

DANIEL CHALONGE AND THE PROBLEM OF THE
ABUNDANCE OF HYDROGEN

S. CHANDRASEKHAR

*Enrico Fermi Institute, The University of Chicago
933 East 56th Street, Chicago, Illinois 60637, U.S.A.*

I feel greatly privileged to give this opening address at this, the first course of the International School of Physics dedicated to Daniel Chalonge, and to pay homage to a scientist whom I admired and whose friendship I cherished. Chalonge was one of the architects who fashioned the astrophysical concepts of today: he played a crucial role in establishing a basic fact of current astronomy: that hydrogen is by far the most abundant element in the cosmos.

It is now assumed without question or doubt that

Hydrogen:Helium:all other atoms = 25000:4500:80 (by numbers)

and

Hydrogen:Helium:all other atoms = 25000:18000:1400 (by weight).

But I believe that it is not generally appreciated that the abundance of hydrogen was a matter of great controversy and intense debate for nearly two decades during the thirties and forties: Henry Norris Russell championing a hydrogen-to-metal ratio of 1000:1 and Albrecht Unsöld championing 10-50:1. The reason for this discrepancy is clearly explained in Unsöld's *post facto* statement in the second edition of his great treatise *Physik der Sternatmosphären* (Springer-Verlag, 1954, pp. 164-65). He wrote:

These ideas (relating to the determination of the abundances of the elements in stellar atmospheres) were incorporated in numerous calculations by H. N. Russell (1933), L. Biermann (1933), A. Unsöld (1935), and A. Pannekoek (1935). In these calculations, the following physical processes were taken into account: the "bound-free" and the "free-free" transitions of the electron in the hydrogen atom (H); and similar transitions of the outer (loosely bound) electrons in metals (M) evaluated more or less as in the case of hydrogen.

The principal focus of the controversy was the abundance ratio, H:M, by numbers.

H.N. Russell (1933) determined this ratio by the positions of the maxima of certain of the "spark lines" of the metals (by arguments by no means convincing) and by the observed intensities of several hydride bands in the solar spectrum. He derived a value for H:M in the range 1000 to 2000. A. Pannekoek (1935) took over into his calculations the ratio 1000:1.

A. Unsöld, on the other hand, considered the dependence on temperature of the discontinuity, $D = \Delta \log I$, at the

limit ($\lambda 3700\text{\AA}$) of the Balmer series, particularly in stars with spectral types in the range *A* to *G*. If the atmospheres of the stars were purely of hydrogen, the discontinuity *D* must continuously *increase* with decreasing temperature, since the absorption from the second quantum state of hydrogen must decrease much less rapidly than the background absorption from the higher quantum states 3, 4, . . . The observed decrease of *D* below $T = 9000^\circ\text{K}$ must then be accounted for by the enhancement, at lower temperatures, of the continuous absorption by the metals relative to that of hydrogen. (On this picture, *D* is directly related to the ratio of the number of hydrogen atoms in the second quantum state to the number of metal atoms contributing to the background absorption.) A quantitative evaluation of these effects led to a value H:M = 14-50 -- the spread in the ratio resulting only from the uncertainty in the quantal estimates of the absorption coefficient by the metals.

But there was another strand of equal significance which Unsöld does not mention: the source of the continuous opacity in the stars, determining the relation between their color and effective temperatures, and more importantly, the source of the continuous opacity of the solar atmosphere.

It is now known that both these problems -- the origin of the discrepancy between Russell's and Unsöld's views on the abundance of hydrogen and the source of the continuous opacity of the solar atmosphere -- are different aspects of the same problem. The solution of both these problems was provided in one stroke by Rupert Wildt in 1938 who pointed out that the negative ion of hydrogen must be present in sufficient numbers in the atmospheres of the sun and the stars to resolve the vexed issues. I shall return to this identification of H^- presently. But first I should draw attention to the fact that the discontinuity at the head of the Balmer series, the relation between the color and the effective temperatures of stars, and the source of solar opacity, are all areas to which Chalonge and his collaborators made major contributions at the time.

Let me digress then to recall that in 1931 Chalonge, Barbier, and their colleagues established the Scientific Station on the Jungfrauoch (Switzerland) at an altitude of 3457 meters. The high altitude of the site was ideally suited to ultraviolet spectroscopy; and Chalonge initiated a carefully planned program for measuring the *gradients*, Φ_1 and Φ_2 , in the continuous spectrum on either side of the head of the Balmer series (at 3647\text{\AA}) as well as the size of the *discontinuity* measured by the logarithm of the ratio of the intensities on the two sides (see Fig. 1). You will notice that the quantities they measured are precisely those to which I have already drawn attention, and which were to play crucial roles in resolving the issues that were intensely debated during the thirties and forties.

By 1939, Barbier and Chalonge had observed some 200 stars, and measured the discontinuity at the head of the Balmer series in their spectra as well as the gradients Φ_1 and Φ_2 on either side of the discontinuity. Figures 2 and 3 are examples of the discontinuities they observed and measured. And in Figures 4 and 5, their observations on the discontinuity

D and the gradients Φ_1 and Φ_2 are presented as they assembled them in their classic paper "Etude du Rayonnement Continue de Quelques E'toiles Metre 3100 et 2699Å" (Daniel Barbier et Daniel Chalonge, *Ann. D'astro-physique*, 1939, 4, 30-67).

Now let me return to the question why the discontinuity at the head of the Balmer series was so essential to Unsöld's argument for determining the abundance of hydrogen. The reason is simple. It is clear that the absorption increases by a large factor as we go to the short wavelength side of the Balmer discontinuity, because only radiation with wavelengths shorter than 3647Å can ionize the plentiful hydrogen atoms in their second quantum state; and on the long wavelength side, the absorption must almost entirely be due to the other sources of opacity. Before Wildt's discovery of the role of the negative ion of hydrogen, the absorption on the redward side of the Balmer discontinuity was naturally attributed to the "metals", meaning the ionizable atoms such as sodium, potassium, iron, magnesium, etc. In other words, the Balmer discontinuity is very directly a measure of the hydrogen-to-metal ratio and there was no way to avoid this conclusion, as Unsöld rightly insisted (prior to Wildt's discovery!).

The problem was of course resolved in the early forties by Wildt's discovery of the role of H^- ; but what led to the definitive identification of the negative ion of hydrogen was not these facts but rather, studies on the continuous spectrum of the sun, to which also Chalonge and Kourganoff were later to make their fundamental contributions.

I therefore turn now to the problem presented by the continuous spectrum of the sun. The basic measurements for this analysis are provided by Abbot's measurements on the distribution with wavelength of the net flux of radiation and the darkening, across the solar disk. The fundamental character of these measurements was clearly stated by E. A. Milne in 1921 and by Minnaert in 1924.

Milne, 1921:

The sun is the only star for which the intensity distribution is at present known in any detail: the observations of Abbot, Fowle, and Aldrich, as well as those of earlier investigators, have determined the darkening both for integrated radiation and for separate wavelengths.

Minnaert, 1924:

Without discussion, we make use of the excellent observations of Abbot and his collaborators made in the Mt. Wilson Observatory.

The exact determination of the solar radiation for different wavelengths and for different points on the disk is fundamental as a test of solar theories.

Fig. 6 presents Minnaert's reduction of the measurements of Abbot and others to a uniform system; and Fig. 7, taken from a paper by Mulders, exhibits corrections needed to allow for the 'blanketing' by the crowding of the absorption lines.

Let me explain briefly why these measurements on the darkening of the sun towards the limb provided so important a base for deductions.

It is clear in a general way that darkening towards the limb is simply an expression of the angular dependence of the intensity of the emergent radiation; and further that the radiation from all depths contributes to the emergent radiation: only the radiation from the deeper layers is increasingly attenuated by the opacity (i.e., the fogginess) of the overlying layers. We may say then that the radiation which emerges from the surface is characteristic of the radiation prevailing at a certain average depth below the surface (see Fig. 8).

Since radiation emerging at an angle traverses a path which is slanting through the atmosphere, it is manifest that such radiation will be representative of the radiation prevailing at a level not as deep as the level which is representative of the radiation which emerges normally from the surface. Since we should expect the deeper layers to be at a higher temperature, radiation emerging at an angle will be characteristic of a lower temperature than the temperature characteristic of the radiation emerging normally. Therefore, the intensity of the radiation emerging at an angle must be less than that emerging normally. In other words, there must be darkening towards the limb.

It is also clear why the relative darkening towards the limb in the different wave lengths should relate directly with the continuous opacity of the sun's outer layers: at the limb we always see the surface layers; but at the center of the disk we see relatively deeper in those wave lengths in which the opacity is less. And if there is a temperature gradient in the outer layers, then in spectral regions with less opacity, we see higher temperatures and therefore greater intensities. Conversely, if the opacity is high, we effectively see only the surface layers at all points of the disk and therefore very little contrast, i.e., little darkening.

From the foregoing description, it is clear that the principal problem, which requires solution before we can account for the phenomenon of darkening, is the distribution of the temperature in the outer layers. Once the temperature distribution has been ascertained, the emergent intensity at any given angle can be directly related to the variation of the opacity (i.e., the absorption coefficient or the absorptive power) of the material for light of different wavelengths.

Suppose $\bar{\kappa}$ is some mean absorption coefficient and let τ be the optical depths measured in terms of $\bar{\kappa}$. Let T_τ be the temperature that prevails at depth τ . Then at the depth τ , the spectral distribution of the radiation will be determined by the Planck distribution,

$$B_\nu(T_\tau) = \frac{2h\nu^3}{c^2} \frac{1}{\exp(h\nu/kT_\tau) - 1}, \quad (1)$$

where ν , c , and h denote the frequency, the velocity of light, and Planck's constant, respectively. Accordingly, the intensity of the emergent radiation, at an angle θ to the normal and with a frequency ν , will be given by

$$I_{\nu}(\theta) = \int_0^{\infty} d\tau B_{\nu}(T_{\tau}) \frac{\kappa_{\nu}}{\bar{\kappa}} \exp\left[-(\kappa_{\nu}/\bar{\kappa})\tau \sec\theta\right] \sec\theta, \quad (2)$$

where κ_{ν} is the absorption coefficient at the particular frequency considered.

It is clear that from a comparison of the observed intensities with those which would follow from equation (2), we can deduce the variation of the absorption coefficient with wavelength (as determined by $\kappa_{\nu}/\bar{\kappa}$). E.A. Milne, who developed the basic ideas that we have outlined, was the first to deduce the variation of the absorption coefficient with wavelength of the solar continuum. The results of a later more refined analysis by G. Mulders is shown in Fig. 9.

Two features of the deduced variation of κ_{λ} that are to be particularly noted are: *first*, that κ_{λ} increases gradually over the entire visual part of the spectrum and attains a very well defined maximum at about 9000Å; and *second*, that beyond 9000Å it decreases to a very deep minimum at about 16000Å. The directness and the simplicity of the analysis leading to these two features of the variation of κ_{λ} with wavelength leaves little doubt that a fundamental constituent of the solar atmosphere is here involved. The unresolved problem of the twenties and the thirties was to identify this constituent. As we have stated already, it was Rupert Wildt who pointed out with rare perception that the constituent in question is the negative ion of hydrogen.

By an application of the Rayleigh-Ritz variational principle, which enables one to set upper bounds to the ground-state of atomic systems, Hylleraas and Bethe, independently, showed in 1930 that a hydrogen atom can be bound to an electron to form a stable negative ion with a binding-energy (i.e., *electron affinity*) of 3/4 e.v. But to confirm Wildt's identification, one had to evaluate by reliable theoretical methods the photo-detachment cross-section, as a function of the wavelength, of H^{-} , whose ground state was known only through a variational calculation. The problem turned out to be more difficult than one had expected. Figure 10 shows the improvement in the calculations that was effected between 1940 and 1946. A comprehensive and a charming account of these and related investigations will be found in Bates's "Other Men's Flowers" in *Physics Reports* 35, 306 (1978); Figure 11 is taken from this report.

By 1946, the continuous absorption coefficient of H^{-} was known with sufficient precision for Chalonge and Kourganoff to provide a detailed confirmation of H^{-} as the dominant source of opacity in the solar atmosphere. Their method was the following. They considered levels in the solar atmosphere at various assigned temperatures, T , and asked for the opacity of the overlying layers at various wavelengths. The results of their analysis are shown in Figure 12. (Their analysis did not include the contribution to the absorption coefficient by the 'free-free' transitions of electrons in the field of a neutral hydrogen atom; their inclusion will account for the 'balance' of the absorption). The analysis of Chalonge and Kourganoff leaves no doubt that the source of the

continuous absorption in the solar atmosphere has indeed been identified.

I return briefly to the interpretation of the results of Barbier and Chalonge on the Balmer discontinuity and on the color-effective temperature-relation assembled in Figures 4 and 5. Since the photo detachment cross-sections of H^- were known with sufficient precision in 1946, it was a relatively simple matter to revise Unsöld's earlier analysis for a renewed determination of the abundance of hydrogen. Thus, expressing the mean absorption-coefficients of H^- and H in the forms,

$$\left. \begin{aligned} \bar{\kappa}(H^-) &= \frac{1-x_H}{m_H} p_e \langle a(H^-) \rangle, \\ \bar{\kappa}(H) &= \frac{1-x_H}{m_H} \langle a(H) \rangle. \end{aligned} \right\} \quad (3)$$

and

$$\bar{\kappa} = \bar{\kappa}(H^-) + \bar{\kappa}(H) = \frac{1-x_H}{m_H} \left[p_e \langle a(H^-) \rangle + \langle a(H) \rangle \right], \quad (4)$$

where x_H denotes the degree of ionization of hydrogen, m_H the mass of the hydrogen atom, p_e the electron pressure (determined principally of the 'metals' i.e., by the H:M-ratio) and $\langle a(H^-) \rangle$ and $\langle a(H) \rangle$ suitably averaged absorption coefficients of H^- and H . The Balmer discontinuity, D , and the color-effective temperature relation can be readily evaluated in terms of the known distribution, $\kappa_\lambda(H^-)/\bar{\kappa}$. Figures 13 and 14 present early attempts at interpreting the results of Chalonge and his associates, principally, Barbier, Kourganoff, and Mlle. René Cannavaglia.

This concludes my account of the crucial role that Daniel Chalonge played in resolving one of the great problems of modern astrophysics; and I am grateful for this opportunity to give an account of that role.

Fig. 1. Definitions of the Balmer discontinuity D and the gradient ϕ (slope of D) and ψ (slope of D).



Fig. 2. Comparison of the positions and the magnitudes of the discontinuity at the head of the Balmer series for a star with an extended atmosphere (ϵ Tau), a normal star (δ Cen) and a super-giant star (θ Cen).

continuous absorption in the solar atmosphere has indeed been identified. I return briefly to the interpretation of the results of Balmer and Chalmers on the Balmer discontinuity in an appendix to this report. The present investigation is based on the data of Balmer and Chalmers and cross-sections of H were known with sufficient precision in 1946. It was a relatively simple matter to revise Unsöld's earlier analysis for H γ and to determine the abundance of hydrogen from the analysis of the mean absorption-coefficients of H and H in the form:

$$\frac{1}{\kappa} = \frac{1}{\kappa_0} + \frac{1}{\kappa_1} \left(\frac{\lambda}{\lambda_0} \right)^2 + \frac{1}{\kappa_2} \left(\frac{\lambda}{\lambda_0} \right)^4$$

It is to be noted that from a comparison of mean values with the variation of the absorption-coefficients of H and H in the form:

$$\frac{1}{\kappa} = \frac{1}{\kappa_0} + \frac{1}{\kappa_1} \left(\frac{\lambda}{\lambda_0} \right)^2 + \frac{1}{\kappa_2} \left(\frac{\lambda}{\lambda_0} \right)^4$$

we can see that the variation of the absorption-coefficients of H and H in the form:

$$\frac{1}{\kappa} = \frac{1}{\kappa_0} + \frac{1}{\kappa_1} \left(\frac{\lambda}{\lambda_0} \right)^2 + \frac{1}{\kappa_2} \left(\frac{\lambda}{\lambda_0} \right)^4$$

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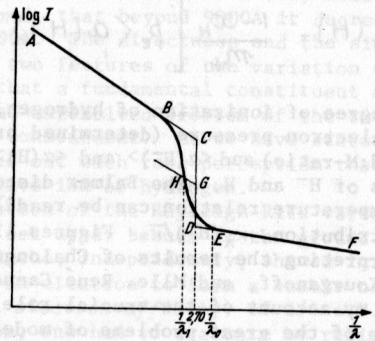
$$\frac{1}{\kappa} = \frac{1}{\kappa_0} + \frac{1}{\kappa_1} \left(\frac{\lambda}{\lambda_0} \right)^2 + \frac{1}{\kappa_2} \left(\frac{\lambda}{\lambda_0} \right)^4$$


Fig. 1. Definitions of the Balmer discontinuity $D(= CD)$ and the gradients ϕ_1 (slope of AC) and ϕ_2 (slope of DF).

By 1946, the mean absorption-coefficients of H γ were known with sufficient precision to provide a detailed comparison of the source of opacity in the solar atmosphere. Their method was the following. They considered levels in the solar atmosphere at various temperatures, T , and asked for the opacity of the overlying layers at various wavelengths. The results of their analysis are shown in Figure 12. (Their analysis did not include the contribution to the absorption coefficient by the "free-free" transition of electrons in the field of a neutral hydrogen atom; their analysis will account for the "balance" of the absorption). The analysis of the mean absorption-coefficients of H and H leaves no doubt that the source of the

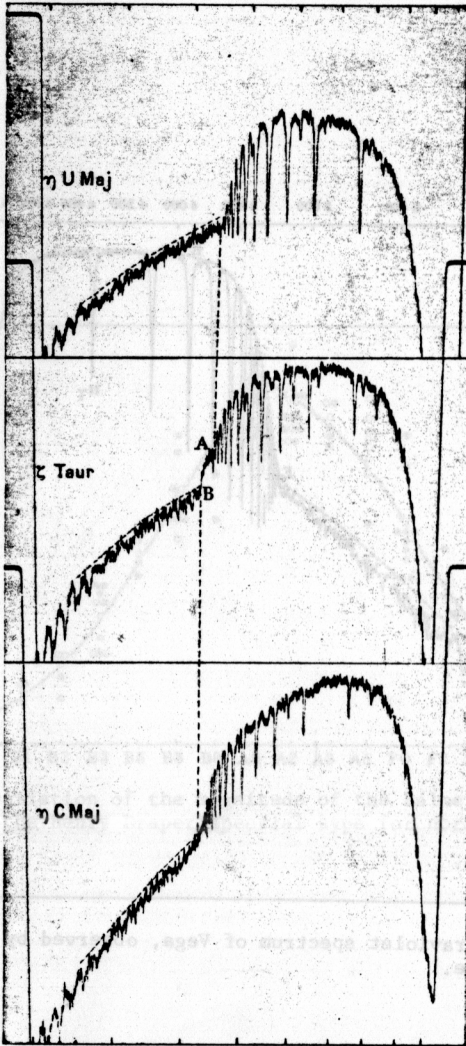


Fig. 2. Comparison of the positions and the magnitudes of the discontinuity at the head of the Balmer series for a star with an extended atmosphere (ζ Taur), a normal star (η U Maj) and a super-giant star (η C Maj).

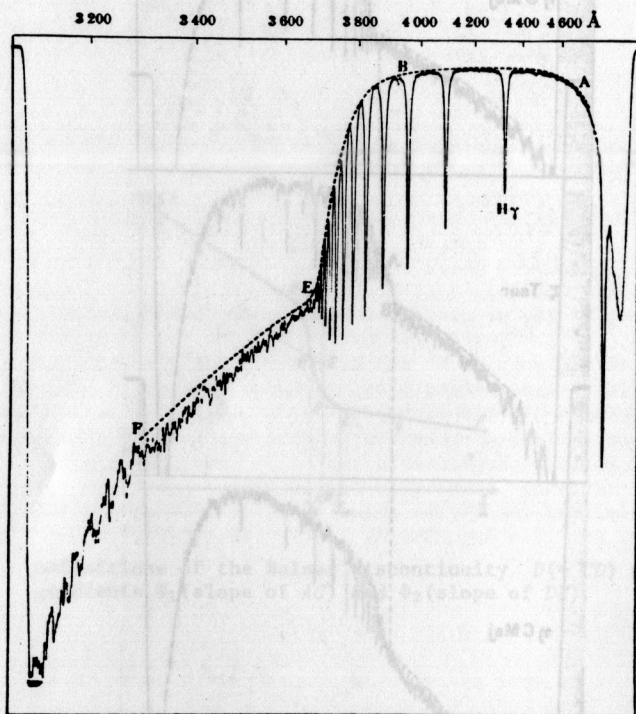


Fig. 3. The ultraviolet spectrum of Vega, observed by Barbier and Chalonge.

Fig. 2. Comparison of the positions and the magnitudes of the lines at the head of the Balmer series for a star with an extended atmosphere (Taur), a normal star (Ursa) and a super-giant star (Orion).

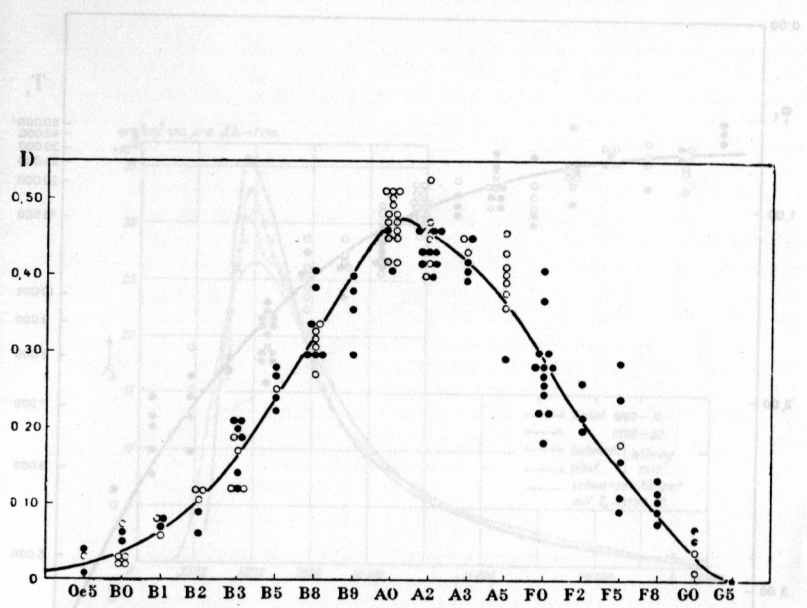


Fig. 4. The variation of the magnitude of the Balmer discontinuity, D , with the Henry Draper spectral type for normal stars.

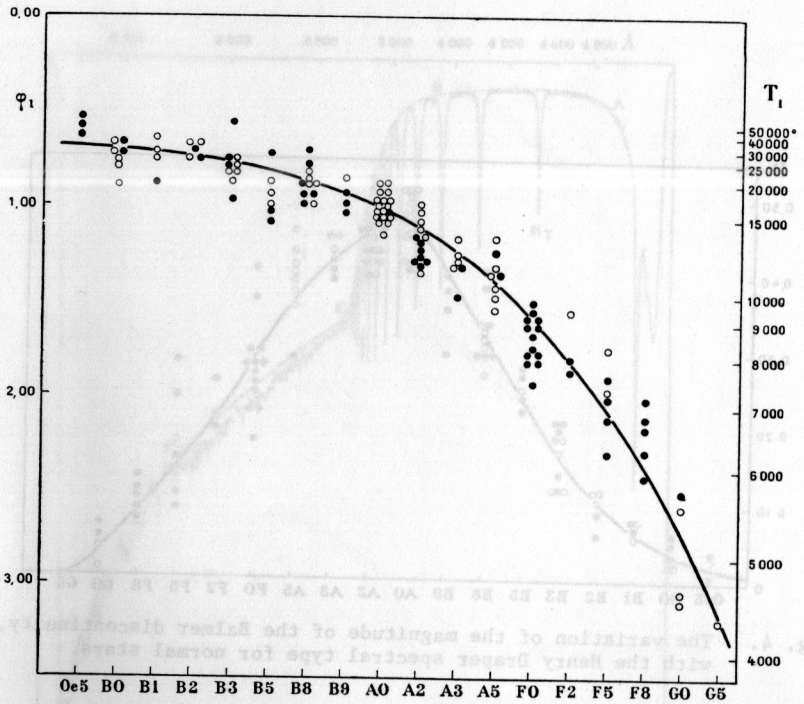


Fig. 5. The variation of the color temperatures of normal stars (in the spectral region 4600 to 3700Å) with the spectral type. (For the spectral types A to F the color temperatures were determined in the spectral region 4600 to 4000Å.)

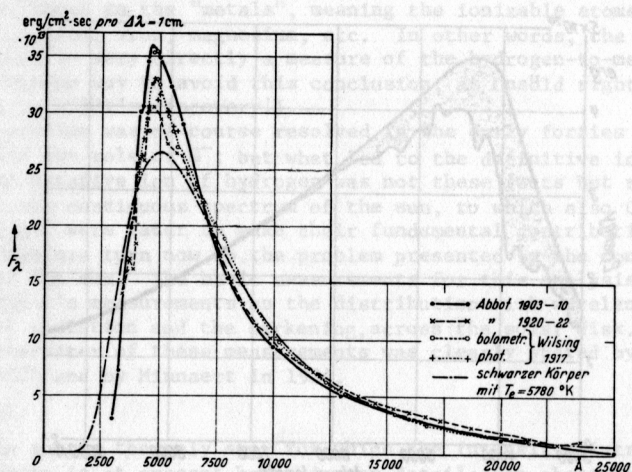


Fig. 6. Minnaert's reduction of Abbot's measurements (1903-1910 and 1920-1922) and Wilsing's bolometric and photometric measurements to a uniform system. The comparison with the black body curve is for $T = 5780$ K.

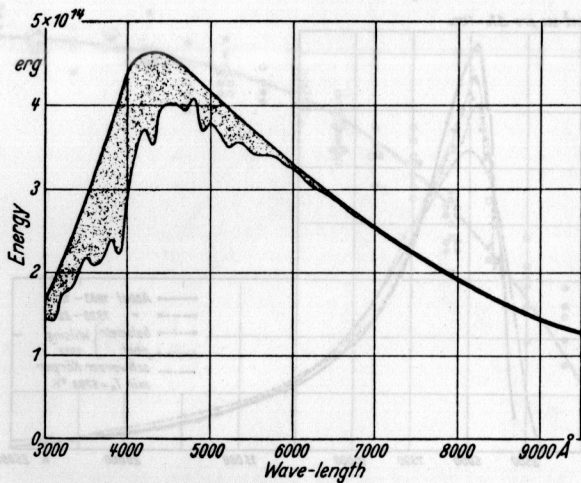


Fig. 7. Mulders's correction for line blanketing: the shaded area represents the flux removed by the crowding of the absorption lines in the solar spectrum.

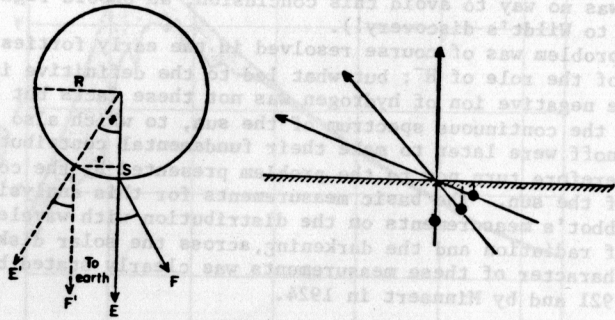


Fig. 8. Illustrating the fact that the darkening results from the angular dependence of the emergent radiation and on the temperature gradient in the atmosphere.

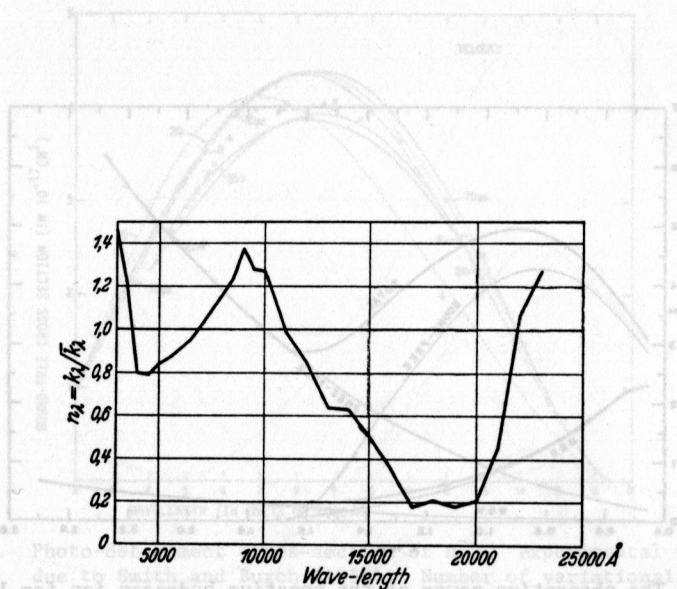


Fig. 9. Mulders's deduced variation of the solar continuous absorption-coefficient with wavelength.

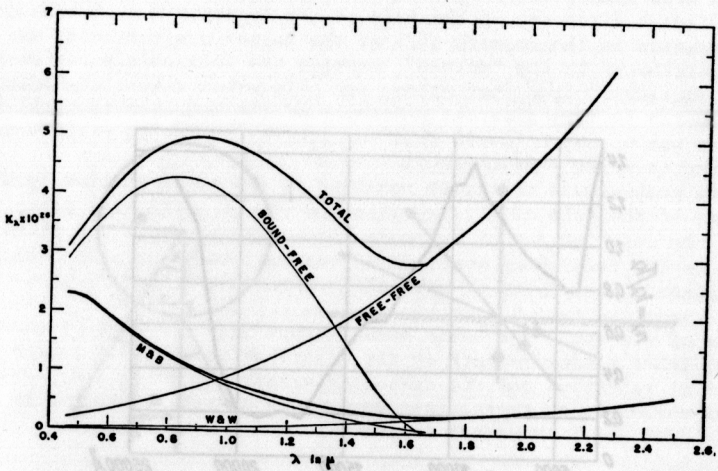


Fig. 10. The absorptive power of the negative hydrogen ion for light of various colors derived from physical theory. The curves are drawn appropriately for an atmosphere at a temperature of 6300° Kelvin and an electron pressure of one dyne/cm.² and per neutral hydrogen atom. The curves *M* and *B*, *W* and *W*, and their sum are the results of earlier determinations by Massey and Bates (1940) and Wheeler and Wildt (1942). The results of later determinations by Chandrasekhar and Breen (1946) are illustrated by the remaining curves.

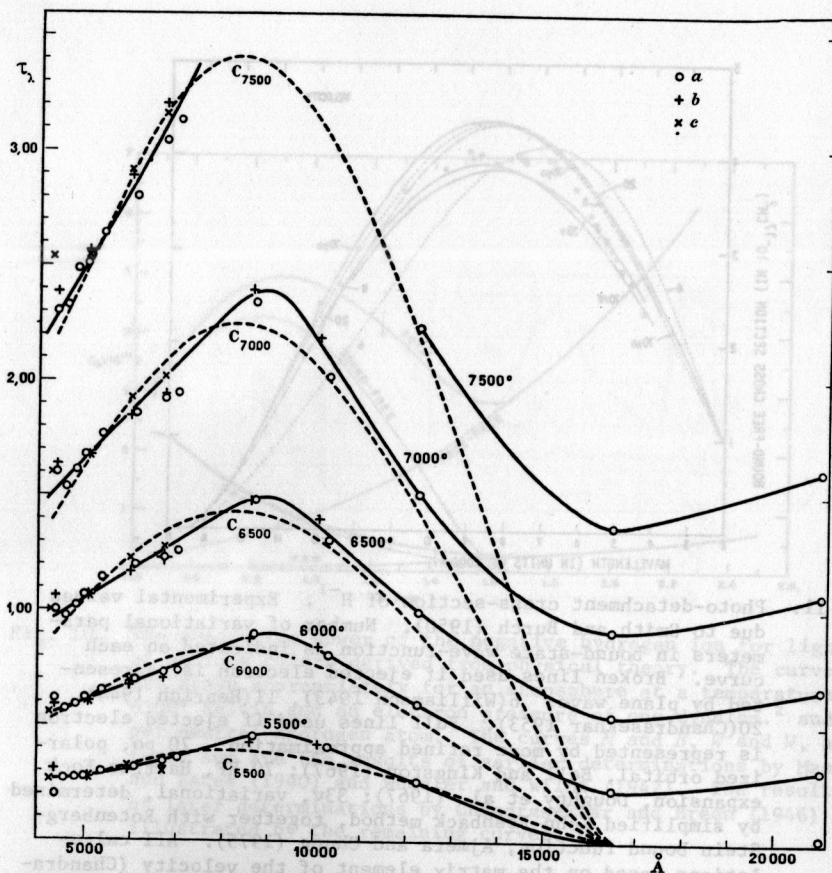


Fig. 12. Optical depth (τ_λ) of photospheric layers at different wavelengths (Chalange and Kourganoff 1946). Dotted lines are H⁻ absorption; right part of graph explained by free-free transitions.

II. The Early Universe

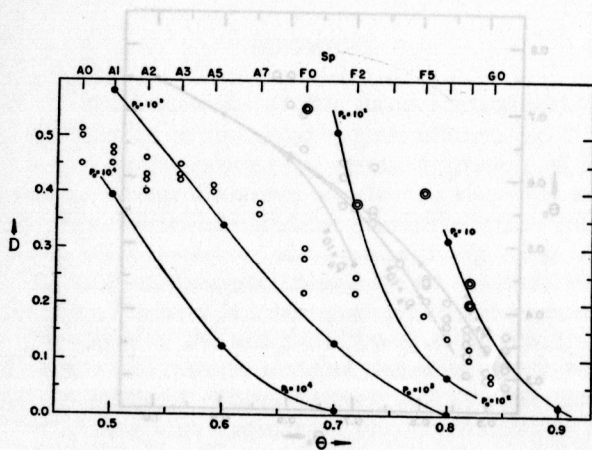


Fig. 13. The predicted discontinuities (D) at the head of the Balmer series for various effective temperatures and electron pressures. The circles represent the discontinuities as measured by Barbier and Chalonge. (The double circles represent the observations for supergiants.)

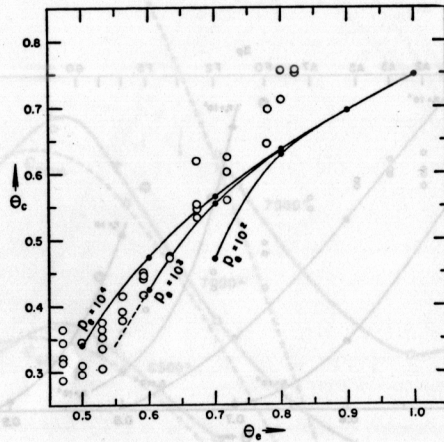


Fig. 14. Comparison of the observed and the theoretical color effective-temperature relations for the wave-length interval 4000-4600Å. The ordinates denote the reciprocal color temperatures, and the abscissae denote the reciprocal $\theta = 5040/T$. The circles represent the color determinations of Barbier and Chalonge for the wave-length interval 4000-4600 Å, reduced to the Morgan, Keenan, and Kellman system of spectral classification.

Fig. 12. Optical depth (τ_0) of photospheric layers at different wave-lengths (Chalonge and Keenan, 1946). Dotted lines are H absorption; right part of graph explained by free-free transitions.