KeV scale dark matter from theory and observations. Galaxy properties from linear primordial fluctuations

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DARK MATTER IN THE UNIVERSE AND UNIVERSAL PROPERTIES OF GALAXIES:

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Standard Cosmological Model: Λ CDM

 Λ CDM = Cold Dark Matter + Cosmological Constant

- Begins by the inflationary era. Slow-Roll inflation explains horizon and flatness.
- Gravity is described by Einstein's General Relativity.
- Particle Physics described by the Standard Model of Particle Physics: $SU(3) \otimes SU(2) \otimes U(1) =$ qcd+electroweak model.
- CDM: dark matter is cold (non-relativistic) during the matter dominated era where structure formation happens. DM is outside the SM of particle physics.
- Dark energy described by the cosmological constant Λ .

Standard Cosmological Model: Λ CDM

 \triangle CDM = Cold Dark Matter + Cosmological Constant begins by the Inflationary Era. Explains the Observations:

- Seven years WMAP data and further CMB data
- Light Elements Abundances
- Large Scale Structures (LSS) Observations. BAO.
- Acceleration of the Universe expansion: Supernova Luminosity/Distance and Radio Galaxies.
- Gravitational Lensing Observations
- Lyman α Forest Observations
- Hubble Constant (H_0) Measurements
- Properties of Clusters of Galaxies
- Measurements of the Age of the Universe

Dark Matter

DM must be non-relativistic by structure formation (z < 30) in order to reproduce the observed small structures at $\sim 2 - 3$ kpc.

DM particles can decouple being ultrarelativistic (UR) at $T_d \gg m$ or non-relativistic $T_d \ll m$.

Consider particles that decouple at or out of LTE (LTE = local thermal equilibrium).

Distribution function: $F_d[p_c]$ freezes out at decoupling.

 $p_c = \text{comoving momentum.}$ $P_f(t) = p_c/a(t) = \text{Physical momentum,}$

Velocity fluctuations:

$$y = P_f(t)/T_d(t) = p_c/T_d$$

$$\langle \vec{V}^2(t) \rangle = \langle \frac{\vec{P}_f^2(t)}{m^2} \rangle = \left[\frac{T_d}{m \, a(t)} \right]^2 \, \frac{\int_0^\infty y^4 F_d(y) dy}{\int_0^\infty y^2 F_d(y) dy}$$

Dark Matter density and DM velocity dispersion

Energy Density: $\rho_{DM}(t) = \frac{m \ g}{2\pi^2} \ \frac{T_d^3}{a^3(t)} \int_0^\infty y^2 \ F_d(y) \ dy$,

g: # of internal degrees of freedom of the DM particle, $1 \le g \le 4$. For $z \le 30 \Rightarrow$ DM particles are non-relativistic:

Using entropy conservation: $T_d = \left(\frac{2}{g_d}\right)^{\frac{1}{3}} T_{cmb}$,

 $g_d = \text{effective } \# \text{ of UR degrees of freedom at decoupling,}$ $T_{cmb} = 0.2348 \ 10^{-3} \text{ eV, and}$

 $\rho_{DM}(\text{today}) = \frac{m \, g}{\pi^2 \, g_d} \, T_{cmb}^3 \int_0^\infty y^2 \, F_d(y) \, dy = 1.107 \, \frac{\text{keV}}{\text{cm}^3} \, (1)$

We obtain for the primordial velocity dispersion:

$$\sigma_{prim}(z) = \sqrt{\frac{1}{3}} \langle \vec{V}^2 \rangle(z) = 0.05124 \ \frac{1+z}{g_d^{\frac{1}{3}}} \left[\frac{\int_0^\infty y^4 \ F_d(y) \ dy}{\int_0^\infty y^2 \ F_d(y) \ dy} \right]^{\frac{1}{2}} \frac{\text{keV}}{m} \frac{\text{km}}{\text{s}}$$

Goal: determine m and g_d . We need TWO constraints.

The Phase-space density $Q= ho/\sigma^3$ and its decrease factor Z

The phase-space density $Q \equiv \rho/\sigma^3$ is invariant under the cosmological expansion and can only decrease under self-gravity interactions (gravitational clustering).

The phase-space density today follows observing dwarf spheroidal satellite galaxies of the Milky Way (dSphs)

 $\frac{\rho_s}{\sigma_s^3} \sim 5 \times 10^3 \ \frac{\text{keV/cm}^3}{(\text{km/s})^3} = (0.18 \text{ keV})^4$ Gilmore et al. 07 and 08.

During structure formation ($z \leq 30$), $Q = \rho/\sigma^3$ decreases by a factor that we call Z:

$$Q_{today} = \frac{1}{Z} Q_{prim}$$
, $Q_{prim} = \frac{\rho_{prim}}{\sigma_{prim}^3}$, (2) $Z > 1$.

The spherical model gives $Z \simeq 41000$ and N-body simulations indicate: 10000 > Z > 1. Z is galaxy dependent.

Constraints: First ρ_{DM} (today), Second $Q_{today} = \rho_s / \sigma_s^3$

Mass Estimates for DM particles

Combining the previous expressions lead to general formulas for m and g_d :

$$m = \frac{2^{\frac{1}{4}} \sqrt{\pi}}{3^{\frac{3}{8}} g^{\frac{1}{4}}} Q_{prim}^{\frac{1}{4}} I_4^{\frac{3}{8}} I_2^{-\frac{5}{8}}, \quad g_d = \frac{2^{\frac{1}{4}} g^{\frac{3}{4}}}{3^{\frac{3}{8}} \pi^{\frac{3}{2}} \Omega_{DM}} \frac{T_{\gamma}^3}{\rho_c} Q_{prim}^{\frac{1}{4}} [I_2 I_4]^{\frac{3}{8}}$$

where: $Q_{prim}^{\frac{1}{4}} = Z^{\frac{1}{4}} \ 0.18$ keV using the dSphs data, $T_{\gamma} = 0.2348 \text{ meV}$, $\Omega_{DM} = 0.228$, $\rho_c = (2.518 \text{ meV})^4$ $I_{2n} = \int_0^\infty y^{2n} F_d(y) dy$, n = 1, 2.

These formulas yield for relics decoupling UR at LTE:

 $m = \left(\frac{Z}{g}\right)^{\frac{1}{4}} \text{ keV } \begin{cases} 0.568 \\ 0.484 \end{cases}, \ g_d = g^{\frac{3}{4}} Z^{\frac{1}{4}} \begin{cases} 155 \text{ Fermions} \\ 180 \text{ Bosons} \end{cases}.$ Since g = 1 - 4, we see that $g_d \gtrsim 100 \Rightarrow T_d \gtrsim 100$ GeV. $1 < Z^{\frac{1}{4}} < 10$ for 1 < Z < 10000. Example: for DM Majorana fermions $(g = 2) \ m \simeq 0.85$ keV.

Out of thermal equilibrium decoupling

Results for m and g_d on the same scales for DM particles decoupling UR out of thermal equilibrium.

For a specific model of sterile neutrinos where decoupling is out of thermal equilibrium:

 $0.56 \text{ keV} \lesssim m_{\nu} Z^{-\frac{1}{4}} \lesssim 1.0 \text{ keV} \quad , \quad 15 \lesssim g_d Z^{-\frac{1}{4}} \lesssim 84$

Relics decoupling non-relativistic: similar bounds: keV $\leq m \leq$ MeV

D. Boyanovsky, H. J. de Vega, N. Sanchez, Phys. Rev. D 77, 043518 (2008), arXiv:0710.5180.

H. J. de Vega, N. G. Sanchez, MNRAS 404, 885 (2010), arXiv:0901.0922.

Linear primordial power today P(k) vs. k Mpc h



 $\log_{10} P(k)$ vs. $\log_{10}[k \text{ Mpc } h]$ for WIMPS, 1 keV DM particles and 10 eV DM particles. $P(k) = P_0 k^{n_s} T^2(k)$.

P(k) cutted for 1 keV DM particles on scales ≤ 100 kpc. Transfer function in the MD era from Gilbert integral eq

Galaxies

Physical variables in galaxies:

- a) Nonuniversal quantities: mass, size, luminosity, fraction of DM, DM core radius r_0 , central DM density ρ_0 , ...
- b) Universal quantities: surface density $\mu_0 \equiv r_0 \rho_0$ and DM density profiles. M_{BH}/M_{halo} (or the halo binding energy).
- The galaxy variables are related by universal empirical relations. Only one variable remains free.
- Universal quantities may be attractors in the dynamical evolution.

Universal DM density profile in Galaxies:

 $\rho(r) = \rho_0 F\left(\frac{r}{r_0}\right), F(0) = 1, x \equiv \frac{r}{r_0}, r_0 = \text{DM core radius.}$

Empirical cored profiles: $F_{Burkert}(x) = \frac{1}{(1+x)(1+x^2)}$.

Cored profiles do reproduce the astronomical observations.

The constant surface density in DM and luminous galaxies

The Surface density for dark matter (DM) halos and for luminous matter galaxies defined as: $\mu_{0D} \equiv r_0 \rho_0$,

 r_0 = halo core radius, ρ_0 = central density for DM galaxies

 $\mu_{0D} \simeq 120 \ \frac{M_{\odot}}{\mathrm{pc}^2} = 5500 \ (\mathrm{MeV})^3 = (17.6 \ \mathrm{Mev})^3$

5 kpc < r_0 < 100 kpc. For luminous galaxies $\rho_0 = \rho(r_0)$. Donato et al. 09, Gentile et al. 09

Universal value for μ_{0D} : independent of galaxy luminosity for a large number of galactic systems (spirals, dwarf irregular and spheroidals, elliptics) spanning over 14 magnitudes in luminosity and of different Hubble types.

Similar values $\mu_{0D} \simeq 80 \frac{M_{\odot}}{\text{pc}^2}$ in interstellar molecular clouds of size r_0 of different type and composition over scales $0.001 \text{ pc} < r_0 < 100 \text{ pc}$ (Larson laws, 1981).

DM surface density from linear Boltzmann-Vlasov eq

The distribution function of the decoupled DM particles:

$$f(\vec{x}, \vec{p}; t) = g f_0(p) + F_1(\vec{x}, \vec{p}; t)$$

 $f_0(p)$ = thermal equilibrium function at temperature T_d .

We evolve the distribution function $F_1(\vec{x}, \vec{p}; t)$ according to the linearized Boltzmann-Vlasov equation since the end of inflation where the primordial inflationary fluctuations are:

$$|\phi_k| = \sqrt{2} \pi \frac{|\Delta_0|}{k^{\frac{3}{2}}} \left(\frac{k}{k_0}\right)^{\frac{n_s-1}{2}}$$
 where
 $|\Delta_0| \simeq 4.94 \ 10^{-5}, \ n_s \simeq 0.964, \ k_0 = 2 \ \mathrm{Gpc}^{-1}.$

We Fourier transform over \vec{x} and integrate over momentum $\Delta(k,t) \equiv m \int \frac{d^3p}{(2\pi)^3} \int d^3x \ e^{-i \vec{x} \cdot \vec{k}} F_1(\vec{x}, \vec{p}; t)$

The matter density fluctuations $\rho_{lin}(r)$ are given today by $\rho_{lin}(r) = \frac{1}{2\pi^2 r} \int_0^\infty k \ dk \ \sin(k r) \ \Delta(k, t_{\text{today}})$

Linear density fluctuations today

 $-\Delta(k,z) \stackrel{z \to 0}{=} \frac{3}{5} T(k) (1 + z_{eq}) \Delta(k, z_{eq}) , \quad _{eq} = \text{equilibration},$ T(k) = transfer function during the matter dominated era $T(0) = 1 , \quad T(k \to \infty) = 0 \quad \text{and} \quad 1 + z_{eq} \simeq 3200.$ T(k) decreases with k with the characteristic free streaming $\text{scale } k_{fs} = \sqrt{2}/r_{lin},$

$$r_{lin} = 2 \sqrt{1 + z_{eq}} \left(\frac{3 M_{Pl}^2}{H_0 \sqrt{\Omega_{DM}} Q_{prim}} \right)^{\frac{1}{3}}$$
 and $\gamma \equiv k r_{lin}$.

The linear profile today results:

$$\rho_{lin}(r) = \frac{27\sqrt{2}}{5\pi} \frac{\Omega_M^2 M_{Pl}^2 H_0}{\sigma_{DM}^2} b_0 b_1 9.6 |\Delta_0| (k_{eq} r_{lin})^{\frac{3}{2}} \times (k_0 r_{lin})^{\frac{1-n_s}{2}} \frac{1}{r} \int_0^\infty d\gamma N(\gamma) \sin\left(\gamma \frac{r}{r_{lin}}\right)$$
where $N(\gamma) \equiv \gamma^{n_s/2-1} \log\left(\frac{c \gamma}{k_{eq} r_{lin}}\right) T(\gamma)$, $c \simeq 0.11604$.

Transfer function T(k)



Density profiles in the linear approximation



Profiles $\rho_{lin}(r)/\rho_{lin}(0)$ vs. $x \equiv r/r_{lin}$. These are universal profiles as functions of x. r_{lin} depends on the galaxy. Fermions and Bosons decoupling ultrarelativistically and particles decoupling non-relativistically (Maxwell-Boltzmann statistics)

Matching the observed and the theoretical surface density

Surface density: $\mu_0 \equiv r_0 \ \rho(0)$ where $r_0 = \text{core radius}$.

Linear approximation: $r_{lin} = \alpha r_0$. α follows fitting the linear profile $\rho_{lin}(r)$ to the Burkert profile with core radius r_0 .

 α -values: $\alpha_{BE} = 0.805$, $\alpha_{FD} = 0.688$, $\alpha_{MB} = 0.421$.

Theoretical result: $\mu_{0 lin} = r_{lin} \rho_{lin}(0)/\alpha$.

Fermions:

$$\mu_{0\,lin} = 8261 \, \left[\frac{Q_{prim}}{(\text{keV})^4} \right]^{0.161} \left[1 + 0.0489 \, \ln \frac{Q_{prim}}{(\text{keV})^4} \right] \,\text{MeV}^3$$

Here: $0.161 = n_s/6$.

Matching the observed values $\mu_{0\,obs}$ with this $\mu_{0\,lin}$ gives $Q_{prim}/(\text{keV})^4$ and the mass of the DM particle as $m = m_0 Q_{prim}^{\frac{1}{4}}/\text{keV}$, data from spiral galaxies. BE: $m_0 = 2.6462$ keV, FD: $m_0 = 2.6934$ keV.

$Q_{prim}/(\text{keV})^4$ from the observed surface density



- p. 17/3

Density profiles in the linear approximation

Density profiles turn to be cored at scales $r \ll r_{lin}$. Intermediate regime $r \gtrsim r_{lin}$:

$$\rho_{lin}(r) \stackrel{r \gtrsim r_{lin}}{=} \left(\frac{36.45 \text{ kpc}}{r}\right)^{1+n_s/2} \ln\left(\frac{7.932 \text{ Mpc}}{r}\right) \times \left[1+0.2416 \ln\left(\frac{m}{\text{keV}}\right)\right] \ 10^{-26} \frac{\text{g}}{\text{cm}^3} \quad , \quad 1+n_s/2 = 1.482.$$

 $\rho_{lin}(r)$ scales with the primordial spectral index n_s .

The theoretical linear results agree with the universal empirical behaviour $r^{-1.6\pm0.4}$: M. G. Walker et al. (2009) (observations), I. M. Vass et al. (2009) (simulations).

The agreement between the linear theory and the observations is remarkable.

Non-universal galaxy properties.

 _	Observed Values	Linear Theory
r_0	5 to 52 kpc	46 to 59 kpc
$ ho_0$	$1.57 \text{ to } 19.3 \times 10^{-25} \frac{\text{g}}{\text{cm}^3}$	$1.49 \text{ to } 1.91 \times 10^{-25} \frac{\text{g}}{\text{cm}^3}$
$\sqrt{\overline{v^2}}_{halo}$	79.3 to 261 km/sec	260 km/sec

Dark matter particle mass: 1.6 < m < 2 keV.

The larger and less denser are the galaxies, the better are the results from the linear theory for non-universal quantities.

The linear approximation turns to improve for larger galaxies (i. e. more diluted).

Therefore, universal quantities can be reproduced by the linear approximation.

Halo radius in the linear approximation vs. observations



The halo radius in the linear approximation $r_{0 lin}$ in kpc in broken green line, the halo radius r_0 from the data in kpc in solid red line vs. the galaxy virial mass $M_{virial}/10^{11} M_{\odot}$.

$$r_{0\,lin} = 52.5 \, \left(rac{1.8 \, {
m keV}}{m}
ight)^{rac{4}{3}} \, {
m kpc}$$

The linear approximation turns to improve for larger galaxies (i. e. more diluted).

Density Contrast

Ratio between the maximum DM mass density $\rho_{lin}(0)$ and the average DM mass density $\bar{\rho}_{DM}$ in the universe

contrast $\equiv \frac{\rho_{lin}(0)}{\bar{\rho}_{DM}} = \frac{\mu_{0\,lin}}{\Omega_{DM}\,\rho_c\,r_0}$



The linear contrast turns to be between 1/3 and 1/2 of the observed value $\sim 3 \times 10^5$ (Salucci & Persic, 1997). Linear galaxies are less dense and larger than the observations. Universal quantities take the right values.

Wimps vs. galaxy observations

 _	Observed Values	Wimps in linear theory
r_0	5 to 52 kpc	0.045 pc
$ ho_0$	$1.57 \text{ to } 19.3 \times 10^{-25} \frac{\text{g}}{\text{cm}^3}$	$0.73 \times 10^{-14} \frac{\text{g}}{\text{cm}^3}$
$\sqrt{\overline{v^2}}_{halo}$	79.3 to 261 km/sec	0.243 km/sec

The wimps values strongly disagree by several order of magnitude with the observations.



 $ho_{lin}(r)_{wimp}$ in g/cm^3 vs. r in pc. Exhibits a cusp behaviour -for $r \gtrsim 0.03$ pc.

Particle physics candidates for DM

No particle in the Standard Model of particle physics (SM) can play the role of DM.

Many extensions of the SM can be envisaged to include a DM particle with mass in the keV scale and weakly enough coupled to the Standard Model particles to fulfill all particle physics experimental constraints.

Main candidates in the keV mass scale: sterile neutrinos, gravitinos, light neutralino, majoron ...

Particle physics motivations for sterile neutrinos:

There are both left and right handed quarks (with respect to the chirality).

It is natural to have right handed neutrinos ν_R besides the known left-handed neutrino. Quark-lepton similarity.

Sterile Neutrinos in the SM of particle physics

SM symmetry group: $SU(3)_{color} \otimes SU(2)_{weak} \otimes U(1)_{hypercharge}$

Leptons are color singlets and doublets under weak SU(2).

Sterile neutrinos ν_R do not participate to weak interactions. Hence, they must be singlets of color, weak SU(2) and hypercharge.

The SM Higgs Φ is a SU(2) doublet with a nonzero vacuum expectation value Φ_0 . It can couple Yukawa-type with the left and right handed leptons:

$$L_{Yuk} = y \ \bar{\nu}_L \ \nu_R \ \Phi_0 + h.c. ,$$

$$y = \text{Yukawa coupling}, \ \Phi_0 = \begin{pmatrix} 0 \\ v \end{pmatrix} , \ v = 174 \text{ GeV}.$$

This induces a mixing (bilinear) term between ν_L and ν_R which produces transmutations of $\nu_L \Leftrightarrow \nu_R$.

Sterile Neutrinos Mixing

Mixing and oscillations of particle states are typical of low energy particle physics !

- Flavor mixing: e- μ neutrino oscillations (explain solar neutrinos).
- $K \overline{K}, B \overline{B}$ and $D \overline{D}$ meson oscillations in connection with CP-violation.

Neutrino mass matrix:
$$(\bar{\nu}_L \ \bar{\nu}_R) \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}$$

 $M = \text{right-neutrino mass}, m_D = y v, M \gg m_D$. Seesaw.

Mass eigenvalues: $\frac{m_D^2}{M}$ and M, with eigenvectors:

- active neutrino: $\nu_{active} \simeq \nu_L \frac{m_D}{M} \nu_R$.
- sterile neutrino: $\nu_{sterile} \simeq \nu_R + \frac{m_D}{M} \nu_L$, $M \gg m_D^2/M$.

Sterile Neutrinos

Choosing $M \sim 1$ keV and $m_D \sim 0.1$ eV is consistent with observations.

Mixing angle: $\theta \sim \frac{m_D}{M} \sim 10^{-4}$ is appropriate to produce enough sterile neutrinos accounting for the observed DM.

Smallness of θ makes very difficult detection of steriles.

Precise measure of nucleus recoil in tritium beta decay: $H_1^2 \implies He_2^1 + e^- + \bar{\nu}$ can show the existence of steriles!!

A sterile neutrino per lepton family is expected.

Only the lightest one (electron family) has lifetime \sim Hubble time and can describe the DM.

Conclusion: the empty slot of right-handed neutrinos in the Standard Model of particle physics can be filled by keV-scale sterile neutrinos describing the DM.

Summary: keV scale DM particles

- Reproduce the phase-space density observed in dwarf satellite galaxies and spiral galaxies (dV S 2009).
- Provide cored universal galaxy profiles in agreement with observations (dV S 2009,dV S S 2010). (Review on cores vs. cusps by de Blok 2010).
- Reproduce the universal surface density μ₀ of DM dominated galaxies (dV S S 2010). WIMPS simulations give 10⁹ times the observed value of μ₀ (Hoffman et al. 2007).
- Alleviate the satellite problem which appears when wimps are used (Avila-Reese et al. 2000, Götz & Sommer-Larsen 2002)
- Alleviate the voids problem which appears when wimps are used (Tikhonov et al. 2009).

Summary: keV scale DM particles

All direct searches of DM particles look for $m \gtrsim 1$ GeV.
DM mass in the keV scale explains why nothing has been found ...

 e^+ and \bar{p} excess in cosmic rays may be explained by astrophysics: P.L. Biermann, et al. (2009), P. Blasi, P. D. Serpico, (2009).

- Peculiar velocities in galaxy clusters. Wimp simulations give velocities below observations by factors 4 10 (Kashlinsky et al. 2008, Watkins et al. 2009, Lee & Komatsu 2010). keV scale DM should alleviate this.
- Galaxies from Wimps simulations are too small (Ryan Joung et al. 2009, Holz & Perlmutter 2010). keV scale DM may alleviate this problem.

Simulations with keV mass DM are urgently needed to clarify all these issues.

Summary and Conclusions

Combining theoretical evolution of fluctuations through the Boltzmann-Vlasov equation with galaxy data points to a DM particle mass 1 - 2 keV. T_d may be > 100 GeV. This is independent of the DM particle physics model.

• Universal Surface density in DM galaxies $[\mu_{0D} \simeq (18 \text{ MeV})^3]$ explained by keV mass scale DM. Density profile scales and decreases for intermediate scales with the spectral index n_s : $\rho(r) \sim r^{-1-n_s/2}$.

H. J. de Vega, P. Salucci, N. G. Sanchez, 'Universal galaxy properties and the mass of the dark matter particle from theory and observations: the power of the linear approximation', arXiv:1004.1908.

H. J. de Vega, N. G. Sanchez, 'Constant surface density in dark matter galaxies', arXiv:0907.0006 and 'Model independent analysis of dark matter points to a particle mass at the keV scale', arXiv:0901.0922, MNRAS 404, 885 _____ (2010).

Future Perspectives

The Golden Age of Cosmology and Astrophysics continues.

Galaxy and Star formation. DM properties from galaxy observations. Better upper bounds on DM cross-sections.

DM in planets and the earth. Flyby and Pioneer anomalies?

Chandra, Suzaku X-ray data: keV mass DM decay?

Sun models well reproduce the sun's chemical composition but not the heliosismology (Asplund et al. 2009). Can DM inside the Sun help to explain the discrepancy?

Nature of Dark Matter? 83% of the matter in the universe.

Light DM particles are strongly favoured $m_{DM} \sim \text{keV}$. Sterile neutrinos ? Other particle in the keV mass scale?

Precision determination of DM properties (mass, T_d , nature) from better galaxy data combined with theory (Boltzmann-Vlasov and simulations).

The Universe is our ultimate physics laboratory

THANK YOU VERY MUCH FOR YOUR ATTENTION!!

ρ/σ^3 vs. r for different z from $\Lambda {\rm CDM}$ simulations



Phase-space density $Q \equiv \rho/\sigma^3$ vs. $r/r_{vir}(z=0)$ dot-dashed line for different redshifts: $0 \le z \le 9$.

We see that from z = 9 to z = 0 the *r*-average of ρ/σ^3 decreases by a factor $Z \sim 10$.

I. M. Vass et al. MNRAS, 395, 1225 (2009).

The self-gravity decreasing factor ${\cal Z}$ for spirals.



$$Q_{today} = \frac{1}{Z} Q_{prim} \quad , \quad Q \equiv \frac{\rho}{\sigma^3}$$

 $\log_{10} Z$ in solid red line and the common logarithm of the observed phase-space density $\log_{10} Q_{halo}/(keV^4)$ in broken green line vs. $M_{virial}/[10^{11}M_{solar}]$.

The value of Z depends on the type of galaxy. Z is larger for spirals than for dSphs.

The observed surface density



 $\log_{10} \mu_{0 obs}$ in $(MeV)^3$ vs. the common logarithm of the core radius $\log_{10} r_0$ in kpc from spiral galaxies.

Both r_0 and ρ_0 vary by a factor thousand while μ_0 varies only by about $\pm 10\%$.

Relics decoupling non-relativistic

 $F_d^{NR}(p_c) = \frac{2^{\frac{5}{2}} \pi^{\frac{7}{2}}}{45} g_d Y_{\infty} \left(\frac{T_d}{m}\right)^{\frac{3}{2}} e^{-\frac{p_c^2}{2m T_d}} = \frac{2^{\frac{5}{2}} \pi^{\frac{7}{2}}}{45} \frac{g_d Y_{\infty}}{x^{\frac{3}{2}}} e^{-\frac{y^2}{2x}}$ $Y(t) = n(t)/s(t), \ n(t) \text{ number of DM particles per unit}$ volume, s(t) entropy per unit volume, $x \equiv m/T_d, \ T_d < m$.

$$Y_{\infty} = \frac{1}{\pi} \sqrt{\frac{45}{8}} \frac{1}{\sqrt{g_d} T_d \sigma_0 M_{Pl}}$$
 late time limit of Boltzmann.

 σ_0 : thermally averaged total annihilation cross-section times the velocity.

From our previous general equations for m and g_d :

$$m = \frac{45}{4 \pi^2} \frac{\Omega_{DM} \rho_c}{g T_{\gamma}^3 Y_{\infty}} = \frac{0.748}{g Y_{\infty}} \text{ eV} \quad \text{and} \quad m^{\frac{5}{2}} T_d^{\frac{3}{2}} = \frac{45}{2 \pi^2} \frac{1}{g g_d Y_{\infty}} Z \frac{\rho_s}{\sigma_s^3}$$

Finally: $\sqrt{m T_d} = 1.47 \left(\frac{Z}{g_d}\right)^{\frac{1}{3}} \text{ keV}$

We used ρ_{DM} today and the decrease of the phase space density by a factor Z.

Relics decoupling non-relativistic 2

Allowed ranges for m and T_d .

 $m>T_d>b~{\rm eV}$ where $b>1~{\rm or}~b\gg 1$ for DM decoupling in the RD era

$$\left(\frac{Z}{g_d}\right)^{\frac{1}{3}} 1.47 \text{ keV} < m < \frac{2.16}{b} \text{ MeV} \left(\frac{Z}{g_d}\right)^{\frac{2}{3}}$$

 $g_d \simeq 3$ for $1 \text{ eV} < T_d < 100 \text{ keV}$ and $1 < Z < 10^3$

 $1.02 \text{ keV} < m < \frac{104}{b} \text{ MeV}$, $T_d < 10.2 \text{ keV}$.

Only using ρ_{DM} today (ignoring the phase space density information) gives one equation with three unknowns: m, T_d and σ_0 ,

$$\sigma_0 = 0.16 \text{ pbarn } \frac{g}{\sqrt{g_d}} \frac{m}{T_d}$$
 http://pdg.lbl.gov

WIMPS with m = 100 GeV and $T_d = 5$ GeV require $Z \sim 10^{23}$.

Linear results for μ_{0D} and the profile vs. observations

Since the surface density $r_0 \rho(0)$ should be universal, we can identify $r_{lin} \rho_{lin}(0)$ from a spherically symmetric solution of the linearized Boltzmann-Vlasov equation.

The comparison of our theoretical values for μ_{0D} and the observational value indicates that $Z \sim 10 - 1000$. Recalling the DM particle mass:

$$m = 0.568 \left(\frac{Z}{g}\right)^{\frac{1}{4}}$$
 keV for Fermions.

This implies that the DM particle mass is in the keV range.

Remarks:

1) For larger scales nonlinear effects from small k should give the customary r^{-3} tail in the density profile.

2) The linear approximation describe the limit of very large galaxies with typical inner size $r_{lin} \sim 100$ kpc.