Observable Consequences of Quantum Inflation

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What is inflation?

- It is a stage of *accelerated expansion* in the very early Universe.

- It explains the *homogeneity*, *isotropy* and *flatness* of the Universe; and the observed characteristics of the *cosmic microwave background*.

FRW space: $ds^2 = dt^2 - a^2(t)d\vec{x}^2$
Accelerated expansion, $[\ddot{a} = -(4\pi/3)(\epsilon + 3p)a]$

$$\ddot{a} > 0 \quad \Leftrightarrow \quad p/\epsilon < -\frac{1}{3} \quad \Leftrightarrow \quad 2\,\epsilon_{kinetic} < \epsilon_{potential}$$

In a quasi-De Sitter space ($a \simeq e^{H\,t}$), the present homogeneity, isotropy and flatness imply

$$\text{number of efolds:} \quad N_e \equiv ln\left(\frac{a_f}{a_0}\right) > 60$$

Some open questions in inflation

- Start: How inflation begins?
  = Which states lead to inflation?

- During: Is it valid the classical description of inflation? When?
  Which are the effects of the quantum corrections?, and which are their observable consequences?

- End: How inflation ends?
  Transition from inflation to radiation dominated epoch (reheating).

Classical Inflation: background

Homogeneous classical scalar field (matter) in a FRW-space,

$$\begin{aligned} \epsilon &= \dot\eta^2/2 + V(\eta) \qquad\qquad (\epsilon_0 \sim M_{Pl}^4) \\ p &= \dot\eta^2/2 - V(\eta) \\ \ddot\eta &+ 3\,H\,\dot\eta + V'(\eta) = 0\,; \qquad H^2 = \frac{8\pi}{3\,M_{Pl}^2}\,\epsilon \end{aligned}$$

- Classical chaotic inflation:

large V

. $\Rightarrow$ large $|\eta_0|$ (equivalent to a dense homogeneous "sea" of *zero* momentum particles).

. $\Rightarrow$ initial state breaks the symmetry $\eta \to -\eta$.

- Classical new inflation:

large $V \Rightarrow \eta$ close to an unstable equilibrium point of $V(\eta)$.

Slow roll condition, $|\dot\eta| \ll |\eta|$

. $\Rightarrow$ there is inflation and it last long.

Classical Inflation: perturbations

Metric and field backgrounds classical,
only the perturbations are quantized.

Evolution of coupled metric and field quantum perturbations in the classical background $\rightarrow$ Spectrum of primordial scalar and tensor perturbations for classical inflation

Quantum Field Foundations of Inflation

- Give a consistent quantum field treatment to inflation

- Find an initial state as general as possible that leads to inflation.

  Generalized slow-roll condition
  (and beyond).

- When the inflaton can be described by an effective classical field?

- Which are the effects of the quantum corrections?, and which are their observable consequences?

The Model

- Inflaton treated as a full quantum field
- Classical gravity with $\langle T^{\mu\nu} \rangle$ as source term (semiclassical gravity).

Large $\epsilon \Rightarrow$ quantum non-perturbative methods needed

N scalar fields $\vec{\Phi}$ with $V(\vec{\Phi}) = \frac{m^2}{2}\,\vec{\Phi}^2 + \frac{\lambda}{8\,N}\,\vec{\Phi}^4$ in the large N limit.

Homogeneous expectation value:

. $\langle \vec{\Phi} \rangle = (\sqrt{N}\,\eta(t),\ \vec{0})$

$\vec{\Phi}(\vec{x},t) = \langle \vec{\Phi} \rangle(t) + \vec{\pi}(\vec{x},t)$ where

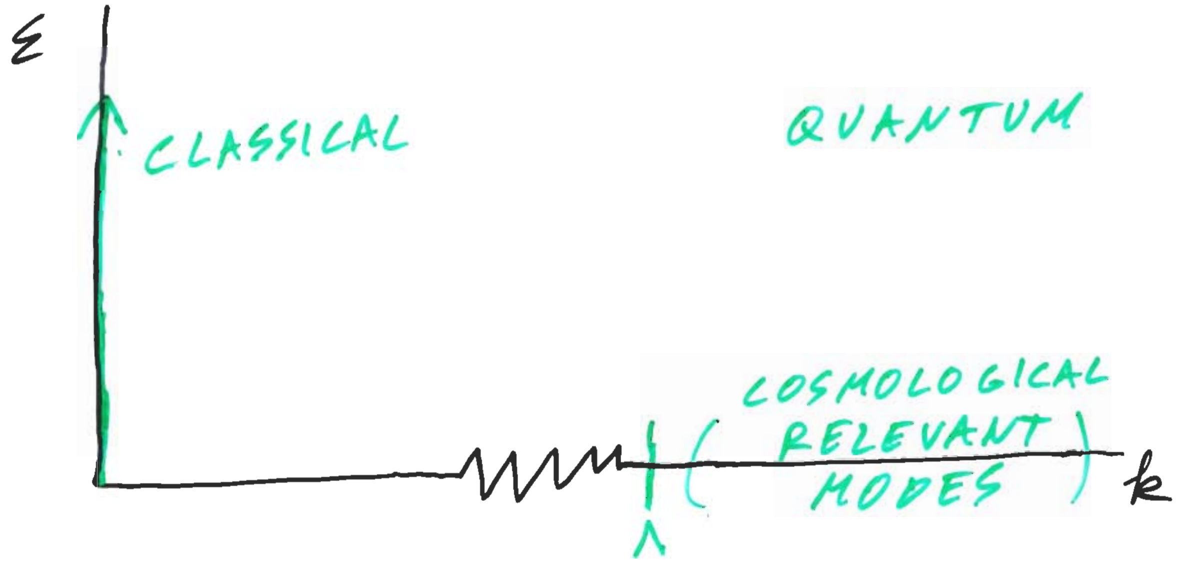
$\vec{\pi}(\vec{x},t) = \int \frac{d^3k}{\sqrt{2}(2\pi)^3} \left[\vec{a}_k\ f_k(t)\ e^{i\vec{k}\cdot\vec{x}} + \vec{a}_k^\dagger\ f_k^*(t)\ e^{-i\vec{k}\cdot\vec{x}} \right]$

f_k are the modes of the quantum fluctuations (they can be large).

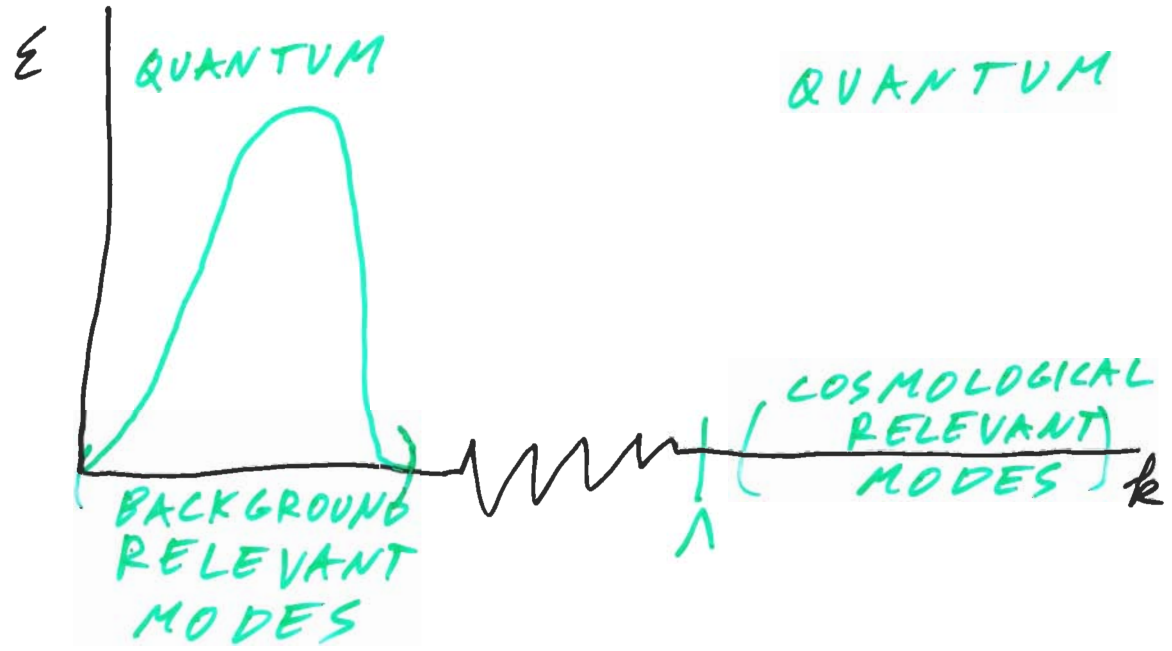
Allows the presence of homogeneous "seas" of *non-zero momentum* particles.

FRW space: $ds^2 = dt^2 - a^2(t) d\vec{x}^2$

CLASSICAL INFLATION



QUANTUM INFLATION



Evolution equations

$$\ddot{\eta} + 3\,H\,\dot{\eta} + \mathcal{M}^2\,\eta = 0$$

$$\underline{\underline{\ddot{f}_k + 3\,H\,\dot{f}_k + \left(\frac{k^2}{a^2} + \mathcal{M}^2\right)\,f_k = 0}}$$

(similar to damped oscillators)

with $$\mathcal{M}^2 = m^2 + \frac{\lambda}{2}\,\eta^2 + \underline{\underline{\frac{\lambda}{2}\int \frac{d^3k}{2(2\pi)^3}\,|f_k|^2}}$$

$$H^2 = \frac{8\pi}{3\,M_{Pl}^2}\,\epsilon$$

$$\begin{aligned}\frac{\epsilon}{N} \;=\; & \frac{1}{2}\,\dot{\eta}^2 + \frac{\mathcal{M}^4 - m^4}{2\lambda} + \frac{m^4}{2\lambda}\,\frac{1-\alpha}{2} \\ & \underline{\underline{+\frac{1}{4}\int \frac{d^3k}{(2\pi)^3}\left(|\dot{f}_k|^2 + \frac{k^2}{a^2}|f_k|^2\right)}}\end{aligned}$$

$$\alpha = \mathrm{sign}(m^2)$$

Which states gives rise to efficient inflation?

Generalized slow roll condition

$$\dot{\eta}^2+\int\frac{d^3k}{2(2\pi)^3}|\dot{f}_k|^2 \ll m^2\left(\eta^2+\int\frac{d^3k}{2(2\pi)^3}|f_k|^2\right)$$

$\Rightarrow$ there is inflation ($\ddot{a}>0$) and it last long.

(Includes the classical one: $|\dot{\eta}|\ll m|\eta|$)

Example in chaotic inflation: initial state with $\eta=0$ and energy concentrated in modes with $k\simeq k_0$ and

$$|\dot{f}_{k_0}(0)|\ll m|f_{k_0}(0)|$$

with a large total energy density, that makes the damping dominate ($H^2\propto\epsilon$).

The background dynamics

Two inflationary epochs in quantum inflation.

1) The pre-condensate epoch
During this epoch the term

$$D \equiv \frac{1}{4} \int \frac{d^3k}{(2\pi)^3} \frac{k^2}{a^2} |f_k|^2$$

gives and important contribution to the energy density.
Redshift: $k/a \to 0$ (dominant process)
This epoch ends when D becomes negligible.

2) The post-condensate quasi-De Sitter epoch
The enormous redshift assembles the modes into a zero mode condensate,

$$\eta_{eff}(t) = \sqrt{\eta^2 + \int \frac{d^3k}{2(2\pi)^3} |f_k|^2}$$

$$\ddot{\eta}_{ef} + 3\,H\,\dot{\eta}_{ef} + m^2\,\eta_{ef} + \frac{\lambda}{2}\,\eta_{ef}^3 = 0$$

Number of efolds

For fixed initial energy the number of efolds is less or equal than in classical inflation.

Ex. in chaotic inflation: When the quadratic term in the potential dominates ($V(\vec{\Phi}) \simeq \frac{m^2}{2}\vec{\Phi}^2$),

$$N_e \equiv ln\left(\frac{a_f}{a_0}\right) \simeq \frac{4\pi}{M_{Pl}^2 m^2}\frac{\epsilon_0}{1+(k_0/m)^2}$$

Number of efolds decrease with increasing k_0 (for fixed initial energy)
For typical values ($\epsilon_0 \sim 10^{-2}M_{Pl}^4$, $m \sim 10^{-4}M_{Pl}$) we have enough efolds even for hard momentum ($k \sim 80m$)

Primordial scalar perturbations in quantum inflation

Scalar metric perturbations are tightly coupled to the inflaton perturbations.

In more natural scenarios, the last 60 efolds in postcondensate epoch
$\Rightarrow$ dynamics of cosmologically relevant perturbations well approximated by that given by the effective classical inflaton background.

(Recall: form of the effective classical potential, and of the initial classical state determined by the underlying quantum field description.)

In this first approximation the classical inflation results for the spectrum of primordial scalar perturbations are recovered.

Corrections to this first approximation can be estimated.

Quantum corrections to the primordial scalar perturbations

- non-vacuum initial conditions after the pre-condensate epoch
- corrections to the mass from the modes $k > \Lambda$, and the $1/N$ corrections

$$\frac{|f_k(t)|^2}{|f_k^{cl\ inf}(t)|^2} \overset{k/a \gg m}{=} 1 + \overbrace{\mathcal{O}\left(\frac{a^2(t)\ m^2}{k^2}\right)}^{\text{ini. cond.}} + \\ + \overbrace{\mathcal{O}\left(\frac{a^2(t)}{k^2}\,\delta\mathcal{M}^2_\Lambda(t)\right)}^{k>\Lambda\ \text{contribution}} + \overbrace{\mathcal{O}\left(\frac{a^2(t)}{k^2}\,\delta\mathcal{M}^2_N(t)\right)}^{1/N\ \text{corrections}}$$

when the modes exited the horizon $k/a \sim H$

$$= 1 + \mathcal{O}\left(\frac{m^2}{H^2}\right) + \mathcal{O}\left(\frac{\lambda\ a^2(t)\ m^4}{H^2\ \Lambda^2}\right) + \mathcal{O}\left(\frac{m^2}{N\ H^2}\right)$$

50 efolds before the end of inflation $m/H \sim 1/5$

$$= 1 + \overbrace{0.04}^{4\%} + (\ll 10^{-9}) + \overbrace{0.04}^{4\%}\ \ (\text{for } N = 1)$$

Tensor perturbations in quantum inflation

The amplitude of tensor perturbations is determined only by the background evolution, which after the condensate formation has an *effective classical description*.

This gives a correspondence between the quantum and the classical inflation results for the spectrum of primordial tensor perturbations.

Conclusions: new answers found

• Start: How inflation begins?
= Which states lead to inflation?

Generalized slow-roll condition (the initial states can be more general than was thinked before)

• During: Is it valid the classical description of inflation? When?

Yes, during the the postcondensate epoch

Which are the effects and the observable consequences of the quantum corrections?

They can be up to 4% due to $1/N$ corrections to the large N approximation, and also up to 4% due to initial conditions.

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