



# The effects of free streaming on hot-warm dark matter haloes



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# Motivation

- Warm dark matter (WDM) provides an interesting alternative to cold dark matter (CDM) candidates since it can be tested with astrophysical observations on small scales where the CDM model is challenged.
- WDM has a non-negligible velocity dispersion which dampens the small scale fluctuation spectrum and sets a phase space limit to cosmic structures.
- More recently we have seen renewed interest in warm dark matter since a candidate may occur naturally within extensions to the standard model of particle physics.
- The sterile neutrino can explain some key physical phenomena including neutrino oscillations, the dark matter and the baryon asymmetry of the universe.

# The effects of free streaming on warm dark matter haloes: a test of the Gunn-Tremaine limit

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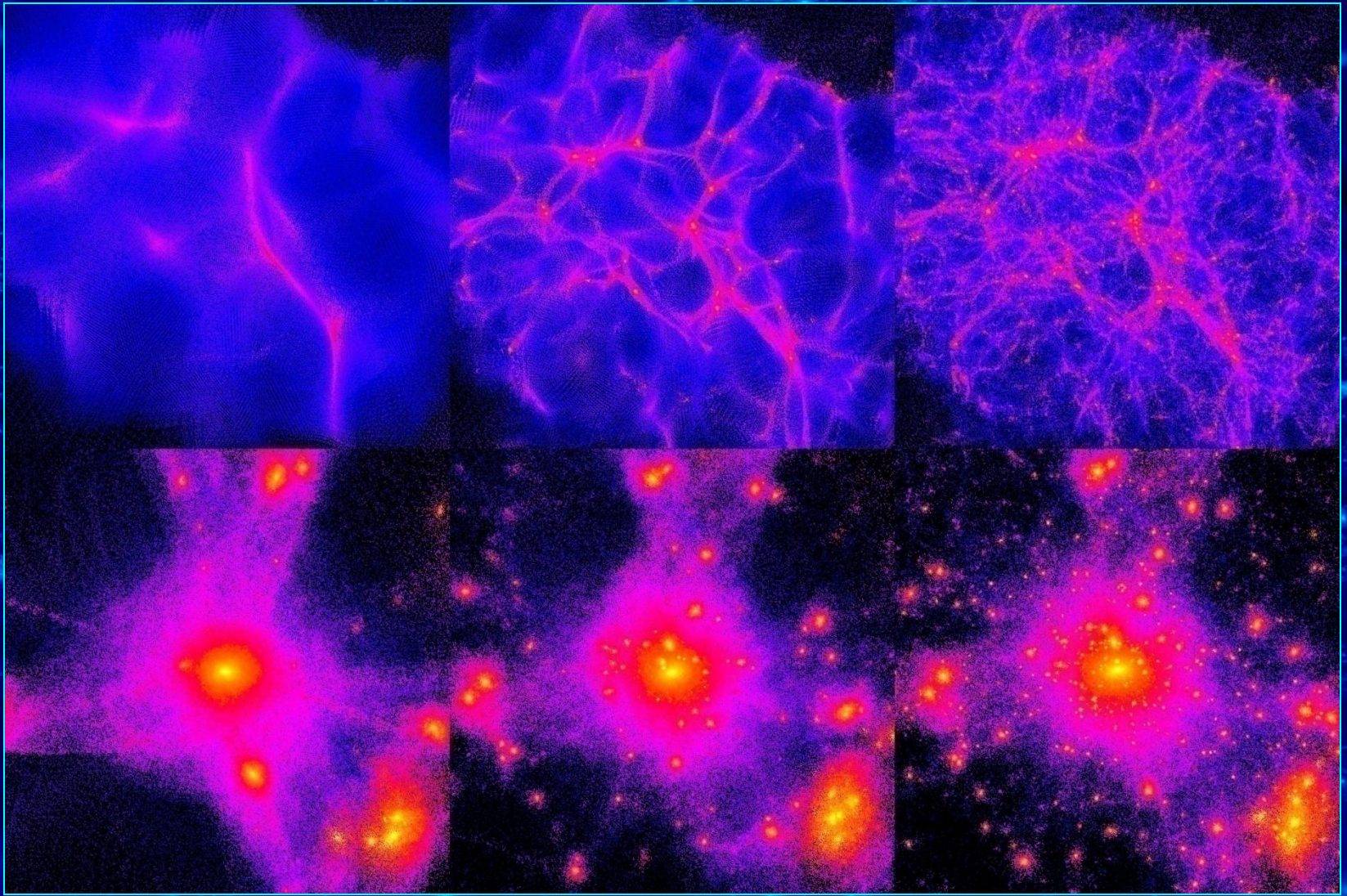
23 June 2009

## ABSTRACT

The free streaming of warm dark matter particles dampens the fluctuation spectrum, flattening the mass function of haloes and imprinting a fine grained phase density limit for dark matter structures. We explore these effects using high resolution simulations of structure formation in a warm dark matter universe. The Gunn-Tremaine limit is expected to imprint a constant density core at the halo center and we verify this with our simulations which we show preserve Liouville's theorem. The maximum phase space density is conserved and the size of the core is in good agreement with the simple expectations. However, to create a significant core, say about 1 kpc in a dwarf galaxy, one requires a thermal neutrino of  $\sim 20$ eV which would erase all structures that form at that mass scale and below. We find that structure formation in these warm dark models occurs top-down on galactic scales where we find the lowest mass haloes collapsing first. The halo mass-concentration mass-redshift formation relations are thus reversed with respect to cold dark matter.

**Key words:**

Dark matter: N-body simulations – galaxies, haloes.



$\langle 1 \text{ keV DM}$

$>1 \text{ keV DM}$

CDM

# Simulating the WDM...

$$P(k) = k^n * T^2(k, z) \quad n \sim 1$$

$$k_s \approx \left( \frac{0.3}{\Omega_X} \right)^{0.15} \left( \frac{m_X}{\text{keV}} \right)^{1.15} \text{ Mpc}^{-1}$$

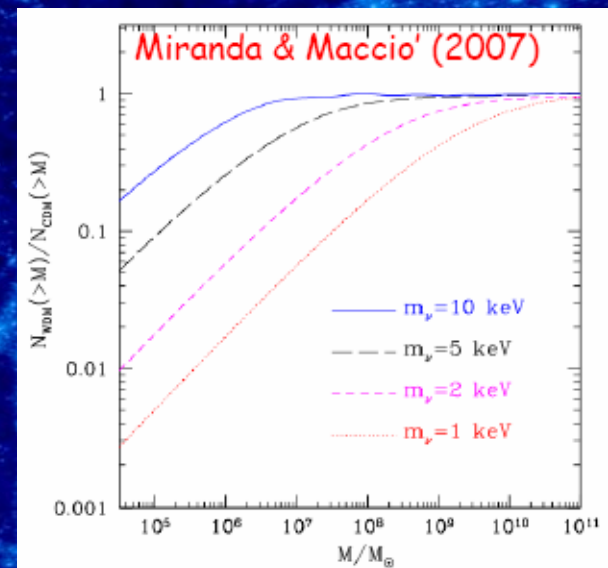
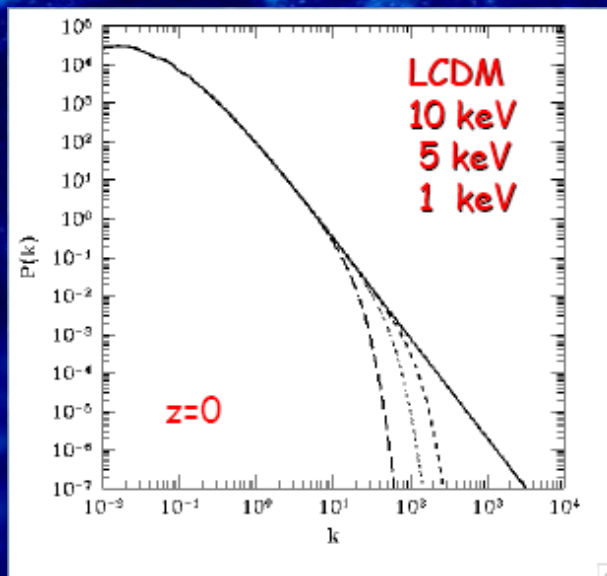
$$T^2(k) = \frac{P^{WDM}}{P^{CDM}} = [1 + (\alpha k)^{2\nu}]^{-10/\nu}$$

$$\alpha = 0.049 \cdot \left( \frac{m_x}{1 \text{keV}} \right)^{-1.11} \cdot \left( \frac{\Omega_\nu}{0.25} \right)^{0.11} \cdot \left( \frac{h}{0.7} \right)^{1.22} h^{-1} \text{ Mpc.}$$

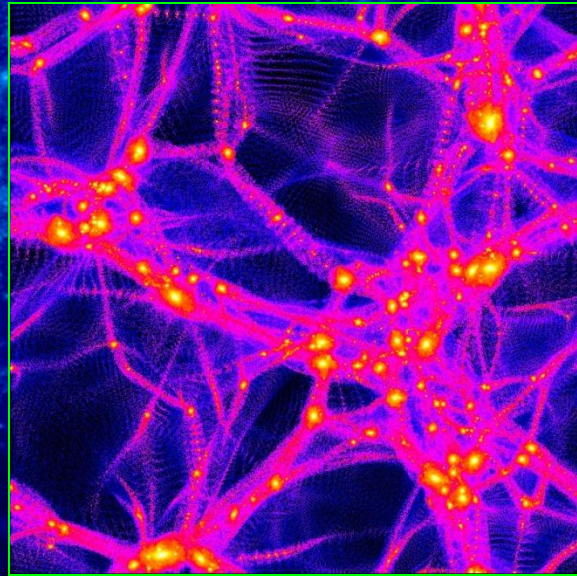
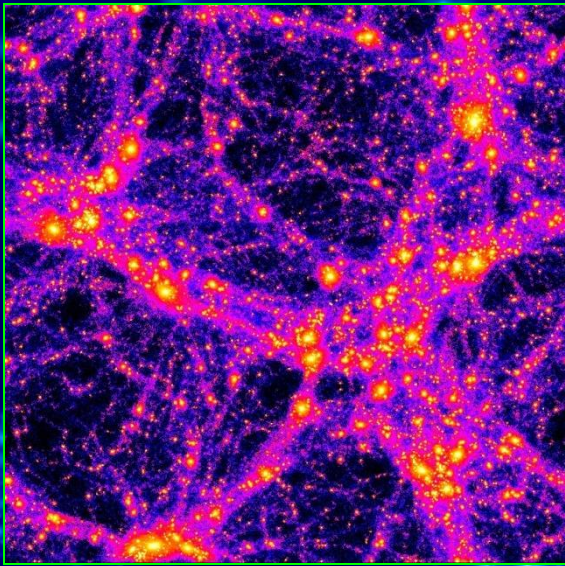
Bode, Turok, and Ostriker (2001)

Viel et. Al 2005

$$\frac{v_0(z)}{1+z} = .012 \left( \frac{\Omega_X}{0.3} \right)^{\frac{1}{3}} \left( \frac{h}{0.65} \right)^{\frac{2}{3}} \left( \frac{1.5}{g_X} \right)^{\frac{1}{3}} \left( \frac{\text{keV}}{m_X} \right)^{\frac{4}{3}} \text{ km s}^{-1}$$

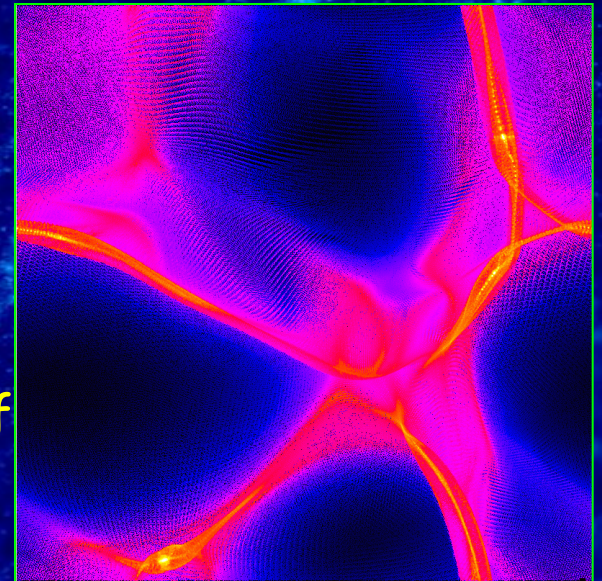


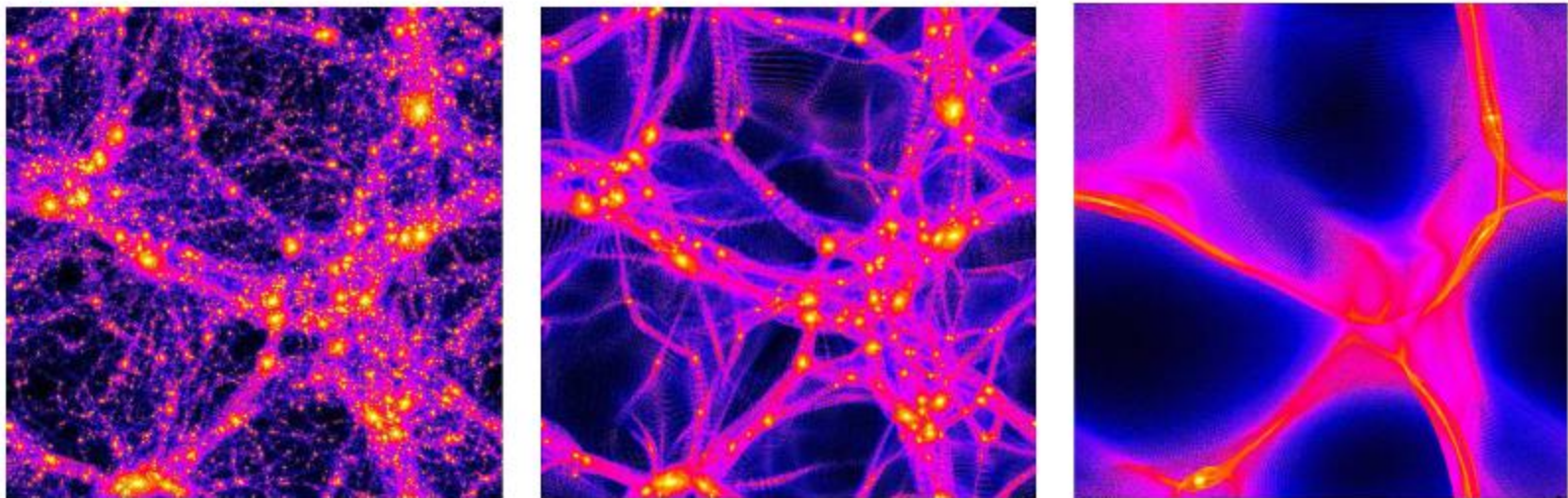
CDM



200eV power spectrum cutoff  
20eV velocities

50eV power spectrum cutoff  
50eV velocities





**Figure 1.** Three snapshots of different simulations at redshift  $z = 0$ . CDM, WDM3 and WDM4 are shown from left to right.

**Table 1.** Details of the simulations

Label	particle mass	velocities	box size	no.of particles	softening ( $r_{200}$ )	halo mass	$r_{200}$ (kpc)	$N(< r_{200})$
CDM	-	no	40 Mpc	$160^3$	$2.6 \times 10^{-3}$	$7 \times 10^{11}$	160	$3.6 \times 10^6$
WDM1	200 eV	no	40 Mpc	$160^3$	$2.6 \times 10^{-3}$	$7 \times 10^{11}$	140	$2.7 \times 10^6$
WDM2	200 eV	100 eV	40 Mpc	$160^3$	$2.6 \times 10^{-3}$	$7 \times 10^{11}$	140	$1.7 \times 10^6$
WDM3	200 eV	20 eV	40 Mpc	$160^3$	$2.6 \times 10^{-3}$	$7 \times 10^{11}$	132	$2.7 \times 10^6$
WDM4	50 eV	no	40 Mpc	$160^3$	$2.6 \times 10^{-3}$	-	-	-
WDM5	200 eV	no	42.51 Mpc	$300^3$	$0.66 \times 10^{-3}$	$10^{13}$	425	$18.67 \times 10^6$
WDM6	200 eV	200 eV	42.51 Mpc	$300^3$	$0.66 \times 10^{-3}$	$10^{13}$	425	$18.66 \times 10^6$

## Assumptions in determining the core radius:

- Isothermal spheres
- Liouville - Phase space density (PSD) is conserved
- Pauli exclusion principle
- PSD constant as mixing occurs
- Velocity dispersion in central halo = constant
- Density profile in central halo = constant



## Halo core radii resulting from a phase space limit

$$\rho_0 m_\nu^{-4} (2\pi\sigma^2)^{-3/2}$$

Maximum phase space density for a Maxwellian velocity distribution – Tremaine & Gunn (1979)

$$r_c^2 = 9\sigma^2 / 4\pi G\rho_0.$$

The core radius

$$Q_{\max} = 5 \times 10^{-4} \beta \left(\frac{g}{2}\right) \left(\frac{m_x}{1\text{keV}}\right)^4 M_\odot \text{pc}^{-3} (\text{km s}^{-1})^{-3}$$

Maximum phase space density

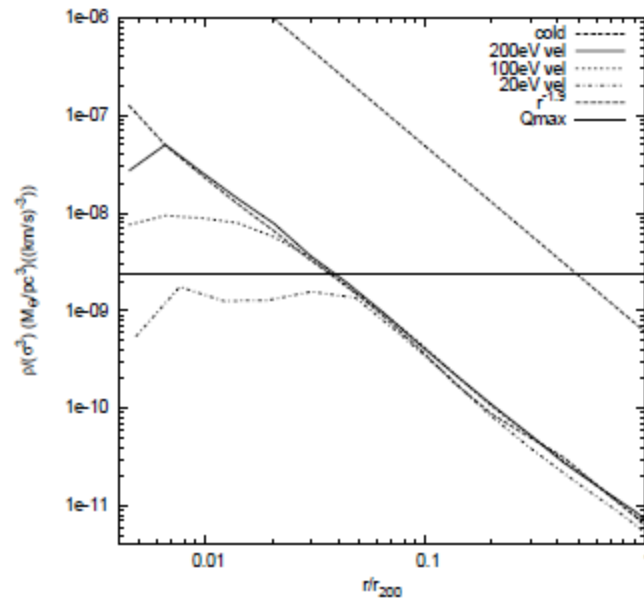
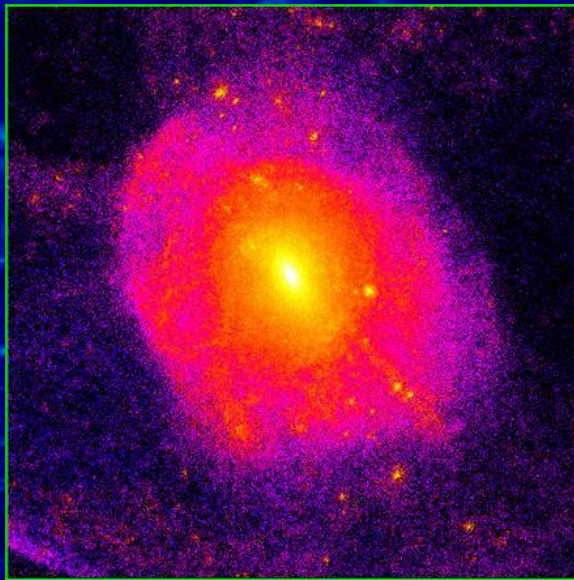
$$Q_0 = \frac{\rho}{\sigma^3} = \rho_{\text{crit}} \Omega \left(\frac{m_\nu c^2}{KTe}\right)^3$$

PSD in our simulations, for a constant density in the initial conditions

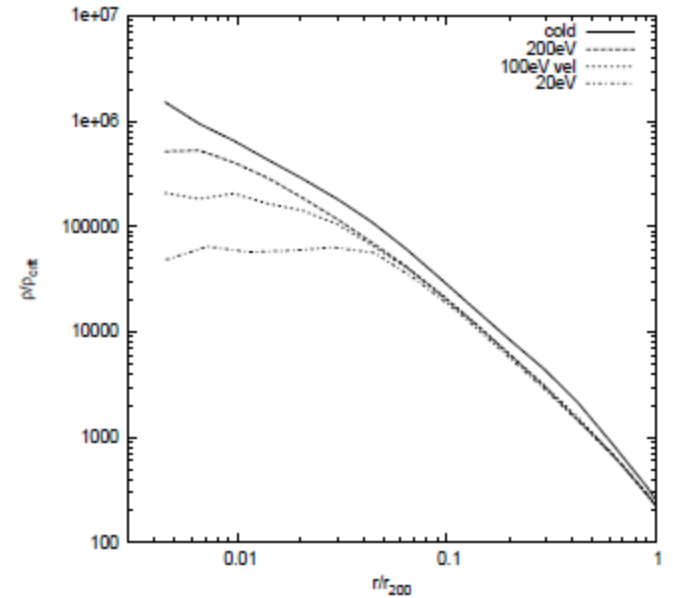
$$Q_0 = 3 \times 10^{-4} M_\odot \text{pc}^{-3} (\text{km/s})^{-3} \left(\frac{m_x}{1\text{keV}}\right)^3$$

The core radius

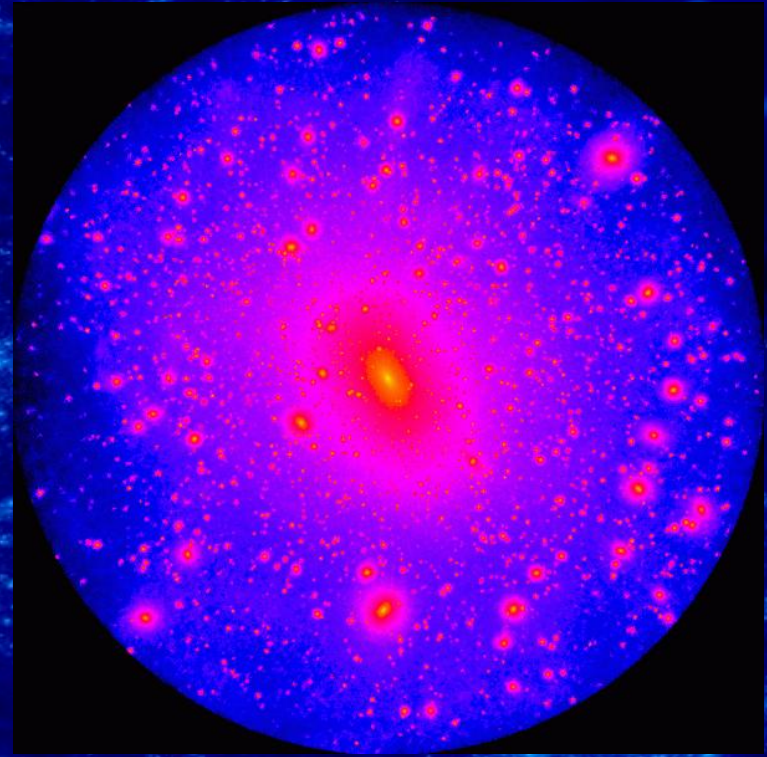
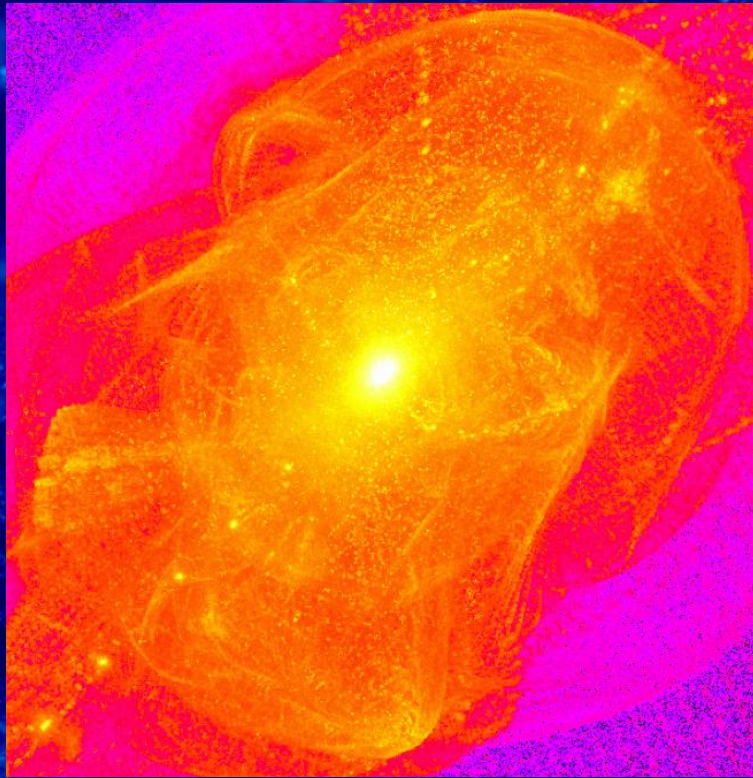
$$r_{c,\min}^2 = \frac{\sqrt{3}}{4\pi G Q_0} \frac{1}{\langle \sigma^2 \rangle^{1/2}}$$



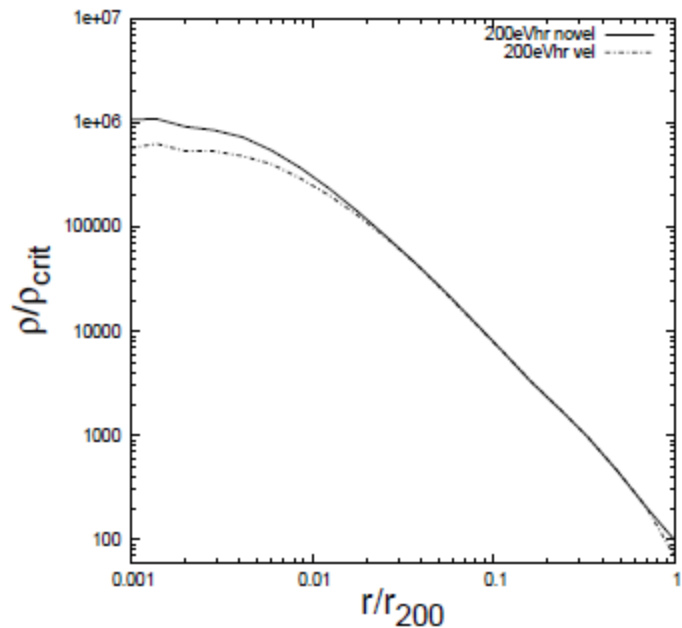
**Figure 3.** “Phase-space density” (PSD) profiles of the same haloes shown in Figure 1, calculated using  $\rho/\sigma^3$ .



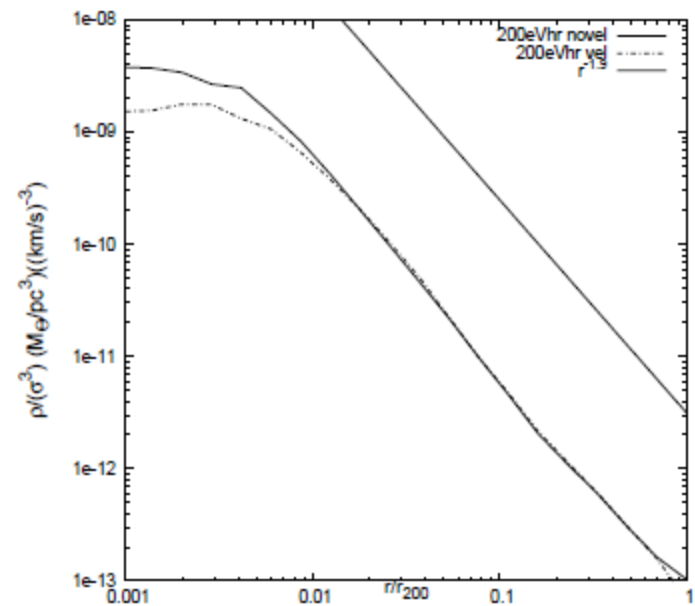
**Figure 2.** The spherically averaged density profiles for CDM, WDM1, WDM2 & WDM3 haloes. The resolution limit is at approximately 0.5% of the virial radius (the softening radii are a 0.26% of the virialized radius).



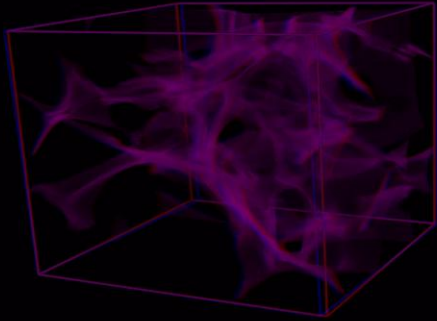
	Number of part.	$r_{vir}$ [kpc]	mass p. part. [ $M_{\odot}$ ]	boxsize [Mpc]
$m < 200\text{eV}$	18 mil.	630	$10^5$	42.5
$m > 2\text{keV}$	50 mil.	200	$10^7$	40



**Figure 4.** The density profiles for a high resolution halo in the 200 eV simulation WDM5 and WDM6



**Figure 5.** The PSD profiles for the 200eV halo from WDM5 and WDM6. A power law line of slope  $r^{-1.9}$  is shown for reference.

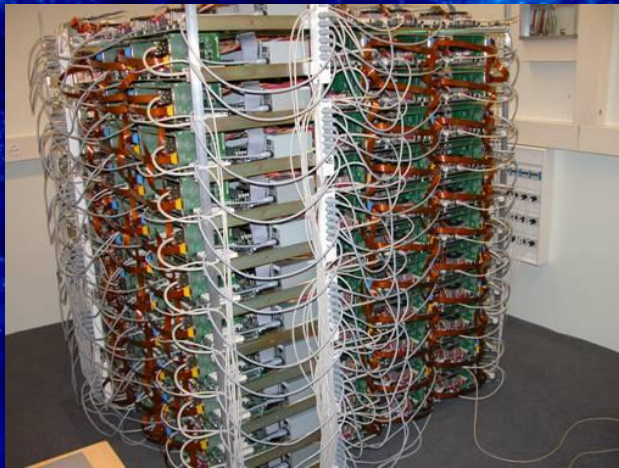


Simulations movies of structure formation available at:

[http://www.itp.uzh.ch/research\\_groups/astrophysics/movies.html](http://www.itp.uzh.ch/research_groups/astrophysics/movies.html)

<http://obswww.unige.ch/~paduroiu/>

All simulations were performed with PkdGrav code developed by Joachim Stadel



zBox2 (2006)

500 quad Opteron 852's, 580Gb memory,  
65Tb disk, 3d-SCI low latency network.



zBox1: (Stadel & Moore 2002)

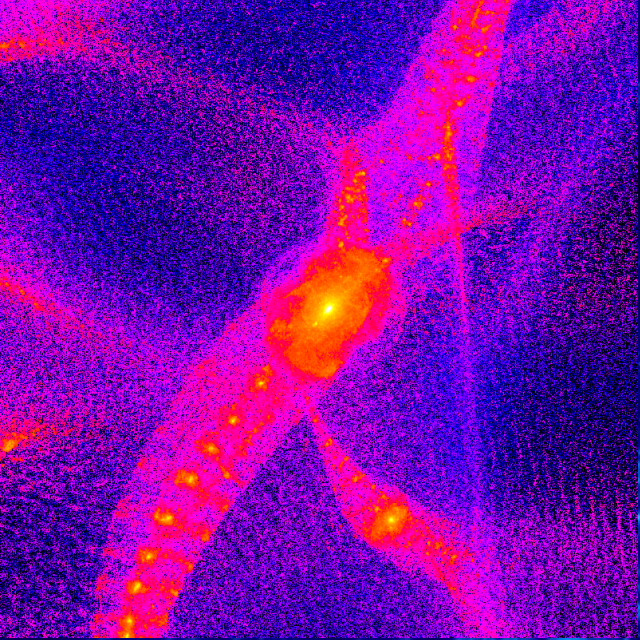
288 AMD MP2200+ processors, 144 Gigs ram

Compact, easy to cool and maintain

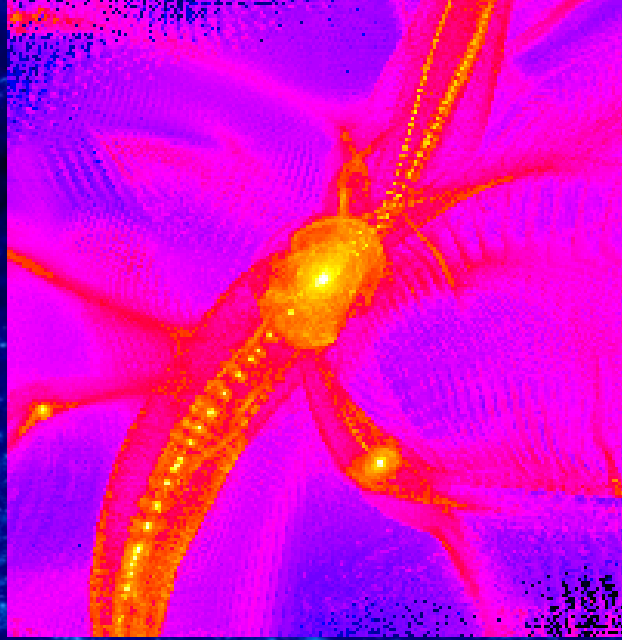
Very fast Dolphin/SCI interconnects - 4 Gbit/s,  
microsecond latency

A teraflop supercomputer for \$500,000

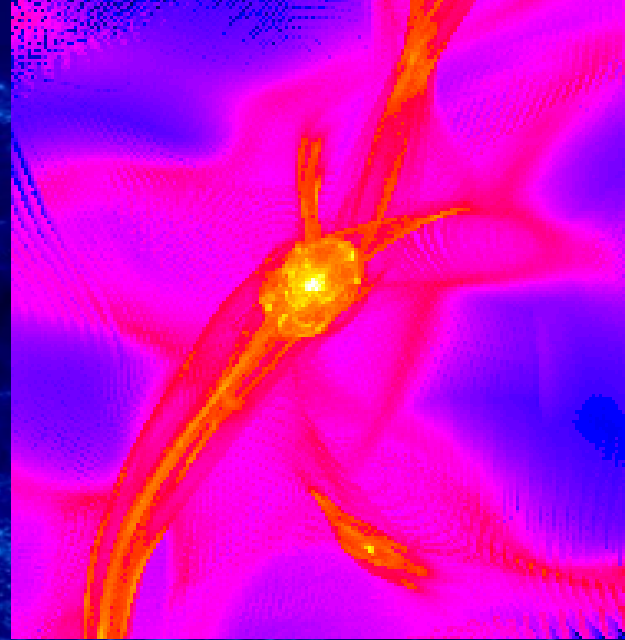
Roughly one cubic meter, one ton and requires 40  
kilowatts of power



"Low" resolution

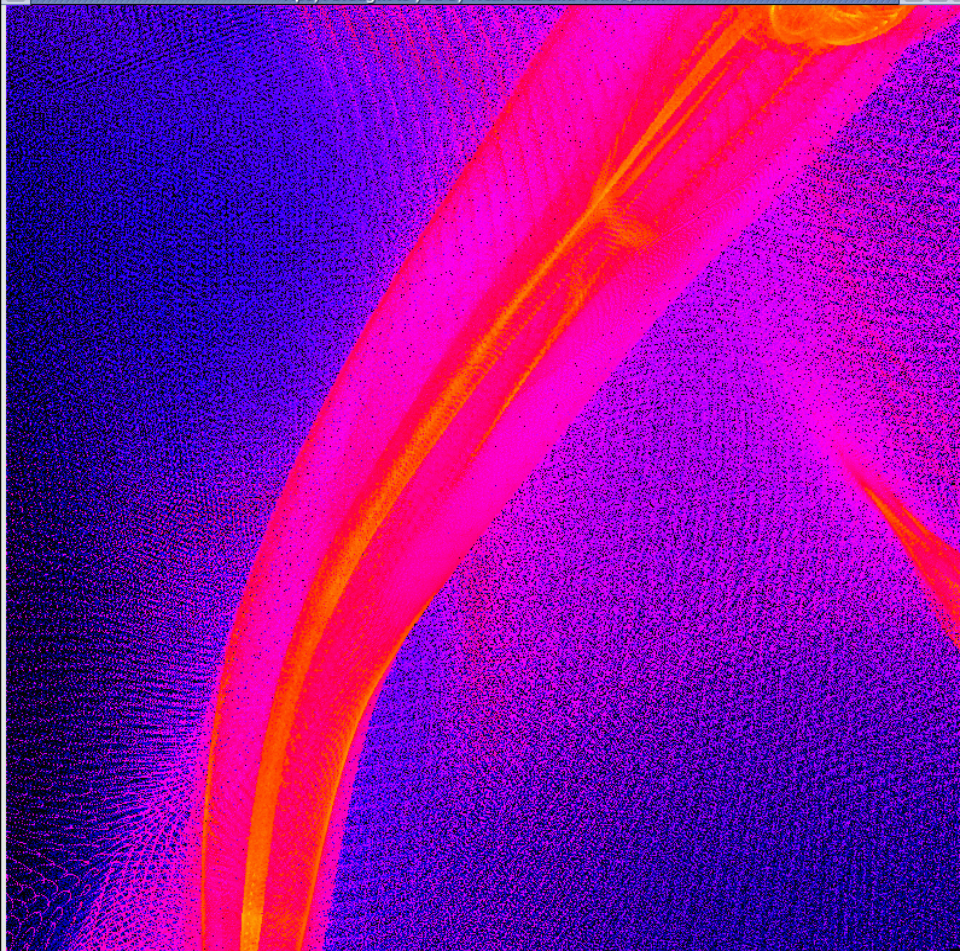
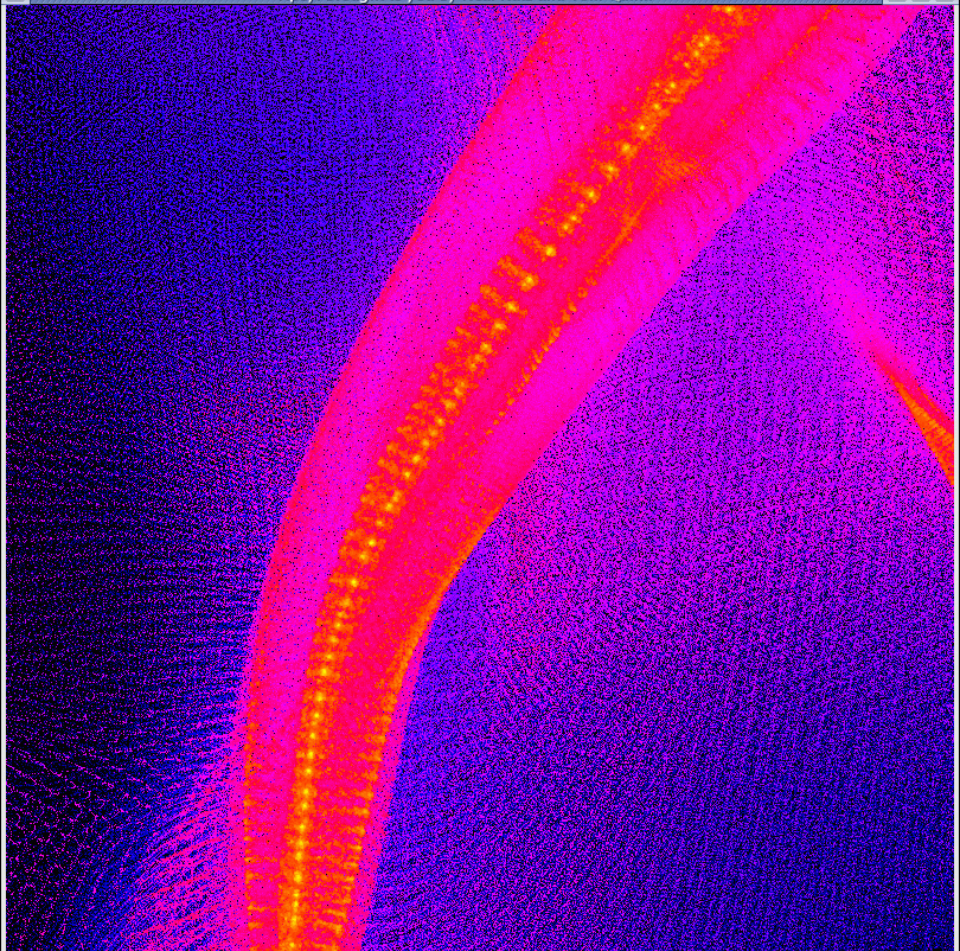


"High" resolution



"High" resolution  
(Large softening)





Color

zoom  unzoom

8.00x

Down  Up

Left  Right

Cntr  Clock

Color

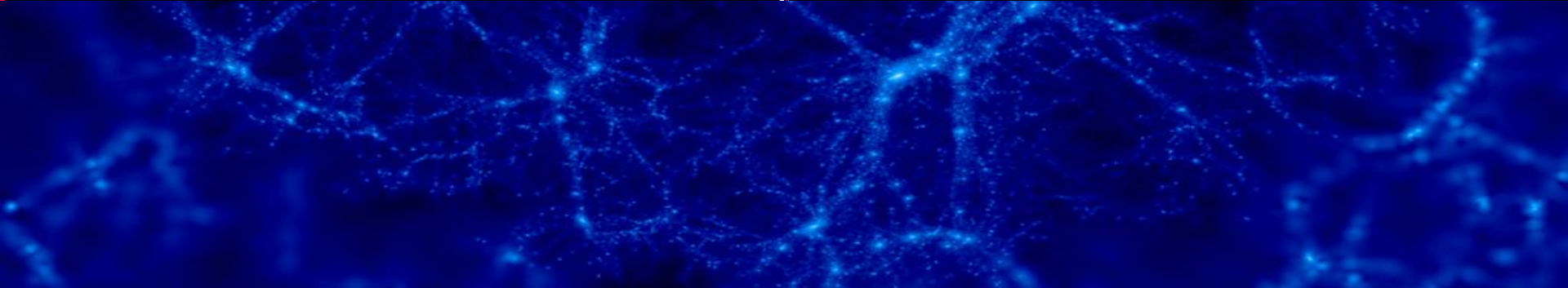
zoom  unzoom

8.00x

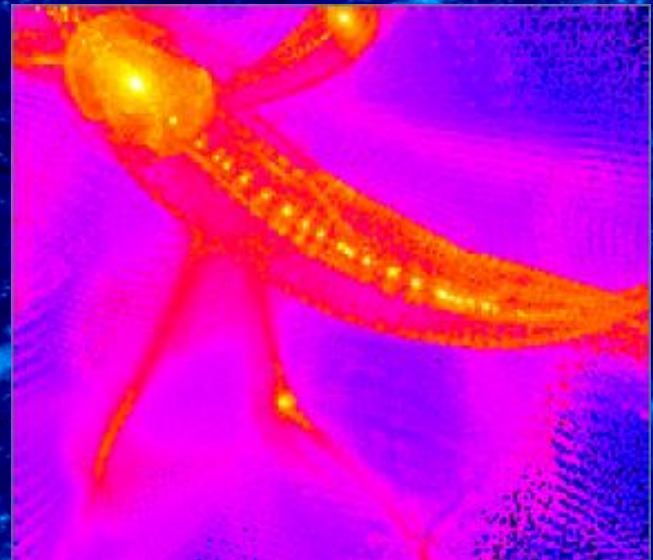
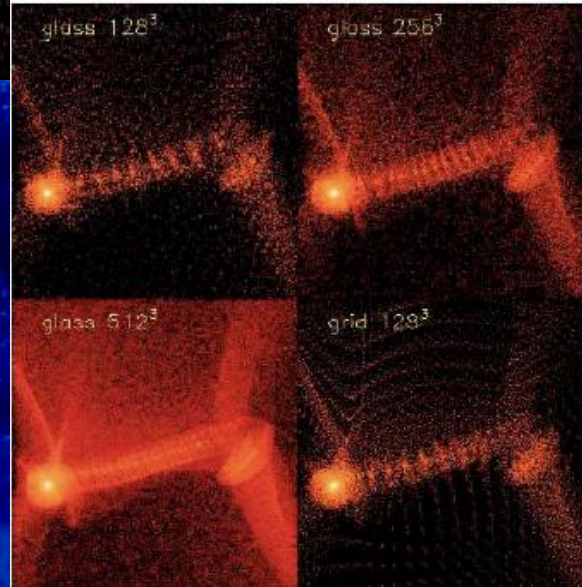
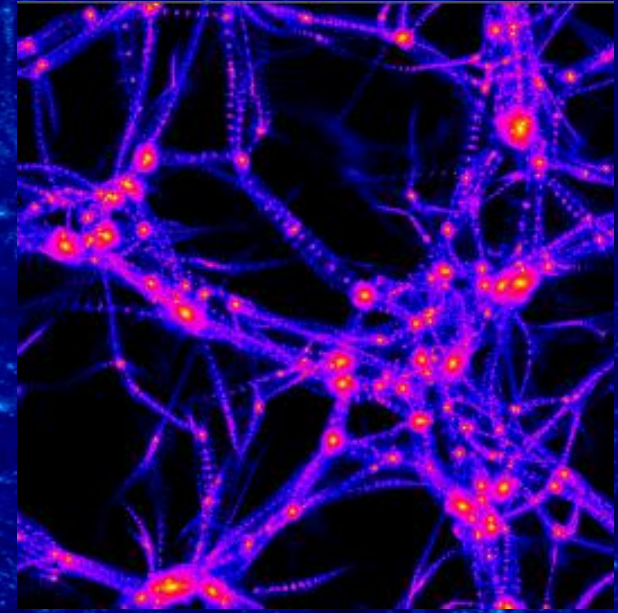
Down  Up

Left  Right

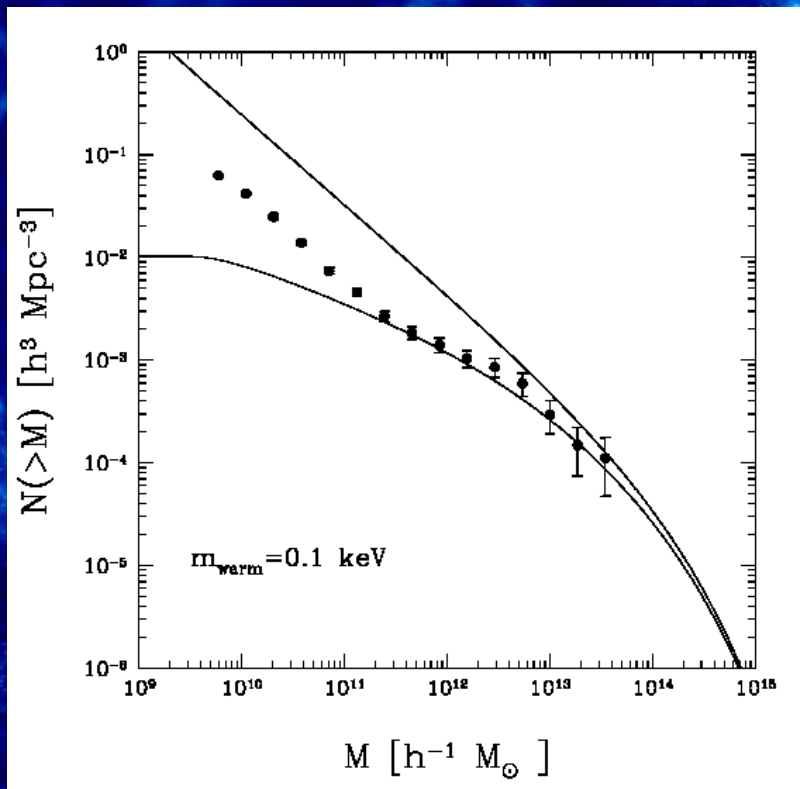
Cntr  Clock



grid  $128^3$   
Wang & White 2007

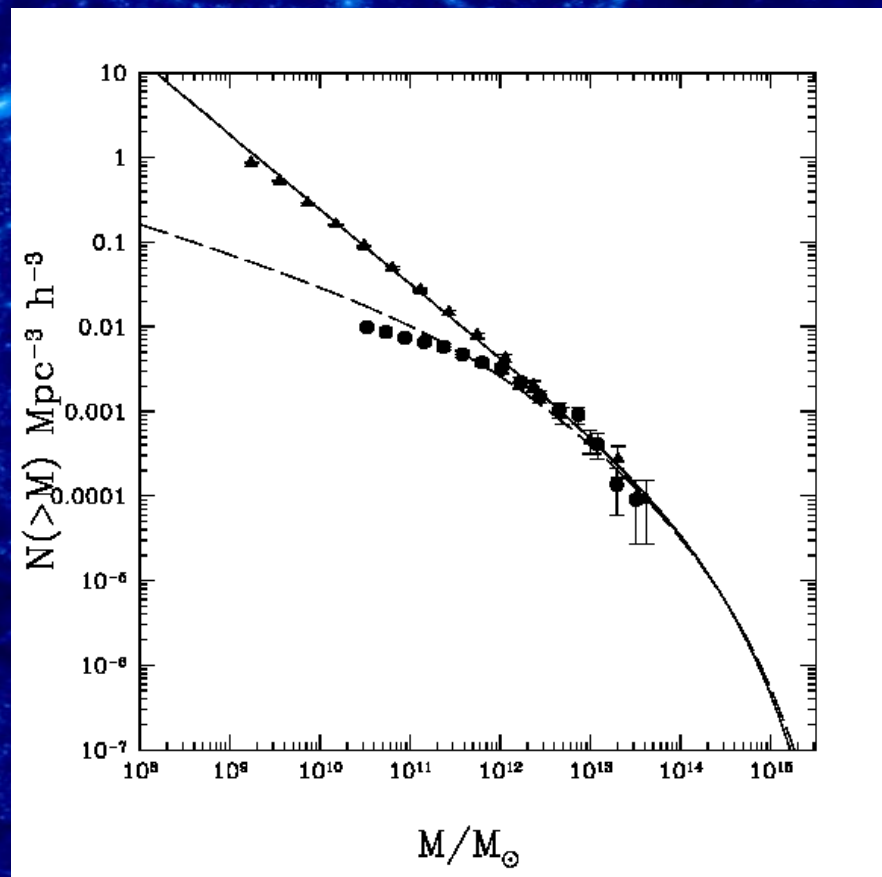


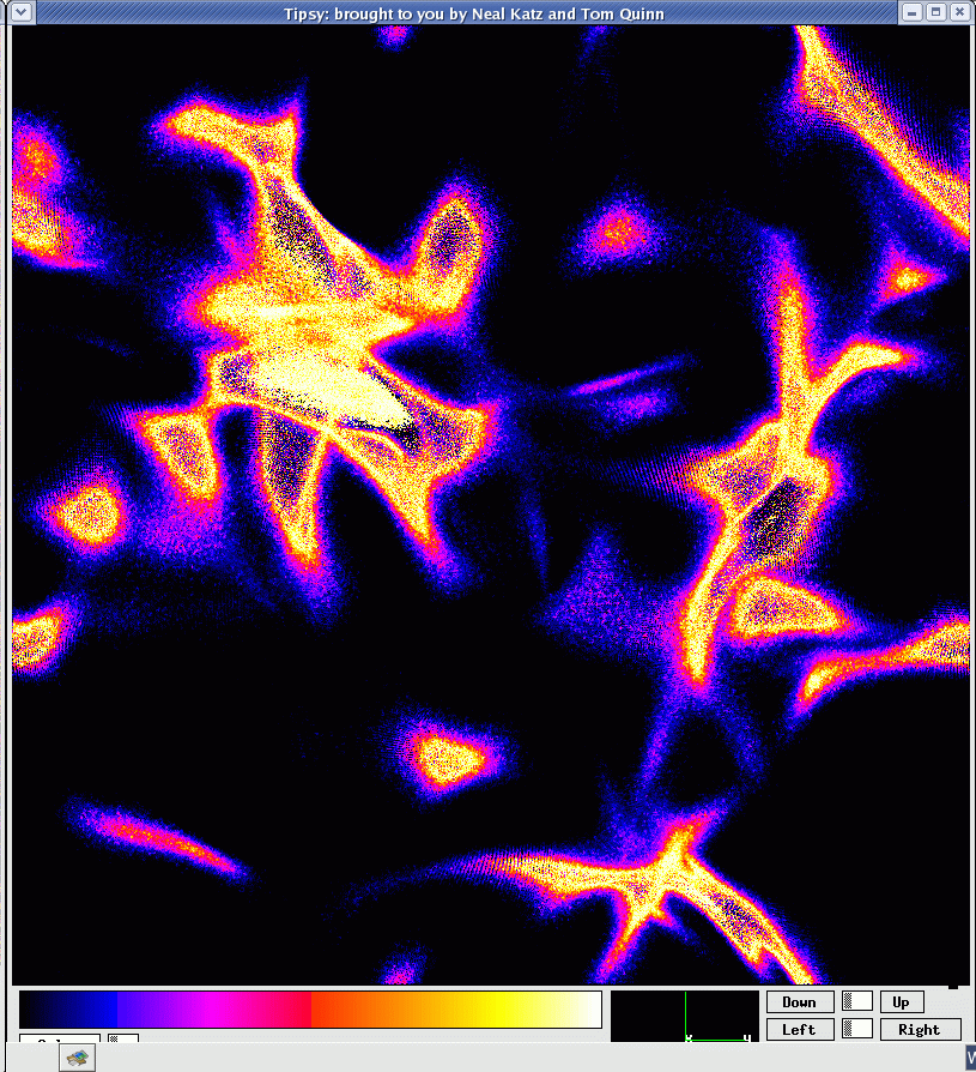
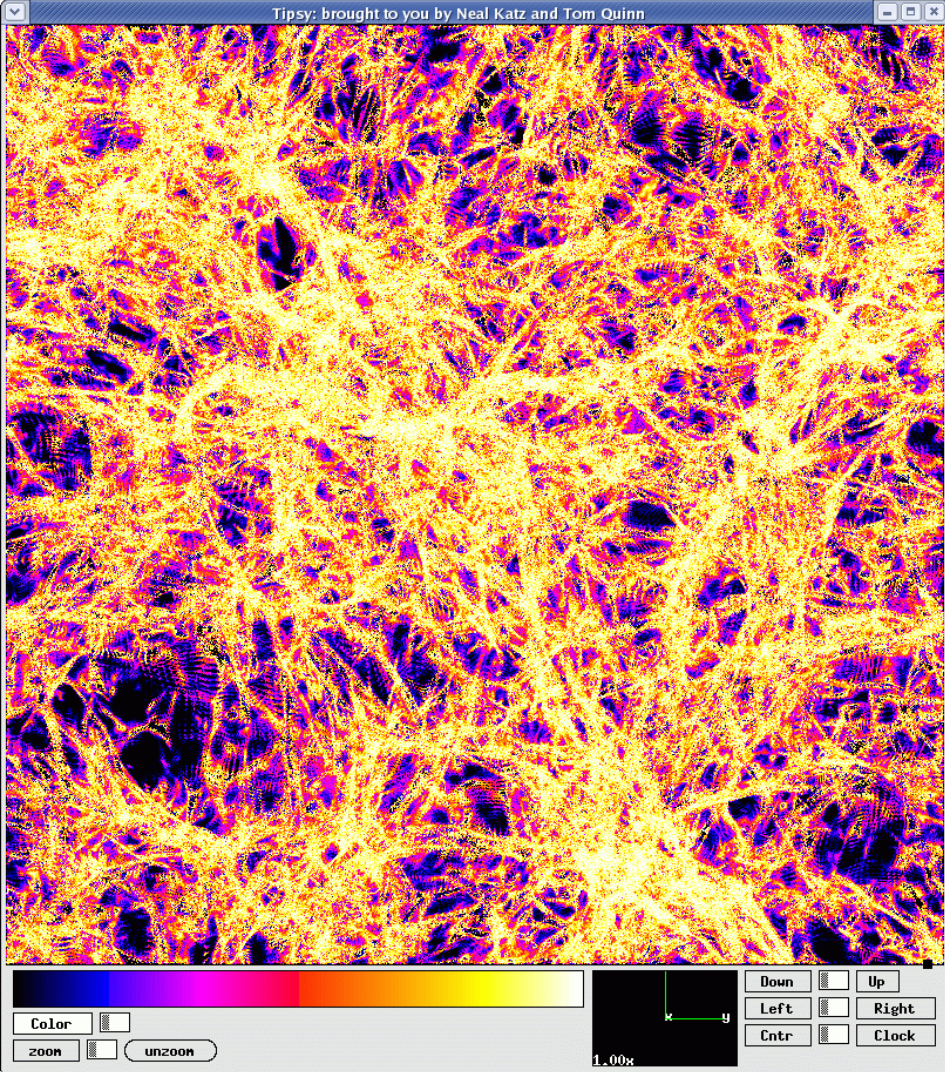




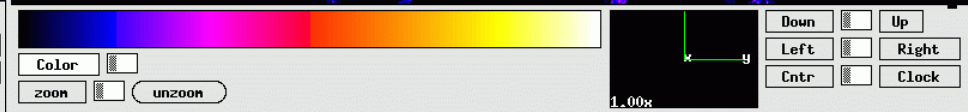
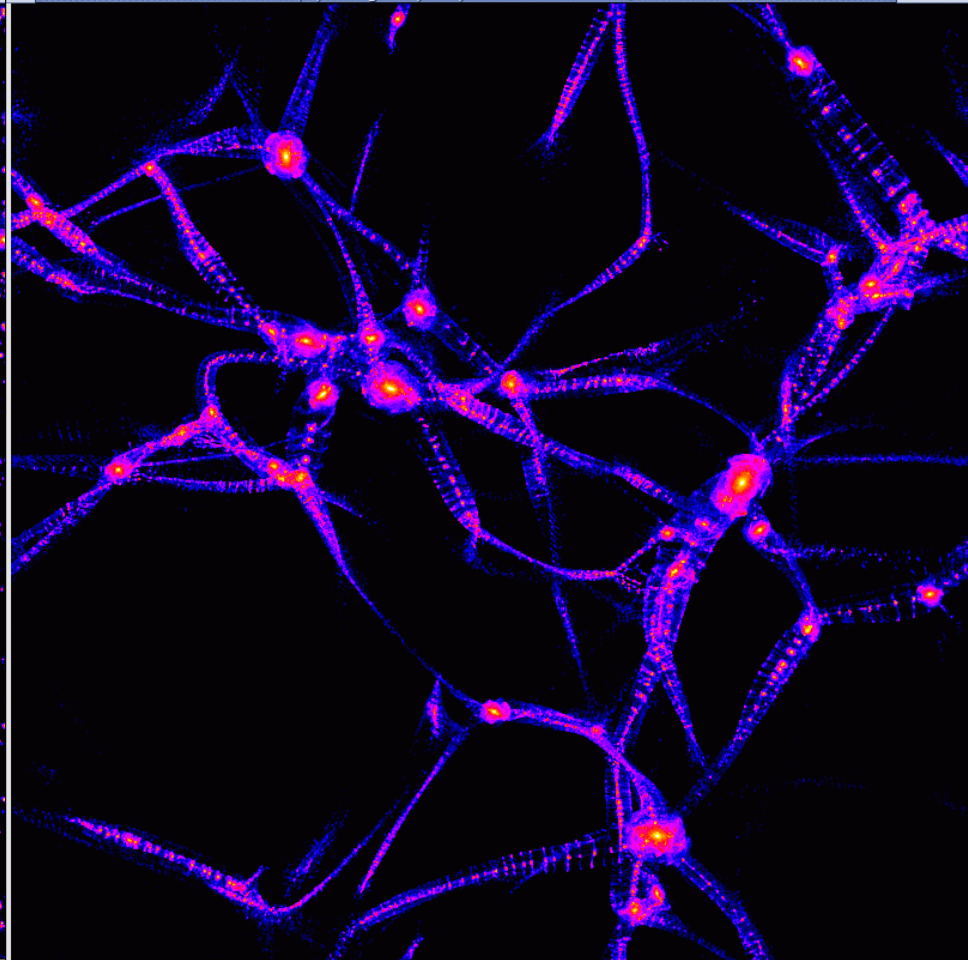
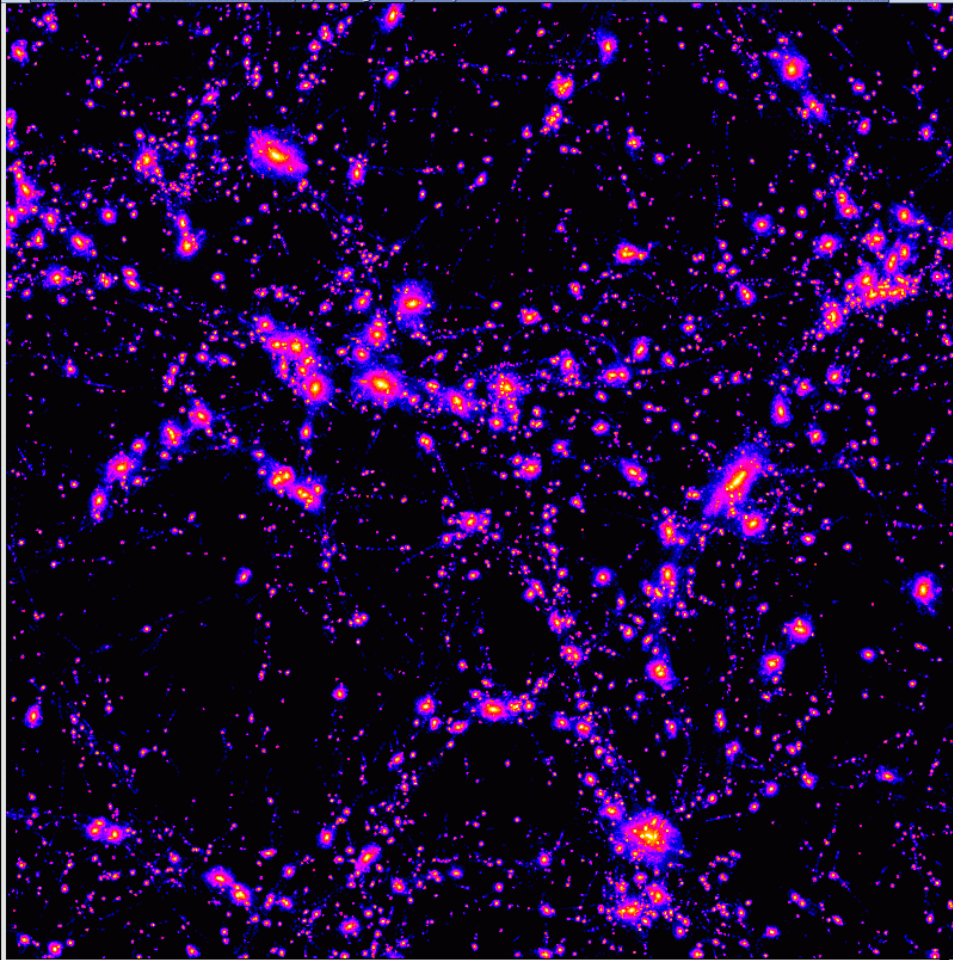
WDM  $N_p > 50$

CDM & WDM  $N_p > 500$

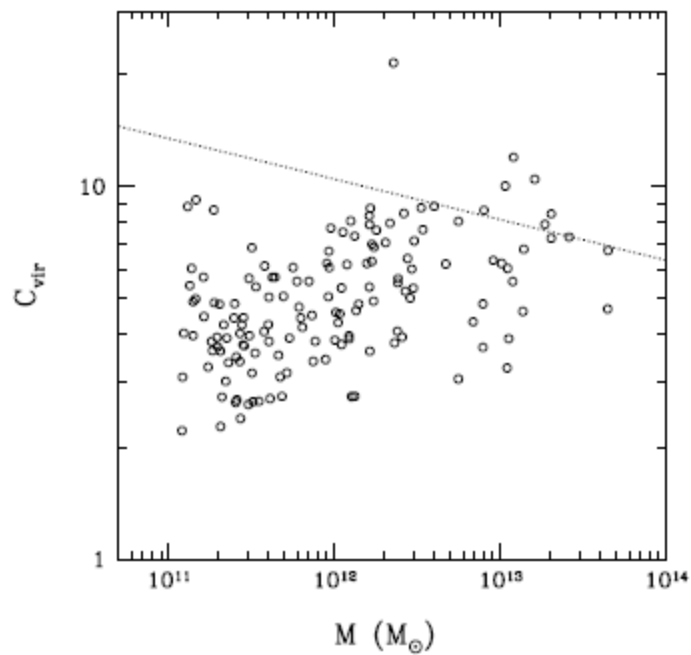




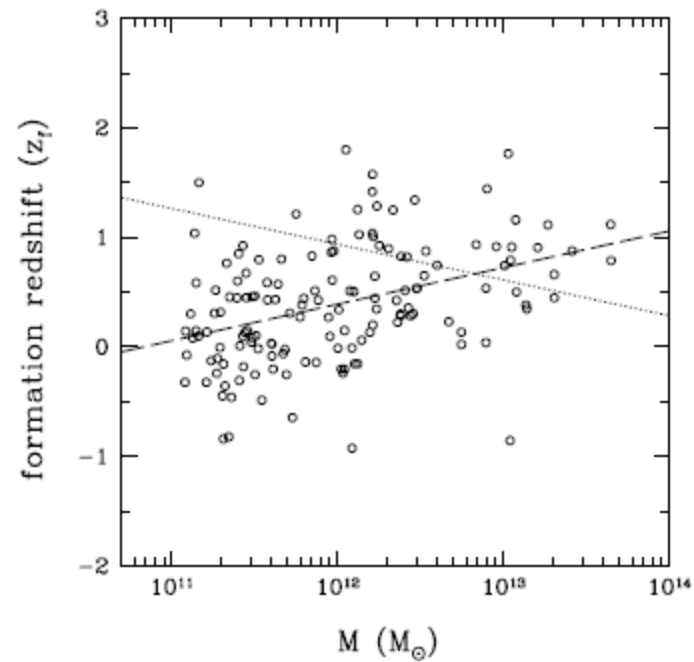
Mildly non-linear regions at  $z=3$  in CDM and WDM (200eV)  
i.e. overdensities between 1 and 5 w.r.t. mean



Virialised regions at  $z=3$  in CDM and WDM (200ev)  
i.e. overdensities higher than 100 w.r.t. mean



**Figure 7.** The mass concentration relation for haloes in the WDM6 simulation. The dotted line shows results for the  $\Lambda$ CDM model from Macciò et al. (2007).



**Figure 8.** The relation between the formation redshift and mass of haloes in the WDM6 simulation. The dashed line is a fit to the WDM data, the dotted line shows the expected behaviour in a  $\Lambda$ CDM scenario.

# CONCLUSIONS

- The finite initial fine grained PSD is also a maximum of coarse grained PSD.
- The turn over in PSD results in constant density core with characteristic size.
- Formation of haloes in WDM models differs from CDM. Large haloes form first from the fragmentation of intersecting filaments
- The concentration – mass relation for WDM haloes is reversed with respect to that found for CDM
- Spurious fragmentation below the free streaming scale hard to overcome – in case of infinite resolution a filament collapses into a two dimensional line
- If the primordial velocities are large enough to produce a significant core in dwarf galaxies, then the free streaming erases all perturbations on that scale so that the haloes can not form.
- Warm dark matter haloes contain visible caustics and shells.