Warm Dark Matter Galaxies with Central Supermassive Black Holes

Universe 2022, keV Warm Dark Matter Special Issue in Tribute to Hector de Vega

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https://www.mdpi.com/journal/universe/special_issues/kWDM

Norma G Sanchez, SISSA-Trieste- IFPU Workshop

The Nature of Dark Matter, 16 June 2022

WDM properties

WDM is characterized by

- its initial power spectrum cutted off for scales below ~ 50 kpc. Thus, structures are not formed in WDM for scales below ~ 50 kpc.
- its initial velocity dispersion. However, this is negligible for $z < 20$ where the non-linear regime starts.
- Classical N-body simulations break down at small distances ($\sim$ pc). Need of quantum calculations to find WDM cores.

Structure formation is hierarchical in CDM.

WDM simulations show in addition top-hat structure formation at large scales and low densities but hierarchical structure formation remains dominant.

WDM free streaming scale

The scale $l_{1/2}$ is where the WDM power spectrum is one-half of the CDM power spectrum:

$$l_{1/2} = 1/k_{1/2} = 207 \text{ kpc } \left(\text{keV}/m_{FD}\right)^{1.12}$$

This scale reproduces the sizes of the observed DM galaxy cores when the WDM mass is in the keV scale !!

$l_{1/2}$ is similar but more precise than the free streaming scale (or Jeans' scale):

$$r_{Jeans} = 210 \, \text{kpc} \, \frac{\text{keV}}{m_{FD}} \, \left(\frac{100}{g_d}\right)^{\frac{1}{3}} \; ,$$

$g_d =$ number of UR degrees of freedom at decoupling.

Summary Warm Dark Matter, WDM: $m \sim$ keV

- Large Scales, structures beyond ~ 100 kpc: WDM and CDM yield identical results which agree with observations
- Intermediate Scales: WDM simulations give the correct abundance of substructures.
- Inside galaxy cores, below ~ 100 pc: N-body classical physics simulations are incorrect for WDM because of important quantum effects.
- Quantum calculations (Thomas-Fermi) give galaxy cores, galaxy masses, velocity dispersions and densities in agreement with the observations.
- Direct Detection of the main WDM candidate: the sterile neutrino. Beta decay and electron capture. ^{3}H, Re, Ho.

So far, not a single valid objection arose against WDM.
Baryons (<16%DM) expected to give a correction to WDM

UPDATE and CLARIFICATIONS

→ΛCDM agrees with CMB + LSS **BUT**
ΛCDM **DOES NOT agree with SSS (GALAXIES)**

→ΛWDM agrees with CMB + LSS + SSS (GALAXIES)
The Standard Model of the Universe is LWDM =
{GR, Newtonian Gravity, Field Theory, QFT}

Sentences like « CMB confirms the ΛCDM model … »
Must be completed by adding: « in the large scales" »
and must be updated with the sentence:
→ **CMB confirms the ΛWDM model in large scales**

NEW: Gravity and Quantum Mechanics in Galaxies. Newton, Fermi and Dirac meet together in Galaxies because of keV WDM

DARK MATTER UPDATE

- **THERE IS NO CUSP/CORE problem:**
- **Observed Galaxy density profiles are cored.**
- **WDM Galaxy density profiles are cored**

- **THERE IS NO satellite problem**
- **WDM abundance of structures agrees with observations**
- **In addition, these are not fundamental problems.**

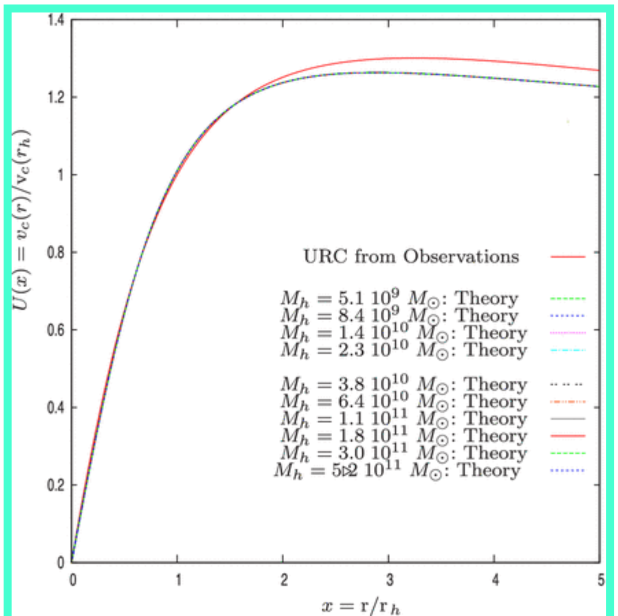
NO CDM Wimps, NO DM annhilation,

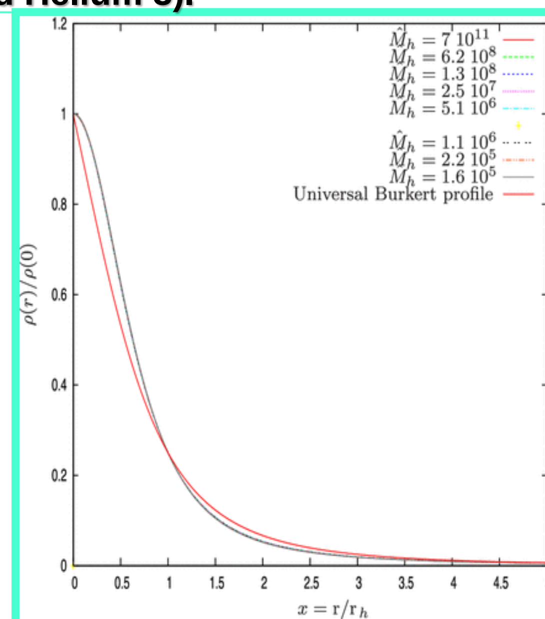
The Total DM cannot be bosons (Axions)

Newton, Fermi and Dirac, meet together in Galaxies through keV Warm Dark Matter

Rotation curves (left panel): The theoretical curves for 10 different galaxy masses all fall one into each other providing an Universal Rotation Curve (URC) which remarkably coincides with the observed universal curve (displayed in red) . Small deviations show up only at distances outside twice the *radius.*

The right panel the density profiles for the 10 galaxy masses: All fall into the same and universal density profile which reproduces the observed universal density profile and its size (in red). Interestingly enough, small deviations show up for compact dwarf galaxies as a manifestation of the quantum macroscopic effects predicted in these galaxies, and which can be further tested observations. (Examples of other macroscopic objects in nature are dwarf stars, neutron stars and the liquid Helium 3).





Universal rotation curves and Universal density profiles: The same for all large galaxies

The theoretically obtained galaxy rotation curves and density profiles reproduce extremely well the observational curves from
ten different and independent sets of data for galaxy
Masses from 5×10^{9} Msun till 5×10^{11} Msun.

Remarkably enough, the normalized circular velocities and density profiles are universal (URC): they are the same for all galaxies of different types, sizes and masses, and agree extremely well with the observational curves described by cored profiles (flat smooth profiles at the center) and their sizes.

Interestingly enough, small deviations from the exact scaling relations show up for compact dwarf galaxies as a manifestation of the quantum macroscopic effects present in these galaxies.

Robust Results

independent of any particular WDM particle physics model, they only follow from the self-gravitation of the WDM particles and their fermionic nature. Ability of this approach to describe the galaxy structures. Baryonic corrections are not very important to WDM, consistent with the fact that DM is in average at least six times more abundant than baryons.

Reference:

H.J. de Vega; P. Salucci; N. G.Sanchez MNRAS 442 (2): 2717 (2014)

For self-gravitating systems, the potential $\mu(r)$ is proportional to the gravitational potential $\phi(r)$: $\mu(r) = \mu 0 - m \ \phi(r)$, $\mu 0$ being a constant, and obeys the **self-consistent** and **non -linear Poisson equation**

$$\nabla^2 \mu(r) = - 4\pi g \ G \ m^2 \int d^3p \ (2\pi/\hbar)^3 \ f \left(p^2 / 2m - \mu(r) \right)$$

We extend **The boundary condition** of the chemical or gravitational potential at the center ***to allow for the presence of center the central black hole***, namely:

$$\mu(r)_{\ r\to 0} = \frac{G \ m \ M_{BH}}{r} + \text{Const.} + O(r).$$

- $\mu(r) > 0$ → self-gravitating quantum gas regime
- $\mu(r) < 0$ → self-gravitating classical (Boltzmann) regime .
- $\mu(r) = 0$ → transition from quantum to classical regime
- *Results : In Galaxies possessing central black holes,*
- *both regimes do appear.*
- The strong gravitational field of the central BH → WDM $\mu(r)$ large and > 0 near the center. → WDM behaves quantum mechanically inside a small quantum core with a nearly constant density.
- *rA : transition radius : $\mu(r_A) = 0$, rA >> r schw;*

r < rA quantum core gets formed , rho >> 0 high, exhibits a constant plateau.

- r > rA: rho drops several orders , WDM classical Boltzmann

ri : BH influence radius: Force BH (r = ri) = Force WDM (r=ri)

The total gravitational potential V(r) and its derivative V'(r) are :

$$V(r) = -GM_{BH} / r + \varphi(r), \qquad V'(r) = GM_{BH} / r^2 - \mu'(r) / m$$

ri : solution : $r_i^2 \, \mu'(r_i) = G \, m \, M_{BH}$

Core size rh of the halo: $\rho(r_h) / \rho_0 = 1/4$, $r_h = l_0 \xi_h$.

Surface density : $\Sigma_0 \equiv r_h \rho_0$,

Σ_0 **:** ***nearly Constant*****,** ***indep.of luminosity in diff. gal. systems*** (spirals, dwarfs irregular and spheroidal ellipticals) **over 14 order of magnitudes in lum. and over different Hubble types.**

All galaxies: $\Sigma_0 \simeq$ **120 M⊙ / pc^2** **up to 10 - 20%**

Remarkable: Σ_0 **is the only dimensionful quantity practically constant among the different galaxies.**

Other quantities: r_h, ρ_0, **baryon fraction, galaxy mass M vary by orders of magnitude from one galaxy to another.**

$\Sigma_0 \simeq$ **Const. → physical scaling relation between the M and halo size** r_h **.**

→ Σ_0 **: useful physical scale to express the galaxy**

$$M(r) + M_{BH} = 4\pi \left(\frac{9\,\pi\,\hbar^6}{2^9\,G^3\,m^8} \right)^{\frac{2}{5}} \left[\frac{\Sigma_0}{\xi_h\,I_2(\nu_0)} \right]^{\frac{3}{5}} \xi^2\,|\nu'(\xi)|\,,$$

$$M(r) + M_{BH} = \frac{27312\,\xi^2}{[\xi_h\,I_2(\nu_0)]^{\frac{3}{5}}}\,|\nu'(\xi)|\,\left(\frac{2\,\mathrm{keV}}{m} \right)^{\frac{16}{5}} \left(\frac{\Sigma_0\,\mathrm{pc}^2}{120\,M_\odot} \right)^{\frac{3}{5}} M_\odot$$

$$\sigma^2(r) = \frac{1}{3}\,v^2(r) = \frac{11.0402}{[\xi_h\,I_2(\nu_0)]^{\frac{4}{5}}}\,\frac{I_4(\nu(\xi))}{I_2(\nu(\xi))} \left(\frac{2\,\mathrm{keV}}{m} \right)^{\frac{8}{5}} \left(\frac{\Sigma_0\,\mathrm{pc}^2}{120\,M_\odot} \right)^{\frac{4}{5}} \left(\frac{\mathrm{km}}{\mathrm{s}} \right)^2\,,$$

$$P(r) = \frac{8\,\pi}{5}\,G \left[\frac{\Sigma_0}{\xi_h\,I_2(\nu_0)} \right]^2 I_4(\nu(\xi))\,,$$

$$P(r) = \frac{200.895}{[\xi_h\,I_2(\nu_0)]^2}\,I_4(\nu(\xi)) \left(\frac{\Sigma_0\,\mathrm{pc}^2}{120\,M_\odot} \right)^2 \frac{M_\odot}{\mathrm{pc}^3} \left(\frac{\mathrm{km}}{\mathrm{s}} \right)^2\,.$$

That is, $M(r) + M_{BH}$ is the total mass inside a sphere of radius *r* including the mass
tral black hole.

Notice that both $M(r)$ and M_{BH} at fixed Σ_0 *do scale* with the WDM particle mass as $m^{-\frac{16}{5}}$.
In particular, the halo galaxy mass M_h follows from Equation (50) at $r = r_h$:

$$M_h \equiv M(r_h) + M_{BH} = \frac{27312\, \xi_h^{\frac{7}{5}}}{[I_2(\nu_0)]^{\frac{3}{5}}} \, |\nu'(\xi_h)| \left(\frac{2\,\mathrm{keV}}{m}\right)^{\frac{16}{5}} \left(\frac{\Sigma_0\,\mathrm{pc}^2}{120\,M_\odot}\right)^{\frac{3}{5}} M_\odot \,.$$

The phase–space density $Q(r)$ follows from Equations (48) and (51) as

$$Q(r) \equiv \frac{\rho(r)}{\sigma^3(r)} = 3\sqrt{3}\,\frac{\rho(r)}{<v^2>^{\frac{3}{2}}(r)} = \frac{\sqrt{125}}{3\,\pi^2}\,\frac{m^4}{\hbar^3}\,\frac{I_2^{\frac{5}{2}}(\nu(\xi))}{I_4^{\frac{3}{2}}(\nu(\xi))} \,.$$

e can express the black hole mass as

$$M_{BH} = 2.73116 \times 10^4 \, M_\odot \, \frac{\xi_0}{\left[\xi_h \, I_2(\nu_0)\right]^{\frac{3}{5}}} \left(\frac{\Sigma_0 \, \mathrm{pc}^2}{120 \, M_\odot}\right)^{\frac{3}{5}} \left(\frac{2 \, \mathrm{keV}}{m}\right)^{\frac{16}{5}}$$

ensionless central radius and $I_2(\nu_0)$ the 2nd momentum of the distribution function Equatior

en ξ_0 grows. Notice that M_{BH} does not simply grow linearly with ξ_0 due to the presence of the

- small size (mass) galaxies,

- intermediate size (mass) galaxies,

- and large size (mass) ,

Two characteristic parameters: the dimensionless central radius ξ0 and the constant *A* characteristic of the chemical

e derive an illuminating expression for the central radius r_0 for $M_h \gtrsim 10^6\ M_\odot$ explicitly in terms of the black hole mass M_{BH}, the halo ma[ss] [and ref]erence surface density Σ_0. It follows from Equations (41), (43) and (66) that

$$r_0 = l_0\, \xi_0 = 126.762 \sqrt{\frac{10^6\ M_\odot}{M_h}}\ \frac{M_{BH}}{10^6\ M_\odot} \sqrt{\frac{120\ M_\odot}{\Sigma_0\ \mathrm{pc}^2}}\ \mathrm{pc}$$

$$\xi_0 \equiv \frac{G\, m\, M_{BH}}{T_0\, l_0} = \frac{r_0}{l_0}\ .$$

We find from our extensive numerical calculations that the halo is thermalized at the uniform temperature T_0 and matches the circular temperature $T_c(r)$ by $r \sim 3\, r_h$. This picture is similar to the picture found in the absence of the central black hole which follows from the observed density profiles in the Eddington-like approach to galaxies [13]. We obtain here in the Thomas–Fermi approach and in the presence of a central supermassive black hole that the halo is thermalized at a uniform temperature T_0 inside $r \lesssim 3\, r_h$, which tends to the circular temperature $T_c(r)$ at $r \sim 3\, r_h$ as illustrated in **Figure 4**. The circular temperature is defined in terms of the circular velocity as $T_c(r) = \frac{m}{2}\, v_c^2(r)$. The circular temperature is

$$l_0 = \left(\frac{9\,\pi}{2^9}\right)^{\frac{1}{5}} \left(\frac{\hbar^6}{G^3\,m^8}\right)^{\frac{1}{5}} \left[\frac{\xi_h\,I_2\,(\nu_0)}{\Sigma_0}\right]^{\frac{1}{5}} ,$$

$$l_0 = \; 4.2557\,[\xi_h\,I_2\,(\nu_0)]^{\frac{1}{5}} \left(\frac{2\,\mathrm{keV}}{m}\right)^{\frac{8}{5}} \left(\frac{120\,M_\odot}{\Sigma_0\,\mathrm{pc}^2}\right)^{\frac{1}{5}} \;\mathrm{pc}$$

$$T_0 = \left(18\,\pi^6\,\frac{\hbar^6\,G^2}{m^3}\right)^{\frac{1}{5}} \left[\frac{\Sigma_0}{\xi_h\,I_2\,(\nu_0)}\right]^{\frac{4}{5}} ,$$

$$T_0 = \; \frac{7.12757\,10^{-3}}{[\xi_h\,I_2\,(\nu_0)]^{\frac{4}{5}}} \left(\frac{2\,\mathrm{keV}}{m}\right)^{\frac{3}{5}} \left(\frac{\Sigma_0\,\mathrm{pc}^2}{120\,M_\odot}\right)^{\frac{4}{5}} \mathrm{K}\,.$$

SOLUTIONS for the GALAXY – Central SMBH SYSTEM

Small size galaxy :

$$r_i = 221 \text{ pc} \quad , \quad r_h = 452 \text{ pc} \quad , \quad T_0 = 0.0978 \text{ K},$$
$$\sqrt{<v^2>}(r \gtrsim r_A) = 35.48 \text{ km/s}, \qquad \sqrt{<v^2>}(r \lesssim r_A) = 383.75 \text{ km/s},$$
$$M_h = 7.678 \times 10^7 \, M_\odot, \quad M_{vir} = 8.582 \times 10^8 \, M_\odot,$$
$$M_{BH} = 1.947 \times 10^5 \, M_\odot, \quad r_{BH}^{Schw} = 1.863 \times 10^{-8} \text{ pc},$$
$$\rho_0 = 1.797 \times 10^{-23} \text{ g/cm}^3,$$
$$\rho_A = 0.9878 \times 10^{-19} \text{ g/cm}^3, \quad M_A = 8.767 \times 10^4 \, M_\odot, \quad r_A = 1.91 \text{ pc}.$$

Medium size galaxy :

$$r_i = 54.3 \text{ pc} \quad , \quad r_h = 210 \text{ kpc} \quad , \quad T_0 = 26.97 \text{ K},$$
$$\sqrt{<v^2>}(r \gtrsim r_A) = 559.8 \text{ km/s}, \qquad \sqrt{<v^2>}(r \lesssim r_A) = 6370.9 \text{ km/s},$$
$$M_h = 9.022 \times 10^{12} \, M_\odot, \qquad M_{vir} = 8.222 \times 10^{13} \, M_\odot,$$
$$M_{BH} = 9.224 \times 10^7 \, M_\odot, \qquad r_{BH}^{Schw} = 8.828 \times 10^{-6} \text{ pc},$$
$$\rho_0 = 3.867 \times 10^{-26} \text{ g/cm}^3,$$
$$\rho_A = 7.182 \times 10^{-15} \text{ g/cm}^3, \quad M_A = 1.932 \times 10^7 \, M_\odot, \quad r_A = 0.2 \text{ pc}.$$

Large size galaxy :

$$r_i = 21.66 \text{ pc}, \quad r_h = 8.237 \times 10^3 \text{ kpc} \quad , \quad T_0 = 1061 \text{ K},$$
$$\sqrt{<v^2>}(r \gtrsim r_A) = 3511.2 \text{ km/s}, \qquad \sqrt{<v^2>}(r \lesssim r_A) = 39591 \text{ km/s},$$
$$M_h = 1.3753 \times 10^{16} \, M_\odot, \qquad M_{vir} = 3.3482 \times 10^{16} \, M_\odot,$$
$$M_{BH} = 1.8632 \times 10^9 \, M_\odot, \qquad r_{BH}^{Schw} = 1.783 \times 10^{-4} \text{ pc},$$
$$\rho_0 = 0.9860 \times 10^{-27} \text{ g/cm}^3,$$
$$\rho_A = 2.9163 \times 10^{-12} \text{ g/cm}^3, \quad M_A = 3.873 \times 10^8 \, M_\odot, \quad r_A = 0.074 \text{ pc}.$$

$$M_h^{min} = 6.892 \times 10^7 \left(\frac{2\,\mathrm{keV}}{m}\right)^{\frac{16}{5}} \left(\frac{\Sigma_0\,\mathrm{pc}^2}{120\,M_\odot}\right)^{\frac{3}{5}} M_\odot \text{ , with central black hole.}$$

$$M_h^{min} = 3.0999 \times 10^4 \left(\frac{2\,\mathrm{keV}}{m}\right)^{\frac{16}{5}} \left(\frac{\Sigma_0\,\mathrm{pc}^2}{120\,M_\odot}\right)^{\frac{3}{5}} M_\odot \text{ , } T_0^{min} = 0 \text{ , without central black ho}$$

Mh min with BH = 2.2233 × 10^3 times Mh min without BH

Galaxies with a central BH are in the classical dilute Boltzmann regime: large mass: Mh > 10^6 M☉ > Mh min.

On the contrary, compact galaxies, eg, ultracompact galaxies in the quantum regime Mh < 2.3×10^6 M☉ ,

cannot harbor central BH :

Mh min with BH is always > 2.3 × 10^6 M☉.

→***Ultra compact dwarfs* Mh < Mminh,**

***necessarily do not possess central* BH**

The dimensionless quantum radius of the galaxy ξ_0 Equation (**28**) can be expressed as

$$\xi_0 = \left(\frac{2^8}{3^4\,\pi^7}\right)^{\frac{1}{5}} \left[\frac{\xi_h\, I_2\left(\nu_0\right)}{\Sigma_0}\right]^{\frac{3}{5}} G^{\frac{6}{5}}\, m^{\frac{16}{5}}\, M_{BH}\ ,$$

$$\xi_0 = 36.6145 \left[\xi_h\, I_2\left(\nu_0\right)\, \frac{120\ M_\odot}{\Sigma_0\ \mathrm{pc}^2}\right]^{\frac{3}{5}} \left(\frac{m}{2\ \mathrm{keV}}\right)^{\frac{16}{5}} \frac{M_{BH}}{10^6\ M_\odot}\ .$$

$$\rho\left(r\right) = \left(\frac{2^9\, G^3\, m^8}{9\,\pi\,\hbar^6}\right)^{\frac{1}{5}} \left[\frac{\Sigma_0}{\xi_h\, I_2\left(\nu_0\right)}\right]^{\frac{6}{5}} I_2\left(\nu\left(\xi\right)\right),$$

$$\rho\left(r\right) = 18.1967\ ;\ \frac{I_2\left(\nu\left(\xi\right)\right)}{\left[\xi_h\, I_2\left(\nu_0\right)\right]^{\frac{6}{5}}} \left(\frac{m}{2\ \mathrm{keV}}\right)^{\frac{8}{5}} \left(\frac{\Sigma_0\ \mathrm{pc}^2}{120\ M_\odot}\right)^{\frac{6}{5}} \frac{M_\odot}{\mathrm{pc}^3}$$

UNIVERSAL SCALING RELATIONS
IN THE PRESENCE OF CENTRAL SMBH

At fixed ξ_0, the BH mass M_{BH} scales with the halo mass M_h as

$$M_{BH} = D(\xi_0)\, M_h^{3/8},$$

where $D(\xi_0)$ is an increasing function of ξ_0.

The halo mass M_h grows when T_0 increases.

→ *Colder galaxies are smaller.*

Warmer galaxies are larger.

- M_h *accurately scales as* r_h*^2:*

- $M_h = 1.75\ \Sigma_0\ r_h^2$ without BH

$M_h = b\ \Sigma_0\ r_h^2$ with BH

- The proportionality factor in this scaling relation is

$1.6 \leq b \leq 4$

- *b changes from 1/2 up to 2 while Mh varies by ten orders of magnitude.*

- *b is independent of the precise value of the WDM particle mass m,* because the scaling relation emerges in the classical Boltzman regime of the galaxy ($M_h \gtrsim 10^6 M_\odot$).

In summary, the scaling relation and the coefficient b turn out to be remarkably robust.

T0 increases when the absolute value of A increases at fixed $\xi 0 > 0$

Without SMBH, the galaxy temperature T0 tends to zero for $A \to \infty$

(the exact Fermi degenerate state),

With SMBH, we find that T0 is always =/ 0 >T0 min

is always larger than the non-zero minimal value:

$T0\ min = 0.069\ (2keV/\ m)^{3/5}\ (\Sigma 0\ pc^2/\ 120\ M_\odot)^{4/5}\ K$

The SMBH heats up the DM gas and prevents it from becoming exactly degenerate at zero temperature. The Mmin and size and most compact galaxy state with a SMBH is a nearly degenerate state at very low temperature T0 min.

No MBH min mass emerges

QUANTUM PHYSICS IN GALAXIES

The de Broglie wavelength of DM particles in a galaxy :

$$\lambda dB(r) = h / m\, v(r)$$

the average interparticle distance d at r can be estimated as

$$d(r) = (\, m\rho(r)\,)^{1/3}$$

$\rho(r)$ is the local density in the galaxy core.

We can measure the classical or quantum character of the system by

considering the ratio $\mathcal{R}(r) \equiv \lambda dB(r) / d(r)$

For $\mathcal{R} \lesssim 1$, classical dilute regime,

For $\mathcal{R} \gtrsim 1$, it is a macroscopic quantum regime.

Phase–space density $Q(r) = \rho(r) / \sigma^3(r)$,

$$\mathcal{R}(r) = (2\pi\, \hbar / \sqrt{3})\, [\, Q(r) / m^4)\,]^{1/3}.$$

$$\mathcal{R}(r) = 1.513\ \ [\, I_2\,(\, \nu(\xi)\,)\,]^{5/6} \ / \ [\, I_4(\, \nu(\xi)\,)\,]^{1/2}$$

$\nu(\xi)$ changes sign, indicating the transition from **the quantum to the classical galaxy regime precisely at the same point where $R \simeq 1$, as it**

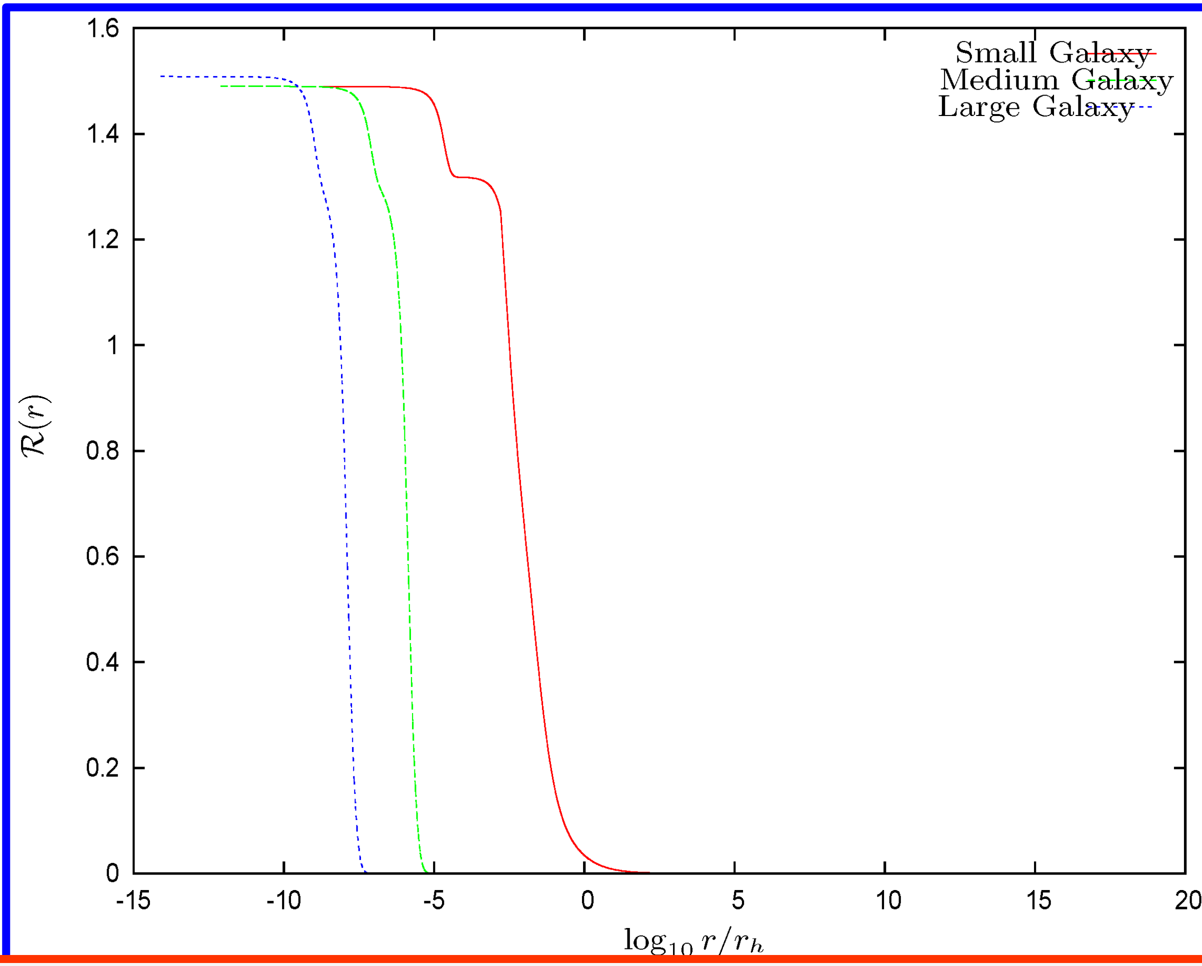


Figure 5. The ratio $\mathscr{R}$ of the particle de Broglie wavelength to the interparticle distance in the galaxy as a function of r for the three representative galaxy solutions with central SMBH: small galaxy (red), medium galaxy (green), and large galaxy (blue). For $\mathscr{R} \lesssim 1$, the galaxy plus SMBH system is of a classical nature, while for $\mathscr{R} \gtrsim 1$, the system is quantum. The transition from the quantum to the classical regime occurs precisely at **the same point** r_A where the chemical potential vanishes (see **Figure 1**), showing the consistency and power of our treatment. This point defines the transition from the quantum to the classical behavior.

CIRCULAR VELOCITIES

$$v_c(r) = 5.2537 \, \frac{\sqrt{-\xi \, \nu'(\xi)}}{[\xi_h \, I_2(\nu_0)]^{\frac{2}{5}}} \left(\frac{2\,\text{keV}}{m}\right)^{\frac{4}{5}} \left(\frac{\Sigma_0 \, \text{pc}^2}{120 \, M_\odot}\right)^{\frac{2}{5}} \frac{\text{km}}{\text{s}} \, .$$

For $r \to 0$, the circular velocity $v_c(r)$ grows due to the black hole field as

$$v_c(r) \overset{r \to 0}{=} \sqrt{\frac{T_0}{m} \, \frac{r_0}{r}} \, ,$$

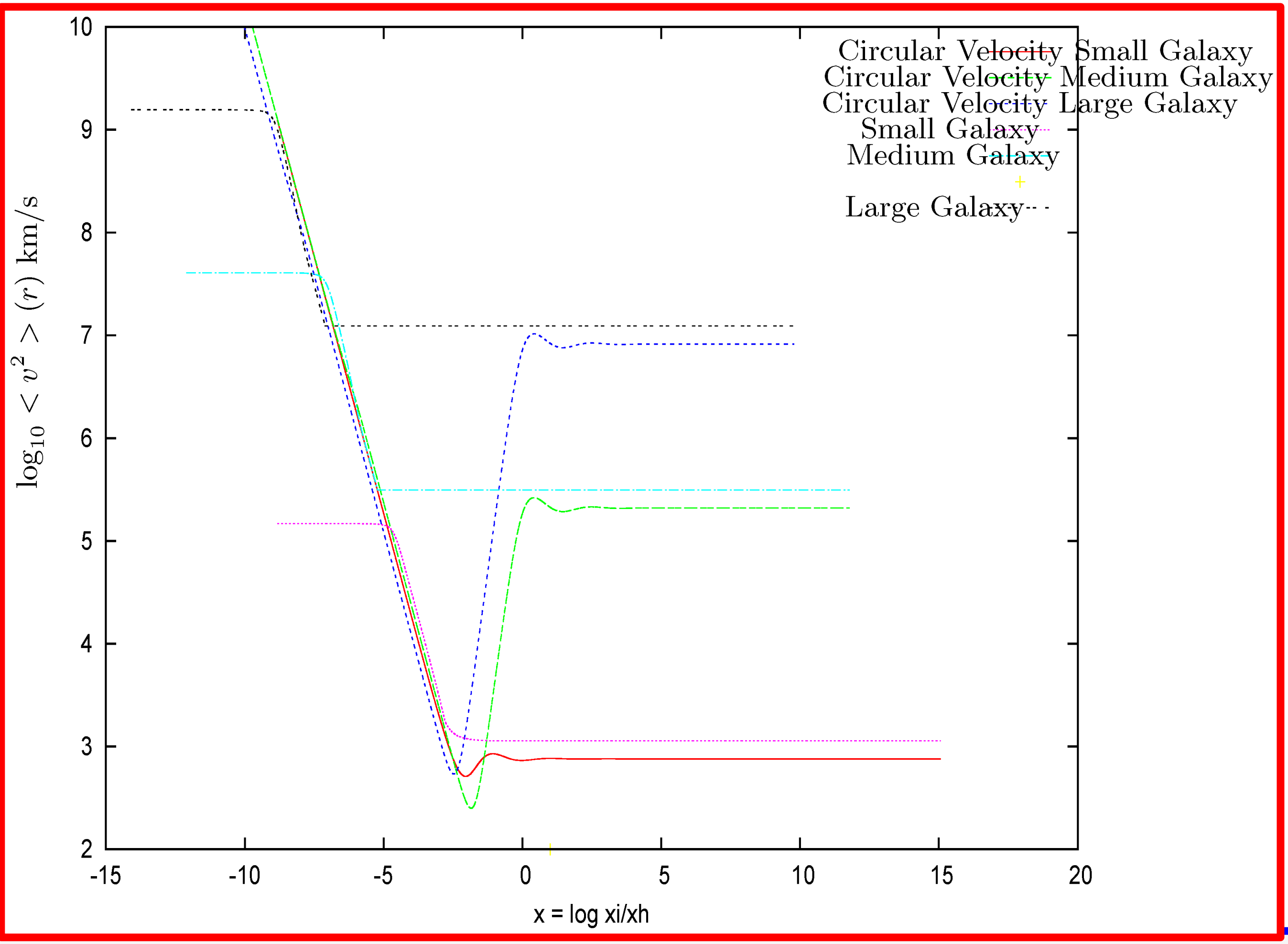


Figure 4. The velocity dispersion $< v^2 > (r)$ and the circular velocity $v_c^2\,(r)$ for the three representative galaxy solutions with central SMBH vs. $\log_{10}\ (r/r_h)$. The velocity dispersion is constant in the Boltzmann and in the quantum regions, indicating a thermalized WDM with two different temperatures, $T_0 = \frac{1}{3}\ m\ \ < v^2 > (r)$. For $r > r_h$, the circular velocity tends to the velocity dispersion [13]. These results are in agreement with the DM thermalization found in the absence of a central BH [12,13].

The EQUATION OF STATE

is in very good approximation *a local (r-dependent)*

perfect gas equation of state

$$P(r) = \sigma^2 \ \rho(r),$$

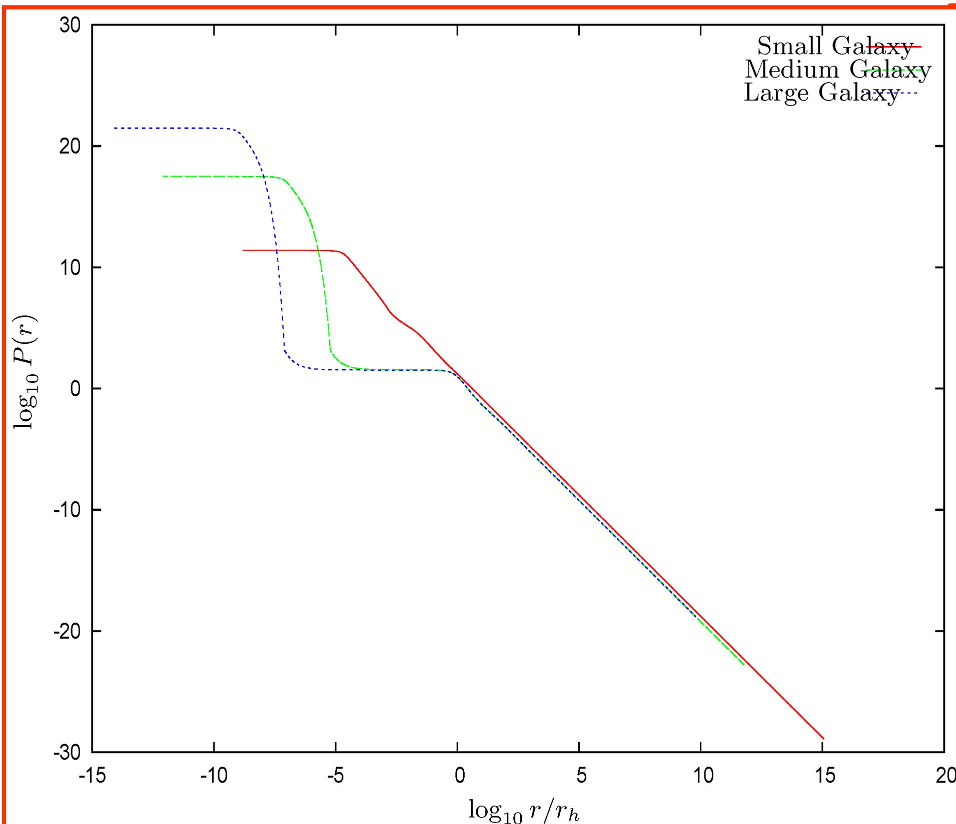
which stems from the fact that galaxies with central black holes have halo masses

$$Mh \ > \ Mh \ \gtrsim \ 10^6 \, M_\odot \ > \ Mhmin,$$

and therefore necessarily belong to the

***dilute* Boltzmann classical regime**

The WDM is in a quantum (highly compact) regime inside the quantum radius rA is in the parsec scale or smaller, the bulk of the WDM is in the Boltzmann classical regime, which is consistently reflected in the perfect gas equation of state behavior.



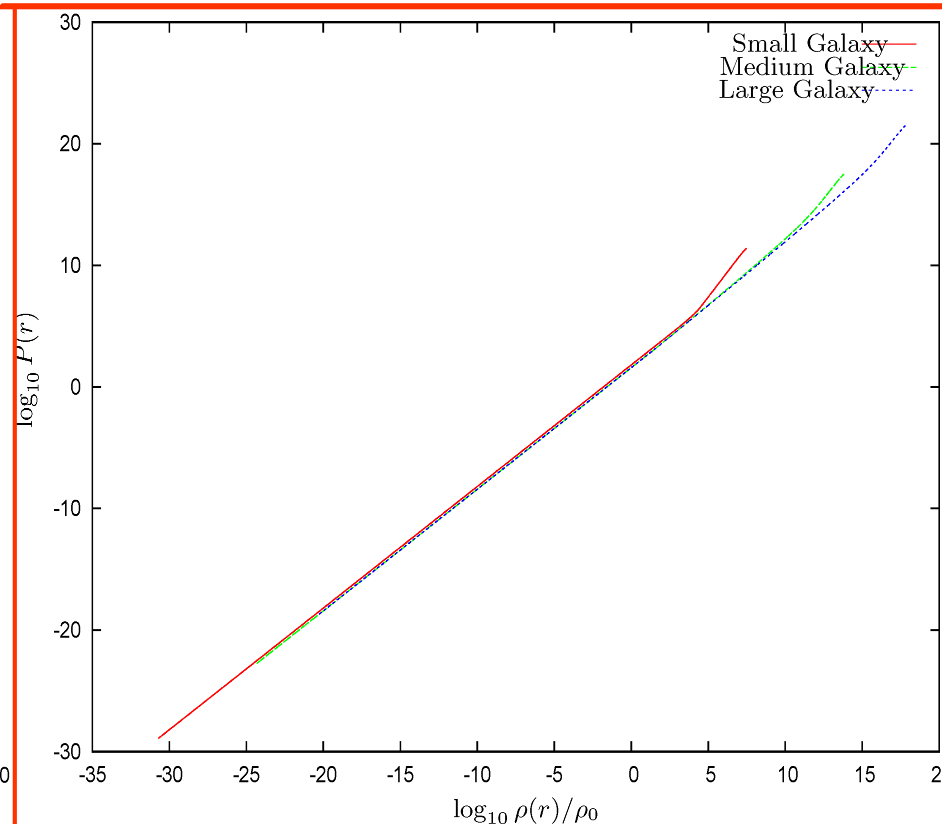
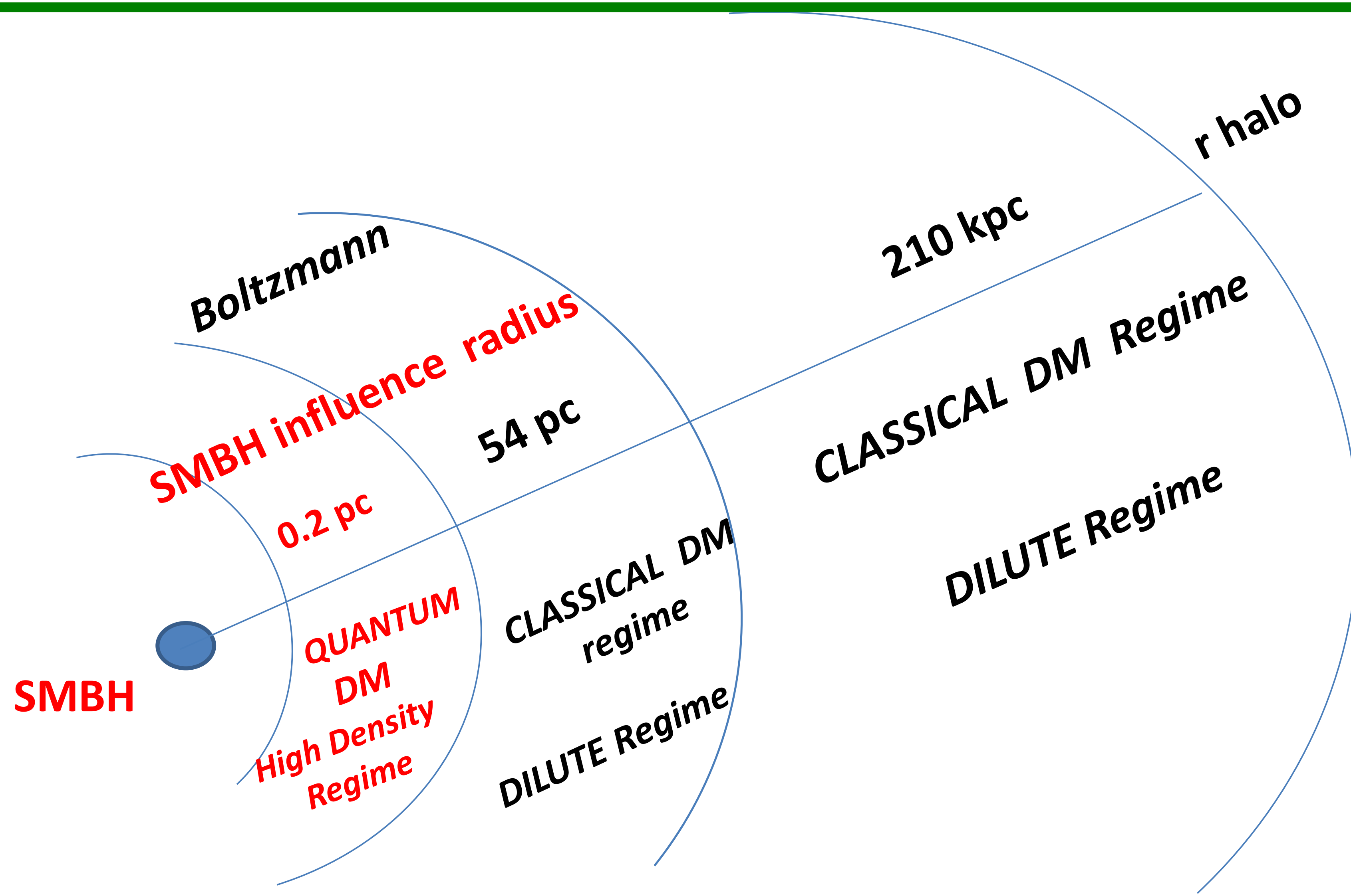


Figure 13. The logarithm of the local pressure $\log_{10} P(r)$ vs. $\log_{10}(r/r_h)$ for the three galaxy solutions with central SMBH. Notice the huge values of $P(r)$ in the quantum (high density) region $r < r_A$ and its sharp decrease entering the classical (dilute) region $r > r_A$.

Figure 14. The obtained equation of state of the galaxy plus central SMBH system: the logarithm of the local pressure $\log_{10} P(r)$ vs. $\log_{10} \rho(r)/\rho_0$. In all the cases, we find almost straight lines of unit slope. The equation of state is a perfect gas equation of state in the Boltzmann classical region. In the quantum gas (dense) region, the equation of state becomes steeper than the perfect gas. Galaxies with central black holes are in the dilute Boltzmann regime because their halo masses are $M_h > M_h^{min}$, Equation (83). This explains the perfect gas equation of state.



WDM Thomas-Fermi Galaxy Theory with SMBH

SMBH: Super Massive Black holes

H.J. de Vega & N.G. Sanchez

In *Conclusion*, these results show *the power and cleanliness of the Thomas–Fermi approach and WDM*

to properly describe the galaxy structures and the galaxy physical states with and without SMBHs

Adding baryons will introduce corrections,
but the picture of galaxy structures with central SMBHs
should not change essentially.

This approach is independent of any WDM particle physics model.
It depends only on the fermionic WDM nature and gravity.

All these results do not depend on the precise value of the WDM particle mass m but only on the fact that m is in the keV scale,

namely 2 keV $\lesssim$ m $\lesssim$ 10 keV, for example.

This framework stresses the key role of Gravity and Warm Dark Matter in structuring galaxies with their central SMBHs and provides correctly the major physical quantities to be first obtained for the galaxy–black-hole system:

Masses, sizes, densities, velocity dispersions,

and their internal physical states.

Whether they are compact, ultracompact, low density or large dilute galaxies, encompassed with their classical physics and quantum gas physical properties.

We thus found different regions structuring internally the halo of the galaxy from the vicinity of the central SMBH region to the external regions or virial radius.

For all galaxies harboring a central SMBH, there is a transition from the quantum to the classical regime going from the more compact inner regions, which are in a quantum gas state,

to the classical dilute regions in a Boltzmann-like state.

This is accompanied by a decreasing in the local temperature from the central warmer regions to the colder external ones.

The SMBH heats the DM near around and prevents it from becoming exactly degenerated at zero temperature. Although the inner DM quantum core is highly compact in a nearly degenerate quantum gas state, it is not at zero temperature.

Inside r ≲ 3rh, the halo is thermalized at a uniform or slowly varying local temperature T0, which tends to the circular temperature Tc(r) at r~3rh.

(1) The realistic astrophysical masses of supermassive black holes are naturally obtained in this framework.

(2) With SMBH, both the quantum and classical behaviors of the dark matter gas do co-exist generically in any galaxy from the

small compacts to the dilute large ones,

and a novel galaxy halo structure with three regions show up

(3) *The transition* from the quantum to classical occurs at rA, where the chemical potential vanishes and which is, in addition, precisely and consistently, the point where the particle λ and the interparticle d are equal (*their ratio being a measure of the Q or classical system)*.

(4) rA is larger for the smaller and more compact galaxies and diminishes with increasing galaxy and black hole masses for the large dilute galaxies

Many Ongoing WDM Directions of Research :

- Particle Models, Sterile neutrinos, Production mechanisms. WDM decay, Experimental searches.

- WDM Numerical Simulations: structure formation -Constraints on WDM m_x, mvs: Analytical, numerical, small scales, velocity dispersions

- WDM Astrophysics & Cosmological: reionization, 21 cm line, prospects for SKA. High z supernova lensing, HST, JWST,WDM Star Formation, WDM BHs

- WDM CMB: WDM decay, CMB Spectrum distortions, ...

THANK YOU

FOR YOUR ATTENTION