{2}}{\pi^2}\;\frac{m^{\frac{5}{2}}\;T_0^{\frac{3}{2}}}{\rho_0}\;\int_{\nu(\infty)}^{\nu} d\nu'\;\sqrt{\nu - \nu'}\;\;\Psi(-\nu')$$

This is an Abel integral equation and its solution, the Eddington formula:

$$\Psi(-\nu) = \sqrt{2}\;\pi\;\frac{\rho_0}{m^{\frac{5}{2}}\;T_0^{\frac{3}{2}}}\;\int_{\nu(\infty)}^{\nu}\frac{d\nu'}{\sqrt{\nu-\nu'}}\;\frac{d^2F}{d\nu'^2}$$

Boundary condition: Ψ and $d\Psi/d\nu$ vanish at infinite distance.

The Distribution Function in terms of the Density Profile

We explicitly find the distribution function $\Psi(q)$ in terms of the density profile $F(q)$ in H. J. de Vega, N. G. Sanchez, arXiv:1401.0726.

$$\Psi(q) = \frac{1}{G^{\frac{3}{2}} \; r_h^3 \; m^4 \; \sqrt{\rho_0}} \;\; \mathcal{D}(q) \; , \; \mathcal{D}(q) \equiv \frac{1}{\sqrt{32 \, \pi}} \int_q^\infty \frac{\mathcal{J}(q') \; dq'}{\sqrt{\varepsilon(q) - \varepsilon(q')}}$$

$$\mathcal{J}(q) \equiv \frac{1}{\left(-\frac{d\varepsilon}{dq}\right)} \left[\frac{d^2 F}{dq^2} - \frac{\frac{d^2\varepsilon}{dq^2}}{\frac{d\varepsilon}{dq}} \; \frac{dF}{dq} \right]$$. Notice that $\left(-\frac{d\varepsilon}{dq} > 0\right)$.

We explicitly find the velocity dispersion and the pressure in terms of the density profile $F(q)$:

$$v^2(r) = 6 \, \pi \; G \; \rho_0 \; r_h^2 \; \frac{1}{F(q)} \int_q^\infty dq' \; \left[\varepsilon(q) - \varepsilon(q')\right]^2 \; \mathcal{J}(q')$$

$$P(r) = 2 \, \pi \; G \; \Sigma_0^2 \; \int_q^\infty dq' \; \left[\varepsilon(q) - \varepsilon(q')\right]^2 \; \mathcal{J}(q')$$

Physical results from the Distribution Function

Cored density profiles behaving quadratically for small distances $\rho(r) \stackrel{r\to 0}{=} \rho(0) - K\, r^2$ produce finite and positive distribution functions at the halo center while cusped density profiles always produce divergent distribution functions at the center.

We explicitly compute the phase–space distribution function and the equation of state for the family of α-density profiles

$$\rho(r) = \frac{\rho_0}{\left[1+\left(\frac{r}{r_h}\right)^2\right]^\alpha} \quad , \quad 1 \leq \alpha \leq 2.5$$

This cored density profile generalizes the pseudo-thermal profile and with $\alpha \sim 1.5$, it is perfectly appropriate to fit galaxy observations.

For $\alpha = 5/2$ this is the Plummer profile describing the density of stars in globular clusters.

Halo Thermalization from the Distribution Function

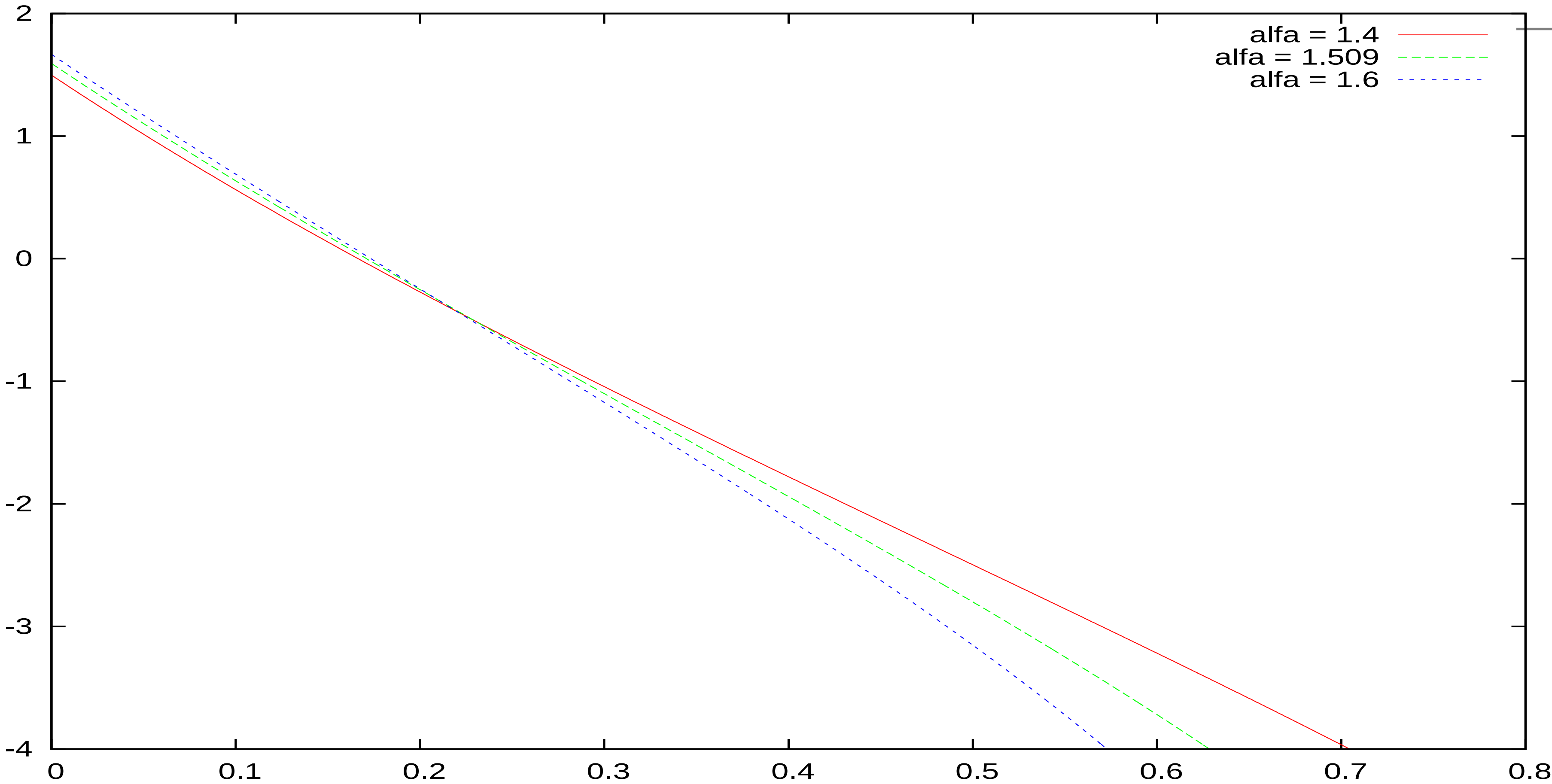
The obtained distribution function $\Psi(q)$ is **positive** for all values of q in the whole range $1 \leq \alpha \leq 2.5$. Therefore, the $\alpha-$profiles are physically meaningful. [In general, there is no guarantee that $\Psi(q)$ from the Eddington formula will be nowhere negative.]

$\ln \mathcal{D}(-\varepsilon)$ is approximately a linear function of the energy $-\varepsilon$ for $\alpha \sim 1.5$ and $0 < -\varepsilon \lesssim 0.6$ which corresponds to $0 < r \lesssim 7\, r_h$.

Therefore, the distribution function corresponding to $\alpha-$profiles for $\alpha \sim 1.5$ is approximately a thermal Boltzman distribution in this interval. These are realistic halo galaxy density profiles.

The galaxy halos are therefore thermalized, supporting and confirming the Thomas-Fermi WDM approach.

Halo Thermalization



The distribution function $\ln \mathcal{D}(-\varepsilon)$ vs. the energy $-\varepsilon$.

This linear behaviour of $\ln \mathcal{D}(-\varepsilon)$ indicates a Boltzman distribution function for $0 \leq -\varepsilon \lesssim 0.7$ and $0 < r \lesssim 7\, r_h$. No assumption about the DM particle nature is made here.

he Halo Dark Matter equation of state from the density prof

From the density profile we obtained the pressure and therefore the DM equation of state

$$\frac{P(r)}{\rho(r)} = \frac{1}{3}\, v^2(r) = G\, \Sigma_0\, r_h\, \frac{\Pi(q)}{F(q)}$$

The local temperature $T(r)$ is given by $T(r) = \frac{1}{3}\, m\, v^2(r)$.

Hence, the dark matter obeys locally an ideal gas equation of state

$$P(r) = \frac{T(r)}{m}\, \rho(r)\ ,\ T(r) \equiv m\, G\, \Sigma_0\, r_h\, t(q)\ ,\ t(q) \equiv \frac{\Pi(q)}{F(q)}$$

The temperature $T(r)$ turns to be approximately constant inside the halo radius $r \lesssim r_h\ : t(q) \simeq 1.419$.

$$T(r) = 8.238\, t(q)\, \frac{m}{2\ \mathrm{keV}} \sqrt{\frac{\Sigma_0\ \mathrm{pc}^2}{120\ M_\odot}\, \frac{M_h}{10^6\ M_\odot}} \text{ m } {}^o\text{K}$$

The temperature grows as the square root of the galaxy halo mass.

Circular velocity and circular temperature

The circular velocity and the circular temperature are defined by the virial theorem:

$$v_c^2(r) \equiv \frac{G\,M(r)}{r} \; , \; T_c(r) \equiv \frac{1}{3}\, m \, v_c^2(r) = \frac{G\,m\,M(r)}{3\,r}$$

$$T_c(r) = m\,G\,\rho_0\,r_h^2\,t_c(q)$$

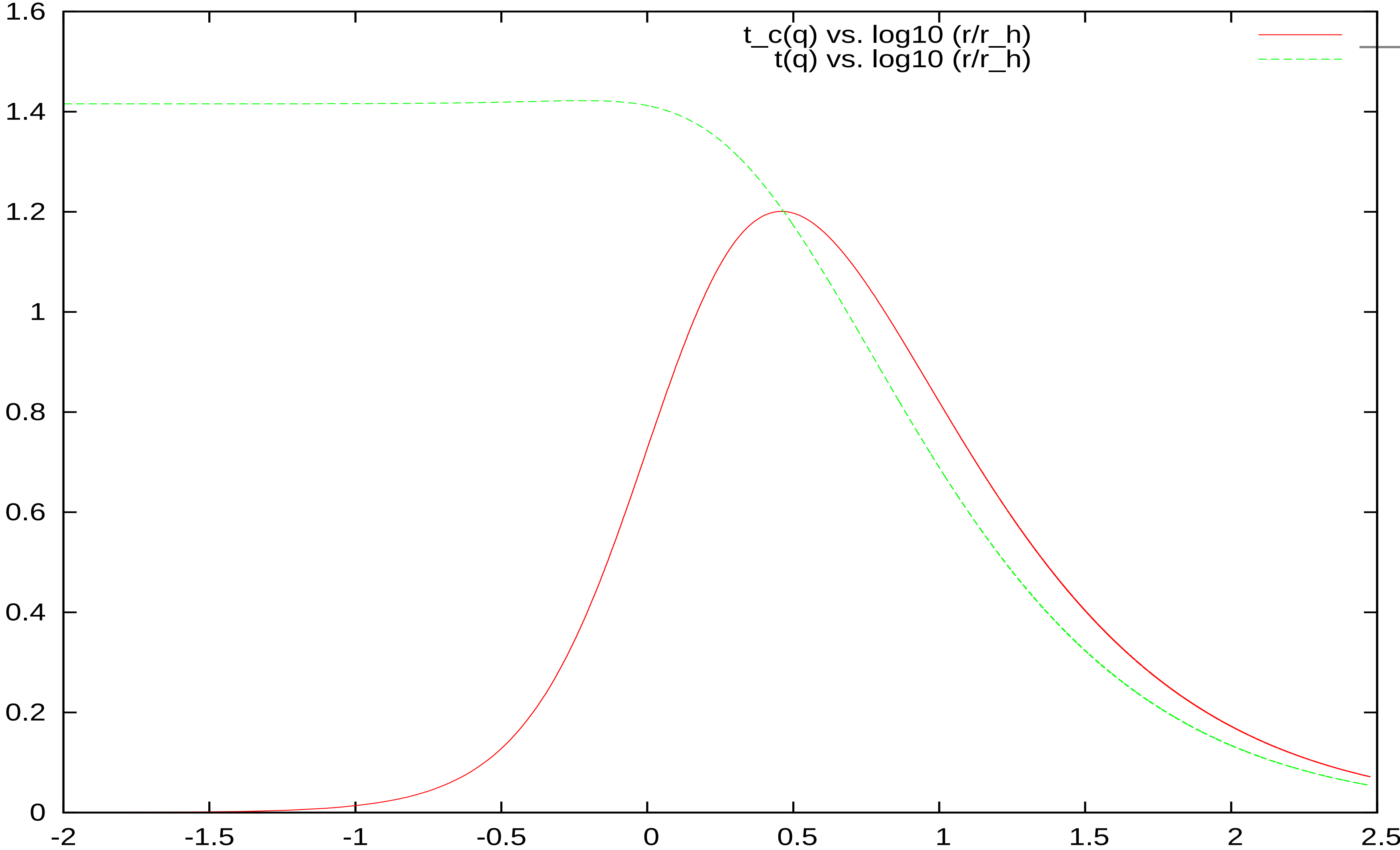
The local temperature $t(q)$ turns to follow the decrease of the circular temperature $t_c(q)$ for $r \gtrsim r_h$.

Conclusion:

- Halo thermalization for $r < r_h$.
- Halo virialization for $r > r_h$.

H. J. de Vega, N. G. Sanchez, arXiv:1401.0726

Thermalization and Virialization



The normalized temperature $t(r/r_h)$ and the circular temperature $t_c(r/r_h)$ vs. $\log_{10}(r/r_h)$ for $\alpha = 1.509$.

For $r \gtrsim r_h$, the local temperature decreases slowly with r.

Axions are ruled out as dark matter

Hot Dark Matter (eV particles or lighter) are ruled out because their free streaming length is too large $\gtrsim$ Mpc and hence galaxies are not formed.

A Bose-Einstein condensate of light scalar particles evades this argument because of the quantum nature of the BE condensate. $r_{Jeans} \sim 5$ kpc implies $m_{axion} \sim 10^{-22}$ eV.

The phase-space density $Q = \rho/\sigma^3$ decreases during structure formation: $Q_{today} < Q_{primordial} \propto m^4$.
Computing $Q_{primordial}$ for a DM BE condensate we derived lower bounds on the DM particle mass m using the data for Q_{today} in dwarf galaxies:

TE: $m \geq 0.155 \text{ MeV } \left(\frac{25}{g_d}\right)^{5/3}$. Out of TE: $m \geq 14 \text{ eV } \left(\frac{25}{g_d}\right)^{5/3}$

Axions with $m \sim 10^{-22}$ eV are ruled out as DM candidates.

D. Boyanovsky, H. J. de Vega, N. G. Sanchez, PRD 77, 043518 (08). H. de Vega, N. Sanchez, arXiv:1401.1214

Dark Energy

$76 \pm 5\%$ of the present energy of the Universe is Dark!
Current observed value:
$\rho_\Lambda = \Omega_\Lambda \, \rho_c = (2.39 \text{ meV})^4 \quad , \quad 1 \text{ meV} = 10^{-3} \text{ eV}$.
Equation of state $p_\Lambda = -\rho_\Lambda$ within observational errors.
Quantum zero point energy. Renormalized value is finite.
Bosons (fermions) give positive (negative) contributions.
Mass of the lightest particles ~ 1 meV is in the right scale.
Spontaneous symmetry breaking of continuous symmetries produces massless scalars as Goldstone bosons. A small symmetry breaking provide light scalars: axions,majorons...
Observational Axion window $10^{-3} \text{ meV} \lesssim M_{\text{axion}} \lesssim 10 \text{ meV}$.
Dark energy can be a cosmological zero point effect. (As the Casimir effect in Minkowski with non-trivial boundaries).
We need to learn the physics of light particles (< 1 MeV), also to understand dark matter !!

Effective Theory of Inflation (ETI) confirmed by Planck

Quantity	ETI Prediction	Planck 2013
Spectral index $1 - n_s$	order $1/N = 0.02$	0.04
Running $dn_s/dlnk$	order $1/N^2 = 0.0004$	< 0.01
Non-Gaussianity f_{NL}	order $1/N = 0.02$	< 6
	ETI + WMAP+LSS	
tensor/scalar ratio r	r > 0.02	< 0.11see BICEP
inflaton potential curvature $V''(0)$	$V''(0) < 0$	$V''(0) < 0$

ETI + WMAP+LSS means the MCMC analysis combining the ETI with WMAP and LSS data. Such analysis calls for an inflaton potential with negative curvature at horizon exit. The double well potential is favoured (new inflation).
D. Boyanovsky, C. Destri, H. J. de Vega, N. G. Sanchez, arXiv:0901.0549, IJMPA 24, 3669-3864 (2009).

THANK YOU VERY MUCH
FOR YOUR ATTENTION!!