

# The low quadrupole: Theoretical issues and MCMC data analysis

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*Physics of the Standard Model of the Universe: Theory and Observations*, Colloquium at the Colegio de España, Cité Internationale Universitaire de Paris, 75014 Paris

## Outline

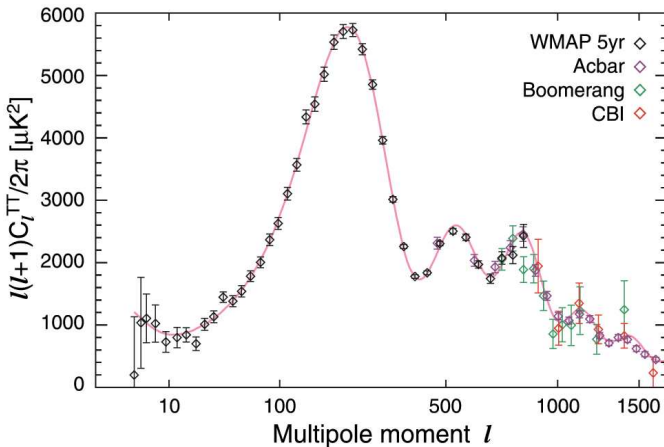
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  - Observational data
  - Cosmic variance
  - Independent random variables
- 2 Theoretical setup
  - EFT of Inflation
  - New inflation
  - On the issue of initial conditions
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  - Cosmological MCMC
  - MCMC likelihoods
  - Best fit comparisons

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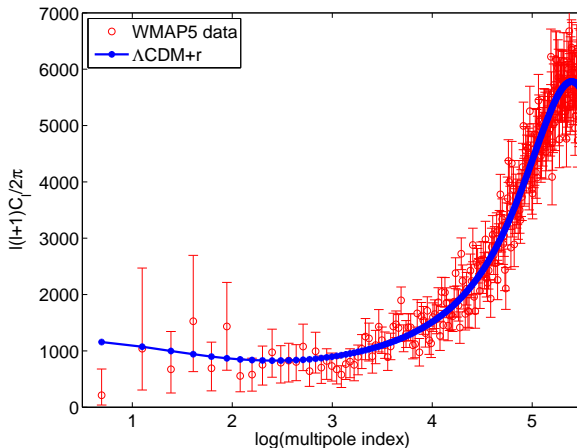
## The WMAP+small scale TT multipoles (binned)

from "M. R.olta et al.", arXiv:0803.0593 [astro-ph] 5 Mar 2008



$C_2 = 223 \mu K^2$  (WMAP5 ML value) ,  $C_2 \simeq 1200 \mu K^2$  ( $\Lambda$ CDM)

## WMAP5 unbinned $C_\ell$ for $\ell \leq 250$



(stat. exp. error)/(cosmic variance) = 20% at  $\ell = 250$

## Other analysis of WMAP5 data

- ...
- P.K. Samal, R. Saha, J. Delabrouille, S. Prunet, P. Jain, T. Souradeep, *"CMB Polarization and Temperature Power Spectra Estimation using Linear Combination of WMAP 5-year Maps"*, arXiv:0903.3634

$$C_2 = 557 \mu K^2 \text{ (WMAP5+150\%)} \quad , \quad C_3 = 306 \mu K^2 \text{ (WMAP5-40\%)}$$

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$$C_2, C_3, C_4 \rightarrow 0 \quad , \quad C_2, C_3, C_4, C_5, C_6 \rightarrow \text{(WMAP5-50\%)}$$

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## Neglecting all uncertainties but cosmic variance:

Let  $X_\ell = C_\ell^{(data)} / C_\ell^{(model)}$ ; then

$Pr(X_\ell = x | model) \propto \frac{1}{x} (xe^{-x})^{\ell+1/2}$  (reduced chi-square distribution) is the probability density for  $C_\ell^{(data)}$  given the model, with

$$\langle X_\ell \rangle = 1 \text{ and } (X_\ell)_{ML} = \frac{2\ell - 1}{2\ell + 1}$$

At the same time, if  $Y_\ell = 1/X_\ell = C_\ell^{(model)} / C_\ell^{(data)}$ , then

$Pr(Y_\ell = y | data) \propto \left( e^{-1/y} / y \right)^{\ell+1/2}$  is the probability density for  $C_\ell^{(model)}$  given the data (assuming flat priors), with

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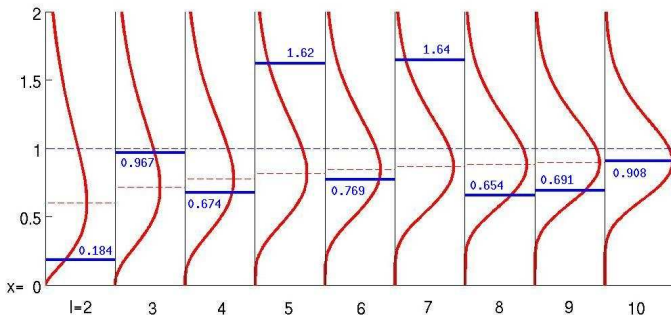
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## An example: lowest 9 TT multipoles

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probability curves from best fit  $\Lambda$ CDM  
 WMAP5 data



$$\text{Prob}[C_2^{(data)} < 0.184 C_2^{(model)}] \simeq 0.031 \dots$$

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Let  $p_\ell = \Pr(X_\ell \leq x | \text{model})$  (recall  $X_\ell = C_\ell^{(\text{data})} / C_\ell^{(\text{model})}$ ), then

all  $p_\ell$  are independent random numbers flatly distributed in  $(0, 1)$

$$\Pr[\text{there are } k \text{ of the first } n p_\ell \text{ in } (0, p)] = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\langle k \rangle = pn \quad (\Delta k)^2 = p(1-p)n$$

In the first 250 multipoles we expect (to  $1\sigma$ ) up to 15  $C_\ell^{(\text{data})}$  so low w.r.t.  $C_\ell^{(\text{model})}$  to have a probability less than 0.031

$$p_\ell < 0.031$$

2 22 48 54 72 84 98 105 113 114  
 120 124 149 181 195 209 228 234 249

$$p_\ell > 1 - 0.031$$

69 73 83 117

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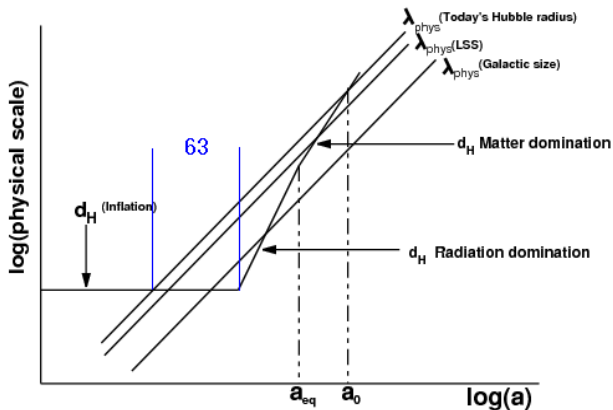
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## Inflation essentials

### Early accelerated expansion of the Universe

$$ds^2 = dt^2 - a(t)dx^2, \quad \ddot{a} > 0$$



## EFT of (single field) inflation à la Ginsburg-Landau

D. Boyanowski, C.D., H.J. de Vega, N. Sanchez, arXiv:0901.0549, to appear on IJMP

Inflaton potential ( $\hbar = 1$ ,  $c = 1$ ,  $M_{PL} = 2.4 \times 10^{18}$  GeV)

$$V(\varphi) = M^4 v(\varphi), \quad \varphi = \frac{\phi}{M_{PL}}$$

Energy scale of inflation and inflaton mass

$$M \simeq 0.57 \times 10^{16} \text{ GeV} \sim M_{\text{GUT}}, \quad m = M^2/M_{PL} \sim 1.3 \times 10^{13} \text{ GeV}$$

Hubble parameter and quantum corrections

$$H \sim 7m \ll M_{PL}, \quad \text{loops} \rightarrow (H/M_{PL})^2 \sim 10^{-9}$$

Number of inflation e-folds since horizon exit

$$N = \log \frac{a(t_{\text{end}})}{a(t_{\text{exit}})}, \quad v(\phi_{\text{end}}) = v'(\phi_{\text{end}}) = 0$$

 $t_{\text{exit}}$ : the mode with comoving  $k_0$  becomes superhorizon ( $\rightarrow N = N(k_0)$ )

$$\text{WMAP: } k_0 = 2 \text{ Gpc}^{-1}, \quad N \simeq 61$$

$$\text{CosmoMC: } k_0 = 50 \text{ Gpc}^{-1}, \quad N \simeq 57$$

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The number of e-folds  $N$  (coarse-grained: constant  $H$ , sharp  $\Delta D \rightarrow RD$ )

Friedmann's equation right after inflation

$$\frac{H^2}{H_0^2} = \Omega_\Lambda + \frac{\Omega_m}{a^3} + \frac{\Omega_r}{a^4} \simeq \frac{\Omega_r}{a^4}, \quad \frac{H}{H_0} \simeq 4.2 \times 10^{55}, \quad (H \simeq 2.8 \times 10^{-5} M_{Pl})$$

$$\Rightarrow \quad \log \frac{1}{a} \simeq \frac{1}{2} \log \frac{H}{H_0 \sqrt{\Omega_r}}$$

Inflation e-folds for the quadrupole scale  $k_Q$

(today  $k_Q = 1.014 H_0 = 0.238 \text{ Gpc}^{-1} = 1.52 \times 10^{-42} \text{ GeV}$ )

$$N_Q \simeq \log \frac{H}{H_0} - \log \frac{1}{a} \simeq \log \frac{H \sqrt{\Omega_r}}{H_0} \simeq 62$$

$$N_Q \simeq 63 + \frac{1}{2} \log \left( \frac{10^{-4} M_{Pl}}{H} \right)$$

$$N = N_Q - \log \frac{k_0}{k_Q} \simeq 57, \quad k_0 = 0.05 \text{ Mpc}^{-1} \text{ (CosmoMC)}$$

The number of e-folds  $N$  (coarse-grained: constant  $H$ , sharp  $\Delta D \rightarrow RD$ )

Friedmann's equation right after inflation

$$\frac{H^2}{H_0^2} = \Omega_\Lambda + \frac{\Omega_m}{a^3} + \frac{\Omega_r}{a^4} \simeq \frac{\Omega_r}{a^4}, \quad \frac{H}{H_0} \simeq 4.2 \times 10^{55}, \quad (H \simeq 2.8 \times 10^{-5} M_{Pl})$$

$$\Rightarrow \quad \log \frac{1}{a} \simeq \frac{1}{2} \log \frac{H}{H_0 \sqrt{\Omega_r}}$$

Inflation e-folds for the quadrupole scale  $k_Q$

(today  $k_Q = 1.014 H_0 = 0.238 \text{ Gpc}^{-1} = 1.52 \times 10^{-42} \text{ GeV}$ )

$$N_Q \simeq \log \frac{H}{H_0} - \log \frac{1}{a} \simeq \log \frac{H \sqrt{\Omega_r}}{H_0} \simeq 62$$

$$N_Q \simeq 63 + \frac{1}{2} \log \left( \frac{10^{-4} M_{Pl}}{H} \right)$$

$$N = N_Q - \log \frac{k_0}{k_Q} \simeq 57, \quad k_0 = 0.05 \text{ Mpc}^{-1} \text{ (CosmoMC)}$$

Dimensionless setup:  $t$  in units of  $m^{-1}$ ,  $H = hm$

### Equations of motion

$$h^2 = \frac{1}{3} \left[ \frac{1}{2} \dot{\phi}^2 + v(\phi) \right], \quad \ddot{\phi} + 3h\dot{\phi} + v'(\phi) = 0, \quad \dot{h} = -\frac{1}{2} \dot{\phi}^2$$

### Energy density and pressure

$$\varepsilon = M^4 \left[ \frac{1}{2} \dot{\phi}^2 + v(\phi) \right], \quad p = M^4 \left[ \frac{1}{2} \dot{\phi}^2 - v(\phi) \right]$$

### Fast-roll vs. slow-roll

$$\frac{1}{2} \dot{\phi}^2 \sim v(\phi), \quad \frac{1}{2} \dot{\phi}^2 \ll v(\phi)$$

which potential  $v(\phi)$  ?

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## Outline

- 1 Is the low CMB TT quadruple too low?
  - Observational data
  - Cosmic variance
  - Independent random variables
- 2 **Theoretical setup**
  - EFT of Inflation
  - **New inflation**
  - On the issue of initial conditions
- 3 MCMC analysis
  - Cosmological MCMC
  - MCMC likelihoods
  - Best fit comparisons



MCMC analysis of current data plus Ginsburg-Landau stability point to double-well type potentials with the inflaton  $\phi$  rolling from a region of negative curvature near  $\phi = 0$  (the “false vacuum”) toward the true absolute minimum  $\phi_{min}$  of the potential where  $v(\phi_{min}) = v'(\phi_{min}) = 0$ .

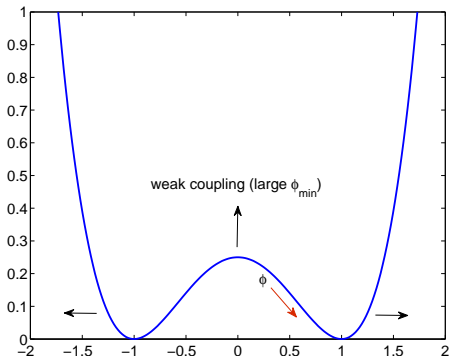
In general

$$v(\phi) = \phi_{min}^2 F(\phi/\phi_{min})$$

with  $F'(x) \simeq x$  as  $x \rightarrow 0$ .

For instance BNI  
(Binomial New Inflation)

$$F(x) = \frac{1}{4}(x^2 - 1)^2$$

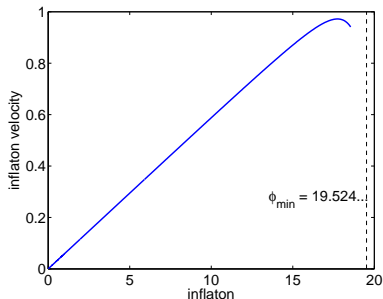


## The extreme slow-roll solution (a sort of half de Sitter)

$$\ddot{\phi} + 3h\dot{\phi} + \phi = 0$$

$$\phi \propto \exp(\alpha t), \quad t \rightarrow -\infty$$

$$\alpha = \frac{1}{2} \left[ (\sqrt{3v(0)+4} - \sqrt{3v(0)}) \right]$$



	start	$a = 1$	end: $\ddot{a} = 0^+$
$t$	$-344.9514017\dots$	0	$17.40482446\dots$
$\phi$	$10^{-8}$	$6.7484118\dots$	$18.5586530\dots$
$\dot{\phi}$	$\alpha 10^{-8} = 5.89371084\dots 10^{-10}$	$0.3973384\dots$	$0.94150557\dots$
$\log a$	$-1938.4867948\dots$	0	60
$h$	$(12g)^{-1/2} = 5.6361006\dots$	$4.9653973\dots$	$0.6657449\dots$
$\eta$	$-\infty$ (f.a.p.p)	$-0.2020610\dots$	0

Singular solutions as  $t \rightarrow t_*^+$ 

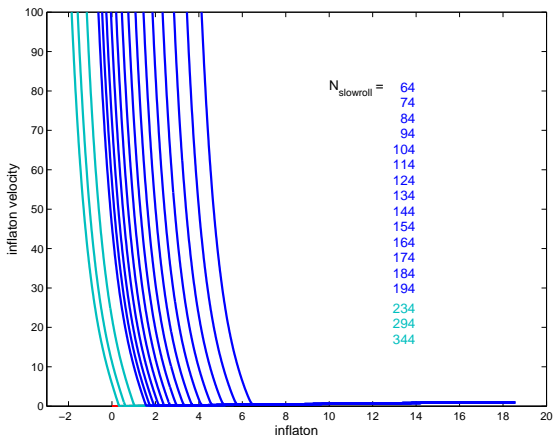
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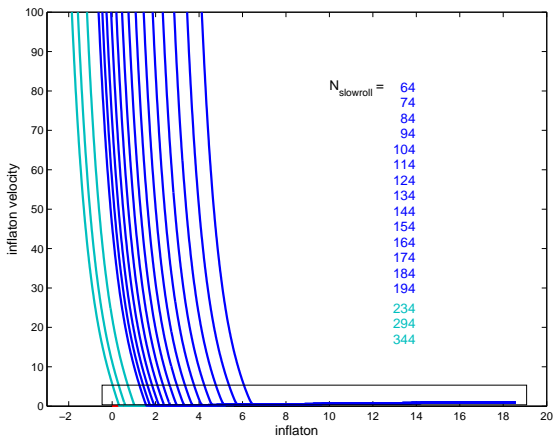
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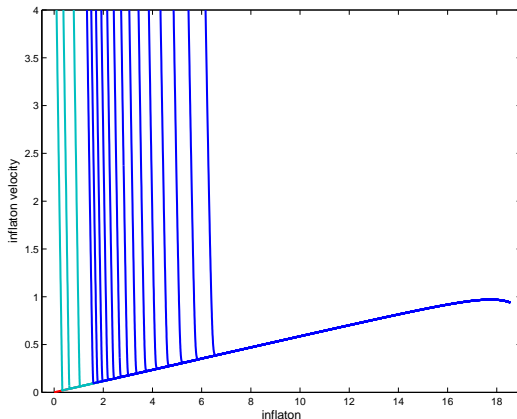
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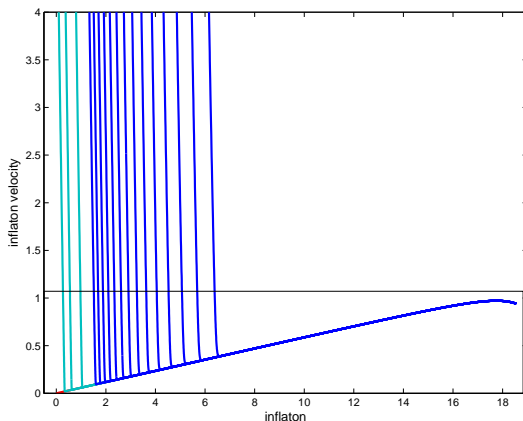
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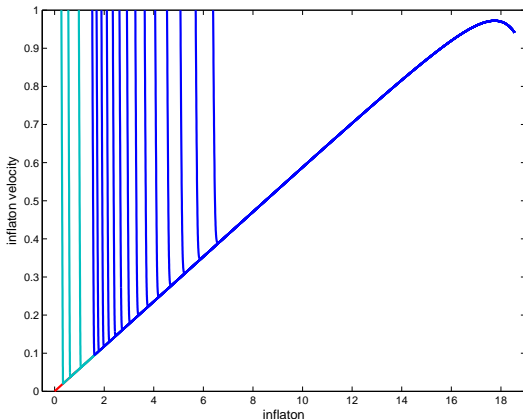
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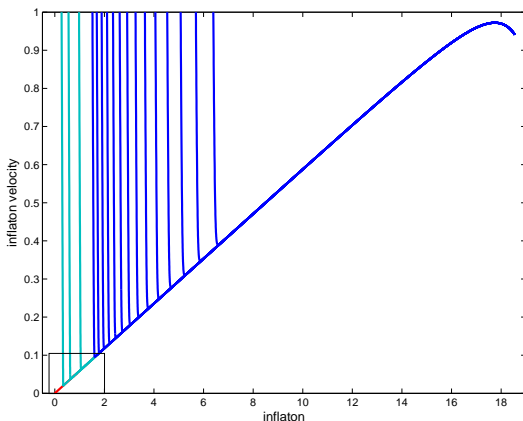




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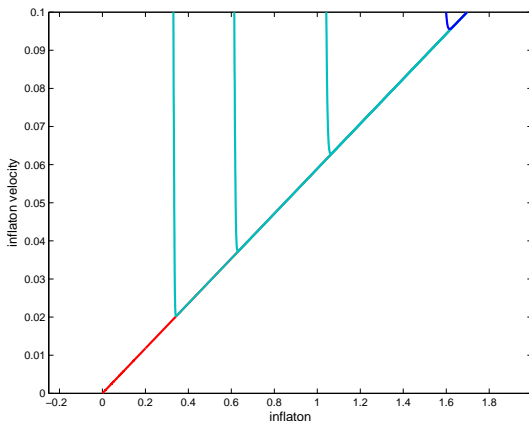
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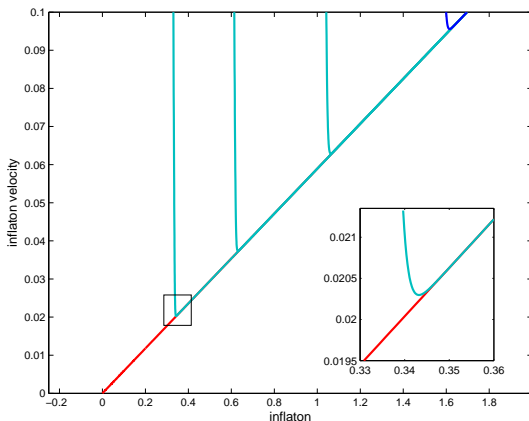
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## Scalar fluctuations

### Gauge-invariant quantum perturbation field

$$u(x, t) = -\xi(t) R(x, t) = \int \frac{d^3k}{(2\pi)^{3/2}} \left[ \alpha_k S_k(\eta) e^{ik \cdot x} + \alpha_k^\dagger S_k^*(\eta) e^{-ik \cdot x} \right]$$

$$[\alpha_k, \alpha_{k'}^\dagger] = \delta^{(3)}(k - k'), \quad \xi(t) = \frac{a(t)}{H(t)} \dot{\phi}(t), \quad \eta = \int \frac{dt}{a(t)}$$

### Schroedinger-like dynamics

$$\left[ \frac{d^2}{d\eta^2} + k^2 - W(\eta) \right] S_k = 0, \quad W(\eta) = \frac{1}{\xi} \frac{d^2 \xi}{d\eta^2}$$

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### Standard parametrization in dimensionless setup

$$U(t) = h^2(2 - 7\varepsilon_V + 2\varepsilon_V^2) - 2\dot{\phi} \frac{V'(\phi)}{h} - \eta_V V(\phi), \quad \varepsilon_V = \frac{\dot{\phi}^2}{2h^2}, \quad \eta_V = \frac{V''(\phi)}{V(\phi)}$$

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$$\epsilon_V \simeq \frac{1}{2} \left[ \frac{v'(\phi_{\text{exit}})}{v(\phi_{\text{exit}})} \right]^2, \quad \eta_V \simeq \frac{v''(\phi_{\text{exit}})}{v(\phi_{\text{exit}})}, \quad W(\eta) = \frac{v^2 - 1/4}{\eta^2}, \quad v = \frac{3}{2} + 3\epsilon_V - \eta_V$$

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$$P(k) = \lim_{\eta \rightarrow 0} \left( \frac{m}{M_{\text{PL}}} \right)^2 \frac{k^3}{2\pi^2} \left| \frac{S_k \eta}{\xi(\eta)} \right|^2$$

Bunch–Davies vacuum at  $t \rightarrow -\infty$  in extreme slow-roll

$$S_k(\eta \rightarrow -\infty) = \frac{e^{ik\eta}}{\sqrt{2k}}, \quad P_\infty = A_s \left( \frac{k}{k_0} \right)^{n_s - 1}, \quad A_s = \left( \frac{m}{M_{\text{PL}}} \right)^2 \frac{N^2}{12\pi^2} \mathcal{O}(1)$$

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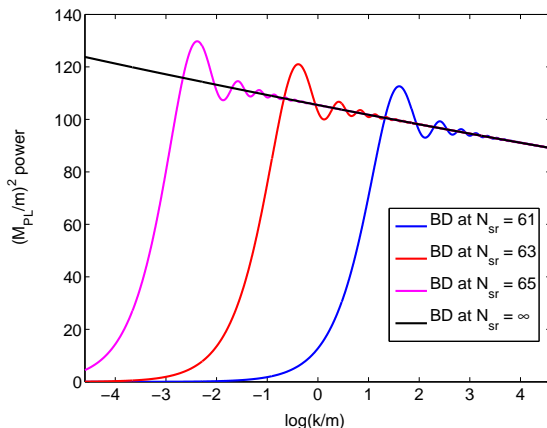
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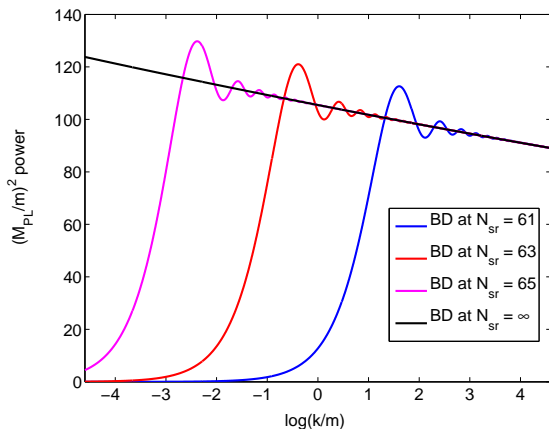
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Compare the small  $k$ - behavior of BD and quasi-De Sitter modes

$$S_k(\eta) = \frac{e^{ik\eta}}{\sqrt{2k}}, \quad g_\nu(k; \eta) = \frac{1}{2} i^{\nu+1/2} \sqrt{-\pi\eta} H_\nu^{(1)}(-k\eta) \simeq \frac{\Gamma(\nu)}{\sqrt{2\pi k}} \left( \frac{2}{ik\eta} \right)^{\nu-1/2}$$

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$$P(k) = P_{\infty}(k) \left[ 1 + D(k) \right]$$

more formally ...

Effect on quadratic observables of linear combinations of the solutions of linear differential equations of second order, or Bogoliubov transformations on creation–annihilation operators.

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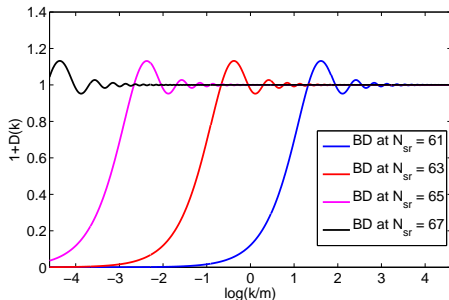
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$$D(k) \simeq D(k\eta_0)$$

$$D(k) \sim k^{-2}, \quad k \rightarrow \infty$$

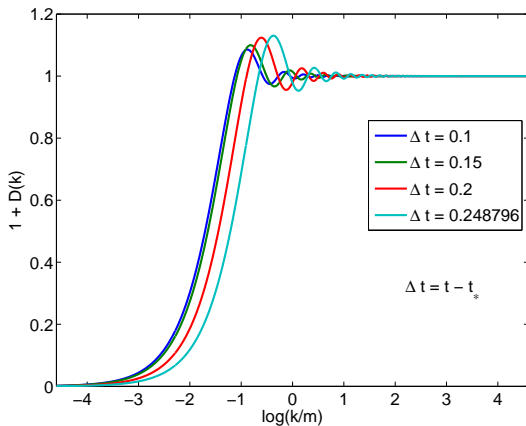
to have a negligible  
back–reaction on the  
metric





## Transfer function for fast-roll trajectories

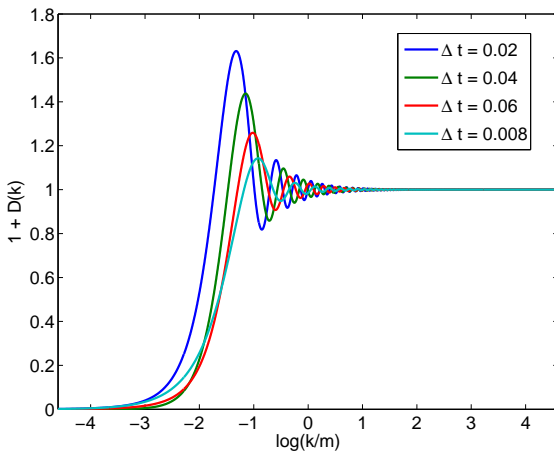
C.D., H.J. de Vega and N. Sanchez, in preparation



depression of lowest multipoles

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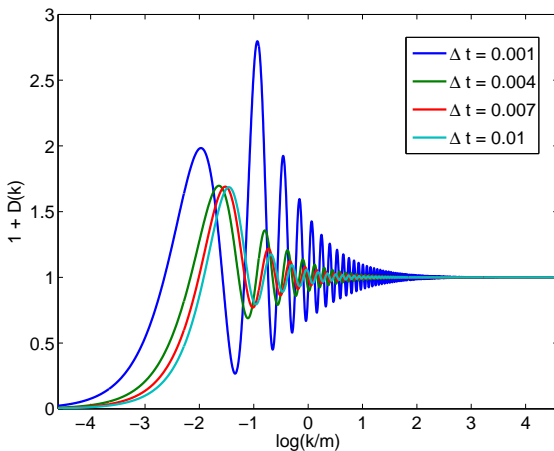
C.D., H.J. de Vega and N. Sanchez, in preparation



up and down with little change on average

## Transfer function for fast-roll trajectories

C.D., H.J. de Vega and N. Sanchez, in preparation



up and down with net overall enhancement

## Outline

- 1 Is the low CMB TT quadruple too low?
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- Allows adiabatic Bunch–Davies vacuum of de Sitter spacetime
- Gravity always semiclassical ( $H \ll M_{PL}$  at any time)

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## Once upon a time ...

in the matter dominated era there was a very low quadrupole that would later (now) become a very low  $\ell = 22$  multipole.

The argument based on fastroll to explain such a low quadrupole would have given  $N_{\text{slowroll}} = 54$  but would have been proven wrong when more superhorizon modes reentered. Except that ...

The entropy lower bound

$$N_{\text{slowroll}} \geq 62.4 - \frac{1}{2} \log \left( \frac{10^{-4} M_{\text{PL}}}{H} \right) - \frac{1}{12} \log \frac{g_{\text{rh}}}{1000} \simeq 62$$

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## Binomial New Inflation with sharpcut or (simplified) fastroll

C.D., H.J. de Vega, N. Sanchez, Phys. Rev. D78

### Simplification

- Born's approximation for  $k$  not too small.
- $k_{tran} = -1/\eta_0$  is the comoving wavenumber that exits the horizon when fast-roll ends and slow-roll starts.

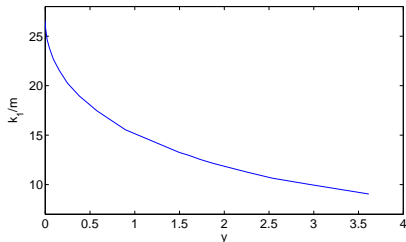
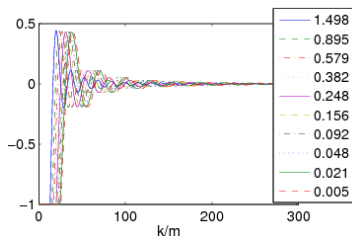
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For BNI,  $v(\phi) = \frac{1}{4}g(\phi^2 - 1/g)^2$ ,  $g = y/(8N)$ ,  $y = z - 1 - \log(z)$





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## BNI+sharpcut vs. BNI+fastroll

MCMC parameters:  $\omega_b, \omega_c, \theta, \tau$ , (slow),  $A_s, z, k_{tran}$  (fast)  
Context:  $N = 60, \Omega_v = 0, \dots$  ; standard priors,  
no SZ, lensed CMB, linear mpk, ...  
Datasets: WMAP5, SDSS, ACBAR08

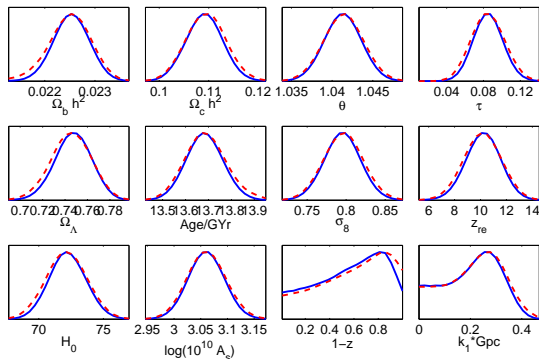
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param	best fit
$100\Omega_b h^2$	2.256
$\Omega_c h^2$	0.110
$\theta$	1.041
$100\tau$	8.83
$H_0$	71.82
$\sigma_8$	0.803
$\log[10^{10} A_s]$	0.307
$z$	0.162
$k_1$	0.260
$-\log(L)$	1253.96

sharp-cut

flat  $0 < z < 1$  prior



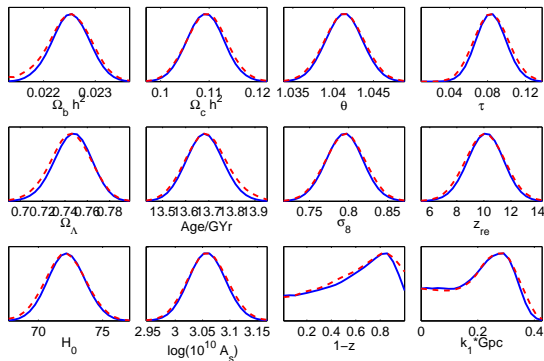
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param	best fit
$100\Omega_b h^2$	2.253
$\Omega_c h^2$	0.109
$\theta$	1.041
$100\tau$	8.42
$H_0$	72.00
$\sigma_8$	0.794
$\log[10^{10} A_s]$	0.306
$z$	0.102
$k_1$	0.284
$-\log(L)$	1253.82

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flat  $0 < z < 1$  prior



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### $\Delta\chi^2$ w.r.t. $\Lambda\text{CDM}+r$

	WMAP5	+SDSS+ACBAR08	+SDSS+SN
BNI+sharpcut	-1.07	-0.71	-1.02
BNI+fastroll	-1.15	-0.99	-1.45

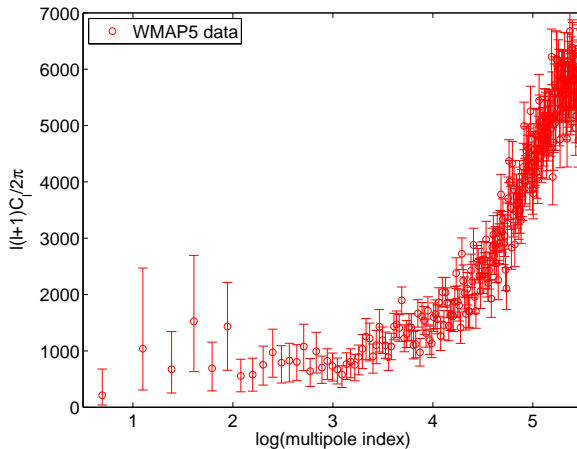
### 95% lower bound on $r$

	WMAP5	+SDSS+ACBAR08	+SDSS+SN
BNI+sharpcut	0.025	0.033	0.022
BNI+fastroll	0.024	0.032	0.023

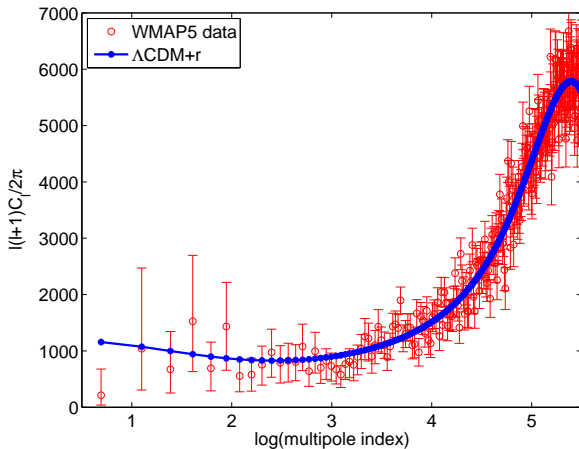
### most likely value of $k_{tran}$ (in $\text{Gpc}^{-1}$ )

	WMAP5	+SDSS+ACBAR08	+SDSS+SN
BNI+sharpcut	0.258	0.260	0.244
BNI+fastroll	0.298	0.284	0.291

## Comparing TT multipoles

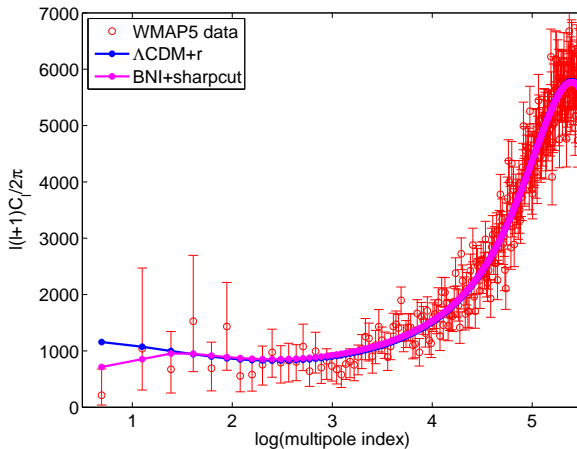


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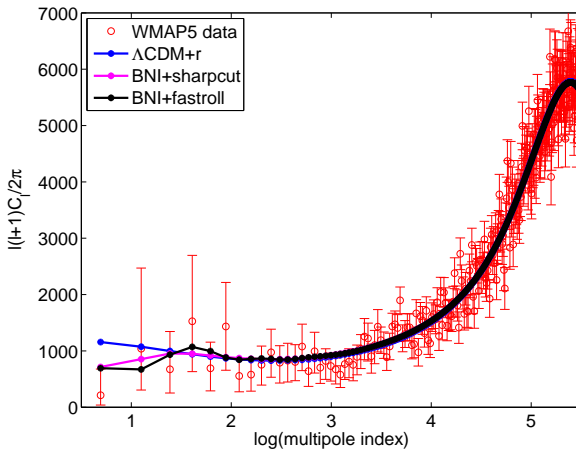




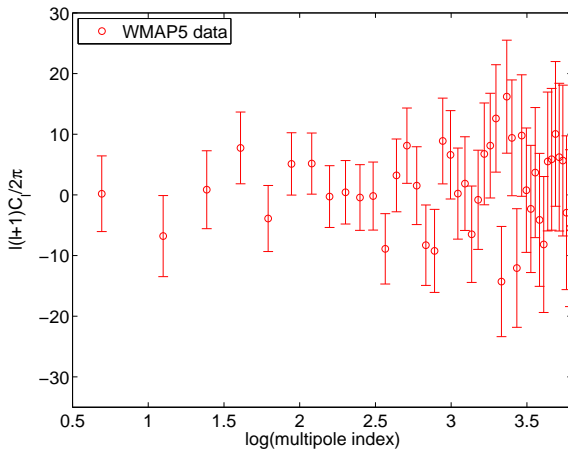
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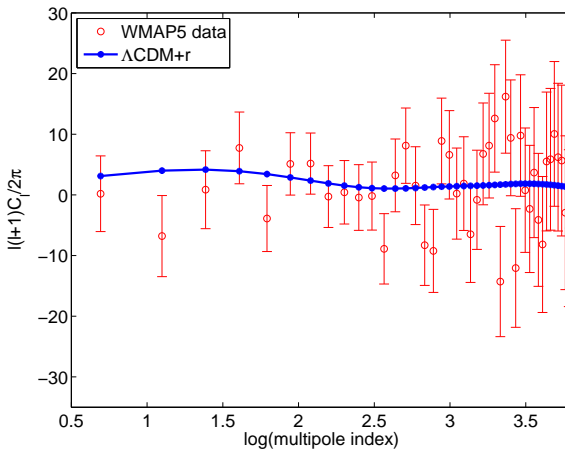
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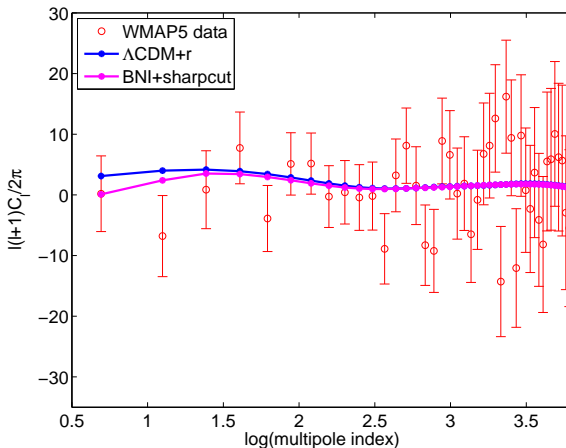
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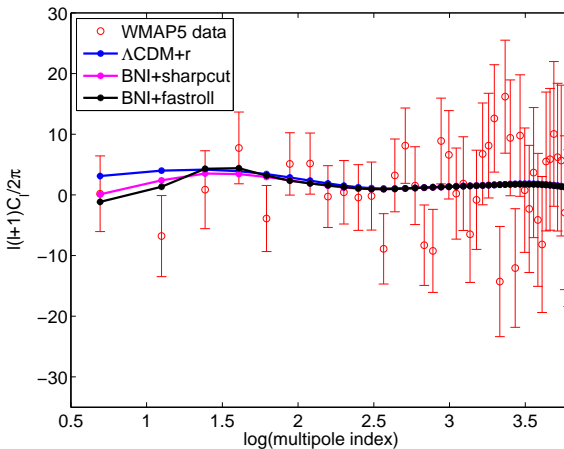
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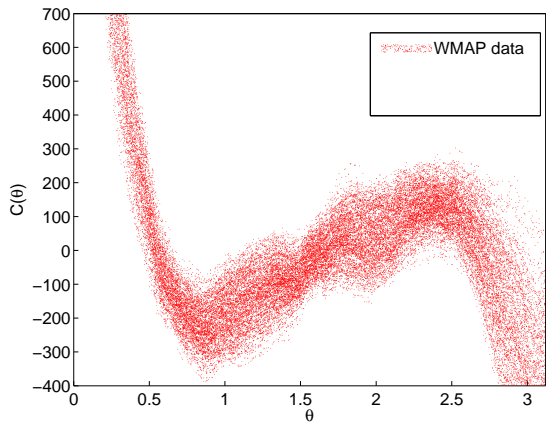
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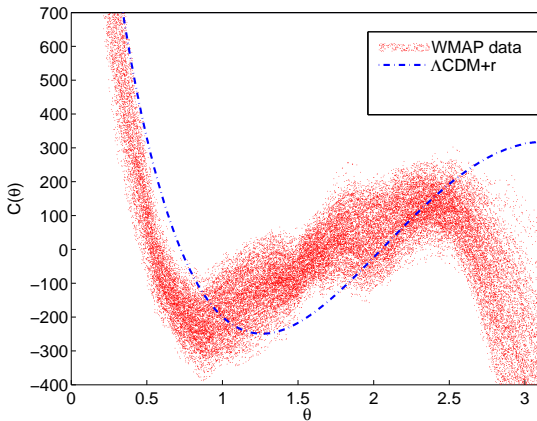
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# Comparing real-space TT correlations

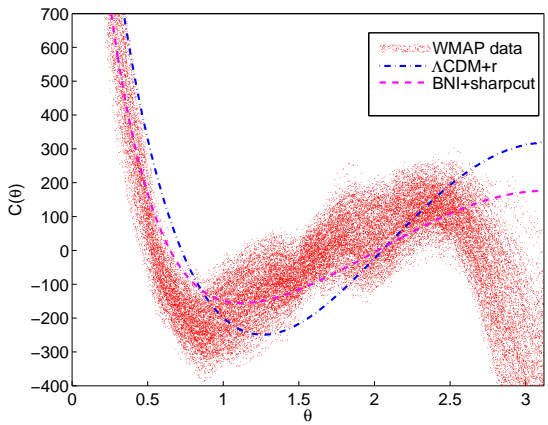


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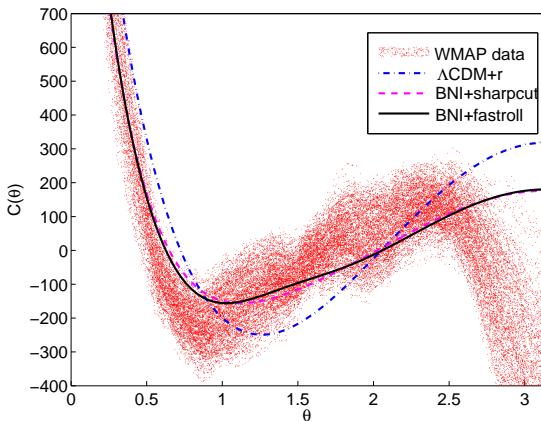




# Comparing real-space TT correlations



## Comparing real-space TT correlations



## Highlight

### The quadrupole wavenumber

$k_Q \simeq 0.83 k_{tran}$  and exits roughly 1/10 of an efold before  $k_{tran}$   
 $N_{slowroll} \simeq 63$

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- Early fast-roll inflation is generic and provides a mechanism for lowest multipoles depression.
- BNI+fastroll significantly improves the fit w.r.t.  $\Lambda$ CDM+r.
- BNI+fastroll improves the fits also w.r.t. BNI+sharpcut.
- Fast-roll sets to 63 the number of slow-roll inflation e-folds.
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  - Improve the EFT of inflations (entropy, reheating, ...)
  - Wait for better data (Planck, Atacama, ...)
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