Galaxy phase-space density data preclude Bose-Einstein condensate be the total Dark Matter

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Ultralight scalars with typical mass of the order $m \sim 10^{-22}$ eV and light scalars forming a Bose-Einstein condensate (BEC) exhibit a Jeans length in the kpc scale and were therefore proposed as dark matter (DM) candidates. Our treatment here is generic, independent of the particle physics model and applies to all DM BEC, in both : in or out of equilibrium situations. Two observed quantities crucially constrain DM in an inescapable way: the average DM density ρ_{DM} and the phase-space density Q. The observed values of ρ_{DM} and Q in galaxies today, constrain both the possibility to form a BEC and the DM mass m. These two constraints robustly exclude axion DM that decouples after inflation. Moreover, the value $m \sim 10^{-22}$ eV can only be obtained with a number of ultrarelativistic degrees of freedom at decoupling in the trillions which is impossible for decoupling in the radiation dominated era. In addition, we find for the axion vacuum misalignment scenario that axions are produced strongly out of thermal equilibrium and that the axion mass in such scenario turns to be **17 orders of magnitude** too large to reproduce the observed galactic structures. Moreover, we also consider inhomogenous gravitationally bounded BEC's supported by the bosonic quantum pressure independently of any particular particle physics scenario. For a typical size $R \sim$ kpc and compact object masses $M \sim 10^7\ M_\odot$they remarkably lead to the same particle mass $m \sim 10^{-22}$ eV as the BEC free-streaming length. However, the phase-space density for the gravitationally bounded BEC's turns to be more than **sixty orders of magnitude** smaller than the galaxy observed values. We conclude that the BEC cannot be the total DM. The axion can be candidate to be only part of the DM of the universe. Besides, an axion in the mili-eV scale may be a relevant source of dark energy through the zero point cosmological quantum fluctuations.

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I. INTRODUCTION

Deciphering the nature of dark matter (DM) is nowadays one of the most active domains in astrophysics, cosmology and particle physics. Cold dark matter (CDM) particles heavier than a GeV spectacularly succeed to reproduce the observations for large scales beyond the Mpc. DM particles with mass m below the eV (HDM-hot DM) are ruled out because their too large Jeans lengths exclude the formation of the observed galaxies. There is a way out for scalar particles if they form Bose-Einstein condensates (BEC) where the Jeans length can be estimated as [1, 2]

$$\lambda_J \sim 4 \sqrt{\frac{10^{-22}\ \text{eV}}{m}}\ \text{kpc} \simeq 1.2 \times 10^{17} \sqrt{\frac{10^{-22}\ \text{eV}}{m}}\ \text{km} \ . \tag{1}$$

Hence, in BEC dark matter one should have typically

$$m \sim 10^{-22}\ \text{eV} \tag{2}$$

in order to reproduce the observed galactic structures. The same requirement but for non-BEC dark matter gives m in the keV scale, that is warm dark matter (WDM) [3], [4],[5],[6],[7],[8],[9],[10],[11],[12],[13],,[14],[15],[16],[17]. CDM and WDM yield identical results for large scales beyond the Mpc; WDM provides too the correct medium, galactic and small scales in agreement with observations [3] to [17] and references therein.

BEC of alkali atoms, BEC of molecules and BEC of magnons have been observed experimentally in the laboratory [18].

The galactic phase space density is an important physical quantity and its analysis is crucial to constrain the nature of Dark Matter from the by now robust observational data for it, as described in Section III and IV here below and references therein.

In this paper, we study the density in physical space ρ and the density in phase-space Q in order to constrain for the first time with these two observables the Bose-Einstein condensates (BECs) as Dark Matter candidates, mainly the mass of generic light scalars forming a DM BEC and other relevant properties of such BECs.

Axion cosmological scenarii and BECs as Dark matter have a wide literature from many years see for example [19],[20], [21],[22],[23],[24], and is not our aim here to review all them, this is not a review paper, and our aim here is to provide constraints never considered before for DM BECs. We stress that previous DM BEC literature have not introduced the modern DM galactic phase density and the constraints imposed by the real galaxy data for the DM phase density, and none of previous DM BEC papers relate to the aspects of the BEC DM constraints we are treating for the first time in BEC DM here.

Building particle physics models of axion scenarii although interesting in its own, is not the aim of this paper. The DM BEC phase space density constraints and the galaxy data for them are not treated in the previous DM BEC literature.

Moreover, the phase space density and its galactic data to constraint DM is a treatement rather generic and universal , independent of the details of such models. This apply to any kind kind of DM and is manifest in the astrophysical and dark matter galactic literature.

In this paper we consider and constraint with galaxy data different observables as the density, surface density, free streaming length, the number of effective degrees of freedom, the mass range of the different ultraligh mass particles in the BECs, in different situations, in thermal and out of thermal equilibrium, for homogeneous as well as gravitational non homogeneous BECs. Cross-correlation and self-consistency of all them is performed which make the results of this paper strongly robust and far beyond the current literature in the field.

The *two* observables: the average DM density ρ_{DM} *and* the galactic phase space density Q robustly constrain in an inescapable way both: the possibility to form a BEC eg T_d/T_c and the DM particle mass m ruling out BEC DM in general, and the BEC axion DM in particular. Moreover, the typical value $m \sim 10^{-22}$ eV can only be obtained with a number of ultrarelativistic degrees of freedom at decoupling in **the trillions** which is impossible for decoupling in the radiation dominated era. The situation for lighter DM particles is even worst and makes the exclusion result even stronger.

This paper is organized as follows:

In Section II we analyze the Bose Einstein Condensate as a Dark Matter candidate. Our treatment applies to any shape of the distribution function and is valid for any particle physics model. A detailed and updated analysis of the BEC phase space density and BEC coarse-grained phase space density is provided in Sections III and IV, both in theory and observations.

Section III provides an updated synthesis and clarification on the phase density in DM and a useful state-of-the art. From such analysis, robust DM BECs constraints are derived in Section IV.

DM BEC decoupling at Thermal Equilibrium is treated in Section V including its implications for the DM axion. DM BEC in decoupling out of Thermal Equilibrium is treated in Section VI, including its implications for the Jeans- Lengths and for the BEC number of ultra-relativistic degrees of freedom.

In Section VII we discuss inhomogeneous, gravitationally bounded BE condensates of finite size. The constraints from the galactic phase space density remarkably provide similar values to the homogeneous BEC constraints found in Sections IV and V and confirm the generic character and robutsness of the results.

In Section VIII we analyze DM axions in the canonical axion vacuum misalignment scenario [25], [26], [27] and we constraint it with the DM phase space density and galactic data. The exclusion constraints obtained for the axion as a DM candidate confirm the universal constraints obtained in this paper for the DM BECs in general.

Section IX summarizes our results, conclusions and remarks. We notice that axions with masses in the meV $= 10^{-3}$ eV range can play an important role in astrophysics and cosmology, [28], [29], [30], [32], [33], not for dark matter but for dark energy as we proposed and studied in ref [28], and the misalignment scenario could produce axions with mass in such meV scale.

II. THE BOSE EINSTEIN CONDENSATE (BEC) AS A DARK MATTER CANDIDATE

- After decoupling, the DM distribution function freezes out and is a function of the covariant momentum p. We consider **generic** distribution functions f_d out of thermal equilibrium or thermal. The specific form of f_d in the non-thermal cases depends on the details of the interactions before decoupling.
- Our treatment applies to **any shape** of f_d and is valid for **any** particle physics model. For convenience and without losing generality, we choose f_d as a function of p/T_d : $f_d(p/T_d)$, where T_d is the covariant decoupling temperature.

In a BEC a sizeable fraction of the particles is in the zero momentum state while the rest is on excited states. We call ρ_0 the zero-momentum comoving contribution to the mass density. The contribution from the excited states $\rho - \rho_0$ follows as usual by integrating the distribution function. When the particles became nonrelativistic, we thus have

$$\rho - \rho_0 = m \int_0^\infty \frac{p^2 \, dp}{2\,\pi^2} f_d\left(\frac{p}{T_d}\right) = \frac{m}{2\,\pi^2} \, T_d^3 \, U \quad , \tag{3}$$

$$U \equiv \int_0^\infty y^2 \; f_d(y) \; dy \; , \tag{4}$$

where we consider neutral scalars. The case where the particles remain ultra-relativistes (UR) is considered in eq.(29) below.

The BEC density ρ_0 vanishes at the BEC covariant critical temperature T_c. Therefore, the BEC can be present if $T_d < T_c$ and we have from eq.(3) [34],

$$\rho = \frac{m\,U}{2\,\pi^2}\,T_c^3 \quad , \quad \rho_0 = \frac{m\,U}{2\,\pi^2}\,\left(T_c^3 - T_d^3\right) \quad , \quad T_d < T_c \; . \tag{5}$$

[T_c is defined by the above equation even in the out of thermal equilibrium case]. T_d and T_c are related to the respective effective number of UR degrees of freedom g_d and g_c, and to the photon temperature today T_γ by entropy conservation [25]:

$$T_c = \left(\frac{2}{g_c}\right)^{1/3} T_\gamma \; , \quad T_d = \left(\frac{2}{g_d}\right)^{1/3} T_\gamma \; , \quad T_\gamma = 0.2348 \; 10^{-3}\text{eV} \; . \tag{6}$$

The DM density ρ must reproduce the observed average DM in the universe $\Omega_{DM}\,\rho_{\rm crit}$. Hence,

$$\rho_{\rm DM} \;\equiv\; \Omega_{DM}\;\rho_{\rm crit} \;=\; \frac{m\,U}{\pi^2\;g_d}\;\left(\frac{T_c}{T_d}\right)^3\;T_\gamma^3, \tag{7}$$

$$\rho_{\rm DM} = 0.9259\ 10^{-23}\ {\rm keV}^4$$

Therefore, the DM particle mass m can be related to T_d/T_c as

$$m = \pi^2 \ \frac{\rho_{DM}\ g_d}{T_\gamma^3\ U} \ \left(\frac{T_d}{T_c}\right)^3 = 7.059\ {\rm eV}\ \frac{g_d}{U} \ \left(\frac{T_d}{T_c}\right)^3 \ . \tag{8}$$

The value of g_d depends on the detailed particle physics of the light scalar particle. For QCD axions decoupling soon after the QCD phase transition one has $g_d \sim 25$. The covariant critical temperature T_c as well as g_c are parameters that depend on the BEC state.

It must be $g_c < g_d$ (and hence $T_d < T_c$) in order to have a BEC.

The continuous and bounded function $f_d\,(p/T_d)$ stands for the excited states of the distribution function. The total distribution function can be written as a Dirac delta function representing the zero momentum BEC plus the excited states piece $f_d\,(p/T_d)$ as follows

$$f_d^{total}(p) = (2\ \pi)^3 \ \frac{\rho_0}{m} \ \delta\,(\vec{p}) + f_d\left(\frac{p}{T_d}\right) = 2\ \pi^2 \ \frac{\rho_0}{m} \ \frac{\delta(p^2)}{p^2} + f_d\left(\frac{p}{T_d}\right) \tag{9}$$

The continuum Dirac delta notation is convenient for calculations but in reality the BEC is in a finite comoving volume V_c and the wavenumbers $\vec{p}$ are discretized:

$$\vec{p} = \frac{2\ \pi}{V_c^{1/3}}\ \vec{n} \quad , \quad \vec{n} \ \epsilon\ \mathcal{Z}^3 \quad , \quad \delta\,(\vec{p}) = \frac{V_c}{(2\ \pi)^3}\ \delta_{\vec{p},\vec{0}} \ .$$

Therefore, $\delta(\vec{0}) = \dfrac{V_c}{(2\ \pi)^3}$ is finite as well as $f_d^{total}(0)$:

$$f_d^{total}(0) = \frac{\rho_0}{m}\ V_c + f_d(0) \ .$$

The comoving squared velocity $< v^2 >$ can be then expressed as

$$< v^2 > = \frac{< p^2 >}{m^2} \ = \ \frac{1}{m^2} \ \frac{\int_0^\infty p^4\ f_d\,(p/T_d)\ dp/(2\ \pi^2)}{(\rho_0/m) + \int_0^\infty p^2\ f_d\,(p/T_d)\ dp/(2\ \pi^2)} \tag{10}$$

$$< v^2 > = \frac{T_d^5}{m^2\ T_c^3} \ \frac{V}{U} \ , \qquad V \equiv \int_0^\infty y^4\ f_d(y)\ dy \tag{11}$$

and U is given by eq.(4). The BEC does not contribute to the integral in the numerator of $< v^2 >$ in eq.(10) because the integral of the Dirac delta function in f_d^{total} eq.(9) vanishes upon integration over $\vec{p}$ due to the extra p^2 factor in the numerator of eq.(10) with respect to the denominator.

As noticed in ref. [34] the Bose-Einstein distribution, for massless particles is of the order of the comoving volume V_c as discussed above. The BEC distribution function is well defined. This is also the case for massive particles because the chemical potential at decoupling μ_d must be equal to the particle mass m in order to form a BEC [34]. Even the part of the distribution function that describes the particles outside the condensate is of the order of V_c.

Therefore, the two pieces of the total distribution function: the BEC part and the excited states part $f_d\,(p/T_d)$ are of the order of the comoving volume V_c which in this case is very small. This is so because for the QCD phase transition $g_d \sim 25$ and $(z_d + 1) \sim 1.7\ 10^{12}$, giving for V_c,

$$V_c \;=\; \frac{V_{today}}{(z_d+1)^3} \;\sim\; 2 \times 10^{-37}\; V_{today}\ . \tag{12}$$

Taking for example $V_{today}^{1/3} = 1$ kpc, yields $V_c^{1/3} \sim 18200$ km.

III. THE BEC PHASE SPACE DENSITY

Let us discuss now the phase space density which is defined by [35]:

$$Q \equiv \frac{\rho}{<\sigma^2>^{3/2}} \;=\; \sqrt{27}\,\frac{\rho}{<v^2>^{3/2}} \;=\; \sqrt{27}\,\frac{m^3\,\rho}{\langle \vec{P}_f^2\rangle^{3/2}}\ , \tag{13}$$

where $\sigma^2 =< v^2 > /3$ is the velocity dispersion, and $\vec{P}_f$ is the physical momentum.

Including explicitly the BEC, using eqs.(5) and (10), the phase space density Q eq.(13) is given by

$$Q = \sqrt{27}\; m^3\; \frac{\left[(\rho_0/m) + \int_0^\infty p^2\; f_d\,(p/T_d)\;\; dp/(2\,\pi^2)\right]^{5/2}}{\left[\int_0^\infty p^4\; f_d\,(p/T_d))\;\; dp/(2\,\pi^2)\right]^{3/2}} \tag{14}$$

$$Q = \frac{3\,\sqrt{3}}{2\,\pi^2}\; m^4\; \frac{U^{5/2}}{V^{3/2}}\; \left(\frac{T_c}{T_d}\right)^{15/2}$$

- In the absence of self-gravity Q is Liouville invariant because both ρ and $\langle \vec{P}_f^2\rangle^{3/2}$ redshift as $(z+1)^3$.
- Because the distribution function is frozen and is a solution of the collisionless Boltzmann (Liouville) equation, it is clear that Q is a *constant*, namely a Liouville invariant, in the absence of self-gravity [34].

- The value of Q given by eq.(14) is valid after decoupling and before structure formation when Q is invariant under the universe expansion.

IV. THE BEC COARSE-GRAINED PHASE DENSITY CONSTRAINT

The expression of Q eq.(13) provides a good approximation to *the coarse-grained phase-space density*. Given the central role played by the phase-space density in the galaxy DM context, we derive here the connection between the coarse-grained phase-space distribution function $F(\mathbf{r}, \mathbf{v})$ and $Q(\mathbf{r})$.

The matter density express in terms of $F(\mathbf{r}, \mathbf{v})$ as

$$\rho(\mathbf{r}) = \int d^3v \; F(\mathbf{r}, \mathbf{v}) \; = < v^2 >^{3/2} \int d^3w \; F(\mathbf{r}, \mathbf{w}) \; , \tag{15}$$

where we change the integration variable to $\mathbf{w} \equiv \mathbf{v}/\sqrt{< v^2 >}$.

We find from eq.(15) the inequality

$$\frac{\rho(\mathbf{r})}{< v^2 >^{3/2}} = \int d^3w \; F(\mathbf{r}, \mathbf{w}) \;\; \geq \int_{|\mathbf{w}| \leq 1} d^3w \; F(\mathbf{r}, \mathbf{w})$$

where we used the positiveness of the distribution function $F(\mathbf{r}, \mathbf{w}) \geq 0$.

Now we can apply the first mean value theorem [36] to the last integral over $\mathbf{w}$,

$$Q(\mathbf{r}) \equiv \frac{\rho(\mathbf{r})}{< v^2 >^{3/2}} \;\; \geq \int_{|\mathbf{w}| \leq 1} d^3w \; F(\mathbf{r}, \mathbf{w}_1) = \frac{4\,\pi}{3} \; F(\mathbf{r}, \mathbf{w}_1) \; , \tag{16}$$

where $\mathbf{w}_1$ is a point inside the unit sphere.

- We find that Q approximates the coarse-grained phase-space distribution function by excess by a factor of order one. Previous estimations [37] of Q yielded values similar to the rigorous derivation presented here.

- Therefore, eq.(13) provides an appropriate coarse-grained phase-space density Q. The phase-space density expressions eq.(13) are always valid: in the primordial universe where it is constant as well as afterwards in the presence of self-gravity when structure formation occurs.

Tremaine and Gunn [38] argued that the value of the coarse grained phase space density is *always smaller* than, or equal to, the maximum value of the (fine grained) microscopic phase space density, which is the distribution function.

Such argument relies on the theorem that states that the coarse grained phase space density can only diminish by collisionless phase mixing or violent relaxation by gravitational dynamics [39].

A similar argument was presented by Dalcanton and Hogan [35], and confirmed by numerical studies.

In ref [40] Q was implemented in a model independent manner to evaluate the DM particle mass scale in non-BEC DM.

The phase-space density Q is a *constant* in the absence of self-gravity, and Q can only **decrease** by collisionless phase mixing or self-gravity dynamics [35], [38], [39], [41]. For these reasons, Q^{-1} behaves as an entropy that can only increase or stay constant during the universe expansion.

Therefore, necessarily :

$$Q_{\rm today} \leq Q \;, \quad \text{where} \quad Q_{\rm today} \equiv \frac{Q}{Z} \tag{17}$$

being $Z \geq 1$ a numerical constant, namely *the decreasing factor* Z introduced in Ref [40]). The value of $Q_{\rm today}$ can be computed with galaxy data today for ρ and $< \sigma^2 >$, namely

$$Q_{\rm today} = \frac{\rho_{\rm today}}{< \sigma^2_{\rm today} >^{3/2}} \tag{18}$$

Normally, ρ and σ^2 are averaged over the galaxy core.

$Q_{\rm today}$ has been well measured by different galaxy observations and it is galaxy dependent. $Q_{\rm today}$ is the largest for ultracompact dwarf galaxies and the smallest for large and dilute spiral galaxies [42], [43], [44]. From the compilation of different and well established sets of galaxy data, eg in Table 1 of ref. [11] we have

$$5 \times 10^{-6} \; < \; \left(\frac{Q_{\rm today}}{\rm keV^4}\right)^{2/3} \; < 1.4 \; . \tag{19}$$

Now, from eqs.(8) and (14) we express m and (T_d/T_c) in terms of Q, U and V with the result:

$$m \; = \; \frac{2^{2/3}}{3\,\pi^2} \, V \, Q^{2/3} \, \frac{T_\gamma^5}{(\rho_{DM} \; g_d)^{5/3}} \; = \; 44.62 \; {\rm keV} \, \left(\frac{Q}{\rm keV^4}\right)^{2/3} \left(\frac{25}{g_d}\right)^{5/3} V \; ,$$

$$\left(\frac{T_d}{T_c}\right)^3 \; = \; \frac{2^{2/3}}{3\,\pi^4} UV \, \frac{T_\gamma^8}{(\rho_{DM} \; g_d)^{8/3}} \, Q^{2/3} \; = \; 248.43 \, \left(\frac{Q}{\rm keV^4}\right)^{2/3} UV \left(\frac{25}{g_d}\right)^{8/3} . \tag{20}$$

$g_d \sim 25$ corresponds to DM decoupling just after the QCD phase transition as it is the case for axions.

V. DECOUPLING AT THERMAL EQUILIBRIUM

For decoupling at thermal equilibrium (TE), from eqs.(3) and (10) we have

$$U^{TE} \; = \; 2\,\zeta(3) \; = \; 2.404114, \quad V^{TE} \; = \; 24\,\zeta(5) \; = \; 24.88627$$

From these values, eq.(20) yields

$$\frac{T_d}{T_c} = 24.58709 \left(\frac{Q}{\text{keV}^4}\right)^{2/9} \left(\frac{25}{g_d}\right)^{8/9} , \tag{21}$$

$$m = 1.110 \text{ MeV} \left(\frac{Q}{\text{keV}^4}\right)^{2/3} \left(\frac{25}{g_d}\right)^{5/3} : \text{ TE} .$$

Therefore, from eqs.(21) and (19) the condition $Q \geq Q_{\rm today}$ implies

$$\frac{T_d}{T_c} \geq 27.5 \left(\frac{25}{g_d}\right)^{8/9} , \quad m \geq 1.55 \text{ MeV} \left(\frac{25}{g_d}\right)^{5/3} \quad : \text{ TE} . \tag{22}$$

We see that for $g_d \sim 25$, is always $T_d > T_c$ and hence **no BEC forms for TE decoupling**. In order to form a BEC is necessary that the decoupling temperature T_d be below the critical temperature T_c. The condition

$$T_d \leq T_c$$

yields from eq.(22)

$$g_d \geq 1040 \quad : \quad \text{BEC, TE} . \tag{23}$$

- This requires particle models possesing a **huge number** of particle states and where DM decouples presumably in the Grand Unification scale (GUT) where the number of ultrarelativistic degrees of freedom is in the hundreds (well above the electroweak scale).
- Recall that at the TeV scale in the standard model of particle physics, $g_d \sim 100$ [25]. In addition, for $g_d = 1040$ eq.(22) yields

$$m > 3.10 \text{ keV} \quad , \quad g_d = 1040 \ : \quad \text{BEC, TE} . \tag{24}$$

This particle mass value is much larger than the DM particle mass appropriate for BEC DM eq.(2) $m \sim 10^{-22}$ eV. **This is a huge difference** of orders of magnitude.

A. Implications for the Axions

- **QCD axions** can decouple well before the QCD phase transition, at temperatures $T_d \sim 10^{11}$ GeV.

For $T_d \sim 10^{11}$ GeV we can have g_d in the hundreds and from eq.(22) the axion mass m turns to be **in the keV scale**, a huge difference of orders of magnitude above the mass range values for the axion mass given by the present experimental limits [2]:

$$6 \times 10^{-6}\ \text{eV}\ <\ m_a\ <\ 2 \times 10^{-3}\ \text{eV},$$

and even a more huge difference with respect to the typically ultra-light BEC mass value eq.(2) $m \sim 10^{-22}$ eV.

Besides, it must be noticed that an isolated system which is not integrable can thermalize because is an ergodic system both classically and quantum mechanically [45]. Namely, the particle trajectories explore ergodically the constant energy manifold in phase-space, covering it uniformly according to precisely the microcanonical measure and yielding to a thermal situation [45]. This is the case for **axions** and more generally for **QCD systems**. Also, it is generally the case of self-gravitating DM particles in galaxies [46] [14], [47].

In order to determine how the thermalization happens, as well as the thermalization time scale and further physical features, specific calculations must be performed in the corresponding model, see refs. [2], [48], [49], [50], [51], [52]. The methods used in these calculations are appropriately chosen for the system considered: Boltzmann equations, classical evolution equations, Schwinger-Keldysh approach for quantum fields, expansions in 1/N in field theory and others methods.

VI. DECOUPLING OUT OF THERMAL EQUILIBRIUM

For **decoupling out of TE** we recall that typically, thermalization is reached by the mixing of the particle modes and the scattering between particles which redistribute the particles in phase space as following:

The higher momentum modes are populated by a *cascade* whose wave front moves towards the ultraviolet region akin to a direct cascade in turbulence, leaving in its wake a state of nearly local TE but with a temperature lower than that of equilibrium [48].

Hence, when the dark matter particles at decoupling are not at thermodynamical equilibrium, their momentum distribution is expected to be peaked at smaller momenta than in the TE case because the ultraviolet cascade is not yet completed [48]. Therefore, the distribution function at decoupling out of TE can be written as

$$f_d^{out\ TE}(p) = \frac{f_0}{e^{\frac{p}{\xi\, T_d}} - 1}\ \theta(p^0 - p)\ , \tag{25}$$

where $\xi = 1$ at TE and $\xi \lesssim 1$ before thermodynamical equilibrium is attained; $f_0 \sim 1$ is a normalization factor and p^0 cuts the spectrum in the ultraviolet region not yet reached by the cascade.

- The above features and the distribution function out of TE eq.(25) are generic and universal, the result is unique irrespective of the different ways the massive bosons forming a BEC can be out of TE, because the formation of a BEC is a unique process requiring one universal condition $T_d \leq T_c$.

- Eq.(25) describes out of equilibrium massive scalar particles [48]. Out of equilibrium massless scalar particles were studied in refs. [49], [50], [51].

The distribution function eq.(25) yields for U and V through eqs.(3) and (10),

$$U^{out\ TE} = f_0\ \xi^3\ U(s)\ , \quad V^{out\ TE} = f_0\ \xi^5\ V(s)\ , \quad s \equiv \frac{p^0}{\xi\ T_d}\ ,$$

$$U(s) \equiv \int_0^s \frac{y^2\ dy}{e^y - 1} \quad , \quad V(s) \equiv \int_0^s \frac{y^4\ dy}{e^y - 1}\ .$$

Then, from eq.(20) the condition $Q \geq Q_{\rm today}$ implies for **decoupling out of TE**:

$$m\ \geq\ 44.62\ {\rm keV} \left(\frac{Q_{\rm today}}{{\rm keV}^4}\right)^{2/3} \left(\frac{25}{g_d}\right)^{5/3} V^{out\ TE} \tag{26}$$

$$m\ \geq\ 1.110\ {\rm MeV} \left(\frac{Q_{\rm today}}{{\rm keV}^4}\right)^{2/3} \left(\frac{25}{g_d}\right)^{5/3} f_0\ \xi^5\ \frac{V(s)}{V(\infty)} \quad ,$$

$$\left(\frac{T_d}{T_c}\right)^3 \geq 1.48635\ 10^4 \left(\frac{Q_{\rm today}}{{\rm keV}^4}\right)^{2/3} f_0^2\ \xi^8\ \frac{U(s)\ V(s)}{U(\infty)\ V(\infty)} : \quad \text{out of TE}\ . \tag{27}$$

Typically, out of TE we have

$$s = \mathcal{O}(1)\ , \quad f_0 = \mathcal{O}(1)\ , \quad U(1)/U(\infty) = 0.147,$$

$$V(1)/V(\infty) = 0.00658$$

From the bound eq.(26), the limiting condition $T_d \sim T_c$ for the presence of a BEC is satisfied for $s \sim 1,\ \xi \sim 0.7,\ f_0 \sim 1$. As a consequence, in the BEC limiting case of **decoupling out of TE**, we find:

$$T_d \sim T_c\ ,\ f_0\ \xi^5\ \frac{V(s)}{V(\infty)} \sim 10^{-3}\ , \quad m \geq 14\ {\rm eV} \left(\frac{25}{g_d}\right)^{\frac{5}{3}} : \quad \text{BEC out of TE}\ . \tag{28}$$

We conclude that:

- BEC DM decoupling at thermal equilibrium requires a particle model with a **huge number** $g_d \geq 1040$ of particle states ultrarelativistic at DM decoupling [eq.(23)]. For $g_d = 1040$ the particle mass must be $m > 3$ keV [eq.(24)], that is, **twenty-five orders of magnitude larger** than the appropriate BEC mass value eq.(2).

- BEC DM decoupling out of thermal equilibrium requires for $g_d \sim 25$ a particle mass m of at least 0.03 eV [eq.(28)]. For $T_d \sim 10^{11}$ GeV, g_d is in the hundreds and we obtain from eq.(28) m at least **twenty orders of magnitude larger** than the appropriate BEC mass value eq.(2).

A. Implications for the BEC Jeans Lengths

From eq.(1) the Jeans lengths corresponding to the above cases eqs.(24) and (28) are:

$$\lambda_J(\text{keV}) = 3.8 \times 10^4 \text{ km} \quad \text{for TE} ,$$

$$\lambda_J(10 \text{ eV}) = 6.9 \times 10^6 \text{ km} \quad \text{for out of TE} .$$

- These BEC Jeans-length values are **unrealistically small** by eleven to thirteen orders of magnitude [see eq.(1)] in order to form the observed galaxy structures. Namely, DM structures of **all** sizes above these minuscule Jeans lengths will be formed in contradiction with astronomical observations. These Jeans length values are even worse than the cold DM Jeans length which is $\sim 3 \times 10^{12}$ km.

- Therefore, the BEC particle masses compatible with the DM average density and the DM galaxy phase-space density constraints, namely:

  $$m \; > \; 3 \, \text{keV} \quad (\text{in TE}) \quad \text{and} \quad m \; > \; 0.03 \, \text{eV} \quad (\text{out of TE}),$$

  have **exceedingly small** Jeans lengths, results which **strongly disfavour BEC DM**.

B. Implications for the BEC number of ultra-relativistic degrees of freedom

It is interesting to see which value of g_d corresponds to the particle mass value $m \sim 10^{-22}$ eV appropriate for galaxy structure formation. We find from eqs.(2), (22) and (28) that g_d must take the values

$$g_d \sim 2 \times 10^{11} \quad \text{TE} \quad ; \quad g_d \sim 2 \times 10^{14} \quad \text{out of TE} .$$

- These gigantic values of g_d are totally impossible for decoupling in the radiation dominated era. Namely, these values of degrees of freedom are absolutely unrealistic for whatever particle physical model one considers. Hence, there is no way to realize a tiny DM mass $m \sim 10^{-22}$ eV.

In the case DM stays ultra-relativistic till today, eq.(5) for the DM density becomes

$$\rho = \frac{T_c^4}{2\,\pi^2}\, W \quad , \quad W \equiv \int_0^\infty y^3 \; f_d(y)\; dy \tag{29}$$

This equation is valid both in the BEC case and in the absence of a BEC.

For out of thermal equilibrium decoupling we have from eq.(25) that

$$W^{\;out\;TE} \; < \; W^{TE} \; = \; \pi^4/15$$

We thus obtain from eqs.(6) and (29),

$$g_d^{TE} \; = \; 0.4443 \; > \; g_d^{out\;TE} \tag{30}$$

- Equation (30) **cannot** be satisfied because it must always be $g_d \geq 2$ due to the existence of the photon. Therefore, **scalar particles which are ultra-relativistic today cannot describe the DM**.

- The treatment we presented here is **independent** of the particle physics model describing the DM particle and applies to all DM BEC. All the results found here only follow from the gravitational interaction of the particles, their bosonic nature and the robust DM observational constraints from the average DM density ρ_{DM} and the DM phase-space density Q.

VII. GRAVITATIONALLY BOUNDED BOSE-EINSTEIN CONDENSATES OF FINITE SIZE

So far, we have here considered homogeneous Bose Einstein condensate (BEC) i.e. a BEC in all the space. Gravitationally bounded BEC's with a finite size R can also exist. That is to say, a two phase situation in which the BEC is inside the radius R and the normal phase is outside. This gravitationally bounded BEC can be considered as the final stationary state, dynamically produced by a gravitational BEC phase transition.

The results provided by this gravitational BEC study are robust irrespective of any particular particle physics model. A gravitationally bounded object formed by a BEC can be obtained by equating the bosonic quantum pressure and the gravitational pressure.

The BEC quantum pressure is the flux of the quantum momentum, $P_Q = n\ v\ p$, p being the minimum momentum from the Heinsenberg principle $p \sim \hbar/R$ and $n = \rho/m$ is the number density of particles. Therefore,

$$P_Q = \rho \left(\frac{\hbar}{R\ m}\right)^2 = \frac{3}{4\ \pi}\ \frac{\hbar^2\ M}{R^5\ m^2}\ , \tag{31}$$

where $\rho = (3\ M)/(4\ \pi\ R^3)$ and M is the mass of the BEC.

For an object of radius R and mass M the gravitational pressure is

$$P_G = \frac{G\ M^2}{4\ \pi\ R^4}\ . \tag{32}$$

Thus, $P_Q = P_G$ implies for the BEC size R,

$$R = \frac{3}{G\ M}\ \left(\frac{\hbar}{m}\right)^2 = 2.861\ 10^{-36}\ \frac{M_\odot}{M}\ \left(\frac{\text{eV}}{m}\right)^2\ \hbar\ \text{kpc}\ . \tag{33}$$

The DM particle mass becomes from eq.(33)

$$m = 5.349\ 10^{-22}\ \text{eV}\ \sqrt{\frac{10^7\ M_\odot}{M}\ \frac{\text{kpc}}{R}}\ . \tag{34}$$

That is, for a typical compact BEC object of kpc size and mass $M \sim 10^7\ M_\odot$ we obtain the BEC DM particle mass in agreement with eq.(2). This remarkable result shows the consistency of the self-gravitating BEC quantum estimate eq.(31)-(33) with the BEC free-streaming length eq.(2).

The phase space density $Q = \rho/\sigma^3$ can be estimated following similar lines as above, namely

$$Q = \sqrt{27}\ \rho\ \left(\frac{m\ R}{\hbar}\right)^3 = \frac{\sqrt{27}}{4\ \pi}\ \left(\frac{m}{\hbar}\right)^3\ M\ , \tag{35}$$

with the result

$$\frac{Q}{\text{keV}^4} = 0.461\ 10^{-68}\ \left(\frac{m}{10^{-22}\ \text{eV}}\right)^3\ \left(\frac{M}{10^7\ M_\odot}\right)\ . \tag{36}$$

- BEC objects would correspond to compact halos ie typically M about $10^7\ M_\odot$, thus $Q \sim 10^{-68}$ for the typical $m \sim 10^{-22}$ eV. That is, Q turns out **more than sixty orders of magnitude smaller** than the observed values eq.(19).

- Although $m \sim 10^{-22}$ eV provides reasonable BEC free-streaming lengths [eq.(2)], the corresponding BEC phase-space density turns to be **ridiculously small**.

  Notice that the value eq.(35) is a **maximal** value for Q as evaluated from the minimum saturated quantum value of the momentum using the Heisenberg principle. That is to say, eq.(36) is a *robust* result. In conclusion, a gravitationnally bounded BEC **cannot be the DM**.

VIII. THERMAL AND NON-THERMAL AXIONS

The main DM candidate for a scalar particle forming a BEC condensate is the axion [26, 53]. For a report on axionic DM and axion like particles ('ALPs') see for example [54].

In the usual scenario of DM axions, axions decouple soon after the QCD phase transition ($g_d \sim 25$) and then they are assumed (i) to become nonrelativistic, (ii) to thermalize and (iii) to form a BEC [2]. (Ref. [52] recently criticized this scenario). Hence, the bound eq.(22) clearly shows that **no DM axion-like BEC can be formed.**

For non-thermal axions, the canonical scenario is the axion vacuum misalignment [25–27] in which case the average axion DM density is given by eq.(10.49) in [25]

$$\rho_{DM} = \rho_{crit} \; 0.13 \times 10^{\pm 0.4} \; \Lambda_{200}^{-0.7} \; F(\bar{\theta}_1) \; \bar{\theta}_1^2 \; \left(\frac{m}{10^{-5} \text{ eV}}\right)^{-1.18} \tag{37}$$

where $\Lambda_{200} \equiv \Lambda_{QCD}/200$ MeV, θ_1 is the value of the axion phase field when axion oscillations begin. Its canonical value is $\theta_1 = \pi/\sqrt{3}$. $F(\bar{\theta}_1)$ accounts for anharmonic effects, $F(\pi/\sqrt{3}) \simeq 1.3$.

$\Omega_{DM} = 0.22$ in Eq.(37) yields for the axion mass:

$$m = 3.77 \; 10^{-5} \; J^{-0.85} \text{ eV} \quad , \quad J \equiv \frac{3 \; \bar{\theta}_1^2 \; 10^{\pm 0.4} \; F(\bar{\theta}_1)}{\pi^2 \; \Lambda_{200}^{-0.7} \; 1.3} \sim 1 \; . \tag{38}$$

Matching eqs.(7)-(8) with eqs.(37)-(38) yields:

$$U \left(\frac{T_c}{T_d}\right)^3 = 4.681 \; 10^6 \; \frac{g_d}{25} \; J^{0.85} \; . \tag{39}$$

Eq. (39) and comparison with the thermal case $U^{TE} = 2.404\ldots$ suggest that axions in the vacuum misalignment scenario are strongly out of equilibrium.

Moreover, we can also estimate the phase-space density Q using the DM density and DM velocity estimates from [25] with the result

$$\frac{Q}{\text{keV}^4} = 2.253 \times 10^8 \, \Lambda_{200}^{2.59} \left(\frac{g_d}{25}\right)^{0.648} \left(\frac{\text{eV}}{m}\right)^{1.295} \tag{40}$$

The constraint $Q_{\text{today}} \leq Q$ yields the following upper bound on the axion mass which is easy to fulfil:

$$m \ \leq \ 2.818 \, \text{MeV} \ \ \Lambda_{200}^2 \sqrt{\frac{g_d}{25}} \left(\frac{\text{keV}^4}{Q_{\text{today}}}\right)^{0.772} \tag{41}$$

We see from eqs.(2) and (38) that the axion mass in the vacuum misalignment scenario is **17 orders of magnitude too large** to reproduce the observed galactic structures.

IX. CONCLUSIONS

Present experimental limits leave as available window for the axion mass [2]

$$6 \times 10^{-6} \, \text{eV} \ < \ m_a \ < \ 2 \times 10^{-3} \, \text{eV} \tag{42}$$

- The window eq.(42) **disagrees** by many orders of magnitude both with the galaxy phase-space density constraint eq.(22) and with the Jeans length constraint eq.(1) in order for the axion to be DM.

- The existence of the axion particle is well motivated from QCD [26, 53]. But, as we have seen, the axion **c**annot be the DM particle. The two observables: the average DM density ρ_{DM} and the phase space density Q robustly constrain in an inescapable way both: the possibility to form a BEC eg T_d/T_c and the DM particle mass m ruling out BEC DM in general, and the BEC axion DM in particular.

- Moreover, the value $m \sim 10^{-22}$ eV can only be obtained with a number of ultrarelativistic degrees of freedom at decoupling **in the trillions** which is impossible for decoupling in the radiation dominated era.

- In addition, we have also considered inhomogenous gravitationally bounded BEC's supported by the bosonic quantum pressure independently of any particular particle physics scenario. For a typical size $R \sim$ kpc and compact object masses $M \sim 10^7 \, M_\odot$ they remarkably lead to the same particle mass $m \sim 10^{-22}$ eV as the BEC free-streaming length. However, the phase-space density for the gravitationally bounded BEC's turns out to be **more than sixty orders of magnitude smaller** than the galaxy observed values.

- We have provided here a generic treatment, independent of the particle physics model and which applies to all DM BEC, in both : in or out of equilibrium situations. We conclude that the BEC cannot be the total DM. The axion can be candidate to be only part of the DM of the universe.

- In all the DM BEC discussion here it is assumed that axions represent the whole DM in the universe, as is usually the case to investigate the feasibility of a DM candidate. In mixed scenarios where particles other than axions could form a large part of the DM, one could have an axion DM BEC constituing a part of the universe DM.

- In supersymmetric models the supersymmetric partner of the axion is a fermion called axino, degenerate in mass with the axion. An **axino** with mass **in the keV scale** would be a good warm dark matter (WDM) candidate. Actually, an axion (and hence an axino) with particle mass in the keV scale naturally appeared for a decoupling temperature $T_d \sim 10^{11}$ GeV [see eqs.(22)-(24)].

- We would like to stress that although not being the DM, the axion may play a crucial role in cosmology. The observed dark energy density $\rho_\Lambda = (2.35 \text{ meV})^4$ indicates an energy scale in the meV $= 10^{-3}$ eV. This energy value is in the allowed window of axion masses. Therefore, the axion may be the source of the dark energy through the zero point cosmological quantum fluctuations as we derived in ref. [28]. In addition, white dwarf stars observations would suggest axions in the range of 2-8 meV [29], [30].

- Overall, a robust conclusion of this paper is that the BEC in general, and the BEC axion in particular, cannot be the total Dark Matter of the Universe. However, they can play an important role in astrophysics and cosmology. We see indications for an axion mass in the meV range from dwarf stars observations eg [29],[30], and mainly from the dark energy scale as we studied in ref [28]. In addition, the misalignment scenario [25], [26], [27] may be able to produce axions with mass in the meV range.

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