Sterile Neutrinos for Warm Dark Matter in Flavor Symmetry Models

CIAS, Paris, 08 June 2012

Contents:

- Neutrino masses and mixing
- Phenomena of light sterile neutrinos ν_s
- Mechanisms for light ν_s
- Models based on flavor symmetry & Froggatt-Nielsen mechanism

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FÜR KERNPHYSIK

$m_\nu = m_L - m_D^T M_R^{-1} m_D$

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MANITOP
Massive Neutrinos: Investigating their Theoretical Origin and Phenomenology

In collaboration with **J. Barry** and **W. Rodejohann**

Based on JHEP07 (2011) 091; arXiv:1110.6382, arXiv:1110.6838

Neutrinos are massless in the SM as a result of the model's simple structure:

--- $SU(2)_L \times U(1)_Y$ gauge symmetry and Lorentz invariance;

Fundamentals of the model, mandatory for its consistency as a QFT.

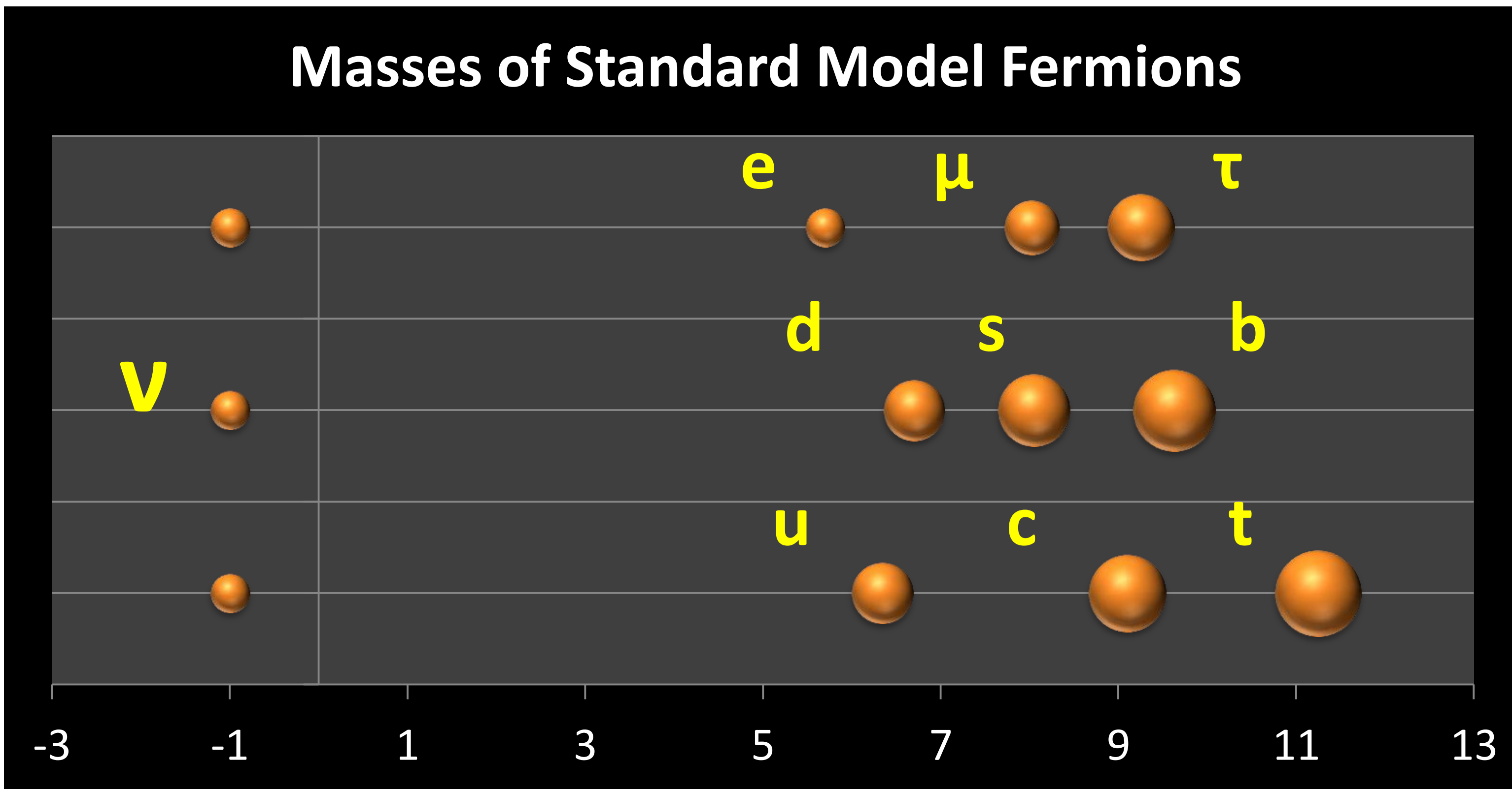
--- Economical particle content:

No right-handed neutrinos --- a Dirac mass term is not allowed.

Only one Higgs doublet --- a Majorana mass term is not allowed.

--- Renormalizability:

No dimension ≥ 5 operators --- a Majorana mass term is forbidden.



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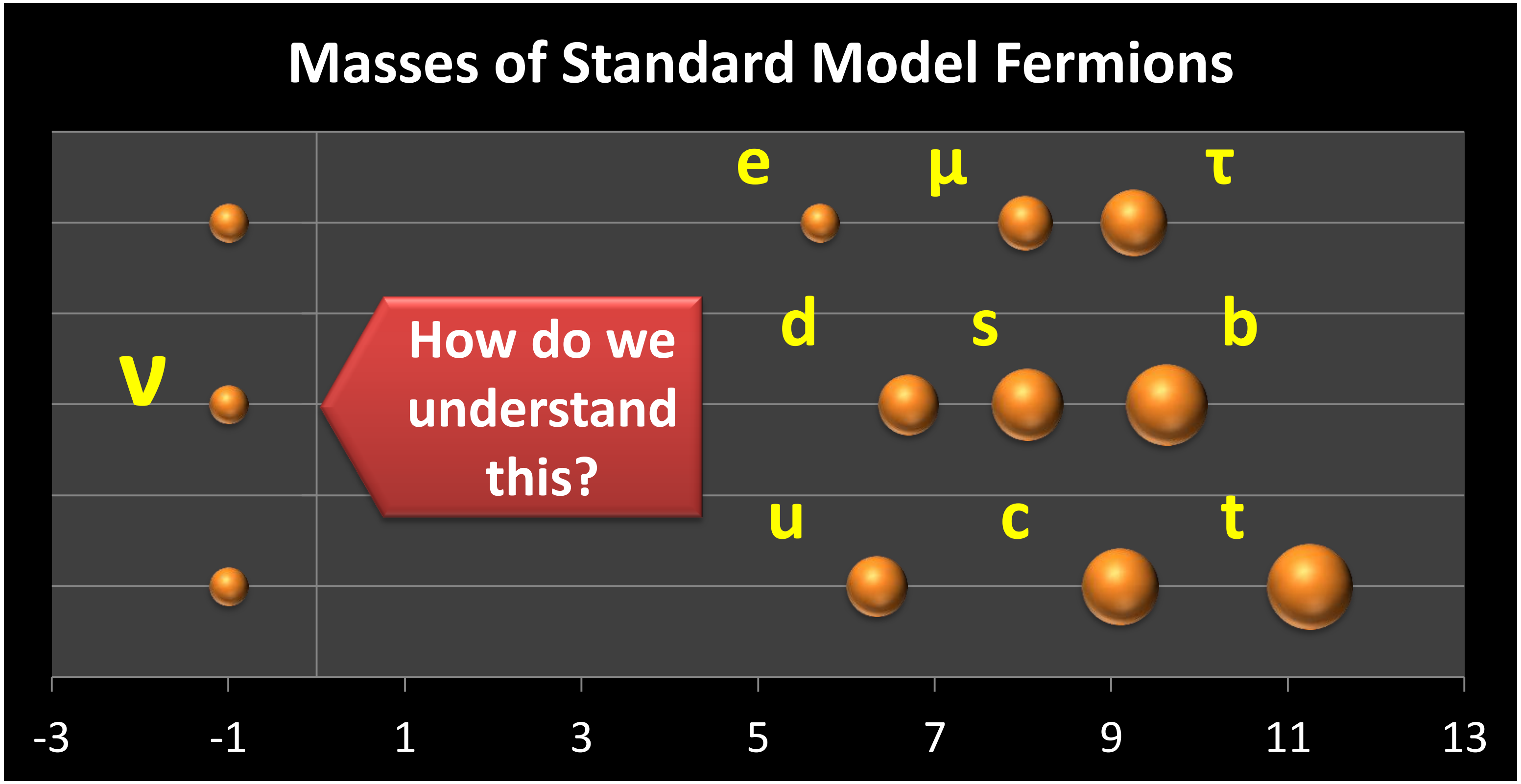
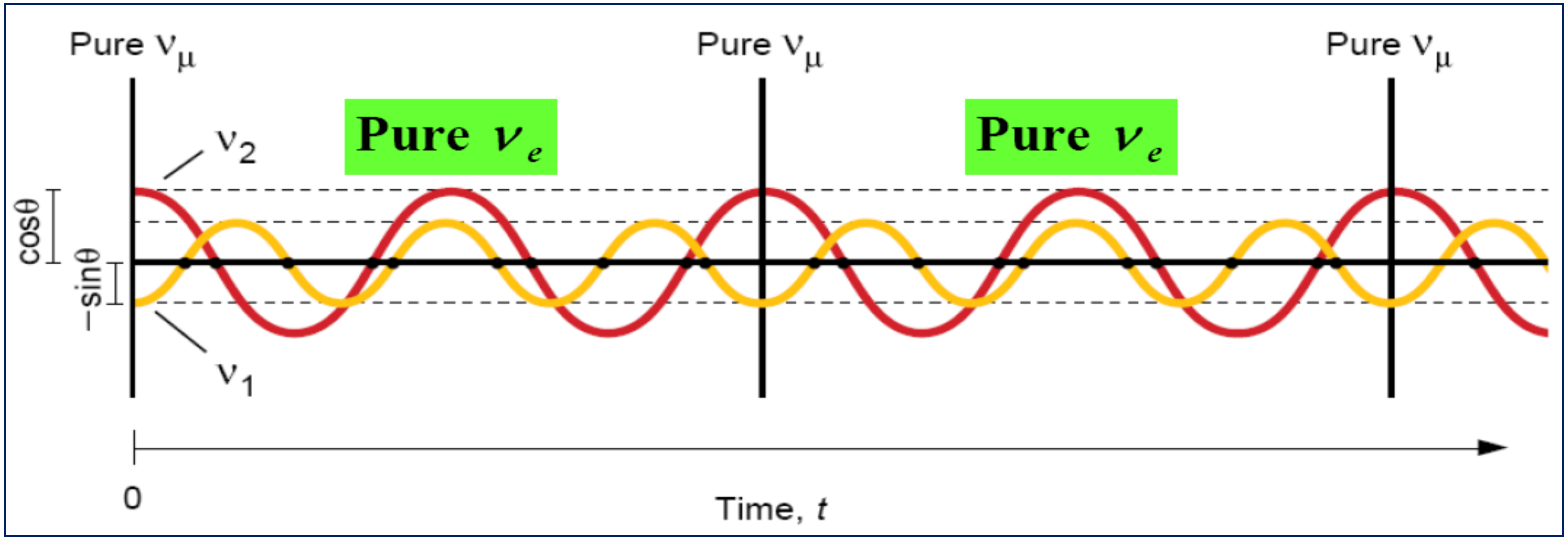


Neutrino mixing: ***two flavors***

Neutrinos have (different) masses $\Rightarrow$ $Dm^2 = m_1^2 - m_2^2$

The ***Weak Eigenstates*** are a mixture of ***Mass Eigenstates***

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$



$$\boxed{P\left(\nu_\mu \to \nu_e\right) = \sin^2 2\theta \sin^2 \frac{1.27 \Delta m^2 L}{E}}$$

Lepton flavor mixing: three flavors

Weak interaction eigenstates

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Mass eigenstates

Standard parametrization

Majorana CP-violating phases

$$V_{MNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -\sin\theta_{13}e^{i\delta} & 0 & \cos\theta_{13} \end{pmatrix} \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ \sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Dirac CP phase δ

Neutrino oscillation parameters

TABLE I: Results of the global 3ν oscillation analysis, in terms of best-fit values and allowed 1, 2 and 3σ ranges for the 3ν mass-mixing parameters. We remind that Δm^2 is defined herein as $m_3^2 - (m_1^2 + m_2^2)/2$, with $+\Delta m^2$ for NH and $-\Delta m^2$ for IH.

Parameter	Best fit	1σ range	2σ range	3σ range
$\delta m^2/10^{-5}$ eV^2 (NH or IH)	7.54	7.32 – 7.80	7.15 – 8.00	6.99 – 8.18
$\sin^2\theta_{12}/10^{-1}$ (NH or IH)	3.07	2.91 – 3.25	2.75 – 3.42	2.59 – 3.59
$\Delta m^2/10^{-3}$ eV^2 (NH)	2.43	2.34 – 2.50	2.26 – 2.58	2.15 – 2.66
$\Delta m^2/10^{-3}$ eV^2 (IH)	2.42	2.32 – 2.49	2.25 – 2.56	2.14 – 2.65
$\sin^2\theta_{13}/10^{-2}$ (NH)	2.45	2.14 – 2.79	1.81 – 3.11	1.49 – 3.44
$\sin^2\theta_{13}/10^{-2}$ (IH)	2.46	2.15 – 2.80	1.83 – 3.13	1.50 – 3.47
$\sin^2\theta_{23}/10^{-1}$ (NH)	3.98	3.72 – 4.28	3.50 – 4.75	3.30 – 6.38
$\sin^2\theta_{23}/10^{-1}$ (IH)	4.08	3.78 – 4.43	3.55 – 6.27	3.35 – 6.58
δ/π (NH)	0.89	0.45 – 1.18	—	—
δ/π (IH)	0.90	0.47 – 1.22	—	—

Fogli, Lisi, Marrone, Montanino, Palazzo, Rotunno, 1205.5254

global-fit $\sin\theta_{12}^2 \approx 0.31$ $\sin\theta_{23}^2 \approx 0.40$ $\sin\theta_{13}^2 \approx 0.024$

Questions to be answered:

1. Sterile neutrino?
2. Sign of Δm_{31}^2
3. Dirac or Majorana ?
4. Absolute mass scale
5. Leptonic CP violation?
6. Non-standard interactions?

Neutrino mixing

Tri-maximal

$$\begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{\omega}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{\omega^2}{\sqrt{3}} \\ \frac{\omega^2}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} \end{pmatrix}$$

Tri-bimaximal

$$\begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

Bi-maximal

$$\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{\sqrt{2}}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

Harrison, Perkins, Scott, 01; Xing, 01

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$\sin\theta_{12}^2 = 0.33$ $\quad\sin\theta_{23}^2 = 0.50$ $\quad\sin\theta_{13}^2=0$

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Is there a flavor symmetry behind the TBM?

$\mu - \tau \quad Z_2 \quad A_4 \quad S_4 \quad D_n \quad \Delta_{27} \quad Q_6 \ \ldots$

Neutrino masses: **Dirac neutrinos**

Neutrinos are Dirac particles

ν_R + a pure Dirac mass term

Extremely tiny Yukawa coupling ~10^{-11}, (hierarchy puzzle)

$$\mathscr{L} = \mathscr{L}_{\mathrm{SM}} + \left\{ Y \bar{l}_{\mathrm{L}} \nu_{\mathrm{R}} \tilde{\phi} + \mathrm{h.c.} \right\}$$

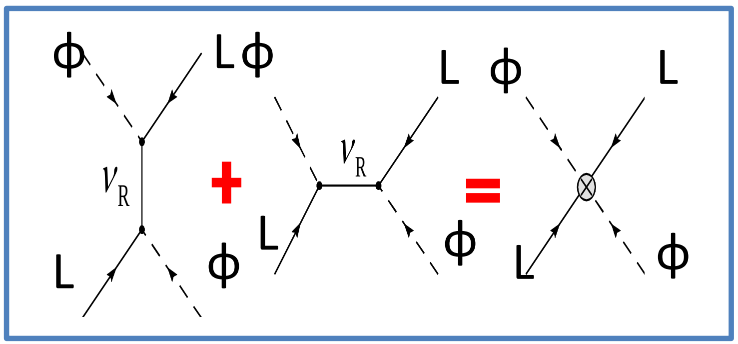
Neutrino masses: **Seesaw**

Neutrinos are Majorana particles

ν_R + Majorana & Dirac masses + seesaw
Natural description of the smallness of ν-masses

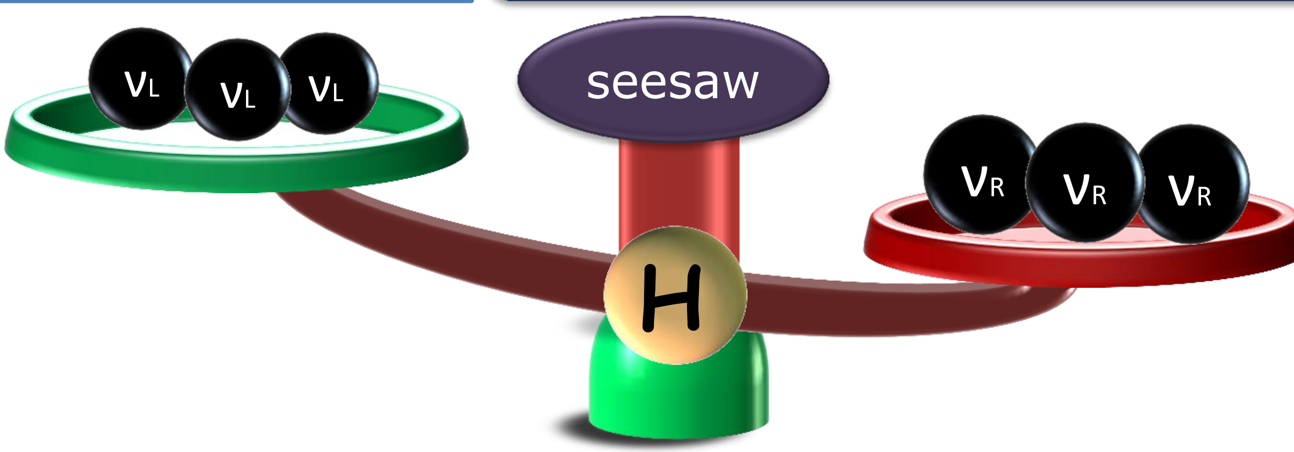
Integrate out right-handed neutrinos

$$\mathscr{L} = \mathscr{L}_{\mathrm{SM}} + \left\{ Y \bar{l}_{\mathrm{L}} \nu_{\mathrm{R}} \tilde{\phi} + \boxed{\frac{1}{2} M_{\mathrm{R}} \bar{\nu}_{\mathrm{R}} \nu_{\mathrm{R}}^{\mathrm{C}}} + \mathrm{h.c.} \right\}$$



$$-iY^T \frac{\not{p} + M_R}{p^2 - M_R^2} Y \left(\varepsilon_{cd}\varepsilon_{ba} + \varepsilon_{ca}\varepsilon_{bd}\right) P_L = i\kappa \left(\varepsilon_{cd}\varepsilon_{ba} + \varepsilon_{ca}\varepsilon_{bd}\right) P_L$$

$$p^2 \ll M_R^2 \Rightarrow \boxed{Y^{\mathrm{T}} M_{\mathrm{R}}^{-1} Y = \kappa} \Rightarrow \boxed{m_\nu = -m_{\mathrm{D}}^{\mathrm{T}} M_{\mathrm{R}}^{-1} m_{\mathrm{D}}}$$

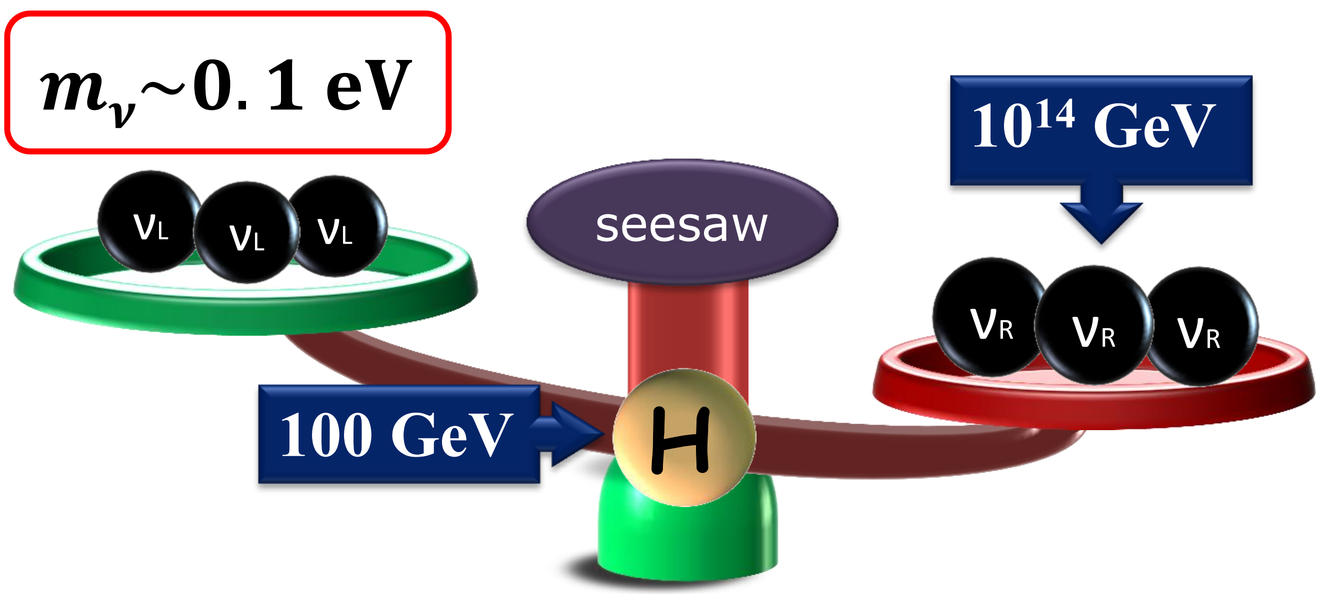


Neutrino masses: **Seesaw scale**

Typical choice of the seesaw scale:

$M_{\mathrm{R}} \sim \Lambda_{\mathrm{GUT}} \gg \Lambda_{\mathrm{EW}}$ & $M_{\mathrm{D}} \sim \Lambda_{\mathrm{EW}}$

Baryogenesis via Leptogenesis



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Low-scale (~ TeV) seesaw

$M_{\mathrm{R}} \sim \Lambda_{\mathrm{EW}}$

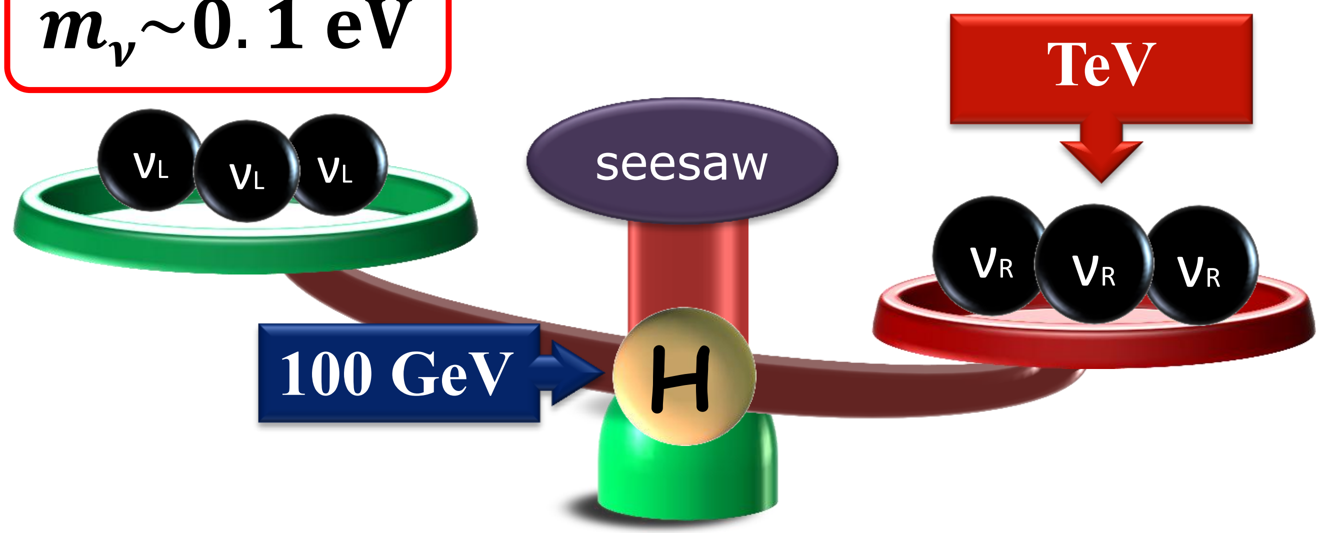
$$m_\nu = M_D\, M_R^{-1} M_D^T$$

(Buchmueller, Greub 91; Ingelman, Rathsman 93; Heusch, Minkowski 94; ……; Kersten, Smirnov 07)

Rich phenomena at colliders and ν-osci. experiments

active-sterile mixing → unitarity violation

$\boldsymbol{m_\nu \sim 0.1\ \mathrm{eV}}$



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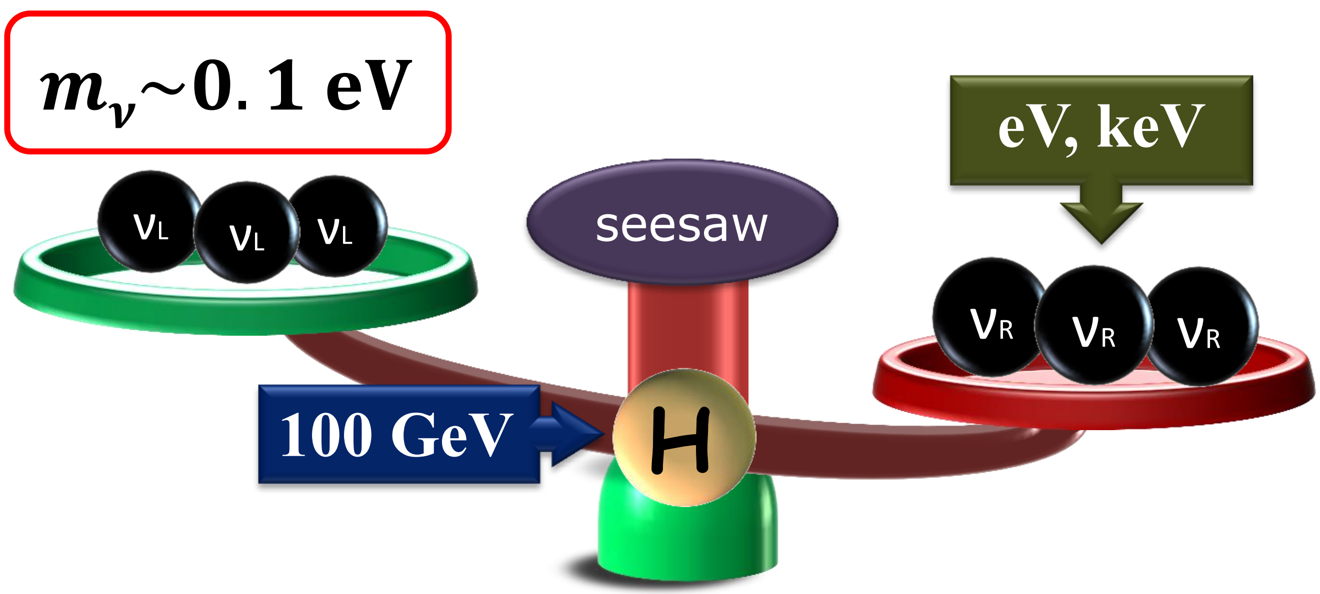

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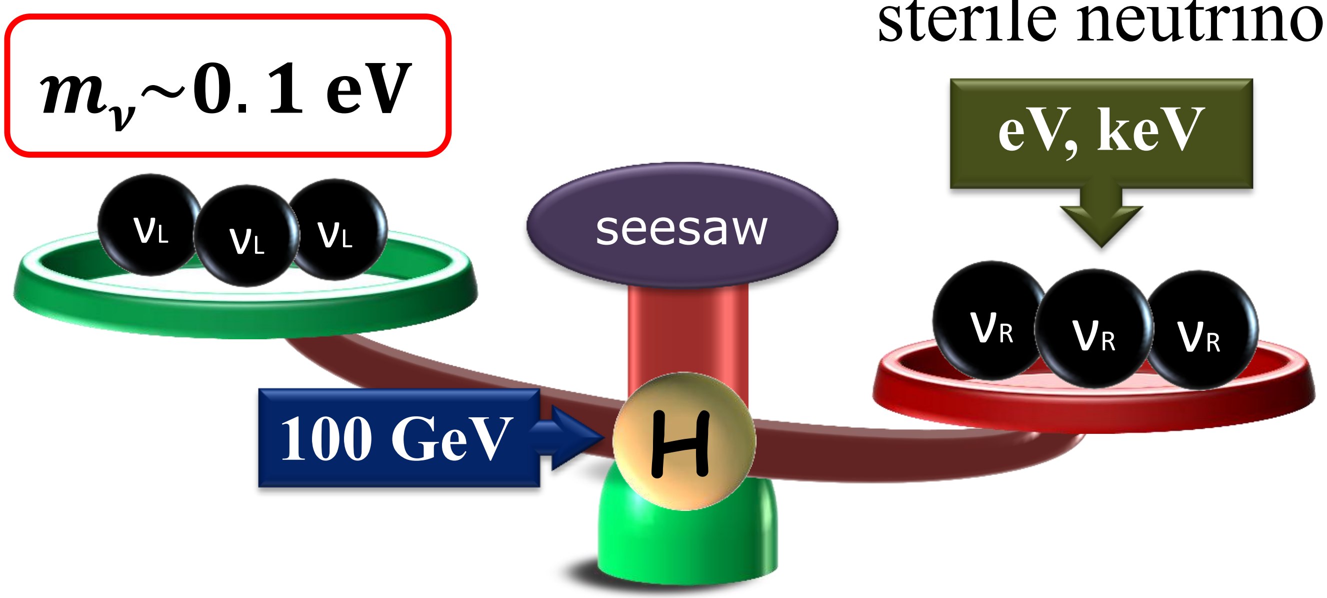
sterile neutrinos: ν_s



Neutrino mixing matrix: with sterile neutrinos

4×4 case: $U = R_{34}\tilde{R}_{24}\tilde{R}_{14}R_{23}\tilde{R}_{13}R_{12}P$

$$R_{34} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_{34} & s_{34} \\ 0 & 0 & -s_{34} & c_{34} \end{pmatrix} \qquad \tilde{R}_{14} = \begin{pmatrix} c_{14} & 0 & 0 & s_{14} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_{14} & 0 & 0 & c_{14} \end{pmatrix}$$

$$P = \mathrm{diag}\left(1,\ e^{i\alpha/2},\ e^{i(\beta/2+\delta_{13})},\ e^{i(\gamma/2+\delta_{14})}\right)$$

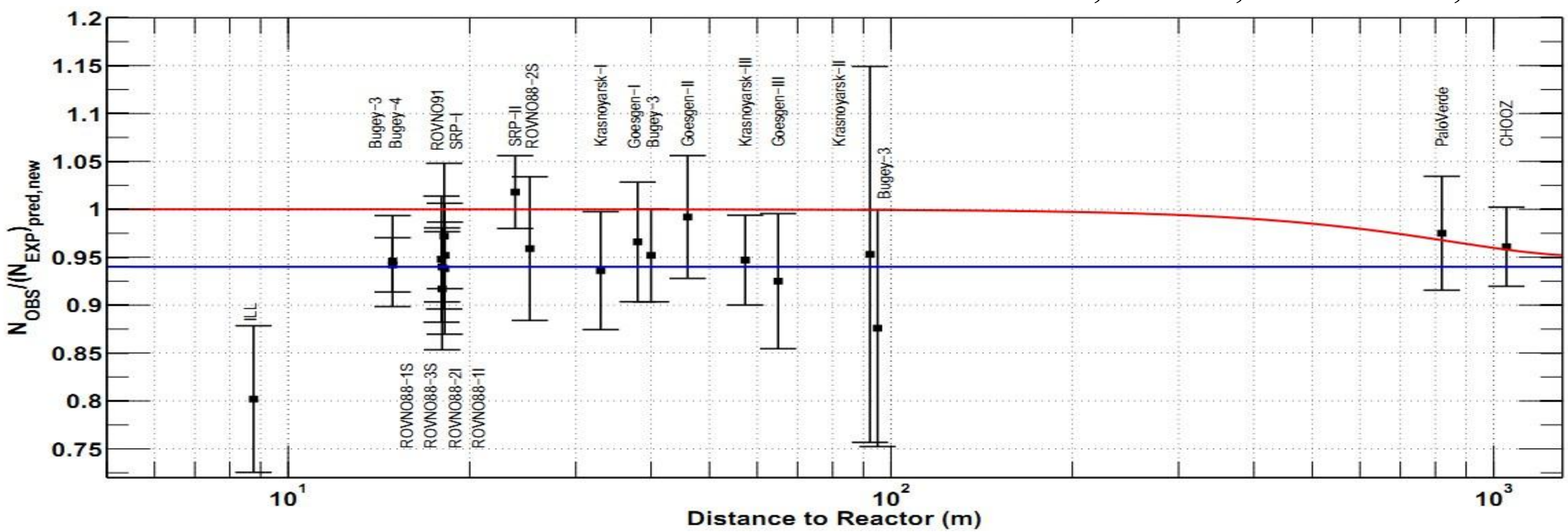
six mixing angles + 3 Dirac phases +3 Majorana phases

5×5 case: $U = \tilde{R}_{25}R_{34}R_{25}\tilde{R}_{24}R_{23}\tilde{R}_{15}\tilde{R}_{14}\tilde{R}_{13}R_{12}P$

Active-sterile neutrino mixing

Reactor neutrino anomaly

Mention, Fechner, Lasserre, Mueller, Lhuillier, Cribier, Letourneau, 11



Recent re-evaluation of the anti-neutrino spectra of nuclear reactors indicates increased fluxes, which be explained by additional sterile neutrinos with masses at the eV scale

$$P_{\overline{\nu}_e \to \overline{\nu}_e} \simeq 1 - 4\left|U_{e4}\right|^2 \sin^2\left(\frac{\Delta_{41} L}{4E}\right) = 1 - \sin^2(2\theta_{14}) \sin^2\left(\frac{\Delta_{41} L}{4E}\right)$$

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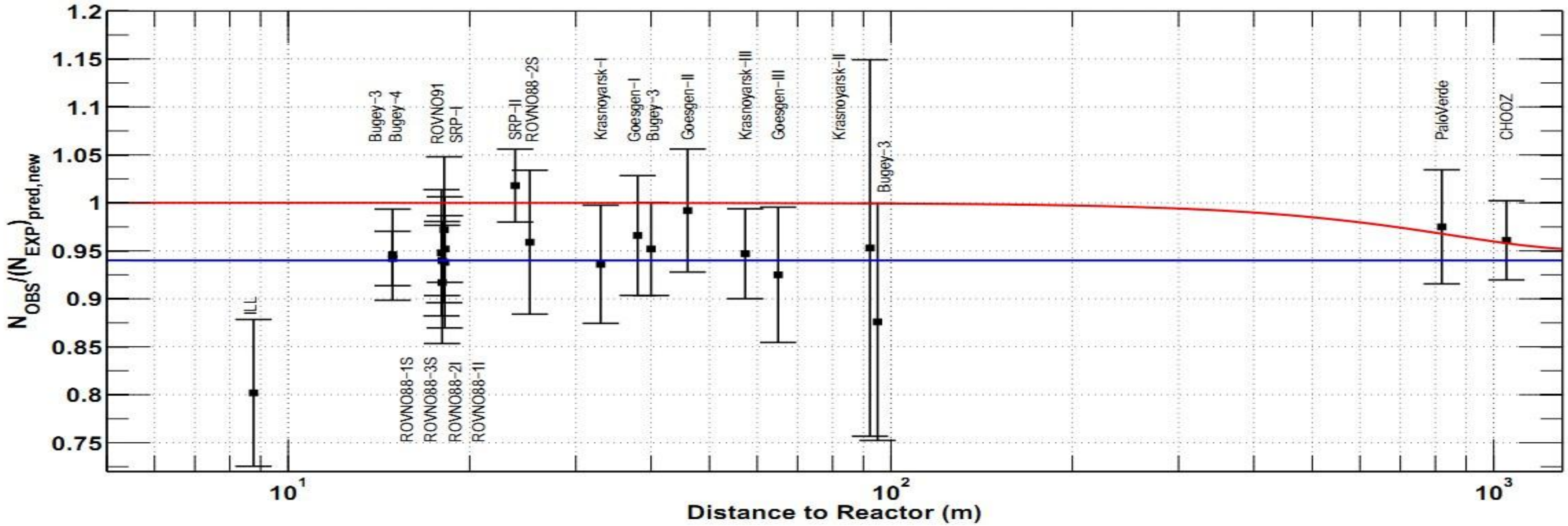

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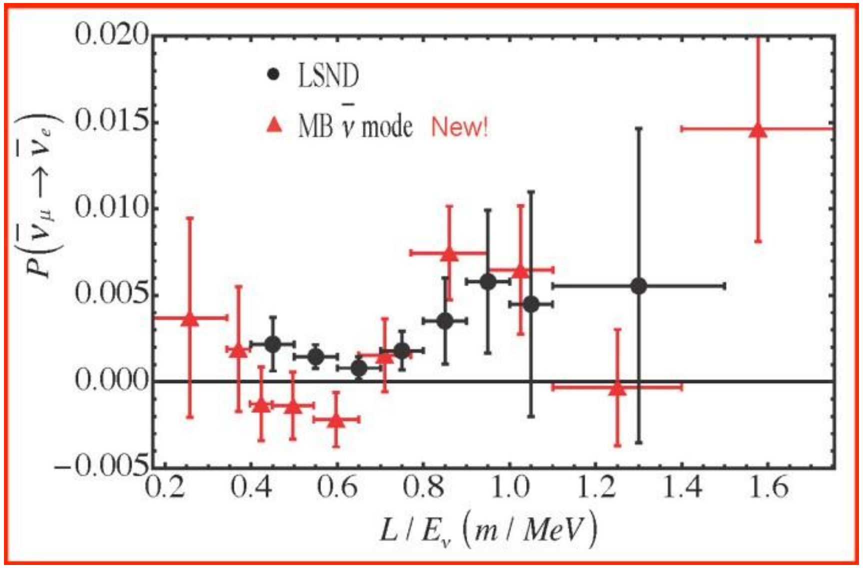
Short-baseline exps:
LSND and MiniBooNE



Active-sterile neutrino mixing

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$$P_{\overline{\nu}_\mu \to \overline{\nu}_e} \simeq 4|U_{e4}|^2|U_{\mu4}|^2 \sin^2\left(\frac{\Delta_{41}L}{4E}\right) + 4|U_{e5}|^2|U_{\mu5}|^2 \sin^2\left(\frac{\Delta_{51}L}{4E}\right)$$
$$+ 8|U_{e4}U_{\mu4}U_{e5}U_{\mu5}| \sin\left(\frac{\Delta_{41}L}{4E}\right) \sin\left(\frac{\Delta_{51}L}{4E}\right) \cos\left(\frac{\Delta_{54}L}{4E} + \delta\right)$$

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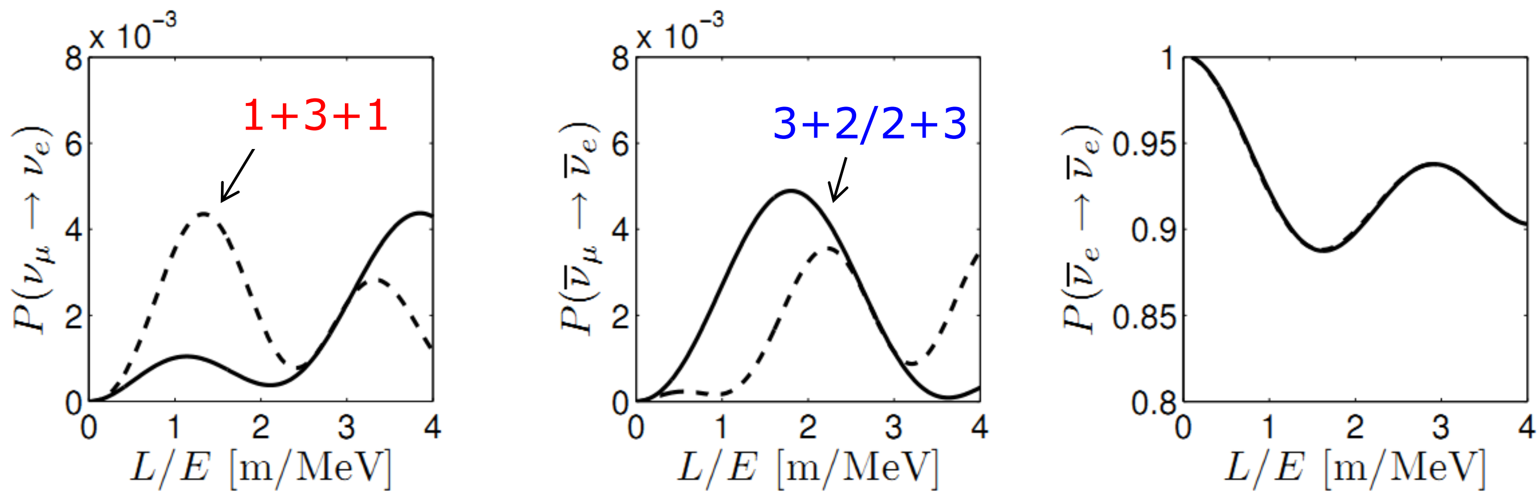
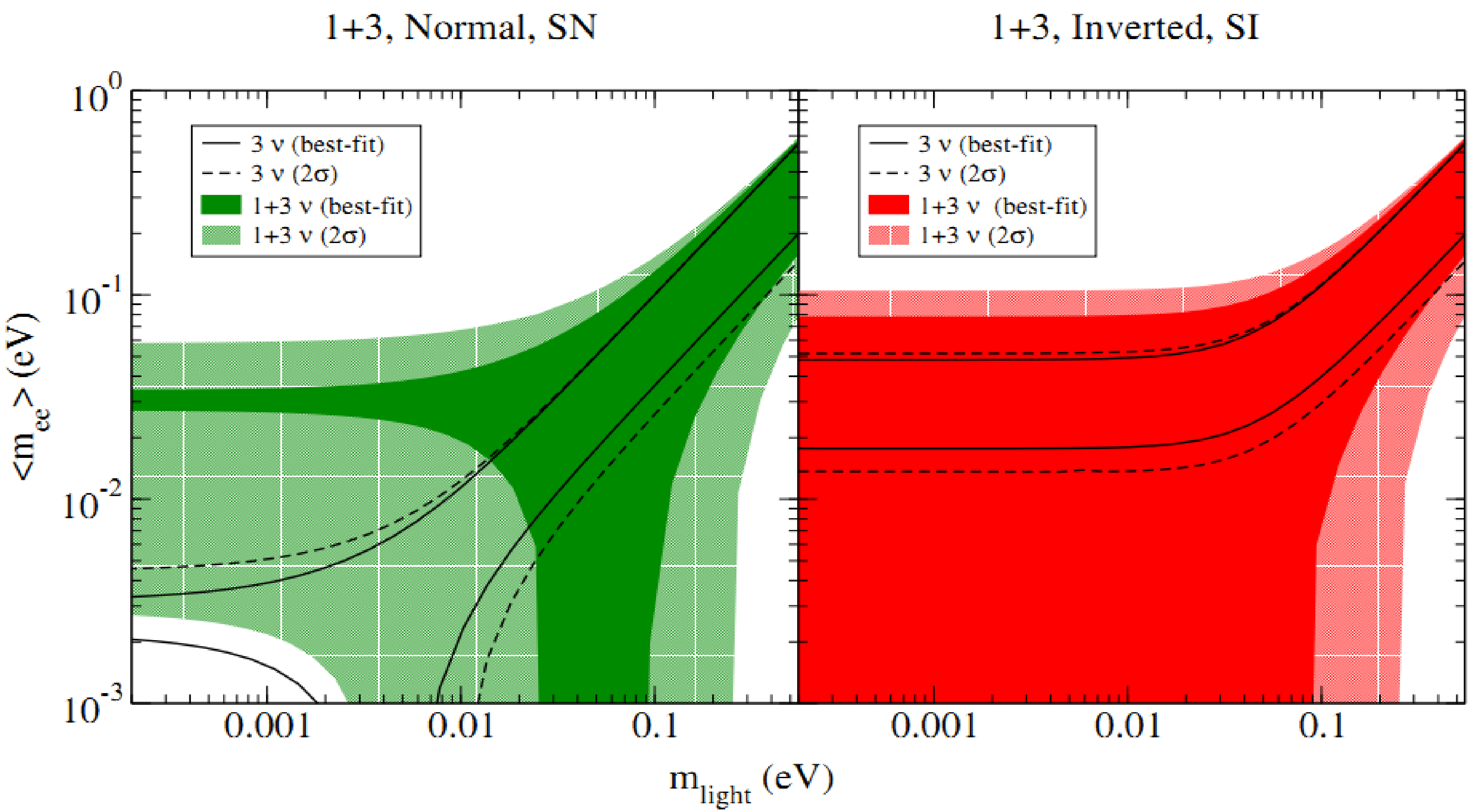


Active-sterile neutrino mixing

neutrino-less double beta decay



The allowed ranges in the $\langle m_{ee} \rangle - m_{\text{light}}$ parameter space

Best-fit and estimated 2σ values of the sterile neutrino parameters.

Kopp, Maltoni, Schwetz, 1103.4570

	parameter	Δm^2_{41} [eV]	$\lvert U_{e4}\rvert^2$	Δm^2_{51} [eV]	$\lvert U_{e5}\rvert^2$
3+1/1+3	best-fit	1.78	0.023		
	2σ	1.61–2.01	0.006–0.040		
3+2/2+3	best-fit	0.47	0.016	0.87	0.019
	2σ	0.42–0.52	0.004–0.029	0.77–0.97	0.005–0.033
1+3+1	best-fit	0.47	0.017	0.87	0.020
	2σ	0.42–0.52	0.004–0.029	0.77–0.97	0.005–0.035

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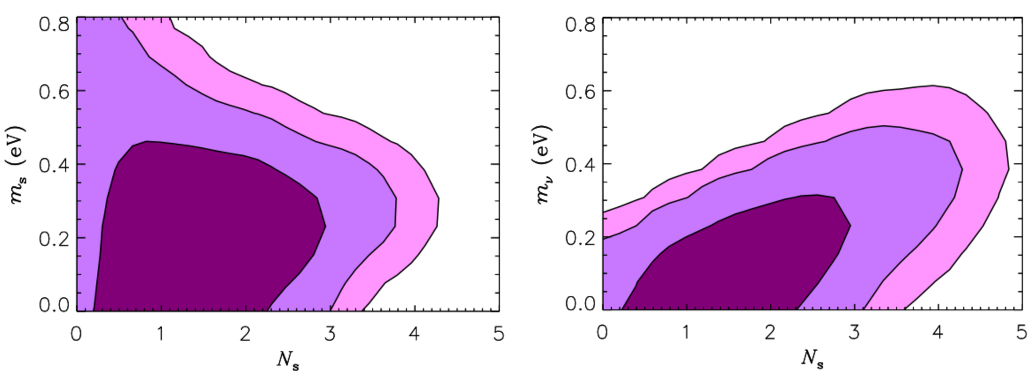
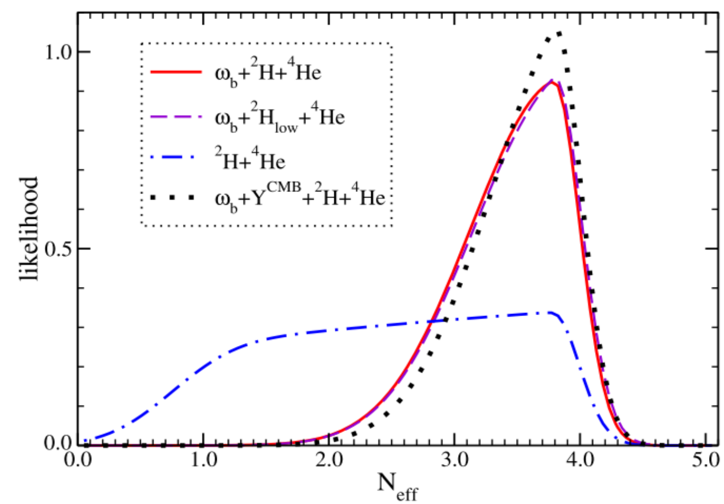
Constraints from cosmology

CMB J. Hamann et al, 1006.5276



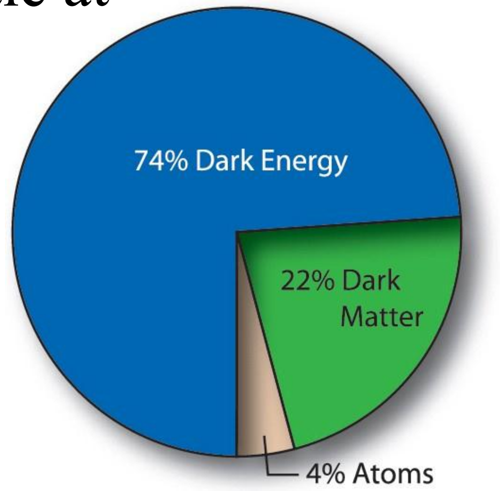
BBN



Mangano, Serpico, 1103.1261

Phenomenology: ν_s Warm Dark Matter

WDM – relativistic at decoupling, non-relativistic at radiation to matter dominance transition



reduces small scale structure:
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- less Dwarf Satellites

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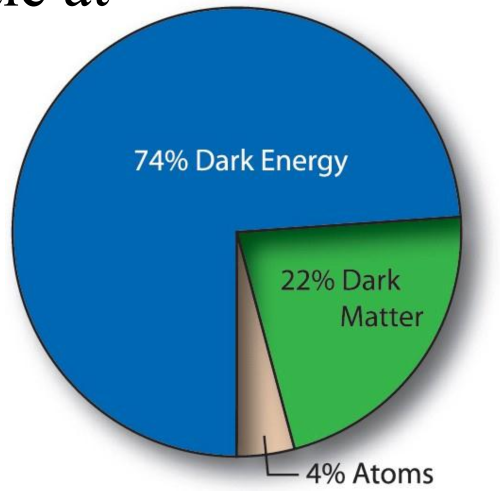

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keV sterile neutrino WDM: works very well

- Right-handed neutrinos probably exist (seesaw)
- $M_R \approx$ keV
- Only one light ν_s is enough, the other two could still be heavy (thermal leptogenesis)

The νMSM

Asaka, Blanchet, Shaposhnikov, 05 Asaka, Shaposhnikov, 05

- A keV ν_{R1} can be WDM   Dodelson, Widrow '93,…

  (production: active-sterile oscillation, etc)
- GeV-scale ν_{R2} & ν_{R3} generate light neutrino masses via seesaw
- Decay of heavy right-handed neutrinos account for the Baryon Asymmtry of the Universe

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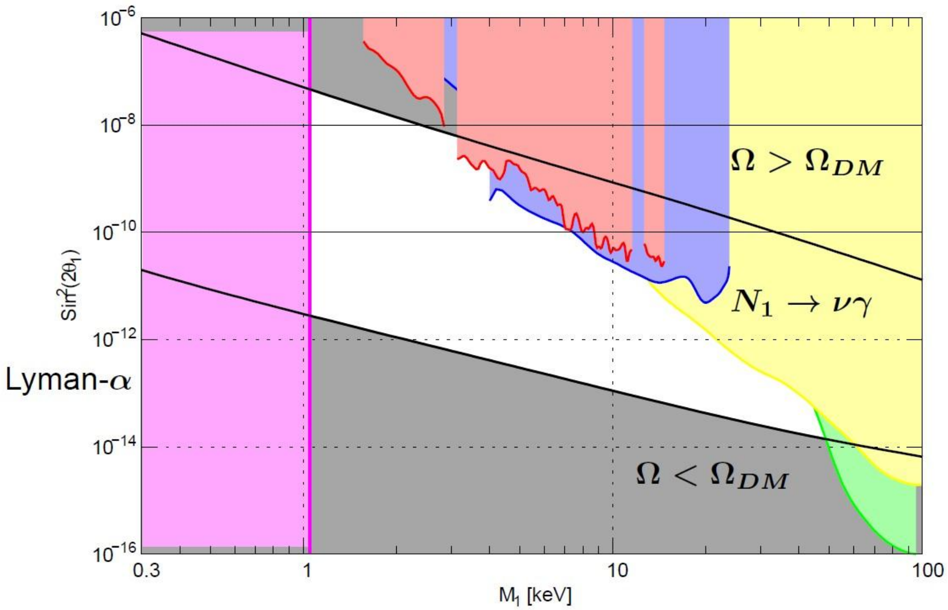

- Weak mixing between ν_{R1} and active neutrinos
- ν_{R2} and ν_{R3} are quasi-degenerate
- one massless active ν
- mass splitting between right-handed neutrinos

The νMSM

Asaka, Blanchet, Shaposhnikov, 05 Asaka, Shaposhnikov, 05

- A keV ν_{R1} can be WDM Dodelson, Widrow '93,…

  (production: active-sterile oscillation, etc)
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Alternative scenarios: ν_R charged under BSM gauge group

Bezrukov, Hettmannsperger, Lindner, 09

- $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
- thermal production of WDM
- entropy productions due to the decay of heavy ν_R
- rich collider phenomena (signatures at the LHC?)

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✓ A mechanism is needed to make ν_s light

How to realize low-scale ν_s

$$\boxed{m_\nu = M_D \, M_R^{-1} M_D^T}$$

0.1 eV   1 keV

- Why are they so light?
- Why do they not form the Dirac particles as heavy as the charged fermions?

How to realize low-scale ν_s

$$\boxed{m_\nu = M_D \, M_R^{-1} M_D^T}$$

0.1 eV 1 keV

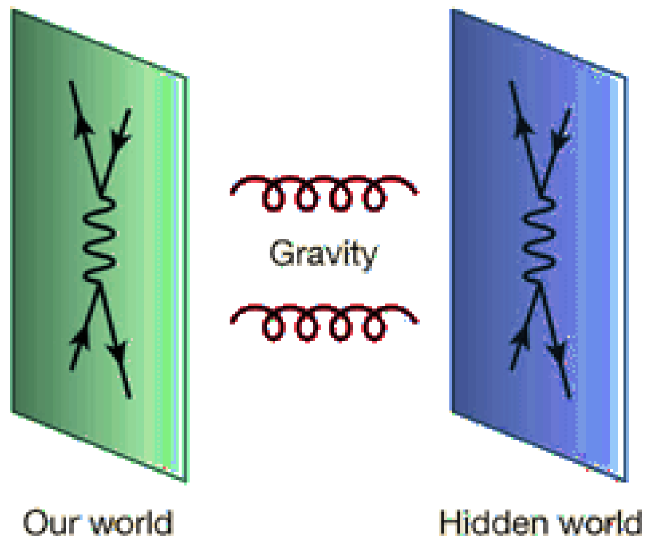
❖ both M_R and M_D are suppressed by symmetries

How to realize low-scale ν_s

Extra dimension theories (Kusenko, Takahashi, Yanagida, **10**)

- Splitting between the SM brane and a hidden brane

active neutrinos



sterile neutrinos

How to realize low-scale ν_s

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- Effects of right-handed neutrinos are exponentially suppressed since they are located on the hidden brane

$$S = \int d^4x\, dy\, M \left(i\bar{\Psi}\Gamma^A\partial_A\Psi + m\bar{\Psi}\Psi \right)$$

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zero mode with an exponential profile in the bulk

$$\Psi_R^{(0)}(y, x) = \sqrt{\frac{2m}{e^{2m\ell} - 1}} \frac{1}{\sqrt{M}} e^{my} \psi_R^{(4D)}(x)$$

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$$M_{Ri} = \kappa_i v_{\mathrm{B-L}} \frac{2m_i}{M(e^{2m_i \ell} - 1)} \qquad \lambda_{i\alpha} = \frac{\tilde{\lambda}_{i\alpha}}{\sqrt{M}} \sqrt{\frac{2m_i}{e^{2m_i \ell} - 1}} = \tilde{\lambda}_{i\alpha} \sqrt{\frac{M_{Ri}}{\kappa_i v_{\mathrm{B-L}}}}$$

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$$\boxed{(m_\nu)_{\alpha\beta} = \left(\sum_i \frac{1}{\kappa_i} \tilde{\lambda}_{i\alpha} \tilde{\lambda}_{i\beta} \right) \frac{\langle \phi^0 \rangle^2}{v_{\mathrm{B-L}}}}$$

How to realize low-scale ν_s

Flavor symmetries

$L_e - L_\mu - L_\tau$ symmetry:

	L_{eL}	$L_{\mu L}$	$L_{\tau L}$	e_R	μ_R	τ_R	N_{1R}	N_{2R}	N_{3R}	ϕ	Δ
$\mathcal{F}$	1	−1	−1	1	−1	−1	1	−1	−1	0	0

(Lindner, Merle, Niro, **10**)

$$\mathcal{M}_\nu = \left(\begin{array}{ccc|ccc} 0 & m_L^{e\mu} & m_L^{e\tau} & m_D^{e1} & 0 & 0 \\ m_L^{e\mu} & 0 & 0 & 0 & m_D^{\mu 2} & m_D^{\mu 3} \\ m_L^{e\tau} & 0 & 0 & 0 & m_D^{\tau 2} & m_D^{\tau 3} \\ \hline m_D^{e1} & 0 & 0 & 0 & M_R^{12} & M_R^{13} \\ 0 & m_D^{\mu 2} & m_D^{\tau 2} & M_R^{12} & 0 & 0 \\ 0 & m_D^{\mu 3} & m_D^{\tau 3} & M_R^{13} & 0 & 0 \end{array}\right)$$

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$$\Downarrow$$

$$\begin{pmatrix} \lambda_+ & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_- & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Lambda_+ & 0 & 0 \\ 0 & 0 & 0 & 0 & \Lambda_- & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

two heavy + one massless right-handed neutrinos

$$\Lambda_\pm = \pm\sqrt{2}M_R$$
$$\lambda_\pm = \pm\sqrt{2}[m_L - m_D^2/M_R]$$

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light sterile neutrino from symmetry breaking

$$\left(\begin{array}{ccc|ccc} s_L^{ee} & m_L^{e\mu} & m_L^{e\tau} & m_D^{e1} & 0 & 0 \\ m_L^{e\mu} & s_L^{\mu\mu} & 0 & 0 & m_D^{\mu 2} & m_D^{\mu 3} \\ m_L^{e\tau} & 0 & s_L^{\tau\tau} & 0 & m_D^{\tau 2} & m_D^{\tau 3} \\ \hline m_D^{e1} & 0 & 0 & S_R^{11} & M_R^{12} & M_R^{13} \\ 0 & m_D^{\mu 2} & m_D^{\tau 2} & M_R^{12} & S_R^{22} & 0 \\ 0 & m_D^{\mu 3} & m_D^{\tau 3} & M_R^{13} & 0 & S_R^{33} \end{array}\right)$$

$$\begin{pmatrix} \lambda_+ & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_- & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Lambda_+ & 0 & 0 \\ 0 & 0 & 0 & 0 & \Lambda_- & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

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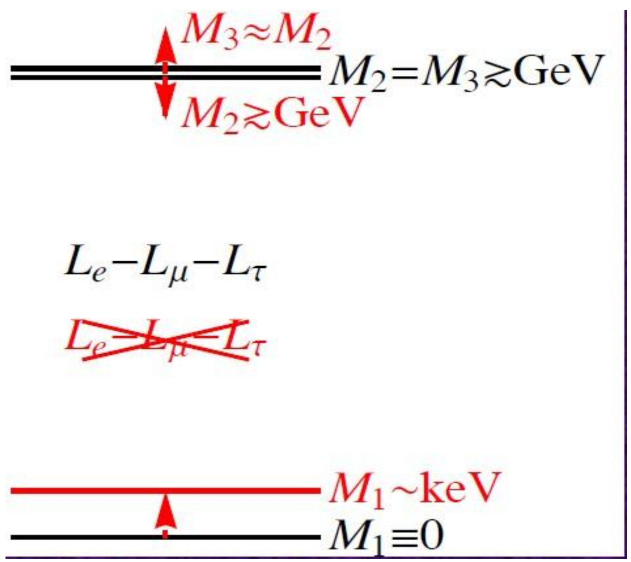

$m_s \sim S \sim \text{keV}$

$\Delta M = M_3 - M_2 \sim m_s$

drawback: maximum solar mixing!

How to realize low-scale ν_s

Flavor symmetries

Friedberg-Lee symmetry: R.Friedberg & T.D.Lee, 2006

Neutrino mass operator is invariant under the transformation

$$\nu_e \to \nu_e + z; \qquad \nu_\mu \to \nu_\mu + z; \qquad \nu_\tau \to \nu_\tau + z;$$

$z \to$ Grassmann number

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$$\overline{M} = \begin{pmatrix} b+c & -b & c \\ -b & a+b & a \\ c & a & c+a \end{pmatrix}$$

❖ Rank 2 → one massless eigenstate

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- Rank 2 → one massless eigenstate

- ✓ Applied to the right-handed neutrino sector
- ✓ One massless sterile neutrino before symmetry breaking

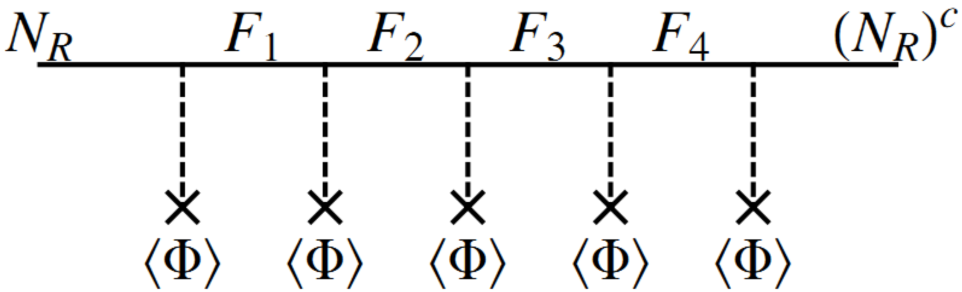
He, Li, Liao, 2009

How to realize low-scale ν_s

Froggatt-Nielsen mechanism

- Fermion flavors are differently charged under a $U(1)_{\mathrm{FN}}$ symmetry
- Right-handed neutrino masses receive a suppression factor

$M \to M\lambda^{2F} \quad (\lambda = \frac{\langle\phi\rangle}{\Lambda} < 1)$



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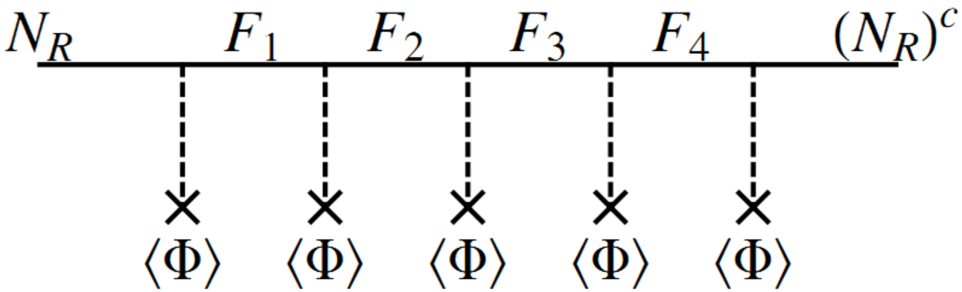

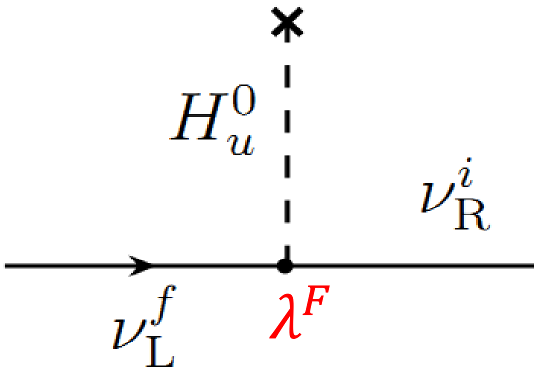

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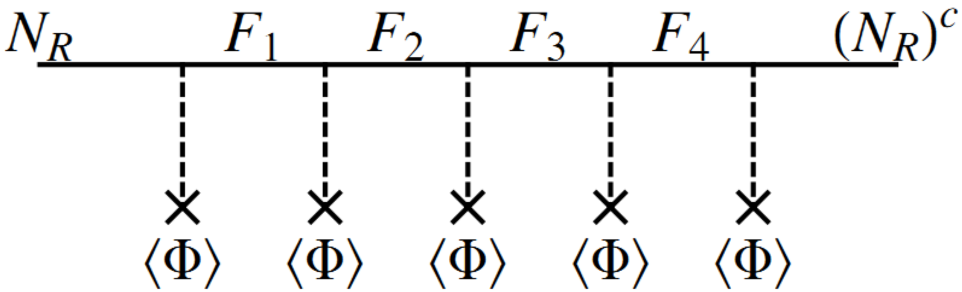

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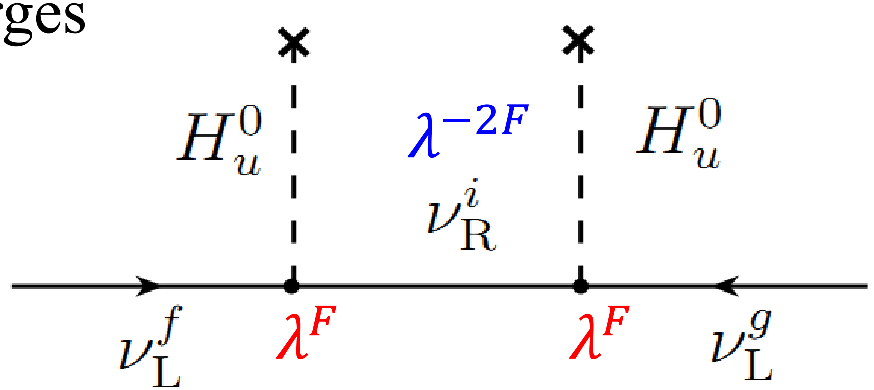

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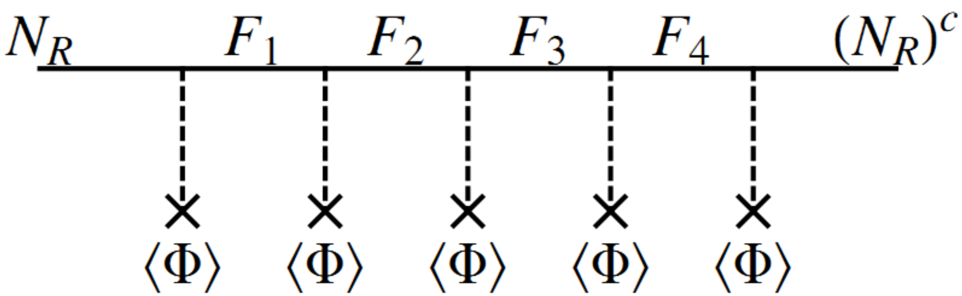
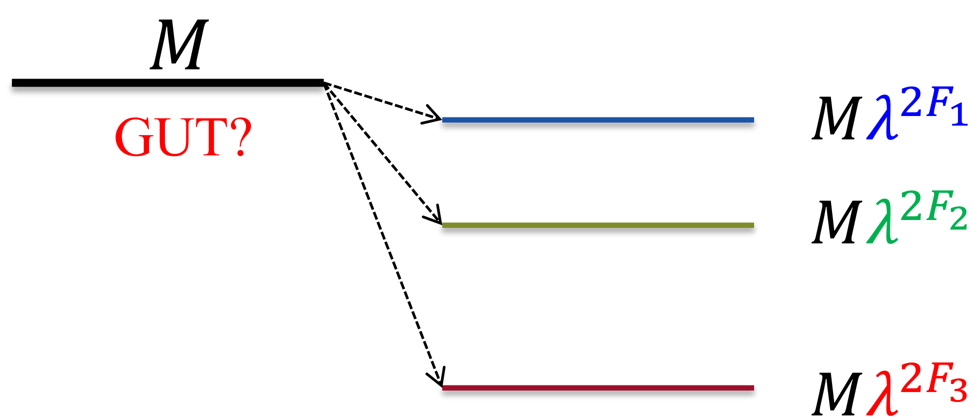


- Seesaw formula and the active neutrino masses are not affected by the FN charges

- sterile neutrino mass spectrum



How to realize low-scale ν_s : Minimal Extended Seesaw

Light sterile neutrinos: suppressed by **seesaw** as well?

(HZ, **1110.6838**)

How to realize low-scale ν_s : Minimal Extended Seesaw

The model: SM + three right-handed neutrinos + one singlet S

$$-\mathcal{L}_m = \overline{\nu_L} M_D \nu_R + \overline{S^c} M_S \nu_R + \frac{1}{2}\overline{\nu_R^c} M_R \nu_R + \text{h.c.}$$

How to realize low-scale ν_s : Minimal Extended Seesaw

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$$M_S = (\times \quad \times \quad \times)$$

- The full 7×7 neutrino mass matrix if of rank 6, and therefore, one active neutrino is massless.

$$M_\nu^{7\times 7} = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & M_R & M_S^T \\ 0 & M_S & 0 \end{pmatrix}$$

How to realize low-scale ν_s : Minimal Extended Seesaw

The model: SM + three right-handed neutrinos + one singlet S

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$$M_\nu^{7\times 7} = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & M_R & M_S^T \\ 0 & M_S & 0 \end{pmatrix}$$

If $M_R \gg M_S , M_D$, we can integrate out ν_R

$$m_\nu \simeq M_D M_R^{-1} M_S^T \left(M_S M_R^{-1} M_S^T\right)^{-1} M_S \left(M_R^{-1}\right)^T M_D^T - M_D M_R^{-1} M_D^T$$

$$m_s \simeq -M_S M_R^{-1} M_S^T$$

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$$m_\nu \simeq \underline{M_D M_R^{-1} M_S^T \left(M_S M_R^{-1} M_S^T\right)^{-1} M_S \left(M_R^{-1}\right)^T} M_D^T - \underline{M_D M_R^{-1} M_D^T}$$

$$m_s \simeq -M_S M_R^{-1} M_S^T$$

Do not cancel with each other

$\Rightarrow$ two massive light neutrinos

How to realize low-scale ν_s : Minimal Extended Seesaw

The model: SM + three right-handed neutrinos + one singlet S

$$-\mathcal{L}_m = \overline{\nu_L} M_D \nu_R + \overline{S^c} M_S \nu_R + \frac{1}{2}\overline{\nu_R^c} M_R \nu_R + \text{h.c.}$$

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$$m_s \simeq -M_S M_R^{-1} M_S^T$$

$M_D \sim 100$ GeV;
$M_S \sim 10^4$ GeV; $M_R \sim 2 \times 10^{14}$ GeV $\Rightarrow$ $m_\nu \sim 0.05$ eV;
$m_s \sim$ keV;

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$$-\mathcal{L}_m = \overline{\nu_L} M_D \nu_R + \overline{S^c} M_S \nu_R + \frac{1}{2}\overline{\nu_R^c} M_R \nu_R + \text{h.c.}$$

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$M_S \sim 10^4$ GeV; $M_R \sim 2\times 10^{14}$ GeV $\Rightarrow$ $m_\nu \sim 0.05$ eV;
$m_s \sim$ keV;

- ✓ No need to artificially insert small mass scales and tiny Yukawa couplings for light neutrino masses.
- ✓ Thermal leptogenesis works.
- ✓ Only one singlet S is allowed (minimal extension).

How to realize low-scale ν_s : Minimal Extended Seesaw

The model: SM + three right-handed neutrinos + one singlet S

$$-\mathcal{L}_m = \overline{\nu_L} M_D \nu_R + \overline{S^c} M_S \nu_R + \frac{1}{2}\overline{\nu_R^c} M_R \nu_R + \text{h.c.}$$

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A similar idea was used with a sterile state of mass $\sim 10^{-3}$ eV introduced in order to explain the solar neutrino problem

(Chun, Joshipura, Smirnov, **95**)

How to realize low-scale ν_s : Non-standard Approach

Mirror model

Berezhiania, Mohapatra 95; Foot, Volkas, 95;
Berezinsky, Narayan, Vissani 02

$$SU(3) \times SU(2) \times U(1) \quad \times \quad SU(3)' \times SU(2)' \times U(1)'$$

Quarks (B=1/3) & Leptons (L=1)   Quarks (B'=1/3) & Leptons (L'=1)

Yukawa interactions
$-L = Y\overline{f_L}Hf_R$

Yukawa interactions
$-L = Y'\overline{f_L}'H'f_R'$

$\langle H \rangle = v$

$\langle H' \rangle = v'$

$m_\nu \sim v^2/M$

$$\boxed{m_{\nu s} \sim \frac{LL'HH'}{\Lambda_{\mathrm{PL}}}}$$

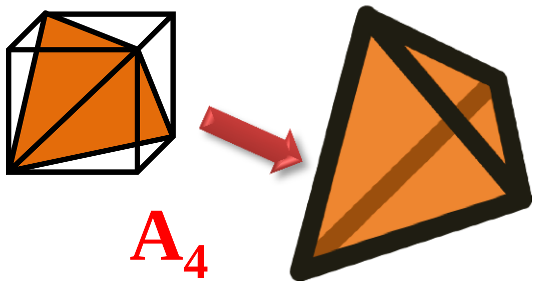
$m_s \sim v'^2/M$

Different inflation, reheating temp

Axino scenario

WDM is the supersymmtry particle of axion

ν_s in flavor symmetry models: A_4 + FN mechanism



- Symmetry group of tetrahedron
- Even permutations of four objects
- Twelve elements
- Four irreducible represents: $1, 1', 1''$, and 3

Tri-bimaximal mixing (TBM)

$$\begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

ν_s in flavor symmetry models: an effective approach

Field	L	e^c	μ^c	τ^c	$h_{u,d}$	φ	φ'	ξ	Θ	ν_s
$SU(2)_L$	2	1	1	1	2	1	1	1	1	1
A_4	$\underline{3}$	$\underline{1}$	$\underline{1}''$	$\underline{1}'$	$\underline{1}$	$\underline{3}$	$\underline{3}$	$\underline{1}$	$\underline{1}$	$\underline{1}$
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	1	1
$U(1)_{\mathrm{FN}}$	-	4	2	0	-	-	-	-	-1	8

Barry, Rodejohann, HZ, 2012

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Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	1	1
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Barry, Rodejohann, HZ, 2012

Invariant Lagrangian under $A_4 \times Z_3 \times U(1)_{FN}$

$$
\begin{aligned}
-\mathcal{L}_{\rm Y} = {} & \frac{y_e}{\Lambda}\lambda^4 \left(\varphi L h_d\right) e^c + \frac{y_\mu}{\Lambda}\lambda^2 \left(\varphi L h_d\right)' \mu^c + \frac{y_\mu}{\Lambda} \left(\varphi L h_d\right)'' \tau^c + \frac{x_a}{\Lambda^2}\xi (L h_u L h_u) + \frac{x_d}{\Lambda^2}(\varphi' L h_u L h_u) \\
& + \frac{x_e}{\Lambda^2}\lambda^8 \xi (\varphi' L h_u)\nu_s + \frac{x_f}{\Lambda^2}\lambda^8 (\varphi'\varphi' L h_u)\nu_s + m_s \lambda^{16} \nu_s^c \nu_s^c + \text{h.c.},
\end{aligned}
$$

$$\boxed{\lambda \equiv \langle \Theta \rangle / \Lambda < 1}$$

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Barry, Rodejohann, HZ, 2012

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$$-\mathcal{L}_{\rm Y} = \frac{y_e}{\Lambda}\lambda^4 (\varphi L h_d) e^c + \frac{y_\mu}{\Lambda}\lambda^2 (\varphi L h_d)' \mu^c + \frac{y_\mu}{\Lambda}(\varphi L h_d)'' \tau^c + \frac{x_a}{\Lambda^2}\xi(L h_u L h_u) + \frac{x_d}{\Lambda^2}(\varphi' L h_u L h_u)$$
$$+\frac{x_e}{\Lambda^2}\lambda^8 \xi(\varphi' L h_u)\nu_s + \frac{x_f}{\Lambda^2}\lambda^8(\varphi'\varphi' L h_u)\nu_s + m_s\lambda^{16}\nu_s^c\nu_s^c + \text{h.c.},$$

$$\boxed{\lambda \equiv \langle\Theta\rangle/\Lambda < 1}$$

vacuum alignments and mass scales

$$\langle\varphi\rangle = (v,0,0)\,, \quad \langle\varphi'\rangle = (v',v',v')\,, \quad \langle\xi\rangle = u\,, \quad \langle h_{u,d}\rangle = v_{u,d}$$

$$u \simeq v' \simeq 10^{10}~\text{GeV}\,, \quad v \simeq 10^{11}~\text{GeV}\,, \quad \Lambda \simeq 10^{12}~\text{GeV}$$
$$v_{u,d} \simeq 10^2~\text{GeV}\,, \quad \langle\Theta\rangle \simeq 10^{11}~\text{GeV}\,,$$

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Barry, Rodejohann, HZ, 2012

$\lambda = 10^{-1} = 0.1$

$\Lambda = 10^{12}$ GeV

Full 4×4 neutrino mass matrix

$$M_\nu^{4\times4} = \begin{pmatrix} a + \frac{2d}{3} & -\frac{d}{3} & -\frac{d}{3} & e \\ \cdot & \frac{2d}{3} & a - \frac{d}{3} & e \\ \cdot & \cdot & \frac{2d}{3} & e \\ \cdot & \cdot & \cdot & m_s \end{pmatrix}$$

$$a \simeq d \simeq 0.1 \left(\frac{u}{10^{10}\ \mathrm{GeV}}\right) \left(\frac{v_u}{10^2\ \mathrm{GeV}}\right)^2 \left(\frac{10^{12}\ \mathrm{GeV}}{\Lambda}\right)^2 \mathrm{eV}\,,$$

$$e \simeq 0.1 \left(\frac{\lambda}{10^{-1}}\right)^8 \left(\frac{u}{10^{10}\ \mathrm{GeV}}\right) \left(\frac{v'}{10^{10}\ \mathrm{GeV}}\right) \left(\frac{v_u}{10^2\ \mathrm{GeV}}\right) \left(\frac{10^{12}\ \mathrm{GeV}}{\Lambda}\right)^2 \mathrm{eV}$$

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$$a \simeq d \simeq 0.1 \left(\frac{u}{10^{10}\ \text{GeV}}\right) \left(\frac{v_u}{10^{2}\ \text{GeV}}\right)^2 \left(\frac{10^{12}\ \text{GeV}}{\Lambda}\right)^2 \text{eV}\,,$$

$$e \simeq 0.1 \left(\frac{\lambda}{10^{-1}}\right)^8 \left(\frac{u}{10^{10}\ \text{GeV}}\right) \left(\frac{v'}{10^{10}\ \text{GeV}}\right) \left(\frac{v_u}{10^{2}\ \text{GeV}}\right) \left(\frac{10^{12}\ \text{GeV}}{\Lambda}\right)^2 \text{eV}$$

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Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	1	1
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Barry, Rodejohann, HZ, 2012

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tri-bimaximal mixing

$$M_\nu^{4\times4} = \begin{pmatrix} a+\frac{2d}{3} & -\frac{d}{3} & -\frac{d}{3} & e \\ \cdot & \frac{2d}{3} & a-\frac{d}{3} & e \\ \cdot & \cdot & \frac{2d}{3} & e \\ \cdot & \cdot & \cdot & m_s \end{pmatrix} \Rightarrow U \simeq \begin{pmatrix} \frac{2}{\sqrt6} & \frac{1}{\sqrt3} & 0 & 0 \\ -\frac{1}{\sqrt6} & \frac{1}{\sqrt3} & -\frac{1}{\sqrt2} & 0 \\ -\frac{1}{\sqrt6} & \frac{1}{\sqrt3} & \frac{1}{\sqrt2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & \frac{e}{m_s} \\ 0 & 0 & 0 & \frac{e}{m_s} \\ 0 & 0 & 0 & \frac{e}{m_s} \\ 0 & -\frac{\sqrt3 e}{m_s} & 0 & 0 \end{pmatrix}$$

$$a \simeq d \simeq 0.1 \left(\frac{u}{10^{10}\ \text{GeV}}\right)\left(\frac{v_u}{10^2\ \text{GeV}}\right)^2\left(\frac{10^{12}\ \text{GeV}}{\Lambda}\right)^2 \text{eV}\,,$$
$$e \simeq 0.1 \left(\frac{\lambda}{10^{-1}}\right)^8 \left(\frac{u}{10^{10}\ \text{GeV}}\right)\left(\frac{v'}{10^{10}\ \text{GeV}}\right)\left(\frac{v_u}{10^2\ \text{GeV}}\right)\left(\frac{10^{12}\ \text{GeV}}{\Lambda}\right)^2 \text{eV}$$

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active-sterile mixing

$$a \simeq d \simeq 0.1 \left(\frac{u}{10^{10}\ \text{GeV}}\right)\left(\frac{v_u}{10^2\ \text{GeV}}\right)^2\left(\frac{10^{12}\ \text{GeV}}{\Lambda}\right)^2 \text{eV}\,,$$

$$e \simeq 0.1 \left(\frac{\lambda}{10^{-1}}\right)^8\left(\frac{u}{10^{10}\ \text{GeV}}\right)\left(\frac{v'}{10^{10}\ \text{GeV}}\right)\left(\frac{v_u}{10^2\ \text{GeV}}\right)\left(\frac{10^{12}\ \text{GeV}}{\Lambda}\right)^2 \text{eV}$$

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active-sterile mixing

Leading order contribution to sterile neutrino mass

$$\left(\frac{x_s}{\Lambda}\varphi\varphi\right)\lambda^{16}\nu_s^c\nu_s^c \Longrightarrow \left(x_s\frac{v^2}{\Lambda}\right)\lambda^{16}$$

$$m_s \simeq \left(\frac{\lambda}{10^{-1}}\right)^{16}\left(\frac{v}{10^{11}\ \text{GeV}}\right)^2\left(\frac{10^{12}\ \text{GeV}}{\Lambda}\right)\ \text{keV}$$

ν_s in flavor symmetry models: an effective approach

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tri-bimaximal mixing

active-sterile mixing

$$\boxed{\theta_s = \frac{e}{m_s} \simeq 10^{-4}}$$

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Barry, Rodejohann, HZ, 2012

$\lambda = 10^{-1} = 0.1$

$\Lambda = 10^{12}$ GeV

Next-to-leading order corrections to the neutrino mass matrix

$$\frac{x_1}{\Lambda^3}(\varphi\varphi')'(Lh_uLh_u)'' , \quad \frac{x_2}{\Lambda^3}(\varphi\varphi')''(Lh_uLh_u)' , \quad \text{and} \quad \frac{x_3}{\Lambda^3}\xi(\varphi Lh_uLh_u)$$

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$$M_\nu = M_\nu^{(0)} + M_\nu^{(1)} = \begin{pmatrix} a+\frac{2d}{3} & -\frac{d}{3} & -\frac{d}{3} \\ \cdot & \frac{2d}{3} & a-\frac{d}{3} \\ \cdot & \cdot & \frac{2d}{3} \end{pmatrix} + \begin{pmatrix} \frac{2}{3}\eta_3 & \eta_2 & \eta_1 \\ \cdot & \eta_1 & -\frac{1}{3}\eta_3 \\ \cdot & \cdot & \eta_2 \end{pmatrix}$$

$$\eta_i \simeq 0.01\left(\frac{v}{10^{11}\ \text{GeV}}\right)\left(\frac{v'}{10^{10}\ \text{GeV}}\right)\left(\frac{v_u}{10^{2}\ \text{GeV}}\right)^2\left(\frac{10^{12}\ \text{GeV}}{\Lambda}\right)^3 \text{eV}$$

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Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	1	1
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$$\frac{x_1}{\Lambda^3}(\varphi\varphi')'(Lh_uLh_u)'' , \quad \frac{x_2}{\Lambda^3}(\varphi\varphi')''(Lh_uLh_u)' , \quad \text{and} \quad \frac{x_3}{\Lambda^3}\xi(\varphi Lh_uLh_u)$$

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$$\boxed{\sin^2\theta_{13} \simeq \frac{(\eta_1-\eta_2)^2}{8a^2}}$$

- Higher dimensional operators → non-zero θ_{13}

ν_s in flavor symmetry models: an effective approach

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Barry, Rodejohann, HZ, 2012

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$\Lambda = 10^{12}$ GeV

Charged lepton mass matrix is diagonal at leading order

$$m_\alpha = y_\alpha v_d \frac{v}{\Lambda}\lambda^{F_\alpha} \simeq 10\left(\frac{v_d}{10^2\ \mathrm{GeV}}\right)\left(\frac{v}{10^{11}\ \mathrm{GeV}}\right)\left(\frac{10^{12}\ \mathrm{GeV}}{\Lambda}\right)\left(\frac{\lambda}{10^{-1}}\right)^{F_\alpha}\ \mathrm{GeV}$$

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Charged lepton mass matrix is diagonal at leading order

Charged lepton mass hierarchy

$$m_\alpha = y_\alpha v_d \frac{v}{\Lambda}\lambda^{F_\alpha} \simeq 10 \left(\frac{v_d}{10^2\ \mathrm{GeV}}\right)\left(\frac{v}{10^{11}\ \mathrm{GeV}}\right)\left(\frac{10^{12}\ \mathrm{GeV}}{\Lambda}\right)\left(\frac{\lambda}{10^{-1}}\right)^{F_\alpha}\ \mathrm{GeV}$$

ν_s in flavor symmetry models: an effective approach

Field	L	e^c	μ^c	τ^c	$h_{u,d}$	φ	φ'	ξ	Θ	ν_s
$SU(2)_L$	2	1	1	1	2	1	1	1	1	1
A_4	$\underline{3}$	$\underline{1}$	$\underline{1}''$	$\underline{1}'$	$\underline{1}$	$\underline{3}$	$\underline{3}$	$\underline{1}$	$\underline{1}$	$\underline{1}$
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	1	1
$U(1)_{\rm FN}$	-	4	2	0	-	-	-	-	-1	8

Barry, Rodejohann, HZ, 2012

$\lambda = 10^{-1} = 0.1$

$\Lambda = 10^{12}$ GeV

Charged lepton mass matrix is diagonal at leading order

Charged lepton mass hierarchy

$$m_\alpha = y_\alpha v_d \frac{v}{\Lambda}\lambda^{F_\alpha} \simeq 10\left(\frac{v_d}{10^2\ {\rm GeV}}\right)\left(\frac{v}{10^{11}\ {\rm GeV}}\right)\left(\frac{10^{12}\ {\rm GeV}}{\Lambda}\right)\left(\frac{\lambda}{10^{-1}}\right)^{F_\alpha}\ {\rm GeV}$$

NLO corrections to the charged lepton mass matrix remains diagonal

$$\frac{1}{\Lambda^2}\left[y'_e\lambda^4\left(\varphi\varphi L h_d\right)e^c + y'_\mu\lambda^2\left(\varphi\varphi L h_d\right)'\mu^c + y'_\tau\left(\varphi\varphi L h_d\right)''\tau^c\right]$$

ν_s in flavor symmetry models: realization in seesaw

- Assigning different FN charges to three right-handed neutrinos

ν_s in flavor symmetry models: realization in seesaw

- Assigning different FN charges to three right-handed neutrinos

Model A: three eV-scale sterile neutrinos.

No neutrinoless double beta decay;
More tension with cosmology

Model B: 2eV + 1keV sterile neutrinos

Reactor & LSND/MiniBooNE anomalies;
WDM candidate

Model C: 1eV + 1keV + 1heavy sterile neutrinos

Neutrinoless double beta decay;
WDM Candidate

Model D: 1eV + 2heavy (>GeV) sterile neutrinos

Neutrinoless double beta decay;
Successful leptogenesis

Model E: 1keV + 2heavy (>GeV) sterile neutrinos (*νMSM*)

Both baryon asymmetry & Warm Dark Matter puzzles can be solved
Failed in explaining the reactor anomaly

ν_s in flavor symmetry models: realization in seesaw

Field	L	e^c	μ^c	τ^c	$h_{u,d}$	φ	φ'	φ''	ξ	ξ'	ξ''	Θ	ν_1^c	ν_2^c	ν_3^c
$SU(2)_L$	2	1	1	1	2	1	1	1	1	1	1	1	1	1	1
A_4	$\underline{3}$	$\underline{1}$	$\underline{1}''$	$\underline{1}'$	$\underline{1}$	$\underline{3}$	$\underline{3}$	$\underline{3}$	$\underline{1}$	$\underline{1}'$	$\underline{1}$	$\underline{1}$	$\underline{1}$	$\underline{1}'$	$\underline{1}$
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω^2	ω^2	ω	1	1	ω^2	ω	1
$U(1)_{\mathrm{FN}}$	-	3	1	0	-	-	-	-	-	-	-	-1	F_1	F_2	F_3

particle assignments

Barry, Rodejohann, H.Z., JHEP07(2011)091

ν_s in flavor symmetry models: realization in seesaw

Field	L	e^c	μ^c	τ^c	$h_{u,d}$	φ	φ'	φ''	ξ	ξ'	ξ''	Θ	ν_1^c	ν_2^c	ν_3^c
$SU(2)_L$	2	1	1	1	2	1	1	1	1	1	1	1	1	1	1
A_4	$\underline{3}$	$\underline{1}$	$\underline{1}''$	$\underline{1}'$	$\underline{1}$	$\underline{3}$	$\underline{3}$	$\underline{3}$	$\underline{1}$	$\underline{1}'$	$\underline{1}$	$\underline{1}$	$\underline{1}$	$\underline{1}'$	$\underline{1}$
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω^2	ω^2	ω	1	1	ω^2	ω	1
$U(1)_{\rm FN}$	-	3	1	0	-	-	-	-	-	-	-	-1	F_1	F_2	F_3

particle assignments

Barry, Rodejohann, H.Z., JHEP07(2011)091

ν_s in flavor symmetry models: realization in seesaw

Field	L	e^c	μ^c	τ^c	$h_{u,d}$	φ	φ'	φ''	ξ	ξ'	ξ''	Θ	ν_1^c	ν_2^c	ν_3^c
$SU(2)_L$	2	1	1	1	2	1	1	1	1	1	1	1	1	1	1
A_4	$\underline{3}$	$\underline{1}$	$\underline{1}''$	$\underline{1}'$	$\underline{1}$	$\underline{3}$	$\underline{3}$	$\underline{3}$	$\underline{1}$	$\underline{1}'$	$\underline{1}$	$\underline{1}$	$\underline{1}$	$\underline{1}'$	$\underline{1}$
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω^2	ω^2	ω	1	1	ω^2	ω	1
$U(1)_{\rm FN}$	-	3	1	0	-	-	-	-	-	-	-	-1	F_1	F_2	F_3

particle assignments

Barry, Rodejohann, H.Z., JHEP07(2011)091

- Right-handed neutrino are A_4 singlets so as to assign different FN charges (mass splitting)
- One of the sterile neutrinos is located at keV scale acting as WDM
- The other two right-handed neutrinos generate active neutrino masses via seesaw
- Tri-bimaximal mixing is obtained at leading order from vacuum alignments of flavons
- Charged-lepton corrections → θ_{13}

ν_s in flavor symmetry models: realization in seesaw

Field	L	e^c	μ^c	τ^c	$h_{u,d}$	φ	φ'	φ''	ξ	ξ'	ξ''	Θ	ν_1^c	ν_2^c	ν_3^c
$SU(2)_L$	2	1	1	1	2	1	1	1	1	1	1	1	1	1	1
A_4	$\underline{3}$	$\underline{1}$	$\underline{1}''$	$\underline{1}'$	$\underline{1}$	$\underline{3}$	$\underline{3}$	$\underline{3}$	$\underline{1}$	$\underline{1}'$	$\underline{1}$	$\underline{1}$	$\underline{1}$	$\underline{1}'$	$\underline{1}$
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω^2	ω^2	ω	1	1	ω^2	ω	1
$U(1)_{\mathrm{FN}}$	-	3	1	0	-	-	-	-	-	-	-	-1	F_1	F_2	F_3

Invariant Lagrangian under $A_4 \times Z_3 \times U(1)_{FN}$

$$
\begin{aligned}
-\mathcal{L}_{\mathrm{Y}} = & \frac{y_e}{\Lambda}\lambda^3\left(\varphi L h_d\right) e^c + \frac{y_\mu}{\Lambda}\lambda\left(\varphi L h_d\right)' \mu^c + \frac{y_\mu}{\Lambda}\left(\varphi L h_d\right)'' \tau^c \\
& + \frac{y_1}{\Lambda}\lambda^{F_1}(\varphi L h_u)\nu_1^c + \frac{y_2}{\Lambda}\lambda^{F_2}(\varphi' L h_u)''\nu_2^c + \frac{y_3}{\Lambda}\lambda^{F_3}(\varphi'' L h_u)\nu_3^c \\
& \boxed{+ \frac{1}{2}\left[w_1\lambda^{2F_1}\xi\nu_1^c\nu_1^c + w_2\lambda^{2F_2}\xi'\nu_2^c\nu_2^c + w_3\lambda^{2F_3}\xi''\nu_3^c\nu_3^c\right]} + \mathrm{h.c.},
\end{aligned}
$$

ν_S in flavor symmetry models: realization in seesaw

	F_1, F_2, F_3	Mass spectrum	$\lvert U_{\alpha 4}\rvert$	$\lvert U_{\alpha 5}\rvert$	$\langle m_{ee}\rangle$ NO	$\langle m_{ee}\rangle$ IO	Phenomenology
I	9, 10, 10	$M_{2,3} = \mathcal{O}(\mathrm{eV})$,	$\mathcal{O}(0.1)$	$\mathcal{O}(0.1)$	0	0	$3+2$ mixing
IIA	9, 10, 0	$M_2 = \mathcal{O}(\mathrm{eV})$ $M_3 = \mathcal{O}(10^{11}\,\mathrm{GeV})$	$\mathcal{O}(0.1)$	$\mathcal{O}(10^{-11})$	0	$\frac{2\sqrt{\Delta m_{\mathrm{A}}^2}}{3}$	$3+1$ mixing
IIB	9, 0, 10	$M_2 = \mathcal{O}(10^{11}\,\mathrm{GeV})$ $M_3 = \mathcal{O}(\mathrm{eV})$	$\mathcal{O}(10^{-11})$	$\mathcal{O}(0.1)$	$\frac{\sqrt{\Delta m_{\mathrm{S}}^2}}{3}$	$\frac{\sqrt{\Delta m_{\mathrm{A}}^2}}{3}$	
III	9, 5, 5	$M_{2,3} = \mathcal{O}(10\,\mathrm{GeV})$	$\mathcal{O}(10^{-6})$	$\mathcal{O}(10^{-6})$	$\frac{\sqrt{\Delta m_{\mathrm{S}}^2}}{3}$	$\sqrt{\Delta m_{\mathrm{A}}^2}$	Leptogenesis

Typical choices of FN charges and phenomenological consequences

ν_s in flavor symmetry models: realization in seesaw

	$F_1,\ F_2,\ F_3$	Mass spectrum	$\lvert U_{\alpha 4}\rvert$	$\lvert U_{\alpha 5}\rvert$	$\langle m_{ee}\rangle$ NO	$\langle m_{ee}\rangle$ IO	Phenomenology
I	9, 10, 10	$M_{2,3} = \mathcal{O}(\mathrm{eV})$,	$\mathcal{O}(0.1)$	$\mathcal{O}(0.1)$	0	0	$3+2$ mixing
IIA	9, 10, 0	$M_2 = \mathcal{O}(\mathrm{eV})$ $M_3 = \mathcal{O}(10^{11}\,\mathrm{GeV})$	$\mathcal{O}(0.1)$	$\mathcal{O}(10^{-11})$	0	$\frac{2\sqrt{\Delta m_{\mathrm{A}}^2}}{3}$	$3+1$ mixing
IIB	9, 0, 10	$M_2 = \mathcal{O}(10^{11}\,\mathrm{GeV})$ $M_3 = \mathcal{O}(\mathrm{eV})$	$\mathcal{O}(10^{-11})$	$\mathcal{O}(0.1)$	$\frac{\sqrt{\Delta m_{\mathrm{S}}^2}}{3}$	$\frac{\sqrt{\Delta m_{\mathrm{A}}^2}}{3}$	
III	9, 5, 5	$M_{2,3} = \mathcal{O}(10\,\mathrm{GeV})$	$\mathcal{O}(10^{-6})$	$\mathcal{O}(10^{-6})$	$\frac{\sqrt{\Delta m_{\mathrm{S}}^2}}{3}$	$\sqrt{\Delta m_{\mathrm{A}}^2}$	Leptogenesis

Scenario-I: F_1=9; F_2=10; F_3=10; two eV-scale sterile neutrinos; vanishing $\langle m_{ee}\rangle$;

neutrino mixing and masses (normal mass ordering)

$$m_2^{(0)} \equiv -\frac{3y_2^2 v'^2 v_u^2}{w_2 u' \Lambda^2}\,, \qquad m_3^{(0)} \equiv -\frac{2y_3^2 v''^2 v_u^2}{w_3 u'' \Lambda^2}$$

$$|U_{e3}|^2 \simeq \frac{r_1^2}{2}\left[\left(\frac{y'_\mu}{y_\mu} - \frac{y'_\tau}{y_\tau}\right)^2\right] + \frac{\chi^2}{2} - \chi r_1 \left(\frac{y'_\mu}{y_\mu} - \frac{y'_\tau}{y_\tau}\right)$$

$$|U_{e2}|^2 \simeq \frac{1}{3}\left[1 - 3\epsilon_1^2 - 2r_1\left(\frac{y'_\mu}{y_\mu} + \frac{y'_\tau}{y_\tau}\right)\right],$$

$$|U_{\mu 3}|^2 \simeq \frac{1}{2}\left[1 - 2\epsilon_2^2 + 2\frac{y'_\tau}{y_\tau} r_1 + \frac{2}{3}\chi R\right],$$

$$\chi \equiv \frac{y_1 v}{y_3 v''}\frac{w'_1}{w_1} r_1 \sim 0.01$$

$$r_1^2 \sim \epsilon_1^2 \sim \epsilon_2^2 \simeq 0.01$$

ν_s in flavor symmetry models: realization in seesaw

	$F_1,\ F_2,\ F_3$	Mass spectrum	$\lvert U_{\alpha 4}\rvert$	$\lvert U_{\alpha 5}\rvert$	$\langle m_{ee}\rangle$ NO	$\langle m_{ee}\rangle$ IO	Phenomenology
I	9, 10, 10	$M_{2,3} = \mathcal{O}(\text{eV})$,	$\mathcal{O}(0.1)$	$\mathcal{O}(0.1)$	0	0	$3+2$ mixing
IIA	9, 10, 0	$M_2 = \mathcal{O}(\text{eV})$ $M_3 = \mathcal{O}(10^{11}\,\text{GeV})$	$\mathcal{O}(0.1)$	$\mathcal{O}(10^{-11})$	0	$\frac{2\sqrt{\Delta m^2_{\text{A}}}}{3}$	$3+1$ mixing
IIB	9, 0, 10	$M_2 = \mathcal{O}(10^{11}\,\text{GeV})$ $M_3 = \mathcal{O}(\text{eV})$	$\mathcal{O}(10^{-11})$	$\mathcal{O}(0.1)$	$\frac{\sqrt{\Delta m^2_{\text{S}}}}{3}$	$\frac{\sqrt{\Delta m^2_{\text{A}}}}{3}$	
III	9, 5, 5	$M_{2,3} = \mathcal{O}(10\,\text{GeV})$	$\mathcal{O}(10^{-6})$	$\mathcal{O}(10^{-6})$	$\frac{\sqrt{\Delta m^2_{\text{S}}}}{3}$	$\sqrt{\Delta m^2_{\text{A}}}$	Leptogenesis

Scenario-I: $F_1=9$; $F_2=10$; $F_3=10$; two eV-scale sterile neutrinos; vanishing $\langle m_{ee}\rangle$;

neutrino mixing and masses
(normal mass ordering)

$$m_2^{(0)} \equiv -\frac{3y_2^2 v'^2 v_u^2}{w_2 u' \Lambda^2}, \quad m_3^{(0)} \equiv -\frac{2y_3^2 v''^2 v_u^2}{w_3 u'' \Lambda^2}$$

$$|U_{e3}|^2 \simeq \frac{r_1^2}{2}\left[\left(\frac{y'_\mu}{y_\mu} - \frac{y'_\tau}{y_\tau}\right)^2\right] + \frac{\chi^2}{2} + \chi r_1 \left(\frac{y'_\mu}{y_\mu} - \frac{y'_\tau}{y_\tau}\right)$$

$$|U_{e2}|^2 \simeq \frac{1}{3}\left[1 + 3\epsilon_1^2 - 2r_1\left(\frac{y'_\mu}{y_\mu} + \frac{y'_\tau}{y_\tau}\right)\right],$$

$$|U_{\mu 3}|^2 \simeq \frac{1}{2}\left[1 + 2\epsilon_2^2 + 2\frac{y'_\tau}{y_\tau} r_1 + \frac{2}{3}\chi R\right],$$

NLO seesaw corrections to TBM

$$\chi \equiv \frac{y_1 v}{y_3 v''}\frac{w'_1}{w_1} r_1 \sim 0.01$$

$$r_1^2 \sim \epsilon_1^2 \sim \epsilon_2^2 \simeq 0.01$$

ν_s in flavor symmetry models: realization in seesaw

	$F_1,\ F_2,\ F_3$	Mass spectrum	$\lvert U_{\alpha 4}\rvert$	$\lvert U_{\alpha 5}\rvert$	$\langle m_{ee}\rangle$ NO	$\langle m_{ee}\rangle$ IO	Phenomenology
I	9, 10, 10	$M_{2,3}=\mathcal{O}(\text{eV})$,	$\mathcal{O}(0.1)$	$\mathcal{O}(0.1)$	0	0	$3+2$ mixing
IIA	9, 10, 0	$M_2=\mathcal{O}(\text{eV})$ $M_3=\mathcal{O}(10^{11}\,\text{GeV})$	$\mathcal{O}(0.1)$	$\mathcal{O}(10^{-11})$	0	$\frac{2\sqrt{\Delta m^2_{\rm A}}}{3}$	$3+1$ mixing
IIB	9, 0, 10	$M_2=\mathcal{O}(10^{11}\,\text{GeV})$ $M_3=\mathcal{O}(\text{eV})$	$\mathcal{O}(10^{-11})$	$\mathcal{O}(0.1)$	$\frac{\sqrt{\Delta m^2_{\rm S}}}{3}$	$\frac{\sqrt{\Delta m^2_{\rm A}}}{3}$	$3+1$ mixing
III	9, 5, 5	$M_{2,3}=\mathcal{O}(10\,\text{GeV})$	$\mathcal{O}(10^{-6})$	$\mathcal{O}(10^{-6})$	$\frac{\sqrt{\Delta m^2_{\rm S}}}{3}$	$\sqrt{\Delta m^2_{\rm A}}$	Leptogenesis

Scenario-I: F_1=9; F_2=10; F_3=10; two eV-scale sterile neutrinos; vanishing $\langle m_{ee}\rangle$;

neutrino mixing and masses (normal mass ordering)

$$m_2^{(0)} \equiv -\frac{3y_2^2 v'^2 v_u^2}{w_2 u' \Lambda^2}, \quad m_3^{(0)} \equiv -\frac{2y_3^2 v''^2 v_u^2}{w_3 u'' \Lambda^2}$$

$$|U_{e3}|^2 \simeq \frac{r_1^2}{2}\left[\left(\frac{y'_\mu}{y_\mu}-\frac{y'_\tau}{y_\tau}\right)^2\right] + \frac{\chi^2}{2} - \chi r_1\left(\frac{y'_\mu}{y_\mu}-\frac{y'_\tau}{y_\tau}\right)$$

$$|U_{e2}|^2 \simeq \frac{1}{3}\left[1 - 3\epsilon_1^2 - 2r_1\left(\frac{y'_\mu}{y_\mu}+\frac{y'_\tau}{y_\tau}\right)\right],$$

$$|U_{\mu 3}|^2 \simeq \frac{1}{2}\left[1 - 2\epsilon_2^2 + 2\frac{y'_\tau}{y_\tau}r_1 + \frac{2}{3}\chi R\right],$$

NLO seesaw corrections to TBM

Charged-lepton corrections: θ_{13}

$$\chi \equiv \frac{y_1 v}{y_3 v''}\frac{w'_1}{w_1}r_1 \sim 0.01$$

$$r_1^2 \sim \epsilon_1^2 \sim \epsilon_2^2 \simeq 0.01$$

ν_s in flavor symmetry models: realization in seesaw

	$F_1,\ F_2,\ F_3$	Mass spectrum	$\lvert U_{\alpha 4}\rvert$	$\lvert U_{\alpha 5}\rvert$	$\langle m_{ee}\rangle$ NO	$\langle m_{ee}\rangle$ IO	Phenomenology
I	9, 10, 10	$M_{2,3} = \mathcal{O}(\text{eV})$,	$\mathcal{O}(0.1)$	$\mathcal{O}(0.1)$	0	0	$3+2$ mixing
IIA	9, 10, 0	$M_2 = \mathcal{O}(\text{eV})$ $M_3 = \mathcal{O}(10^{11}\,\text{GeV})$	$\mathcal{O}(0.1)$	$\mathcal{O}(10^{-11})$	0	$\frac{2\sqrt{\Delta m^2_{\text{A}}}}{3}$	$3+1$ mixing
IIB	9, 0, 10	$M_2 = \mathcal{O}(10^{11}\,\text{GeV})$ $M_3 = \mathcal{O}(\text{eV})$	$\mathcal{O}(10^{-11})$	$\mathcal{O}(0.1)$	$\frac{\sqrt{\Delta m^2_{\text{S}}}}{3}$	$\frac{\sqrt{\Delta m^2_{\text{A}}}}{3}$	
III	9, 5, 5	$M_{2,3} = \mathcal{O}(10\,\text{GeV})$	$\mathcal{O}(10^{-6})$	$\mathcal{O}(10^{-6})$	$\frac{\sqrt{\Delta m^2_{\text{S}}}}{3}$	$\sqrt{\Delta m^2_{\text{A}}}$	Leptogenesis

Scenario-I: F_1=9; F_2=10; F_3=10; two eV-scale sterile neutrinos; vanishing $\langle m_{ee}\rangle$;

neutrino mixing and masses
(normal mass ordering)

$$|U_{e4}|^2 = |U_{\mu 4}|^2 \simeq \epsilon_1^2$$

$$|U_{e5}|^2 \simeq \chi^2\epsilon_2^2\ ,$$

$$|U_{\mu 5}|^2 \simeq \epsilon_2^2$$

$$|U_{e3}|^2 \simeq \frac{r_1^2}{2}\left[\left(\frac{y'_\mu}{y_\mu} - \frac{y'_\tau}{y_\tau}\right)^2\right] + \frac{\chi^2}{2} + \chi r_1\left(\frac{y'_\mu}{y_\mu} - \frac{y'_\tau}{y_\tau}\right)$$

$$|U_{e2}|^2 \simeq \frac{1}{3}\left[1 - 3\epsilon_1^2 - 2r_1\left(\frac{y'_\mu}{y_\mu} + \frac{y'_\tau}{y_\tau}\right)\right],$$

$$|U_{\mu 3}|^2 \simeq \frac{1}{2}\left[1 - 2\epsilon_2^2 + 2\frac{y'_\tau}{y_\tau}r_1 + \frac{2}{3}\chi R\right],$$

NLO seesaw corrections to TBM

Charged-lepton corrections: θ_{13}

$$\chi \equiv \frac{y_1 v}{y_3 v''}\frac{w'_1}{w_1}r_1 \sim 0.01$$

$$r_1^2 \sim \epsilon_1^2 \sim \epsilon_2^2 \simeq 0.01$$

ν_s in flavor symmetry models: realization in seesaw

	$F_1,\ F_2,\ F_3$	Mass spectrum	$\lvert U_{\alpha 4}\rvert$	$\lvert U_{\alpha 5}\rvert$	$\langle m_{ee}\rangle$ NO	$\langle m_{ee}\rangle$ IO	Phenomenology
I	9, 10, 10	$M_{2,3}=\mathcal{O}(\text{eV})$,	$\mathcal{O}(0.1)$	$\mathcal{O}(0.1)$	0	0	$3+2$ mixing
IIA	9, 10, 0	$M_2=\mathcal{O}(\text{eV})$ $M_3=\mathcal{O}(10^{11}\,\text{GeV})$	$\mathcal{O}(0.1)$	$\mathcal{O}(10^{-11})$	0	$\frac{2\sqrt{\Delta m_A^2}}{3}$	$3+1$ mixing
IIB	9, 0, 10	$M_2=\mathcal{O}(10^{11}\,\text{GeV})$ $M_3=\mathcal{O}(\text{eV})$	$\mathcal{O}(10^{-11})$	$\mathcal{O}(0.1)$	$\frac{\sqrt{\Delta m_S^2}}{3}$	$\frac{\sqrt{\Delta m_A^2}}{3}$	$3+1$ mixing
III	9, 5, 5	$M_{2,3}=\mathcal{O}(10\,\text{GeV})$	$\mathcal{O}(10^{-6})$	$\mathcal{O}(10^{-6})$	$\frac{\sqrt{\Delta m_S^2}}{3}$	$\sqrt{\Delta m_A^2}$	Leptogenesis

Scenario-I: F_1=9; F_2=10; F_3=10; two eV-scale sterile neutrinos; vanishing $\langle m_{ee}\rangle$;

neutrino mixing and masses (normal mass ordering)

$$|U_{e3}|^2 \simeq \frac{r_1^2}{2}\left[\left(\frac{y'_\mu}{y_\mu}-\frac{y'_\tau}{y_\tau}\right)^2\right] + \frac{\chi^2}{2} + \chi r_1\left(\frac{y'_\mu}{y_\mu}-\frac{y'_\tau}{y_\tau}\right)$$

$$|U_{e2}|^2 \simeq \frac{1}{3}\left[1 - 3\epsilon_1^2 - 2r_1\left(\frac{y'_\mu}{y_\mu}+\frac{y'_\tau}{y_\tau}\right)\right],$$

$$|U_{\mu 3}|^2 \simeq \frac{1}{2}\left[1 - 2\epsilon_2^2 + 2\frac{y'_\tau}{y_\tau}r_1 + \frac{2}{3}\chi R\right],$$

NLO seesaw corrections to TBM

Charged-lepton corrections: θ_{13}

$$|U_{e4}|^2 = |U_{\mu 4}|^2 \simeq \epsilon_1^2$$

$$|U_{e5}|^2 \simeq \chi^2\epsilon_2^2\,,$$

$$|U_{\mu 5}|^2 \simeq \epsilon_2^2$$

active-sterile mixing: too small for reactor anomaly

$$\chi \equiv \frac{y_1 v}{y_3 v''}\frac{w'_1}{w_1}r_1 \sim 0.01$$

$$r_1^2 \sim \epsilon_1^2 \sim \epsilon_2^2 \simeq 0.01$$

ν_s in flavor symmetry models: realization in seesaw

	$F_1,\ F_2,\ F_3$	Mass spectrum	$\lvert U_{\alpha 4}\rvert$	$\lvert U_{\alpha 5}\rvert$	$\langle m_{ee}\rangle$ NO	$\langle m_{ee}\rangle$ IO	Phenomenology
I	9, 10, 10	$M_{2,3}=\mathcal{O}(\mathrm{eV})$,	$\mathcal{O}(0.1)$	$\mathcal{O}(0.1)$	0	0	$3+2$ mixing
IIA	9, 10, 0	$M_2=\mathcal{O}(\mathrm{eV})$ $M_3=\mathcal{O}(10^{11}\,\mathrm{GeV})$	$\mathcal{O}(0.1)$	$\mathcal{O}(10^{-11})$	0	$\frac{2\sqrt{\Delta m_{\mathrm{A}}^2}}{3}$	$3+1$ mixing
IIB	9, 0, 10	$M_2=\mathcal{O}(10^{11}\,\mathrm{GeV})$ $M_3=\mathcal{O}(\mathrm{eV})$	$\mathcal{O}(10^{-11})$	$\mathcal{O}(0.1)$	$\frac{\sqrt{\Delta m_{\mathrm{S}}^2}}{3}$	$\frac{\sqrt{\Delta m_{\mathrm{A}}^2}}{3}$	
III	9, 5, 5	$M_{2,3}=\mathcal{O}(10\,\mathrm{GeV})$	$\mathcal{O}(10^{-6})$	$\mathcal{O}(10^{-6})$	$\frac{\sqrt{\Delta m_{\mathrm{S}}^2}}{3}$	$\sqrt{\Delta m_{\mathrm{A}}^2}$	Leptogenesis

Scenario-I: F_1=9; F_2=10; F_3=10; two eV-scale sterile neutrinos; vanishing $\langle m_{ee}\rangle$;

neutrino mixing and masses
(inverted mass ordering)

$$|U_{e3}|^2 \simeq \frac{r_1^2}{2}\left(\frac{y'_\mu}{y_\mu}-\frac{y'_\tau}{y_\tau}\right)^2,$$

$$|U_{e2}|^2 \simeq \frac{1}{3}\left[1-3\epsilon_1^2-2r_1\left(\frac{y'_\mu}{y_\mu}+\frac{y'_\tau}{y_\tau}\right)-\frac{2}{3}\chi G\right]$$

$$|U_{\mu 3}|^2 \simeq \frac{1}{2}\left[1+2\frac{y'_\tau}{y_\tau}r_1\right],$$

$$|U_{e4}|^2 = |U_{\mu 4}|^2 \simeq \epsilon_1^2$$

$$|U_{e5}|^2 \simeq 4\,(1-\chi)\,\epsilon_2^2$$

$$|U_{\mu 5}|^2 \simeq \epsilon_2^2\ ,$$

ν_s in flavor symmetry models: realization in seesaw

	F_1, F_2, F_3	Mass spectrum	$\lvert U_{\alpha 4}\rvert$	$\lvert U_{\alpha 5}\rvert$	$\langle m_{ee}\rangle$ NO	$\langle m_{ee}\rangle$ IO	Phenomenology
I	9, 10, 10	$M_{2,3} = \mathcal{O}(\text{eV})$,	$\mathcal{O}(0.1)$	$\mathcal{O}(0.1)$	0	0	$3+2$ mixing
IIA	9, 10, 0	$M_2 = \mathcal{O}(\text{eV})$ $M_3 = \mathcal{O}(10^{11}\,\text{GeV})$	$\mathcal{O}(0.1)$	$\mathcal{O}(10^{-11})$	0	$\frac{2\sqrt{\Delta m^2_{\text{A}}}}{3}$	$3+1$ mixing
IIB	9, 0, 10	$M_2 = \mathcal{O}(10^{11}\,\text{GeV})$ $M_3 = \mathcal{O}(\text{eV})$	$\mathcal{O}(10^{-11})$	$\mathcal{O}(0.1)$	$\frac{\sqrt{\Delta m^2_{\text{S}}}}{3}$	$\frac{\sqrt{\Delta m^2_{\text{A}}}}{3}$	
III	9, 5, 5	$M_{2,3} = \mathcal{O}(10\,\text{GeV})$	$\mathcal{O}(10^{-6})$	$\mathcal{O}(10^{-6})$	$\frac{\sqrt{\Delta m^2_{\text{S}}}}{3}$	$\sqrt{\Delta m^2_{\text{A}}}$	Leptogenesis

Scenario-I: F_1=9; F_2=10; F_3=10; two eV-scale sterile neutrinos; vanishing $\langle m_{ee}\rangle$;

neutrino mixing and masses

(inverted mass ordering)

$$|U_{e4}|^2 = |U_{\mu 4}|^2 \simeq \epsilon_1^2$$
$$|U_{e5}|^2 \simeq 4\,(1-\chi)\,\epsilon_2^2$$
$$|U_{\mu 5}|^2 \simeq \epsilon_2^2\,,$$

$$|U_{e3}|^2 \simeq \frac{r_1^2}{2}\left(\frac{y'_\mu}{y_\mu} - \frac{y'_\tau}{y_\tau}\right)^2,$$
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$$|U_{\mu 3}|^2 \simeq \frac{1}{2}\left[1 + 2\frac{y'_\tau}{y_\tau} r_1\right],$$

NLO seesaw corrections to TBM

Charged-lepton corrections: θ_{13}

ν_s in flavor symmetry models: realization in seesaw

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$$\epsilon_1^2 \sim \epsilon_2^2 \sim [0.01 - 0.05]$$

Reactor & LSND/MiniBooNE √

ν_s in flavor symmetry models: realization in seesaw

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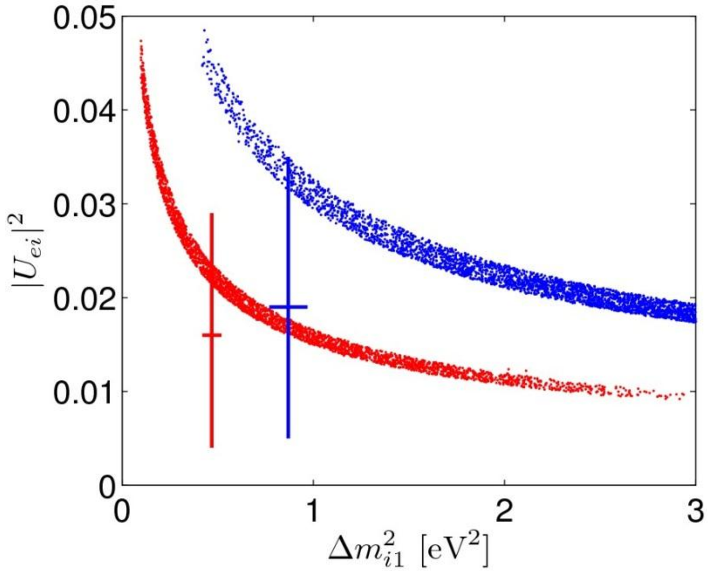
I	9, 10, 10	$M_{2,3} = \mathcal{O}(\text{eV})$,	$\mathcal{O}(0.1)$	$\mathcal{O}(0.1)$	0	0	3 + 2 mixing
IIA	9, 10, 0	$M_2 = \mathcal{O}(\text{eV})$ $M_3 = \mathcal{O}(10^{11}\,\text{GeV})$	$\mathcal{O}(0.1)$	$\mathcal{O}(10^{-11})$	0	$\frac{2\sqrt{\Delta m_A^2}}{3}$	3 + 1 mixing
IIB	9, 0, 10	$M_2 = \mathcal{O}(10^{11}\,\text{GeV})$ $M_3 = \mathcal{O}(\text{eV})$	$\mathcal{O}(10^{-11})$	$\mathcal{O}(0.1)$	$\frac{\sqrt{\Delta m_S^2}}{3}$	$\frac{\sqrt{\Delta m_A^2}}{3}$	3 + 1 mixing
III	9, 5, 5	$M_{2,3} = \mathcal{O}(10\,\text{GeV})$	$\mathcal{O}(10^{-6})$	$\mathcal{O}(10^{-6})$	$\frac{\sqrt{\Delta m_S^2}}{3}$	$\sqrt{\Delta m_A^2}$	Leptogenesis

Scenario-I: F_1=9; F_2=10; F_3=10; two eV-scale sterile neutrinos; vanishing $\langle m_{ee}\rangle$;

Allowed ranges of $|U_{e4}|^2 - \Delta m_{41}^2$ (blue) and $|U_{e5}|^2 - \Delta m_{51}^2$ (red) in the inverted ordering



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ν_S in flavor symmetry models: realization in seesaw

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✓ each column of the Dirac mass term is proportional to a different column of the PMNS matrix – Form Dominance

$$M_D = V_\nu \,\mathrm{diag}\left(\sqrt{-m_1 M_1}, \sqrt{-m_2 M_2}, \sqrt{-m_3 M_3}\right)$$

Chen, King, 09; Choubey, King, Mitra, 10

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- pairwise cancellation in $\langle m_{ee}\rangle$

$$M_\nu^{5\times 5} = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}$$

Vanishing $\langle m_{ee}\rangle$

$$U^2_{e,3+i} M_i = \left[-(V_\nu^2)_{ei}\frac{m_i}{M_i}\right] M_i = -U^2_{ei} m_i$$

ν_s in flavor symmetry models: realization in seesaw

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Scenario-IIB: $F_1=9$; $F_2=0$; $F_3=10$; eV & keV & 10^{11} GeV ν_s; non-vanishing $\langle m_{ee}\rangle$;

- ν_{R2} decouples at low scales $\rightarrow \epsilon_1 \simeq 0$
- corrections to the 4×4 mixing matrix

$$V(\epsilon_2)^{(\text{IO})} \simeq \begin{pmatrix} -\sqrt{6}\epsilon_2^2 & 0 & 0 & 2\epsilon_2 \\ \sqrt{\frac{3}{2}}\epsilon_2^2 & 0 & 0 & -\epsilon_2 \\ \sqrt{\frac{3}{2}}\epsilon_2^2 & 0 & 0 & -\epsilon_2 \\ -\sqrt{6}\epsilon_2 & 0 & 0 & -3\epsilon_2 \end{pmatrix}$$

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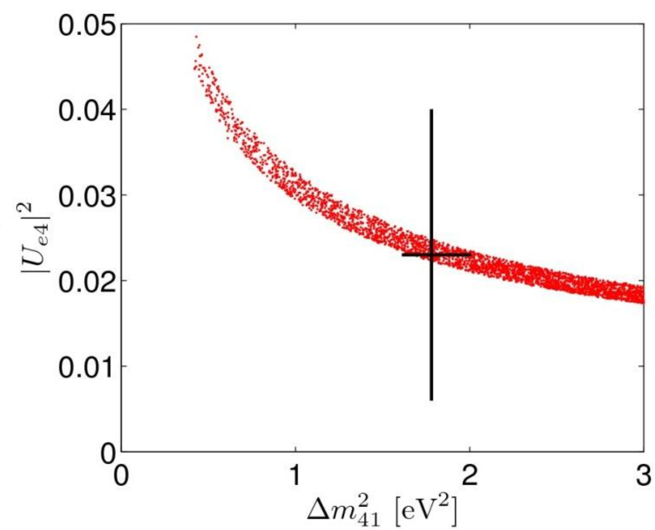

ν_s in flavor symmetry models: realization in seesaw

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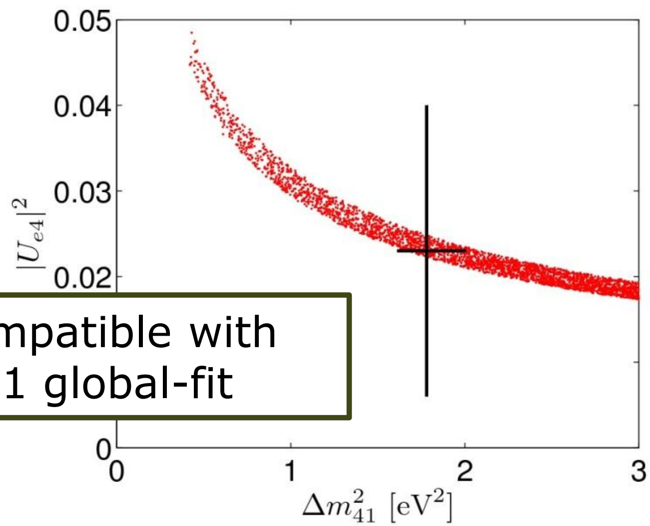

❖ compatible with 3+1 global-fit

ν_S in flavor symmetry models: realization in seesaw

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III	9, 5, 5	$M_{2,3}=\mathcal{O}(10\,\text{GeV})$	$\mathcal{O}(10^{-6})$	$\mathcal{O}(10^{-6})$	$\frac{\sqrt{\Delta m^2_{\text{S}}}}{3}$	$\sqrt{\Delta m^2_{\text{A}}}$	Leptogenesis

Scenario-III: F_1=9; F_2=5; F_3=5; both keV and GeV sterile neutrinos (**νMSM**)

Asaka, Blanchet, Shaposhnikov, 05

- The keV sterile neutrino can be WDM;
- Oscillation of quasi-degenerate heavy sterile neutrinos account for Baryon Asymmetry of the Universe;
- Neutrino-less double beta decay;
- Collider test of right-handed neutrinos

$$\langle m_{ee}\rangle^{(\text{NO})} = \left|\frac{m_2^{(0)}}{3}\right| = \frac{\sqrt{\Delta m^2_{\text{S}}}}{3} \simeq 0.0029\ \text{eV}\,,$$

$$\langle m_{ee}\rangle^{(\text{IO})} = \left|\frac{2m_1^{(0)}}{3} + \frac{m_2^{(0)}}{3}\right| \simeq \sqrt{\Delta m^2_{\text{A}}} \simeq 0.049\ \text{eV}$$

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   a) Suppress M_D and M_R simultaneously via flavor symmetries, warped extra dimensions; FN mechanism; extended seesaw models; …
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   a) Suppress M_D and M_R simultaneously via flavor symmetries, warped extra dimensions; FN mechanism; extended seesaw models; …
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3. A flavor A_4 model using the effective theory approach could give rise to the TBM (at leading order) and accommodate light sterile neutrinos at various energy scales (eV, keV, GeV, …).
   The model can also be realized in the seesaw framework: WDM; active-sterile mixing; deviations from exact TBM; neutrino-less double beta decay; Leptogenesis, et al.

Thanks