

# Numerical Simulations of Structure Formation in Warm Dark Matter Cosmologies

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## Motivation

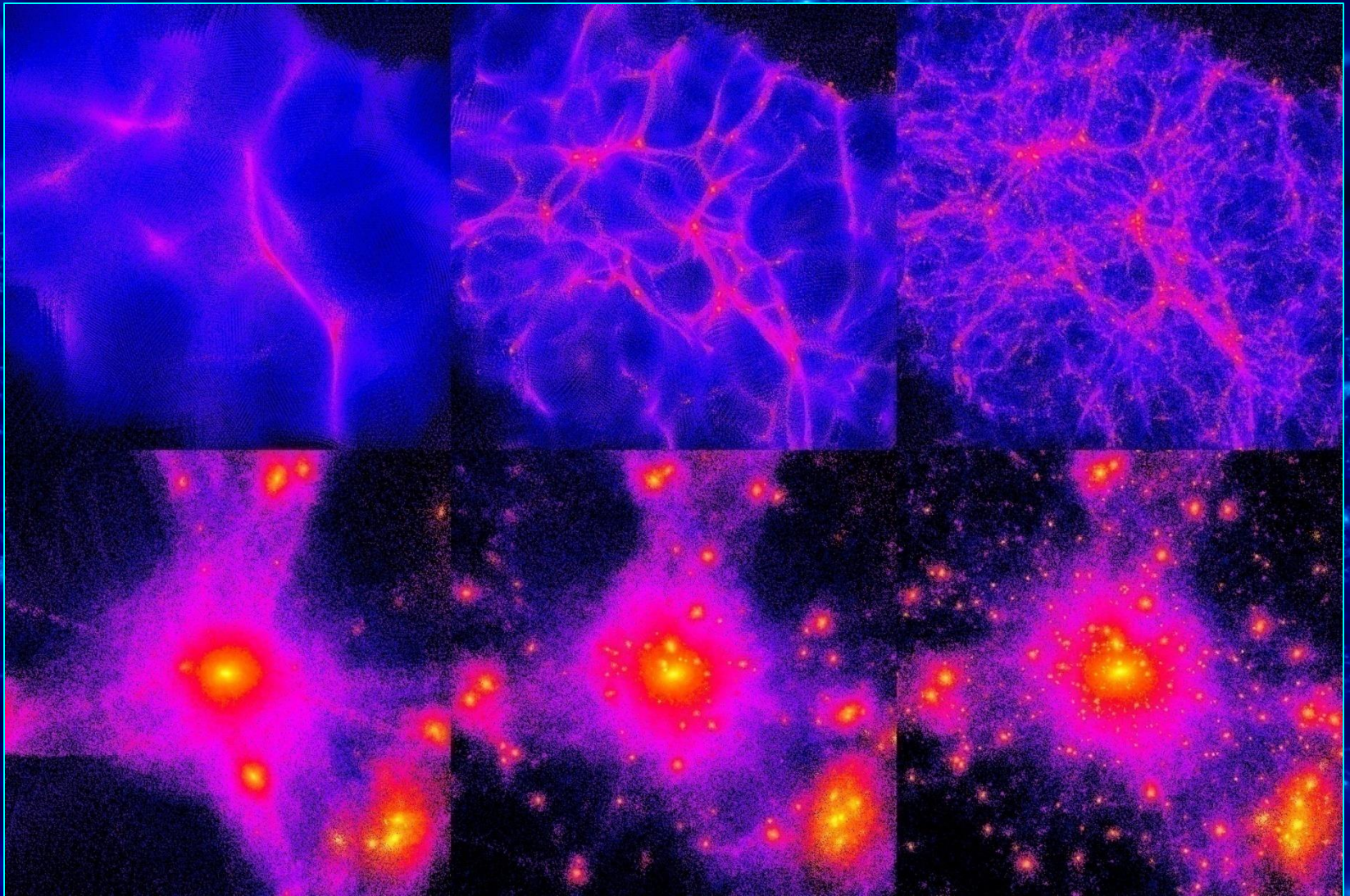
- $\Lambda$ CDM fails to explain observed properties of galaxies and cluster of galaxies
- Missing Satellites Problem
- Cores vs Cusps
- Pure Disk Galaxies
- Merger-free galaxies
- Where is the WIMP?

# Motivation

- Warm dark matter (WDM) provides the best alternative to cold dark matter (CDM) candidates since it can be tested with astrophysical observations on small scales where the CDM model is challenged.
- WDM has a non-negligible velocity dispersion which dampens the small scale fluctuation spectrum and sets a phase space limit to cosmic structures.
- More recently we have seen renewed interest in warm dark matter since a candidate may occur naturally within extensions to the standard model of particle physics.
- The sterile neutrino can explain some key physical phenomena including neutrino oscillations, the dark matter and the baryon asymmetry of the universe.

# Simulating the Warm Dark Matter - The Challenges

- How to treat the particles?
- How to cut the power spectrum?
- What about velocities ? The impact of velocity dispersion.
- How to compare WDM sims with CDM sims and observations
- Resolution and softening
- Fragmentation
- Structure formation and its hidden treasures
- Halo internal structure
- The trustworthy factor and the catch 22

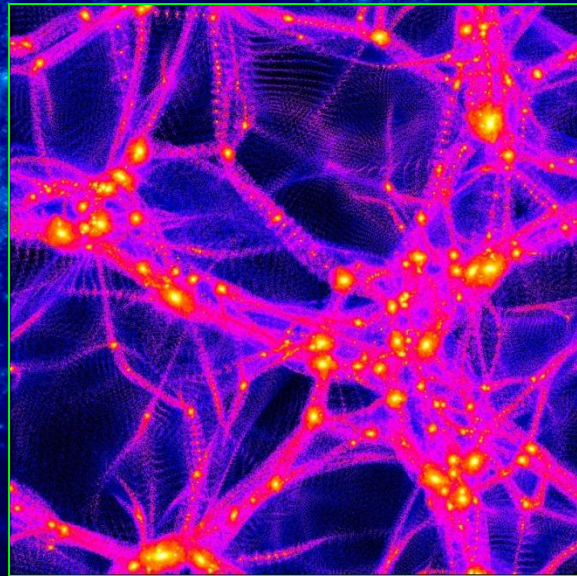
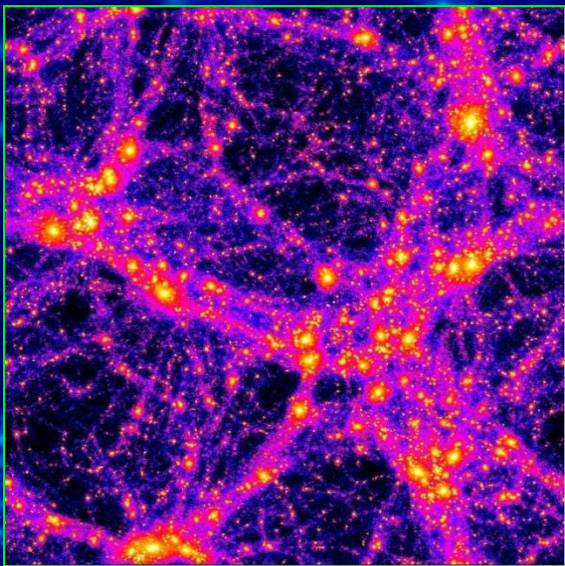


$\langle 1 \text{ keV DM}$

$\rangle 1 \text{ keV DM}$

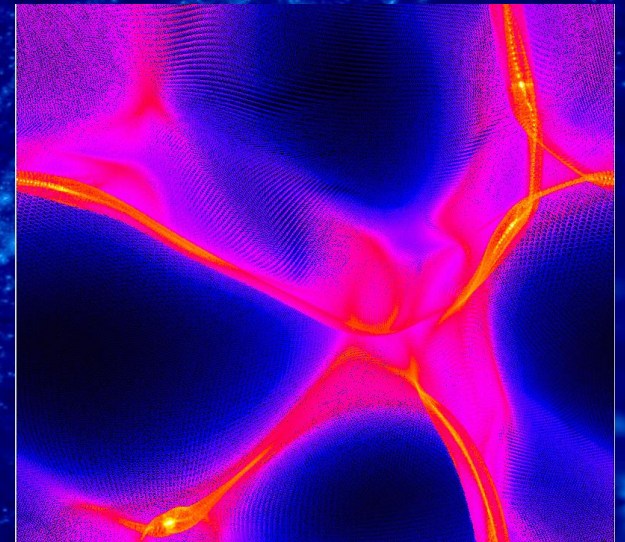
CDM

CDM



200eV power spectrum cutoff  
20eV velocities

50eV power spectrum cutoff  
50eV velocities



# Simulating the WDM...

$$P(k) = k^n * T^2(k, z) \quad n \sim -1$$

$$k_s \approx \left( \frac{0.3}{\Omega_X} \right)^{0.15} \left( \frac{m_X}{keV} \right)^{1.15} Mpc^{-1}$$

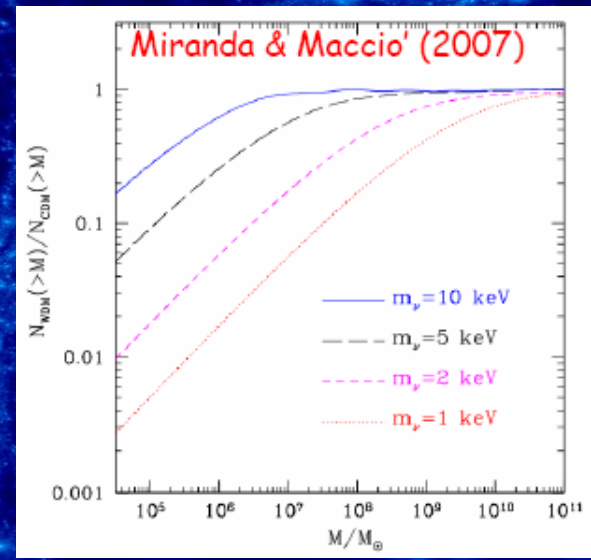
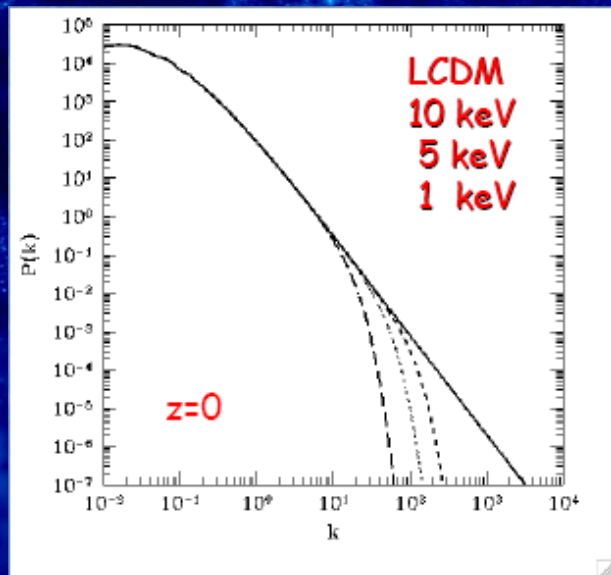
$$T^2(k) = \frac{P^{WDM}}{P^{CDM}} = [1 + (\alpha k)^{2\nu}]^{-10/\nu}$$

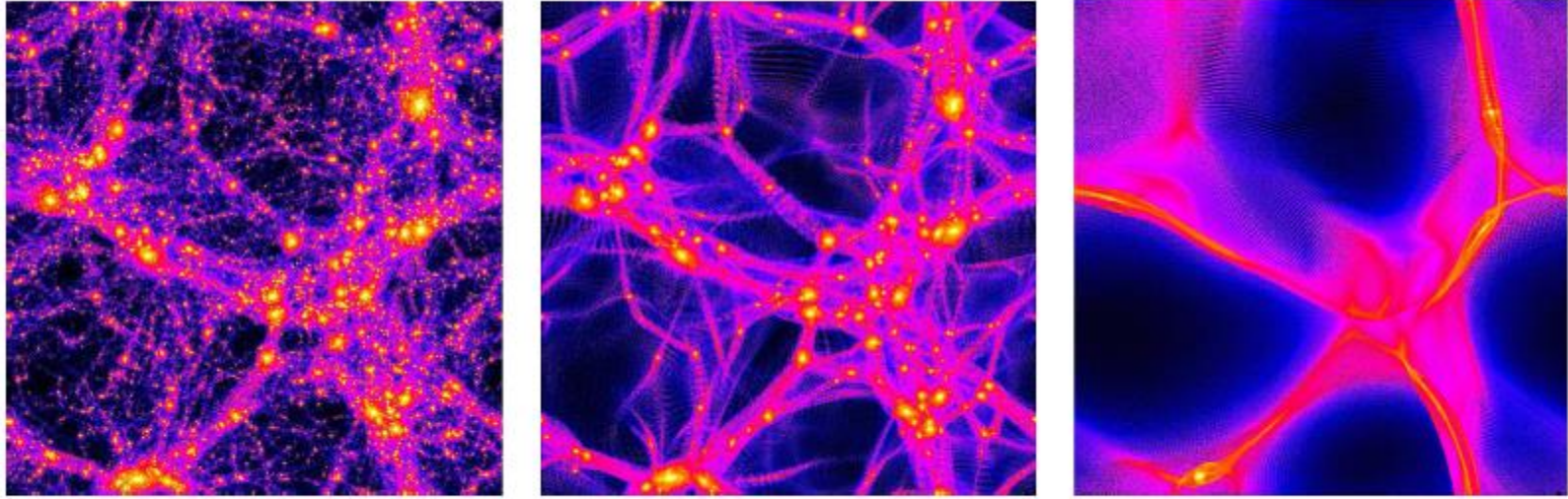
$$\alpha = 0.049 \cdot \left( \frac{m_x}{1keV} \right)^{-1.11} \cdot \left( \frac{\Omega_\nu}{0.25} \right)^{0.11} \cdot \left( \frac{h}{0.7} \right)^{1.22} h^{-1} Mpc.$$

Bode, Turok, and Ostriker (2001)

Viel et. Al 2005

$$\frac{v_0(z)}{1+z} = .012 \left( \frac{\Omega_X}{0.3} \right)^{\frac{1}{3}} \left( \frac{h}{0.65} \right)^{\frac{2}{3}} \left( \frac{1.5}{g_X} \right)^{\frac{1}{3}} \left( \frac{keV}{m_X} \right)^{\frac{4}{3}} km s^{-1}$$





**Figure 1.** Three snapshots of different simulations at redshift  $z = 0$ . CDM, WDM3 and WDM4 are shown from left to right.

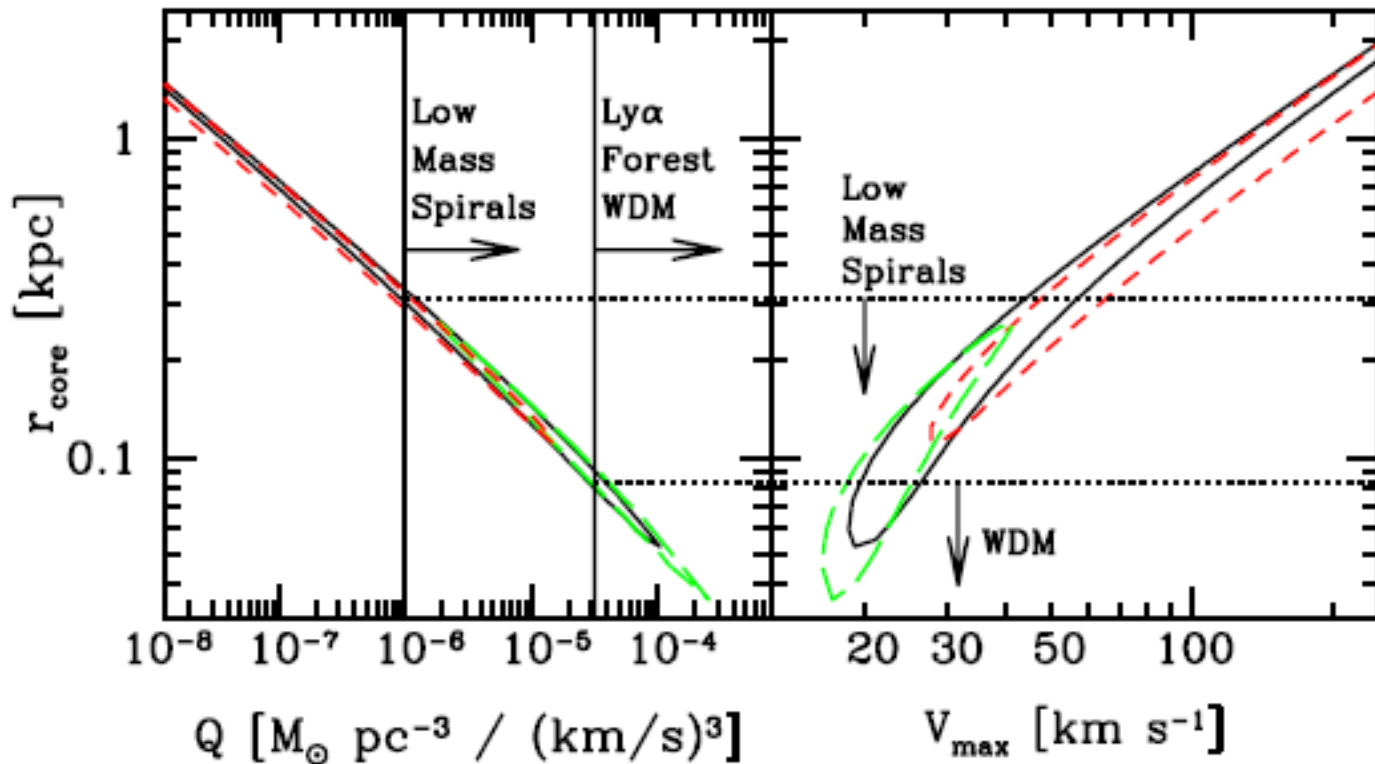
**Table 1.** Details of the simulations

Label	particle mass	velocities	box size	no.of particles	softening ( $r_{200}$ )	halo mass	$r_{200}$ (kpc)	$N(< r_{200})$
CDM	-	no	40 Mpc	$160^3$	$2.6 \times 10^{-3}$	$7 \times 10^{11}$	160	$3.6 \times 10^6$
WDM1	200 eV	no	40 Mpc	$160^3$	$2.6 \times 10^{-3}$	$7 \times 10^{11}$	140	$2.7 \times 10^6$
WDM2	200 eV	100 eV	40 Mpc	$160^3$	$2.6 \times 10^{-3}$	$7 \times 10^{11}$	140	$1.7 \times 10^6$
WDM3	200 eV	20 eV	40 Mpc	$160^3$	$2.6 \times 10^{-3}$	$7 \times 10^{11}$	132	$2.7 \times 10^6$
WDM4	50 eV	no	40 Mpc	$160^3$	$2.6 \times 10^{-3}$	-	-	-
WDM5	200 eV	no	42.51 Mpc	$300^3$	$0.66 \times 10^{-3}$	$10^{13}$	425	$18.67 \times 10^6$
WDM6	200 eV	200 eV	42.51 Mpc	$300^3$	$0.66 \times 10^{-3}$	$10^{13}$	425	$18.66 \times 10^6$



## Assumptions in determining the core radius:

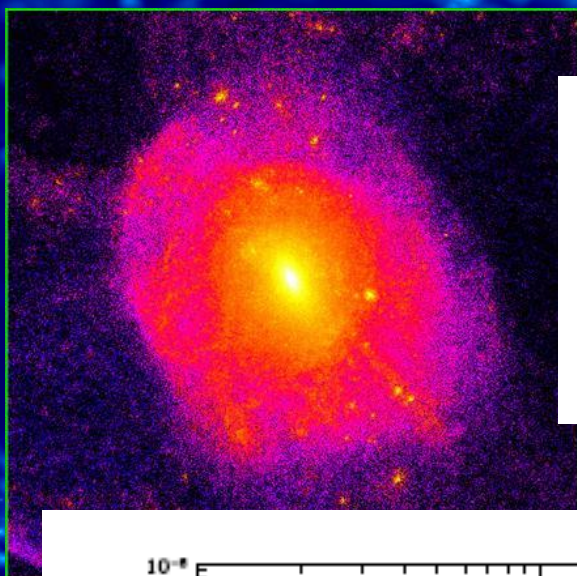
- ⌋ Isothermal spheres
- ⌋ Liouville - Phase space density (PSD) is conserved
- ⌋ Pauli exclusion principle
- ⌋ PSD constant as mixing occurs
- ⌋ Velocity dispersion in central halo = constant
- ⌋ Density profile in central halo = constant



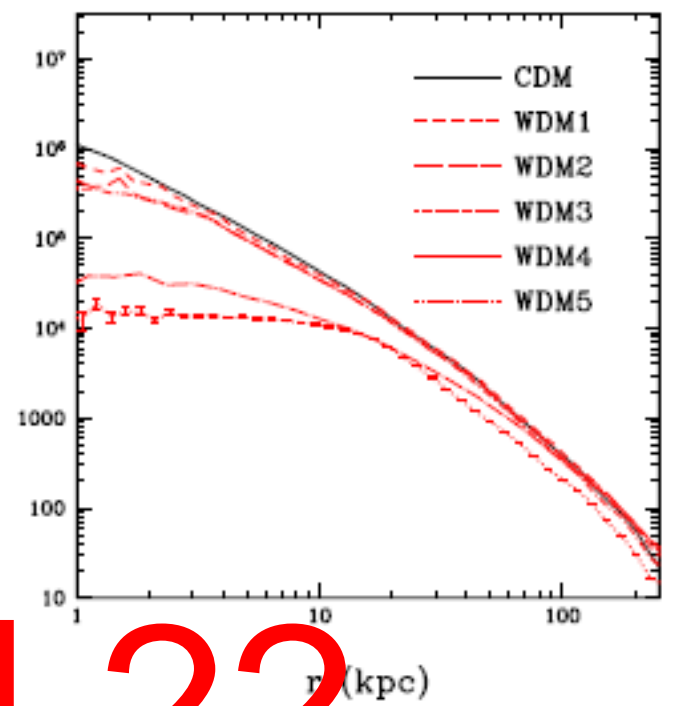
Constraints on the core radius of Fornax as a function of the central phase-space density and maximum circular velocity derived from the velocity dispersion profile

Strigari et al 2006

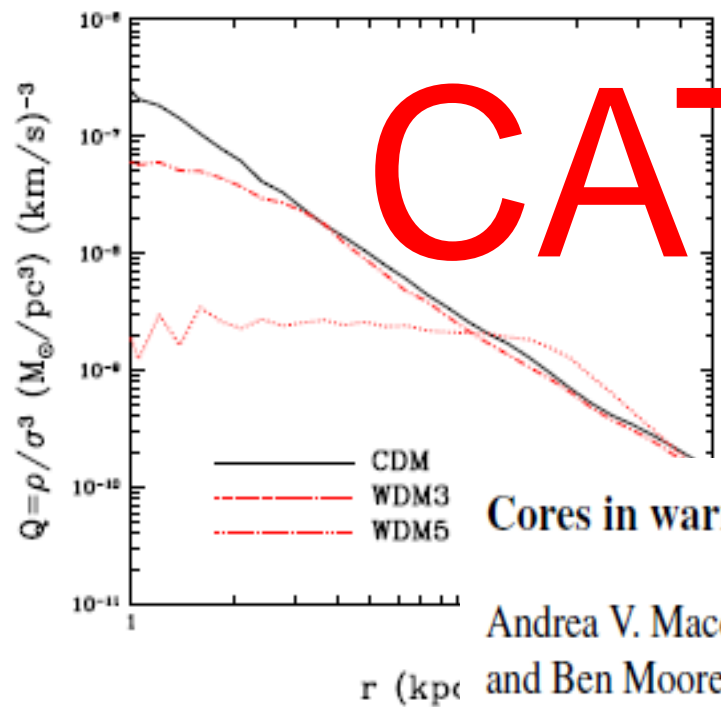
$$\rho(r) = \frac{\rho_0}{[1 + (r/r_0)^\alpha]^{3/\alpha}}$$



Label	$m_\nu$ (keV)	$m_{\nu,vel}$ (keV)	$N_{vir}$ ( $10^6$ )	$M_{vir}$ ( $10^{12} M_\odot$ )
CDM	$\infty$	—	10.2	1.42
WDM1	2.0	1.32	8.6	1.22
WDM2	2.0	0.33	8.4	1.20
WDM3	2.0	0.13	8.5	1.21
WDM4	2.0	0.15	6.7	0.93
WDM5-N	2.0	0.05	4.9	0.71
WDM5	2.0	0.03	5.1	0.82



# CATCH 22



## Cores in warm dark matter haloes: a

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Figure 4. Phase space density profile for WDM5 models at  $z = 0$ .

Figure 2. The spherical averaged density profiles for CDM, WDM1-5 haloes.

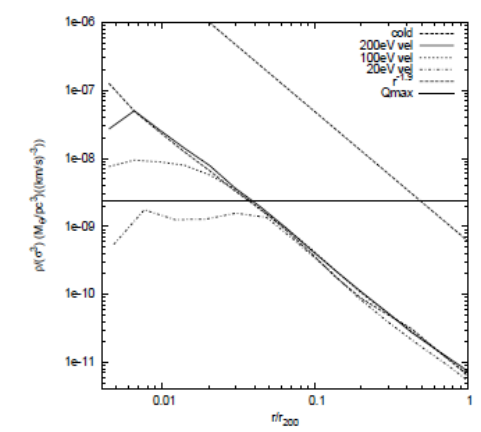
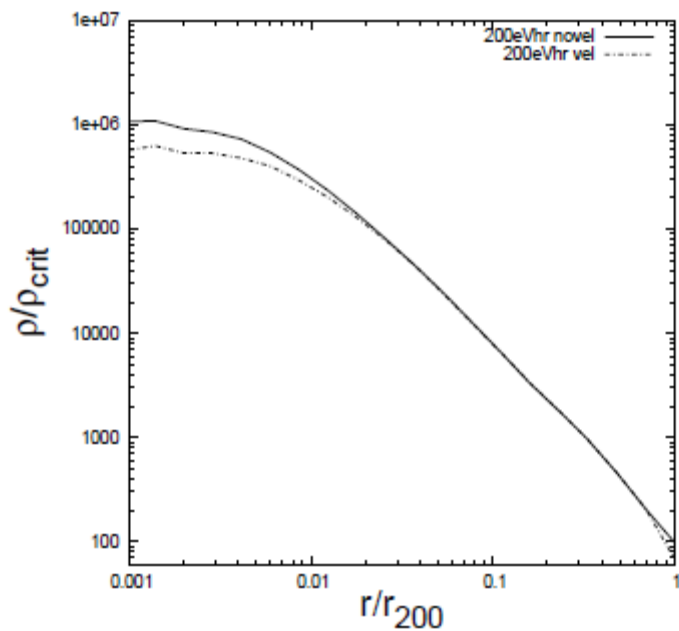
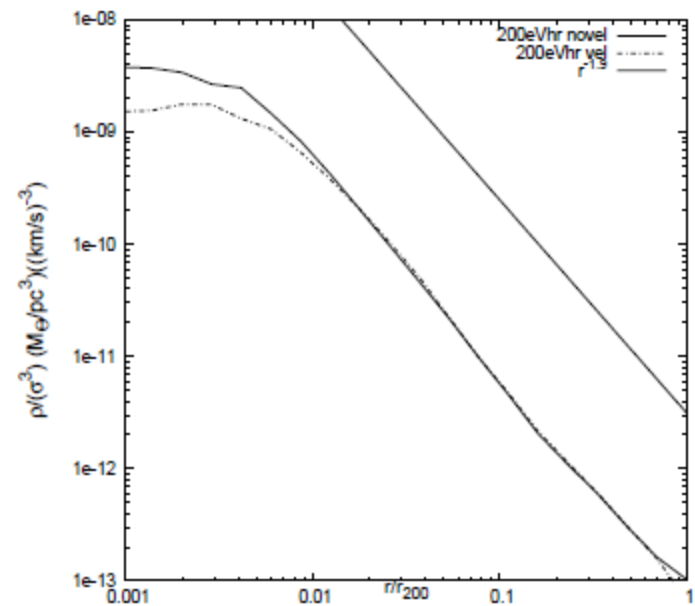


Figure 3. "Phase-space density" (PSD) profiles of the same haloes shown in Figure 1, calculated using  $\rho/\sigma^3$ .

T<sup>3</sup>



**Figure 4.** The density profiles for a high resolution halo in the 200 eV simulation WDM5 and WDM6



**Figure 5.** The PSD profiles for the 200eV halo from WDM5 and WDM6. A power law line of slope  $r^{-1.9}$  is shown for reference.

## How to cook a big core...

Simulations of CDM+gas+AGN feedback+star formation fine-tuned

If one tunes it for solving the missing satellites problem loses the core and vice-versa

Another Catch 22

Unfortunately, proper KeV simulations+baryons haven't been performed yet.

### The formation of disc galaxies in high resolution moving-mesh cosmological simulations

Federico Marinacci<sup>1,2</sup>, Rüdiger Pakmor<sup>1</sup> and Volker Springel<sup>1,2</sup>

<sup>1</sup>Heidelberger Institut für Theoretische Studien, Schloss-Wolfsbrunnengasse 35, 69118 Heidelberg, Germany

<sup>2</sup>Zentrum für Astronomie der Universität Heidelberg, Astronomisches Recheninstitut, Mönchhofstr. 12-14, 69120 Heidelberg, Germany

<<There is no evidence for any dark matter core formation in our simulations, even so they include repeated baryonic outflows by supernova-driven winds and black hole quasar feedback >>

# Fermionic warm dark matter produces galaxy cores in the observed scales because of quantum mechanics

C. Destri<sup>a</sup>, H. J. de Vega<sup>b,c,1</sup>, N. G. Sanchez<sup>c</sup>

<sup>a</sup>*Dipartimento di Fisica G. Occhialini, Università Milano-Bicocca and INFN, sezione di Milano-Bicocca, Piazza della Scienza 3, 20126 Milano, Italia.*

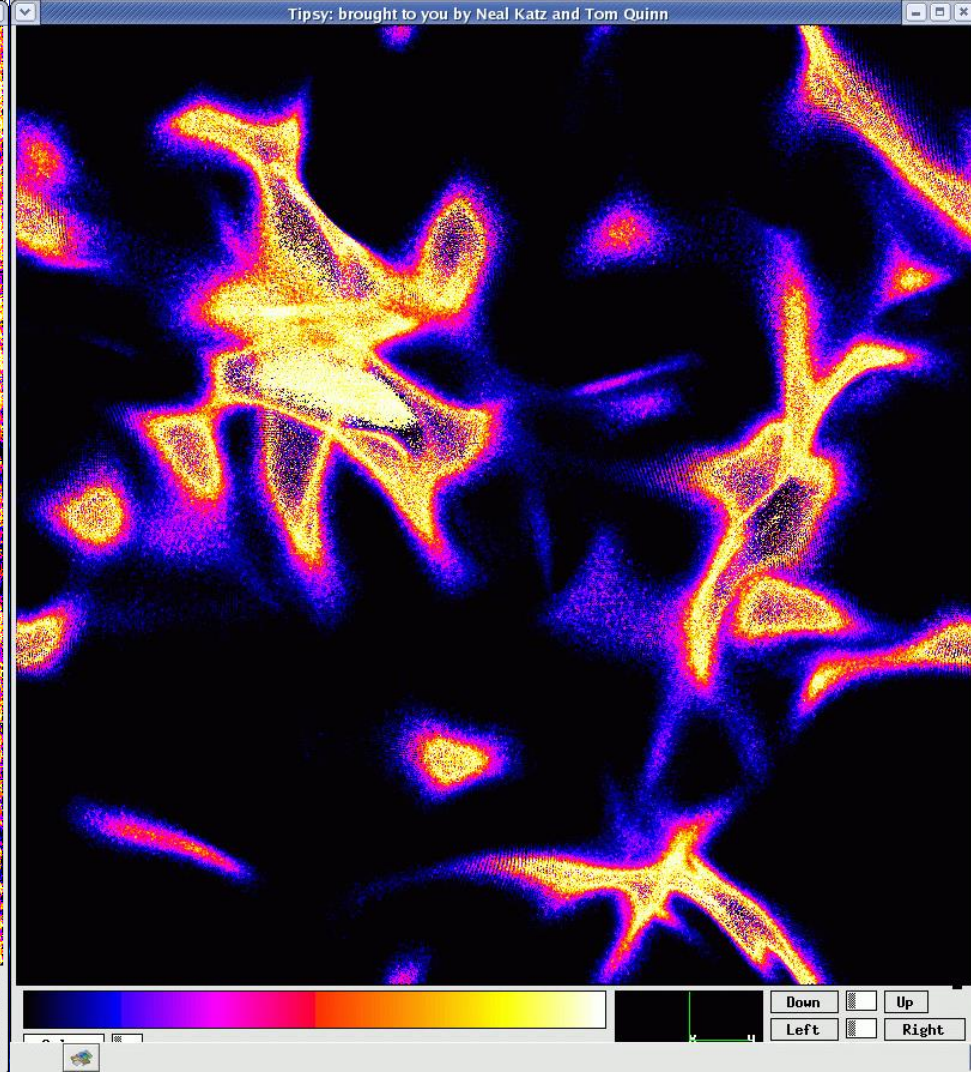
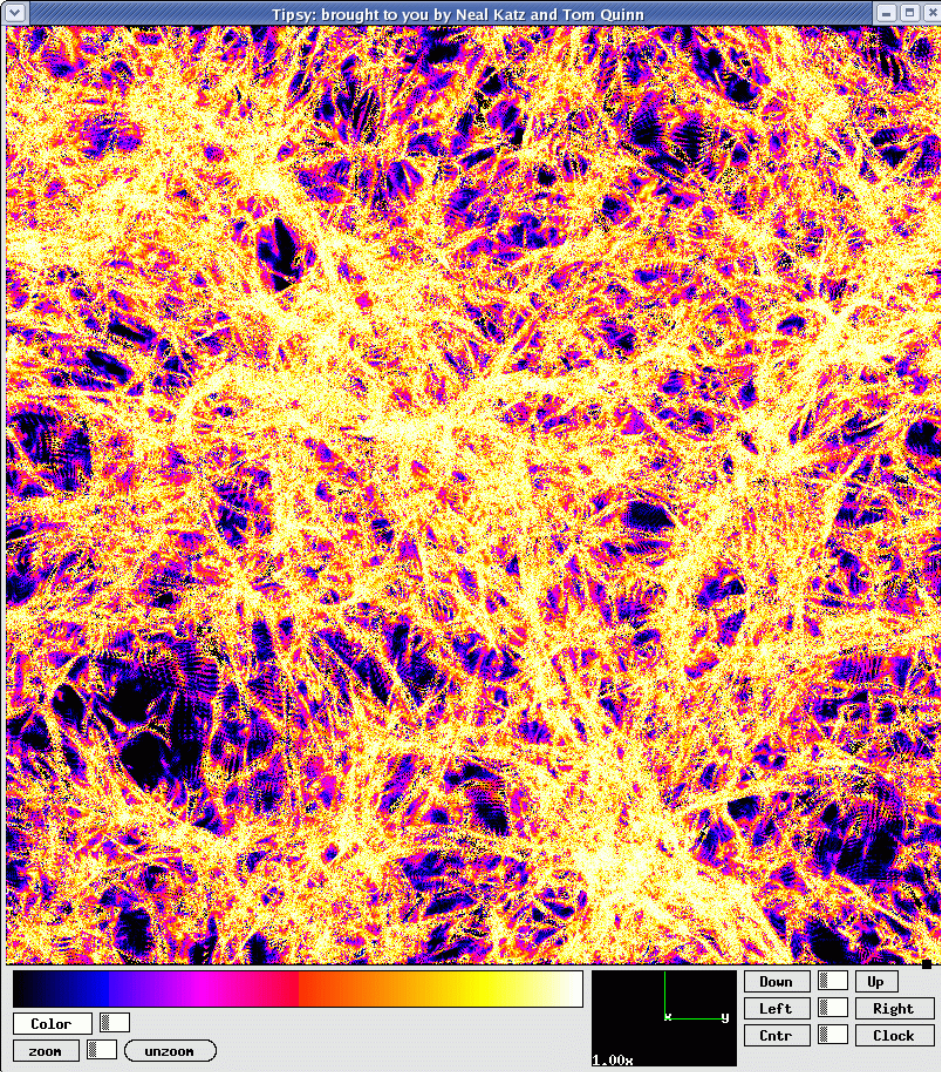
<sup>b</sup>*LPTHE, Université Pierre et Marie Curie (Paris VI), Laboratoire Associé au CNRS UMR 7589, Tour 13-14, 4ème. et 5ème. étage, Boite 126, 4, Place Jussieu, 75252 Paris, Cedex 05, France*

<sup>c</sup>*Observatoire de Paris, LERMA, Laboratoire Associé au CNRS UMR 8112, 61, Avenue de l'Observatoire, 75014 Paris, France*

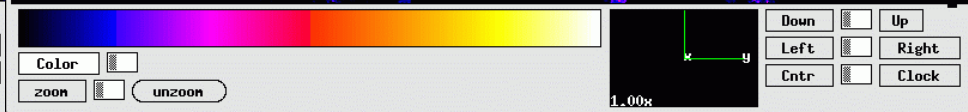
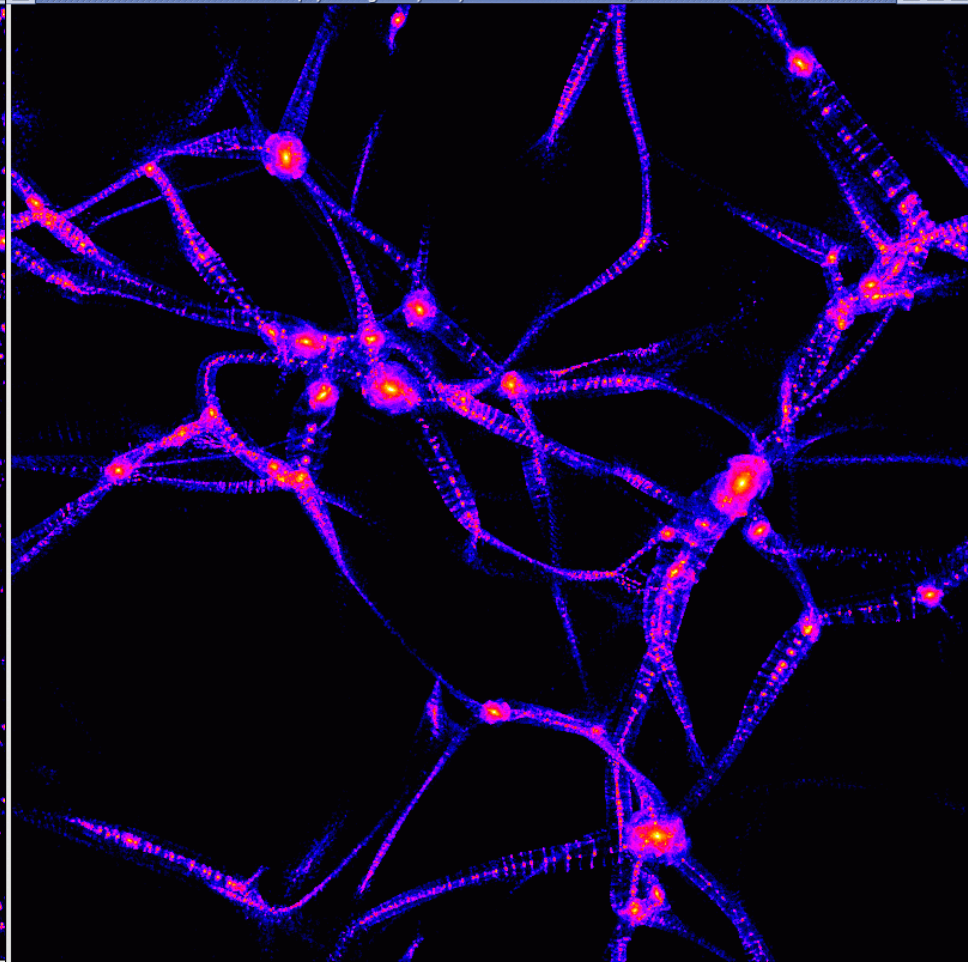
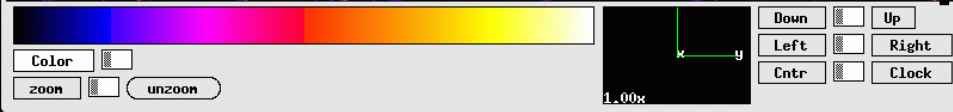
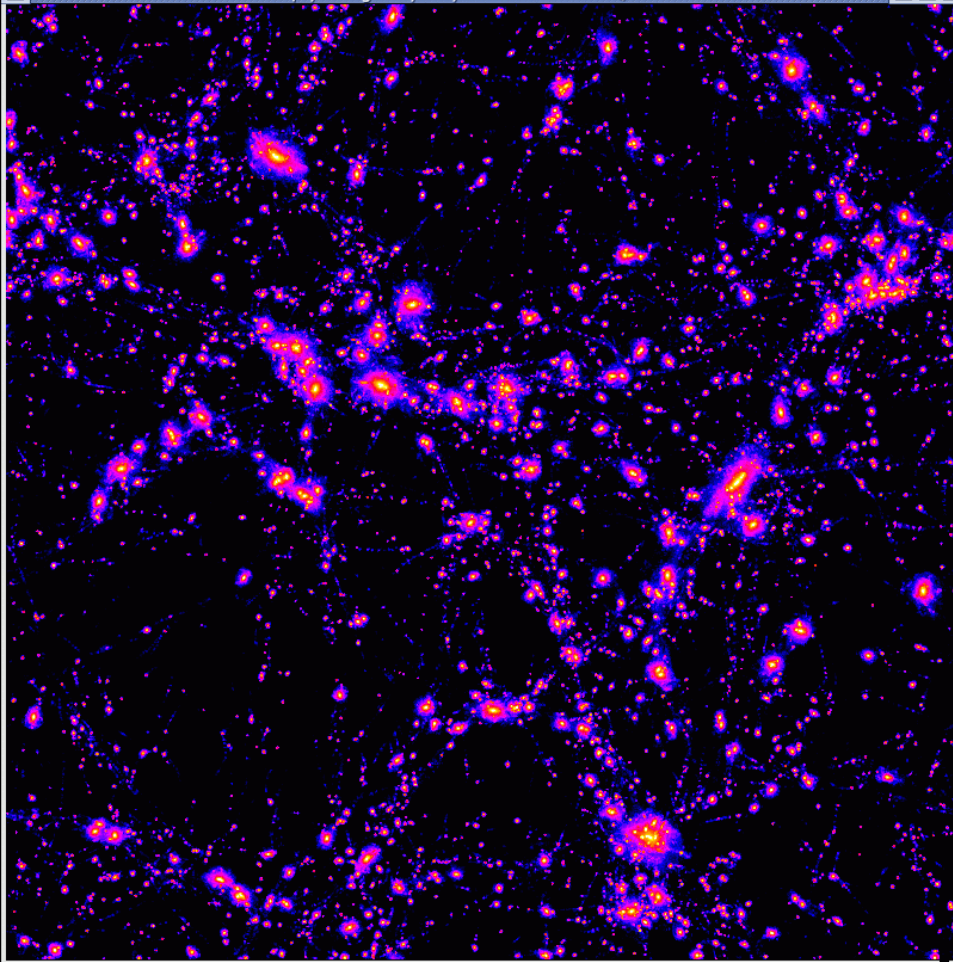
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## Abstract

We derive the main physical galaxy properties: mass, halo radius, phase space density and velocity dispersion from a semiclassical gravitational approach in which fermionic WDM is treated quantum mechanically. They turn out to be fully compatible with observations. The Pauli Principle implies for the fermionic DM phase-space density  $Q(\vec{r}) = \rho(\vec{r})/\sigma^3(\vec{r})$  the quantum bound  $Q(\vec{r}) \leq K m^4/\hbar^3$ , where  $m$  is the DM particle mass,  $\sigma(\vec{r})$  is the DM velocity dispersion and  $K$  is a pure number of order one which we estimate. Cusped profiles from  $N$ -body galaxy simulations produce a divergent  $Q(r)$  at  $r = 0$  violating this quantum bound. The combination of this quantum bound with the behaviour of  $Q(r)$  from simulations, the virial theorem and galaxy observational data on  $Q$  implies lower bounds on the halo radius and a minimal distance  $r_{min}$  from the centre at which classical galaxy dynamics for DM fermions breaks down. For WDM,  $r_{min}$  turns to be in the parsec scale. For cold dark matter (CDM),  $r_{min}$  is between dozens of kilometers and a few meters, astronomically compatible with zero. For hot dark matter (HDM),  $r_{min}$  is from the kpc to the Mpc. In summary, this quantum bound rules out the presence of galaxy cusps for fermionic WDM, in agreement with astronomical observations, which show that the DM halos are cored. We show that compact dwarf galaxies are natural quantum macroscopic objects supported against gravity by the fermionic WDM quantum pressure (quantum degenerate fermions) with a minimal galaxy mass and minimal velocity dispersion. Quantum mechanical calculations which fulfil the Pauli principle become necessary to compute galaxy structures at kpc scales and below. Classical  $N$ -body simulations are not valid at scales below  $r_{min}$ . We apply the Thomas-Fermi semiclassical approach to fermionic WDM galaxies, we resolve it numerically and find the physical galaxy magnitudes: mass, halo radius, phase-space density, velocity

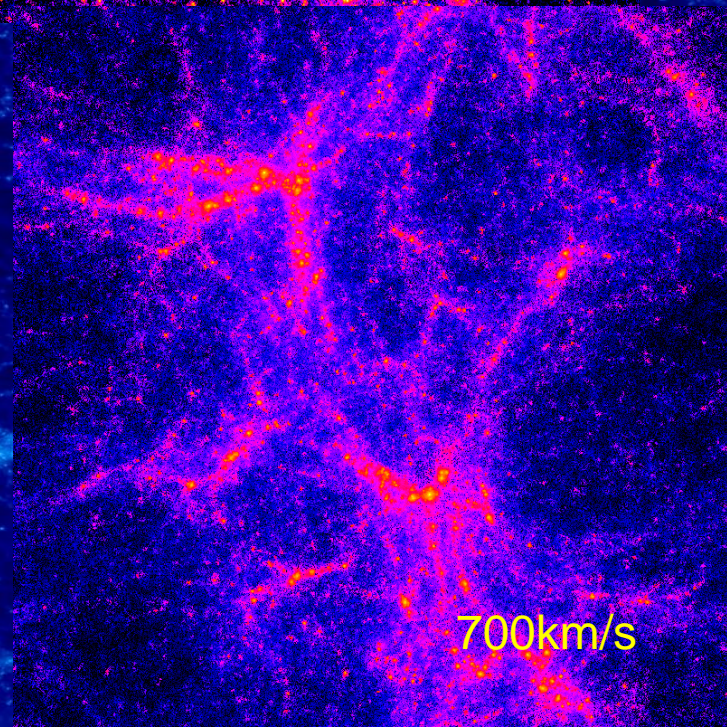
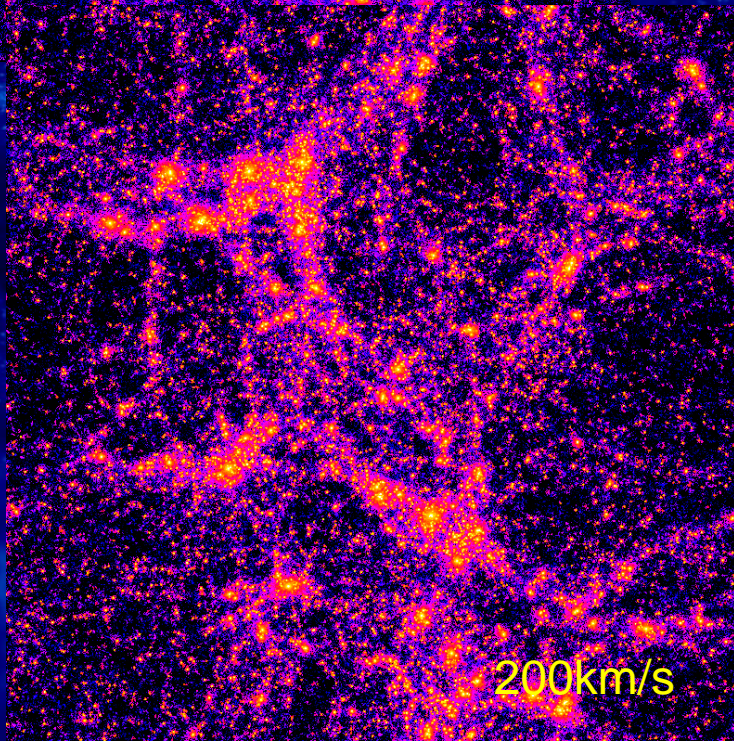
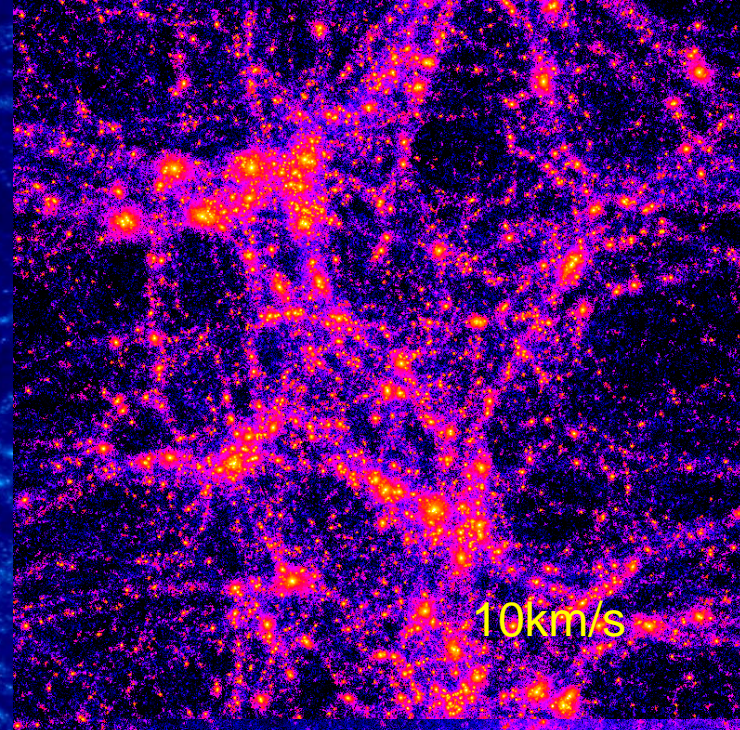
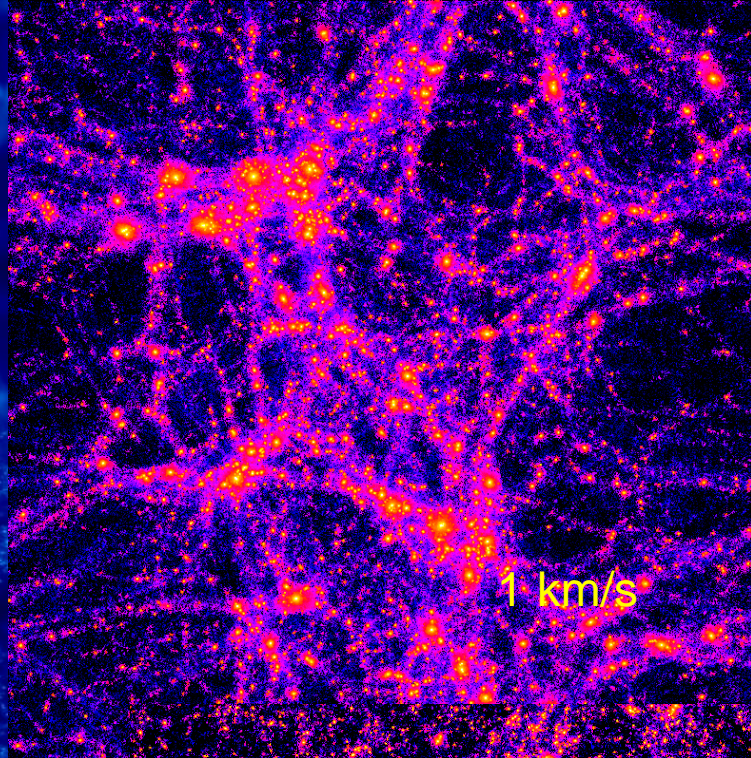


Mildly non-linear regions at  $z=3$  in CDM and WDM (200eV)  
i.e. overdensities between 1 and 5 w.r.t. mean



Virialised regions at  $z=3$  in CDM and WDM (200ev)  
i.e. overdensities higher than 100 w.r.t. mean

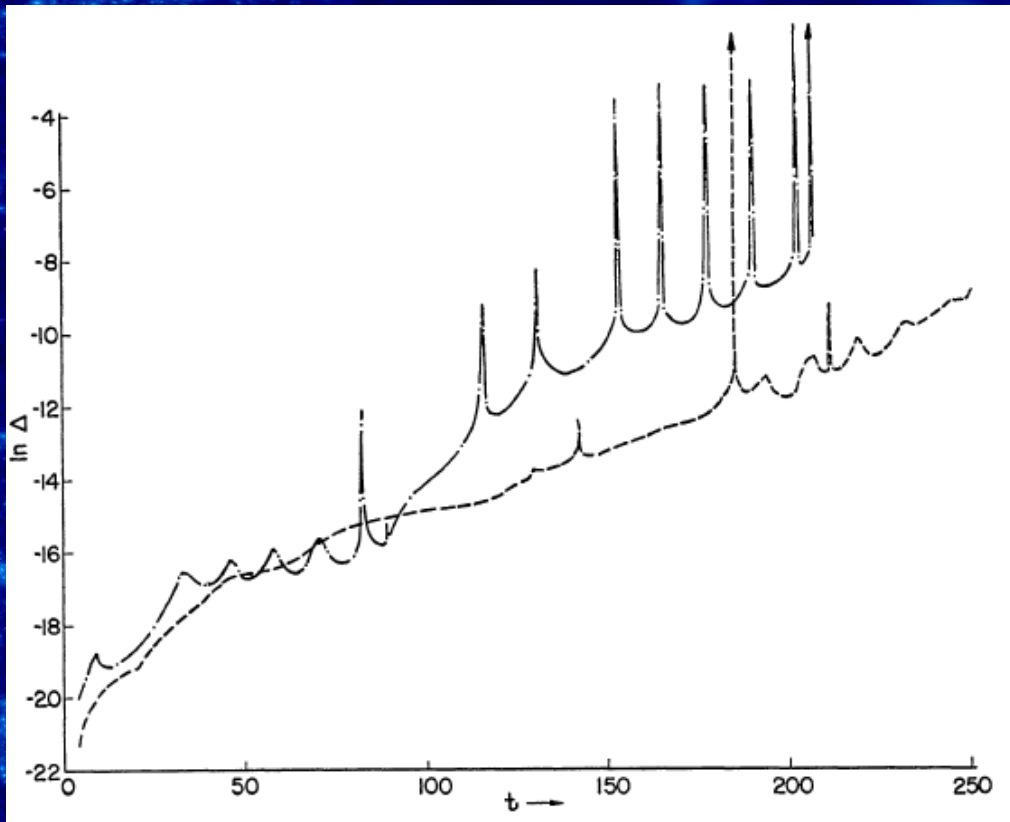




Why velocities?

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{x}} + \vec{g} \cdot \frac{\partial f}{\partial \vec{v}} = 0$$

$$\vec{g} = G \int d^3x' d^3v f(\vec{x}', \vec{v}, t) \frac{(\vec{x} - \vec{x}')}{|\vec{x} - \vec{x}'|^3}$$

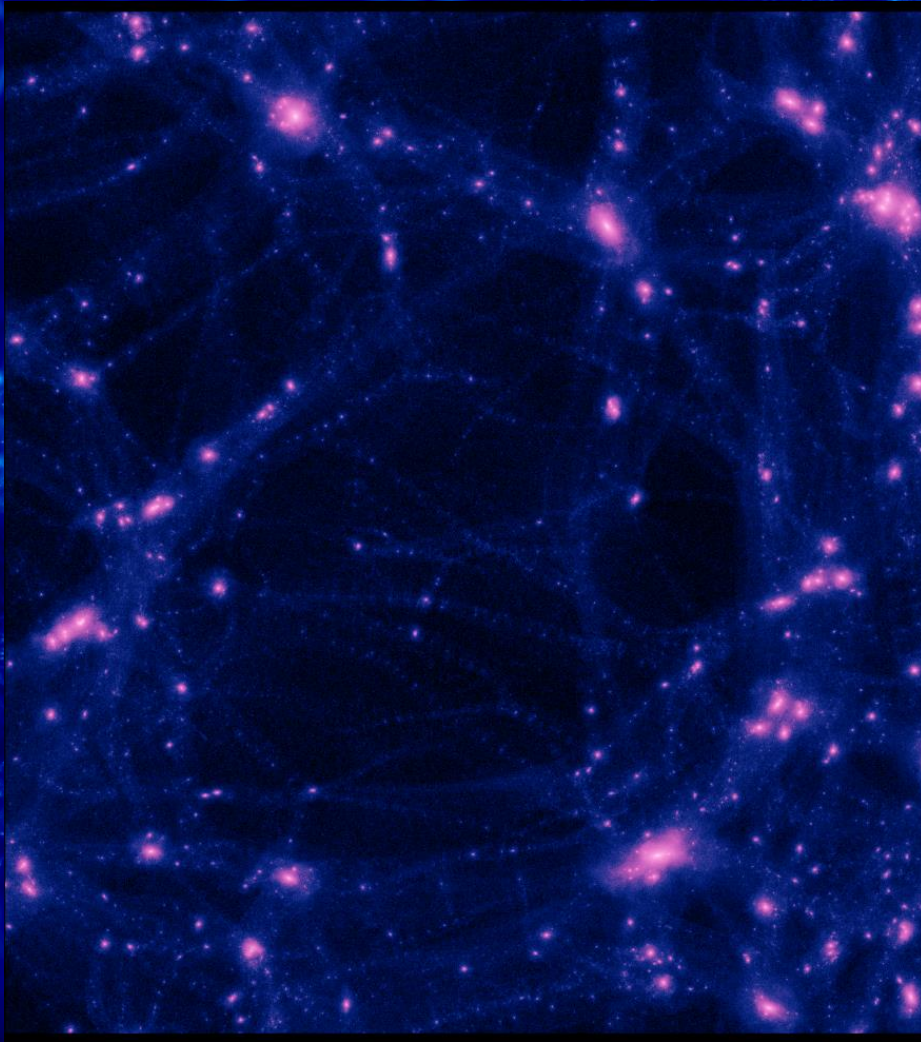


Miller 1964 – Tracks of the separation of phase points

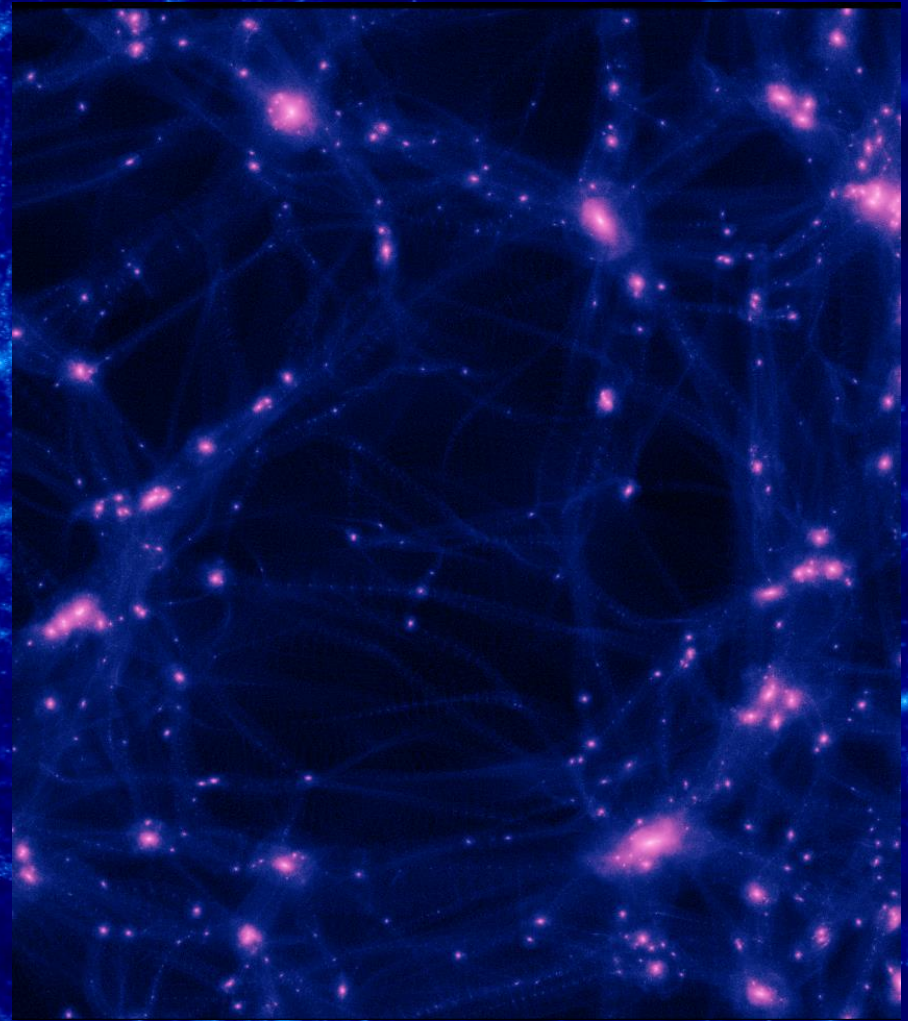
$$\frac{\partial \delta f}{\partial t} + \vec{v} \cdot \frac{\partial \delta f}{\partial \vec{x}} + \vec{g}_0 \cdot \frac{\partial \delta f}{\partial \vec{v}} = -\delta \vec{g} \cdot \frac{\partial f_0}{\partial \vec{v}} - \delta \vec{g} \cdot \frac{\partial \delta f}{\partial \vec{v}}$$

$$\frac{\delta \vec{g}}{|\vec{x} - \vec{x}'|^3} = G \int d^3x' d^3v \delta f(\vec{x}', \vec{v}, t) \frac{(\vec{x} - \vec{x}')}{|\vec{x} - \vec{x}'|^3}$$

$$(\Delta x \Delta v)^3 > m^3 h^3$$



No velocities



Velocities

Label	particle mass	velocities	box size	no.of particles
CDM	-	no	40 Mpc/h	$300^3$
WDM1	200 eV	no	40 Mpc/h	$300^3$
WDM2	200 eV	200 eV	40 Mpc/h	$300^3$
WDM3	1 KeV	no	40 Mpc/h	$300^3$
WDM4	1 KeV	1KeV	40 Mpc/h	$300^3$

Simulations details:  $2.72 \times 10^5 M_{\odot}$  / particle  
 355 pc spline gravitational softening  
 WMAP7 cosmological parameters  
 $z=100$  initial redshift

Label	size	first collapse	average density	highest density
-	box	z	critical	critical
lu.avi	1/4	4.88	$2.64e-1$	$4.77e02$
ld.avi	1/4	4.18	$2.58e-1$	$4.81e02$
ru.avi	1/4	4.18	$2.68e-1$	$4.80e02$
rd.avi	1/4	4.64	$2.58e-1$	$4.74e02$

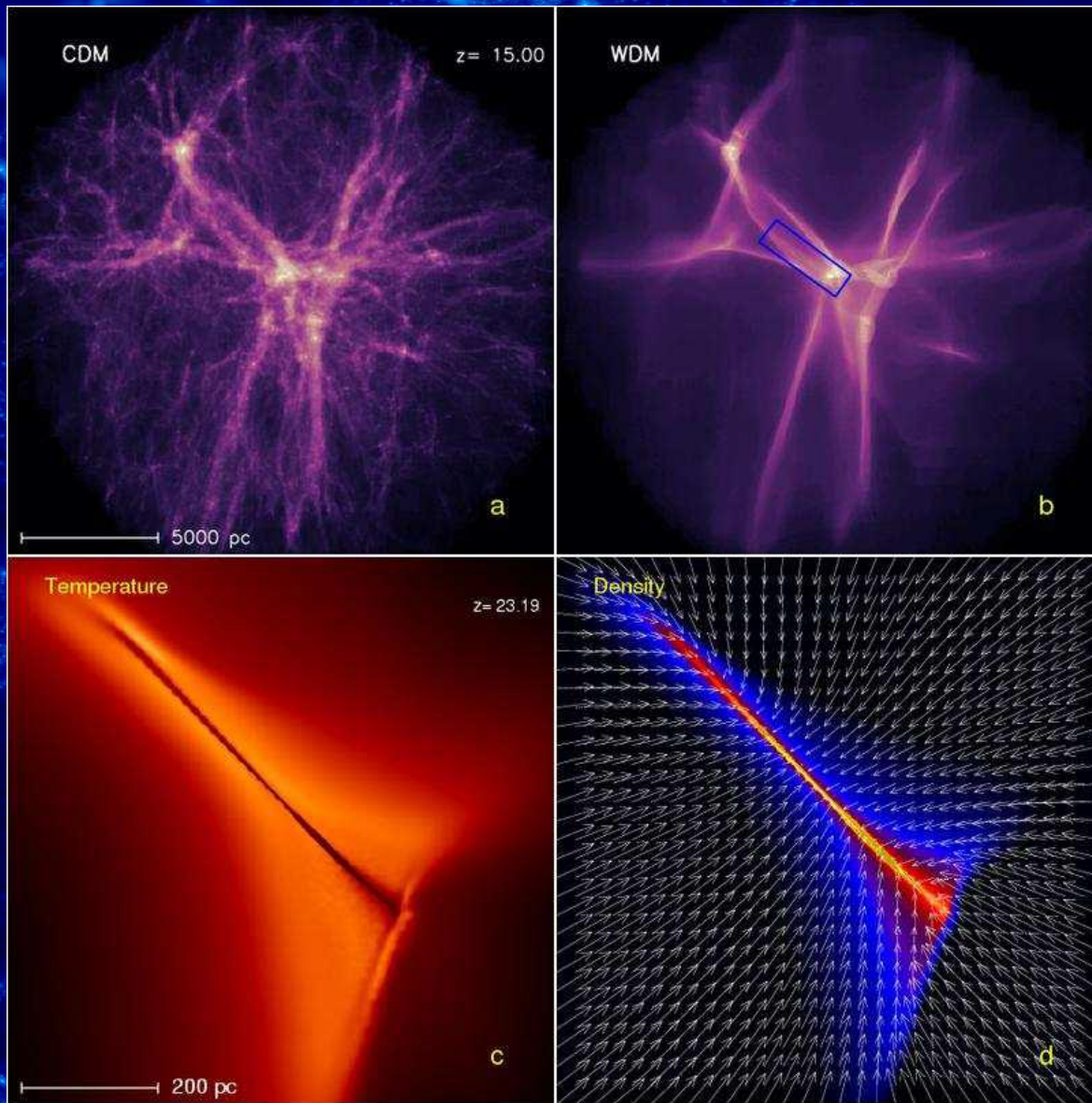
- During the early stages one sees the formation of well contoured filaments.
- In the higher density regions, usually situated at the intersection of such filaments, the first halos are formed through gravitational collapse. These halos continue growing into larger ones by accreting particles from the disrupted filaments.
- In medium density regions, haloes show a hierarchical formation trend. Small haloes collapse first and then merge into bigger haloes.
- In less dense regions, the ones isolated by voids and have a very slow evolution, we have observed filaments that collapse very late. The top down formed halo survives without any mergers until redshift zero.
- Finally there is the more complex scenario in which we observe large haloes formed earlier which merge together.

The movies...

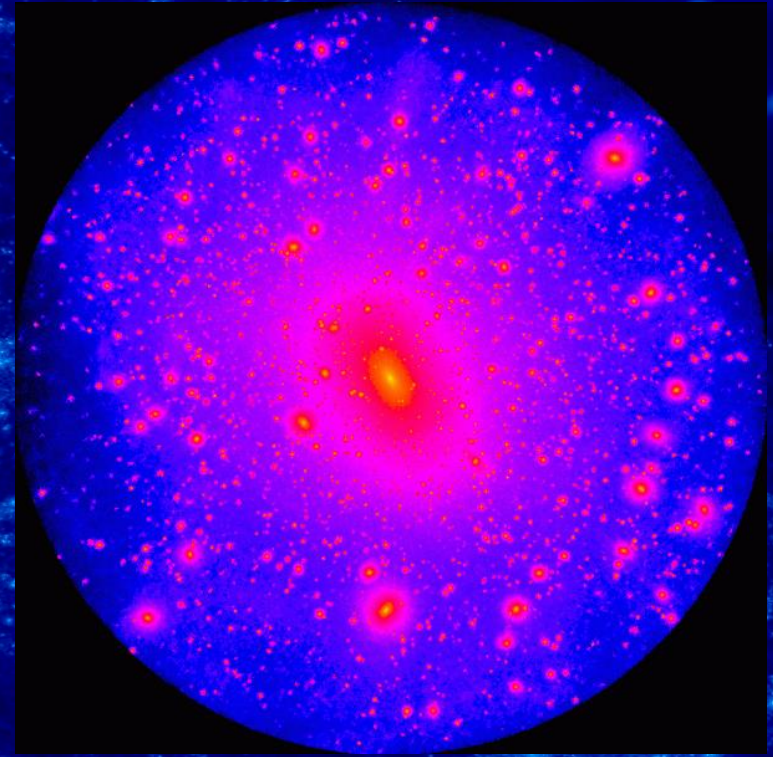
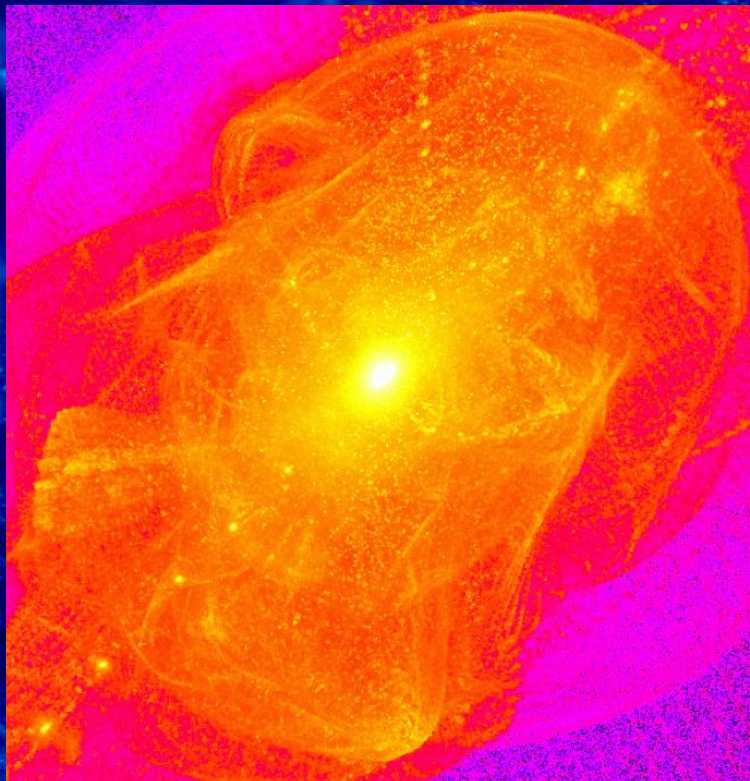
[obswww.unige.ch/~paduroiu](http://obswww.unige.ch/~paduroiu)

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on demand

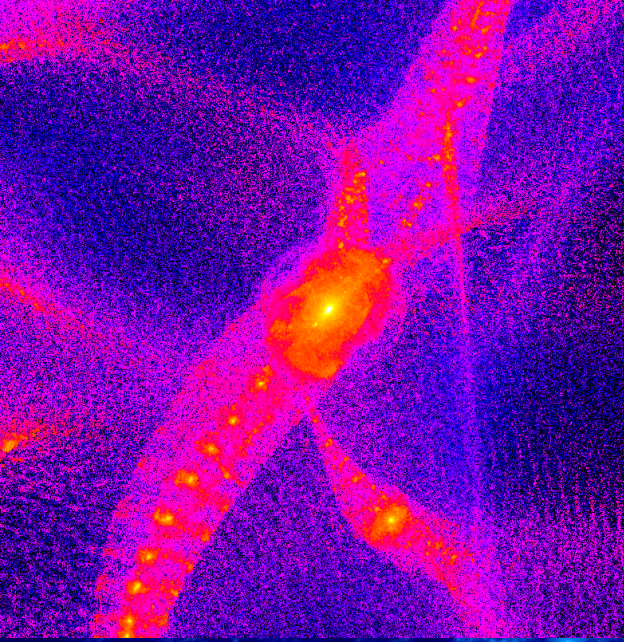


Liang Gao & Tom Theuns, Science sept 2007

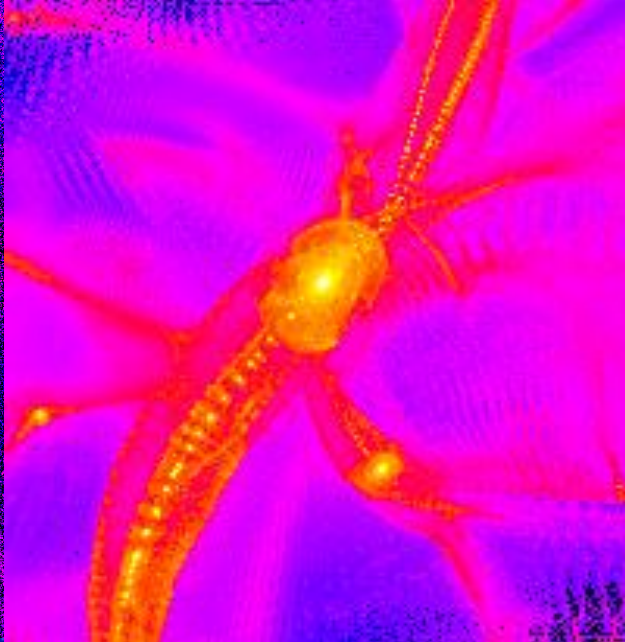


	Number of part.	$r_{vir}$ [kpc]	mass p. part. [ $M_{\odot}$ ]	boxsize [Mpc]
$m < 200\text{eV}$	18 mil.	630	$10^5$	42.5
$m > 2\text{keV}$	50 mil.	200	$10^7$	40

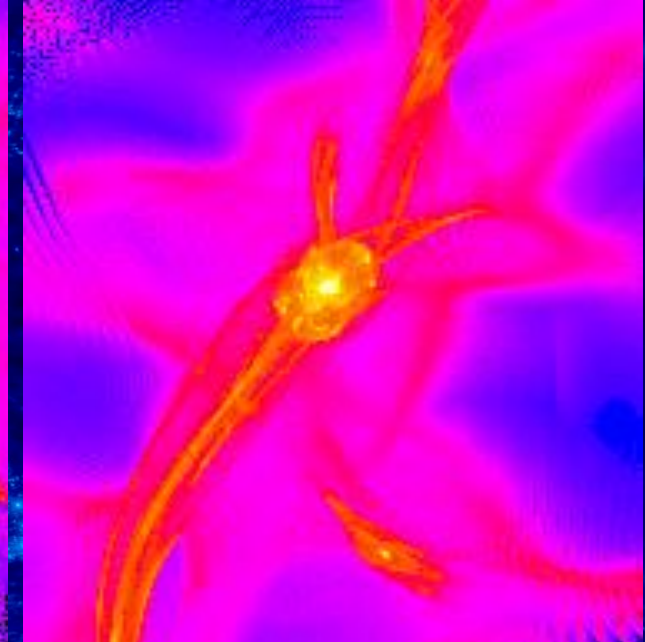




"Low" resolution

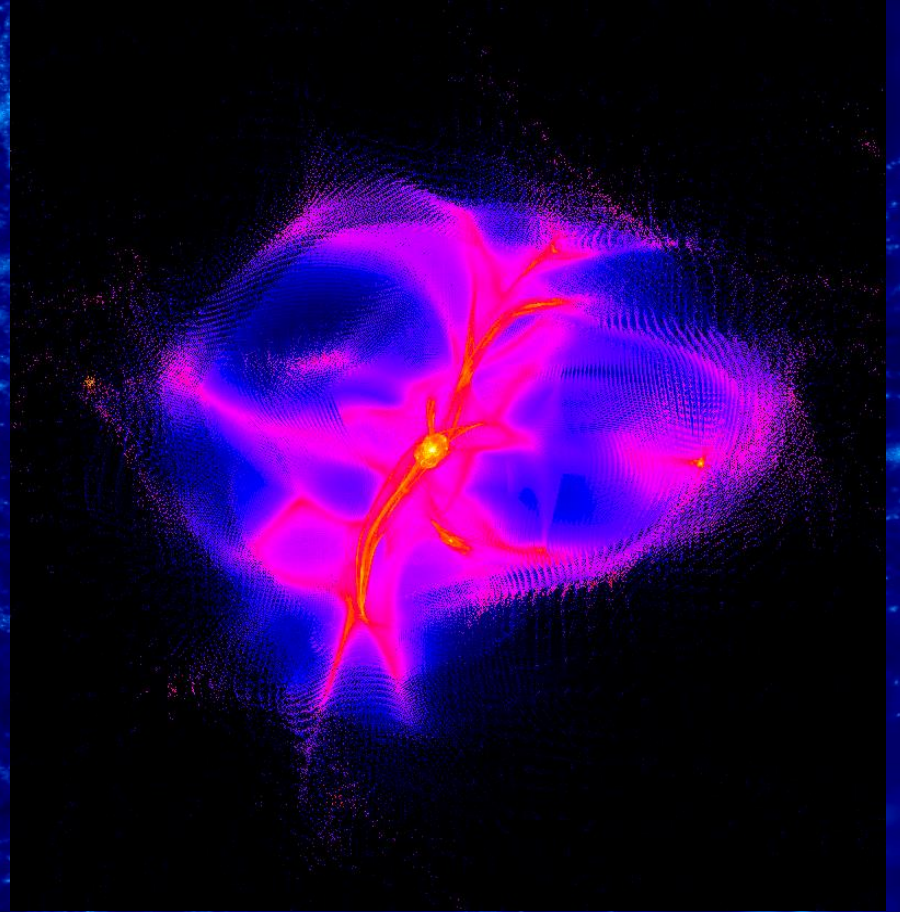
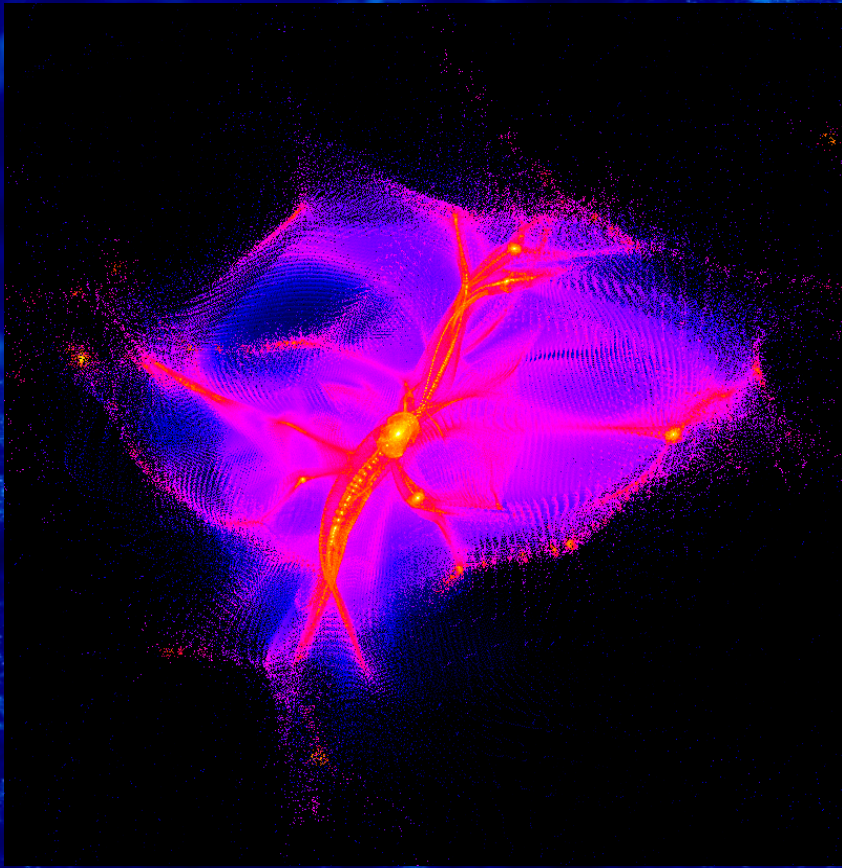


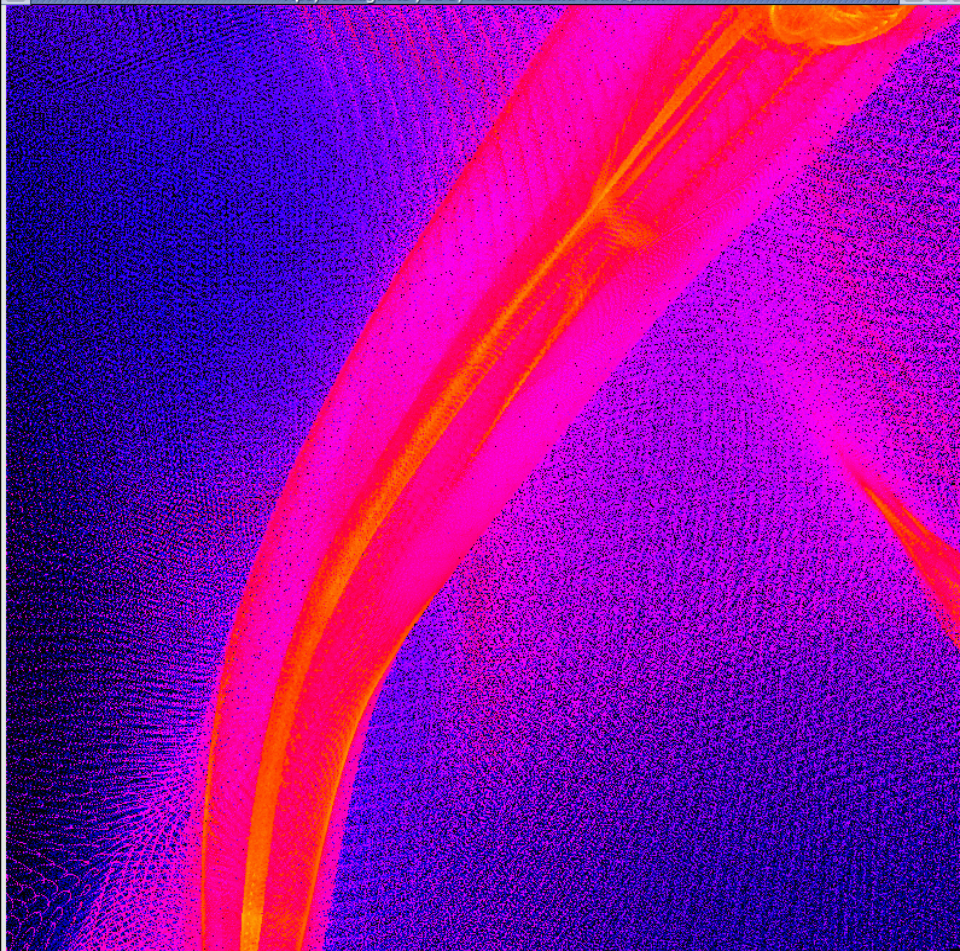
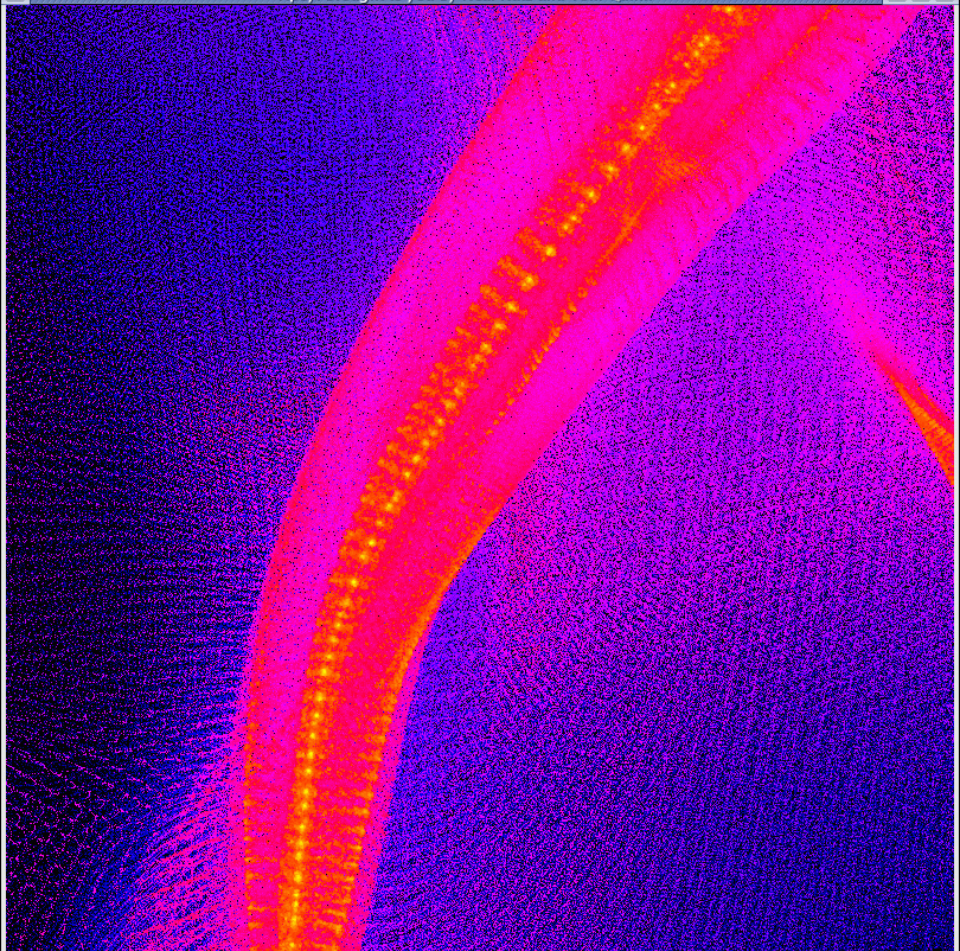
"High" resolution



"High" resolution  
Large Softening







Color

zoon  unzoon

8.00x

Down  Up

Left  Right

Cntr  Clock

Color

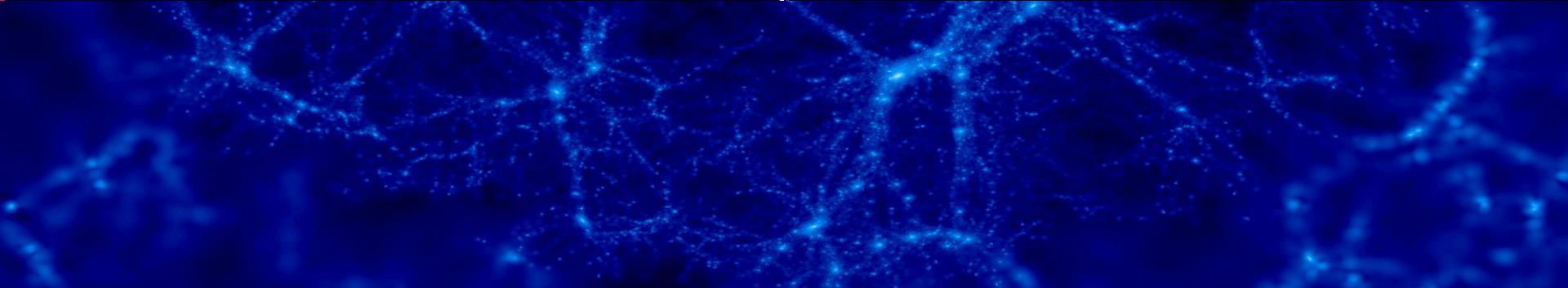
zoon  unzoon

8.00x

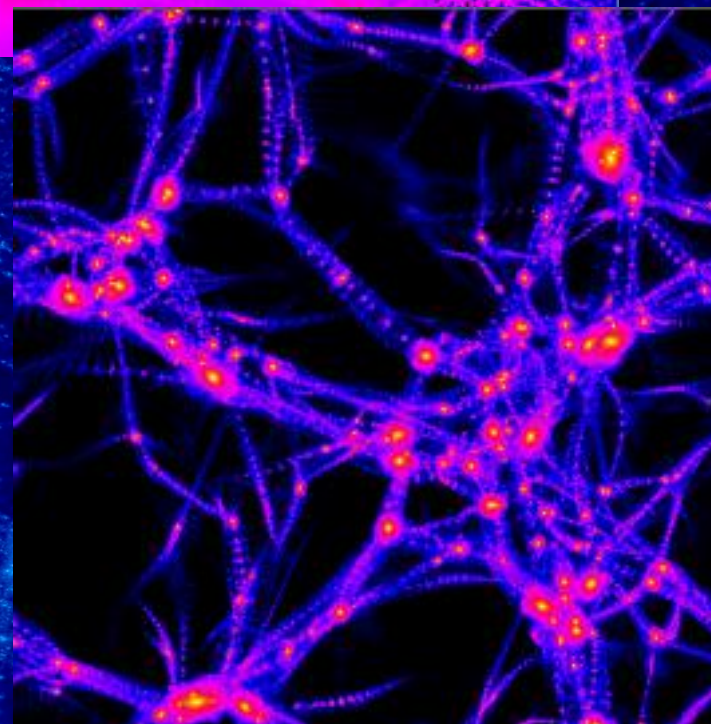
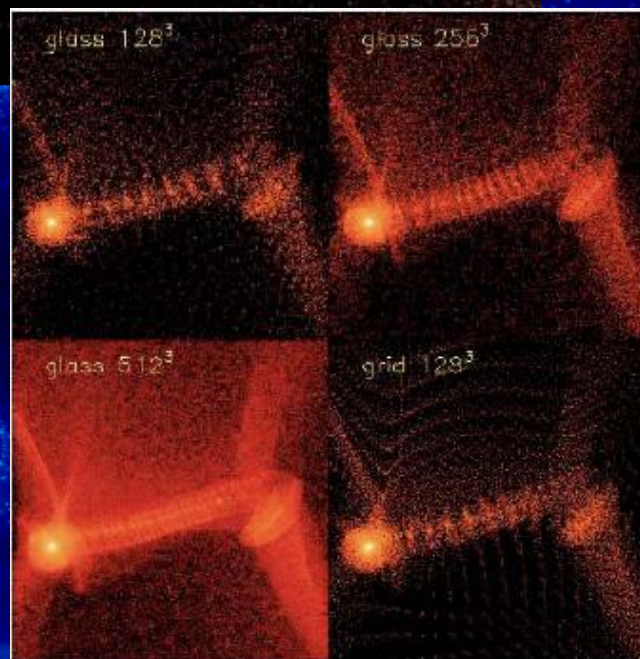
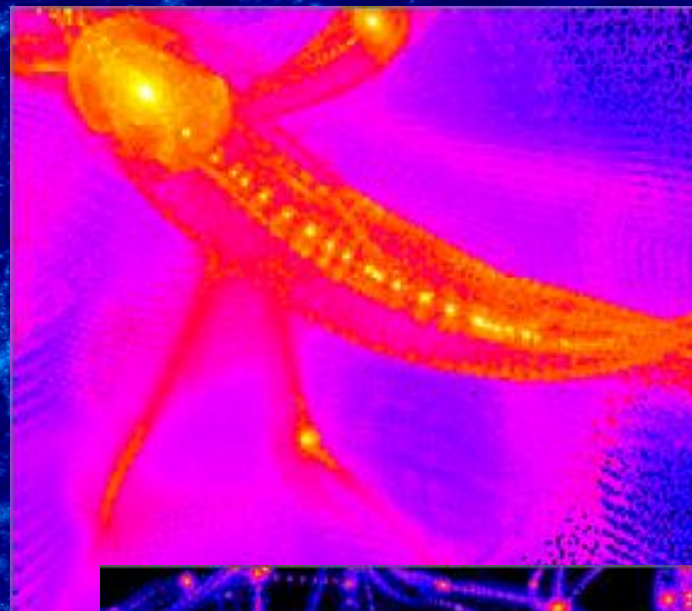
Down  Up

Left  Right

Cntr  Clock



grid  $128^3$   
Wang & White 2007



# CONCLUSIONS and COMPLICATIONS

- ⋈ Formation of haloes in WDM models differs from CDM. Top-Down, Hierarchical, Grid Down. Looking at high redshift galaxies for T-D memory.
- ⋈ The exact recipe for structure formation seems to depend only on the “topology” of the environment
- ⊗ Quantum Pressure; Baryons and their physics
- ⋈ The finite initial fine grained PSD is also a maximum of coarse grained PSD.
- ⋈ The turn over in PSD results in constant density core with characteristic size.
- ⋈ Spurious fragmentation below the free streaming scale hard to overcome – in case of infinite resolution a filament collapses into a two dimensional line
- \$\$ Adaptive softening?
- ⋈ Warm dark matter haloes contain visible caustics and shells.

