Warm (keV) Dark Matter and Galaxy properties from primordial fluctuations and observations

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Chalonge Meudon WDM Workshop

8-10 June 2011

Dark Matter: from microphysics to Galaxies

❖ Cold (CDM): small velocity dispersion: small structures form first, bottom-up hierarchical growth formation, *too heavy (GeV)*

❖Hot (HDM) : large velocity dispersion: big structures form first, top-down, fragmentation, ruled out , *too light* (eV)

Warm (WDM): ``in between'' (keV)

ΛWDM Concordance Model:

CMB + LSS + SSS Observations

DM is WARM and COLLISIONLESS

CDM Problems:

- "clumpy halo problem'', large number of satellite galaxies "satellite problem"
- ρ (r) ~ 1 / r (cusp)
- And other problems…..

[DARK MATTER : FACTS AND STATUS

→ DARK MATTER DOES EXIST

→ ASTROPHYSICAL OBSERVATIONS POINTS TO THE EXISTENCE OF DARK MATTER

→ AFTER MORE THAN TWENTY YEARS OF DEDICATED DARK MATTER PARTICLE EXPERIMENTS, THE DIRECT SEARCH OF DARK MATTER PARTICLES FULLY CONCENTRATED IN "GeV WIMPS" REVEALED SO FAR, UNSUCCEFULL. BUT DARK MATTER DOES EXIST

IN DESPITE OF THAT: PROPOSALS TO REPLACE DARK MATTER DID APPEARED:

PROPOSING TO CHANGE THE LAWS OF PHYSICS (!!!), ADDING OVER CONFUSION, MIXING , POLLUTION...

TODAY, THE DARK MATTER RESEARCH AND DIRECT SEARCH SEEMS TO SPLIT IN THREE SETS:

(1). PARTICLE PHYSICS DARK MATTER: PARTICLE BUILDING MODELS, DEDICATED LAB EXPERIMENTS, ANNHILATING DARK MATTER, (FULLY CONCENTRATED ON “GeV WIMPS”)

(2). ASTROPHYSICAL DARK MATTER: (ASTROPHYSICAL MODELS, ASTROPHYSICAL OBSERVATIONS)

(3). NUMERICAL SIMULATIONS

(1) and (2) DO NOT AGREE IN THE RESULTS
and (2) and (3) DO NOT FULLY AGREE NEITHER

SOMETHING IS GOING WRONG IN THE RESEARCH ON THE DARK MATTER

WHAT IS GOING WRONG ?, [AND WHY IS GOING WRONG]

“FUIT EN AVANT” (“ESCAPE TO THE FUTURE”) IS NOT THE ISSUE

THE SUBJECT IS MATURE

→ THERE EXIST ASTRONOMICAL OBSERVATIONS AND FACILITIES

→ THERE EXIST MODEL /THEORETICAL ASTROPHYSICAL RESULTS WHICH FIT, AGREE WITH THE ASTRONOMICAL OBSERVATIONS

→ THERE EXISTED,THERE EXIST MANY DARK MATTER DEDICATED PARTICLE EXPERIMENTS (ALTHOUGH FULLY CONCENTRATED IN "GeV WIMPS")

→ THERE EXIST COMPUTER AND SUPER COMPUTERS AND DIFFERENT RESEARCHER GROUPS PERFORMING WORK WITH THEM

→ THERE EXIST A CONSIDERABLE AMOUNT OF RESEARCHERS WORKING IN DARK MATTER DURING MORE THAN TWENTY YEARS

" FUITE EN AVANT" ("ESCAPE TO THE FUTURE") IS NOT THE ISSUE

WHAT IS WRONG in the present day subject of Dark Matter?,

(The Answer is Trivial and can be found in these 3 slides)]

CONTENTS OF THIS LECTURE

(0) FRAMEWORK

(I) THE MASS OF THE DARK MATTER PARTICLE

(II) THE BOLTZMAN VLASOV EQUATION: TRANSFERT FUNCTION AND ANALYTIC RESULTS

(III) UNIVERSAL PROPERTIES OF GALAXIES: DENSITY PROFILES, SURFACE DENSITY, AND THE POWER OF LINEAR APPROXIMATION

MASS OF THE DARK MATTER PARTICLE

H. J. De Vega, N.G. Sanchez *Model independent analysis of dark matter points to a particle mass at the keV scale* Mon. Not. R. Astron. Soc. 404, 885 (2010)

D. Boyanovsky, H. J. de Vega, N.G. Sanchez *Constraints on dark matter particles from theory, galaxy observations and N-body simulations* Phys.Rev. D77 043518, (2008)

BOLTZMAN VLASOV EQUATION, TRANSFERT FUNCTION

D. Boyanovsky, H. J. de Vega, N.G. Sanchez *The dark matter transfer function: free streaming, particle statistics and memory of gravitational clustering* Phys. Rev. D78: 063546, (2008)

DENSITY PROFILES, SURFACE DENSITY, DARK MATTER PARTICLE MASS

H. J. de Vega, N.G. Sanchez *Gravity surface density and density profile of dark matter galaxies* IJMPA26:1057 (2011)

H. J. de Vega, P. Salucci, N.G. Sanchez *Universal galaxy properties and the mass of the dark matter particle from theory and observations: the power of the linear approximation* arXiv:1004.1908

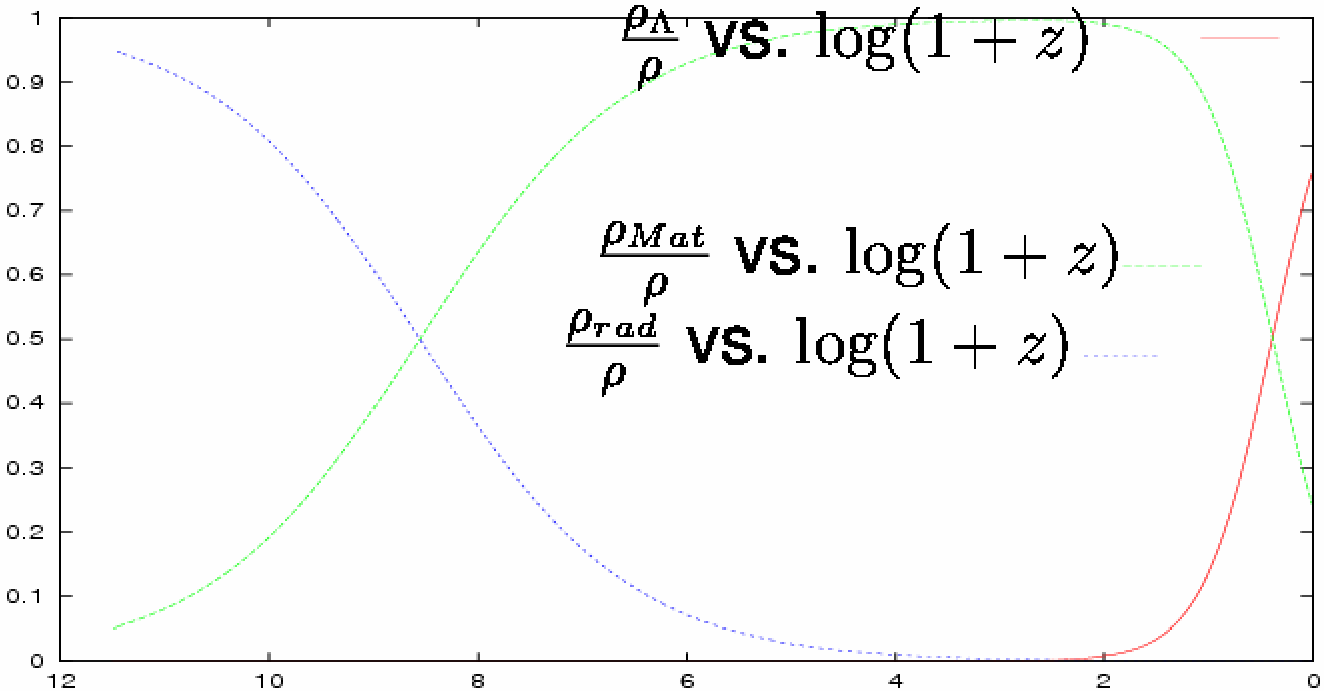
CONTENT OF THE UNIVERSE

ATOMS, the building blocks of stars and planets: represent only the 4.6%

DARK MATTER comprises 23.4 % of the universe. This matter, different from atoms, does not emit or absorb light. It has only been detected indirectly by its gravity.

72% of the Universe, is composed of DARK ENERGY that acts as a sort of an anti-gravity.
This energy, distinct from dark matter, is responsible for the present-day acceleration of the universe expansion, compatible with a cosmological constant

The Universe is made of radiation, matter and dark energy



End of inflation: $z \sim 10^{29}$, $T_{reh} \lesssim 10^{16}$ GeV, $t \sim 10^{-36}$ sec.

E-W phase transition: $z \sim 10^{15}$, $T_{\mathrm{EW}} \sim 100$ GeV, $t \sim 10^{-11}$ s.

QCD conf. transition: $z \sim 10^{12}$, $T_{\mathrm{QCD}} \sim 170$ MeV, $t \sim 10^{-5}$ s.

BBN: $z \sim 10^{9}$, $T \simeq 0.1$ MeV, $t \sim 20$ sec.

Rad-Mat equality: $z \simeq 3200$, $T \simeq 0.7$ eV, $t \sim 57000$ yr.

CMB last scattering: $z \simeq 1100$, $T \simeq 0.25$ eV , $t \sim 370000$ yr.

Mat-DE equality: $z \simeq 0.47$, $T \simeq 0.345$ meV , $t \sim 8.9$ Gyr.

Today: $z = 0$, $T = 2.725$K $= 0.2348$ meV $t = 13.72$ Gyr.

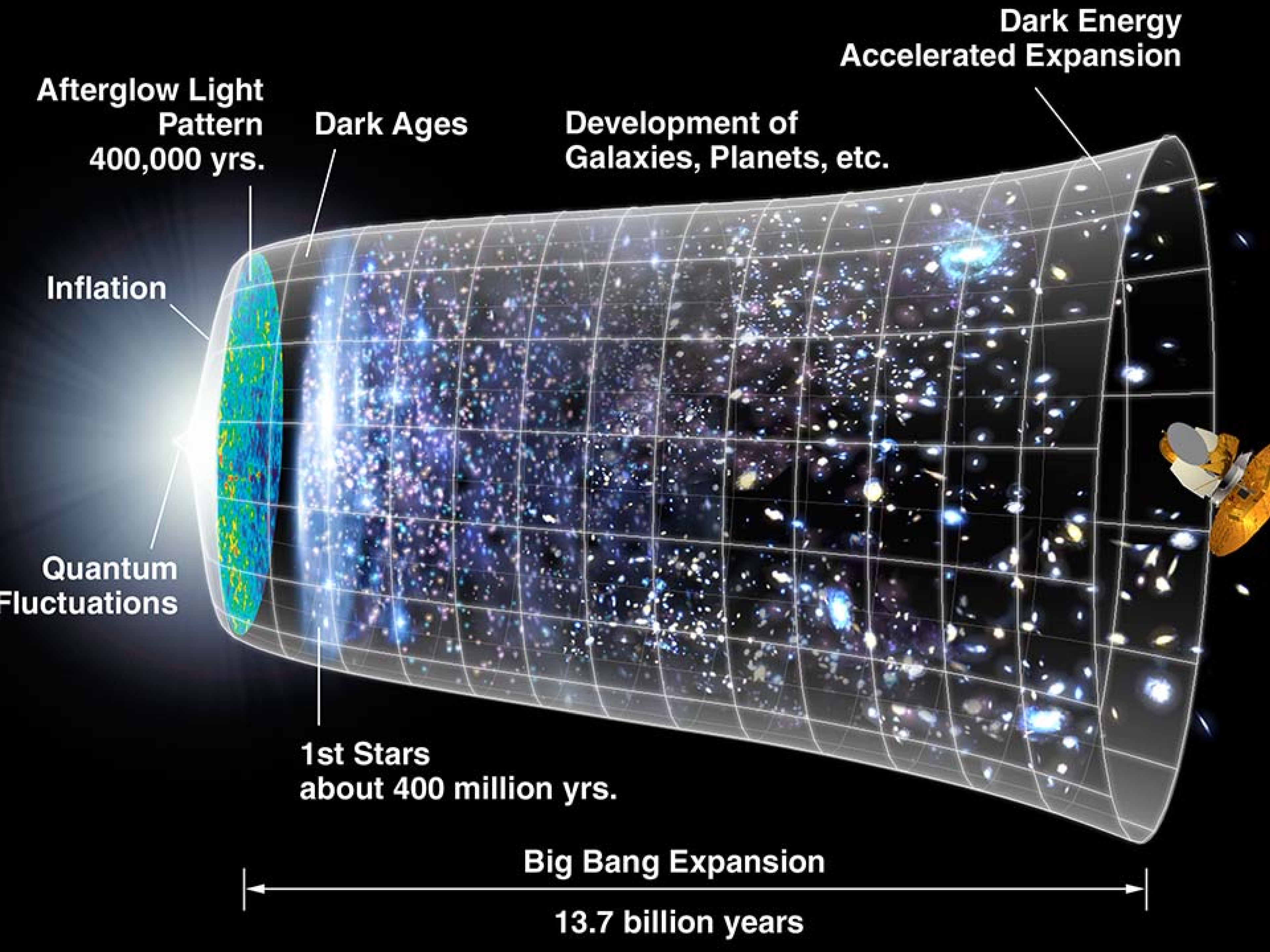
Standard Cosmological Model: DM + Λ + Baryons + Rad

- Begins by the inflationary era. Slow-Roll inflation explains horizon and flatness.
- Gravity is described by Einstein's General Relativity.
- Particle Physics described by the Standard Model of Particle Physics: $SU(3) \otimes SU(2) \otimes U(1) =$ qcd+electroweak model.
- Dark matter is non-relativistic during the matter dominated era where structure formation happens. DM is outside the SM of particle physics.
- Dark energy described by the cosmological constant Λ.

Standard Cosmological Model: ΛCDM $\Rightarrow$ ΛWDM

Dark Matter + Λ + Baryons + Radiation
begins by the Inflationary Era. Explains the Observations:

- Seven years WMAP data and further CMB data
- Light Elements Abundances
- Large Scale Structures (LSS) Observations. BAO.
- Acceleration of the Universe expansion: Supernova Luminosity/Distance and Radio Galaxies.
- Gravitational Lensing Observations
- Lyman α Forest Observations
- Hubble Constant and Age of the Universe Measurements
- Properties of Clusters of Galaxies
- Galaxy structure explained by WDM


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
400,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years

Quantum Fluctuations During Inflation and after

The Universe is homogeneous and isotropic after inflation thanks to the fast and gigantic expansion stretching lenghts by a factor $e^{62} \simeq 10^{27}$. By the end of inflation: $T \sim 10^{14}$ GeV.

Quantum fluctuations around the classical inflaton and FRW geometry were of course present.

These inflationary quantum fluctuations are the seeds of the structure formation and of the CMB anisotropies today: galaxies, clusters, stars, planets, ...

That is, our present universe was built out of inflationary quantum fluctuations. CMB anisotropies spectrum:

$3 \times 10^{-32}\text{cm} < \lambda_{begin\,inflation} < 3 \times 10^{-28}\text{cm}$

$M_{Planck} \gtrsim 10^{18}$ GeV $> \lambda^{-1}_{begin\,inflation} > 10^{14}$ GeV.

total redshift since inflation begins till today $= 10^{56}$:

0.1 Mpc $< \lambda_{today} <$ 1 Gpc , 1 pc $= 3 \times 10^{18}$ cm $= 200000$ AU

Universe expansion classicalizes the physics: decoherence

THE HISTORY OF THE UNIVERSE IS A HISTORY of EXPANSION and COOLING DOWN

THE EXPANSION OF THE UNIVERSE IS THE MOST POWERFUL REFRIGERATOR

INFLATION PRODUCES THE MOST POWERFUL STRETCHING OF LENGTHS

THE EVOLUTION OF THE UNIVERSE IS FROM QUANTUM TO SEMICLASSICAL TO CLASSICAL

From Very Quantum (Quantum Gravity) state to Semiclassical Gravity (Inflation) stage (Accelerated Expansion) to Classical Radiation dominated Era followed by Matter dominated Era (Deccelerated expansion) to Today Era (again Accelerated Expansion)

THE EXPANSION CLASSICALIZES THE UNIVERSE

THE EXPANSION OF THE UNIVERSE IS THE MOST POWERFUL QUANTUM DECOHERENCE MECHANISM

THE MASS OF THE

DARK MATTER PARTICLE

→Compilation of observations of galaxies candidates for DM structure, are compatible with a core of smooth central density and a low mean mass density ~ 0.1 Msun /pc^3 rather than with a cusp.

→Dark matter particles can decouple being ultrarelativistic or non-relativistic. Dark matter must be non-relativistic during structure formation in order to reproduce the observed small structure at ~ 2 – 3 kpc.

→In addition, the decoupling can occurs at local thermal equilibrium or out of local thermal equilibrium. All these cases have been considered in our analysis.

OBSERVATIONS

The observed dark matter energy density observed today has the value $\rho_{DM} = 0.228\ (2.518\ \mathrm{meV})^4$.

In addition, compilation of galaxy observations yield the one dimensional velocity dispersion σ and the radius L in the ranges

$$6.6\ \mathrm{km/s} \leq \sigma \leq 11.1\ \mathrm{km/s}\ , \qquad 0.5\ \mathrm{kpc} \leq L \leq 1.8\ \mathrm{kpc}$$

And the Phase-space Density today (with a precision of a factor 10) has the value :

$$D(0) \sim 5 \times 10^3\ [\mathrm{keV/cm^3}]\ (\mathrm{km/s})^{-3} = (0.18\ \mathrm{keV})^4\ .$$

→**Compute** **from the distribution function of dark matter particles with their different statistics, physical magnitudes as :**

-the dark matter energy density $\rho_{DM}(z)$,

-the dark matter velocity dispersion $\sigma_{DM}(z)$,

-the dark matter density in the phase space D(z)

→ **Confront to their values observed today (z = 0).**

→→ **From them, the mass m of the dark matter particle and its decoupling temperature T_d are obtained.**

The phase-space density today is a factor Z smaller than its primordial value**. The decreasing factor Z > 1 is due to the effect of self-gravity interactions: the range of Z is computed.**

Dark Matter

DM particles can decouple being ultrarelativistic (UR) at $T_d \gg m$ or non-relativistic $T_d \ll m$.

We consider particles that decouple at or out of LTE (LTE = local thermal equilibrium).

Distribution function: $F_d[p_c]$ freezes out at decoupling.

$p_c =$ comoving momentum.

$P_f(t) = p_c/a(t) =$ Physical momentum,

Velocity fluctuations: $y = P_f(t)/T_d(t) = p_c/T_d$

$$\langle \vec{V}^2(t) \rangle = \langle \frac{\vec{P}_f^2(t)}{m^2} \rangle = \left[\frac{T_d}{m\, a(t)} \right]^2 \frac{\int_0^\infty y^4 F_d(y) dy}{\int_0^\infty y^2 F_d(y) dy} .$$

Energy Density: $\rho_{DM}(t) = \frac{m\, g}{2\pi^2} \frac{T_d^3}{a^3(t)} \int_0^\infty y^2\, F_d(y)\, dy$,

g : $\#$ of internal degrees of freedom of the DM particle, $1 \leq g \leq 4$. Formula valid when DM particles are non-relativistic.

The formula for the Mass of the Dark Matter particles

Energy Density: $\rho_{DM}(t) = g \int \frac{d^3 P_f}{(2\pi)^3} \sqrt{m^2 + P_f^2}\; f_d[a(t)\, P_f]$

g : # of internal degrees of freedom of the DM particle, $1 \leq g \leq 4$. For $z \lesssim 30 \;\Rightarrow$ DM particles are non-relativistic:

$\rho_{DM}(t) = m\; g\; \frac{T_d^3}{a^3(t)} \int_0^\infty y^2\; f_d(y)\; \frac{dy}{2\pi^2}\;.$

Using entropy conservation: $T_d = \left(\frac{2}{g_d}\right)^{\frac{1}{3}} T_\gamma\; (1 + z_d)$,

$g_d =$ effective # of UR degrees of freedom at decoupling,
$T_\gamma = 0.2348$ meV , $1 \text{ meV} = 10^{-3}$ eV.
Today $\Omega_{DM} = \rho_{DM}(0)/\rho_c = 0.105/h^2$ and we obtain for the mass of the DM particle:

$$m = 6.986 \text{ eV}\; \frac{g_d}{g \displaystyle\int_0^\infty y^2\; f_d(y)\; dy}\;.$$ Goal: determine m and g_d

Dark Matter density and DM velocity dispersion

Energy Density: $\rho_{DM}(t) = g \int \frac{d^3 P_f}{(2\pi)^3} \sqrt{m^2 + P_f^2}\, F_d[a(t)\, P_f]$

g : # of internal degrees of freedom of the DM particle, $1 \leq g \leq 4$. For $z \lesssim 30 \;\Rightarrow$ DM particles are non-relativistic:

$$\rho_{DM}(t) = \frac{m\, g}{2\pi^2}\, \frac{T_d^3}{a^3(t)} \int_0^\infty y^2\, F_d(y)\, dy \; ,$$

Using entropy conservation: $T_d = \left(\frac{2}{g_d}\right)^{\frac{1}{3}}\, T_{CMB}$,

$g_d =$ effective # of UR degrees of freedom at decoupling, $T_{CMB} = 0.2348\, 10^{-3}$ eV, and

$$\rho_{DM}(\text{today}) = \frac{m\, g}{\pi^2\, g_d}\, T_{CMB}^3 \int_0^\infty y^2\, F_d(y)\, dy = 1.107\, \frac{\text{keV}}{\text{cm}^3} \quad (1)$$

We obtain for the primordial velocity dispersion:

$$\sigma_{DM}(z) = \sqrt{\frac{1}{3}\, \langle \vec{V}^2 \rangle(z)} = 0.05124\, \frac{1+z}{g_d^{\frac{1}{3}}} \left[\frac{\int_0^\infty y^4\, F_d(y)\, dy}{\int_0^\infty y^2\, F_d(y)\, dy}\right]^{\frac{1}{2}} \frac{\text{keV}}{m}\, \frac{\text{km}}{\text{s}}$$

Goal: determine m and g_d. We need TWO constraints.

Phase-space density invariant under universe expansion

Using again entropy conservation to replace T_d yields for the one-dimensional velocity dispersion,

$$\sigma_{DM}(z) = \sqrt{\frac{1}{3}\,\langle \vec{V}^2 \rangle(z)} = \frac{2^{\frac{1}{3}}}{\sqrt{3}}\;\frac{1+z}{g_d^{\frac{1}{3}}}\;\frac{T_\gamma}{m}\;\sqrt{\frac{\int_0^\infty y^4\;F_d(y)\;dy}{\int_0^\infty y^2\;F_d(y)\;dy}} =$$

$$= 0.05124\;\frac{1+z}{g_d^{\frac{1}{3}}}\;\frac{\mathrm{keV}}{m}\;\left[\frac{\int_0^\infty y^4\;F_d(y)\;dy}{\int_0^\infty y^2\;F_d(y)\;dy}\right]^{\frac{1}{2}}\;\frac{\mathrm{km}}{\mathrm{s}}.$$

Phase-space density: $\mathcal{D} \equiv \frac{n(t)}{\langle \vec{P}^2_{phys}(t) \rangle^{\frac{3}{2}}} \overset{\text{non-rel}}{=} \frac{\rho_{DM}}{3\sqrt{3}\,m^4\;\sigma^3_{DM}}$

$\mathcal{D}$ is computed theoretically from freezed-out distributions:

$$\mathcal{D} = \frac{g}{2\,\pi^2}\;\frac{\left[\int_0^\infty y^2 F_d(y)dy\right]^{\frac{5}{2}}}{\left[\int_0^\infty y^4 F_d(y)dy\right]^{\frac{3}{2}}}$$

Theorem: The phase-space density $\mathcal{D}$ can only decrease under self-gravity interactions (gravitational clustering) [Lynden-Bell, Tremaine, Henon, 1986].

he Phase-space density $Q = \rho/\sigma^3$ and its decrease factor

The phase-space density $Q \equiv \rho/\sigma^3$ is invariant under the cosmological expansion and can only decrease under self-gravity interactions (gravitational clustering).

The phase-space density today follows observing dwarf spheroidal satellite galaxies of the Milky Way (dSphs)

$\frac{\rho_s}{\sigma_s^3} \sim 5 \times 10^3 \, \frac{\mathrm{keV/cm^3}}{(\mathrm{km/s})^3} = (0.18 \, \mathrm{keV})^4$ Gilmore et al. 07 and 08.

During structure formation ($z \lesssim 30$), $Q = \rho/\sigma^3$ decreases by a factor that we call Z:

$$Q_{today} = \frac{1}{Z} \, Q_{prim} \quad , \quad Q_{prim} = \frac{\rho_{prim}}{\sigma^3_{prim}} \quad , \qquad (2) \qquad Z > 1.$$

The spherical model gives $Z \simeq 41000$ and N-body simulations indicate: $10000 > Z > 1$. Z is galaxy dependent.

Constraints: First $\rho_{DM}(\mathrm{today})$, Second $Q_{today} = \rho_s/\sigma_s^3$

Mass Estimates for DM particles

Combining the previous expressions lead to general formulas for m and g_d:

$$m = 0.2504\,\mathrm{keV}\ \left(\frac{Z}{g}\right)^{\frac{1}{4}} \frac{\left[\int_0^\infty y^4\ F_d(y)\ dy\right]^{\frac{3}{8}}}{\left[\int_0^\infty y^2\ F_d(y)\ dy\right]^{\frac{5}{8}}}$$

$$g_d = 35.96\, Z^{\frac{1}{4}}\ g^{\frac{3}{4}}\ \left[\int_0^\infty y^4\ F_d(y)\ dy\ \int_0^\infty y^2\ F_d(y)\ dy\right]^{\frac{3}{8}}$$

These formulas yield for relics decoupling UR at LTE:

$$m = \left(\frac{Z}{g}\right)^{\frac{1}{4}}\ \mathrm{keV} \begin{cases} 0.568 \\ 0.484 \end{cases},\ g_d = g^{\frac{3}{4}}\ Z^{\frac{1}{4}} \begin{cases} 155 & \text{Fermions} \\ 180 & \text{Bosons} \end{cases}.$$

Since $g = 1 - 4$, we see that $g_d > 100 \Rightarrow T_d > 100$ GeV.

$1 < Z^{\frac{1}{4}} < 5.6$ for $1 < Z < 1000$. Example: for DM Majorana fermions $(g = 2)$ $m \simeq 0.85$ keV.

Mass Estimates for DM particles

Constraints: First $\rho_{DM}(\text{today})$, Second $Q_{today} = \rho_s/\sigma_s^3$

Combining the previous expressions lead to general formulas for m and g_d:

$$m = \frac{2^{\frac14}\sqrt{\pi}}{3^{\frac38}\, g^{\frac14}}\; Z^{\frac14}\; Q_{today}^{\frac14}\; \frac{I_4^{\frac38}}{I_2^{\frac58}}\ ,\quad g_d = \frac{2^{\frac14}\, g^{\frac34}}{3^{\frac38}\,\pi^{\frac32}\,\Omega_{DM}}\;\frac{T_\gamma^3}{\rho_c}\; Q_{today}^{\frac14}\; Z^{\frac14}\;\left[I_2\, I_4\right]^{\frac38}$$

where: $Q_{today}^{\frac14} = 0.18$ keV from the dSphs data,

$T_\gamma = 0.2348 \text{ meV}\ ,\ \Omega_{DM} = 0.228\ ,\ \rho_c = (2.36 \text{ meV})^4$

These formulas yield for relics decoupling UR at LTE:

$$m = \left(\frac{Z}{g}\right)^{\frac14} \text{keV} \left\{ \begin{array}{l} 0.568 \\ 0.484 \end{array} \right. ,\ g_d = g^{\frac34}\; Z^{\frac14} \left\{ \begin{array}{ll} 155 & \text{Fermions} \\ 180 & \text{Bosons} \end{array} \right. .$$

Since $g = 1-4$, we see that $g_d \gtrsim 100 \Rightarrow T_d \gtrsim 100$ GeV.

$1 < Z^{\frac14} < 10$ for $1 < Z < 10000$. Example: for DM Majorana fermions $(g=2)$ $0.5 \text{ keV} \lesssim m \lesssim 5$ keV.

Out of thermal equilibrium decoupling

Results for m and g_d on the same scales for DM particles decoupling UR out of thermal equilibrium.

For the χ model of sterile neutrinos where decoupling is out of thermal equilibrium:

$$0.56 \text{ keV} \lesssim m_\nu \, Z^{-\frac{1}{4}} \lesssim 1.0 \text{ keV} \quad , \quad 15 \lesssim g_d \, Z^{-\frac{1}{4}} \lesssim 84$$

Therefore, $0.6 \text{ keV} \lesssim m_\nu \lesssim 10 \text{ keV} \quad , \quad 20 \lesssim g_d \lesssim 850.$

Relics decoupling non-relativistic:
similar bounds: keV $\lesssim m \lesssim$ MeV

D. Boyanovsky, H. J. de Vega, N. Sanchez, Phys. Rev. D 77, 043518 (2008), arXiv:0710.5180.

H. J. de Vega, N. G. Sanchez, MNRAS 404, 885 (2010), arXiv:0901.0922.

Relics decoupling non-relativistic

$$F_d^{NR}(p_c) = \frac{2^{\frac{5}{2}}\,\pi^{\frac{7}{2}}}{45}\; g_d\, Y_\infty\; \left(\frac{T_d}{m}\right)^{\frac{3}{2}}\; e^{-\frac{p_c^2}{2\,m\,T_d}} = \frac{2^{\frac{5}{2}}\,\pi^{\frac{7}{2}}}{45}\; \frac{g_d\, Y_\infty}{x^{\frac{3}{2}}}\; e^{-\frac{y^2}{2\,x}}$$

$Y(t) = n(t)/s(t)$, $n(t)$ number of DM particles per unit volume, $s(t)$ entropy per unit volume, $x \equiv m/T_d$, $T_d < m$.

$Y_\infty = \frac{1}{\pi}\sqrt{\frac{45}{8}}\;\frac{1}{\sqrt{g_d}\;T_d\;\sigma_0\;M_{Pl}}$ late time limit of Boltzmann.

σ_0: thermally averaged total annihilation cross-section times the velocity.

From our general equations for m and g_d:

$$m = \frac{45}{4\,\pi^2}\;\frac{\Omega_{DM}\;\rho_c}{g\;T_\gamma^3\;Y_\infty} = \frac{0.748}{g\;Y_\infty}\;\text{eV} \quad \text{and} \quad m^{\frac{5}{2}}\;T_d^{\frac{3}{2}} = \frac{45}{2\,\pi^2}\;\frac{1}{g\;g_d\;Y_\infty}\;Z\;\frac{\rho_s}{\sigma_s^3}$$

Finally:

$$\sqrt{m\,T_d} = 1.47\;\left(\frac{Z}{g_d}\right)^{\frac{1}{3}}\;\text{keV}. \quad m = 3.67\;\text{keV}\;Z^{\frac{1}{3}}\;\frac{g_d^{\frac{1}{12}}}{\sqrt{g}}\;\sqrt{\frac{\sigma_0}{\text{pb}}}$$

We used ρ_{DM} today and the decrease of the phase space density by a factor Z. 1 pb $= 10^{-36}\;\text{cm}^2 = 0.257\;/(10^5\;\text{GeV}^2)$.

Relics decoupling non-relativistic 2

Allowed ranges for m and T_d.

$m > T_d > b$ eV where $b > 1$ or $b \gg 1$ for DM decoupling in the RD era

$$\left(\frac{Z}{g_d}\right)^{\frac{1}{3}} 1.47 \text{ keV} < m < \frac{2.16}{b} \text{ MeV } \left(\frac{Z}{g_d}\right)^{\frac{2}{3}}$$

$g_d \simeq 3 \quad \text{for} \quad 1 \text{ eV} < T_d < 100 \text{ keV}$ and $1 < Z < 10^3$

$1.02 \text{ keV} < m < \frac{104}{b} \text{ MeV} \quad , \quad T_d < 10.2$ keV.

Only using ρ_{DM} today (ignoring the phase space density information) gives one equation with three unknowns:
$m, \ T_d$ and σ_0,

$\sigma_0 = 0.16 \text{ pbarn } \dfrac{g}{\sqrt{g_d}} \ \dfrac{m}{T_d}$ http://pdg.lbl.gov

WIMPS with $m = 100$ GeV and $T_d = 5$ GeV require $Z \sim 10^{23}$.

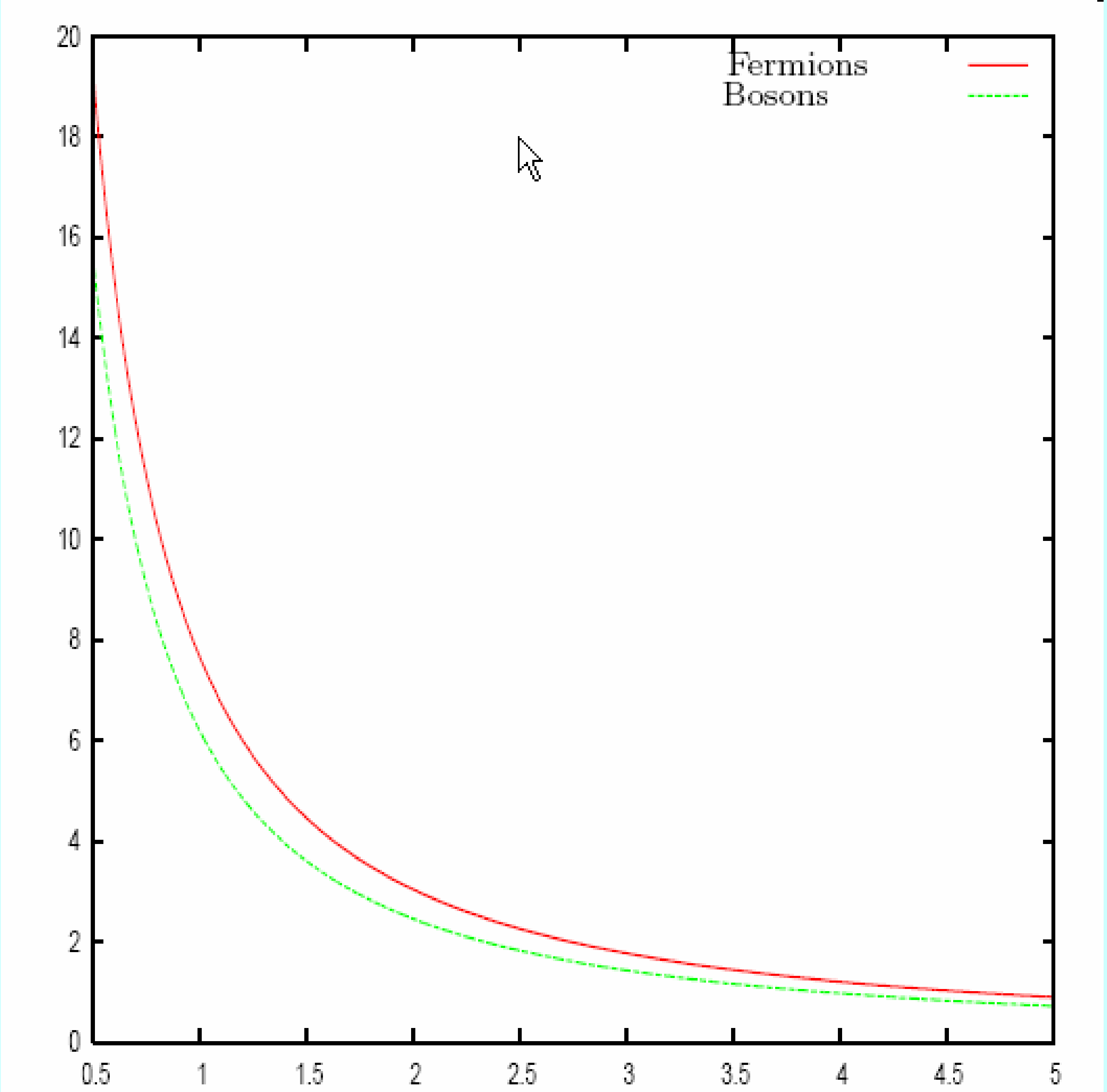
- The comoving **Jeans' (free-streaming) wavelength,** ie the largest wavevector exhibiting gravitational instability , and **the Jeans' mass** (the smallest unstable mass by gravitational collapse) are obtained in the range

$$0.76 \text{ kpc} / (\sqrt{1+z}) < \lambda_{fs}(z) < 16.3 \text{ kpc} / (\sqrt{1+z})$$

$$0.45\ 10^3\ M_{sun} < M_J\ (z)\ (1+z)^{-3/2} < 0.45\ 10^7\ M_{sun}$$

- These values at z = 0 are consistent and of order of the small dark matter structures observed today .

- By the beginning of the matter dominated era $z \sim 3200$, the masses are of the order of galactic masses $10^{12}\ M_{sun}$ and the comoving free-streaming length is of the order of the galaxy sizes today ~ 100 kpc



The free-streaming wavelength today in kpc vs. the dark matter particle mass in keV. *It decreases for increasing mass m and shows little variation with the particle statistics (fermions vs. bosons).*

- The **mass of the dark matter particle,** independent of the particle model, **is in the keV scale** and the **temperature when the dark matter particles decoupled** is in **the 100 GeV scale at least.**

Robust result. No assumption about the particle physics model of the dark matter particle.

keV DM mass much larger than temperature in matter dominated era (which is less than 1 eV)

m and T_d are mildly affected by the uncertainty in the factor Z through a power factor 1/4 of this uncertainty, namely, by a factor $10^{1/4} \sim 1.8$.

- Lower and upper bounds for the dark matter annihilation cross-section σ_0 are derived: $\sigma_0 > (0.239 - 0.956)\ 10^{-9}\ \text{GeV}^{-2}$ and $\sigma_0 < 3200\ m\ \text{GeV}^{-3}$. There is at least five orders of magnitude between them , the dark matter non gravitational self-interaction is negligible (consistent with structure formation and observations, X-ray, optical and lensing observations of the merging of galaxy clusters).

- Typical "wimps" (weakly interacting massive particles) with mass $m = 100$ GeV and $T_d = 5$ GeV would require a huge $Z \sim 10^{23}$, well above the upper bounds obtained and cannot reproduce the observed galaxy properties.

Wimps produce extremely short free-streaming or Jeans length today $\lambda_{fs}(0) = 3.51\ 10^{-4}$ pc = 72.4 AU that would correspond to unobserved structures much smaller than the galaxy structure. Wimps result strongly disfavoured. [TOO cold]

CONSTRAINTS: SUMMARY

- ARBITRARY DECOUPLED DISTRIBUTION FUNCTION

- ABUNDANCE $\longrightarrow$ UPPER BOUND

- dSphs (DM dominated) PHASE SPACE

  $\longrightarrow$ LOWER BOUND

- m ~ keV THERMAL RELICS decoupled when relativistic 100-300 GeV consistent with CORES

- Wimps with m ~ 100 GeV, T_d ~ 10 MeV PSD ~ 10^{18}-10^{15} x (dSphs)

In all cases: DM particles decoupling either ultra-relativistic or non-relativistic, LTE or OTE :

(i) the mass of the dark matter particle is in the keV scale, T_d is 100 GeV at least.

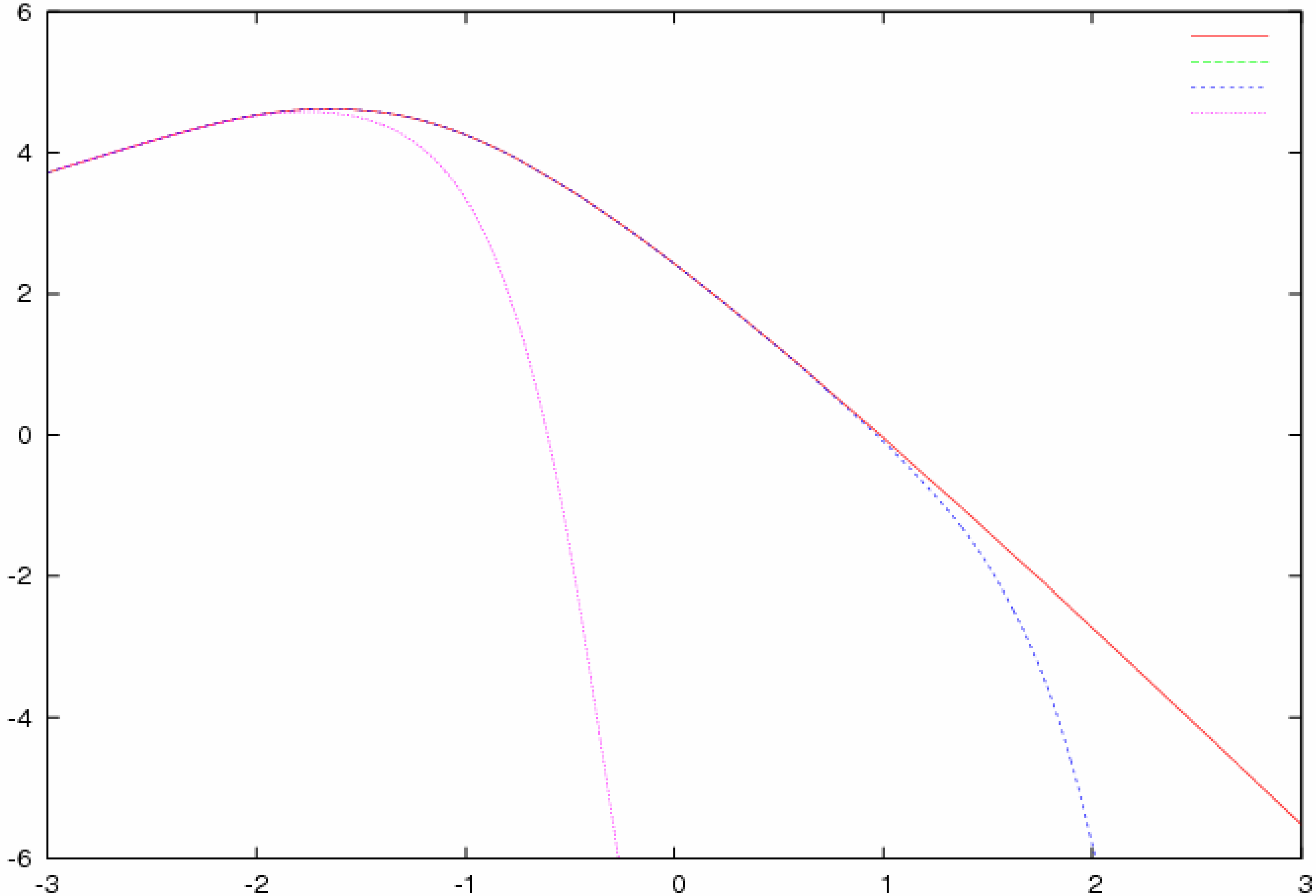
(ii) The free-streaming length today is in the kpc range, consistent with the observed small scale structure and the Jean's mass is in the range of the galactic masses, 10^{12} M_{sun}.

(iii) Dark matter self-interactions (other than grav.) are negligible.

(iv) The keV scale mass dark matter determines cored (non cusped) dark matter halos.

(v) DM candidates with typical high masses 100 GeV (”wimps”) result strongly disfavored.

Linear primordial power today $P(k)$ vs. k Mpc h



$\log_{10} P(k)$ vs. $\log_{10}[k \text{ Mpc } h]$ for WIMPS, 1 keV DM particles and 10 eV DM particles. $P(k) = P_0 \, k^{n_s} \, T^2(k)$.

$P(k)$ cutted for 1 keV DM particles on scales $\lesssim 100$ kpc.

Transfer function in the MD era from Gilbert integral eq

Transfer function and power spectrum:

- ❑ NR Boltzmann-Vlasov eqn for (DM) density + gravitational perturba
- ❑Valid for particles that are NR and modes inside Hubble radiu
- ❑ Matter domination $z \leq z_{eq} \sim 3050$
- ❑All scales relevant for structure formation

What's out?

❖ Photons + Baryons modify T(k) ~ few %

❖BAO on scales ~ 150 Mpc (acoustic horizon) (interested in MUCH smaller scales)

Why?

✓ Study arbitrary distribution functions, couplings, masses

✓Analytical understanding of small scale properties

✓No tinkering with codes

Kinetic Theory in Cosmology

Distribution function in phase-space: $f(t, p_i, x^i)$, $i = 1, 2, 3$

Boltzmann-Vlasov equation:

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{p^i}{p^0}\frac{\partial f}{\partial x^i} + \frac{dp_i}{dt}\frac{\partial f}{\partial p_i} = \text{Collision terms}$$

Geodesic equations:

$$\frac{dx^\alpha}{dt} = \frac{p^\alpha}{p^0} \quad , \quad \frac{dp_\alpha}{dt} = -\frac{1}{2\,p^0}\,p_\beta\,p_\gamma\,\frac{\partial g^{\beta\,\gamma}}{\partial x^\alpha} \quad , \quad 0 \leq \alpha, \beta, \gamma \leq 3$$

The Einstein equations determine the metric $g_{\alpha\,\beta}(t, x^i)$ in terms of the matter+radiation distribution function given by $f(t, p_i, x^i) \Rightarrow$ the Boltzmann-Vlasov equation becomes non-linear.

Collision terms negligible after particle decoupling.

The Boltzmann-Vlasov equation can be linearized around the FRW cosmological geometry before structure formation.

$$f(\vec{p};\vec{x};t) = f_0(p) + F_1(\vec{p};\vec{x};t) \qquad \varphi(\vec{x},t) = \varphi_0(\vec{x},t) + \varphi_1$$

Unperturbed decoupled distribution

(DM) perturbation

Unperturbed grav. Potential (FRW)

Grav. Potential perturbation

Linearized B-V Equation:

$$\frac{1}{a}\frac{\partial F_1}{\partial \tau} + \frac{\vec{p}}{ma^2}\cdot\vec{\nabla}_{\vec{x}} F_1 - m\vec{\nabla}_{\vec{x}}\varphi_1\cdot\vec{\nabla}_{\vec{p}} f_0 = 0$$

Poisson Eqn:

$$\varphi_1(\vec{k};s) = -\frac{4\pi G}{k^2 a(s)}\Delta(\vec{k};s) \qquad \Delta(\vec{k},s) = m\int\frac{d^3p}{(2\pi)^3}F_1(\vec{k},\vec{p};s)$$

``New'' variable s **=** $\dfrac{2u}{H_0\sqrt{\Omega_{DM}a_{eq}}}$; $\quad u = 1 - \left(\dfrac{a_{eq}}{a}\right)^{\frac{1}{2}} \longrightarrow \dfrac{ds}{d\tau} = \dfrac{1}{a}$

Follow the steps…

- Integrate B-V equation (in s)
- Use Poisson's eqn. ⟶ Integral eqn: Gilbert's

- Normalize at initial time (t_{eq}):

$$\Phi(\vec{k},u) = \frac{\varphi_1(\vec{k},u)}{\varphi_1(\vec{k},0)} \qquad \delta(k,u) = \frac{\Delta(k;u)}{\Delta(k;0)}$$

$$\boxed{P_f(k) = T^2(k) P_i(k)}$$

$$T(k) = \frac{5}{3}\Phi(k;1)$$

- Normalize the decoupled
- distribution function:

$$\tilde{f}_0(y) = \frac{f_0(y)}{\int_0^\infty y^2 f_0(y)\,dy}$$

comoving momentum

$$y = \frac{p}{T_{0,d}}$$

decoupling temp.

- Take 2 derivatives w.r.t. u: ⟶

$$\ddot{\delta}(k,u) - \frac{6\,\delta(k,u)}{(1-u)^2} + 3\gamma^2\,\delta(k,u) - \int_0^u du'\,K(u-u')\,\frac{\delta(k,u')}{(1-u')^2} = S_0(k;u)$$

Jeans' **Fluid** equation: replace C^2_s by $\langle V^2 \rangle$

Correction to fluid description: **memory of gravitational clustering**

Free streaming solution in absence of gravity: **INITIAL CONDITIONS**

$$\gamma^2 = \frac{2k^2}{k_{fs}^2(t_{eq})}; \quad k_{fs}(t_{eq}) = \frac{0.0102}{\sqrt{\overline{y^2}}}\left[\frac{g_d}{2}\right]^{\frac{1}{3}}\frac{m}{\mathrm{keV}}[\mathrm{kpc}]^{-1}; \quad \overline{y^2} = \int_0^\infty dy\, y^4\, \tilde{f}_0(y)$$

Free streaming wave vector at matter-radiation equality

$$k_{fs}(t_{eq}) = \begin{cases} \dfrac{5.88}{\mathrm{pc}}\left(\dfrac{g_d}{2}\right)^{\frac{1}{3}}\left(\dfrac{m}{100\,\mathrm{GeV}}\right)^{\frac{1}{2}}\left(\dfrac{T_d}{10\,\mathrm{MeV}}\right)^{\frac{1}{2}} & \text{WIMPs} \\ 0.00284\left(\dfrac{g_d}{2}\right)^{\frac{1}{3}}\dfrac{m}{\mathrm{keV}}[\mathrm{kpc}]^{-1} & \text{FD thermal relics} \\ 0.00317\left(\dfrac{g_d}{2}\right)^{\frac{1}{3}}\dfrac{m}{\mathrm{keV}}[\mathrm{kpc}]^{-1} & \text{BE thermal relics} \end{cases}$$

$$K(u-u') = 6\alpha \int_0^\infty y(\overline{y^2} - y^2)\, \tilde{f}_0(y) \sin[\alpha\, y(u-u')]\, dy \qquad \alpha = \sqrt{\frac{3}{\overline{y^2}}}$$

DECOUPLED DISTRIBUTION FUNCTION: <u>STATISTICS</u>

<u>Properties of K(u-u'):</u>

- **Correction to fluid description**
- **Memory of gravitational clustering** $\longrightarrow$
- **f_0(y) with larger support for small y** $\longrightarrow$
- **<u>longer range of memory</u>**
- **Longer range of memory → → → <u>larger T(k)</u>**
- **Negligible at <u>large</u> scales** $k << k_{fs}(t_{eq})$
- **Important at <u>small</u> scales** $k \geq k_{fs}(t_{eq})$

Exact T(k)

$$T(k)=\frac{10}{\sqrt{3}\,\gamma^{3}}\int_{0}^{1}h_{2}(u)\Big[\frac{I[\alpha u]}{(1-u)^{2}}+\frac{1}{6}S_{NB}[\delta;u]\Big]du$$

Regular solution of Jeans' **Fluid** eqn.

Free streaming solution In absence of gravity: **INITIAL CONDITIONS**

Memory of gravitational clustering: **K(u-u')**

Features:

✓ Systematic Fredholm expansion

✓First TWO terms simple and remarkably accurate

✓Include memory of gravitational clustering

✓**Arbitrary distribution function (statistics + non LTE)**

✓**Arbitrary initial conditions**

Summary: Roadmap

(1) ***Microphysics:*** **Particle physics model independent** →

decoupled, kinetics, decoupling $f(y)$ **distribution function,** $y = p/T_{0,d}$

(2) ***Constrain*** **mass, couplings,** $T_{0,d}$ **from abundance + phase space density**

$$\underbrace{\frac{100\,\mathrm{eV}}{\mathrm{D}^{\frac{1}{4}}}}_{\text{Lower bound from phase Space density of dSphs}} \leq m \leq 6.5\,\mathrm{eV}\,\underbrace{\frac{g_d}{g\int_0^\infty y^2 f(y)\,dy}}_{\text{Upper bound from abundance}} \qquad \mathrm{D} = \frac{g}{2\pi^2}\frac{\left[\int_0^\infty y^2 f(y)dy\right]^{\frac{5}{2}}}{\left[\int_0^\infty y^4 f(y)dy\right]^{\frac{3}{2}}}$$

Lower bound from phase Space density of dSphs

Upper bound from abundance

Thermal relics that decouple relativistically: $\mathrm{D} \sim 2\times 10^{-3}$ → m ~ keV

(3) ***DM T(k): simple + accurate*** →

- **arbitrary f(y) + ini. conds.**
- **corrections to fluid+ memory of grav. clustering.**
- **large f(y) at small y = long memory = affects T(k) at small scales.**

Galaxies

Physical variables in galaxies:

a) Nonuniversal quantities: mass, size, luminosity, fraction of DM, DM core radius r_0, central DM density ρ_0, ...

b) Universal quantities: surface density $\mu_0 \equiv r_0\, \rho_0$ and DM density profiles. M_{BH}/M_{halo} (or the halo binding energy).

The galaxy variables are related by universal empirical relations. Only one variable remains free.

Universal quantities may be attractors in the dynamical evolution.

Universal DM density profile in Galaxies:

$$\rho(r) = \rho_0\, F\left(\frac{r}{r_0}\right) \;,\; F(0) = 1 \;,\; x \equiv \frac{r}{r_0} \;,\; r_0 = \text{DM core radius.}$$

Empirical cored profiles: $F_{Burkert}(x) = \frac{1}{(1+x)(1+x^2)}$.

Cored profiles do reproduce the astronomical observations.

The constant surface density in DM and luminous galaxies

The Surface density for dark matter (DM) halos and for luminous matter galaxies defined as: $\mu_{0D} \equiv r_0 \, \rho_0$,

$r_0 =$ halo core radius, $\rho_0 =$ central density for DM galaxies

$\mu_{0D} \simeq 120 \, \frac{M_\odot}{\mathrm{pc}^2} = 5500 \, (\mathrm{MeV})^3 = (17.6 \, \mathrm{Mev})^3$

5 kpc < r_0 < 100 kpc. For luminous galaxies $\rho_0 = \rho(r_0)$. Donato et al. 09, Gentile et al. 09.[$\mu_{0D} = g$ in the surface].

Universal value for μ_{0D}: independent of galaxy luminosity for a large number of galactic systems (spirals, dwarf irregular and spheroidals, elliptics) spanning over 14 magnitudes in luminosity and of different Hubble types.

Similar values $\mu_{0D} \simeq 80 \, \frac{M_\odot}{\mathrm{pc}^2}$ in interstellar molecular clouds of size r_0 of different type and composition over scales $0.001 \, \mathrm{pc} < r_0 < 100$ pc (Larson laws, 1981).

Scaling of the energy and entropy from the surface density

Total energy using the virial and the profile $F(x)$:

$$E = \frac{1}{2}\langle U \rangle = -\frac{1}{4}\, G \int \frac{d^3r\, d^3r'}{|\boldsymbol{r}-\boldsymbol{r}'|} \langle \rho(r)\, \rho(r') \rangle =$$

$$= -\frac{1}{4}\, G\, \rho_0^2\, r_0^5 \int \frac{d^3x\, d^3x'}{|\boldsymbol{x}-\boldsymbol{x}'|} \langle F(x)\, F(x') \rangle \quad \Rightarrow \quad E \sim G\, \mu_{0D}^2\, r_0^3$$

The energy scales as the volume.

For consistency with the profile, the Boltzmann-Vlasov distribution function must scale as

$$f(\boldsymbol{p}, \boldsymbol{r}) = \frac{1}{m^4\, r_0^3\, G^{\frac{3}{2}}\, \sqrt{\rho_0}}\, \mathcal{F}\left(\frac{\boldsymbol{p}}{m\, r_0\, \sqrt{G\, \rho_0}}, \frac{\boldsymbol{r}}{r_0}\right)$$

Hence, the entropy scales as

$$S = \int f(\boldsymbol{p}, \boldsymbol{r})\; \log f(\boldsymbol{p}, \boldsymbol{r})\; d^3p\, d^3r \sim r_0^3\, \frac{\rho_0}{m} = r_0^2\, \frac{\mu_{0D}}{m}\ .$$

The entropy scales as the surface (as for black-holes).

However, very different proportionality coefficients:

$\frac{S_{BH}/A}{S_{gal}/r_0^2} \sim \frac{m}{\mathrm{keV}}\, 10^{36} \;\Rightarrow$ Much smaller coefficient for galaxies than for black-holes. Bekenstein bound satisfied.

DM surface density from linear Boltzmann-Vlasov eq

The distribution function of the decoupled DM particles:

$f(\vec{x},\vec{p};t) = g\; f_0^{DM}(p) + F_1(\vec{x},\vec{p};t) \quad , \quad f_0^{DM}(p) =$ zeroth order DM distribution function in or out of thermal equilibrium.

We evolve the distribution function $F_1(\vec{x},\vec{p};t)$ according to the linearized Boltzmann-Vlasov equation since the end of inflation. The DM density fluctuations are given by

$$\Delta(t,\vec{k}) \equiv m \int \frac{d^3p}{(2\pi)^3} \int d^3x\; e^{-i\,\vec{x}\cdot\vec{k}}\; F_1(\vec{x},\vec{p};t)$$

Today: $\Delta(\text{today},\vec{k}) = \rho_{DM}\;\bar{\Delta}(z=0,k)\;\sqrt{V}\;|\phi_k|\;g(\vec{k})$,

where $\bar{\Delta}(z,k)$ obeys a Volterra integral equation,
the primordial inflationary fluctuations are:

$|\phi_k| = \sqrt{2}\;\pi\;\frac{|\Delta_0|}{k^{\frac{3}{2}}}\;\left(\frac{k}{k_0}\right)^{\frac{n_s-1}{2}}$, $g(\vec{k})$ is a random gaussian field,

$V =$ phase-space volume at horizon re-entering

$|\Delta_0| \simeq 4.94\;10^{-5},\; n_s \simeq 0.964,\; k_0 = 2\;\text{Gpc}^{-1}$, WMAP7 .

Linear density fluctuations today

The linearized Boltzmann-Vlasov equation can be recasted as a Volterra integral equation for the DM density fluctuations:

$$\bar{\Delta}(z,k) =$$

$$h(z,k) + \frac{6}{(z+1)\, k\, r_{lin}} \int_{s_0}^{s} ds'\, \Pi\{k\, r_{lin}[s(z) - s']\}\, \bar{\Delta}(z(s'),k)$$

$$z(s) + 1 = (z_{eq} + 1)\, \sinh^2 s, \;\; z_{eq} + 1 \simeq 3200, \;\; \bar{\Delta}(\text{initial}, k) = 1$$

$h(z,k) =$ known function: contains the memory from previous UR evolution and the photons gravitational potential.

$$\Pi(x) \equiv \int_0^\infty Q\, dQ\, f_0^{DM}(Q)\, \sin(Q\, x)\;,$$

$f_0^{DM}(Q) =$ zeroth order freezed-out DM distribution.

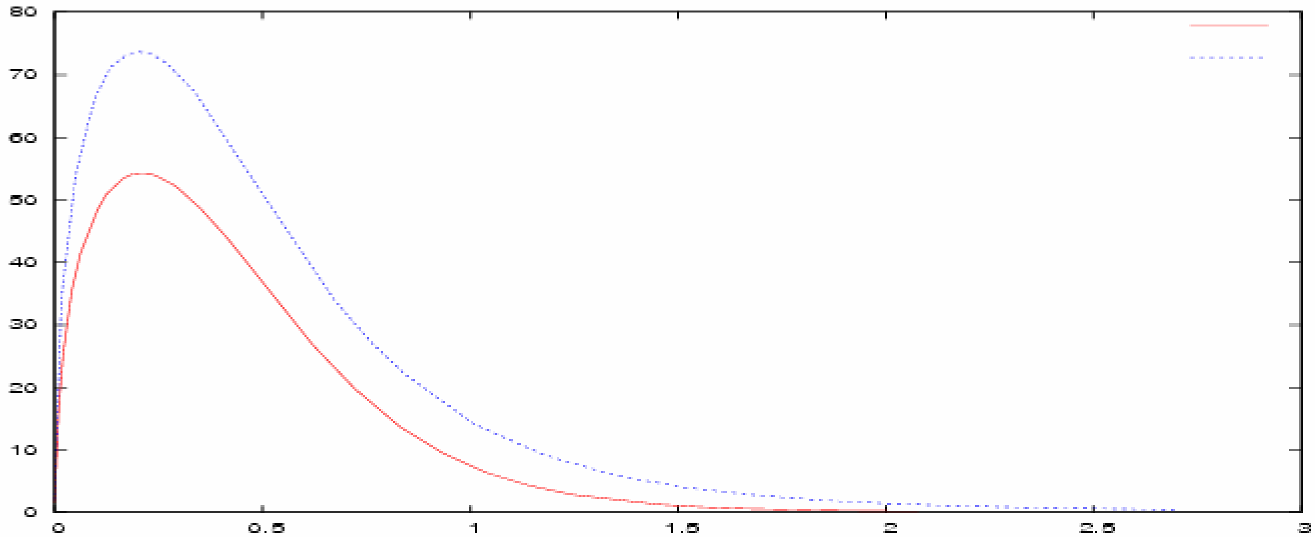
This integral equation is valid both in the RD and MD eras as long as the DM particles are non relativistic. It becomes the Gilbert equation in the MD era (plus memory terms).

The Free Streaming Scale

The characteristic length scale is the free streaming scale (or Jeans' scale)

$$r_{lin} = 2\sqrt{1+z_{eq}}\left(\frac{3\,M_{Pl}^2}{H_0\sqrt{\Omega_{DM}}\,Q_{prim}}\right)^{\frac{1}{3}} = 21.1\,q_p^{\frac{1}{3}}\ \text{kpc}$$

$q_p \equiv Q_{prim}/(\text{keV})^4$. DM particles can freely propagate over distances of the order of the free streaming scale.



DM fluctuations today $\bar{\Delta}(z=0,k)$ vs. $k\,r_{lin}$. Red= thermal FD initial. Blue = χ-sterile neutrinos. Initial value $\bar{\Delta}(k)=1$.

Linear density profile today

The matter density fluctuations $\rho_{lin}(r)$ are given today by

$$\rho_{lin}(r) = \frac{1}{2\,\pi^2\,r} \int_0^\infty k\,dk\ \sin(k\,r)\,\Delta(k, t_{\rm today}) \quad \text{for} \quad g(\vec{k}) = 1$$

The linear profile today results:

$$\rho_{lin}(x) = 14.47\,\rho_{DM}\ \frac{q_p^{\frac{n_s+2}{3}}}{x}\ \frac{I_3}{[(z_i+1)(z_i+1+z_{eq})]^{\frac{3}{4}}} \times$$

$$\times \int_0^\infty \gamma^{n_s/2-1}\,d\gamma\ \sin(\gamma\,x)\,\bar{\Delta}(z=0,\gamma)$$

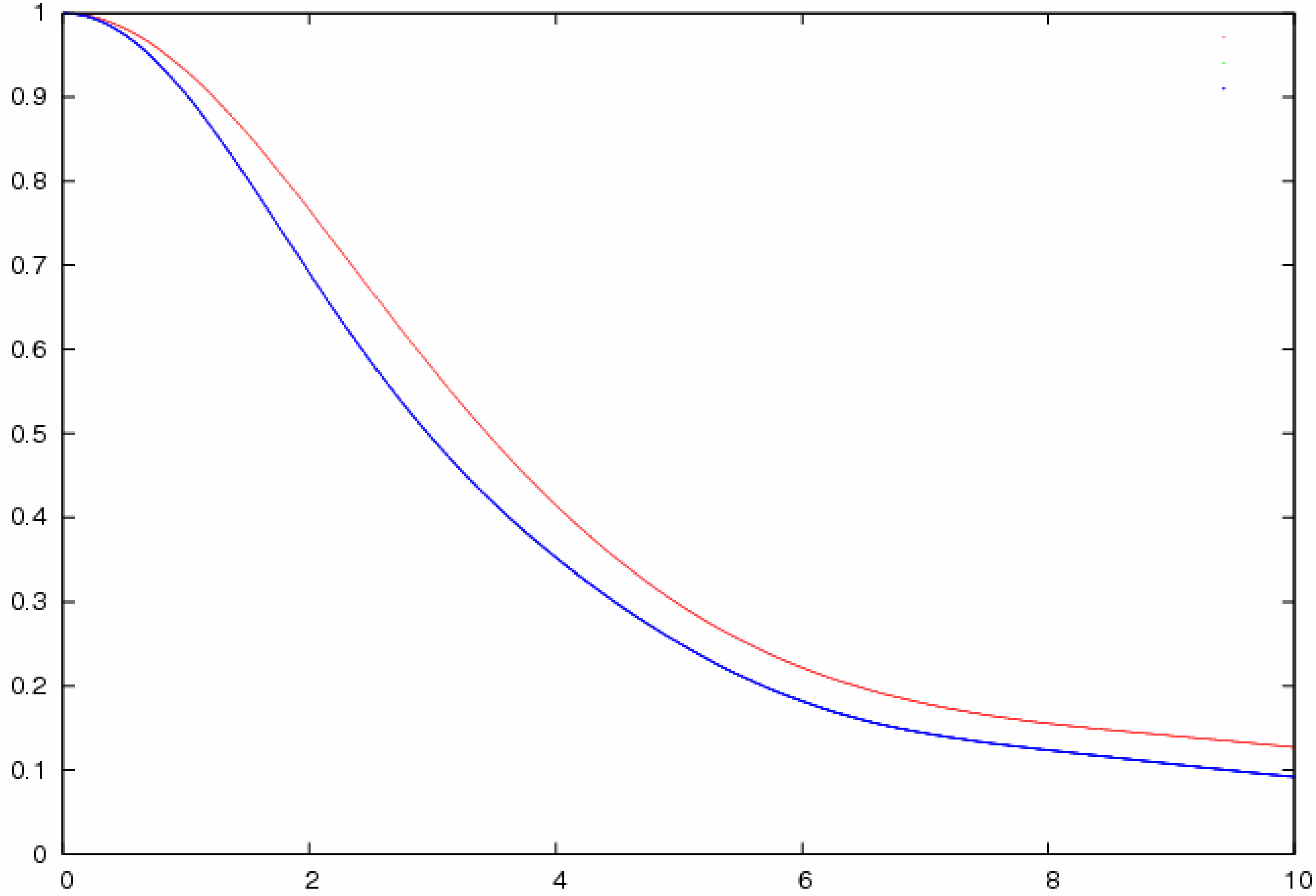
where $\gamma \equiv k\,r_{lin}$ and $x \equiv r/r_{lin}$.

I_n and $\bar{\Delta}(z=0,\gamma)$ depend on the freezed-out DM distribution $f_0^{DM}(Q)$.

Phase-space volume at horizon re-entering by redshift

$$z_i:\ V = \frac{4}{3}\,\pi\left(\frac{2\,\pi}{k_i}\right)^3 \quad , \quad k_i = H_0\sqrt{\Omega_m\,(z_i+1)(1+\frac{z_i+1}{z_{eq}+1})}$$

Density profiles in the linear approximation



Profiles $\rho_{lin}(r)/\rho_{lin}(0)$ vs. $x \equiv r/r_{lin}$.

Fermions decoupling ultrarelativistically in and out of thermal equilibrium. The halo radius r_0 is proportional to r_{lin}: $r_0 = \beta \, r_{lin}$. $\beta_{in\,equil} = 5.565$, $\beta_{out\,equil} = 5.013$.

Matching the observed and the theoretical surface density

Theoretical results:

$$m/\mathrm{keV} = q_p^{\frac{1}{4}} \left\{ \begin{array}{l} 2.646 \ \ \text{Thermal Fermi} - \text{Dirac}, \\ 3.144 < 2.418 \, \tau^{-\frac{1}{4}} < 5.591 \ \chi - \text{sterile}\ \nu, \end{array} \right.$$

[$0.035 < \tau < 0.35$: coupling in the χ sterile neutrino model.]

$$m/\mathrm{keV} = q_p^{\frac{1}{3}} \ 10.447 \ \ ,$$ DW model sterile ν.

Surface density: $\mu_0 \equiv r_0 \, \rho(0)$ where $r_0 =$ core radius.

$$\frac{\mu_{0\,lin}}{(\mathrm{MeV})^3} = \left(\frac{m}{\mathrm{keV}}\right)^{\frac{2}{3}n_s} \frac{\mathcal{N}}{N^{\frac{3}{4}}(z_i)} \times \left\{ \begin{array}{l} 0.2393 \ \text{Thermal FD} \\ 0.2535 \, \tau^{n_s/6} \chi - \text{sterile}\ \nu, \end{array} \right.$$

$$\mathcal{N} = \beta \ I_3 \ \int_0^\infty \gamma^{n_s/2} \, d\gamma \ \bar{\Delta}(\text{today}, \gamma) = \left\{ \begin{array}{l} 348.4 \ \text{Thermal FD} \\ 383.7 \ \chi - \text{sterile}\ \nu, \end{array} \right.$$

$n_s = 0.964$ primordial spectral index,

$N(z_i) \equiv (z_i + 1)(z_i + 1 + z_{eq})$.

The DM particle mass m from the observed surface density

Matching the observed values $\mu_{0\,obs}$ with this $\mu_{0\,lin}$ gives q_p, the mass of the DM particle and g_d.

From spiral galaxies data: $\mu_{0\,obs} = 6000\ (\mathrm{MeV})^3$ and the DM particle mass results,

$$\frac{m}{\mathrm{keV}} = \left[\frac{N(z_i)}{N(100)}\right]^{\frac{9}{8\,n_s}} \times \begin{cases} 5.382 \ \mathrm{Fermi-Dirac} \\ 3.07 < 2.36\ \tau^{-\frac{1}{4}} < 5.46\ \chi - \mathrm{sterile}\,\nu, \end{cases}$$

$$\frac{m}{\mathrm{keV}} = 10.8\ \left[\frac{N(z_i)}{N(100)}\right]^{\frac{3}{2\,n_s}} , \quad \text{DW model sterile } \nu.$$

$$N(z_i) \equiv (z_i + 1)(z_i + 1 + z_{eq})$$

$\#$ UR degrees of freedom ($\Rightarrow$ temperature) at decoupling

$$g_d = \left[\frac{N(100)}{N(z_i)}\right]^{\frac{3}{2\,n_s}} \times \begin{cases} 3293 \ \mathrm{Fermi-Dirac}, \\ 91 < 1124\,\tau^{\frac{3}{4}} < 512\ \chi - \mathrm{sterile}\ \nu, \end{cases}$$

$$g_d = 22.2 \ \mathrm{DW\ model\ sterile}\ \nu\ .$$

Density profiles in the linear approximation

Density profiles turn to be cored at scales $r \ll r_{lin}$.

Intermediate regime $r \gtrsim r_{lin}$:

$$\rho_{lin}(r) \stackrel{r \gtrsim r_{lin}}{=} c_0 \left(\frac{r_{lin}}{r}\right)^{1+n_s/2} \rho_{lin}(0) \quad , \quad 1 + n_s/2 = 1.482.$$

$\rho_{lin}(r)$ scales with the primordial spectral index n_s.

The theoretical linear results agree with the universal empirical behaviour $r^{-1.6 \pm 0.4}$: M. G. Walker et al. (2009) (observations), I. M. Vass et al. (2009) (simulations).

The agreement between the linear theory and the observations is remarkable.

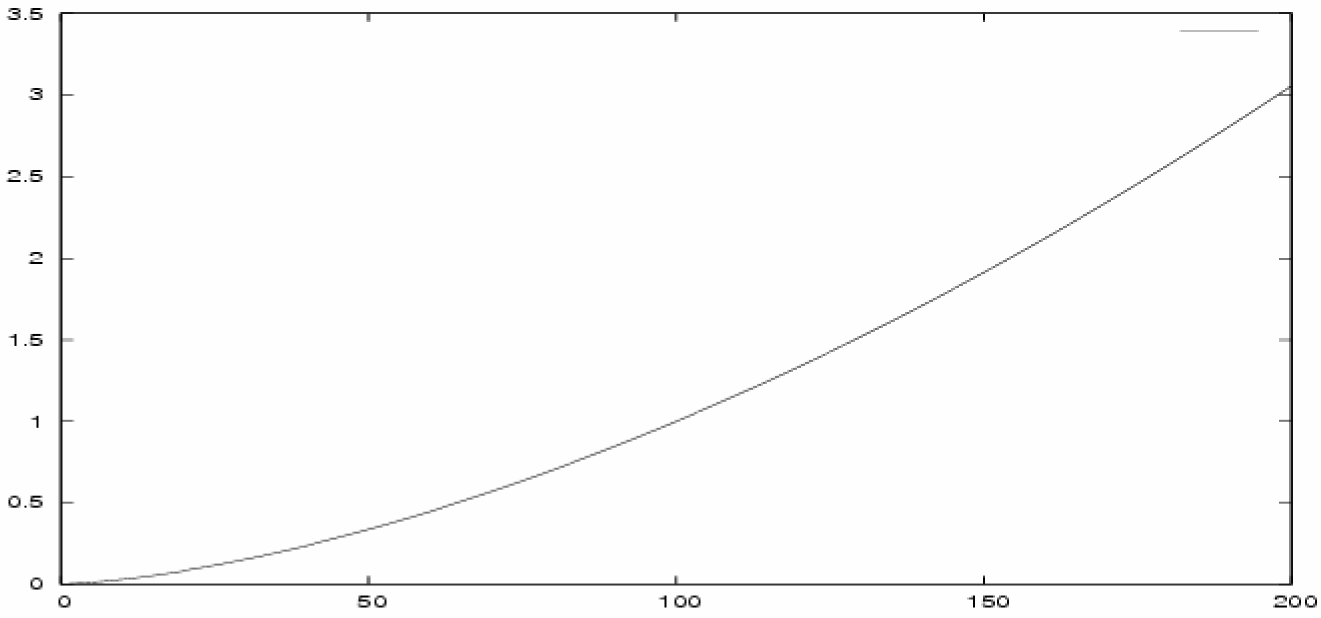
In the asymptotic regime $r \gg r_{lin}$ the small k behaviour of $\Delta(k, t_{\rm today}) \stackrel{k \to 0}{=} c_1 \, (k \, r_{lin})^s$ with $s \simeq 0.5$ implies the presence of a tail: $\rho_{lin}(r) \stackrel{r \gg r_{lin}}{\simeq} c \left(\frac{r_{lin}}{r}\right)^2$.

Non-universal galaxy properties.

	Observed Values
r_0	5 to 52 kpc
ρ_0	1.57 to 19.3 $\times 10^{-25}$ $\frac{\mathrm{g}}{\mathrm{cm}^3}$
$\sqrt{\overline{v^2}_{halo}}$	79.3 to 261 km/sec

	Thermal FD	χ-sterile	DW sterile
$\frac{r_0}{\mathrm{kpc}}\left[\frac{N(z_i)}{N(100)}\right]^{\frac{3}{2\,n_s}}$	36.3	86.9	36.1
$\frac{\rho_0}{10^{-25}\mathrm{g/cm}^3}\left[\frac{N(100)}{N(z_i)}\right]^{\frac{3}{2\,n_s}}$	8.32	3.48	8.37
$\frac{\sqrt{\overline{v^2}_{halo}}}{\mathrm{km/sec}}\left[\frac{N(z_i)}{N(100)}\right]^{\frac{3}{4\,n_s}}$	218	337	217

r_0 and $\overline{v^2}_{halo}$ decrease for increasing initial redshift z_i while ρ_0 increases with z_i. DM particle mass: $3 < m < 11$ keV.



The factor $[N(z_i)/N(100)]^{\frac{3}{2\,n_s}}$ vs. z_i.

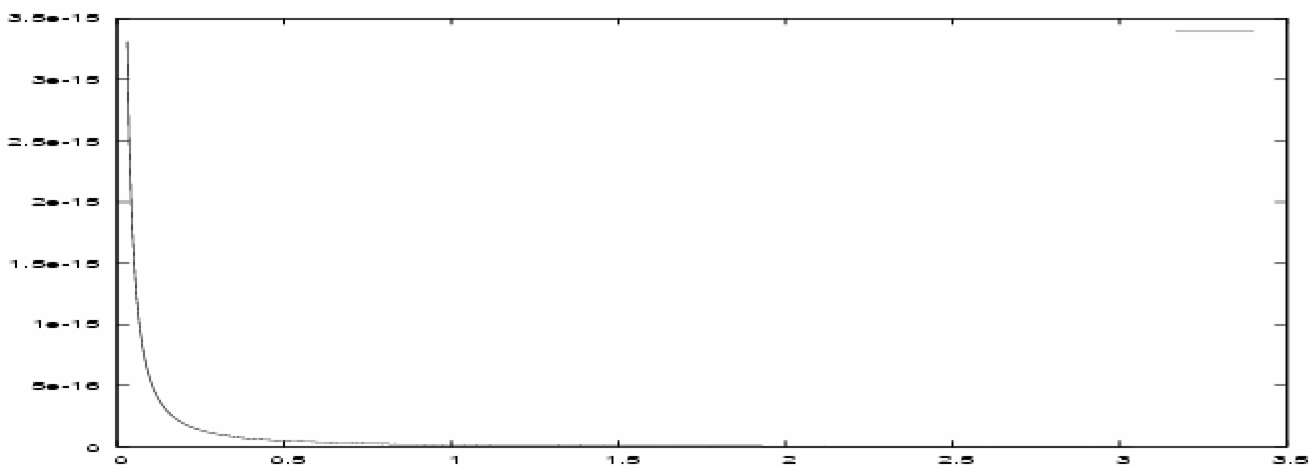
Further work:

- Effects of the random initial field $g(\vec{k})$
- Cluster of galaxies where observations indicate a surface density about eight times larger than in galaxies (Salucci et al. in preparation). This factor eight can be explained theoretically by $z_i^{galaxies} \simeq 16 \; z_i^{clusters}$.

Wimps vs. galaxy observations

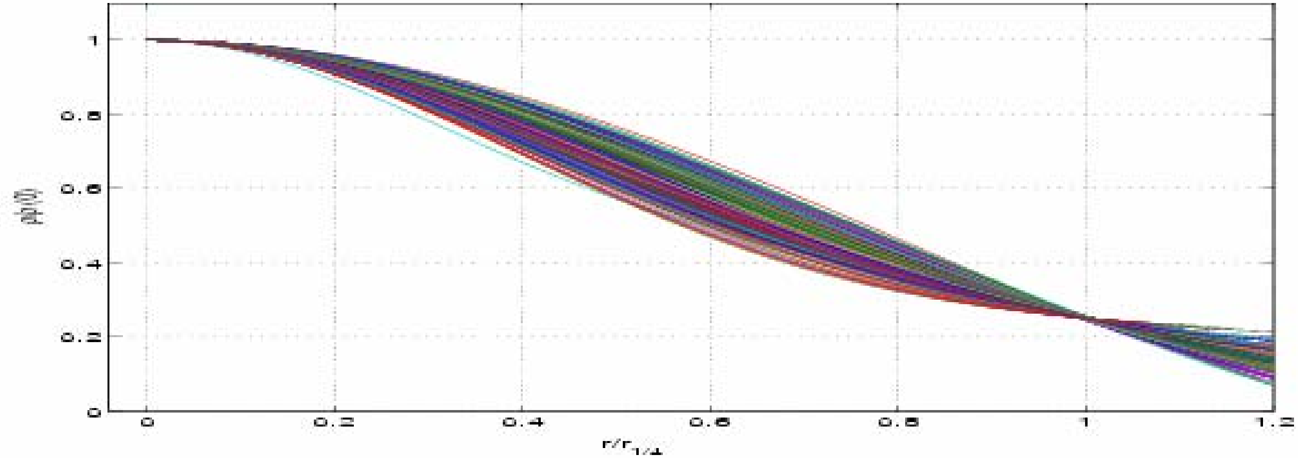
	Observed Values	Wimps in linear theory
r_0	5 to 52 kpc	0.045 pc
ρ_0	1.57 to $19.3 \times 10^{-25}\ \frac{\mathrm{g}}{\mathrm{cm}^3}$	$0.73 \times 10^{-14}\ \frac{\mathrm{g}}{\mathrm{cm}^3}$
$\sqrt{\overline{v^2}_{halo}}$	79.3 to 261 km/sec	0.243 km/sec

The wimps values strongly disagree by several order of magnitude with the observations.

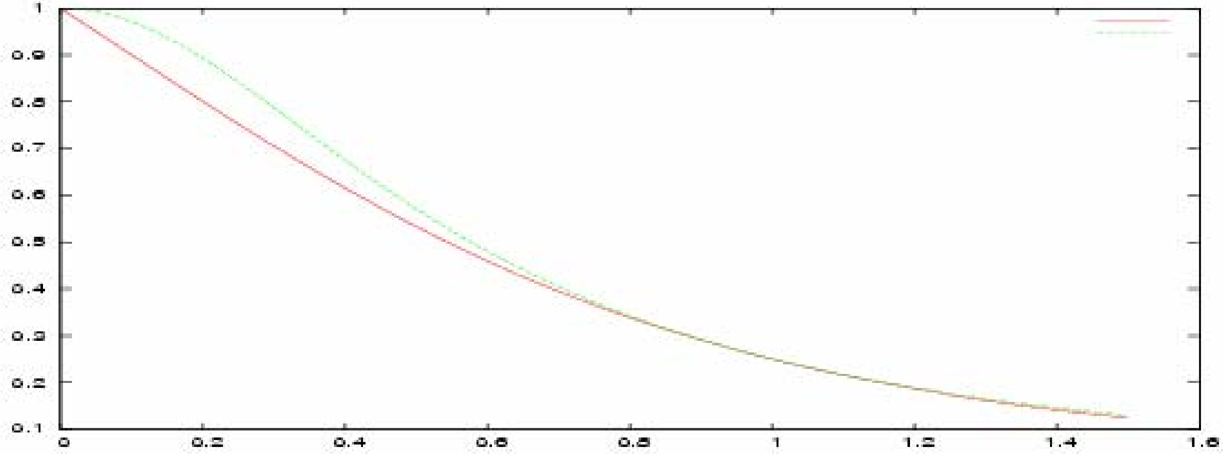


$\rho_{lin}(r)_{wimp}$ in g/cm^3 vs. r in pc. Exhibits a cusp behaviour for $r \gtrsim 0.03$ pc.

Linear evolution from random initial conditions

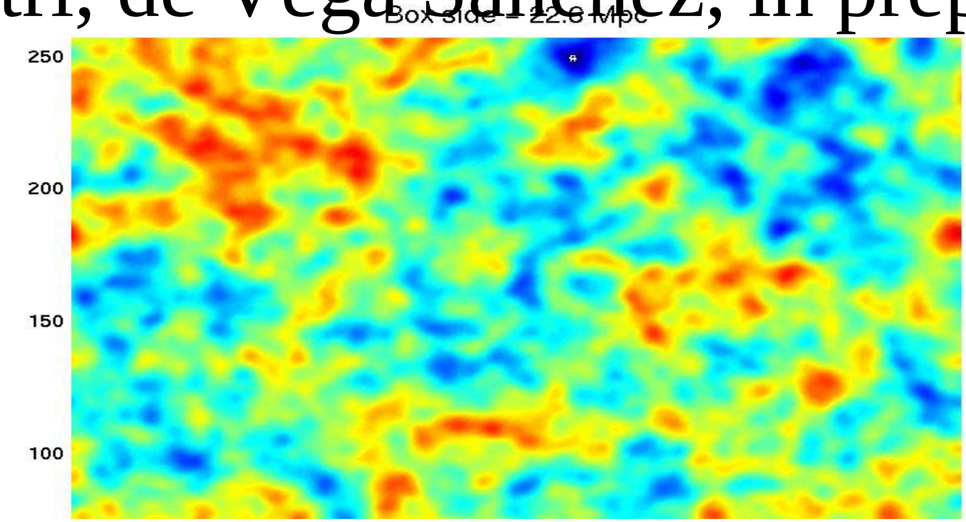


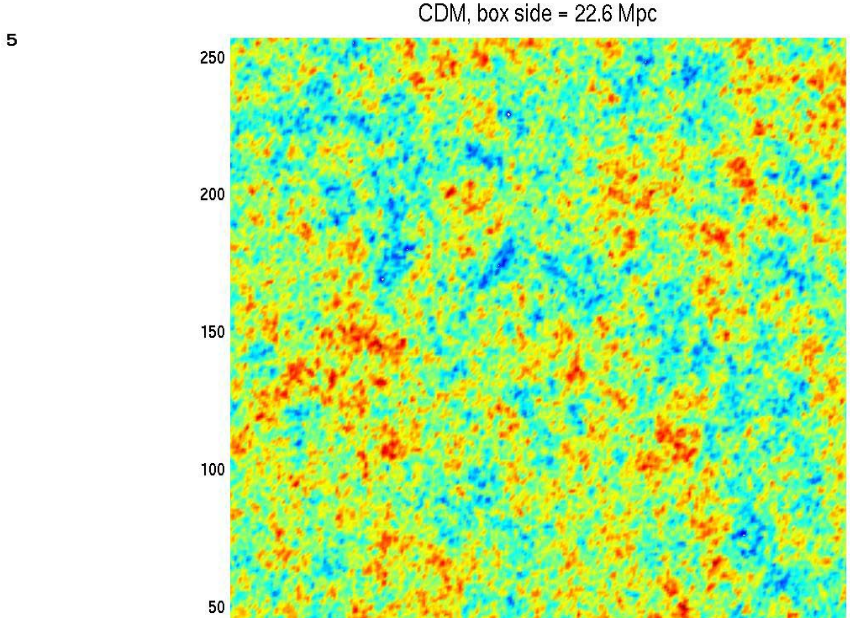
Profiles averaged in the angles for 500 random initial conditions. $\rho(r)/\rho(0)$ vs. $r/r_{1/4}$ [Destri, de Vega, Sanchez, in preparation]. Burkert and Pseudothermal profiles $\rho(r)/\rho(0)$ vs. $r/r_{1/4}$.



WDM vs CDM linear fluctuations Today

Destri, de Vega Sanchez, in preparation





keV SCALE DARK MATTER PARTICLES REPRODUCE:

→OBSERVED GALAXY DENSITIES AND VELOCITY DISPERSIONS

→OBSERVED GALAXY DENSITY PROFILES

→OBSERVED SURFACE DENSITY VALUES OF DARK MATTER DOMINATED GALAXIES

Particle physics candidates for DM

No particle in the Standard Model of particle physics (SM) can play the role of DM.

Many extensions of the SM can be envisaged to include a DM particle with mass in the keV scale and weakly enough coupled to the Standard Model particles to fulfill all particle physics experimental constraints.

Main candidates in the keV mass scale: sterile neutrinos, gravitinos, light neutralino, majoron ...

Particle physics motivations for sterile neutrinos:

There are both left and right handed quarks (with respect to the chirality).

It is natural to have right handed neutrinos ν_R besides the known left-handed neutrino. Quark-lepton similarity.

Summary: keV scale DM particles

- Reproduce the phase-space density observed in dwarf satellite galaxies and spiral galaxies (dV S 2009).
- Provide cored universal galaxy profiles in agreement with observations (dV S 2009,dV S S 2010).
(Review on cores vs. cusps by de Blok 2010, Salucci & Frigerio Martins 2009)
- Reproduce the universal surface density μ_0 of DM dominated galaxies (dV S S 2010). WIMPS simulations give 1000 times the observed value of μ_0 (Hoffman et al. 2007).
- Alleviate the satellite problem which appears when wimps are used (Avila-Reese et al. 2000, Götz & Sommer-Larsen 2002)
- Alleviate the voids problem which appears when wimps are used (Tikhonov et al. 2009).

Summary: keV scale DM particles

- All direct searches of DM particles look for $m \gtrsim 1$ GeV. DM mass in the keV scale explains why nothing has been found ... e^+ and $\bar{p}$ excess in cosmic rays may be explained by astrophysics: P. L. Biermann et al. (2009), P. Blasi, P. D. Serpico (2009).
- Galaxies from Wimps simulations are too small (Ryan Joung et al. 2009, Holz & Perlmutter 2010). keV scale DM may alleviate this problem.
- Velocity widths in galaxies from 21cm HI surveys. ALFALFA survey clearly favours WDM over CDM. Papastergis et al. 2011, Zavala et al. 2009

Reliable simulations with keV mass DM are needed to clarify all these issues.

Summary and Conclusions

- Combining theoretical evolution of fluctuations through the Boltzmann-Vlasov equation with galaxy data points to a DM particle mass 3 - 10 keV. T_d may be $>$ or < 100 GeV. The keV mass scale holds independently of the DM particle physics model.

- Universal Surface density in DM galaxies $[\mu_{0D} \simeq (18 \text{ MeV})^3]$ explained by keV mass scale DM. Density profile scales and decreases for intermediate scales with the spectral index n_s : $\rho(r) \sim r^{-1-n_s/2}$ and $\rho(r) \sim r^{-2}$ for $r \gg r_0$.

H. J. de Vega, P. Salucci, N. G. Sanchez, 'The mass of the dark matter particle from theory and observations', arXiv:1004.1908.

H. J. de Vega, N. Sanchez, 'Model independent analysis of dark matter points to a particle mass at the keV scale', arXiv:0901.0922, MNRAS 404, 885 (2010).

Future Perspectives

The Golden Age of Cosmology and Astrophysics continues.

Galaxy and Star formation. DM properties from galaxy observations. Better upper bounds on DM cross-sections.

DM in planets and the earth. Flyby and Pioneer anomalies?

Chandra, Suzaku X-ray data: keV mass DM decay?

Sun models well reproduce the sun's chemical composition but not the heliosismology (Asplund et al. 2009).
Can DM inside the Sun help to explain the discrepancy?

Nature of Dark Matter? 83% of the matter in the universe.

Light DM particles are strongly favoured $m_{DM} \sim$ keV.
Sterile neutrinos ? Other particle in the keV mass scale?

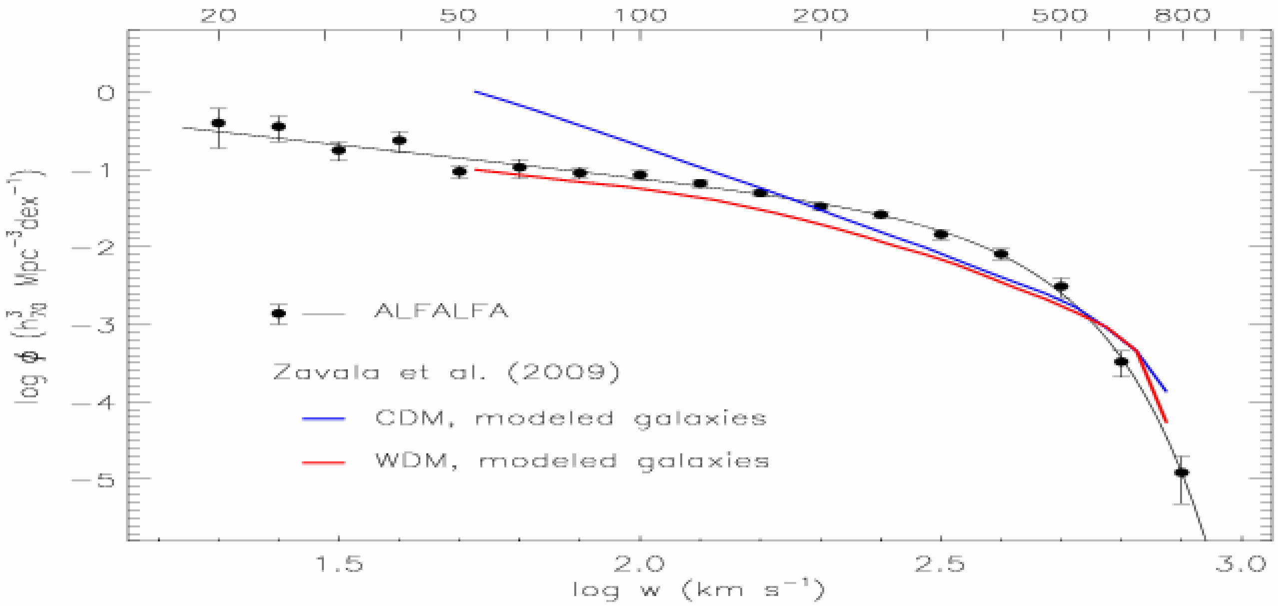
Precision determination of DM properties (mass, T_d, nature) from better galaxy data combined with theory (Boltzmann-Vlasov and simulations).

END

THANK YOU

FOR YOUR ATTENTION

Velocity widths in galaxies



Velocity widths in galaxies from 21cm HI surveys. ALFALFA survey clearly favours WDM over CDM. (Papastergis et al. 2011, Zavala et al. 2009).

Notice that the WDM curve is for $m = 1$ keV.

Recent Chalonge Conferences and Workshops

Highlights and Conclusions of the Chalonge 14th Paris Cosmology Colloquium 2010: 'The Standard Model of the Universe: Theory and Observations'. P Biermann, D Boyanovsky, A Cooray, C Destri, H de Vega, G Gilmore, S Gottlober, E Komatsu, S McGaugh, A Lasenby, R Rebolo, P Salucci, N Sanchez and A Tikhonov present their highlights of the Colloquium.
Conclusions by H. J. de Vega, M.C. Falvella, N. G. Sanchez, arXiv:1009.3494, 58 pages, 20 figures.

Highlights and Conclusions of the Chalonge Meudon Workshop Dark Matter in the Universe. P Biermann, A Cavaliere, H J. de Vega, G Gentile, C Jog, A Lapi, P Salucci, N G. Sanchez, P Serpico, R Stiele, J van Eymeren and M Weber present their highlights of the Workshop.
Conclusions by H. J. de Vega, N. G. Sanchez, arXiv:1007.2411, 41 pages, 10 figures.

Sterile Neutrinos in the SM of particle physics

SM symmetry group: $SU(3)_{color} \otimes SU(2)_{weak} \otimes U(1)^{weak}_{hypercharge}$

Leptons are color singlets and doublets under weak SU(2).

Sterile neutrinos ν_R do not participate to weak interactions. Hence, they must be singlets of color, weak SU(2) and weak hypercharge.

The SM Higgs Φ is a SU(2) doublet with a nonzero vacuum expectation value Φ_0. It can couple Yukawa-type with the left and right handed leptons:

$$L_{Yuk} = y \, \bar{\nu}_L \, \nu_R \, \Phi_0 + h.c. \quad ,$$

$y =$ Yukawa coupling, $\Phi_0 = \begin{pmatrix} 0 \\ v \end{pmatrix} \quad , \quad v = 174$ GeV.

This induces a mixing (bilinear) term between ν_L and ν_R which produces transmutations of $\nu_L \Leftrightarrow \nu_R$.

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Sterile neutrinos ν_R do not participate to weak interactions.

Hence, they must be singlets of color, weak SU(2) and hypercharge.

Mixing (bilinear) terms appear: $\bar{\Phi}_0 \, \bar{\nu}_R \, \nu_L$ and $\bar{\nu}_L \, \nu_R \, \Phi_0$.

They produce transmutations $\nu_L \Leftrightarrow \nu_R$. $(m_D = h_Y \, |\Phi_0|)$.

Neutrino mass matrix: $(\bar{\nu}_L \; \bar{\nu}_R) \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}$

Seesaw mass eigenvalues: $\frac{m_D^2}{M}$ and M, with eigenvectors:

- active neutrino: $\nu_{active} \simeq \nu_L - \frac{m_D}{M} \, \nu_R, \quad M \gg m_D$.
- sterile neutrino: $\nu_{sterile} \simeq \nu_R + \frac{m_D}{M} \, \nu_L, \quad M \gg m_D^2/M$.

Sterile Neutrinos

Choosing $M \sim 1$ keV and $m_D \sim 0.1$ eV is consistent with observations.

Mixing angle: $\theta \sim \frac{m_D}{M} \sim 10^{-4}$ is appropriate to produce enough sterile neutrinos accounting for the observed DM.

Smallness of θ makes the detection of steriles very difficult.

Precise measure of nucleus recoil in tritium beta decay:
$^3H_1 \Longrightarrow {}^3He_2 + e^- + \bar{\nu}$ can show the presence of a sterile instead of the active $\bar{\nu}$ in the decay products.

Rhenium 187 beta decay gives $\theta < 0.095$ for 1 keV steriles [Galeazzi et al. PRL, 86, 1978 (2001)].
Available energy: $Q(^{187}Re) = 2.47$ keV, $Q(^3H_1) = 18.6$ keV.

Conclusion: the empty slot of right-handed neutrinos in the Standard Model of particle physics can be filled by keV-scale sterile neutrinos describing the DM.

Sterile neutrino models

Sterile neutrinos: name coined by Bruno Pontecorvo (1968).

- DW: Dodelson-Widrow model (1994) sterile neutrinos produced by non-resonant mixing from active neutrinos.
- Shi-Fuller model (1998) sterile neutrinos produced by resonant mixing from active neutrinos.
- χ-model (1981)-(2006) sterile neutrinos produced by a Yukawa coupling from a real scalar χ.
- Further models can be proposed...