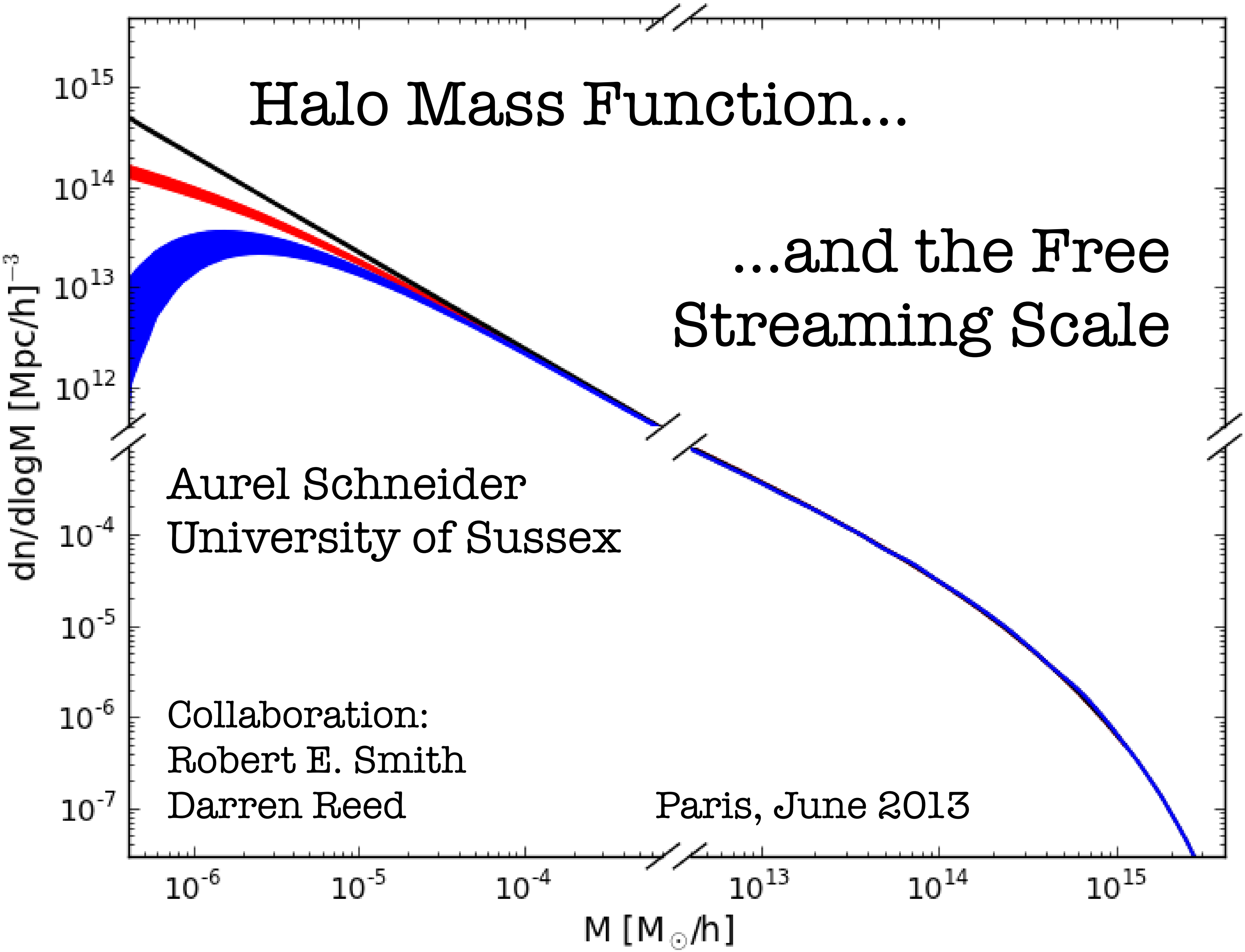
Halo Mass Function...

...and the Free Streaming Scale



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Paris, June 2013

Outline

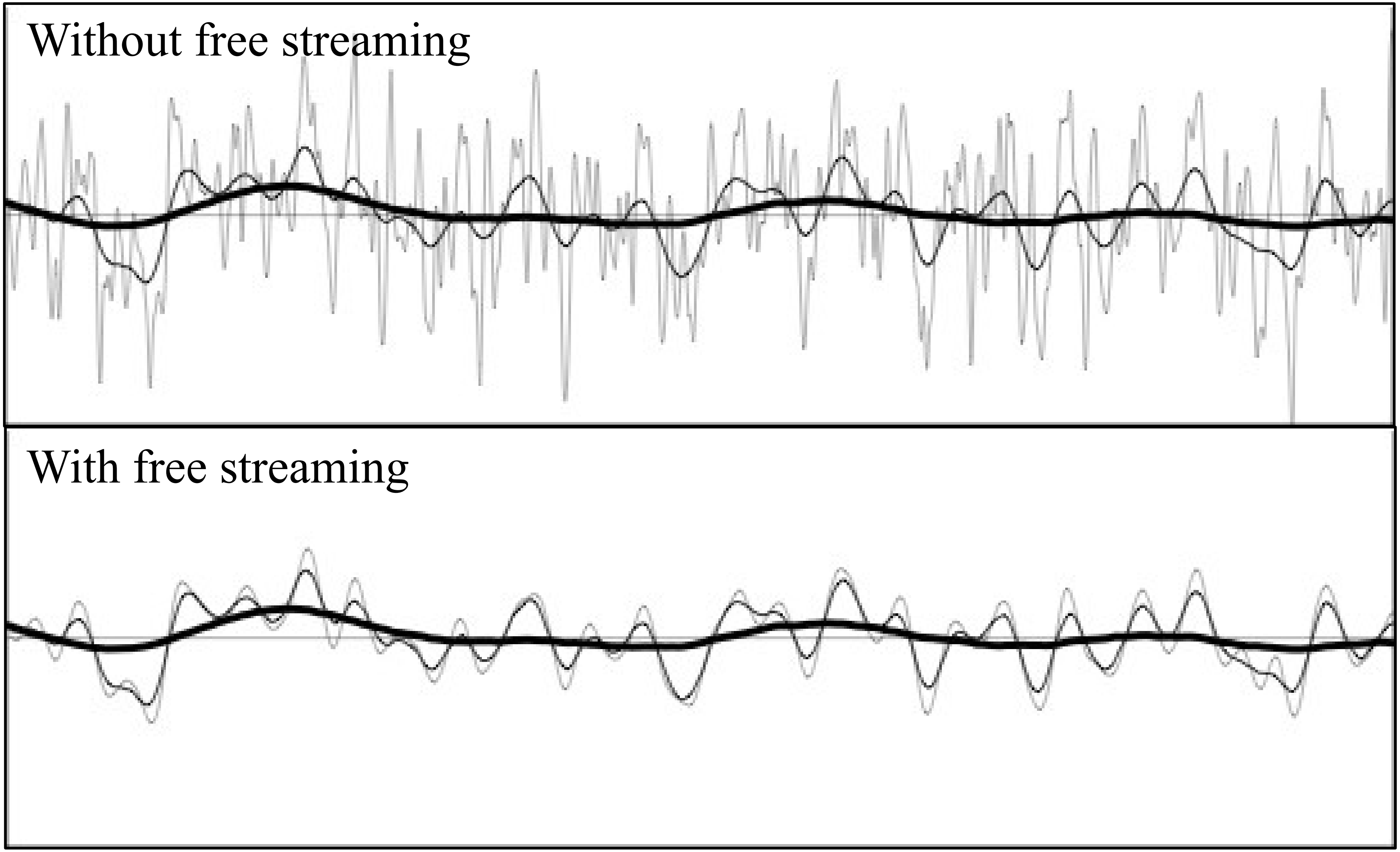
Structure Formation and Free Streaming

Warm Dark Matter - Simulations

Modeling the Halo Mass Function

Predictions for Cold Dark Matter

Structure formation: linear density field



Structure formation: linear density field

Equation of perturbation:

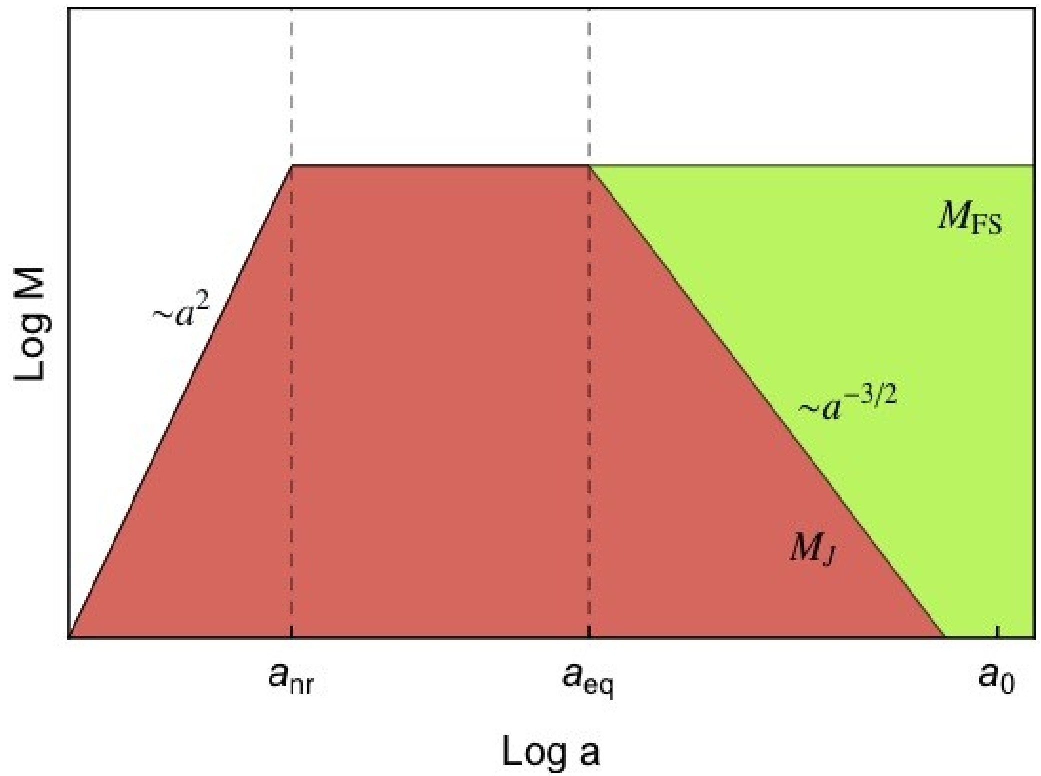
$$\frac{d^2\delta}{dt^2} + 2\frac{\dot{a}}{a}\frac{d\delta}{dt} = \left[4\pi G\rho_\mathrm{b}(t) - \frac{\sigma^2(t)k^2}{a^2}\right]\delta \qquad \delta = \frac{\rho - \rho_\mathrm{b}}{\rho_\mathrm{b}}$$

Jeans criterion:

$$\lambda_\mathrm{J}(t) = \sqrt{\frac{\pi\sigma^2(t)}{G\rho_\mathrm{b}(t)}}$$

Free streaming criterion:

$$\lambda_\mathrm{FS}(t) = \int_0^{t_\mathrm{eq}} \frac{\sigma(t)}{a(t)}dt$$



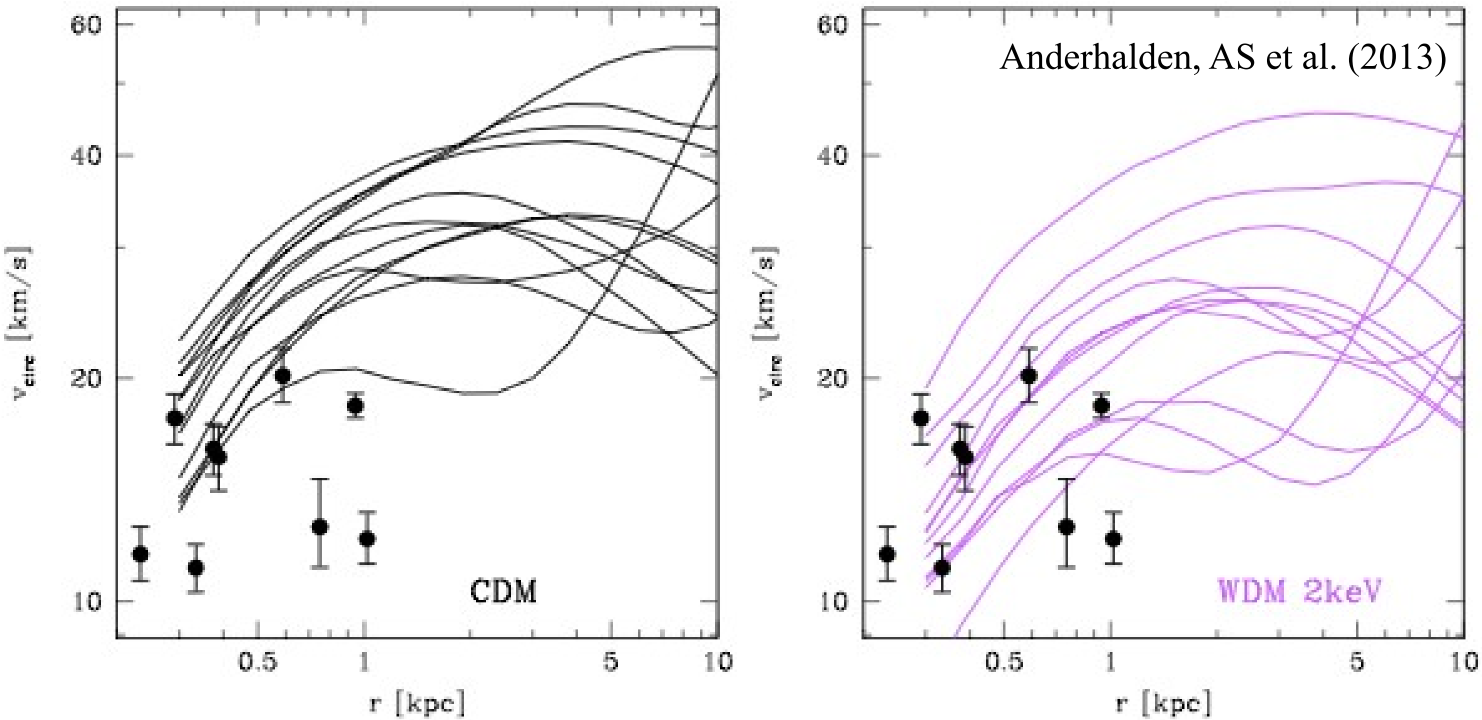
Structure formation: dark matter

Dark Matter Candidates and smallest haloes:

- CDM (WIMP): $\sim 10^{-6} M_{\odot}/h$ (earth-mass microhaloes)
- CDM (Axion): $\sim 10^{-13} M_{\odot}/h$
- WDM (sterile neutrino): $\sim 10^{8} M_{\odot}/h$ (dwarf galaxies)

Structure formation: dark matter

Why should we care...



… because dark matter is a mystery!

… because of inconsistencies in small scales structure formation.

→ WDM effects could be visible in the sky!

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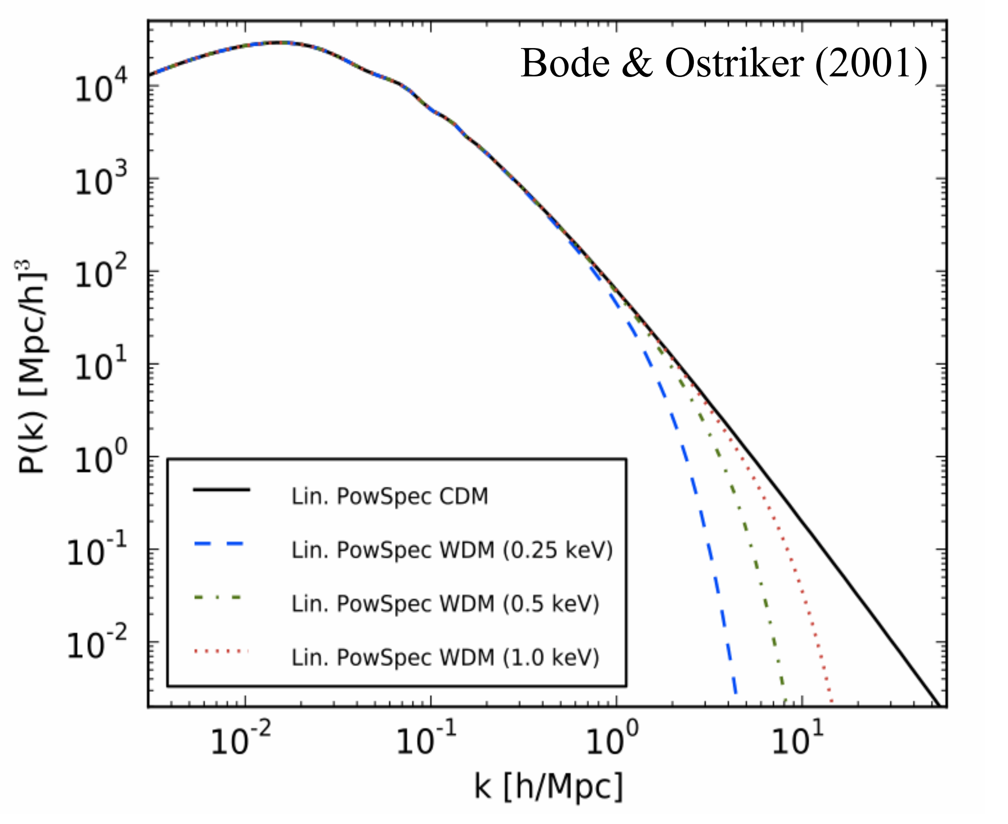
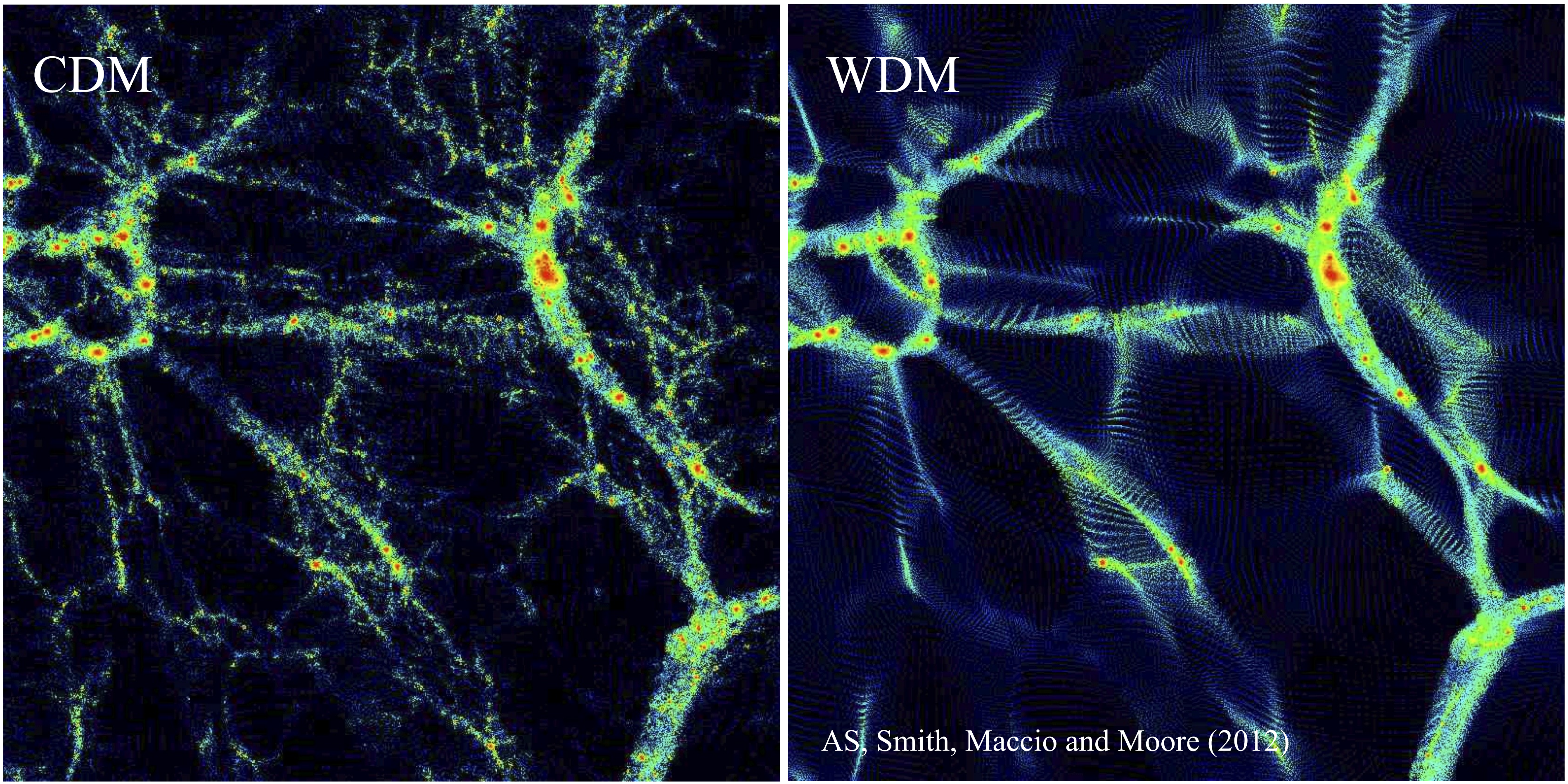
WDM Simulations: Setup

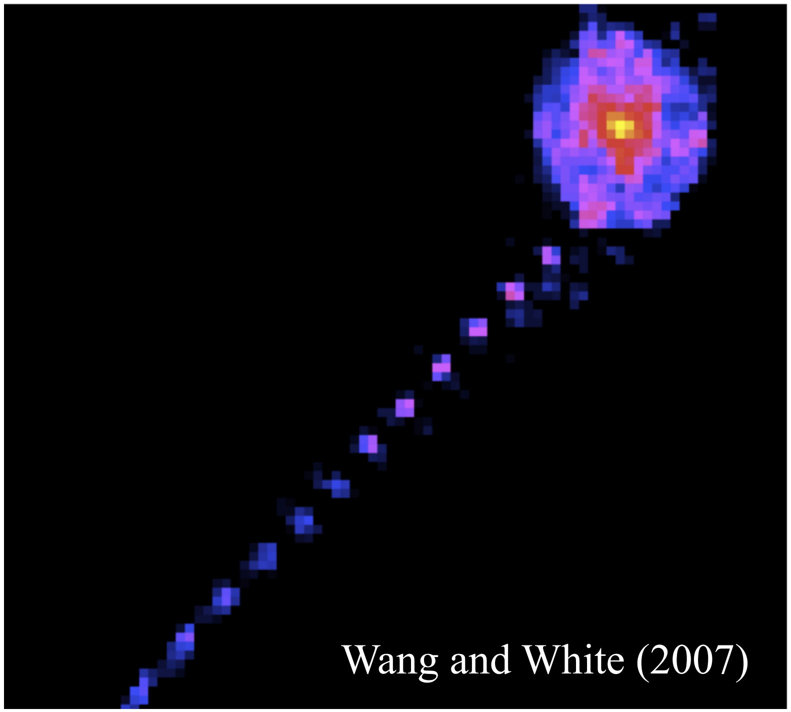
- ICs with WDM power spectrum
- No thermal velocities
- m = {0.25, 0.5, 1, ∞} keV
- L = {16, 64, 256} Mpc/h
- N = {256^3, 512^3, 1024^3}

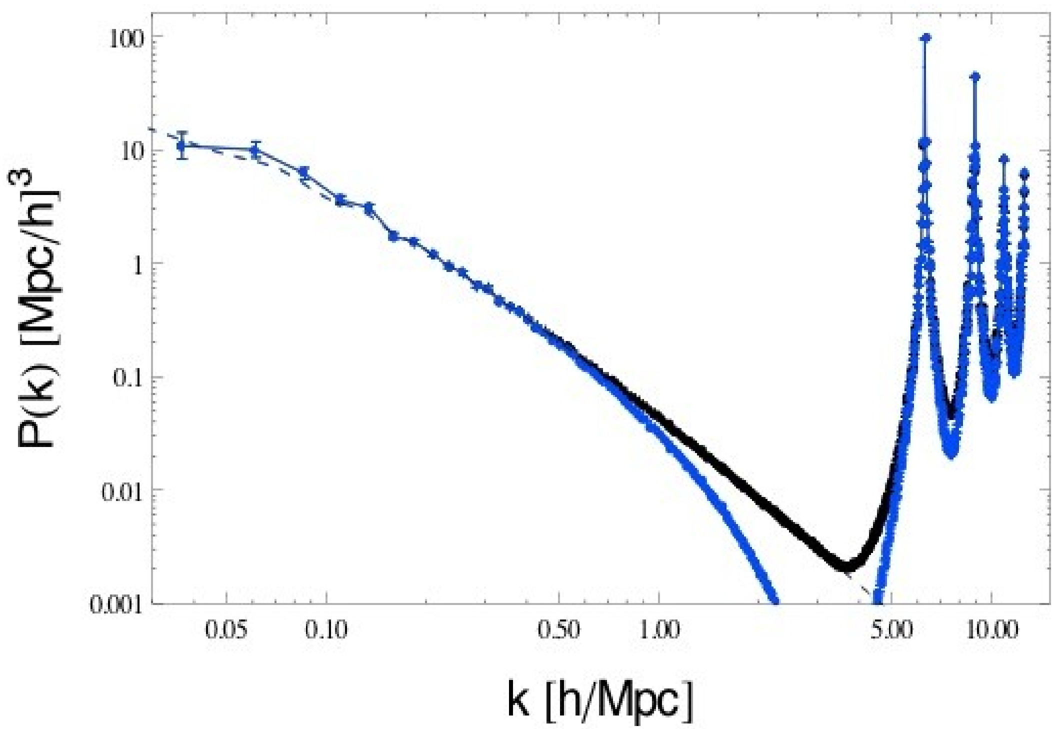


WDM Simulations: Pictures



WDM Simulations: Artifacts

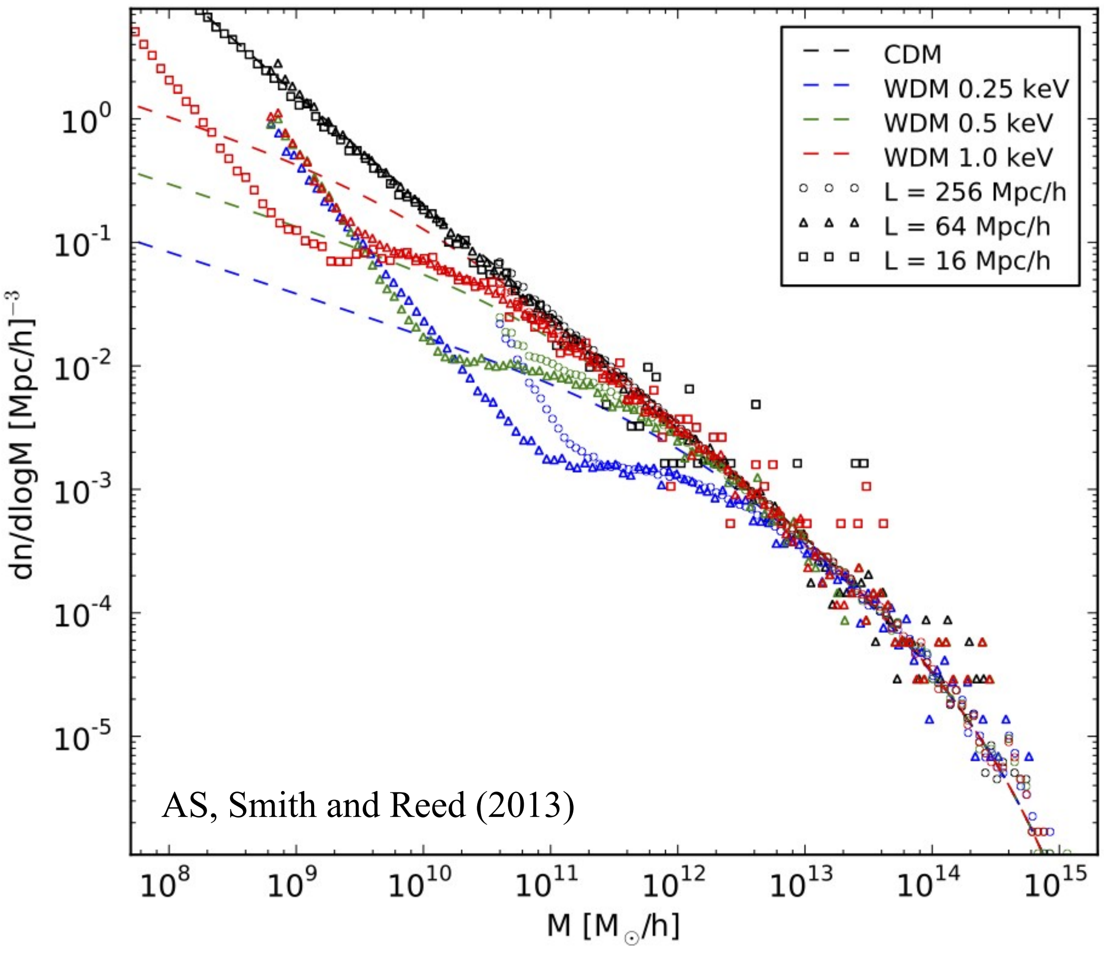




Artificial clumping induced by initial grid.

Simulations do not converge!

WDM Simulations: massfct



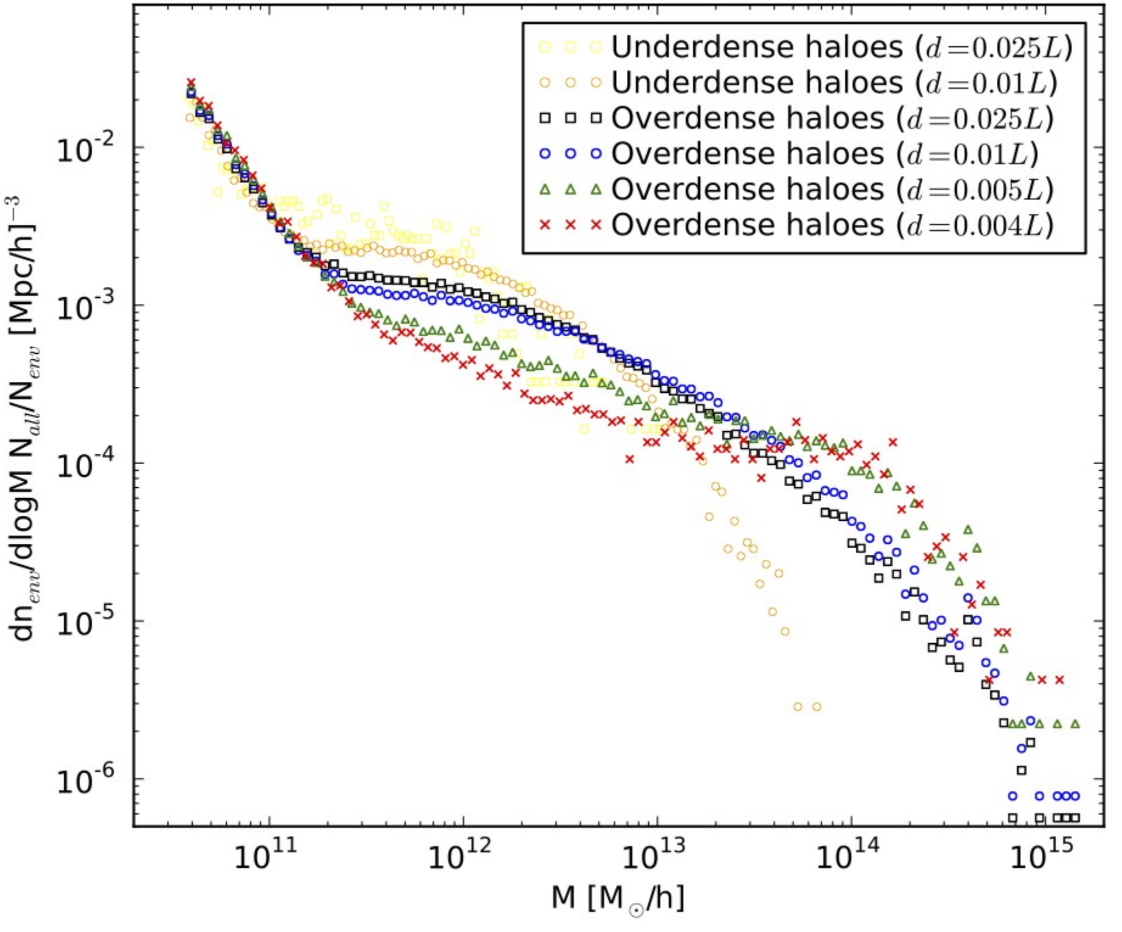
Suppressed WDM mass function.

Artificial upturn (power law).

Resolution ~ $N^{1/3}$.

Can artifacts be removed?

WDM Simulations: conditional massfct



Mass function and environment:

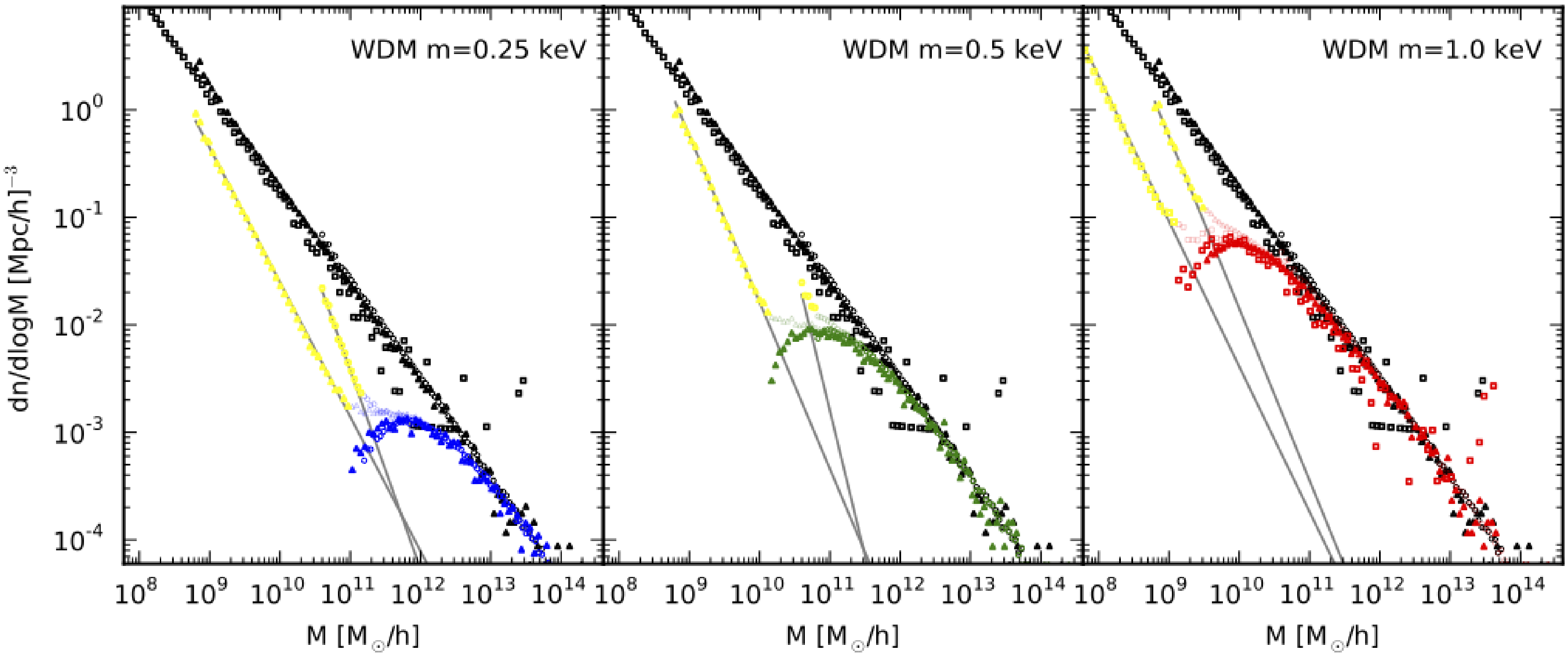
Underdense: no neighbour within distance *d*.

Overdense: at least one neighbour within *d*.

Upturn is shifted to smaller masses in underdense environemnts.

⟶ Artificial upturn is independent of environment!

WDM Simulations: corrected massfct



Subtracting artificial power law.
Mass function turns over!
Roughly convergent.

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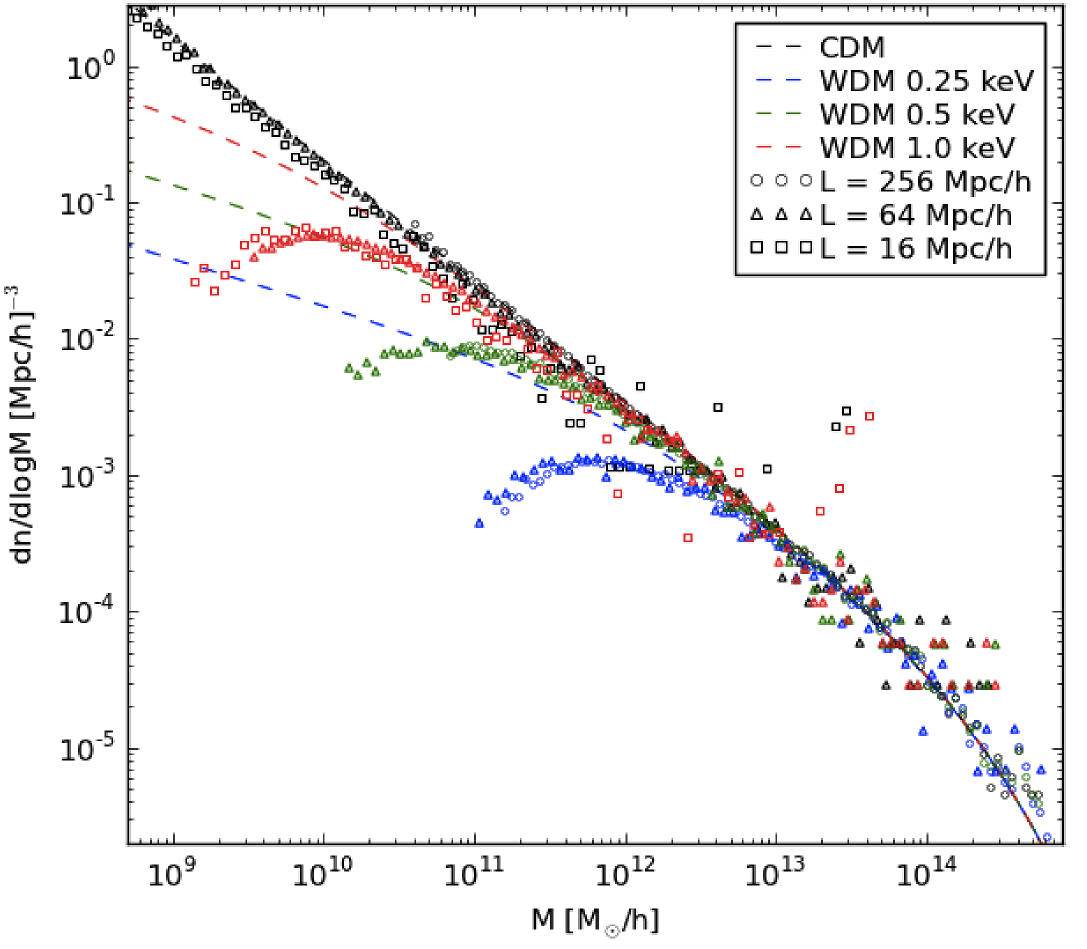
Mass Function: Sheth-Tormen model

$$\frac{dn}{d\log M} = -\frac{1}{2}\frac{\bar{\rho}}{M}f(\nu)\frac{d\log\sigma^2}{d\log M}$$

$$f(\nu) = A\sqrt{\frac{2q\nu}{\pi}}\left[1+(q\nu)^{-p}\right]e^{-q\nu/2},$$

$$\sigma^2(R) = \int \frac{d\mathbf{k}^3}{(2\pi)^3}P_{\rm Lin}(k)W^2(kR)$$

$$\nu = \frac{\delta_c^2}{\sigma^2(M)}$$



Choice of window function:

Tophat: $W_{\rm TH} = 3\left[\sin y - y\cos y\right]/y$ $\qquad \lim_{M\to 0}\frac{dn}{d\log M} \propto M^{-1/3} = \infty$

Sharp-k: $W_{\rm SK} = \Theta(1-y) \qquad y = kR$ $\qquad \lim_{M\to 0}\frac{dn}{d\log M} \propto M^{6-n/3} = 0$

Mass Function: Sharp-k model

$$\frac{dn}{d\log M} = -\frac{1}{2}\frac{\bar{\rho}}{M} f(\nu) \frac{d\log\sigma^2}{d\log M}$$

$$f(\nu) = A\sqrt{\frac{2q\nu}{\pi}}\left[1 + (q\nu)^{-p}\right] e^{-q\nu/2},$$

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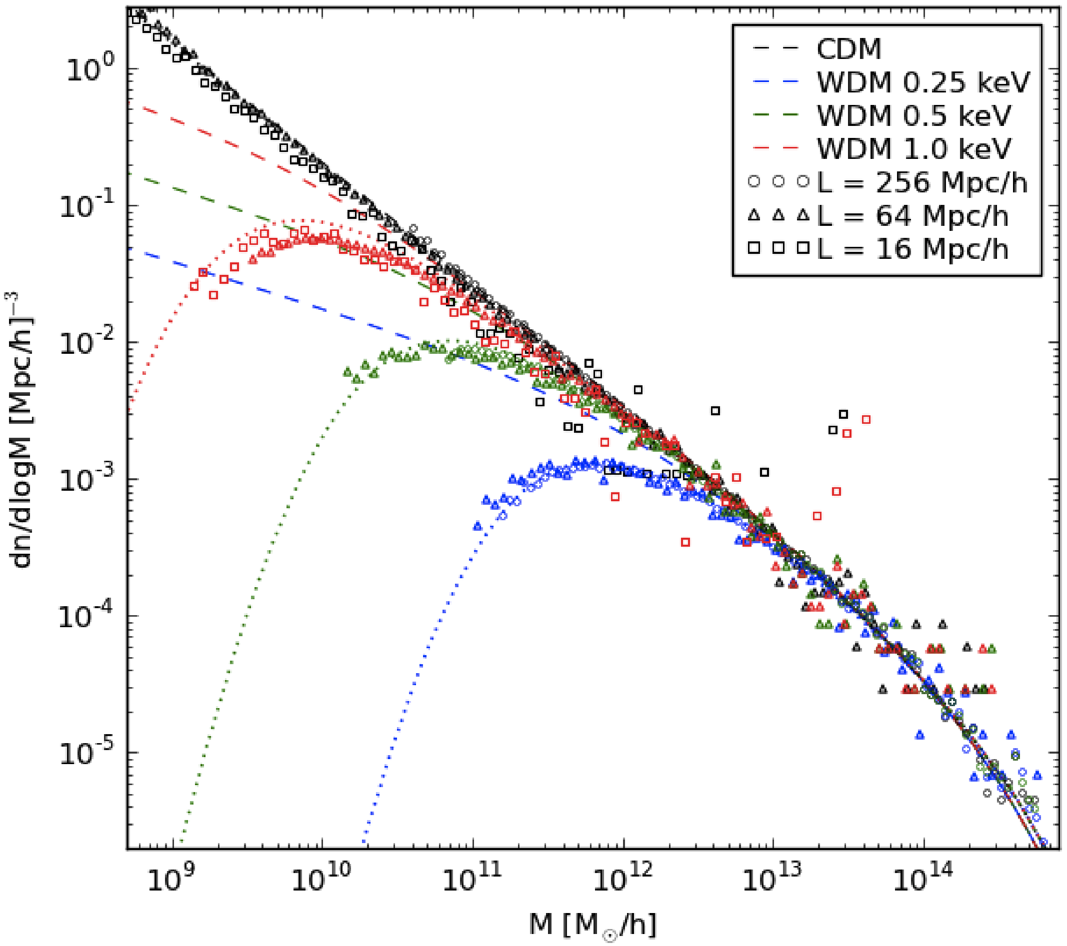

Parameters:

Tophat: $A = 0.322, \quad p = 0.3, \quad q = 0.707.$ $\qquad M = \frac{4\pi}{3}\bar{\rho}R^3$

Sharp-k: $A = 0.322, \quad p = 0.3, \quad q = 1.0.$ $\qquad M = \frac{4\pi}{3}\bar{\rho}[cR]^3, \quad c = 2.7$

Mass Function: Sharp-k model

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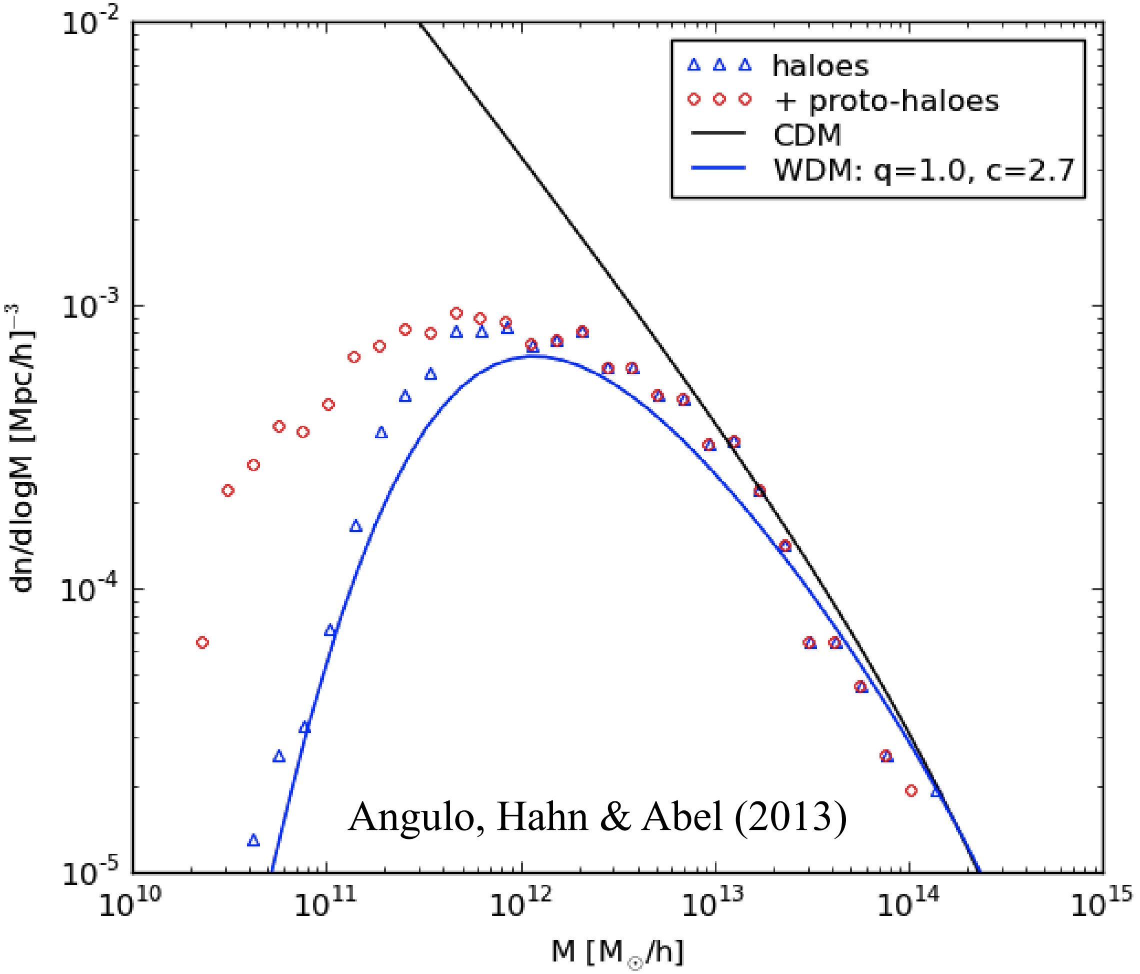

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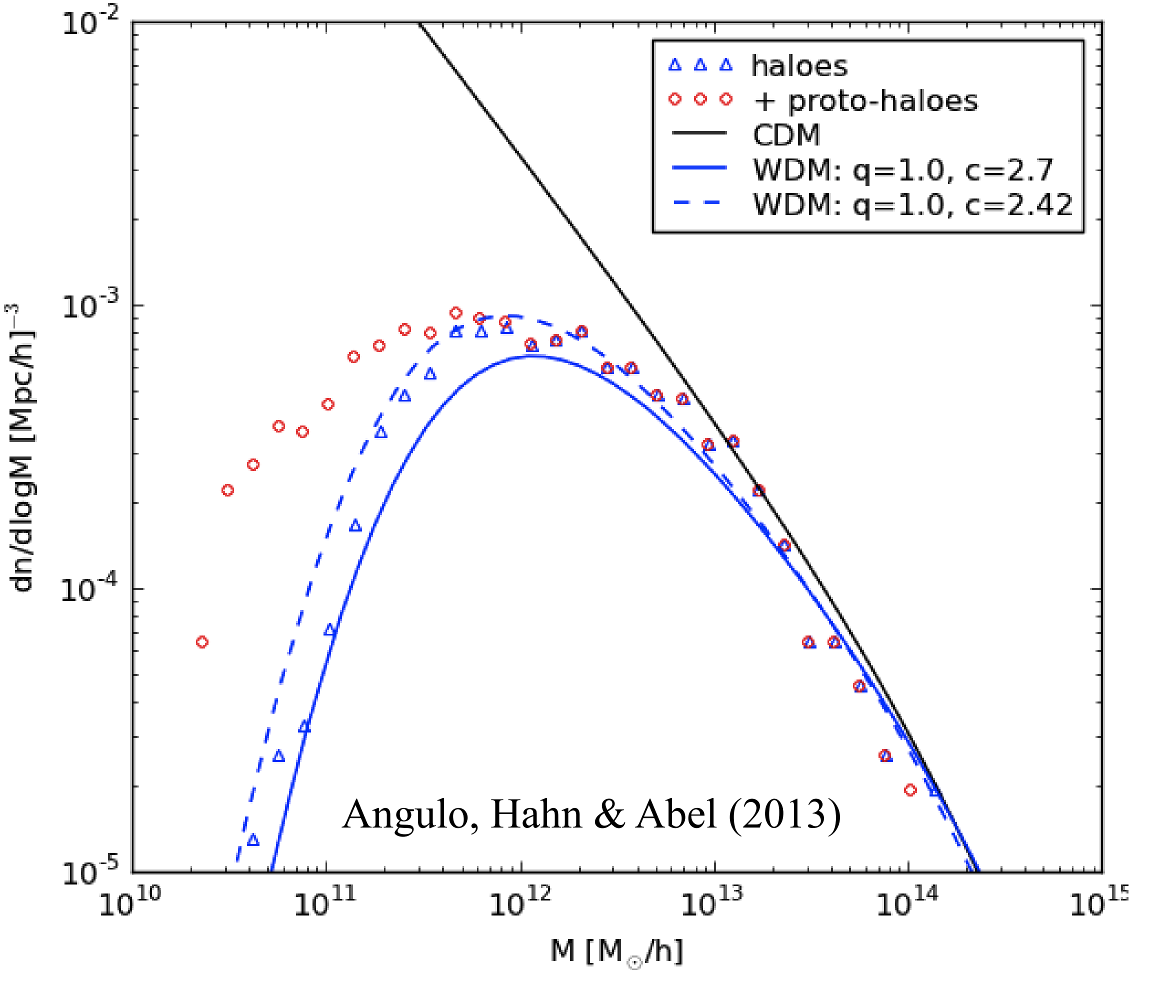

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Mass Function: Sharp-k model

Problem solved ?

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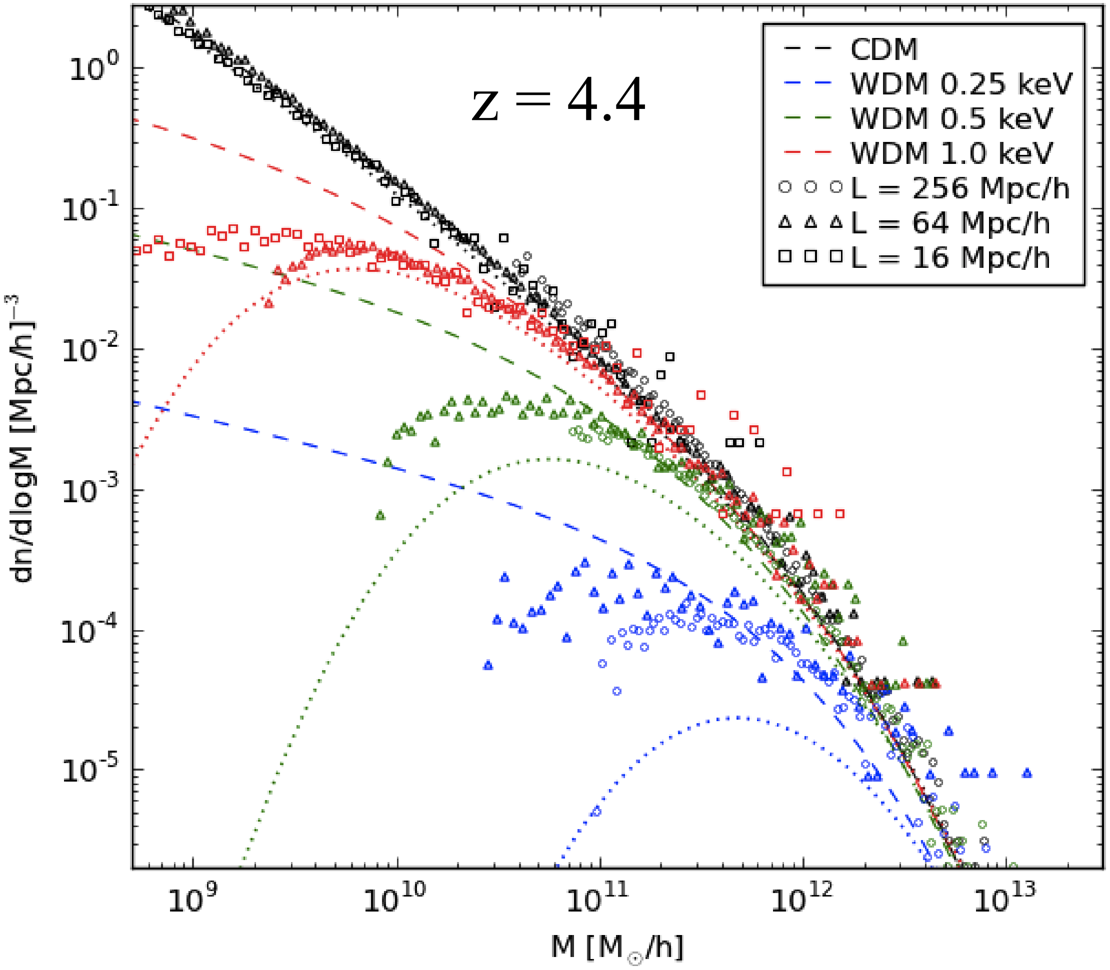


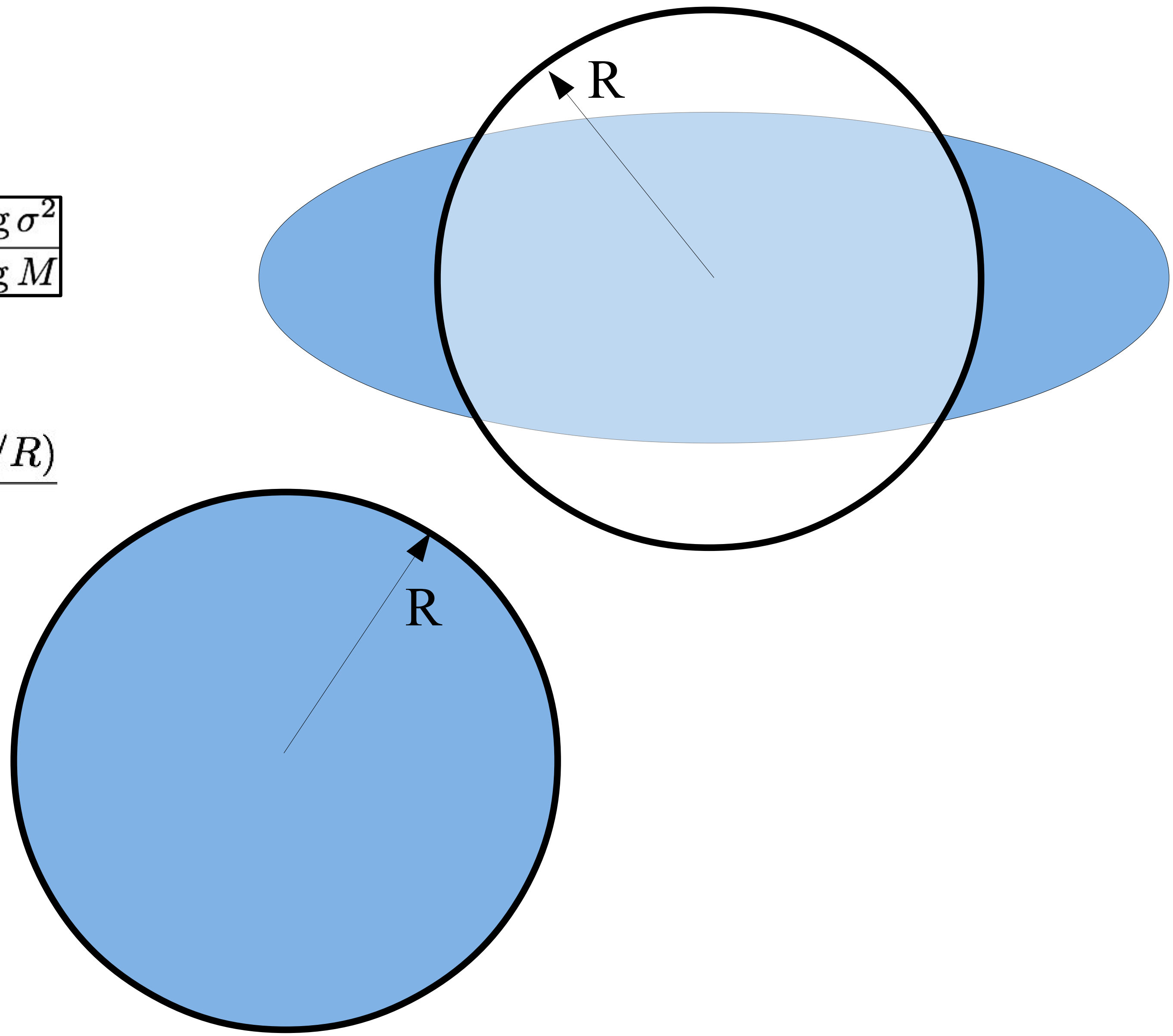
...not at high redshift!

Mass Function: ellipsoidal correction

$$\boxed{\frac{dn}{d\log M} = -\frac{1}{2}\frac{\bar{\rho}}{M}f(\nu)\frac{d\log\sigma^2}{d\log M}}$$

$$\frac{d\log\sigma^2}{d\log M} = \frac{1}{6\pi^2\sigma^2}\frac{P_{\rm Lin}(1/R)}{R^3}$$

$$M = \frac{4\pi}{3}\bar{\rho}[cR]^3$$



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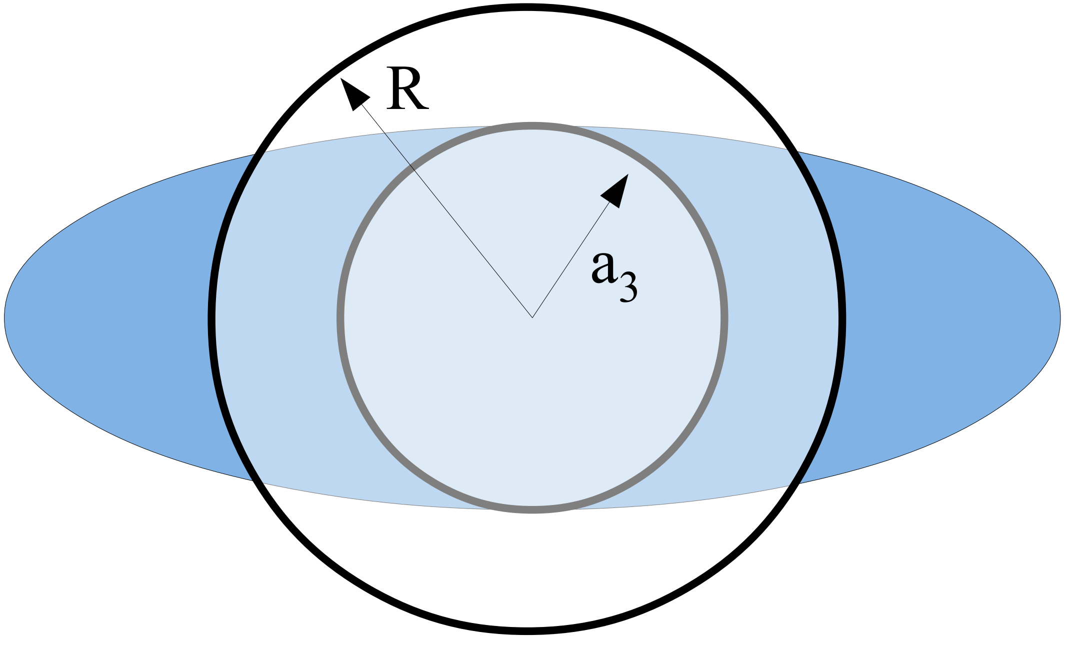
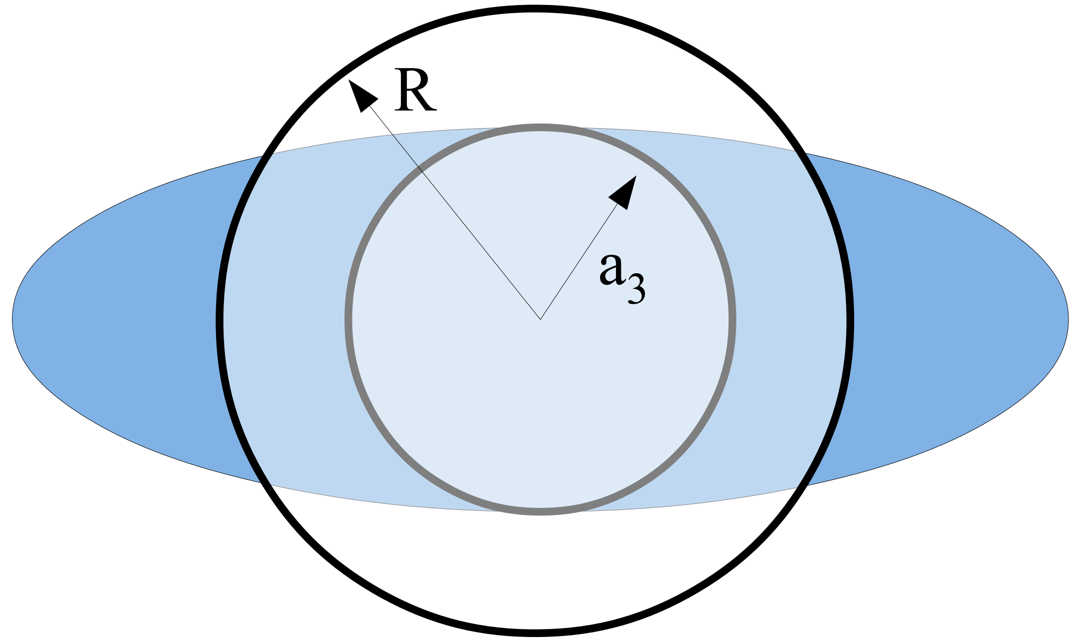


$$\frac{d\log\sigma^2}{d\log M} = \frac{1}{6\pi^2\sigma^2}\frac{P_{\rm Lin}(1/a_3)}{a_3^3}\frac{da_3}{dR}\xi(R)$$

$$R^3 = a_1a_2a_3 = \left(\frac{a_1}{a_3}\frac{a_2}{a_3}\right)a_3 = (\xi a_3)^3, \qquad a_3(R) = \frac{R}{\xi(R)} \qquad M = \frac{4\pi}{3}\bar{\rho}\left[cR(a_3)\right]^3$$

Mass Function: ellipsoidal correction

$$\boxed{\frac{dn}{d\log M} = -\frac{1}{2}\frac{\bar{\rho}}{M} f(\nu) \frac{d\log\sigma^2}{d\log M}}$$



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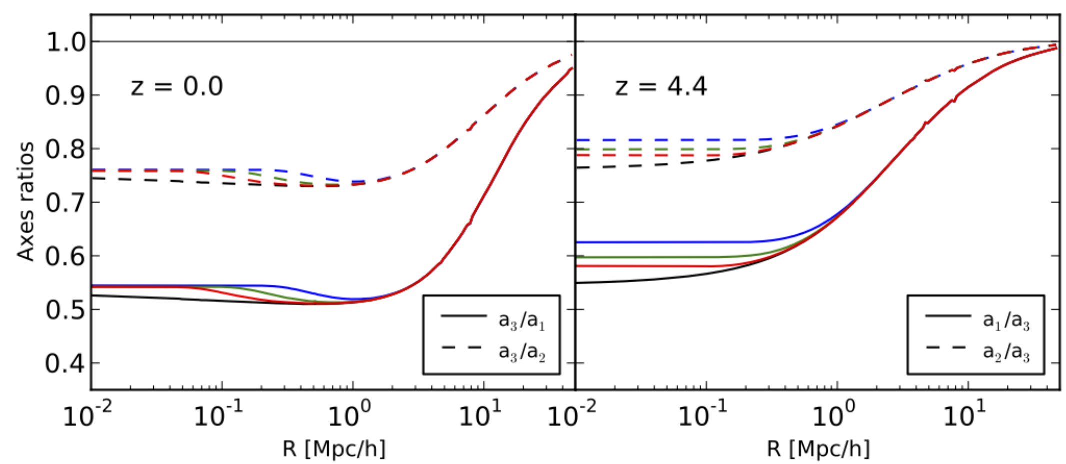
$$R^3 = a_1 a_2 a_3 = \left(\frac{a_1}{a_3}\frac{a_2}{a_3}\right) a_3 = (\xi a_3)^3$$

Shape of initial patches depend on scale and redshift (Bardeen et al 1986)



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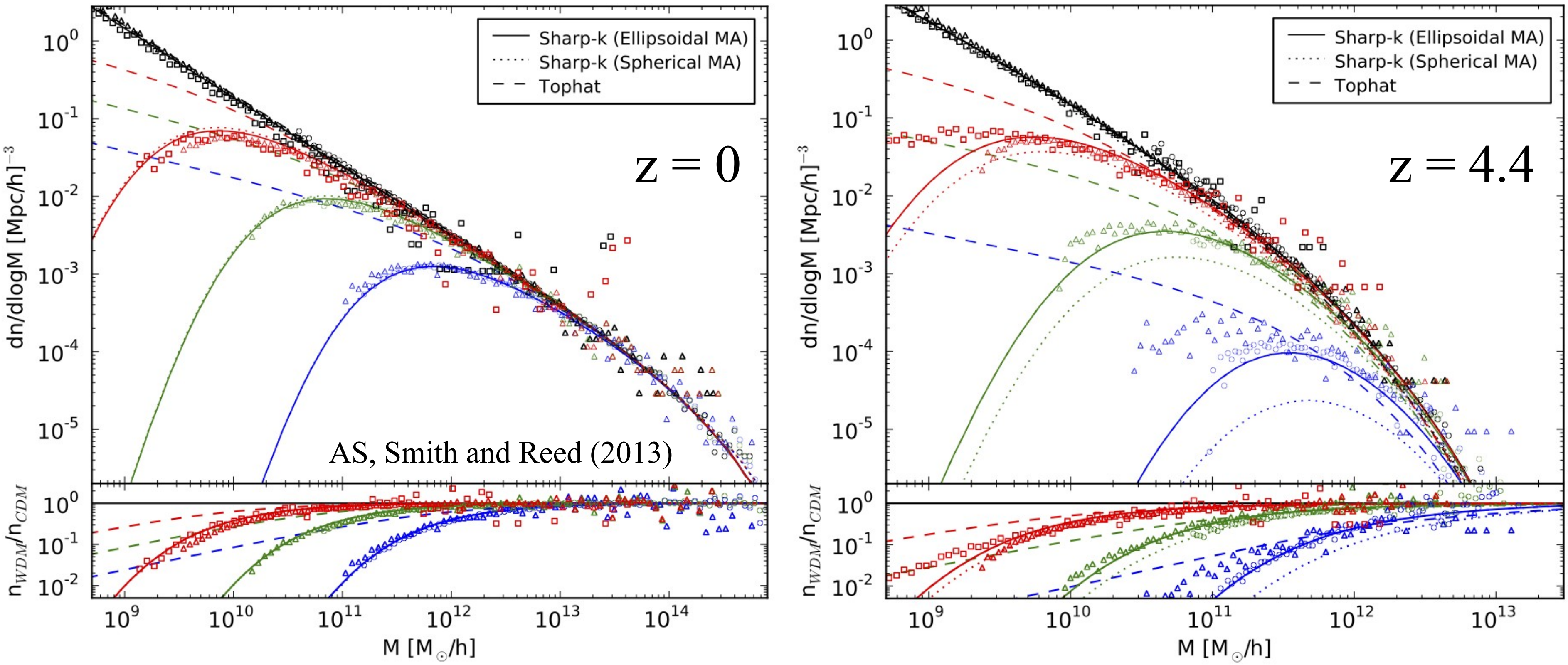

- Spherical sharp-k: $A = 0.322, \quad p = 0.3, \quad q = 1.0.$ $\qquad M = \frac{4\pi}{3}\bar{\rho}[cR]^3, \quad c = 2.7$
- Ellipsoidal sharp-k: $A = 0.322, \quad p = 0.3, \quad q = 0.75$ $\qquad M = \frac{4\pi}{3}\bar{\rho}\,[cR(a_3)]^3\,, \quad c = 2.0$

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Cold Dark Matter: comparison

Does model work for neutralino-CDM?

- Free streaming scale: 10^{15} times smaller!
- Different cutoff

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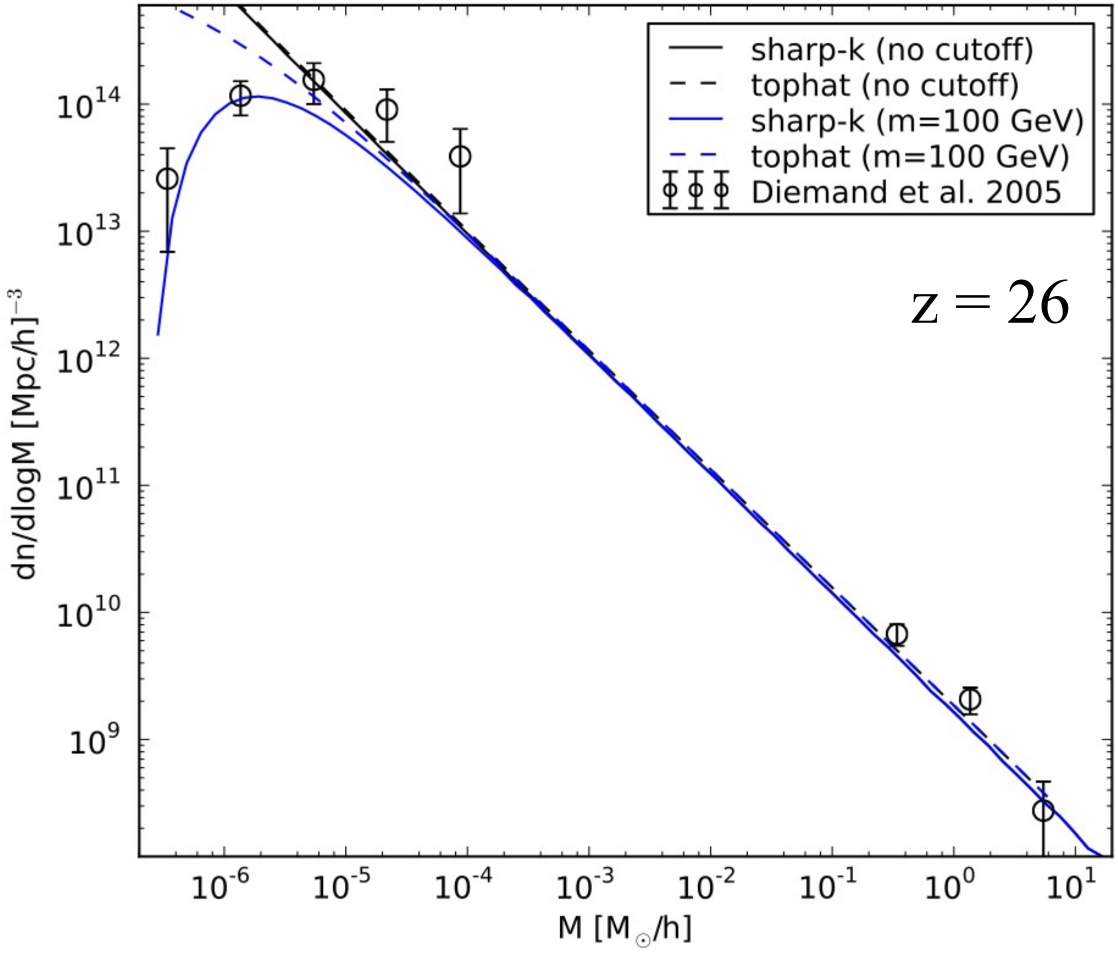
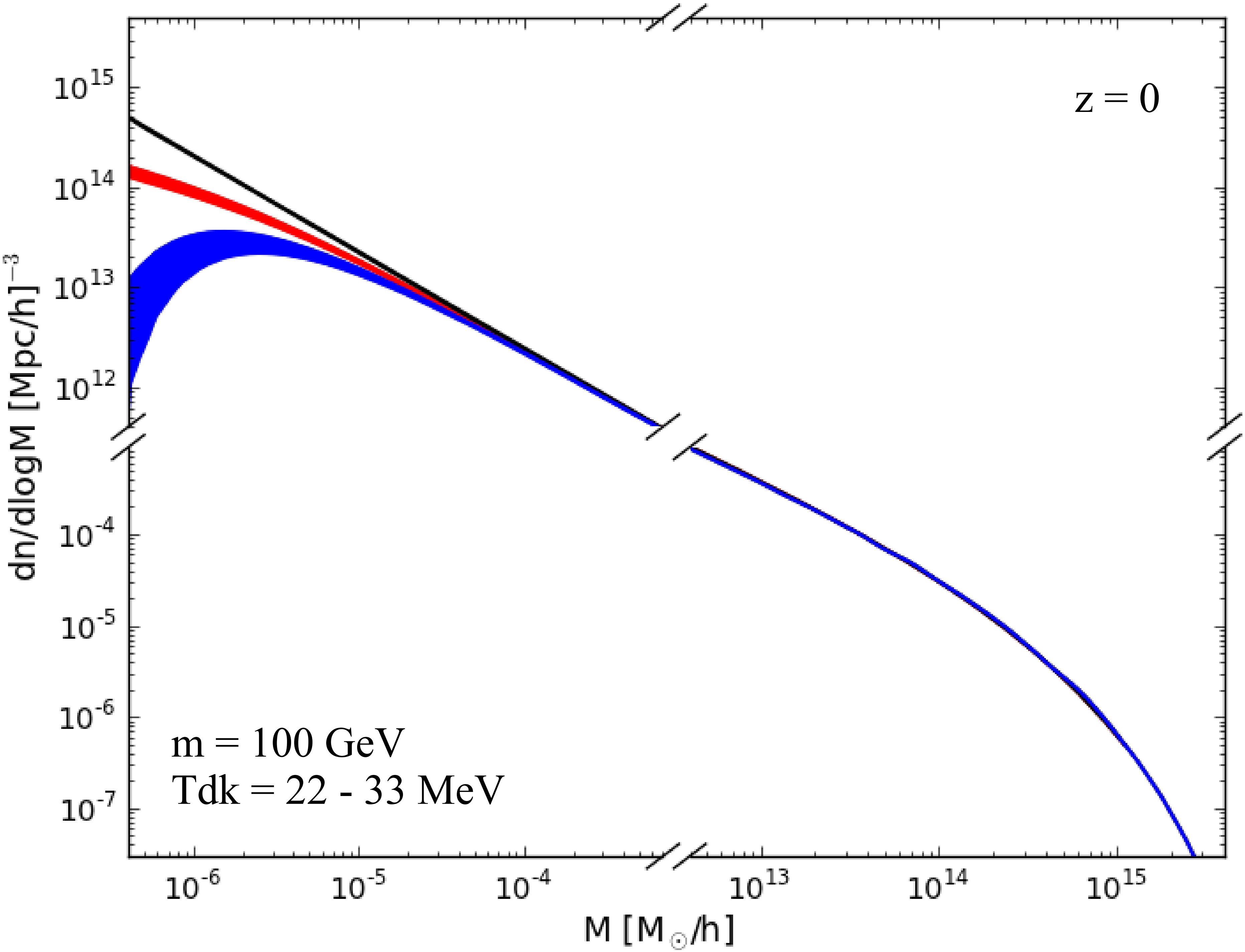


Diemand et al. (2006):
m = 100 GeV, T_{dk} = 28 MeV



z = 0
m = 100 GeV
Tdk = 22 - 33 MeV
dn/dlogM [Mpc/h]$^{-3}$
M [M$_\odot$/h]
10^{15}
10^{14}
10^{13}
10^{12}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{13}
10^{14}
10^{15}

Conclusions:

- Free streaming suppresses small scale halo abundance
- Method to subtract artificial haloes
- Sharp-k mass function with one free parameter
- Ellipticity corrected window for high redshift
- Prediction of neutralino-CDM mass function.

Thank you!