# Quantum slow-roll and quantum fast-roll inflation:

## CMB quadrupole suppression and the low CMB multipoles

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## Outline

## What is inflation?

- It is a stage of *accelerated expansion* in the very early Universe.
- It explains the *homogeneity*, *isotropy* and *flatness* of the Universe; and the observed characteristics of the *cosmic microwave background*.

FRW space: $ds^2 = dt^2 - a^2(t) d\vec{x}^2$
Accelerated expansion, [$\ddot{a} = -(4\pi/3)(\epsilon + 3p)a$]

$$\ddot{a} > 0 \quad \Leftrightarrow \quad p/\epsilon < -\frac{1}{3} \quad \Leftrightarrow \quad 2\ \epsilon_{kinetic} < \epsilon_{potential}$$

In a quasi-De Sitter space ($a \simeq e^{H\,t}$), the present homogeneity, isotropy and flatness imply

$$\text{number of efolds:} \quad N_e \equiv ln\left(\frac{a_f}{a_0}\right) > 60$$

**Some open questions in inflation**

- Start: How inflation begins?
  = Which states lead to inflation?
- During: Is it valid the classical description of inflation? When?
  Which are the effects of the quantum corrections?, and which are their observable consequences?
- End: How inflation ends?
  Transition from inflation to radiation dominated epoch (reheating).

## Classical Inflation: background

Homogeneous classical scalar field (matter) in a FRW-space,

$$\begin{aligned} \rho &= \dot{\varphi}^2/2 + V(\varphi) \qquad (\rho_0 \lesssim 10^{-2} M_{Pl}^4) \\ p &= \dot{\varphi}^2/2 - V(\varphi) \\ \ddot{\varphi} &+ 3\,H\,\dot{\varphi} + V'(\varphi) = 0\ ; \quad H^2 = \frac{8\pi}{3\,M_{Pl}^2}\,\rho \end{aligned}$$

- Classical chaotic inflation:
  large $V \Rightarrow$ large $|\varphi_0|$ (equivalent to a dense homogeneous "sea" of *zero* momentum particles). $\Rightarrow$ initial state breaks the symmetry $\varphi \to -\varphi$.
- Classical new inflation:
  large $V \Rightarrow \varphi$ close to an unstable equilibrium point of $V(\varphi)$.

*Slow roll condition*, $|\dot{\varphi}| \ll |\varphi| \Rightarrow$ there is inflation and it last long.

**Classical Inflation: perturbations**

Metric and field backgrounds classical, only the perturbations are quantized.

Evolution of coupled metric and field quantum perturbations in the classical background $\rightarrow$ Spectrum of primordial scalar and tensor perturbations for classical inflation

## Quantum Field Foundations of Inflation

- Give a consistent quantum field treatment to inflation
- Find an initial state as general as possible that leads to inflation.
  Quantum or generalized slow roll condition
  and beyond: quantum fast roll initial conditions
- When the inflaton can be described by an effective classical field?
- Which are the effects of the quantum corrections?, and which are their observable consequences?

**The Model**

- Inflaton treated as a full quantum field
- Classical gravity with $\langle T^{\mu\nu}\rangle$ as source term (semiclassical gravity).

Large $\rho \Rightarrow$ quantum non-perturbative methods needed

$N$ scalar fields $\vec{\Phi}$ with $V(\vec{\Phi}) = \frac{m^2}{2}\,\vec{\Phi}^2 + \frac{\lambda}{8N}\,\vec{\Phi}^4$ in the large $N$ limit.

Homogeneous expectation value:

. $\langle\vec{\Phi}\rangle = (\sqrt{N}\,\varphi(t),\ \vec{0})$

$\vec{\Phi}(\vec{x},t) = \langle\vec{\Phi}\rangle(t) + \vec{\pi}(\vec{x},t)$ where

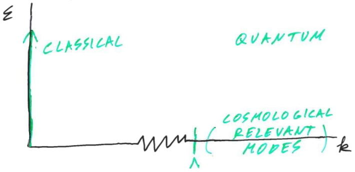
$$\vec{\pi}(\vec{x},t) = \int \frac{d^3k}{\sqrt{2}(2\pi)^3}\left[\vec{a}_k\, f_k(t)\, e^{i\vec{k}\cdot\vec{x}} + \vec{a}_k^\dagger\, f_k^*(t)\, e^{-i\vec{k}\cdot\vec{x}}\right]$$

$f_k$ are the modes of the quantum fluctuations (they can be large).

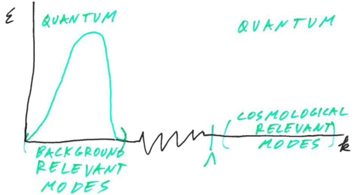
Allows the presence of homogeneous "seas" of *non-zero momentum* particles.

FRW space: $ds^2 = dt^2 - a^2(t)d\vec{x}^2$

CLASSICAL INFLATION




QUANTUM INFLATION



## Evolution equations

$$\ddot{\varphi} + 3\,H\,\dot{\varphi} + \mathcal{M}^2\,\varphi = 0$$

$$\underline{\underline{\ddot{f}_k + 3\,H\,\dot{f}_k + \left(\frac{k^2}{a^2} + \mathcal{M}^2\right)\,f_k = 0}}$$

(similar to damped oscillators)

with $\mathcal{M}^2 = m^2 + \frac{\lambda}{2}\,\varphi^2 + \underline{\underline{\frac{\lambda}{2}\int \frac{d^3k}{2(2\pi)^3}\,|f_k|^2}}$ and $H^2 = \frac{8\pi}{3\,M_{Pl}^2}\,\rho$

$$\frac{\rho}{N} = \frac{1}{2}\,\dot{\varphi}^2 + \frac{\mathcal{M}^4 - m^4}{2\lambda} + \frac{m^4}{2\lambda}\,\frac{1-\alpha}{2} + \underline{\underline{\frac{1}{4}\int \frac{d^3k}{(2\pi)^3}\left(|\dot{f}_k|^2 + \frac{k^2}{a^2}|f_k|^2\right)}}$$

$$\alpha = \text{sign}(m^2)$$

**Which states gives rise to efficient inflation?**
*Generalized slow roll condition*

$$\dot{\varphi}^2 + \int \frac{d^3k}{2(2\pi)^3} |\dot{f}_k|^2 \ll m^2 \left( \varphi^2 + \int \frac{d^3k}{2(2\pi)^3} |f_k|^2 \right)$$

$\Rightarrow$ there is inflation ($\ddot{a} > 0$) and it last long.
(Includes the classical one: $|\dot{\varphi}| \ll m|\varphi|$)
Example in chaotic inflation: initial state with $\varphi = 0$ and energy concentrated in modes with $k \simeq k_0$ and

$$|\dot{f}_{k_0}(0)| \ll m|f_{k_0}(0)|$$

with a large total energy density, that makes the damping dominate ($H^2 \propto \rho$).

**The background dynamics**

*Two inflationary epochs* in quantum inflation.

**1) The pre-condensate epoch**

During this epoch the term $D \equiv \frac{1}{4} \int \frac{d^3k}{(2\pi)^3} \; \frac{k^2}{a^2} |f_k|^2$ gives and important contribution to the energy density.

Redshift: $k/a \to 0$ (dominant process)

This epoch ends when D becomes negligible.

**2) The post-condensate quasi-De Sitter epoch**

The enormous redshift assembles the modes into a zero mode condensate,

$$\varphi_{eff}(t) = \sqrt{\varphi^2 + \int \frac{d^3k}{2(2\pi)^3} \, |f_k|^2}$$

$$\ddot{\varphi}_{eff} + 3\,H\,\dot{\varphi}_{eff} + m^2\,\varphi_{eff} + \frac{\lambda}{2}\,\varphi^3_{eff} = 0$$

**Number of efolds**

For fixed initial energy the number of efolds is less or equal than in classical inflation.

Ex. in chaotic inflation: When the quadratic term in the potential dominates ($V(\vec{\Phi}) \simeq \frac{m^2}{2}\,\vec{\Phi}^2$),

$$N_e \equiv ln\left(\frac{a_f}{a_0}\right) \simeq \frac{4\pi}{M_{Pl}^2\,m^2}\,\frac{\rho_0}{1+(k_0/m)^2}$$

Number of efolds decrease with increasing $k_0$ (for fixed initial energy)

For typical values ($\rho_0 \sim 10^{-2}M_{Pl}^4$, $m \sim 10^{-4}M_{Pl}$) we have enough efolds even for hard momentum ($k \sim 80m$)

## Beyond generalized slow roll initial conditions: Quantum fast roll initial conditions

Quantum fast roll initial conditions = quantum initial conditions that do not verify the generalized slow roll condition.

In general zero and non-zero momentum modes excited, which do not verify slow roll. (Includes classical fast roll as a particular case)

We have found that they can lead to inflation that last long enough, even when their kinetic energy is of the order of their initial potential energy.

If the energy density is large, the kinetic terms are rapidly overdamped ($H^2 \propto \rho$).

In particular, for quantum chaotic inflation fast roll initial conditions are obtained if

$$\dot{f}_k(0) \sim \sqrt{k^2/a^2(0) + \mathcal{M}^2(0)} f_k(0)$$

**Primordial scalar perturbations in quantum inflation**

Scalar metric perturbations are tightly coupled to the inflaton perturbations.

In more natural scenarios, the last 55 efolds in postcondensate epoch

$\Rightarrow$ dynamics of cosmologically relevant perturbations well approximated by that given by the effective classical inflaton background.

(Recall: form of the effective classical potential, and of the initial classical state determined by the underlying quantum field description.)

In this first approximation the classical inflation results for the spectrum of primordial scalar perturbations are recovered.

Corrections to this first approximation can be computed.

## Quantum corrections to the primordial scalar perturbations: order of magnitude estimation

- non-vacuum initial conditions after the precondensate epoch
- corrections to the mass from modes $k > \Lambda$, and $1/N$ corrections

$$\frac{|f_k|^2}{|f_k^{cl\ inf}|^2} \overset{k/a \gg m}{=} 1 + \frac{a^2 m^2}{k^2} [ \overbrace{\mathcal{O}(1)}^{\text{ini. cond.}} + \overbrace{\mathcal{O}\left(\frac{\delta\mathcal{M}_\Lambda^2}{m^2}\right)}^{k>\Lambda \text{ contrib.}} + \overbrace{\mathcal{O}\left(\frac{\delta\mathcal{M}_N^2}{m^2}\right)}^{1/N \text{ corr.}} ]$$

when the modes exited the horizon $k/a \sim H$

$$= 1 + \frac{m^2}{H^2}\left[\mathcal{O}(1) + \mathcal{O}\left(\frac{\lambda\, a^2(t)\, m^2}{\Lambda^2}\right) + \mathcal{O}\left(\frac{1}{N}\right)\right]$$

50 efolds before the end of inflation $m/H \sim 1/5$

$$= 1 + \overbrace{0.04}^{4\%} + (\ll 10^{-9}) + \overbrace{0.04}^{4\%} \ \ (\text{for } N \sim 1)$$

**Tensor perturbations in quantum inflation**

The amplitude of tensor perturbations is determined only by the background evolution, which after the condensate formation has an *effective classical description*.

In first approximation the classical inflation results for the spectrum of primordial tensor perturbations are also recovered. Corrections to this first approximation can be estimated.

## Corrections to the spectrum of perturbations: computation

The evolution equation for the modes of the curvature perturbations $S_{\mathcal{R}}$

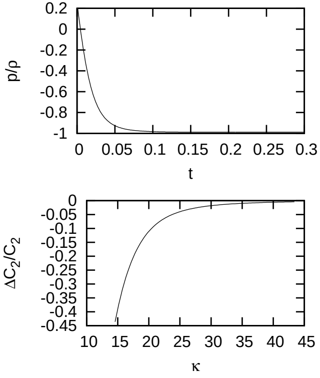
$$\left[\frac{d^2}{d\eta^2} + k^2 - W_{\mathcal{R}}(\eta)\right] S_{\mathcal{R}}(k;\eta) = 0$$

$\eta$ conformal time, $W_{\mathcal{R}} = \frac{1}{z}\frac{d^2 z}{d\eta^2}$, and $z = a\frac{\dot{\varphi}_{eff}}{H}$.
Corrections with respect to classical slow roll due to $\mathcal{V}_{\mathcal{R}} = W_{\mathcal{R}} - W_{\mathcal{R}}^{sr}$. Positive: suppression, negative: enhancement.
Analogously tensor perturbation modes $S_T$ verify

$$\left[\frac{d^2}{d\eta^2} + k^2 - W_T(\eta)\right] S_T(k;\eta) = 0$$

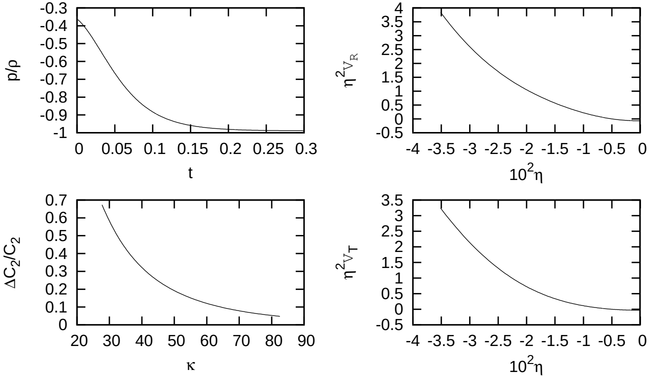
with $W_T = C''(\eta)/C(\eta)$ and $C(\eta) = a(t(\eta))$.
Corrections are given by $\mathcal{V}_T = W_T - W_T^{sr}$

## Corrections due to the fast roll epoch



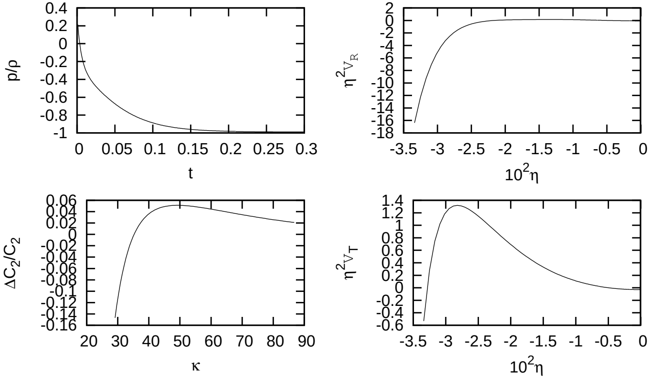

Quantum chaotic inflation with *quantum fast roll initial conditions* with excited modes around $k \ll m$.

## Corrections due to the precondensate epoch



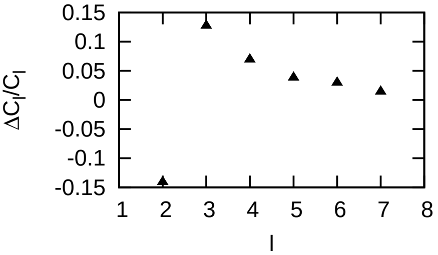

Quantum chaotic inflation with *quantum slow roll initial conditions* with excited modes around $k \sim 8m$ with a characteristic width of $\Delta k \sim 0.8m$.

## Corrections due to the fast roll and precondensate




Quantum chaotic inflation with *quantum fast roll initial conditions* with excited modes around $k \sim 8m$ with a characteristic width of $\Delta k \sim 0.8m$.

## Corrections due to the fast roll and precondensate: effects in the low multipoles of the CMB




Corrections to the CMB multipoles obtained for quantum chaotic inflation with *quantum fast roll initial conditions* with excited modes around $k \sim 8m$ with a characteristic width of $\Delta k \sim 0.8m$.

**Conclusions: new answers found**

• Start: How inflation begins? = Which states lead to inflation?
*The initial states can be more general than was thinked before.*
*Generalized slow roll condition and beyond: quantum fast roll initial conditions*

• During: Is it valid the classical description of inflation? When?
*Yes, during the the postcondensate epoch*
Which are the effects and the observable consequences of the corrections?
*They can be up to the order of a few percent.*
*Those coming from fast roll or precondensate epochs are more important for low multipoles of the CMB, and they can be computed qualitatively.*

• Work in collaboration with H. J. de Vega, N. G. Sanchez, and D. Boyanovsky.