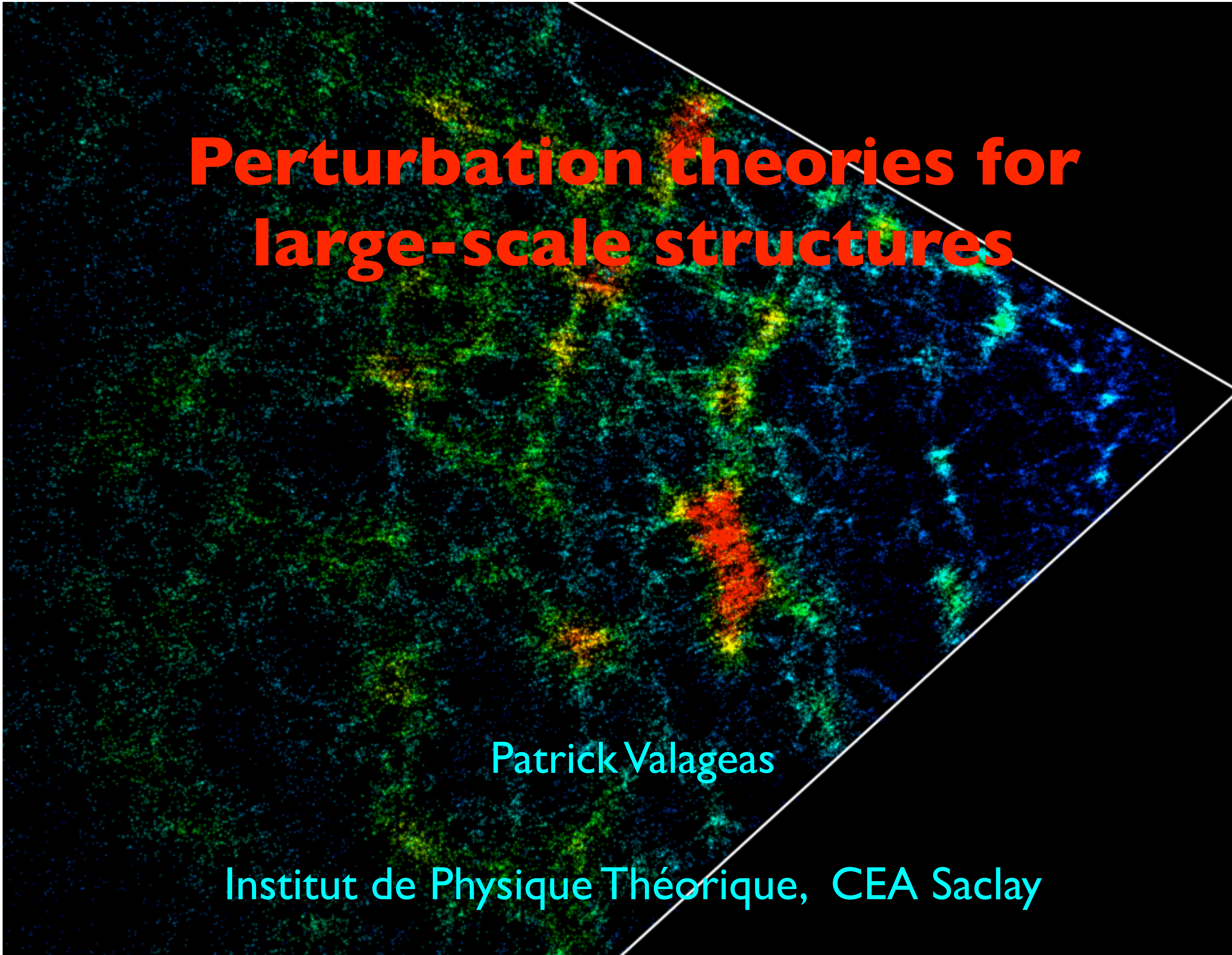

Perturbation theories for large-scale structures

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Outline

- Perturbative approaches

- Exact results for toy models

- Combined models

- Higher-order statistics: the bispectrum

- Other approaches - extensions

Perturbative methods

Scales of interest: $x > 10h^{-1}\mathrm{Mpc}$

$k < 0.4h\mathrm{Mpc}^{-1}$

Linear to weakly nonlinear regime

Future observations require a percent-level accuracy
for theoretical predictions

F. Bernardeau & P.V., 2008 - P.V., 2004; 2007a,b; 2008; 2010a - P.V. & T. Nishimichi, 2010

F. Bernardeau, M. Crocce, E. Sefusatti; 2010 - M. Crocce & R. Scoccimarro, 2006a,b; 2008 - S. Matarrese & M. Pietroni, 2007 - M. Pietroni, 2008 - A. Taruya & T. Hiramatsu, 2008

A- Standard perturbation theory

1) Hydrodynamical approximation: "single-stream approximation"

CDM+baryons: pressureless & irrotational perfect fluid

$$\frac{\partial \delta}{\partial \tau} + \nabla \cdot [(1+\delta)\mathbf{v}] = 0$$

$$\frac{\partial \mathbf{v}}{\partial \tau} + \mathcal{H}\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla\phi$$

$$\Delta\phi = \frac{3}{2}\Omega_{\rm m}\mathcal{H}^2\delta$$

density contrast:

$$\delta(\mathbf{x},t) = \frac{\rho(\mathbf{x},t)-\bar{\rho}}{\bar{\rho}}$$

2) Solution as a perturbative expansion over powers of the linear mode

$$\tilde{\delta}(\mathbf{k},\tau) = \sum_{n=1}^{\infty} \tilde{\delta}^{(n)}(\mathbf{k},\tau) \qquad \text{with} \qquad \tilde{\delta}^{(n)} \propto (\tilde{\delta}_L)^n$$

3) Gaussian average for statistical quantities

$$C_2 = \langle\delta\delta\rangle = \langle\delta^{(1)}\delta^{(1)}\rangle + \langle\delta^{(3)}\delta^{(1)}\rangle + \langle\delta^{(1)}\delta^{(3)}\rangle + \langle\delta^{(2)}\delta^{(2)}\rangle + \dots$$

This yields a perturbative expansion for the density power spectrum:

$$P(k) = \sum_{n=1}^{\infty} P^{(n)}(k) \quad \text{with} \quad P^{(n)} \propto (P_L)^n$$

It can be useful to introduce the variance of the one-dimensional linear displacement or velocity, and to define a "renormalized expansion":

$$P(k) = e^{-k^2 \sigma_v^2} \sum_{n=1}^{\infty} P_{\sigma_v}^{(n)}(k) \quad \text{with} \quad P_{\sigma_v}^{(n)} \propto (P_L)^n \quad \text{and} \quad \sigma_v^2 = \frac{1}{3} \langle |\mathbf{\Psi}_L^2| \rangle$$

4) Example: the Zeldovich dynamics

Particles move on straight lines, according to the linear displacement field (i.e., their initial velocity):

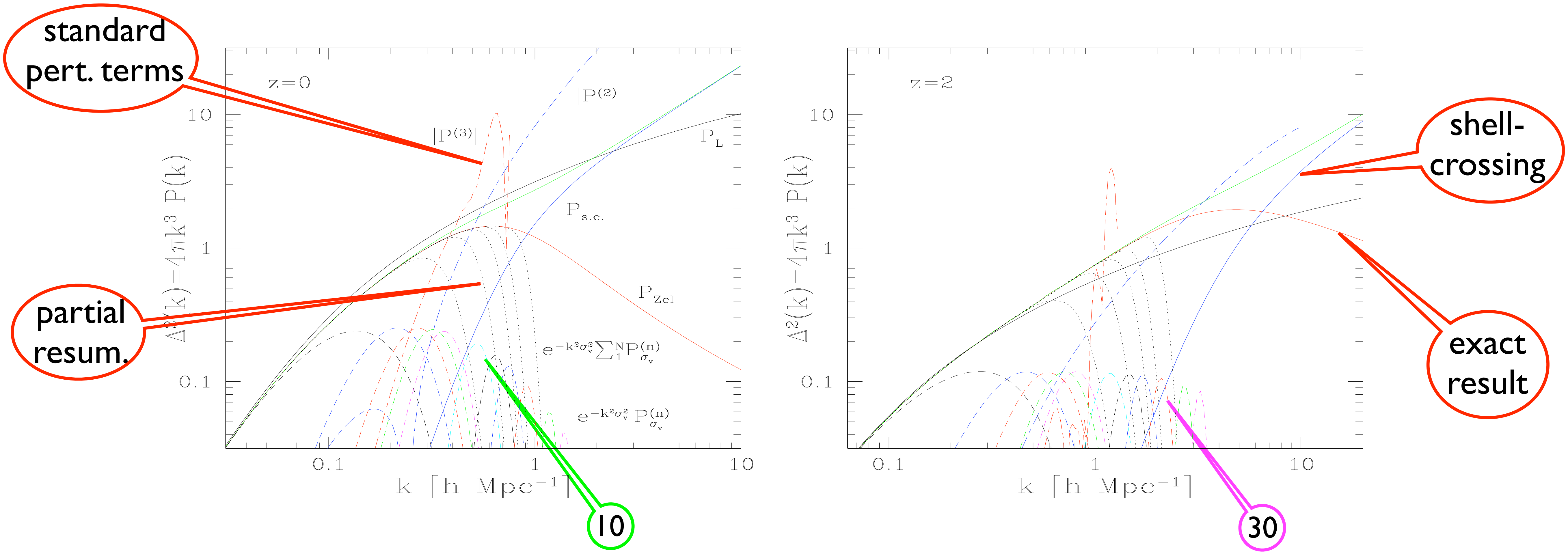
$$\mathbf{x}(\mathbf{q},t) = \mathbf{q} + D_+(t)\, \mathbf{\Psi}_{L0}(\mathbf{q}) \quad \text{with} \quad \nabla \cdot \mathbf{\Psi}_{L0} = -\delta_{L0}$$

$$P(k) = \int \frac{d\mathbf{q}}{(2\pi)^3}\, e^{i\mathbf{k}\cdot\mathbf{q}}\, e^{-\int dw\, [1-\cos(\mathbf{w}\cdot\mathbf{q})] \frac{(\mathbf{k}\cdot\mathbf{w})^2}{w^4} P_L(w)}$$

$$P_{\sigma_v}^{(n)}(k) = n! \int d\mathbf{w}_1 ... d\mathbf{w}_n\, \delta_D(\mathbf{w}_1 + ... + \mathbf{w}_n - \mathbf{k})\, F_n(\mathbf{w}_1, ..., \mathbf{w}_n)^2\, P_L(w_1) ... P_L(w_n)$$

Test on the Zeldovich Dynamics

(particles moving on straight lines according to their initial velocity)



- standard perturbation theory is not well-behaved

- many orders are relevant before the nonperturbative term (shell crossing) dominates

Need for improved (resummation) schemes

Sticky model

This model is identical to the Zeldovich dynamics before shell-crossing,
and only differs afterwards:

Particle pairs do not cross along their longitudinal axis.

$$\text{For} \quad \mathbf{q} = |\mathbf{q}| \, \mathbf{e}_1 : \quad \Delta x_1 = \max(\Delta x_{L1}, 0), \quad \Delta x_2 = \Delta x_{L2}, \quad \Delta x_3 = \Delta x_{L3}$$

Then, the power spectrum is equal to the Zeldovich power spectrum
+ a nonperturbative correction associated with the dynamics beyond shell-crossing:

$$P_{\text{sticky}}(k) = P_{\text{Zel}}(k) + P_{\text{s.c.}}(k)$$

$$P_{\text{s.c.}}(k) = \frac{1}{2} \int \frac{d\mathbf{q}}{(2\pi)^3} \, e^{-k^2(1-\mu^2)[\sigma_v^2 - I_0(q) - I_2(q)]} \, e^{-q^2/(2\sigma_{\parallel}^2(q))} \left\{ w\left(\frac{iq}{\overline{2}\sigma_{\parallel}(q)} \right) \quad w\left(\frac{iq \quad k\mu\sigma_{\parallel}^2(q)}{\overline{2}\sigma_{\parallel}(q)} \right) \right\}$$

B- Path-integral formulation

1) Generating functional of many-body correlations

$$\psi(\mathbf{k},\tau) = \begin{pmatrix} \tilde{\delta}(\mathbf{k},\tau) \\ \tilde{\theta}(\mathbf{k},\tau)/\mathcal{H}f \end{pmatrix}$$

$$Z[j] = \langle e^{j\cdot\psi} \rangle = \int \mathcal{D}\mu_I \; e^{j\cdot\psi[\mu_I] - \frac{1}{2}\mu_I\cdot\Delta_I^{-1}\cdot\mu_I}$$

eq. of motion

Gaussian init. cond.

$$Z[j] = \int \mathcal{D}\psi \, \mathcal{D}\lambda \; e^{j\cdot\psi + \lambda\cdot(-\mathcal{O}\cdot\psi + K_s\cdot\psi\psi) + \frac{1}{2}\lambda\cdot\Delta_I\cdot\lambda}$$

→ auxiliary field $\lambda(x)$

$$\langle \psi(x_1)\lambda(x_2) \rangle = R(x_1,x_2) = \langle \frac{\mathcal{D}\psi(x_1)}{\mathcal{D}\zeta(x_2)} \rangle_{\zeta=0}$$

→ Response function

It measures the sensitivity to perturbations, or to the initial conditions

Memory of the dynamics

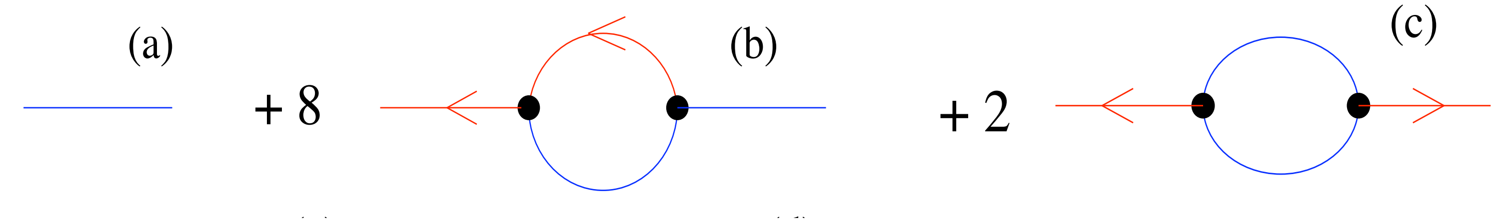
2) Use perturbative schemes to compute Z[j]

→ Power spectrum, bispectrum, ...

1-loop order
(i.e., up to P_L^2)

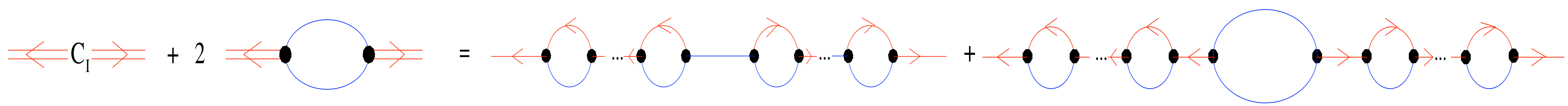
a) Standard perturbation theory

$$P(k) = C_2 =$$

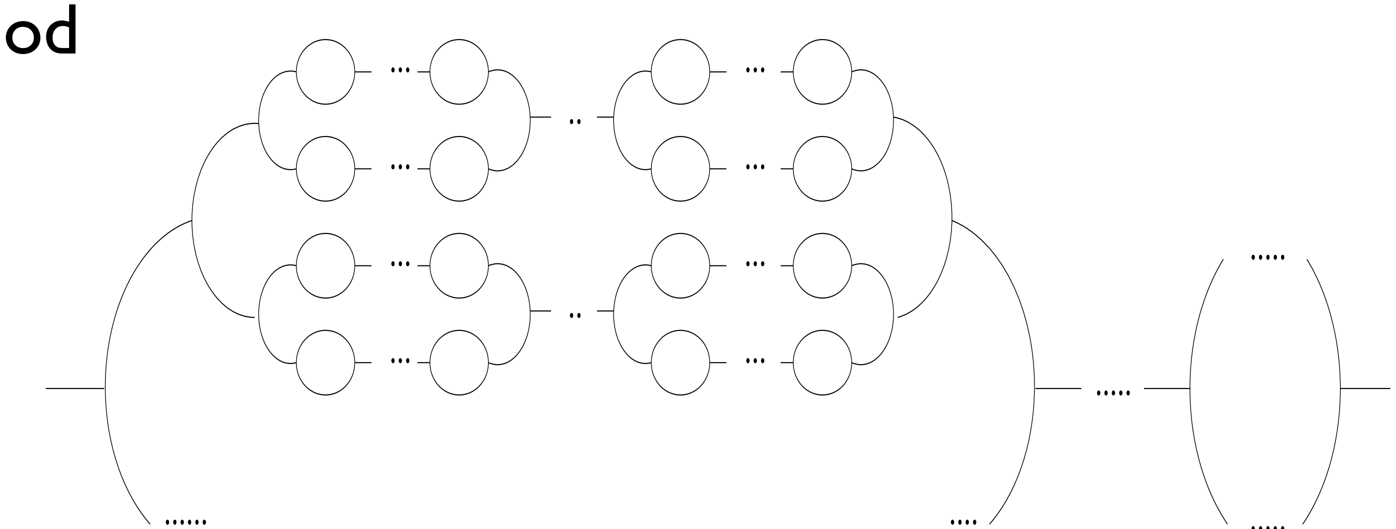


b) Direct steepest-descent method

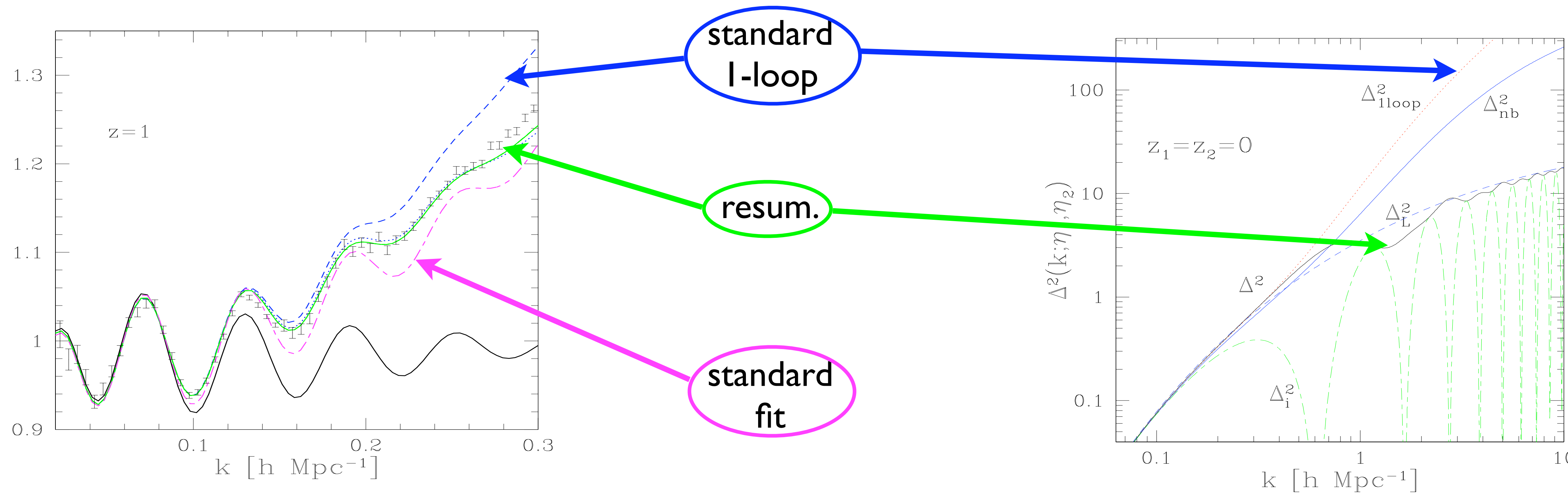
$$P(k) =$$



c) 2PI effective action method



3) Results for the two-point correlation (i.e., power spectrum in Fourier space)



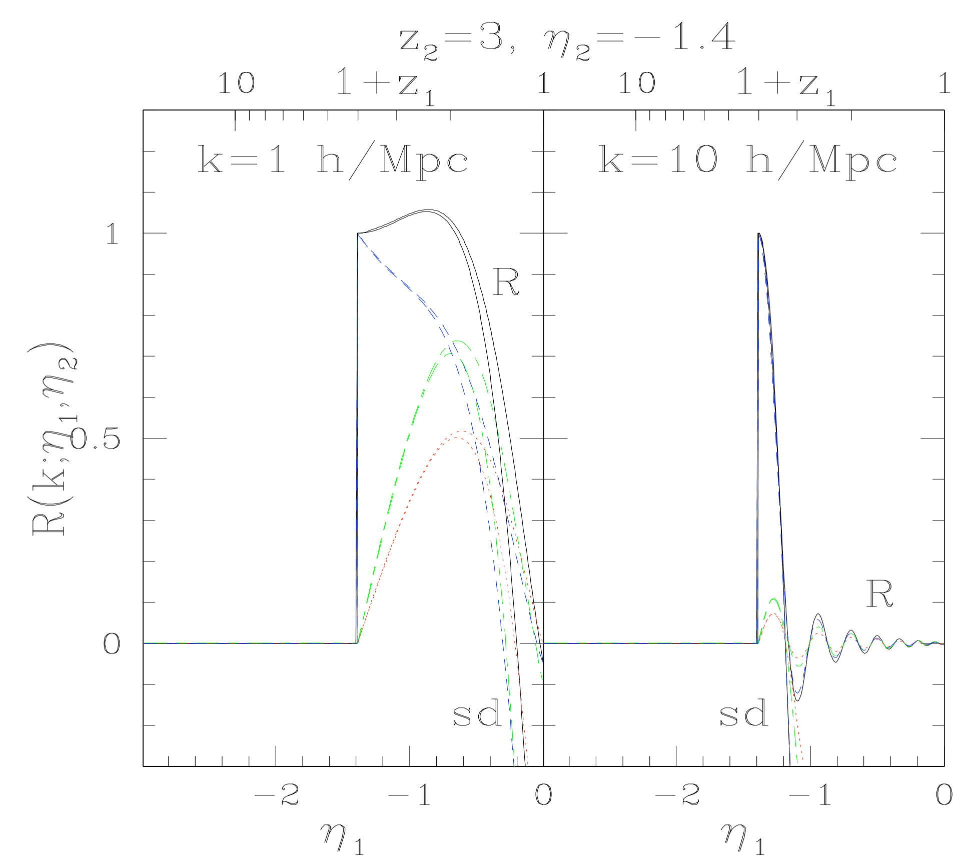
- higher accuracy at large scales

- good behavior at small scales

4) Results for the response function

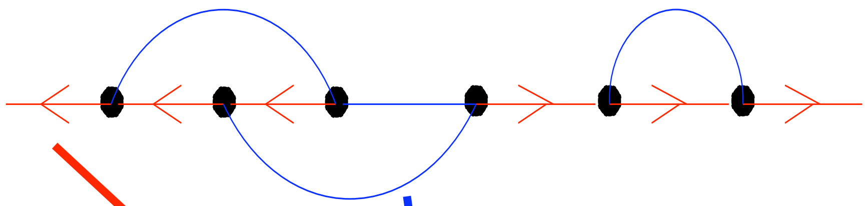
$$R(x_1, x_2) = \langle \frac{\mathcal{D}\psi(x_1)}{\mathcal{D}\zeta(x_2)} \rangle_{\zeta=0}$$

decay at late time & high k



C- High-k resummation

"RPT" (M. Crocce & R. Scoccimarro, 2006a,b)

$P(k) =$  + ...

$$\tilde{R}(\mathbf{k},\tau;\mathbf{k}',\tau_I) = \delta_D(\mathbf{k}-\mathbf{k}')\,\frac{D}{D_I}\,e^{-(D-D_I)^2 k^2 \sigma_v^2/2}$$

Gaussian decay at late times

$$k \gg w$$

Assumption of a wide separation of scales

Linear "effective" equation of motion, with random coefficient, for each k.

$$\frac{\partial\tilde{\delta}}{\partial\tau}(\mathbf{k},\tau) = \int d\mathbf{w_1} d\mathbf{w_2}\,\delta_D(\mathbf{w_1}+\mathbf{w_2}-\mathbf{k})\,(..)\,\tilde{\delta}(\mathbf{w_1},\tau)\tilde{\delta}(\mathbf{w}_2,\tau)$$

$$\begin{aligned}\frac{\partial\tilde{\delta}}{\partial\tau}(\mathbf{k},\tau) &= \tilde{\delta}(\mathbf{k},\tau)\int d\mathbf{w}\,(..)\,\tilde{\delta}_L(\mathbf{w},\tau)\\ &= \hat{\alpha}(\mathbf{k},\tau)\,\tilde{\delta}(\mathbf{k},\tau)\end{aligned}$$

$$\hat{\alpha}(\mathbf{k}) = \int d\mathbf{w}\,\frac{\mathbf{k}\cdot\mathbf{w}}{w^2}\,\tilde{\delta}_{L0}(\mathbf{w})$$

$$\begin{cases}\hat{\tilde{\delta}}(\mathbf{k},D) = D\,\tilde{\delta}_{L0}(\mathbf{k})\,e^{\hat{\alpha}(D-D_I)}\\[2ex] \hat{\delta}(\mathbf{x},D) = D\,\delta_{L0}[\mathbf{x}-\mathbf{s}_L(\mathbf{q}=0,D),D]\end{cases}$$

Sweeping effect

D- Lagrangian framework

Trajectories $\mathbf{x}(\mathbf{q},\tau)$

Divergence of the displacement field $\kappa(\mathbf{q},\tau) = 3 - \frac{\partial \mathbf{x}}{\partial \mathbf{q}}$

→ No longer sensitive to the sweeping effect

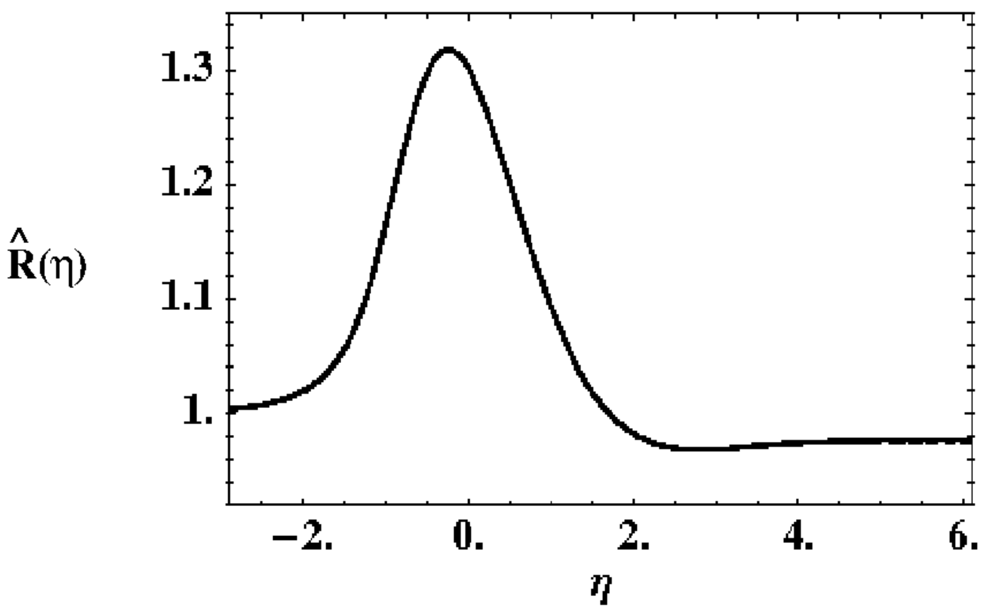
One can apply the same procedure, even without explicit resummation

separation of scale

Linear "effective" equation of motion with random coefficients

$$\kappa''(\mathbf{k},\eta) + \frac{1}{2}\kappa'(\mathbf{k},\eta) - \frac{3}{2}\kappa(\mathbf{k},\eta) = e^{\eta}\,\hat{\alpha}(\mathbf{k})\,\left(\kappa''(\mathbf{k},\eta) + \frac{1}{2}\kappa'(\mathbf{k},\eta)\right) + e^{\eta}\,\hat{\beta}(\mathbf{k})\,\left(\omega''(\mathbf{k},\eta) + \frac{1}{2}\omega'(\mathbf{k},\eta)\right)$$

$$R(\eta) = \int_{-\infty}^{\infty} \hat{\kappa}(\eta;\hat{\alpha},\hat{\beta})\,\mathcal{P}(\hat{\alpha},\hat{\beta})\,d\hat{\alpha}d\hat{\beta}$$



→ No Gaussian decay at late times

Exact results for toy models

Work on firm grounds: beyond perturbative approaches and partial resummations

Include some nonperturbative shell-crossing phenomena

Complete and consistent picture for similar dynamics

Deeper understanding of some processes

Benchmark for approximation schemes

F. Bernardeau & P.V., 2010a,b - P.V., 2009a,b,c - P.V. & F. Bernardeau, 2010

S. N. Gurbatov, A. I. Saichev, S. F. Shandarin, 1989 - S. F. Shandarin 2010 -
M. Vergassola, B. Dubrulle, U. Frisch, A. Noullez, 1994 - D. H. Weinberg & J. E. Gunn, 1990

A- Adhesion model

- Zeldovich approximation

$$\frac{\partial \mathbf{v}}{\partial \tau} + \mathcal{H}\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla\phi$$

replace gravit. potential

by velocity potential

$$\frac{\partial \mathbf{v}}{\partial \tau} + \left(1 - \frac{3\Omega_{\rm m}}{2f}\right)\mathcal{H}\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = 0$$

$$\mathbf{v} = \left(\frac{dD_+}{d\tau}\right)\mathbf{u}$$

$$\frac{\partial \mathbf{u}}{\partial D_+} + (\mathbf{u}\cdot\nabla)\mathbf{u} = 0$$

$$\mathbf{x} = \mathbf{q} + D_+(\tau)\,\mathbf{u}_{L0}(\mathbf{q})$$

- Burgers equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu\frac{\partial^2 u}{\partial x^2} \qquad \nu \to 0^+$$

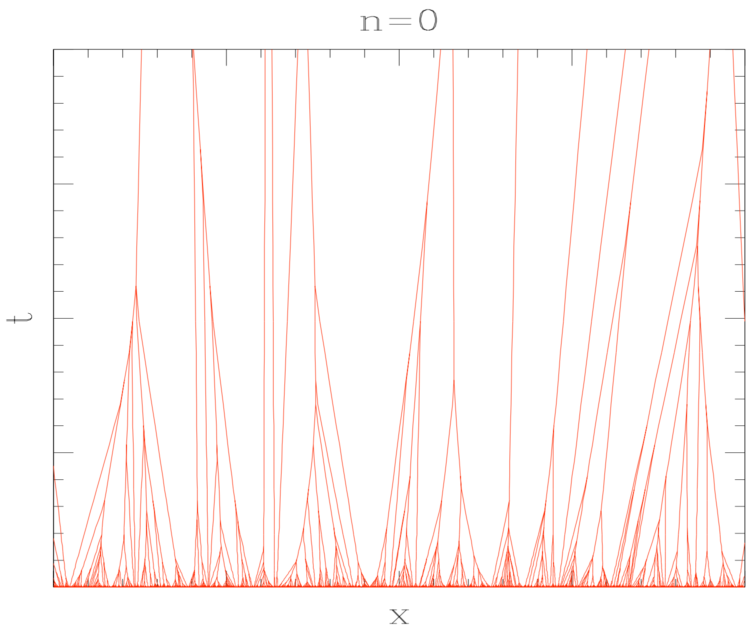
+ continuity equation
for the density field

particles stick together after collision

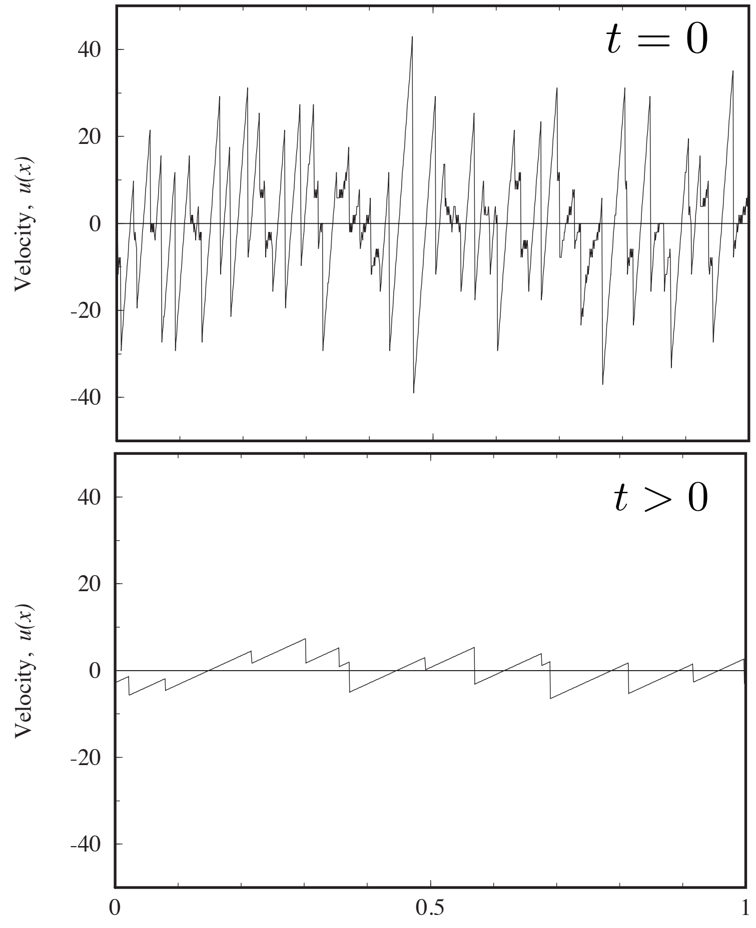
- Gaussian initial conditions with power-law density & velocity power spectrum

$$-3 < n < 1 : \qquad P_{\delta_L}(k,t) \propto t^2\, k^{n+2}$$

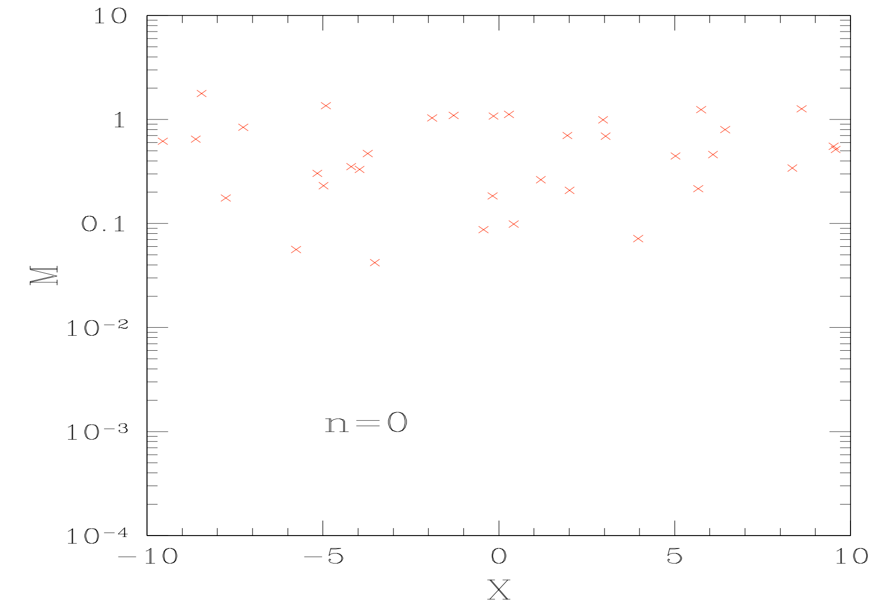
$n = 0:$



particle trajectories



mass

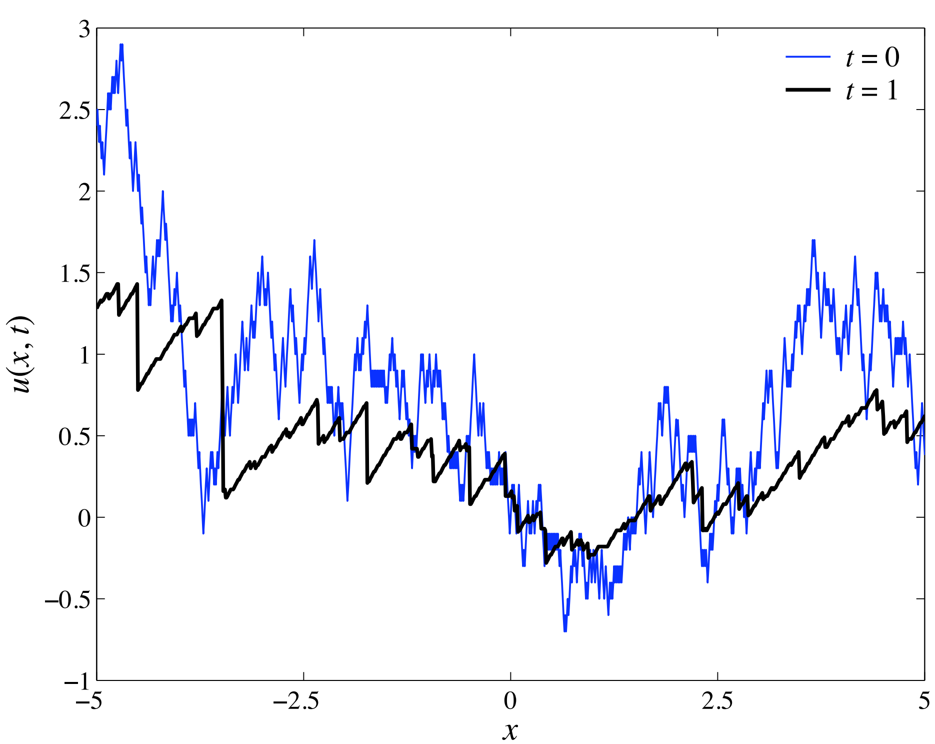


location

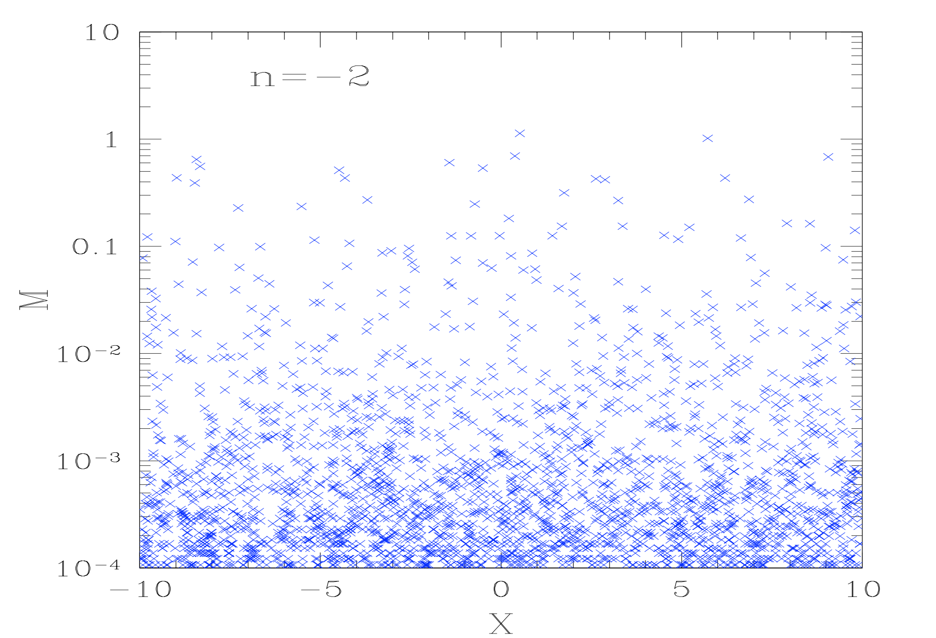
velocity fields

distributions of clusters

$n = -2:$

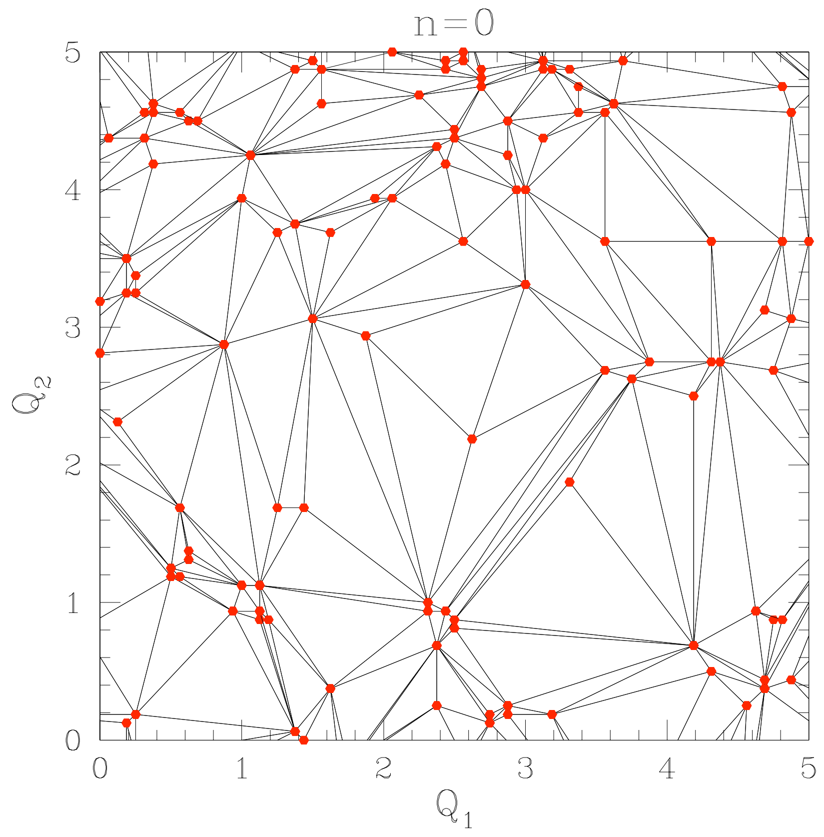


mass

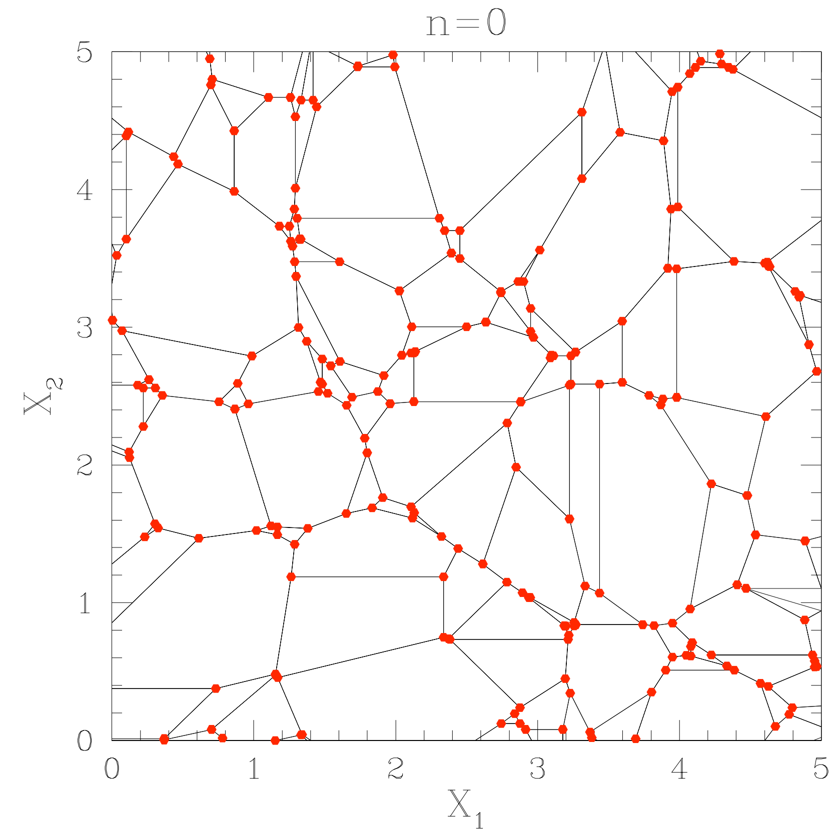


location

in 2D, $n = 0$:



Partition of (initial)
Lagrangian space



Network of filaments and clusters,
in Eulerian space

- Hopf-Cole solution

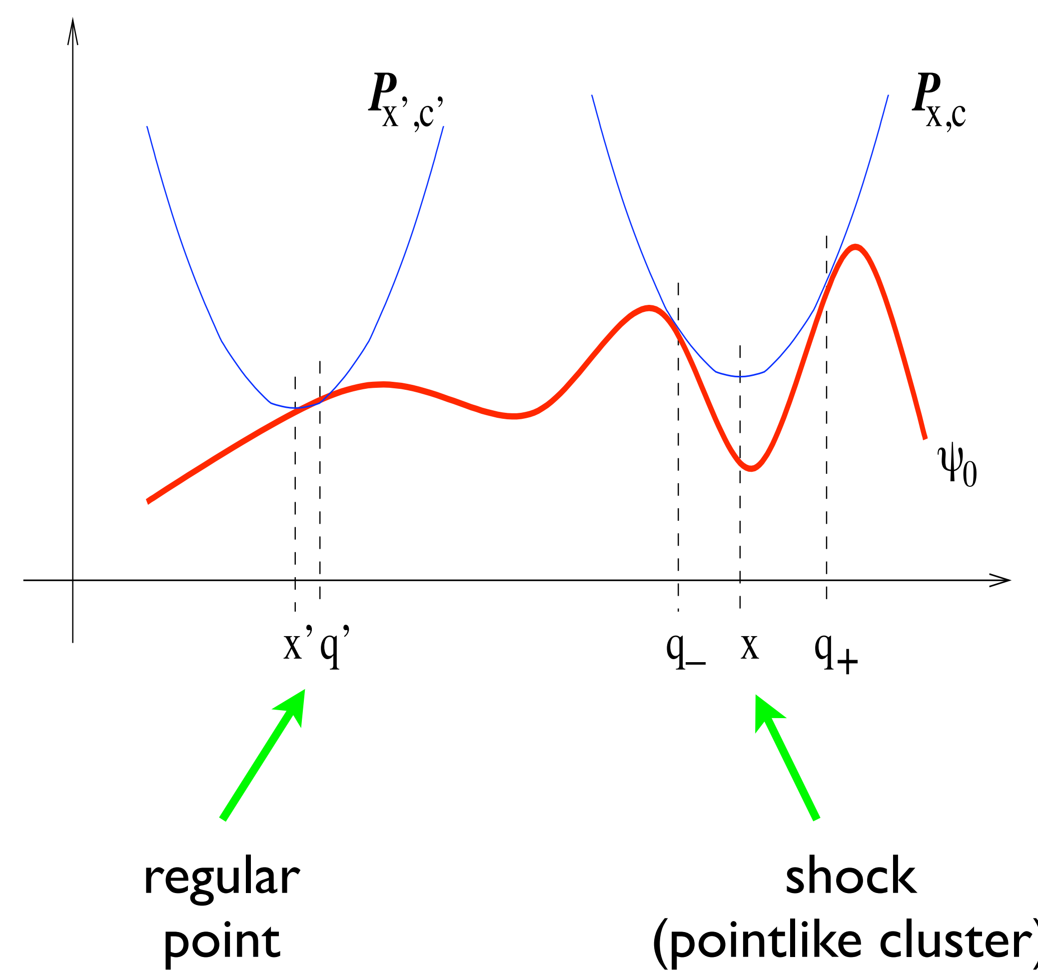
$$u(x,t) = -\frac{\partial \psi}{\partial x} \qquad \psi(x,t) = 2\nu \ln \int_{-\infty}^{\infty} \frac{dq}{\sqrt{4\pi\nu t}} \exp\left[\frac{\psi_0(q)}{2\nu} - \frac{(x-q)^2}{4\nu t}\right]$$

$$\nu \to 0^+ : \quad \psi(x,t) = \max_q \left[\psi_0(q) - \frac{(x-q)^2}{2t}\right], \quad u(x,t) = u_0(q) = \frac{x - q(x,t)}{t}$$

- Geometrical construction

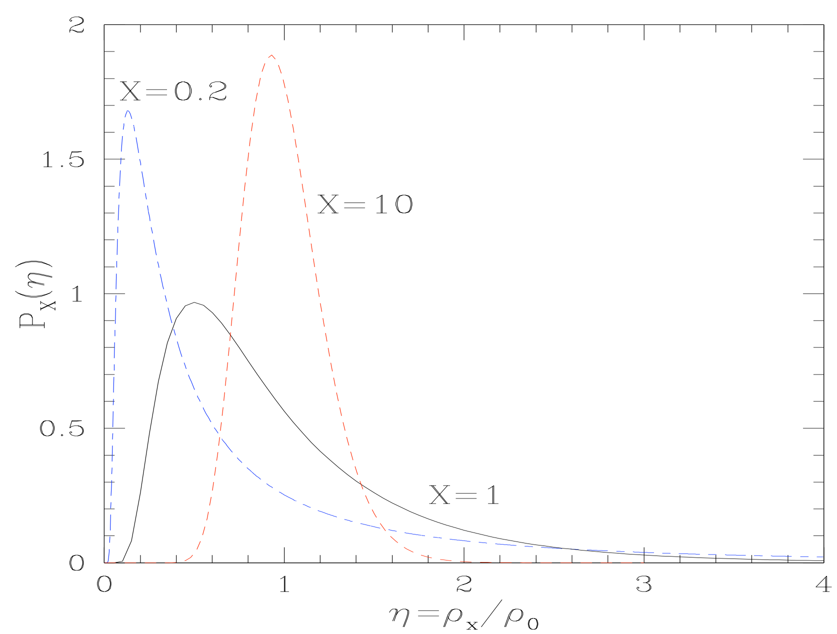
inverse Lagrangian map, $x \mapsto q$,
given by first-contact points

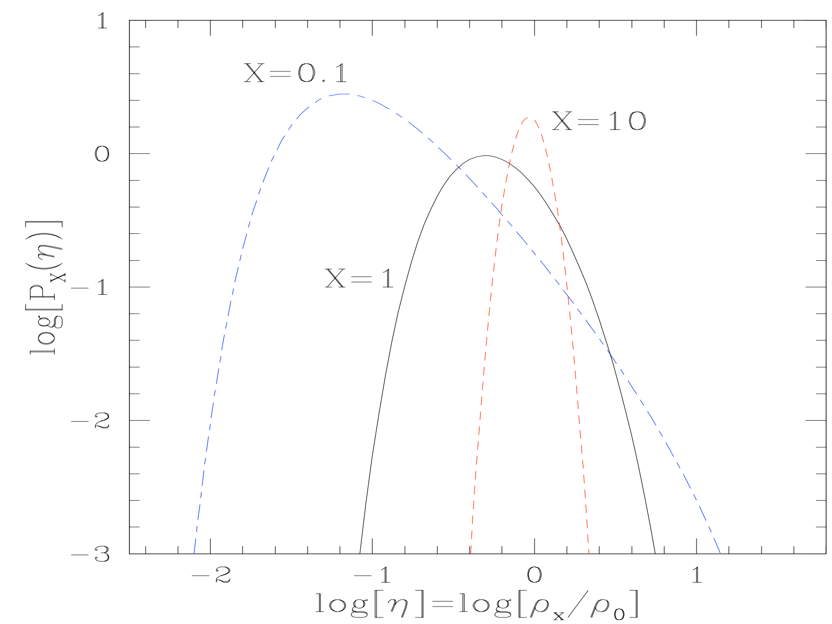
$$\mathcal{P}_{x,c}(q) = \frac{(q-x)^2}{2t} + c$$



B- Brownian initial velocity, n=-2

Probability distribution of the density contrast



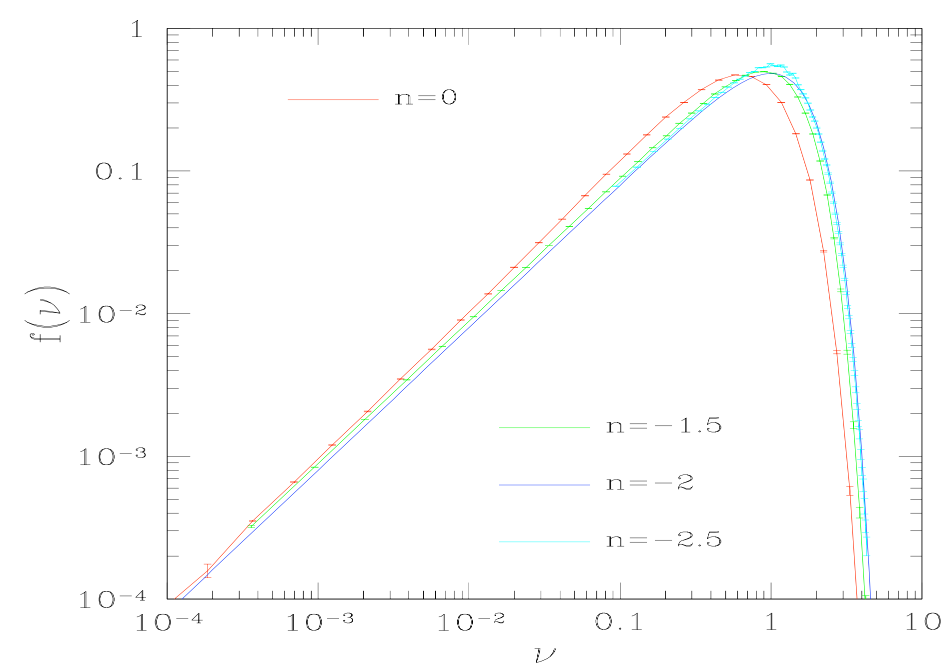


$$\eta = \frac{m}{\bar{\rho}\, x}, \quad \eta \geq 0 : \quad \mathcal{P}_X(\eta) = \sqrt{\frac{X}{\eta}}\, \eta^{-3/2}\, e^{-X(\sqrt{\eta}-1/\sqrt{\eta})^2}$$

$$S_n = \frac{\langle \eta^n \rangle_c}{\langle \eta^2 \rangle_c^{n-1}} = (2n-3)!!$$

→ exact realization of the stable clustering ansatz

Cluster mass function: $$N(M) = \frac{1}{\sqrt{\pi}}\, M^{-3/2}\, e^{-M}$$



→ exact realization of the Press & Schechter ansatz

C- Response functions

1) Eulerian framework

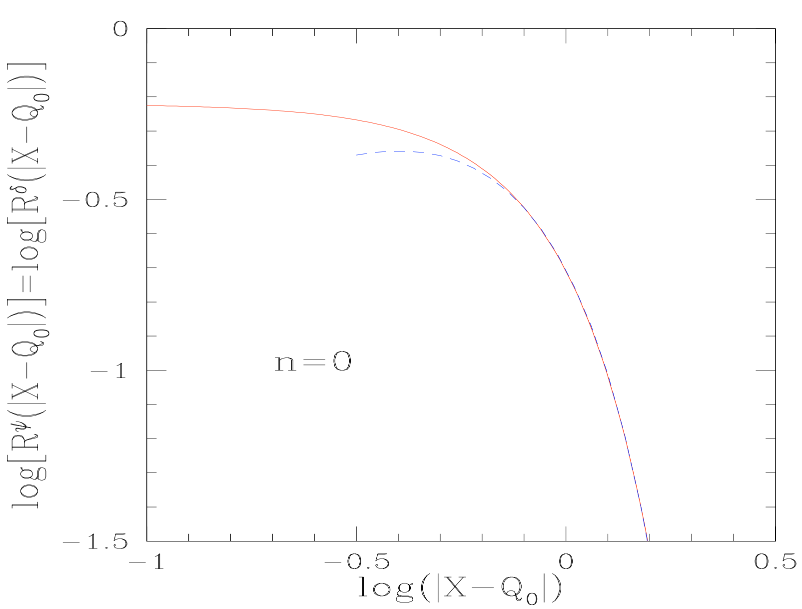
$$R^{\delta}(x,t;q_0) = \langle \frac{\mathcal{D}\delta(x,t)}{\mathcal{D}\delta_{L0}(q_0)} \rangle \qquad R^{\delta}(x,t;q_0) = p_x(u,t) \quad \text{with} \quad u = \frac{x-q_0}{t}$$

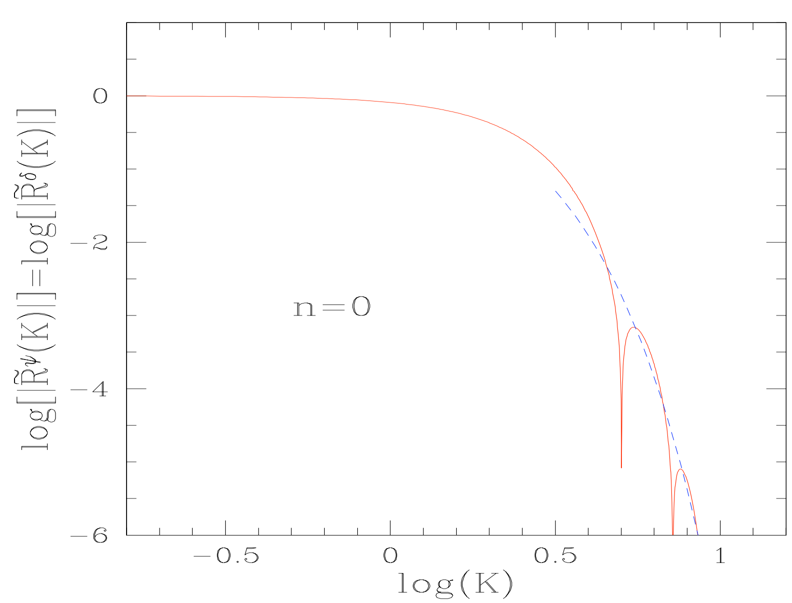
The Eulerian response function is set by the 1-point probability distribution of the velocity.

Linear regime (long wavelengths):

$$R_L^{\delta}(x,t;q_0) = \frac{1}{\sqrt{2\pi}\,\sigma_{u_0}(x)}\, e^{-(x-q_0)^2/(2t^2\sigma_{u_0}^2(x))} \qquad \tilde{R}_L^{\delta}(k,t) = t\, e^{-t^2k^2\sigma_{u_0}^2/2}$$

→ Gaussian decay





$$n = 0 :$$

$$K \gg 1 : \quad \tilde{R}^{\delta}(K) \sim K^{1/4}\, e^{-\frac{\sqrt{2}}{3}K^{3/2} - \frac{\omega_1}{\sqrt{2}}\sqrt{K}} \times \cos\left[\frac{\sqrt{2}}{3}K^{3/2} - \frac{\omega_1}{\sqrt{2}}\sqrt{K} + \frac{\pi}{8}\right]$$

$$-1 < n < 1, \quad |x-q_0| \to \infty \; : \quad R^{\delta}(x,t;q_0) \sim e^{-|x-q_0|^{n+3}/t^2}$$

→ n-dependent behavior

2) Lagrangian framework

$$\kappa(q,t) = 1 - \frac{\partial x}{\partial q} \qquad \rho(x,t) = \frac{\rho_0}{1-\kappa(q,t)} \qquad R^\kappa(q,t;q_0) = \langle \frac{\mathcal{D}\kappa(q,t)}{\mathcal{D}\kappa_{L0}(q_0)} \rangle$$

Using the geometric construction, one obtains

$$R^\kappa(q,t;q_0) = t \int_{\rho_0|q-q_0|}^{\infty} dm\, n(m,t) \left(1 - \frac{|q-q_0|}{m/\rho_0}\right)$$

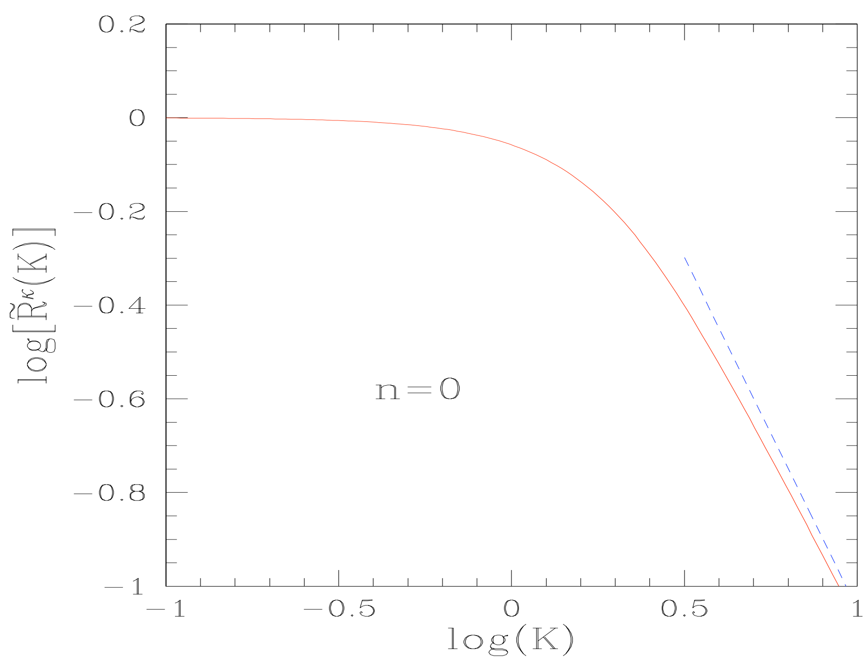
$$\tilde{R}^\kappa(k,t) = 2t \int_0^\infty dm\, n(m,t)\, \frac{1-\cos(km/\rho_0)}{k^2 m/\rho_0}$$

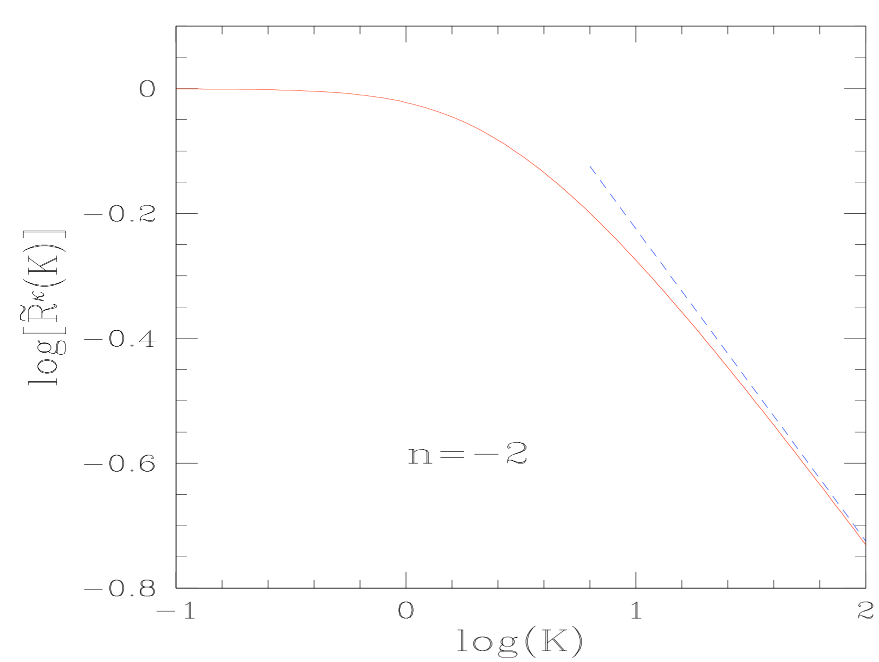
The Lagrangian response function is set by the cluster mass function.

$$\frac{|q-q_0|}{L(t)} \gg 1 : \quad R^\kappa(q,t;q_0) \sim e^{-|q-q_0|^{n+3}/t^2}$$

$$k \gg L(t)^{-1} : \quad \tilde{R}^\kappa(k,t) \sim k^{-(n+3)/2}$$

→ n-dependent power-law decay





Combined models

For some observational probes that are sensitive to both quasilinear and nonlinear regimes (e.g., weak lensing), it is necessary to simultaneously describe both large and small scales.

Combine
the accuracy and systematic character of perturbative approaches one large scales
with
the reasonable description of phenomenological models on small scales

P.V. & T. Nishimichi, 2010

As in the halo model (but from a Lagrangian point of view), decompose the power spectrum as

$$P(k) = P_{2\mathrm{H}}(k) + P_{1\mathrm{H}}(k)$$

"2-halo term"

perturbative contribution

$$P_{2\mathrm{H}}(k) \simeq F_{2\mathrm{H}}(1/k)\; P_{\mathrm{pert}}(k)$$

high-k behavior solved by going beyond standard perturbation theory

resummation schemes

"1-halo term"

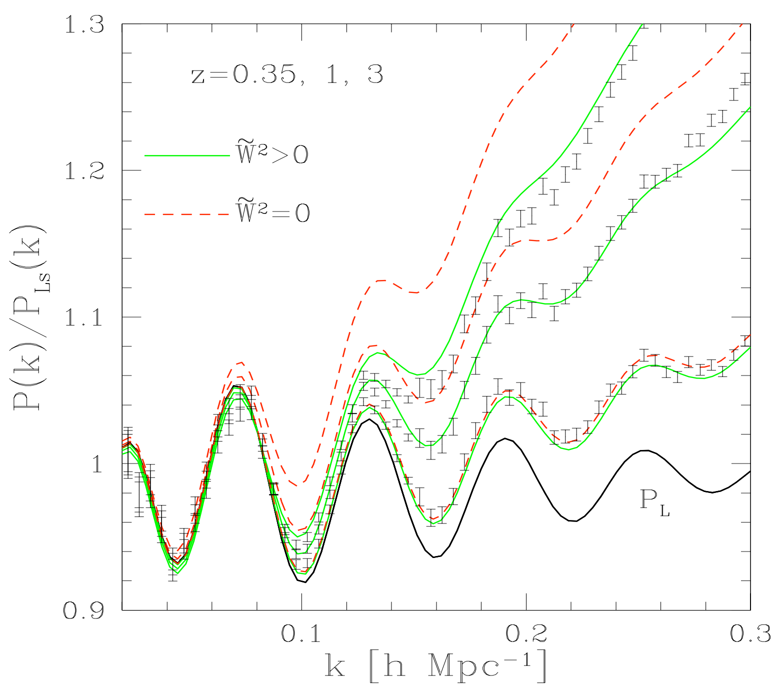
nonperturbative contribution

$$P_{1\mathrm{H}}(k) = \int_0^\infty \frac{d\nu}{\nu}\, f(\nu)\, \frac{M}{\bar{\rho}(2\pi)^3} \left(\tilde{u}_M(k)^2 - \tilde{W}(k\, q_M)^2 \right)$$

halo mass function

halo density profile

low-k behavior solved by counterterm



Using the continuity equation, $\rho(\mathbf{x})\, d\mathbf{x} = \bar{\rho}\, d\mathbf{q}$ $\Rightarrow$ $[1+\delta(\mathbf{x})]\, d\mathbf{x} = d\mathbf{q}$

the density contrast reads in Fourier space as

$$\tilde{\delta}(\mathbf{k}) = \int \frac{d\mathbf{x}}{(2\pi)^3}\, e^{-i\mathbf{k}\cdot\mathbf{x}}\, \delta(\mathbf{x}) = \int \frac{d\mathbf{q}}{(2\pi)^3}\, \left(e^{-i\mathbf{k}\cdot\mathbf{x}(\mathbf{q})} - e^{-i\mathbf{k}\cdot\mathbf{q}}\right)$$

$$\Rightarrow \quad P(k) = \int \frac{d\mathbf{q}}{(2\pi)^3}\, \langle e^{i\mathbf{k}\cdot\Delta\mathbf{x}} - e^{i\mathbf{k}\cdot\mathbf{q}} \rangle \quad \text{with} \quad \Delta\mathbf{x} = \mathbf{x}(\mathbf{q}) - \mathbf{x}(0)$$

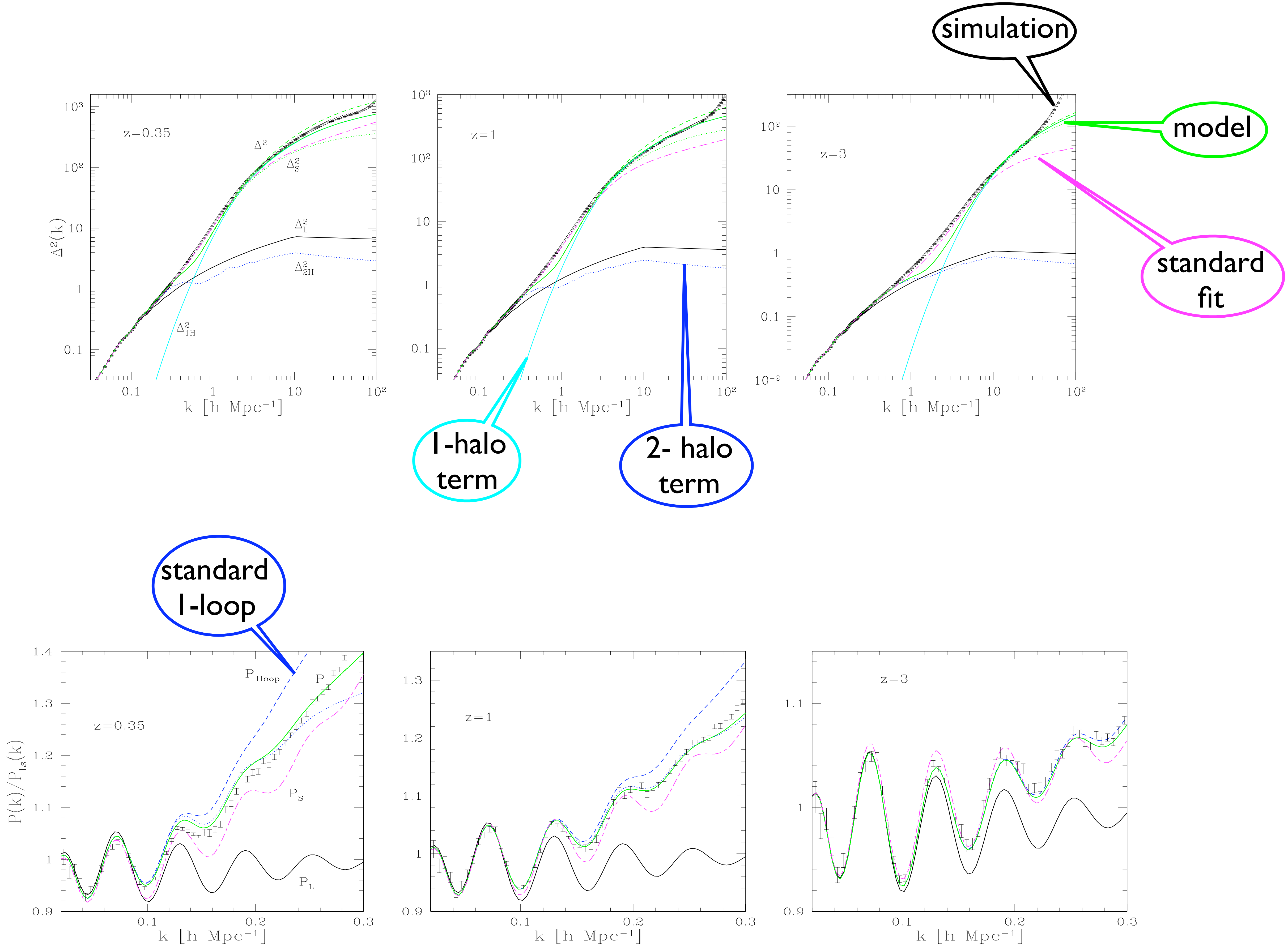
As in the usual halo model, but in Lagrangian space, we can write:

$$P(k) = P_{1\mathrm{H}}(k) + P_{2\mathrm{H}}(k)$$

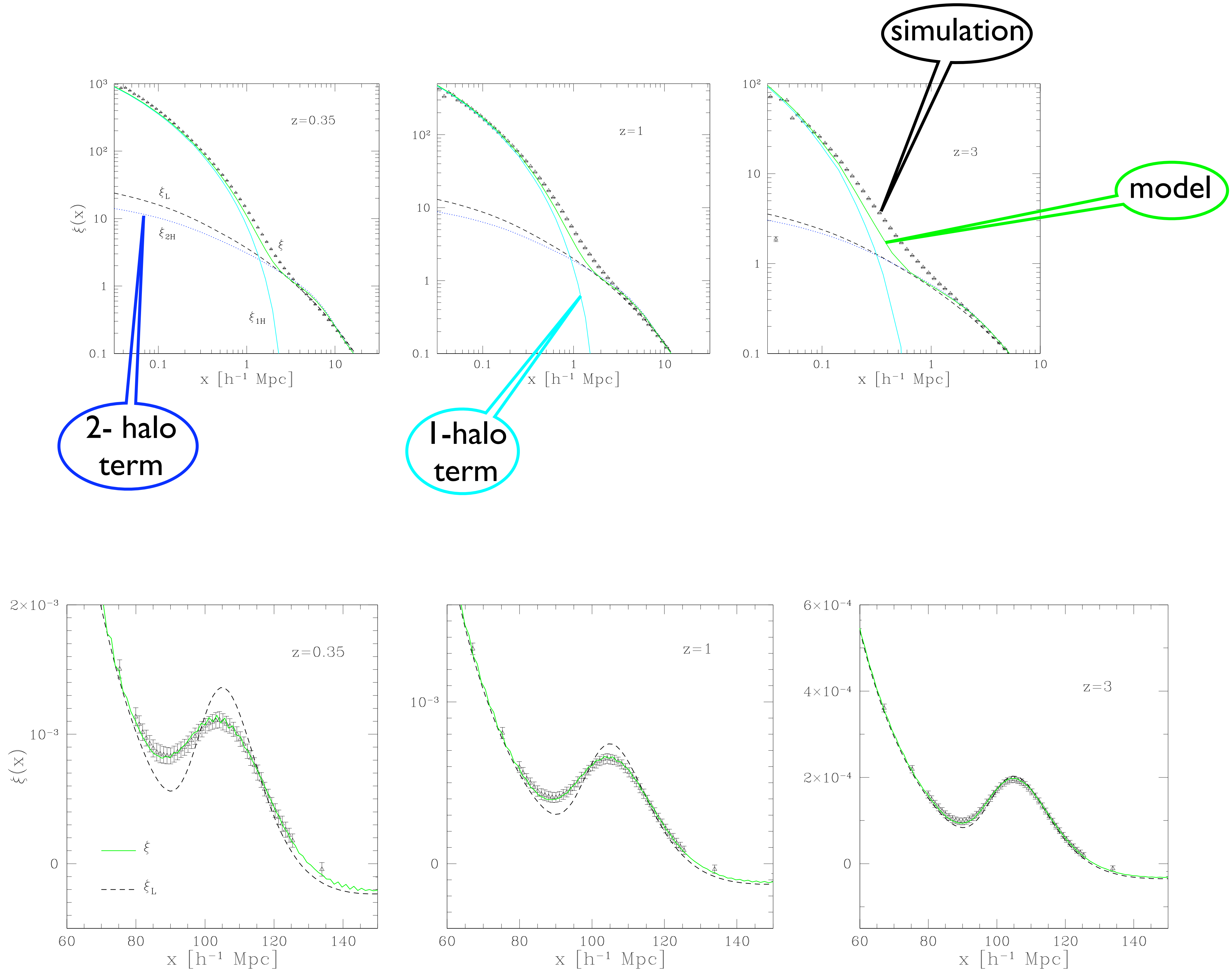
$$P_{1\mathrm{H}}(k) = \int \frac{d\mathbf{q}}{(2\pi)^3}\, F_{1\mathrm{H}}(q)\, \langle e^{i\mathbf{k}\cdot\Delta\mathbf{x}} - e^{i\mathbf{k}\cdot\mathbf{q}} \rangle_{1\mathrm{H}}$$

$$P_{2\mathrm{H}}(k) = \int \frac{d\mathbf{q}}{(2\pi)^3}\, F_{2\mathrm{H}}(q)\, \langle e^{i\mathbf{k}\cdot\ \ \mathbf{x}} - e^{i\mathbf{k}\cdot\mathbf{q}} \rangle_{2\mathrm{H}}$$

Results for the power spectrum



Results for the two-point correlation function



Higher-order statistics: the bispectrum

In order to break degeneracies, it is useful to consider higher-order statistics beyond the power spectrum (i.e., 2-pt correlation).

↓

3-pt correlation / bispectrum

This is also useful to constrain primordial non-Gaussianities

P.V. & T. Nishimichi, 2011

$$\langle\tilde{\delta}(\mathbf{k}_1)\tilde{\delta}(\mathbf{k}_2)\tilde{\delta}(\mathbf{k}_3)\rangle = \delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\, B(k_1,k_2,k_3)$$

Again, as in the halo model (but from a Lagrangian point of view), we decompose the bispectrum as

$$B = B_{1\mathrm{H}} + B_{2\mathrm{H}} + B_{3\mathrm{H}}$$

"1- halo" and "2-halo" terms → nonperturbative contributions

"3-halo term" → perturbative contribution

$$B_{3\mathrm{H}} = B_{\mathrm{pert}}$$

$$B_{1\mathrm{H}} = \int \frac{d\nu}{\nu}\, f(\nu) \left(\frac{M}{\bar{\rho}(2\pi)^3}\right)^2 \prod_{j=1}^{3} \left(\tilde{u}_M(k_j) - \tilde{W}(k_j\, q_M)\right)$$

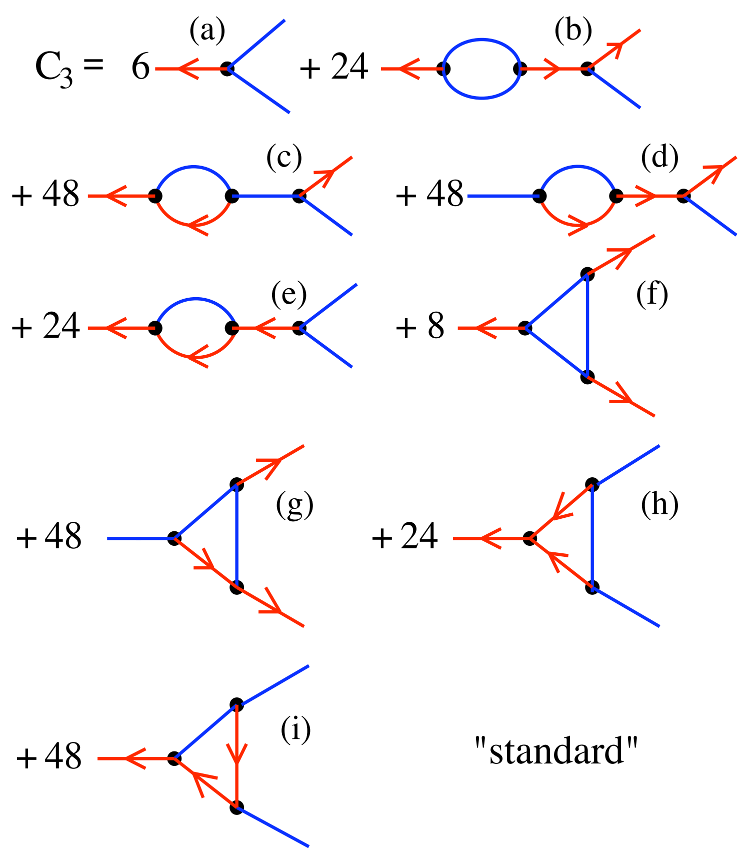
$$B_{2\mathrm{H}} = P_L(k_1) \int \frac{d\nu}{\nu}\, f(\nu)\, \frac{M}{\bar{\rho}(2\pi)^3} \prod_{j=2}^{3} \left(\tilde{u}_M(k_j) - \tilde{W}(k_j\, q_M)\right) + 2\,\mathrm{cyc.}$$

→ counterterms →

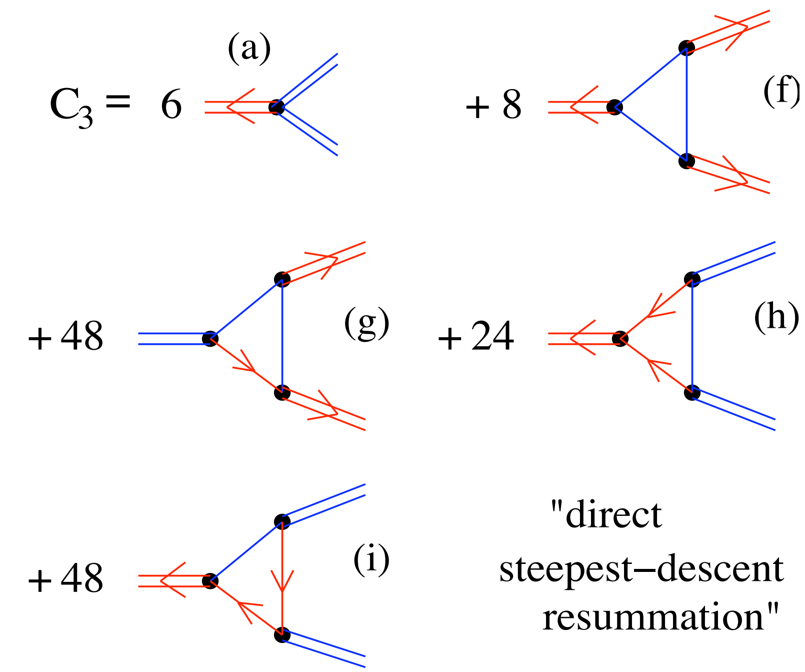
$$B_{1\mathrm{H}} \propto k_j^2 \quad \text{for} \quad k_j \to 0$$

$$B_{2\mathrm{H}} \propto P_L(k_j) \quad \text{for} \quad k_j \to 0$$

The 3-halo term can be computed from any perturbative scheme



standard perturbation theory

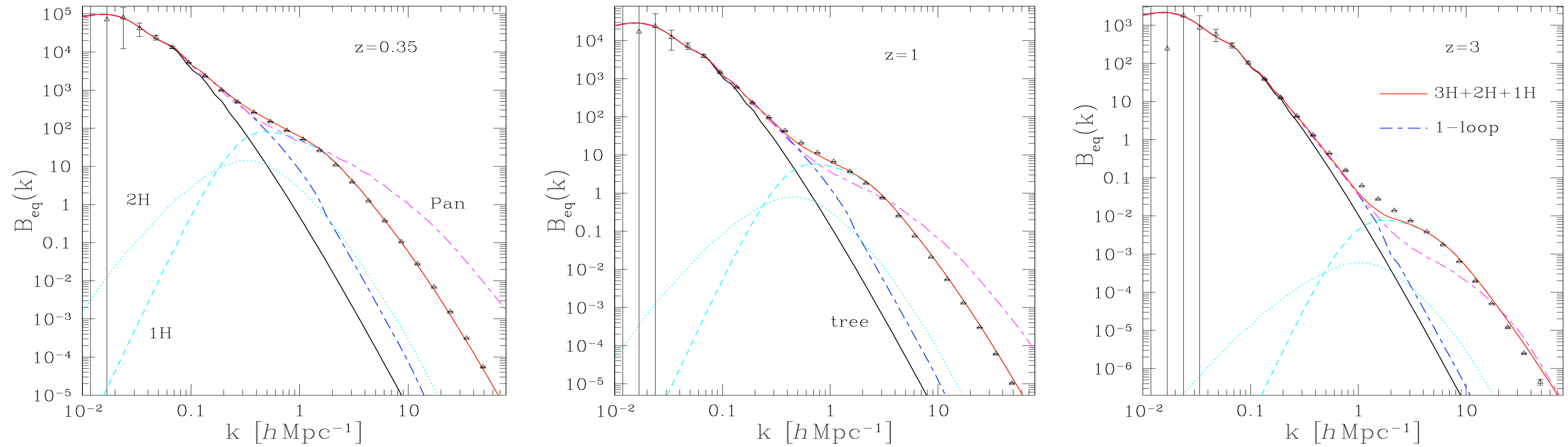


resummation scheme

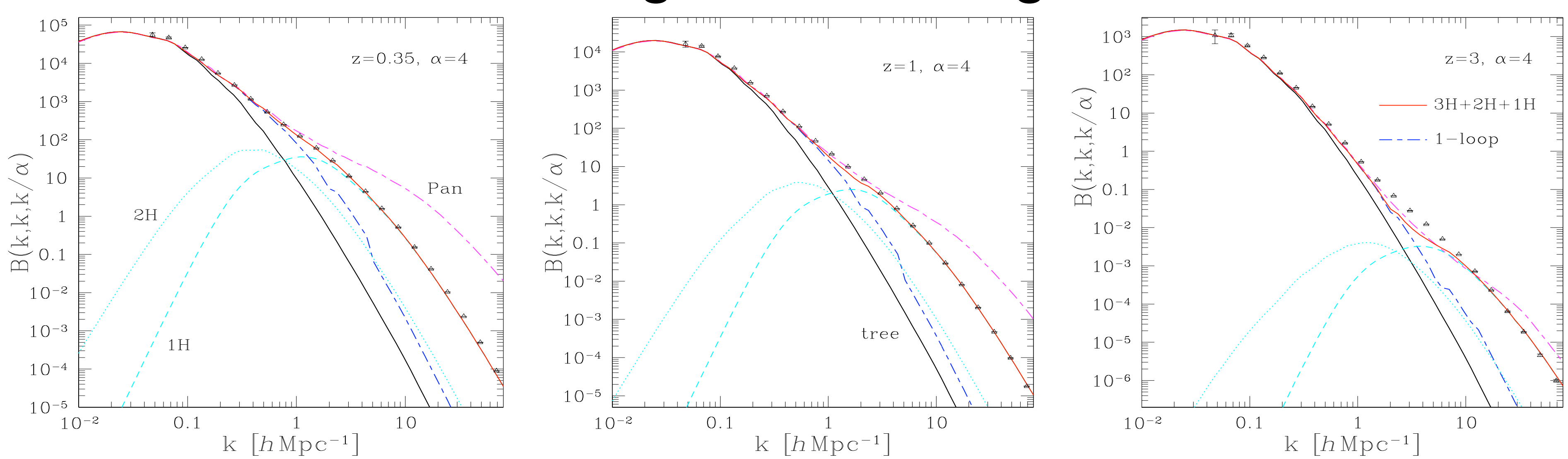
- well-known diagrams
- simpler expressions (time integrals)

- fewer but more complex diagrams
- numerical integrations over time

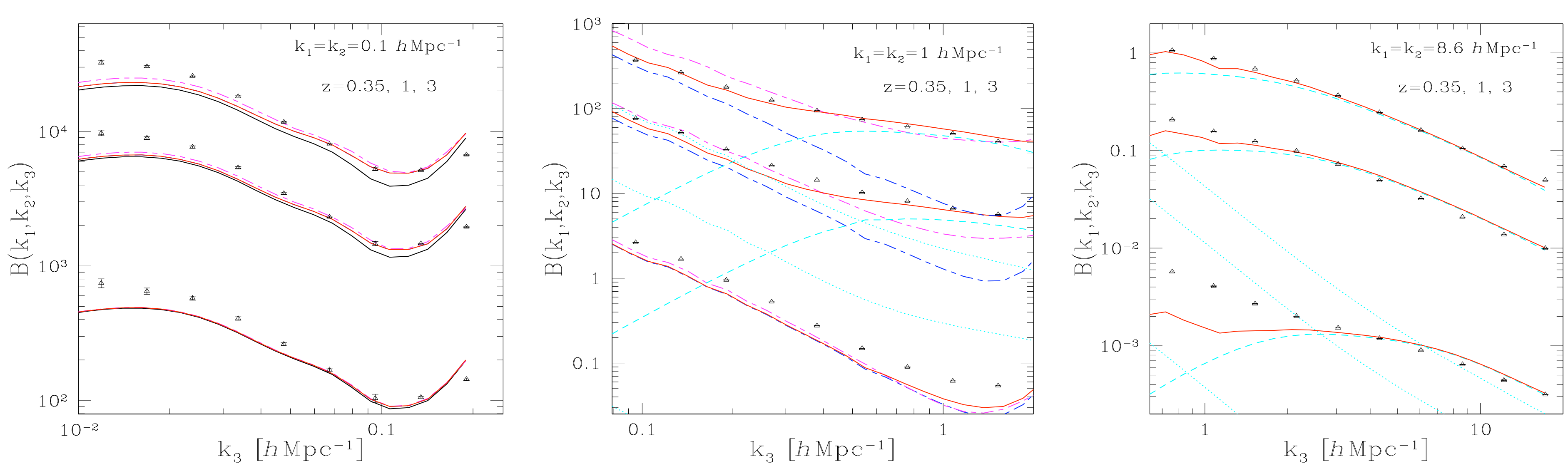
Equilateral triangles



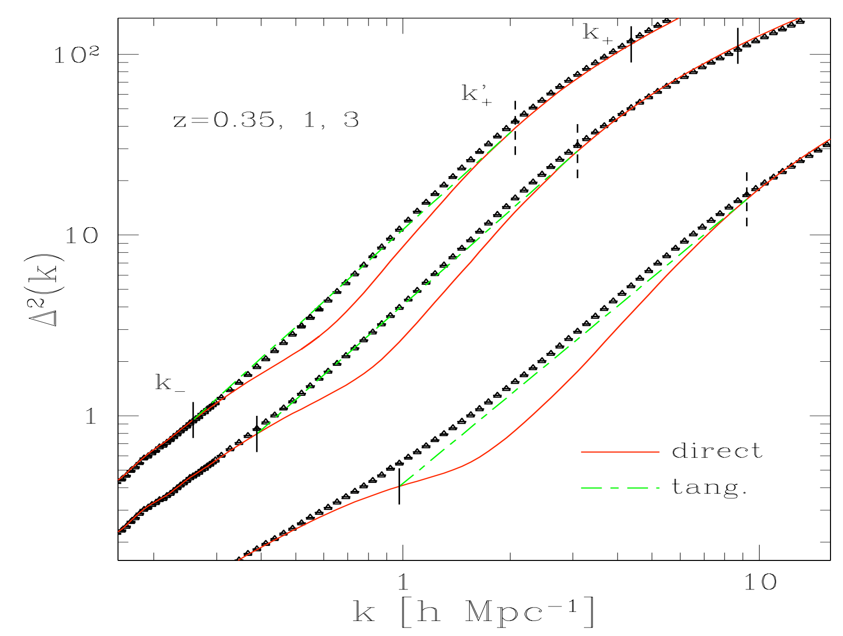
Isosceles triangles at fixed length ratio = 4



Isosceles triangles at fixed equal-sides length

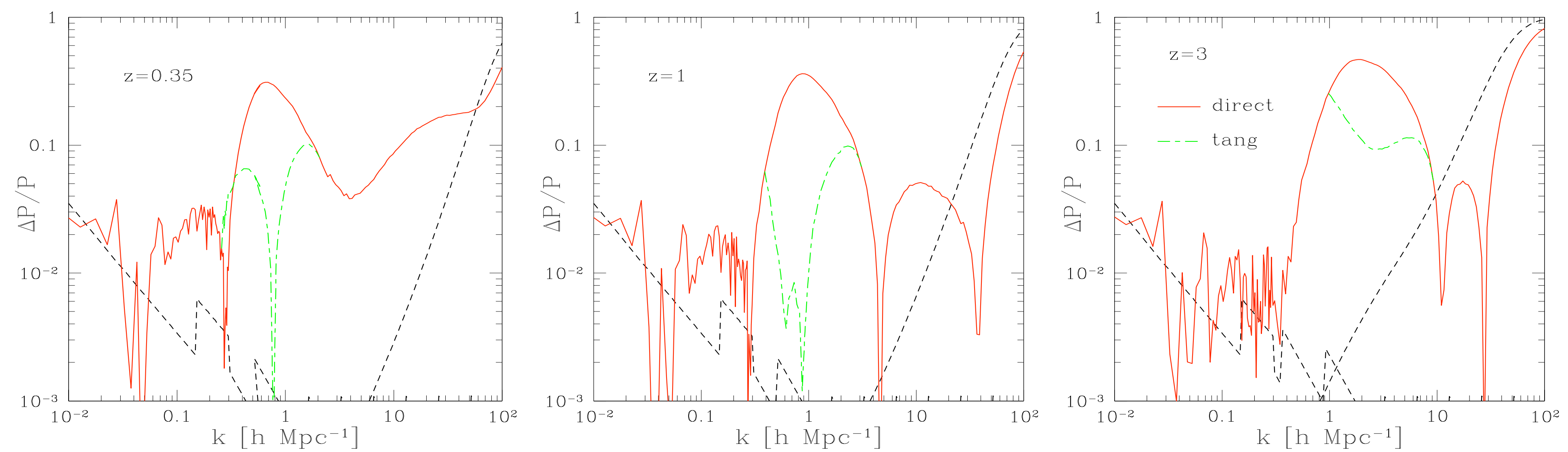


The intermediate regime, at the nonlinear transition, remains the most difficult to predict.

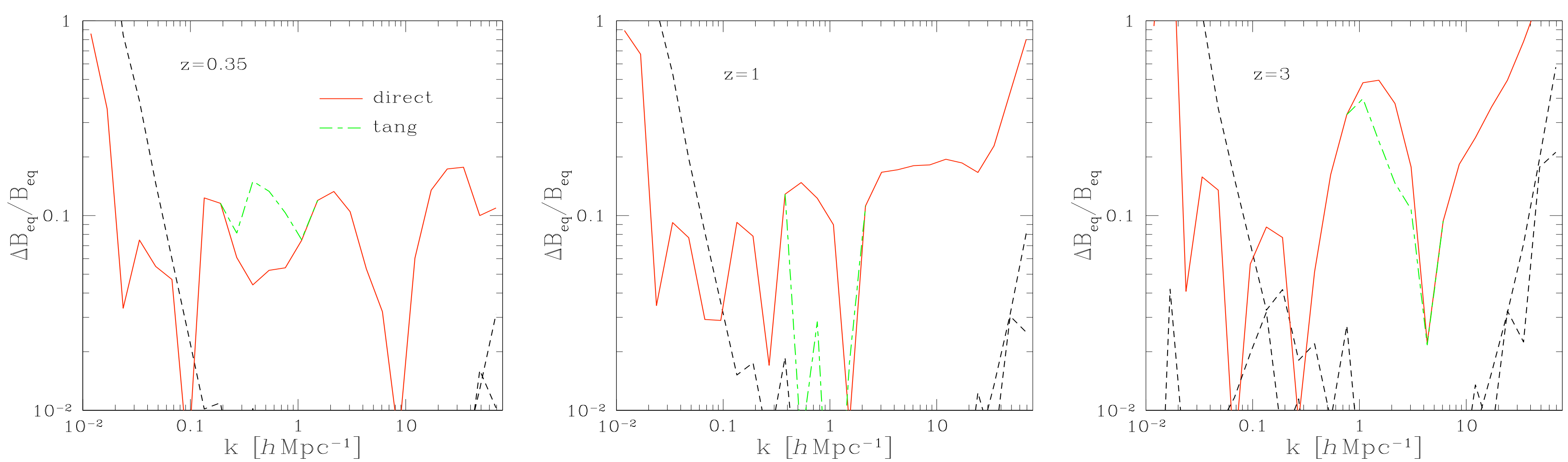


Typical accuracy that is currently reached:

power spectrum:



bispectrum:



Other approaches - extensions

A- Lagrangian perturbation theories

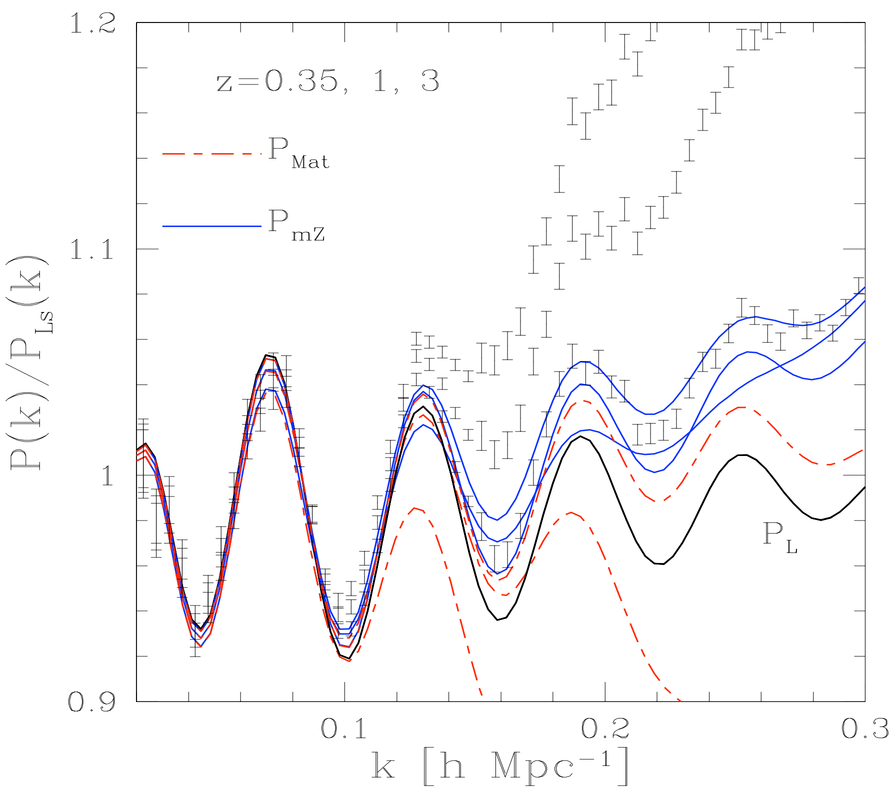
T. Matsubara, 2008

The variance of the linear velocity naturally appears, $$\sigma_v^2 = \frac{4\pi}{3}\int_0^\infty dk\ P_L(k)$$

$$P_{\mathrm{Mat}}(k) = e^{-k^2\sigma_v^2}\ \left(P_L + P_{22} + P_{31} + k^2\sigma_v^2 P_L\right)$$

$$P_{\mathrm{mZ}}(k) = e^{-k^2\sigma_v^2}\ \left(P_L + P^{(2)} + \sum_{n=3}^{\infty} P_Z^{(n)}\right) = P_{\mathrm{Z}}(k) + e^{-k^2\sigma_v^2}\ \left(P^{(2)} - P_Z^{(2)}(k)\right)$$

This Gaussian cutoff is too strong



B-Taking into account the baryons

G. Somogyi & R. E. Smith, 2010

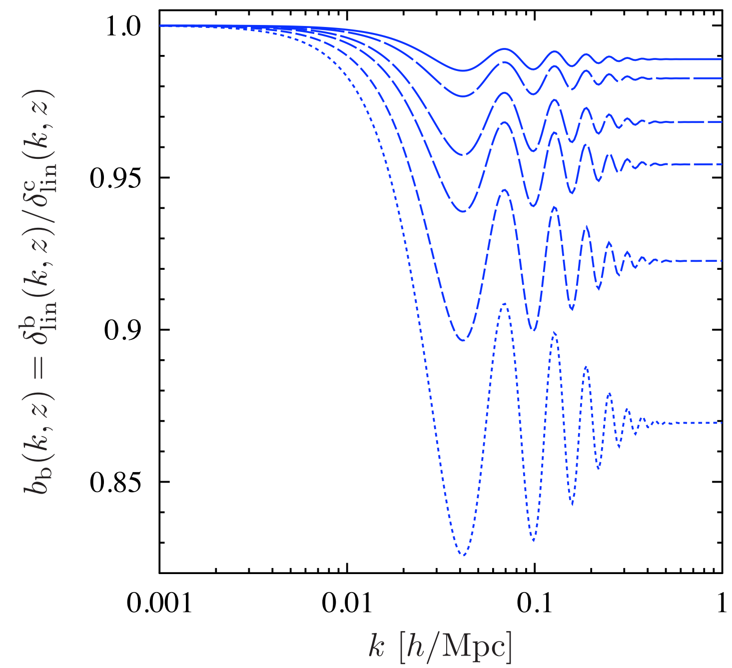
Write the equations of motion for the various components:

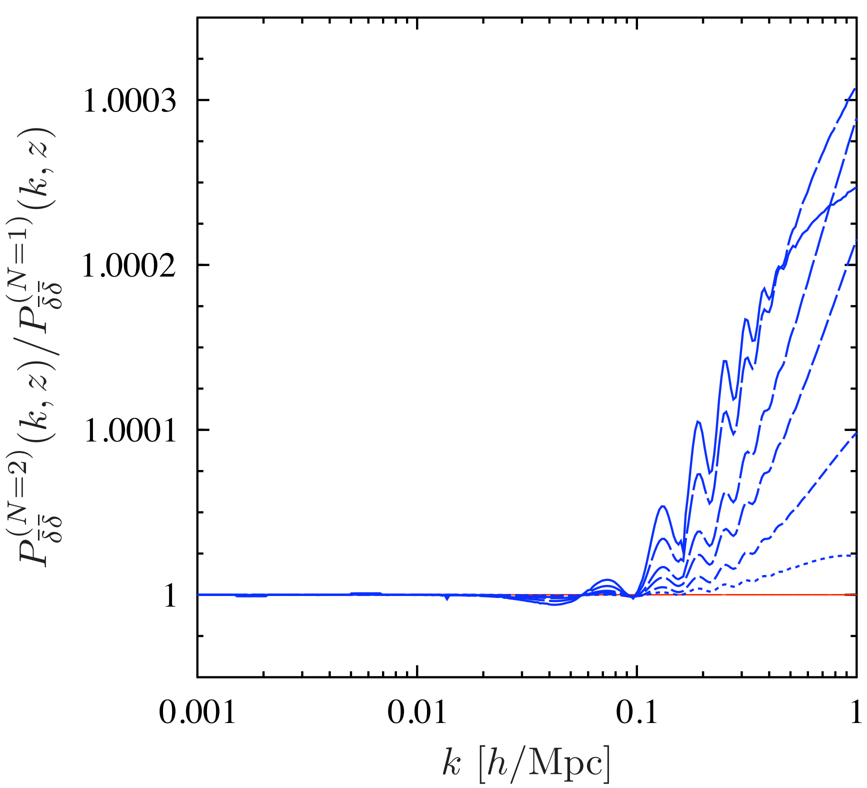
$$\frac{\partial \delta_i}{\partial \tau} + \nabla \cdot [(1+\delta_i)\mathbf{v}_i] = 0$$

$$\frac{\partial \mathbf{v}_i}{\partial \tau} + \mathcal{H}\mathbf{v}_i + (\mathbf{v}_i \cdot \nabla)\mathbf{v}_i = -\nabla\phi$$

$$\Delta\phi = \frac{3}{2}\Omega_\mathrm{m}\mathcal{H}^2 \sum_{i=1}^{N} \delta_i$$

Use the initial conditions specific to each component and solve by perturbation theory





Compare with the single-fluid approximation (up to 1-loop)

C-Truncating the "BBGKY-like" hierarchy

M. Pietroni, 2008

From the equation of motion, which is quadratic, one can derive a hierarchy of equations that relate the n-pt correlation to the (n+1)-pt correlation:

$$\frac{\partial \psi}{\partial \tau}(\mathbf{k}, \tau) = \mathcal{O}(\mathbf{k}, \tau) \cdot \psi(\mathbf{k}, \tau) + K_s(\mathbf{k}; \mathbf{k}_1, \mathbf{k}_2; \tau) \cdot \psi(\mathbf{k}_1, \tau)\psi(\mathbf{k}_2, \tau)$$

$$\frac{\partial}{\partial \tau}\langle\psi\psi\rangle = 2\mathcal{O} \cdot \langle\psi\psi\rangle + 2K_s \cdot \langle\psi\psi\psi\rangle$$

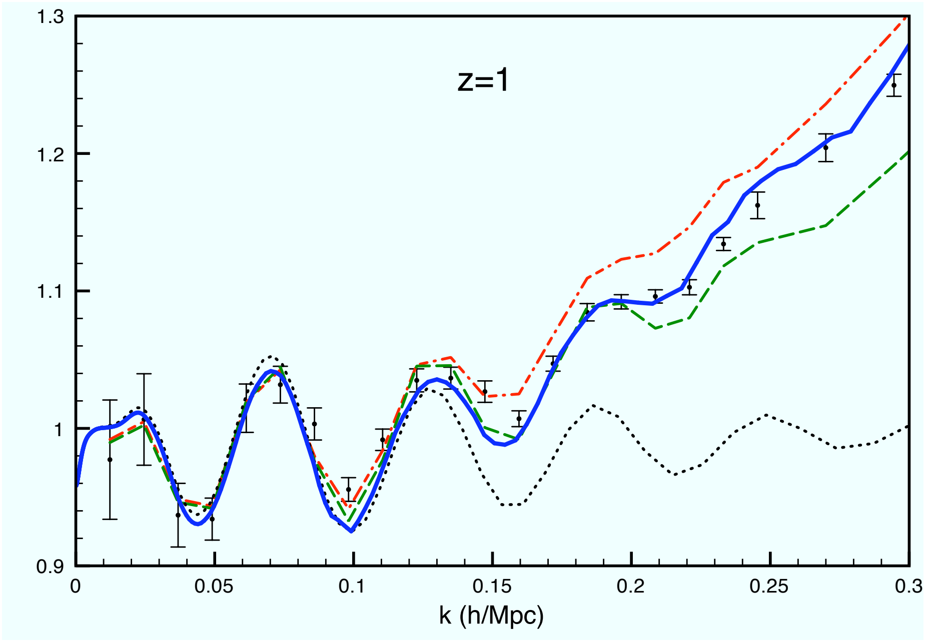
$$\Rightarrow \quad \frac{\partial}{\partial \tau}\langle\psi\psi\psi\rangle = 3\mathcal{O} \cdot \langle\psi\psi\psi\rangle + 3K_s \cdot \langle\psi\psi\psi\psi\rangle$$

$$\frac{\partial}{\partial \tau}\langle\psi\psi\psi\psi\rangle = ...$$

By truncating at a finite order, e.g. setting the connected 4-pt function to zero, one can close the hierarchy and solve it.

This gives a systematic method that agrees with standard perturbation theory and does not involve response functions or different-time statistics.

power spectrum :

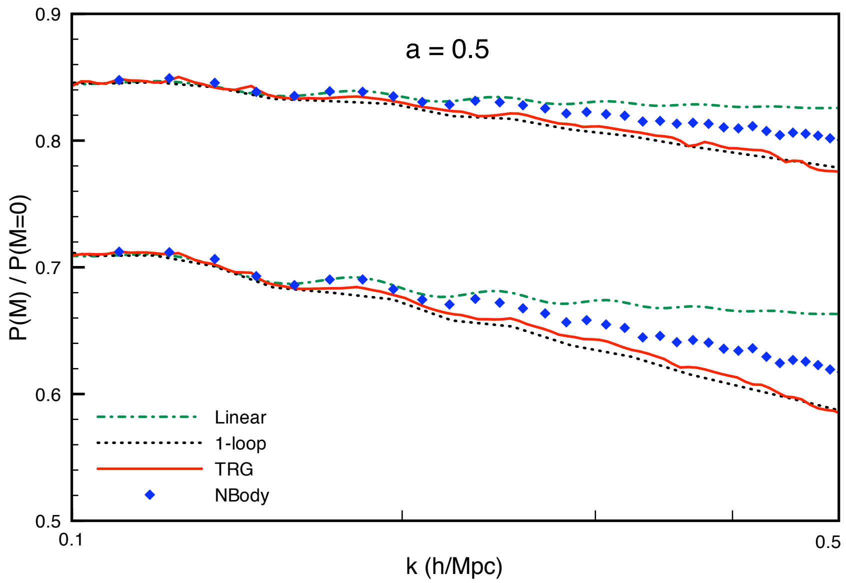


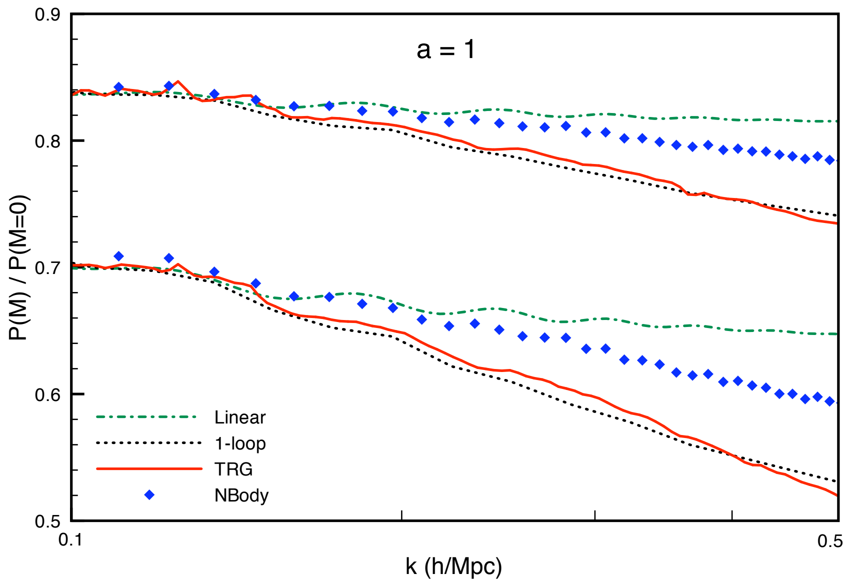
Including neutrinos in an approximate fashion, through a modified Poisson equation where the r.h.s. reads as:

J. Lesgourgues et al., 2009

$$\delta_{\rm m}(\mathbf{k},\tau) \simeq \delta_{\rm cb}(\mathbf{k},\tau) \, \left(1 - f_\nu + f_\nu \frac{\delta_{L\nu}(\mathbf{k},\tau)}{\delta_{L{\rm cb}}(\mathbf{k},\tau)}\right)$$

suppression of the total power spectrum by nonzero neutrino masses:





D-Beyond the fluid approximation: Vlasov eq.

P.V., 2004; S. Tassev 2011

One can work with the full phase-space distribution function: $f(\mathbf{x}, \mathbf{p}; t)$

It obeys the Vlasov-Poisson system:

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{a^2} \cdot \frac{\partial f}{\partial \mathbf{x}} - \frac{\partial \phi}{\partial \mathbf{x}} \cdot \frac{\partial f}{\partial \mathbf{p}} = 0, \qquad \Delta \phi = \frac{4\pi\mathcal{G}}{a} (\rho - \bar{\rho}), \qquad \rho(\mathbf{x}, t) = \bar{\rho} \int f(\mathbf{x}, \mathbf{p}; t) \, d\mathbf{p}$$

This again yields a quadratic equation of motion, that can be solved through perturbative expansions, starting from the linear growing mode:

$$f_L(\mathbf{x}, \mathbf{p}; t) = \delta_D(\mathbf{p}) + \delta_L(\mathbf{x}; t)\delta_D(\mathbf{p}) - \mathbf{p}_L(\mathbf{x}; t) \cdot \frac{\partial \delta_D}{\partial \mathbf{p}}(\mathbf{p})$$

density and velocity fields of the hydrodynamical approximation

The iterative perturbative approach gives back the standard perturbation results of the fluid approximation.

It is possible to consider resummation schemes, for instance using a path-integral reformulation and applying various expansion schemes.

This allows going beyond the fluid approximation. In such cases, it may be useful to start from a regular phase-space distribution, defined for instance from the Zeldovich dynamics, rather than from the singular linear growing mode.

However, no quantitative results have been compared with simulations yet.

Conclusion

Theoretical approach that is complementary to numerical simulations and observations.

- explore regimes that are beyond the reach of numerical simulations (e.g., very large scales)
- understand the main properties of the dynamics (Burgers)

Prospects

- find out which resummation schemes are the most accurate/efficient ?
- improve the modeling of the transition scales
- further develop Lagrangian approaches (useful for redshift-space distortions)
- generalization to warm dark matter ? more complex components (clustering quintessence,...)
- going beyond the fluid approximation: phase-space description ?