Inflation

in the Standard

Model of the Universe:

New understanging after WMAP

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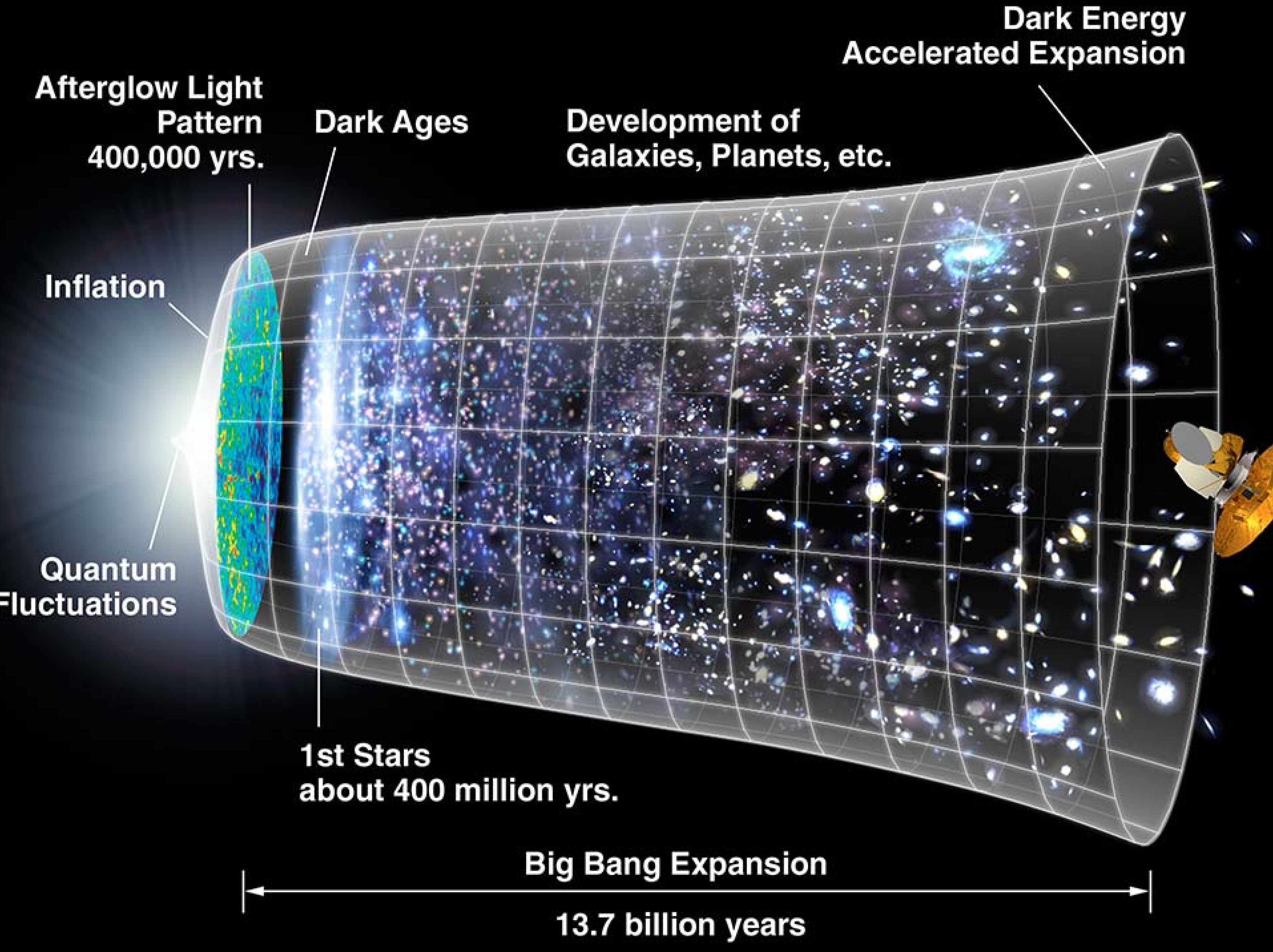


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
400,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years

Standard Cosmological Model: ΛCDM

ΛCDM = Cold Dark Matter + Cosmological Constant
Explains the Observations:

- 5 years WMAP data and previous CMB data
- Light Elements Abundances
- Large Scale Structures (LSS) Observations
- Supernova Luminosity/Distance Relations (Acceleration of the Universe expansion)
- Gravitational Lensing Observations
- Baryonic Acoustic Oscillations
- Hubble Constant (H_0) Measurements
- Properties of Clusters of Galaxies
- ....

Standard Cosmological Model: Concordance Model

$ds^2 = dt^2 - a^2(t)\, d\vec{x}^{\,2}$: spatially flat geometry.

The Universe starts by an **INFLATIONARY ERA**.

Inflation = Accelerated Expansion: $\frac{d^2 a}{dt^2} > 0$.

During inflation the universe expands by at least sixty efolds: $e^{60} \simeq 10^{26}$. Inflation lasts $\simeq 10^{-34}$ sec and ends by $z \sim 10^{28}$ followed by a radiation dominated era.

Energy scale when inflation starts $\sim 10^{16}$ GeV.

This energy scale **coincides** with the GUT scale ($\Longleftarrow$ CMB anisotropies).

Matter can be effectively described during inflation by an Scalar Field $\phi(t, \boldsymbol{x})$: the Inflaton.

Lagrangean: $\mathcal{L} = a^3(t) \left[\frac{\dot{\phi}^2}{2} - \frac{(\nabla\phi)^2}{2\, a^2(t)} - V(\phi) \right]$.

Friedmann eq.: $H^2(t) = \frac{1}{3\, M_{Pl}^2} \left[\frac{\dot{\phi}^2}{2} + V(\phi) \right]$, $H(t) \equiv \dot{a}(t)/a(t)$

Expected CMB constraints on Δ_T *should still improve* this support.

THE SCALE OF SEMICLASSICAL GRAVITY

Δ_T and Δ_R expressed in terms of the semiclassical and quantum Gravity Temperature scales

$$T_{sem} = h\, H /(2\pi\, k_B) \quad , \quad T_{Pl} = M_{Pl}\, c^2/ (2\pi\, k_B)$$

T_{sem} is the semiclassical or Hawking-Gibbons temperature of the initial state (or Bunch-Davies vacuum) of inflation. T_{Pl} is the Planck temparature $10^{32°}$ K.

$$T_{sem} / T_{Pl} = 2\pi\, (2\, \varepsilon_V)^{1/2}\, \Delta_R, \quad T_{sem} / T_{Pl} = \pi\, (2)^{-1/2}\, \Delta_T$$

Therefore, WMAP data yield for the Hawking-Gibbons Temperature of Inflation:

$$\rightarrow \rightarrow \rightarrow \quad T_{sem} \sim (\varepsilon_V)^{1/2}\, 10^{28\,°}\ \mathrm{K}.$$

LOWER BOUND on r
(ON THE PRIMORDIAL GRAVITONS

Our approach (our theory input in the MCMC data analysis of WMAP5+LSS+SN data). [C. Destri, H J de Vega, N G Sanchez, Phys Rev D77, 043509 (2008)].

Besides the upper bound for r (tensor to scalar ratio) $r < 0.22$, we find a clear peak in the r distribution and we obtain a lower bound

$$r > 0.016 \quad \text{at} \;\; 95\% \text{ CL} \quad \text{and}$$

$$r > 0.049 \quad \text{at} \;\; 68\% \text{ CL.}$$

Moreover, we find $r = 0.055$ as the most probable value.

For the other cosmological parameters, both analysis agree.

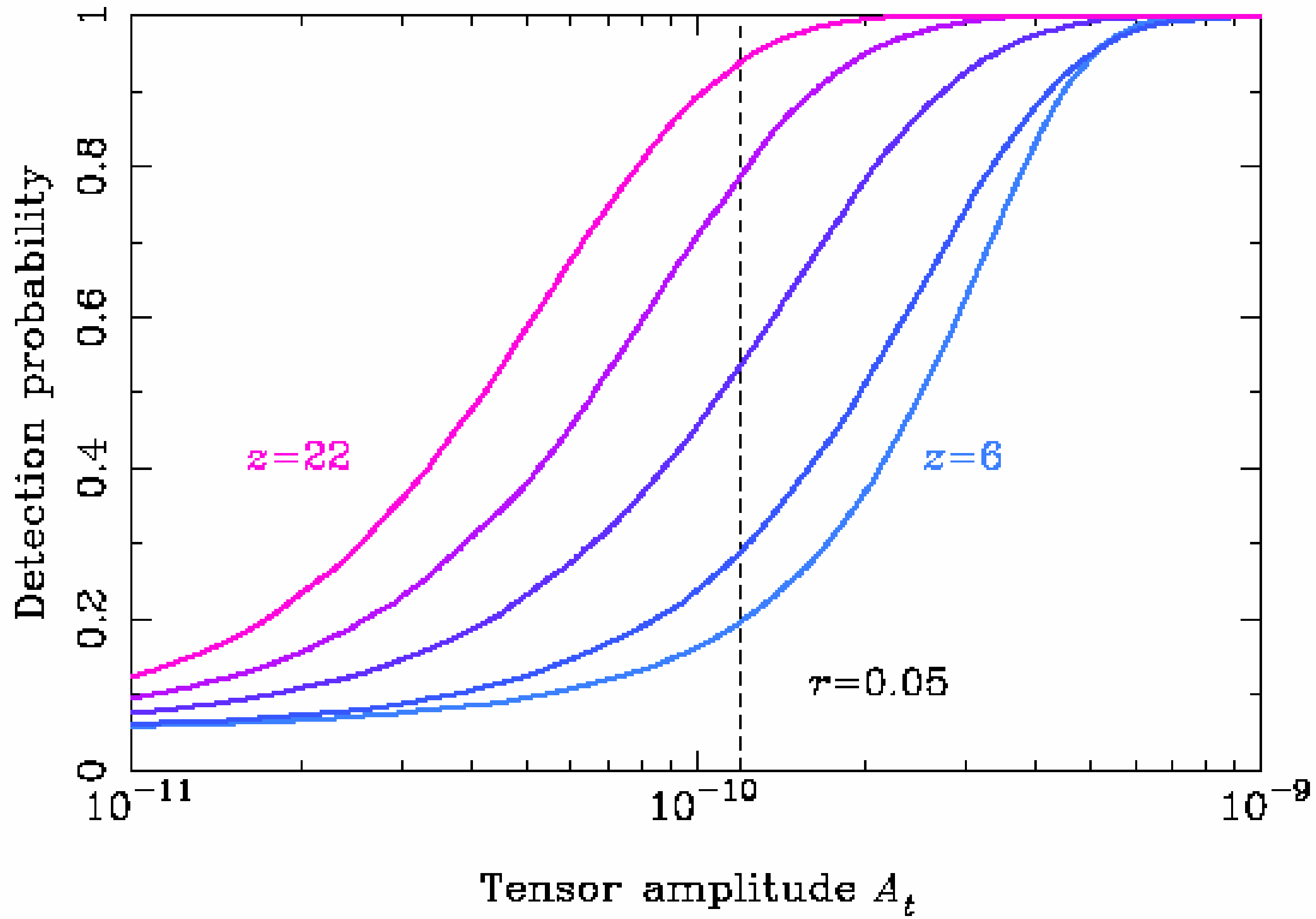


FIG 2.16.—The probability of detecting B-mode polarization at 95% confidence as a function of A_T, the amplitude of the primordial tensor power spectrum (assumed scale-invariant), for *Planck* observations using 65% of the sky. The curves correspond to different assumed epochs of (instantaneous) reionization: $z = 6$, 10, 14, 18 and 22. The dashed line corresponds to a tensor-to-scalar ratio $r = 0.05$ for the best-fit scalar normalisation, $A_S = 2.7 \times 10^{-9}$, from the one-year *WMAP* observations.

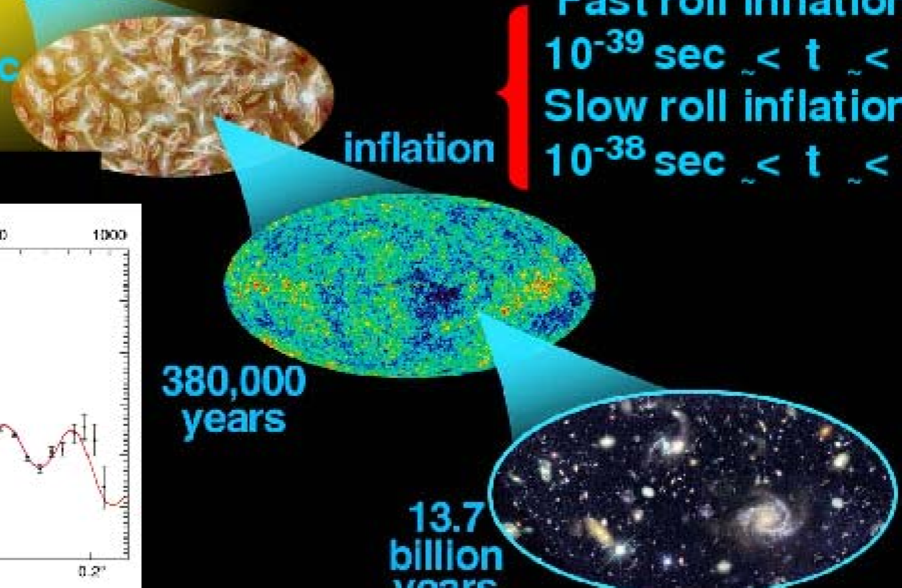
COSMIC HISTORY AND CMB QUADRUPOLE SUPPRESSION

Fast roll inflation produces the CMB quadrupole suppression

DAWN OF TIME

Planck time: $t \sim 10^{-44}$ sec

$t \sim 10^{-39}$ sec

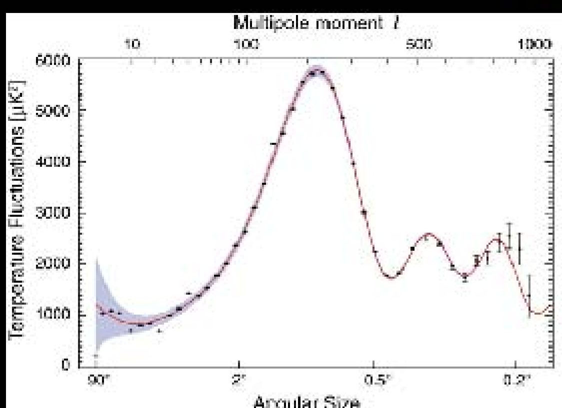


inflation

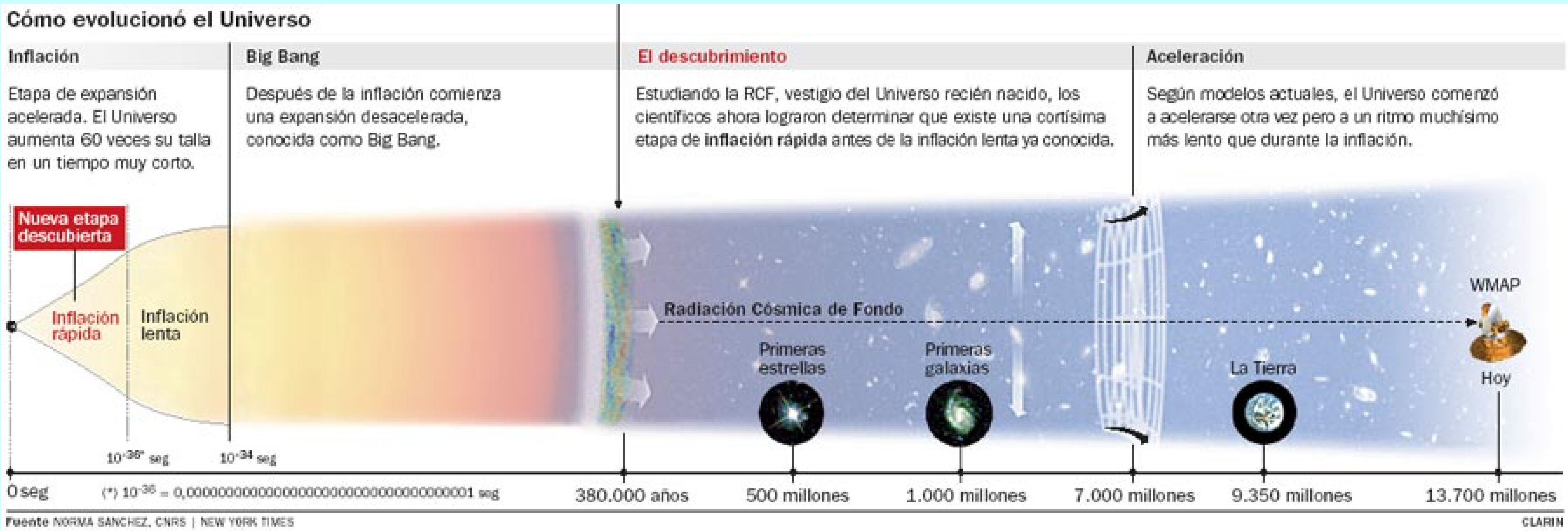
Fast roll inflation
10^{-39} sec $\lesssim t \lesssim 10^{-38}$ sec
Slow roll inflation
10^{-38} sec $\lesssim t \lesssim 10^{-36}$ sec

380,000 years

13.7 billion years



Fast roll Inflation produces the Observed Quadrupole CMB Suppression



D. Boyanovsky, H. J de Vega and N. G. Sanchez,

" CMB quadrupole suppression II : The early fast roll stage "

Phys. Rev. D74 , 123006 (2006)

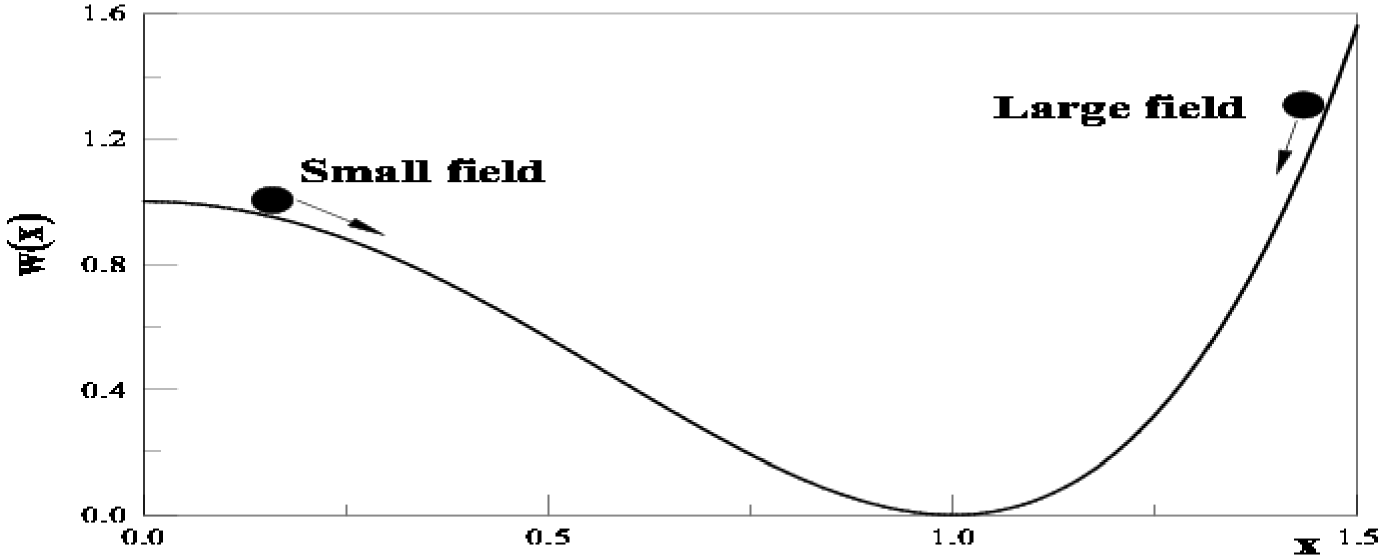
Quadrupole suppression and Fast-roll Inflation

The observed CMB-quadrupole (COBE,WMAP5) is six times smaller than the ΛCDM model value.
In the best ΛCDM fit the probability that the quadrupole is as low or lower than the observed value is 3%.
It is hence relevant to find a cosmological explanation of the quadrupole supression.

Generically, the classical evolution of the inflaton has a brief **fast-roll stage** that precedes the slow-roll regime.
In case the quadrupole CMB mode leaves the horizon during fast-roll, before slow-roll starts, we find that the quadrupole mode gets suppressed.

$P(k) = |\Delta_{k\ ad}^{(S)}|^2\ (k/k_0)^{n_s-1}[1+D(k)]$
The transfer function $D(k)$ changes the primordial power.
$1+D(0)=0, \quad D(\infty)=0$

Slow Roll Inflaton Models



$V(\mathrm{Min}) = V'(\mathrm{Min}) = 0$: inflation ends after a finite number of efolds. Universal form of the slow-roll inflaton potential:

$V(\varphi) = N\ M^4\ w(\chi)\ ,\ \chi \equiv \frac{\varphi}{\sqrt{N}\ M_{Pl}} \quad , \quad \chi$ and $w(\chi) = \mathcal{O}(1)$

$N \sim 60$ number of efolds since horizon exit till end of inflation. $M =$ energy scale of inflation.

Slow-roll is needed to produce enough efolds of inflation.

Slow Roll expansion: a hierarchy of dimensionless parameters:

$$\epsilon_v = \frac{M_P^2}{2}\left[\frac{V'(\phi)}{V(\phi)}\right]^2 \quad , \quad \eta_v = M_P^2 \, \frac{V''(\phi)}{V(\phi)} \cdots$$

as a 1/Ne expansion:

$$N[\phi(t)] = -\frac{1}{M_P^2}\int_{\phi(t)}^{\phi_{end}} V(\phi)\,\frac{d\phi}{dV}\,d\phi$$

$$\phi = \sqrt{N_e}\,M_P\,\chi \longleftarrow$$ Rescale field

$$V(\phi) = N_e\,M^4\,w(\chi)$$

$w(\chi) \longrightarrow \sim \mathcal{O}(1)$

$M^4 \longrightarrow$ energy scale of inflation

$$1 = -\int_{\chi_c}^{\chi_{end}} \frac{w(\chi)}{w'(\chi)}\, d\chi$$

value of χ at the END of inflation

value of χ Ne e-folds before END of inflation

Independent of Ne: an implicit relation between χ and couplings

$$\epsilon_v = \frac{1}{2\,N_e}\left[\frac{w'(\chi_c)}{w(\chi_c)}\right]^2 \quad , \quad \eta_v = \frac{1}{N_e}\,\frac{w''(\chi_c)}{w(\chi_c)}$$

Explicit dependence on Ne: SIMPLE RESCALING

spectral index n_s and the ratio r

$r \equiv$ ratio of tensor to scalar fluctuations.
tensor fluctuations = primordial gravitons.

$$n_s - 1 = -\frac{3}{N}\left[\frac{w'(\chi)}{w(\chi)}\right]^2 + \frac{2}{N}\,\frac{w''(\chi)}{w(\chi)} \quad , \quad r = \frac{8}{N}\left[\frac{w'(\chi)}{w(\chi)}\right]^2$$

$$\frac{dn_s}{d\ln k} = -\frac{2}{N^2}\,\frac{w'(\chi)\,w'''(\chi)}{w^2(\chi)} - \frac{6}{N^2}\,\frac{[w'(\chi)]^4}{w^4(\chi)} + \frac{8}{N^2}\,\frac{[w'(\chi)]^2\,w''(\chi)}{w^3(\chi)} \,,$$

χ is the inflaton field at horizon exit.
$n_s - 1$ and r are always of order $1/N \sim 0.02$ (model indep.)
Running of n_s of order $1/N^2 \sim 0.0003$ (model independent).

D. Boyanovsky, H. J. de Vega, N. G. Sanchez,
Phys. Rev. D 73, 023008 (2006), astro-ph/0507595.

Primordial Power Spectrum

Adiabatic Scalar Perturbations: $P(k) = |\Delta_{k\;ad}^{(S)}|^2 \; (k/k_0)^{n_s - 1}$.

To dominant order in slow-roll:

$$|\Delta_{k\;ad}^{(S)}|^2 = \frac{N^2}{12\,\pi^2} \; \left(\frac{M}{M_{Pl}}\right)^4 \; \frac{w^3(\chi)}{w'^2(\chi)} \; .$$

Hence, for all slow-roll inflation models:

$$|\Delta_{k\;ad}^{(S)}| \sim \frac{N}{2\,\pi\sqrt{3}} \left(\frac{M}{M_{Pl}}\right)^2$$

The WMAP5 result: $|\Delta_{k\;ad}^{(S)}| = (0.494 \pm 0.1) \times 10^{-4}$

determines the scale of inflation M (using $N \simeq 60$)

$$\left(\frac{M}{M_{Pl}}\right)^2 = 0.85 \times 10^{-5} \;\longrightarrow\; M = 0.70 \times 10^{16}\ \text{GeV}$$

The inflation energy scale turns to be the grand unification energy scale !! We find the scale of inflation without knowing the tensor/scalar ratio r !!

The scale M is independent of the shape of $w(\chi)$.

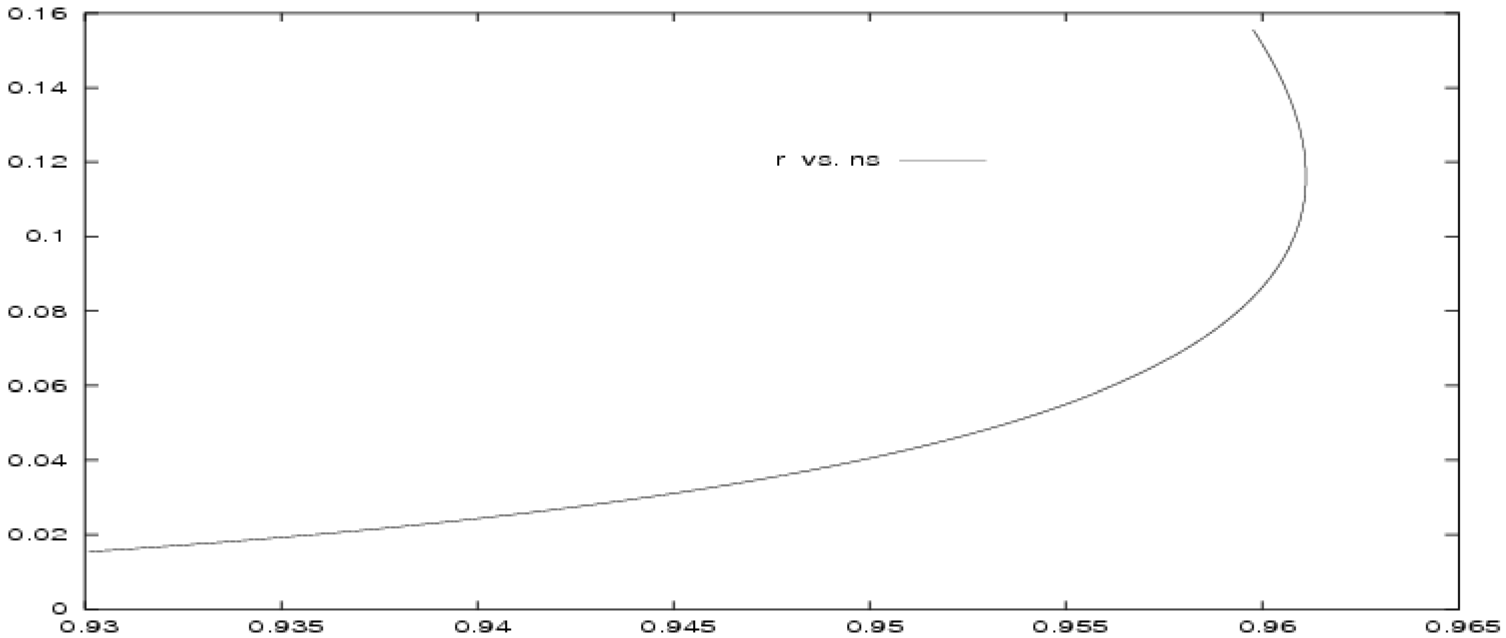
Trinomial Inflationary Models

- Trinomial Chaotic inflation:
  $$w(\chi) = \frac{1}{2}\,\chi^2 + \frac{h}{3}\,\sqrt{\frac{y}{2}}\,\chi^3 + \frac{y}{32}\,\chi^4 \; .$$
- Trinomial New inflation:
  $$w(\chi) = -\frac{1}{2}\,\chi^2 + \frac{h}{3}\,\sqrt{\frac{y}{2}}\,\chi^3 + \frac{y}{32}\,\chi^4 + \frac{2}{y}\,F(h) \; .$$

h = asymmetry parameter. $w(\text{min}) = w'(\text{min}) = 0$,
$y =$ quartic coupling, $F(h) = \frac{8}{3}\,h^4 + 4\,h^2 + 1 + \frac{8}{3}\,|h|\,(h^2+1)^{\frac{3}{2}}$.

H. J. de Vega, N. G. Sanchez, Single Field Inflation models allowed and ruled out by the three years WMAP data. Phys. Rev. D 74, 063519 (2006), astro-ph/0604136.

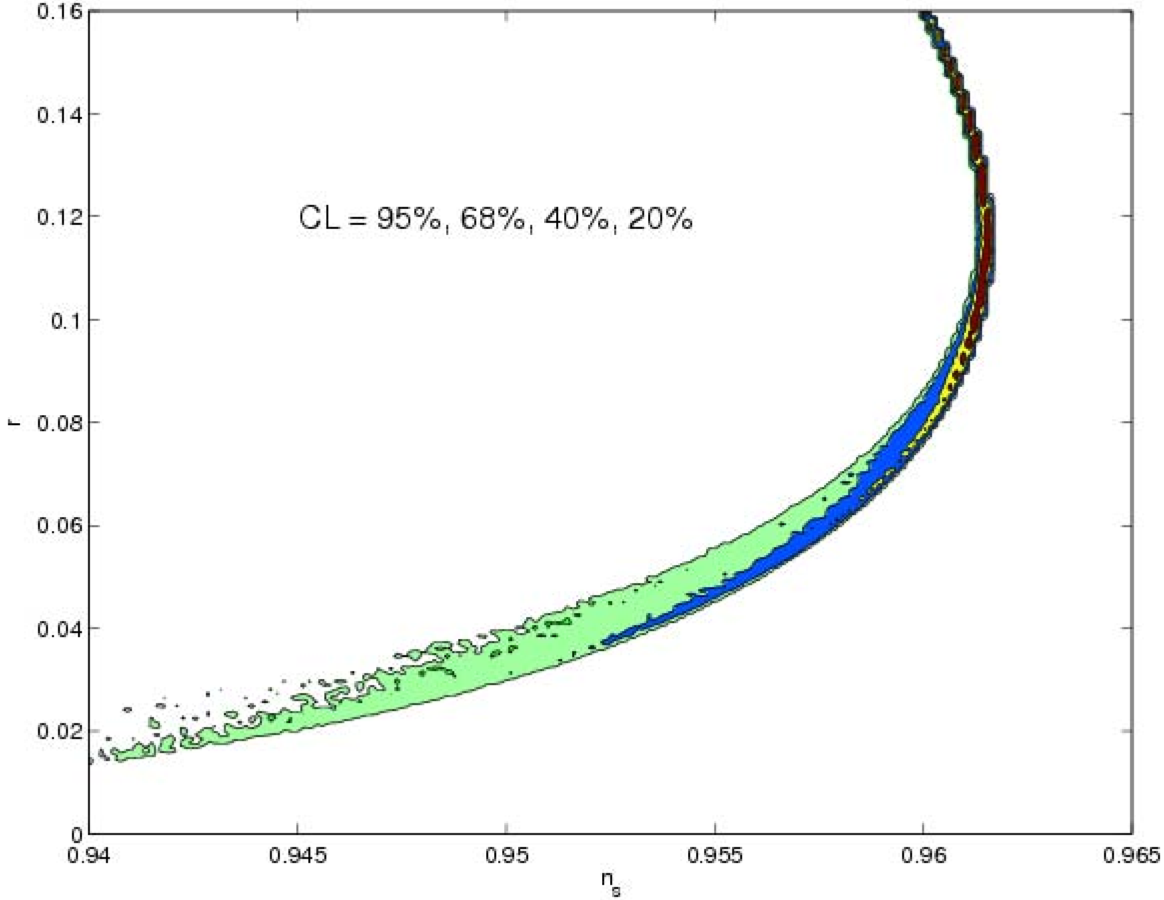
Binomial New Inflation



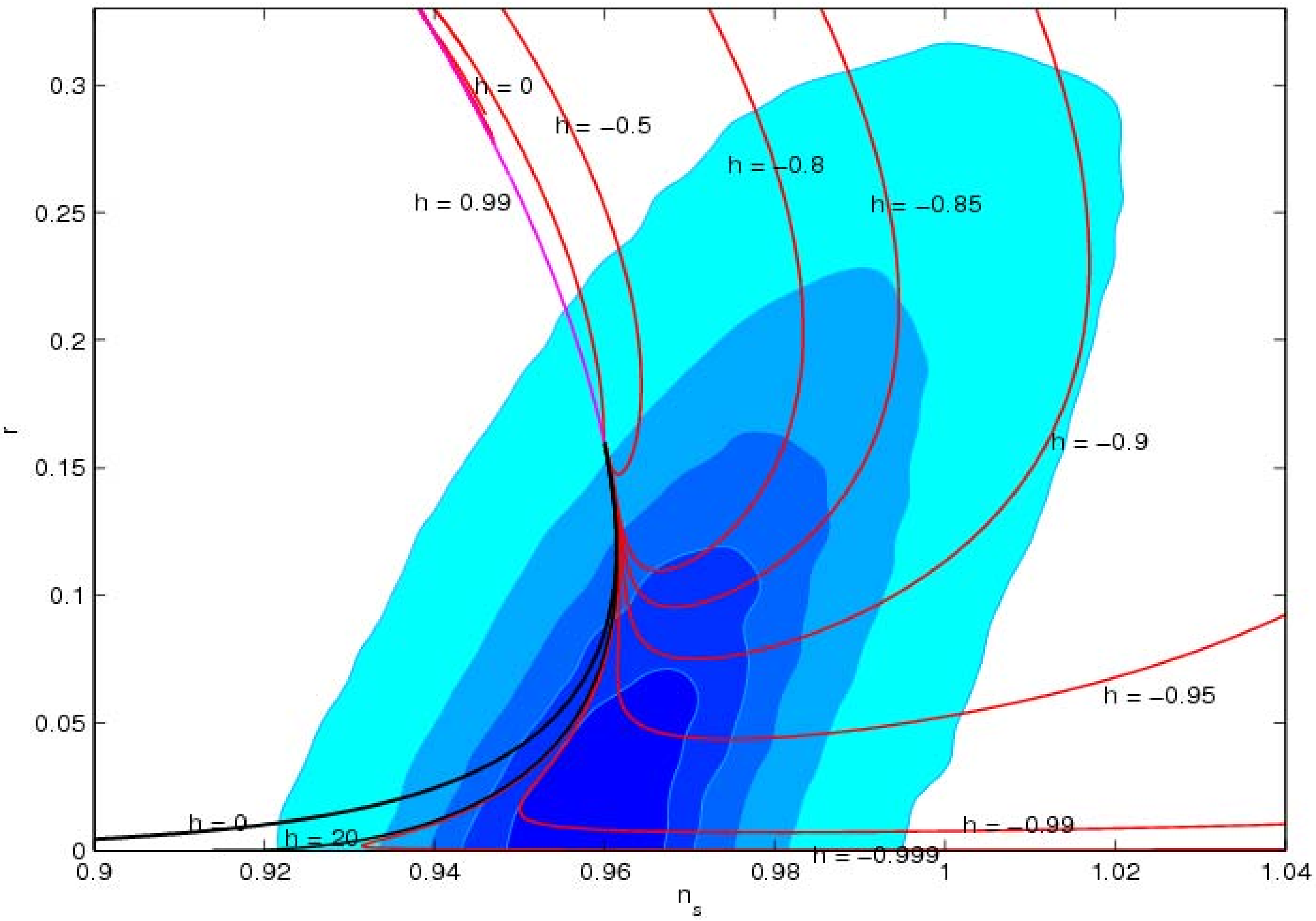
$r = \frac{8}{N} = 0.16$ and $n_s = 1 - \frac{2}{N} = 0.96$ at $y = 0$.

r is a double valued function of n_s.

r vs. n_s data within the Trinomial New Inflation Region.



MCMC Results for Trinomial New Inflation.



Monte Carlo Markov Chains Analysis of Data: MCMC.

MCMC is an efficient stochastic numerical method to find the probability distribution of the theoretical parameters that describe a set of empirical data. We found n_s, the ratio r of tensor to scalar fluctuations and the couplings by MCMC.

NEW: We imposed as a hard constraint that r and n_s are given by the trinomial potential.

Our analysis differs in this crucial aspect from previous MCMC studies.

MCMC Results for Trinomial New Inflation:

Bounds: $r > 0.016$ (95% CL) , $r > 0.049$ (68% CL)

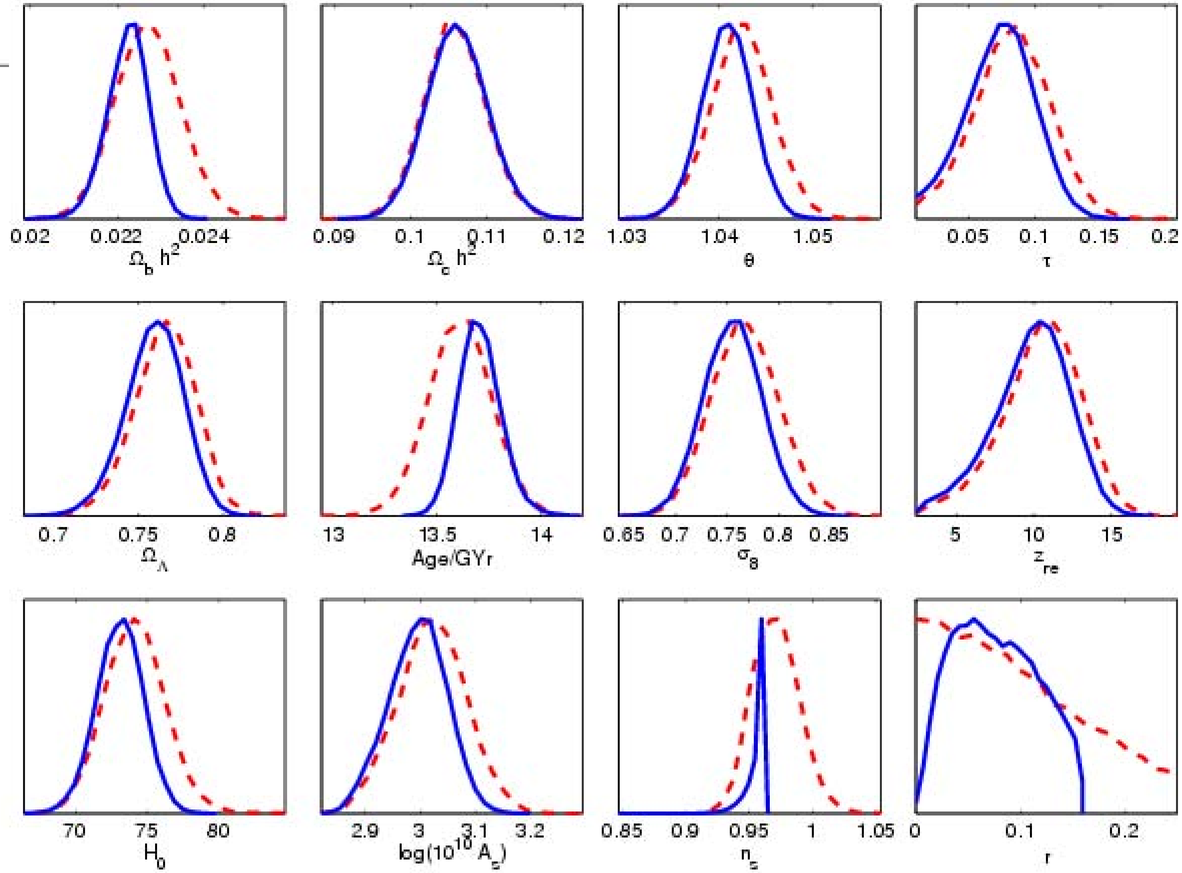
Most probable values: $n_s \simeq 0.956$, $r \simeq 0.055$.

The most probable potential is symmetric and has a moderate nonlinearity:

$$w(\chi) = \frac{y}{32}\left(\chi^2 - \frac{8}{y}\right)^2 \quad , \quad y \simeq 2$$

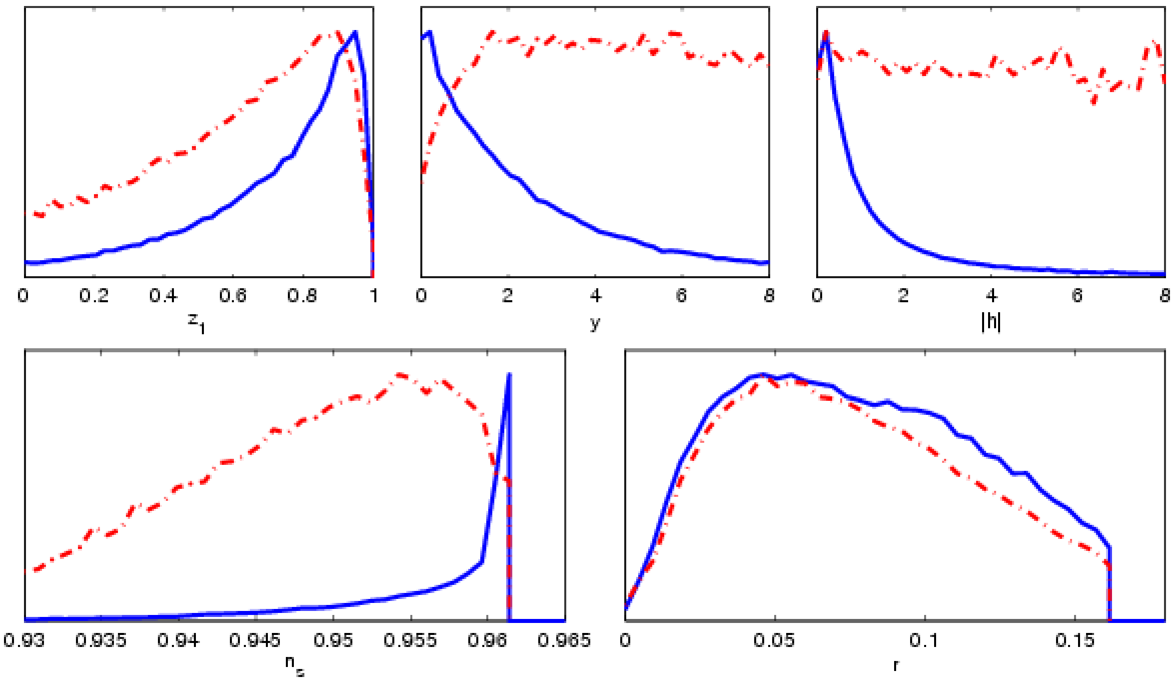
C. Destri, H. J. de Vega, N. Sanchez, astro-ph/0703417, Phys. Rev. **D77**, 043509 (2008).

Marginalized probability distributions. New Inflation.



Imposing the trinomial potential (solid blue curves) and just the ΛCDM+r model (dashed red curves).
(curves normalized to have the maxima equal to one).

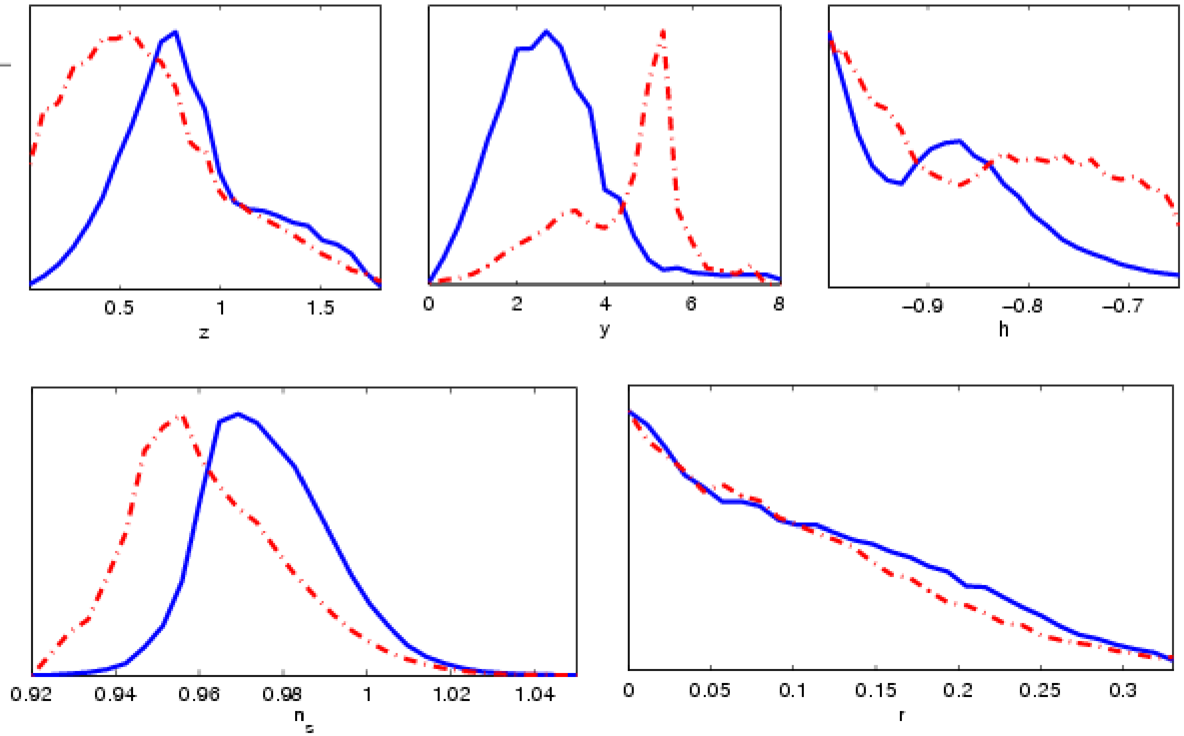
Probability Distributions. Trinomial New Inflation.



Probability distributions: solid blue curves
Mean likelihoods: dot-dashed red curves.

$$z_1 = 1 - \frac{y}{8\left(|h| + \sqrt{h^2+1}\right)^2} \; \chi^2 \; .$$

Probability Distributions. Trinomial Chaotic Inflation.

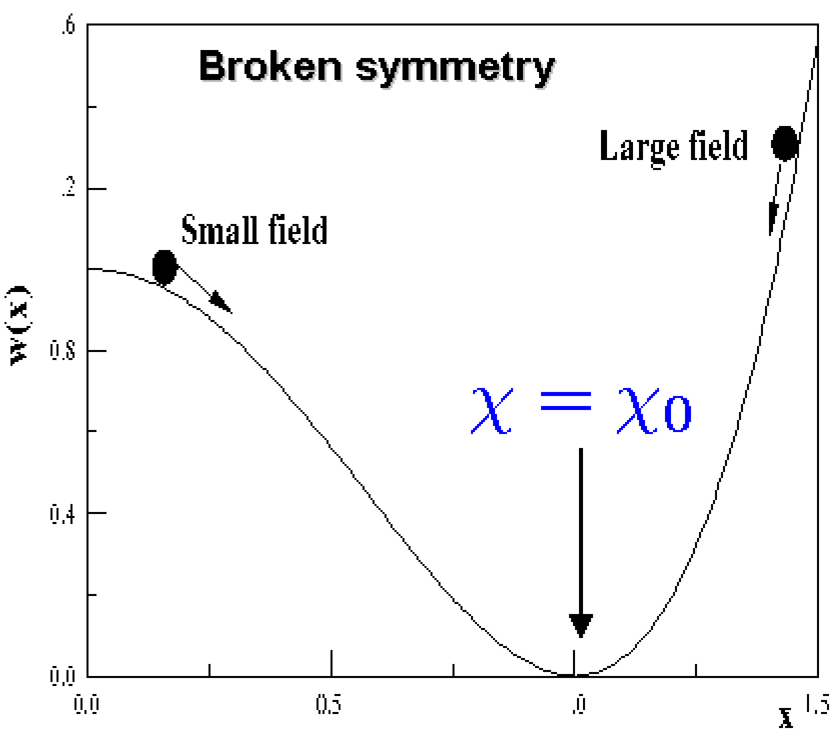


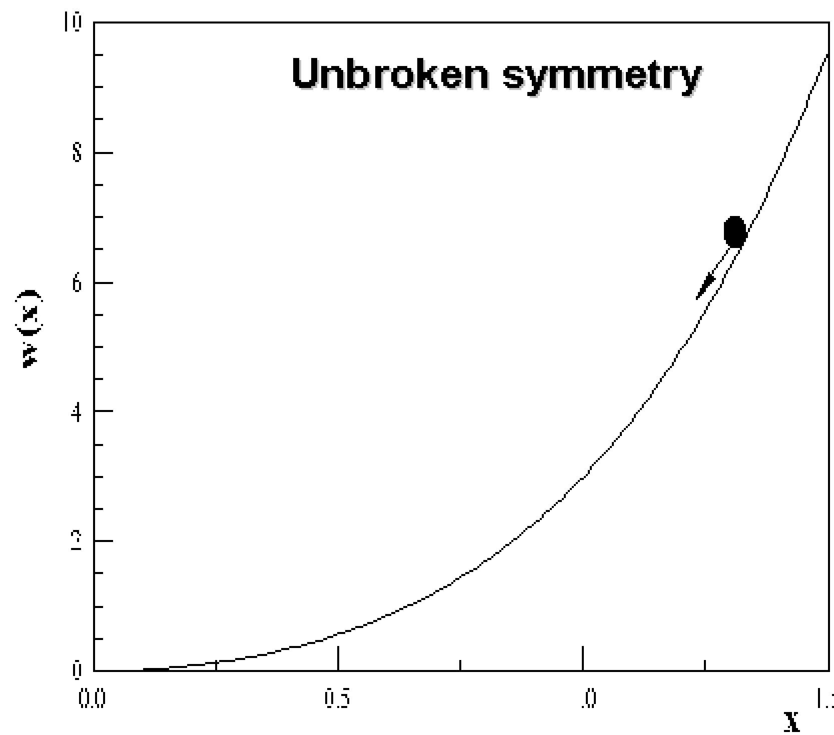
Probability distributions (solid blue curves) and mean likelihoods (dot-dashed red curves).
The data request a strongly asymmetric potential in chaotic inflation almost having two minima. That is, a strong breakdown of the $\chi \to -\chi$ symmetry.

Rescale fields and couplings:

$$\lambda = \frac{m^2\, g}{M_{Pl}^{2\,n-2}\; N_e^{n-1}}\,;\, g = \frac{1}{\chi_0^{2\,n-2}}\,;\, x = \frac{\chi}{\chi_0}$$

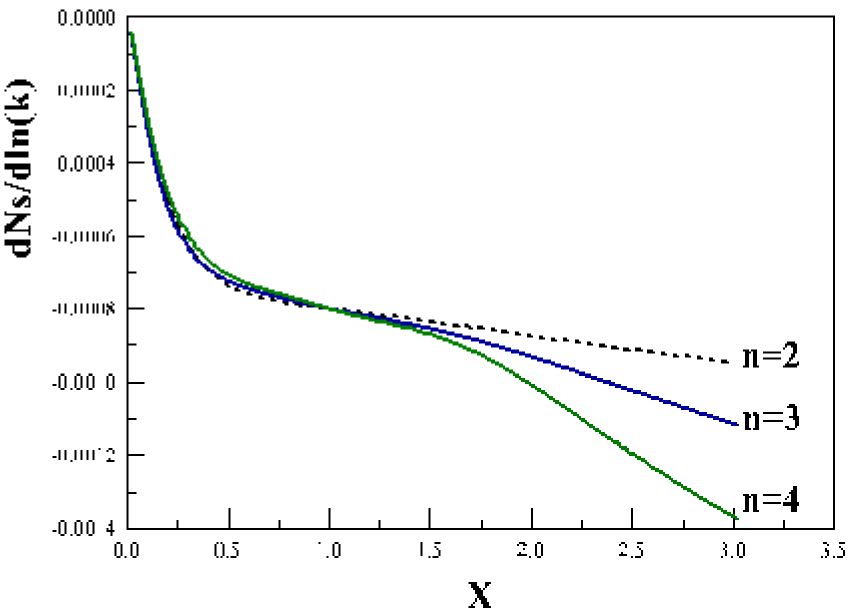
$$w(\chi) = \frac{\chi_0^2}{2\,n}\,\left[n\,\left(1 - x^2\right) + x^{2\,n} - 1\right] \qquad w(\chi) = \frac{\chi_0^2}{2\,n}\,\left[n\,x^2 + x^{2\,n}\right]$$

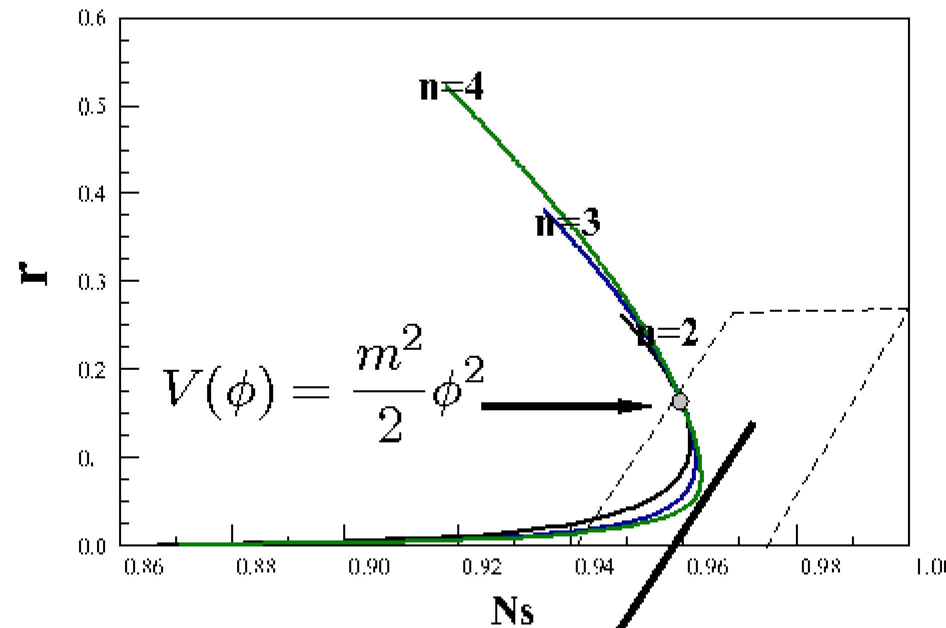




RESULTS

1) New inflation (B.S.) Ne= 50 (change accordingly)

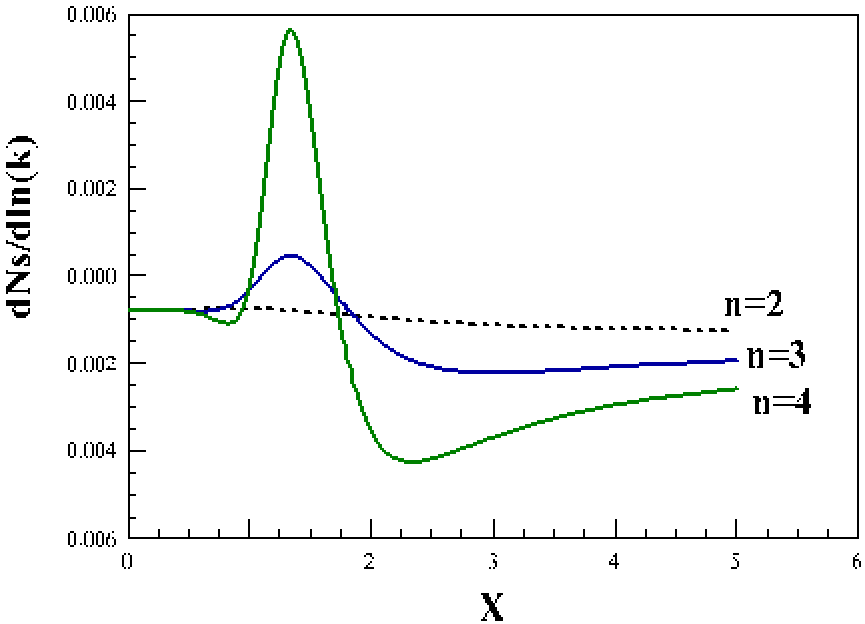


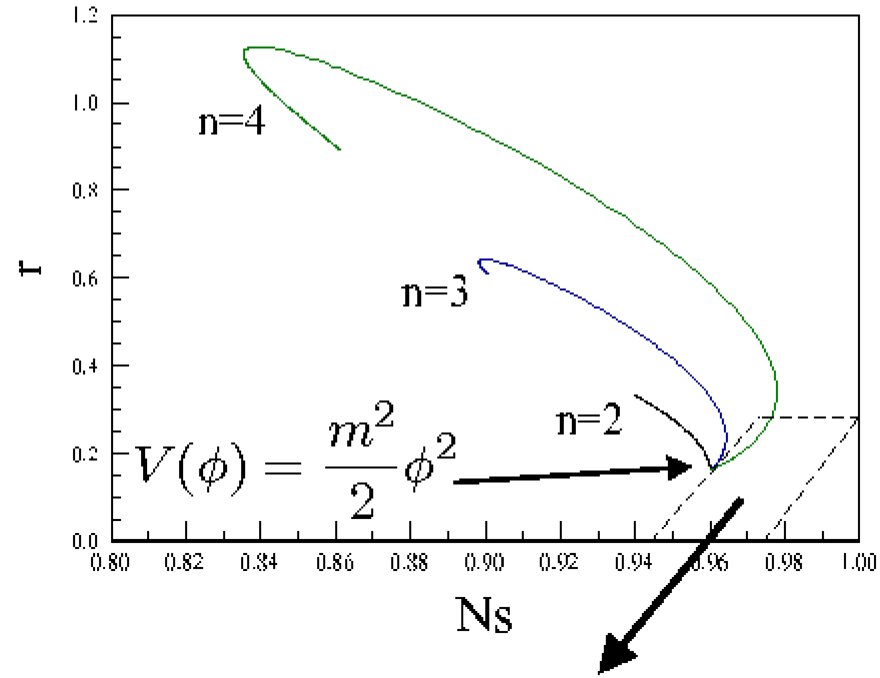


WMAP 3 marginalized region of r- n_s (95% CL)

* *Large region of consistency for small field New Inflation*

2) Chaotic inflation





WMAP 3 marginalized region of r- ns (95% CL)

*** Small region of consistency with WMAP 3**

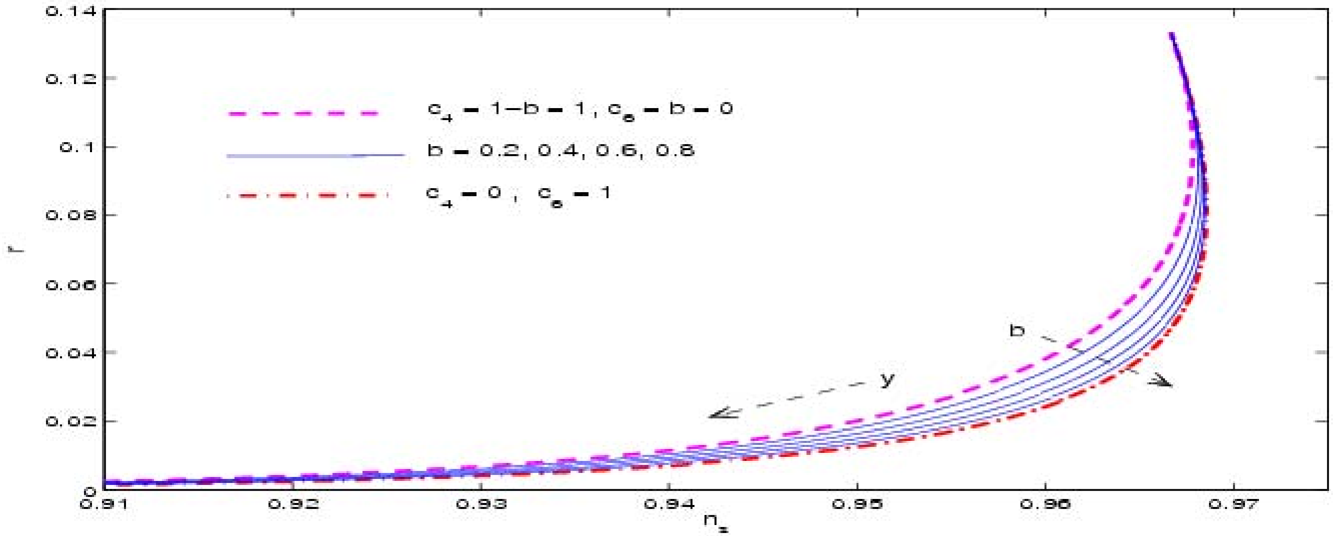
Higher Order Inflaton Potentials

Till here we considered fourth degree inflaton potentials. Can higher order terms modify the physical results and the observable predictions?

We systematically study the effects produced by higher order terms ($n > 4$) in the inflationary potential on the observables n_s and r.
All coefficients in the potential w become order one using the field χ within the Ginsburg-Landau approach:
$w(\chi) = c_0 - \frac{1}{2}\,\chi^2 + \sum_{n=3}^{\infty} \frac{c_n}{n}\,\chi^n \quad , \quad c_n = \mathcal{O}(1)\ .$

All $r = r(n_s)$ curves for double–well potentials of arbitrary high order fall **inside** a universal banana-shaped region $\mathcal{B}$.
Moreover, the $r = r(n_s)$ curves for double–well potentials even for arbitrary positive higher order terms lie inside the banana region $\mathcal{B}$.

The sextic double–well inflaton potential



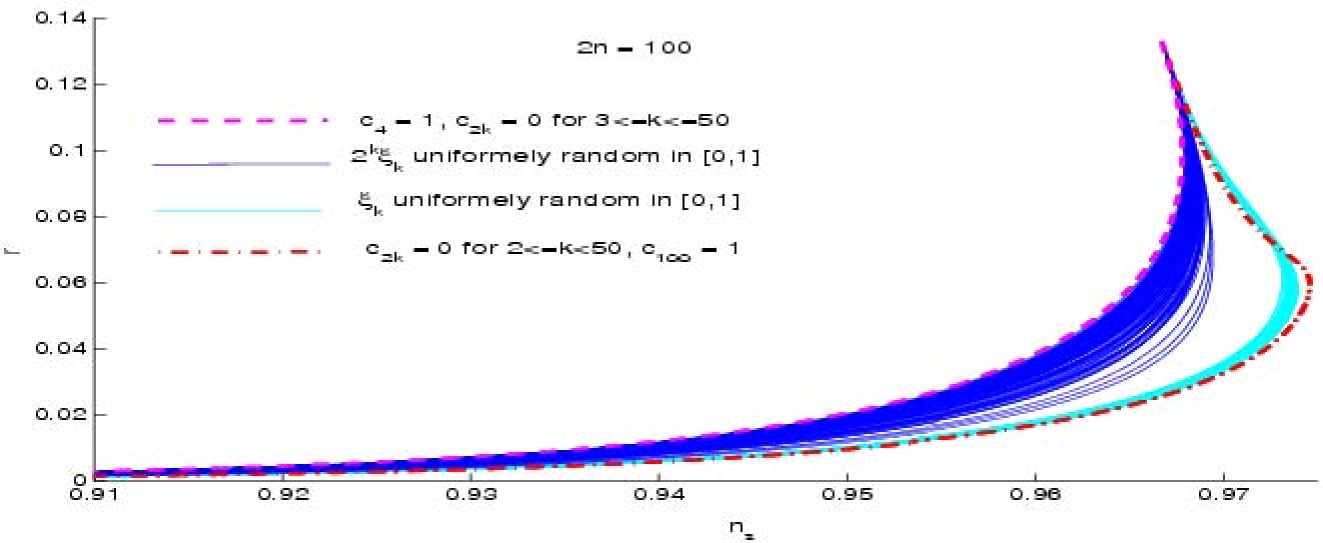
$$w_b(\chi) = \frac{y}{96}\left(\chi^2 - \frac{8}{y}\right)^2 \left(3 + b + \frac{1}{4}\, y\, b\, \chi^2\right) \ .$$

$0 < y < \infty$ coupling. $0 < b < 1$ shape-parameter.

$w_{b=0}(\chi) = \frac{y}{32}\left(\chi^2 - \frac{8}{y}\right)^2$ fourth order double–well.

$w_{b=1}(\chi) = \frac{8}{3\,y} - \frac{1}{2}\,\chi^2 + \frac{y^2}{384}\,\chi^6$ sixth order double–well.

The 100th degree polynomial inflaton potential



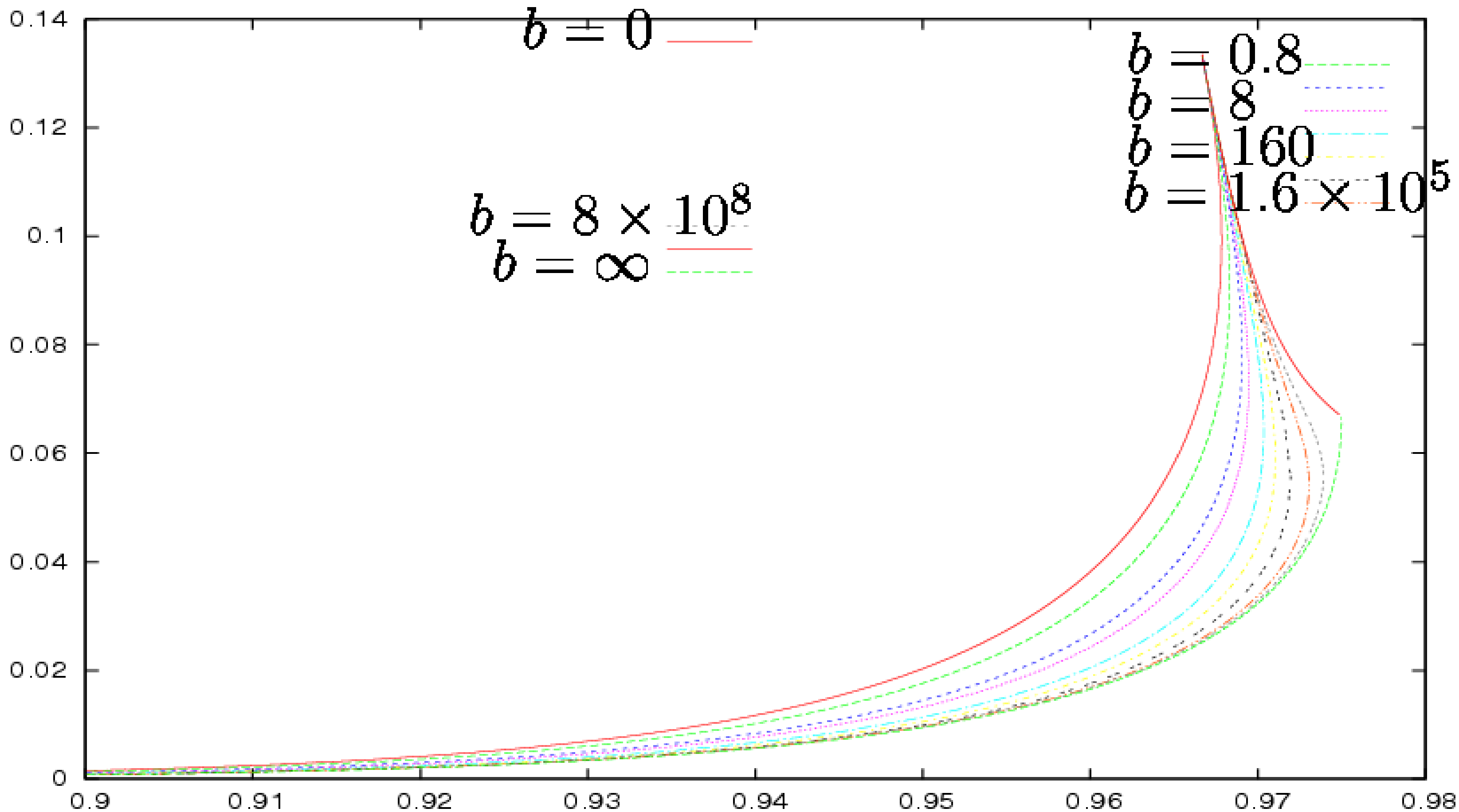
$$w(\chi) = \frac{4}{y} - \frac{1}{2}\chi^2 + \frac{4}{y}\sum_{k=2}^{n} \frac{c_{2k}}{k} \left(\frac{y^k}{8^k}\, u^{2k} - 1\right)$$

The coefficients c_{2k} were extracted at random.

The lower border of the banana-shaped region is given by the potential:

$$w(\chi) = \frac{4}{y} - \frac{1}{2}\chi^2 + \frac{4}{n\,y} \left(\frac{y^n}{8^n}\, u^{2n} - 1\right) \text{ with } n = 50.$$

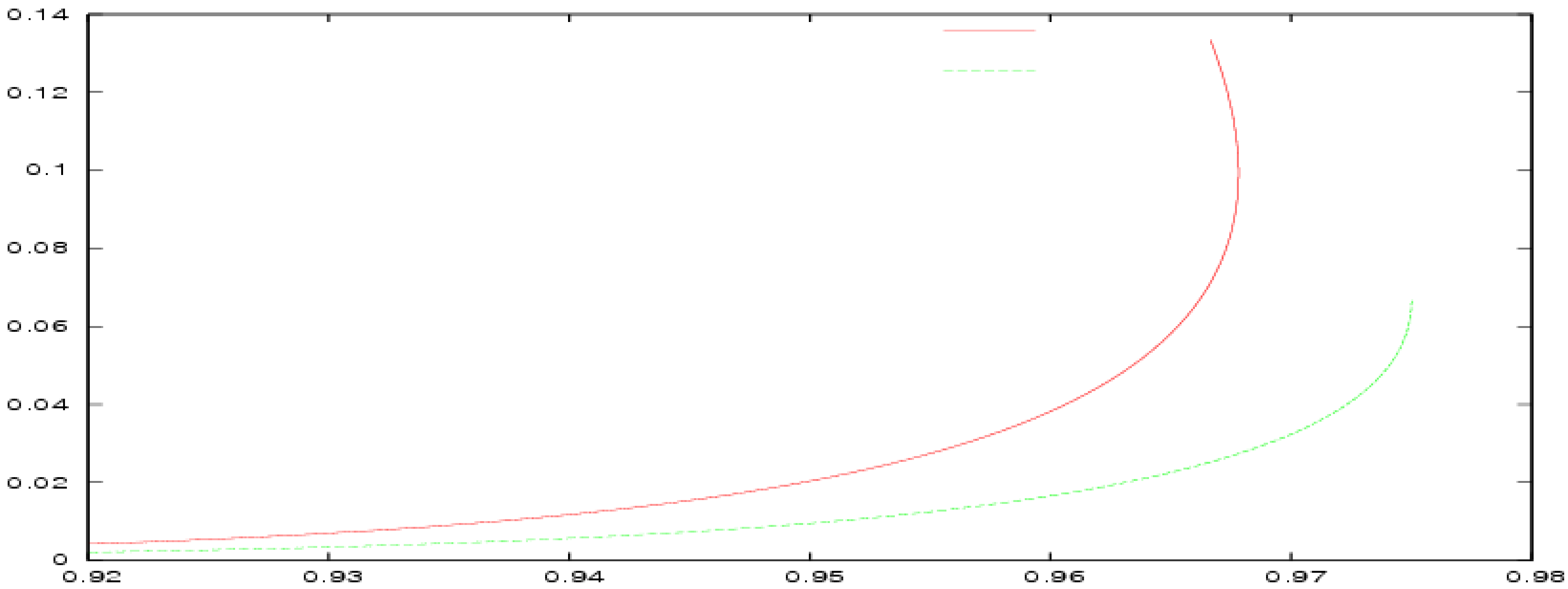
The exponential inflaton potential



$$w(\chi) = \frac{2}{y\, b(e^{2\,b} - 1)} \left[e^{\frac14\, y\, b\, \chi^2} - e^{2\,b} \left(1 - 2\,b + \frac14\, y\, b\, \chi^2\right)\right]$$

The upper border of the banana region is obtained for $b = 0$ and the lower border for $b \to \infty$.

The universal banana region



We find that all $r = r(n_s)$ curves for double–well inflaton potentials in the Ginsburg-Landau spirit fall **inside** the universal banana region $\mathcal{B}$.
The lower border of $\mathcal{B}$ corresponds to the limiting potential:

$$w(\chi) = \frac{4}{y} - \frac{1}{2}\chi^2 \quad \text{for } \chi < \sqrt{\frac{8}{y}} \quad , \quad w(\chi) = +\infty \quad \text{for } \chi > \sqrt{\frac{8}{y}}$$

This gives a lower bound for r for **all** potentials in the Ginsburg-Landau class: $r > 0.021$ for the current best value of the spectral index $n_s = 0.964$.

CONCLUSIONS

Most probable values with the fourth degree double–well inflaton potential: $n_s \simeq 0.964, \; r \simeq 0.051$

Lower bound for r for all potentials in the Ginsburg-Landau class: $r > 0.021$ for the current best value $n_s = 0.964$.

Notice that at $n_s = 0.964$:

$\frac{dr}{dn_s} = 4.9$ on the upper border of $\mathcal{B}$ (fourth degree double–well).

$\frac{dr}{dn_s} = 1.35$ on the lower border of $\mathcal{B}$.

Notice that an improvement δ on the precision of n_s implies an improvement $\simeq 5 \; \delta$ on the precision of r for the fourth degree double–well potential.

PREDICTIONS

From the upper universal curve:

UPPER BOUND $r < 0.053$

From the lower universal curve:

LOWER BOUND $r > 0.021$

$$0.021 < r < 0.053$$

Most probable value: $r \sim 0.051$

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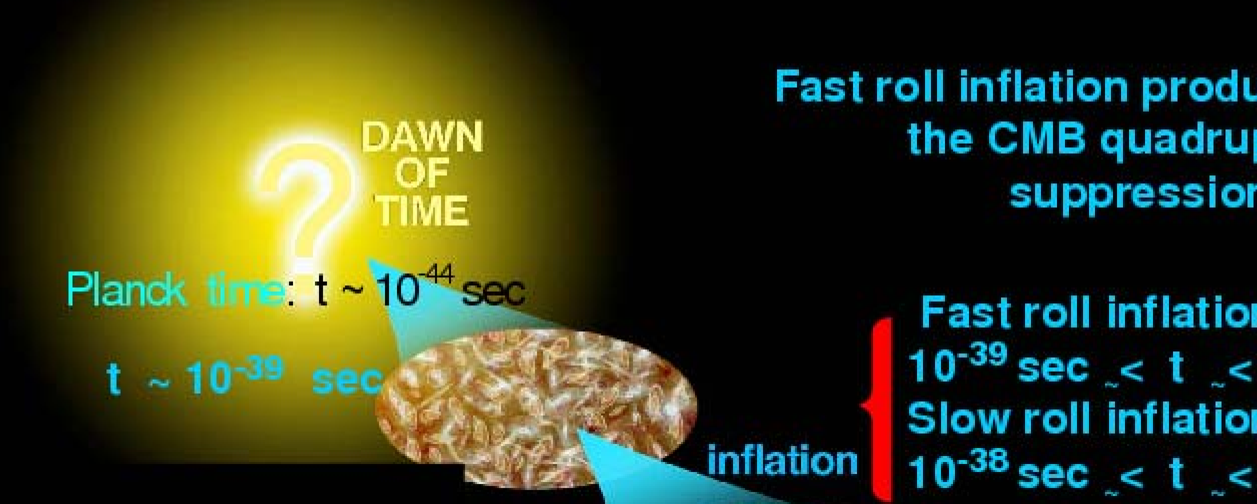
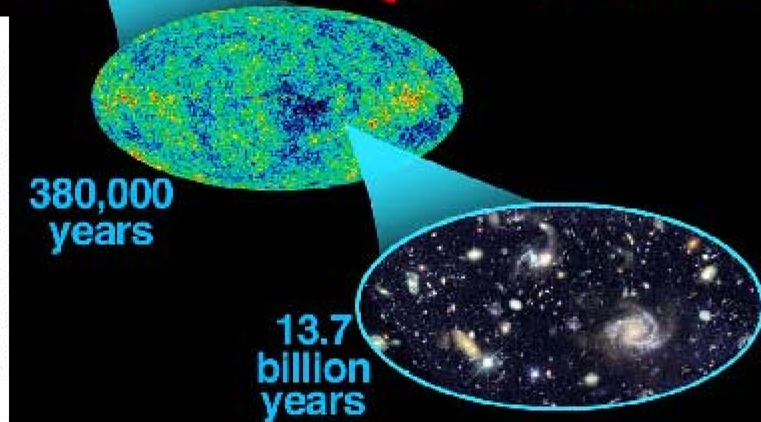
Fast roll inflation produces the CMB quadrupole suppression





Fast roll inflation
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Slow roll inflation
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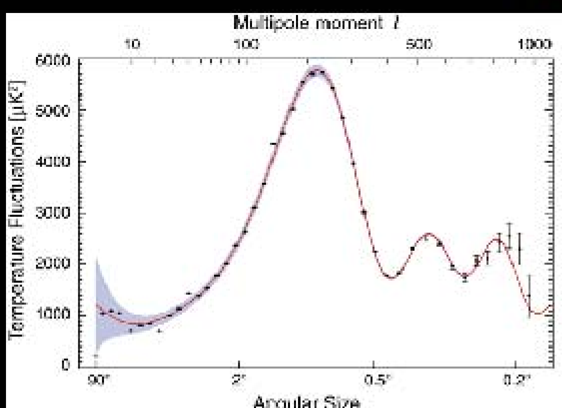


Fast and Slow Roll Inflation

$$H^2 = \frac{1}{3 \, M_{PL}^2} \left[\frac{1}{2} \, \dot{\Phi}^2 + V(\Phi)\right] \, ,$$

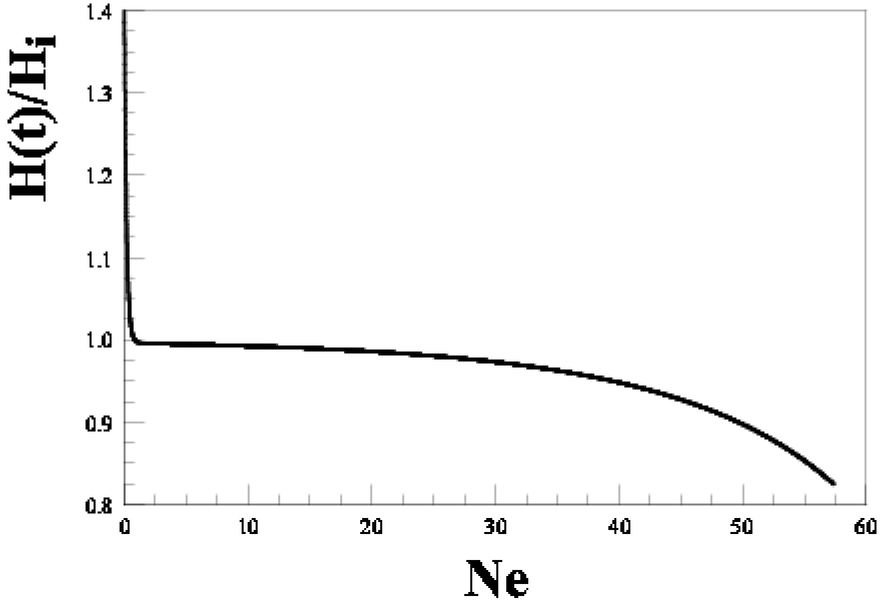
$$\ddot{\Phi} + 3 \, H \, \dot{\Phi} + V'(\Phi) = 0 \, .$$

Slow-roll corresponds to $\dot{\Phi}^2 \ll V(\Phi)$.

Generically, we can have $\dot{\Phi}^2 \sim V(\Phi)$ to start.
That is, FAST ROLL inflation.

However, slow-roll is an attractor with a large basin.

Hubble vs. number of efolds



$H_i =$ Hubble at the beginning of slow-roll.
Fast-roll lasts about one-efold.

Extreme fast roll solution ($y^2 = 3$) in cosmic time:

$$H = \frac{1}{3\,t} \quad , \quad a(t) = a_0 \, t^{\frac{1}{3}} \quad , \quad \Phi = -M_{Pl} \sqrt{\frac{2}{3}} \, \log(m\,t) \, .$$

A: a fast-roll stage PRIOR to slow roll

Allowing for ***RAPID*** variation of the condensate ϕ

$$\left[\frac{d^2}{d\eta^2} + k^2 - \frac{\nu^2 - \frac{1}{4}}{\eta^2} - \mathcal{V}(\eta)\right] v_k = 0$$

$$\downarrow$$

Depends on high(er) derivatives of ϕ : ***neglible in slow roll***, ***large for fast roll***

WHEN? large INITIAL $\dot{\phi}$ but large FRICTION term $\rightarrow$ short fast roll stage

$\longrightarrow \mathcal{V}(\eta) =$ ***LOCALIZED POTENTIAL***

$D(k) \propto T(k)$ = Transmission coeff. of SCATTERING PBM!!

General solution: $S(k;\eta) = A(k)\, g_\nu(k;\eta) + B(k)\, [g_\nu(k;\eta)]^*$

With normalization condition $|A(k)|^2 - |B(k)|^2 = 1$

Quantization:

$$v_k(\eta) = a_k S(k;\eta) + a_k^\dagger S^*(k;\eta)$$

B(k)=0 $\longrightarrow$ B.D. vacuum

Power spectrum

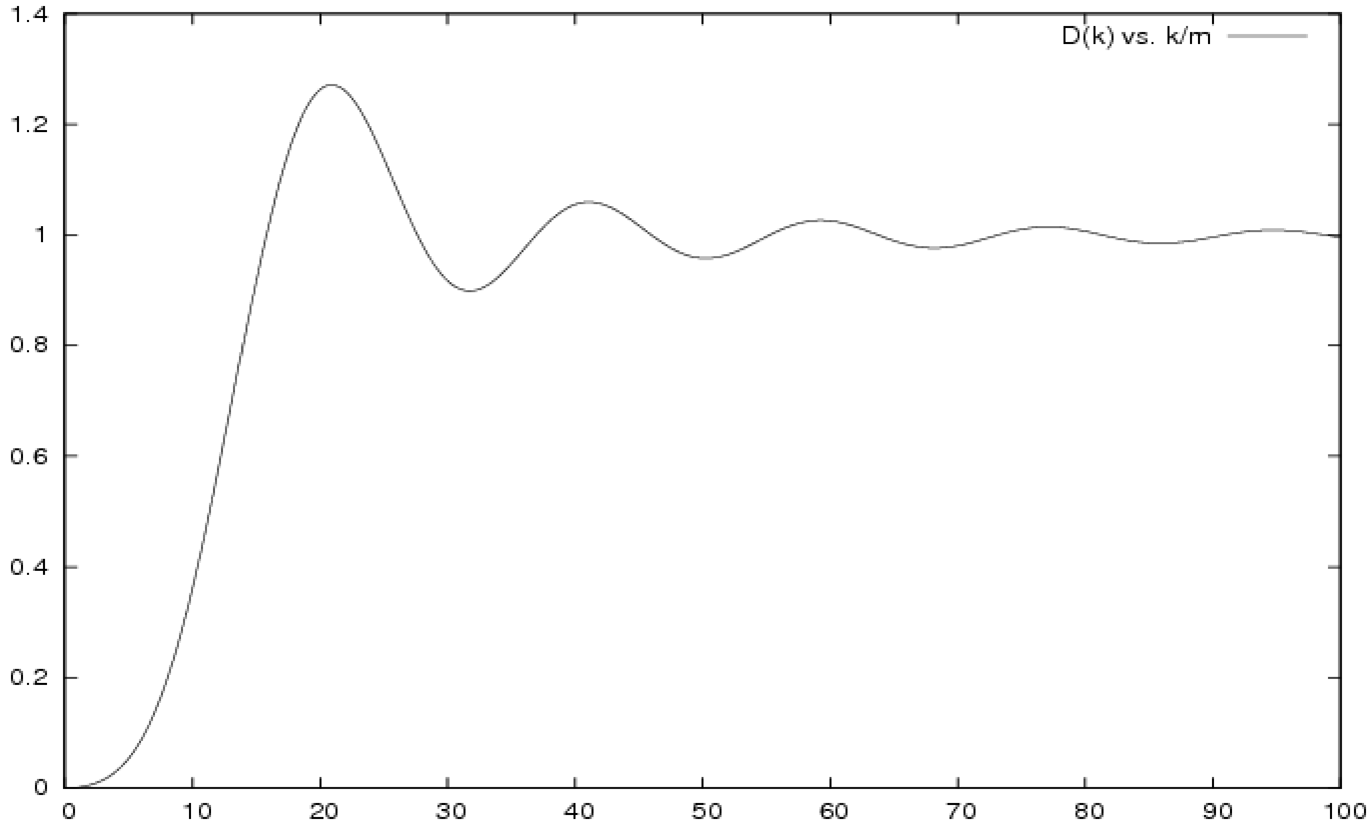
$$\mathcal{P}(k) = \langle 0 || \delta\Psi_k|^2 | 0 \rangle = \mathcal{P}_{BD}(k)\,[1 + D(k)]$$

$$\frac{H^2}{\epsilon_v M_P^2}\left(\frac{k}{k_0}\right)^{n_s - 1}$$

Transfer function for boundary conditions

$$n_s = 1 - 6\epsilon_v + 2\eta_v$$

The Fast-Roll Transfer Function



$k_Q = 11.5\ m,\ k_{fastroll \to slowroll} = 14\ m,\ k_{pivot} = 96.7\ m,$

$m = 1.21\,10^{13}\text{GeV}, k_Q^{today} = 0.238\ \text{Gpc}^{-1} \implies$ redshift at the beginning of inflation $= 0.9 \times 10^{56} \simeq e^{129}$.

Quadrupole suppression and Fast-roll Inflation

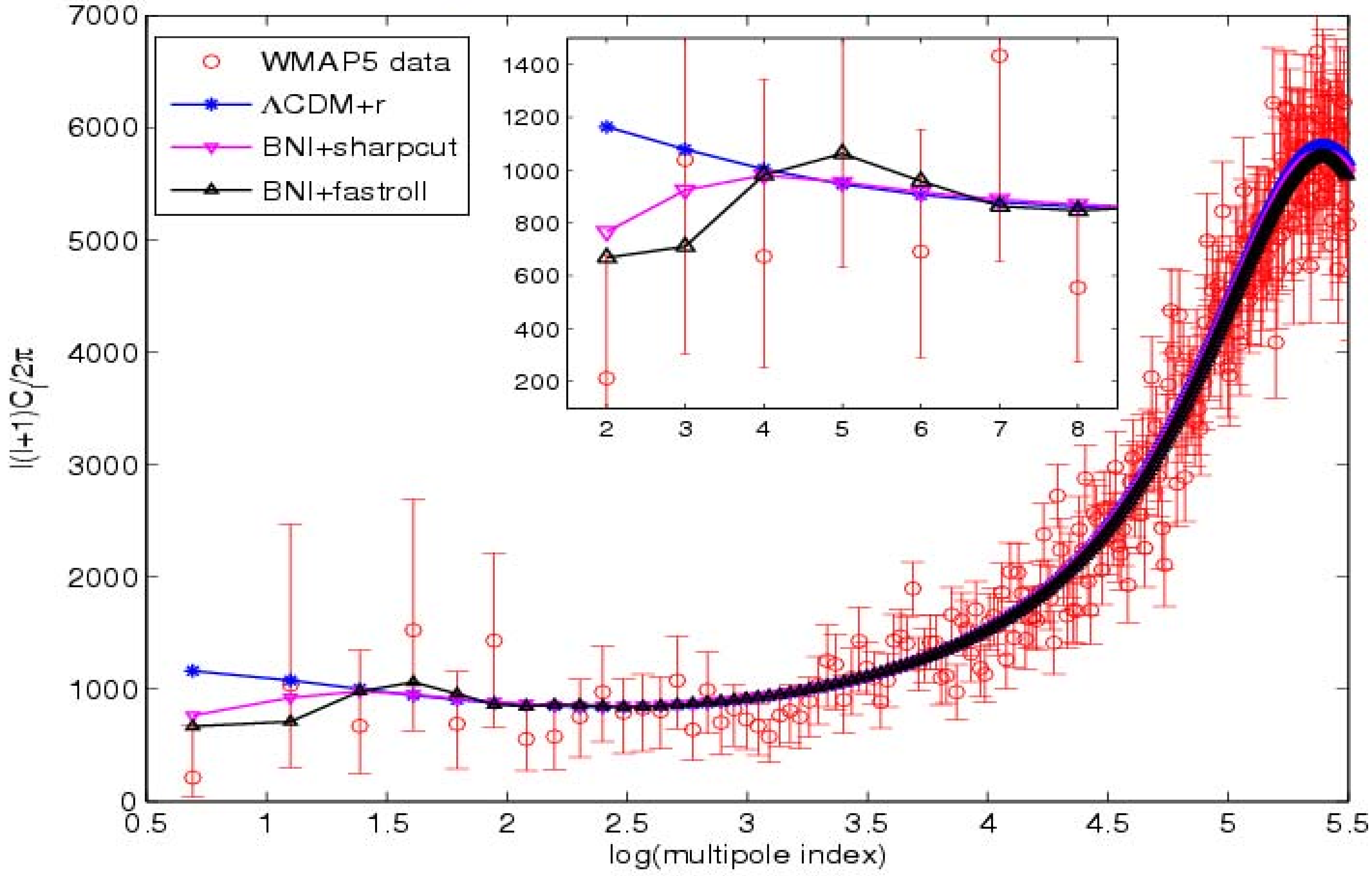
The observed CMB-quadrupole (COBE,WMAP5) is six times smaller than the ΛCDM model value.
In the best ΛCDM fit the probability that the quadrupole is as low or lower than the observed value is 3%.
It is hence relevant to find a cosmological explanation of the quadrupole supression.

Generically, the classical evolution of the inflaton has a brief **fast-roll stage** that precedes the slow-roll regime.
In case the quadrupole CMB mode leaves the horizon during fast-roll, before slow-roll starts, we find that the quadrupole mode gets suppressed.

$P(k) = |\Delta_{k\ ad}^{(S)}|^2\ (k/k_0)^{n_s - 1}[1 + D(k)]$
The transfer function $D(k)$ changes the primordial power.
$1 + D(0) = 0, \quad D(\infty) = 0$

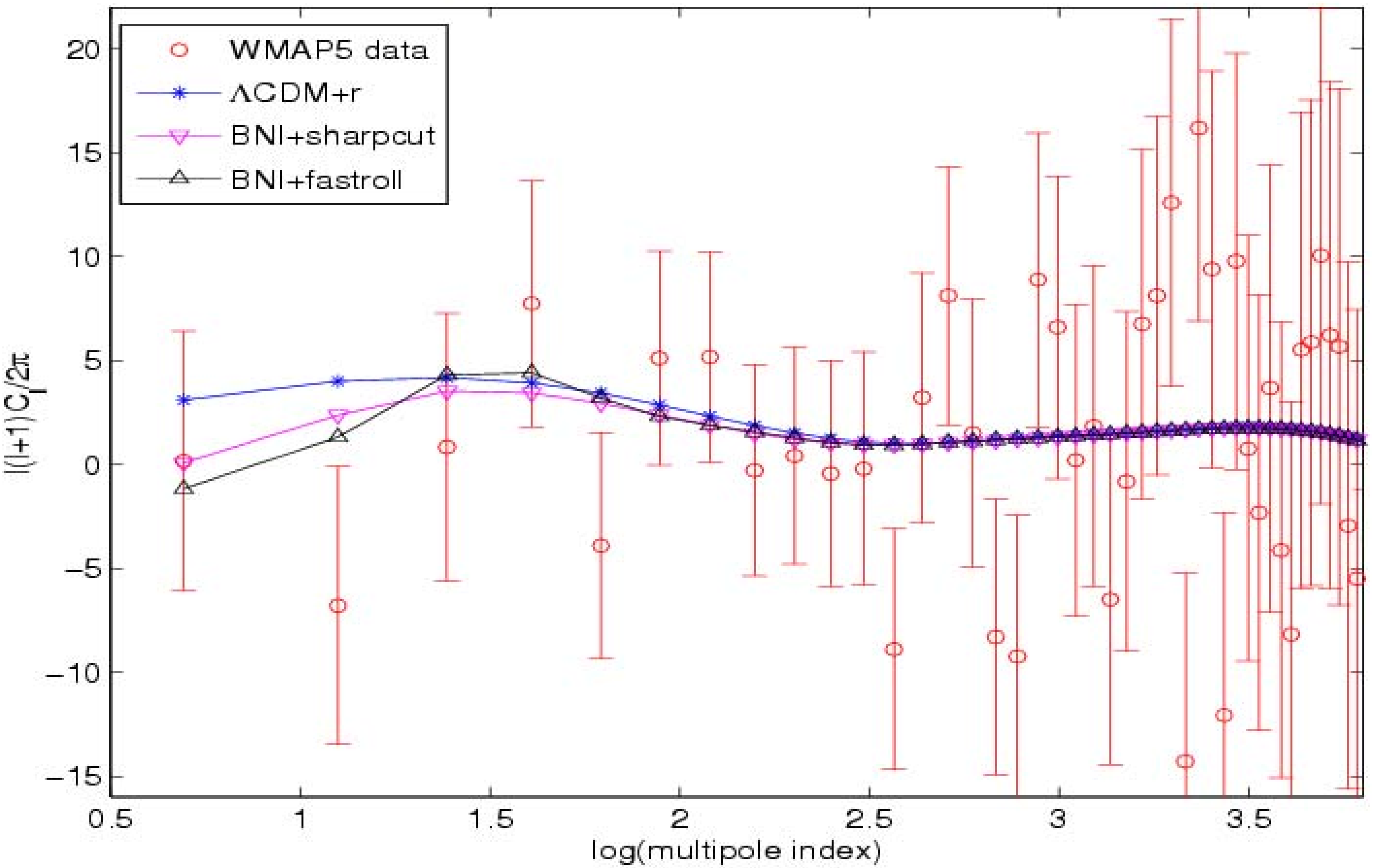
Comparison, with the experimental WMAP5 data

of the theoretical C_ℓ^{TT} multipoles



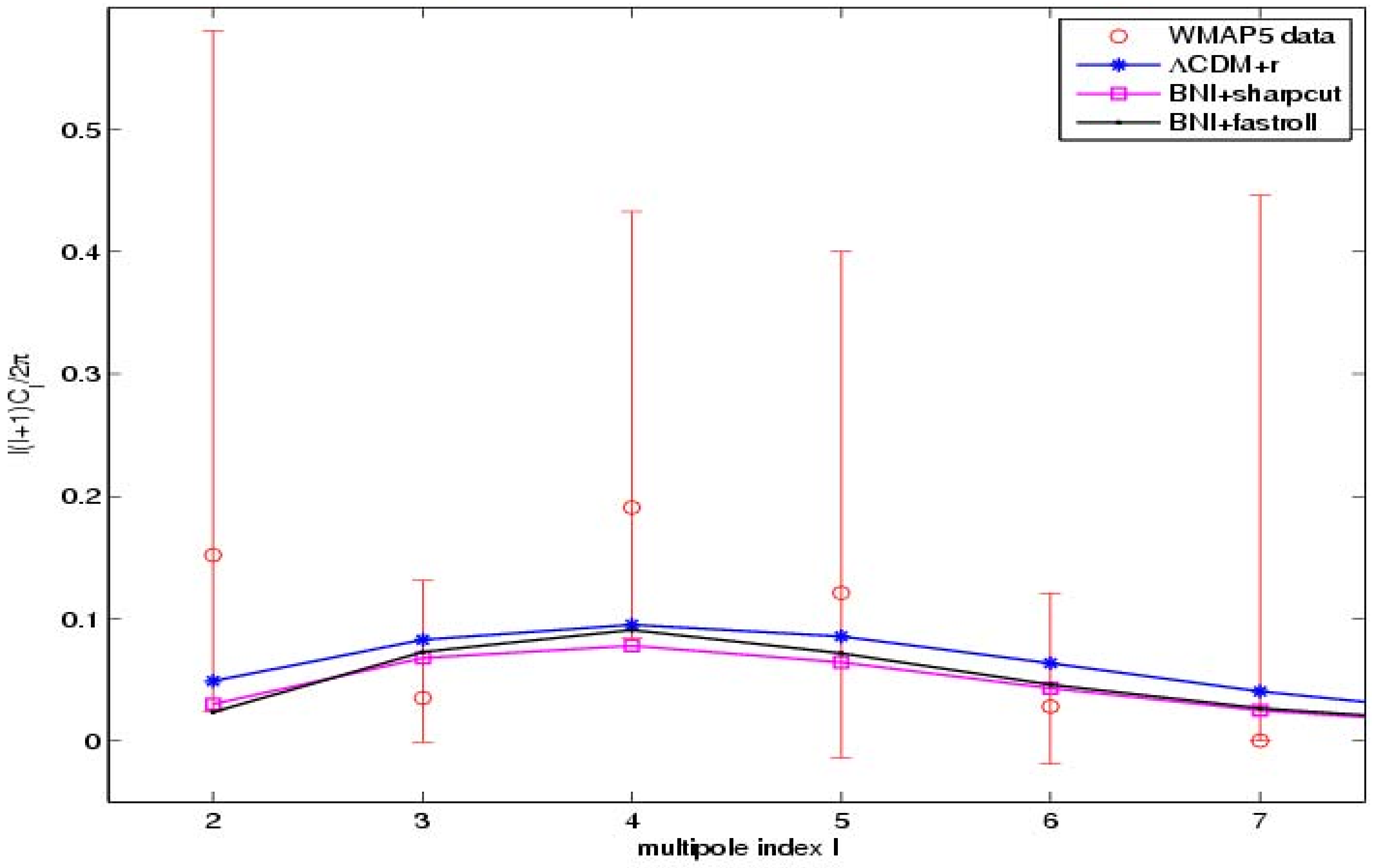
Comparison, with the experimental WMAP5 data

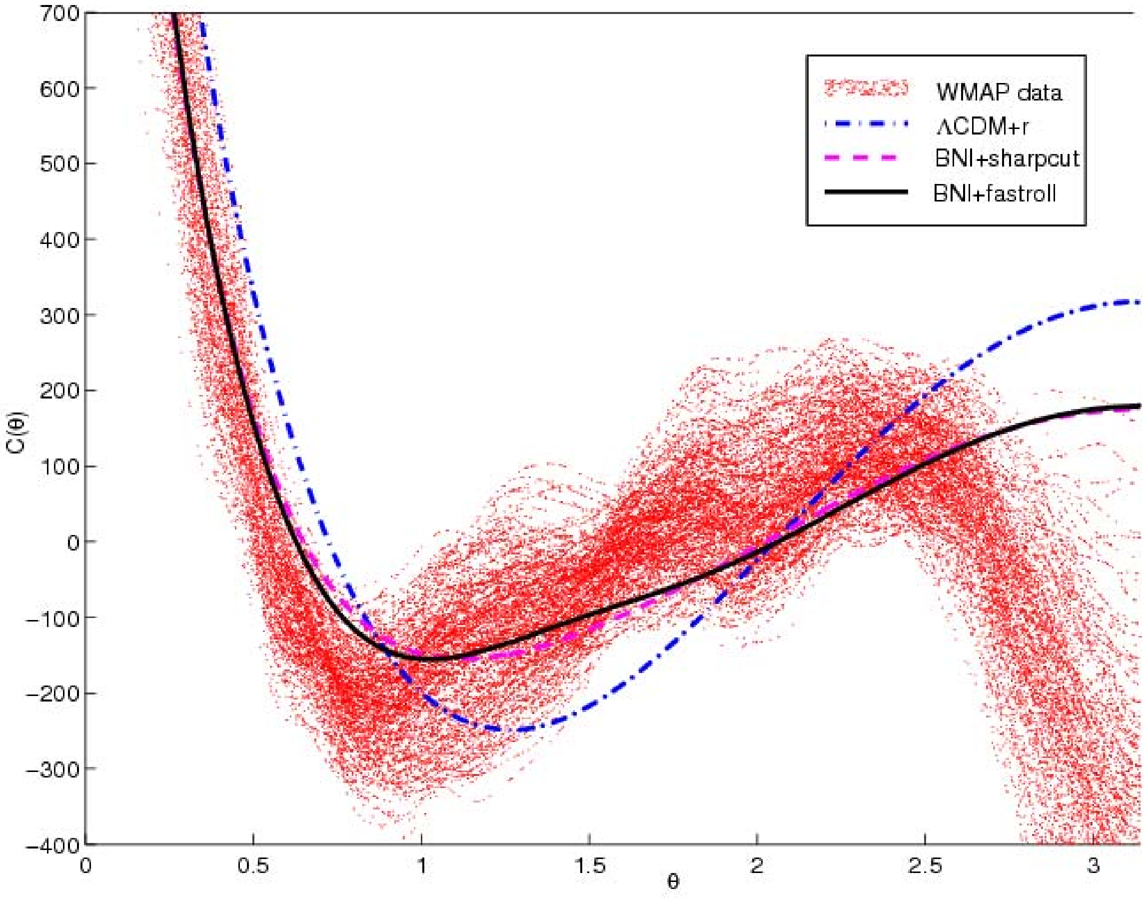
of the theoretical C_ℓ^{TE} multipoles



Comparison, with the experimental WMAP-5 data

of the theoretical C_ℓ^{EE} multipoles





WMAP data
ΛCDM+r
BNI+sharpcut
BNI+fastroll
C(θ)
θ
700
600
500
400
300
200
100
0
−100
−200
−300
−400
0
0.5
1
1.5
2
2.5
3

Summary and Conclusions

- Inflation can be formulated as an effective field theory in the Ginsburg-Landau spirit with energy scale $M \sim M_{GUT} \sim 10^{16}$ GeV $\ll M_{Pl}$.
  Inflaton mass small: $m \sim H/\sqrt{N} \sim M^2/M_{Pl} \ll M$.
  Infrared regime !!
- The slow-roll approximation is a $1/N$ expansion, $N \sim 60$
- MCMC analysis of WMAP+LSS data plus the Trinomial Inflation potential indicates a spontaneously symmetry breaking potential (new inflation): $w(\chi) = \frac{y}{32}\left(\chi^2 - \frac{8}{y}\right)^2$.
- Lower Bounds: $r > 0.016\ (95\%\ \mathrm{CL})$, $r > 0.049\ (68\%\ \mathrm{CL})$.
  The most probable values are $n_s \simeq 0.956$, $r \simeq 0.055$ with a quartic coupling $y \simeq 1.3$.

Summary and Conclusions 2

- CMB quadrupole suppression may be explained by the effect of fast-roll inflation provided the today's horizon size modes exited by the end of fast-roll inflation.
- Quantum (loop) corrections in the effective theory are of the order $(H/M_{Pl})^2 \sim 10^{-9}$. Same order of magnitude as loop graviton corrections.

D. Boyanovsky, H. J. de Vega, N. G. Sanchez,

Quantum corrections to the inflaton potential and the power spectra from superhorizon modes and trace anomalies, Phys. Rev. D 72, 103006 (2005), astro-ph/0507596.

Quantum corrections to slow roll inflation and new scaling of superhorizon fluctuations. Nucl. Phys. B 747, 25 (2006), astro-ph/0503669.

Quantum Fluctuations:

Scalar Curvature, Tensor, Fermion, Light Scalar.

All these quantum fluctuations contribute to the inflaton potential and to the primordial power spectra.

In de Sitter space-time: $< T_{\mu\nu} >= \frac{1}{4} \, g_{\mu\nu} \;\; < T_\alpha^\alpha >$

Hence, $V_{eff} = V_R + < T_0^0 >= V_R + \frac{1}{4} < T_\alpha^\alpha >$

Sub-horizon (Ultraviolet) contributions appear through the trace anomaly and only depend on the spin of the particle. Superhorizon (Infrared) contributions are of the order N^0 and can be expressed in terms of the slow-roll parameters.

$$V_{eff}(\Phi_0) = V(\Phi_0) \left[1 + \frac{H_0^2}{3 \, (4\pi)^2 \, M_{Pl}^2} \left(\frac{\eta_v - 4 \, \epsilon_v}{\eta_v - 3 \, \epsilon_v} + \frac{3 \, \eta_\sigma}{\eta_\sigma - \epsilon_v} + \mathcal{T} \right) \right]$$

$\mathcal{T} = \mathcal{T}_\Phi + \mathcal{T}_s + \mathcal{T}_t + \mathcal{T}_F = -\frac{2903}{20}$ is the total trace anomaly.

$\mathcal{T}_\Phi = \mathcal{T}_s = -\frac{29}{30}, \;\; \mathcal{T}_t = -\frac{717}{5}, \;\; \mathcal{T}_F = \frac{11}{60}$

$\longrightarrow$ the graviton (t) dominates.

Corrections to the Primordial Scalar and Tensor Power

$$|\Delta^{(S)}_{k,eff}|^2 = |\Delta^{(S)}_{k}|^2 \{1+$$

$$+\frac{2}{3}\left(\frac{H_0}{4\,\pi\,M_{Pl}}\right)^2\left[1+\frac{\frac{3}{8}\,r\,(n_s-1)+2\,\frac{dn_s}{d\ln k}}{(n_s-1)^2}+\frac{2903}{40}\right]\right\}$$

$$|\Delta^{(T)}_{k,eff}|^2 = |\Delta^{(T)}_{k}|^2 \left\{1-\frac{1}{3}\left(\frac{H_0}{4\,\pi\,M_{Pl}}\right)^2\left[-1+\frac{1}{8}\,\frac{r}{n_s-1}+\frac{2903}{20}\right]\right\}.$$

The anomaly contribution $-\frac{2903}{20} = -145.15$ DOMINATES as long as the number of fermions less than 783.

The scalar curvature fluctuations $|\Delta^{(S)}_{k}|^2$ are ENHANCED and the tensor fluctuations $|\Delta^{(T)}_{k}|^2$ REDUCED.

However, $\left(\frac{H}{M_{Pl}}\right)^2 \sim 10^{-9}$.

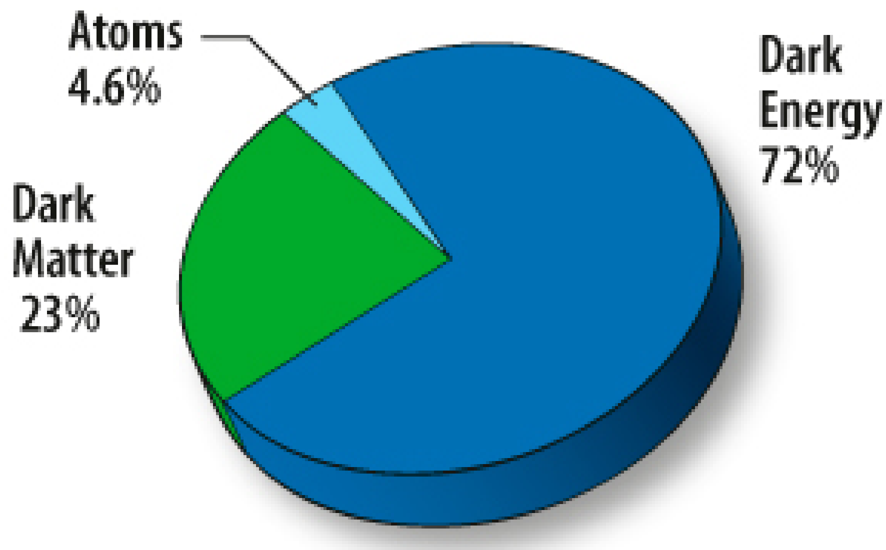
D. Boyanovsky, H. J. de Vega, N. G. Sanchez, Phys. Rev. D 72, 103006 (2005), astro-ph/0507596.

CONTENT OF THE UNIVERSE

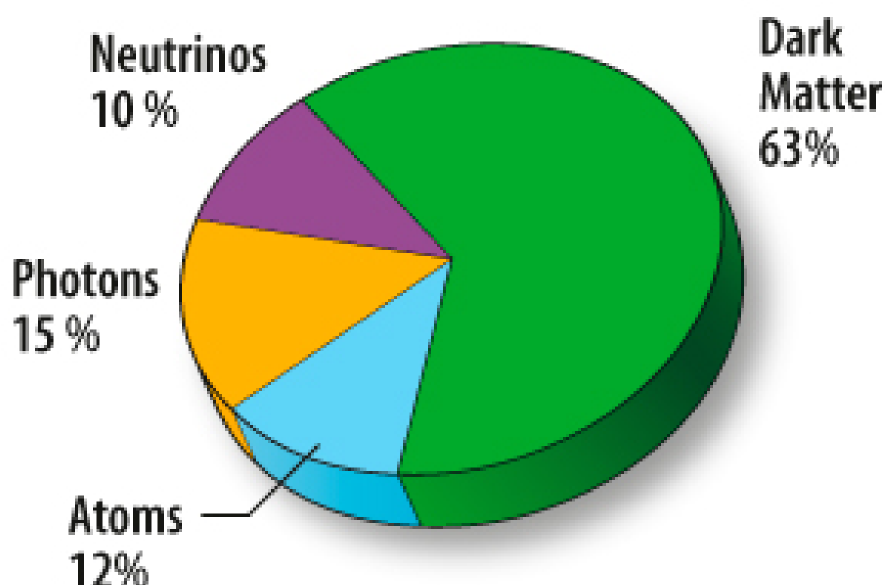
WMAP data reveals that its contents include 4.6% atoms, the building blocks of stars and planets.

Dark matter comprises 23% of the universe.
This matter, different from atoms, does not emit or absorb light.
It has only been detected indirectly by its gravity.

72% of the Universe, is composed of "dark energy", that acts as a sort of an anti-gravity.
This energy, distinct from dark matter, is responsible for the present-day acceleration of the universal expansion.



TODAY



13.7 BILLION YEARS AGO
(Universe 380,000 years old)

Dark Matter

DM must be non-relativistic by structure formation ($z < 30$) in order to reproduce the observed small structures at $\sim 2-3$ kpc. DM particles can decouple being ultrarelativistic (UR) at $T_d \gg m$ or non-relativistic $T_d \ll m$.
Consider particles that decouple at or out of LTE (LTE = local thermal equilibrium).
Distribution function:
$f_d[a(t)\; P_f(t)] = f_d[p_c]$ freezes out at decoupling.
$P_f(t) = p_c/a(t) =$ Physical momentum.
$p_c =$ comoving momentum.

Velocity fluctuations: $y = P_f(t)/T_d(t) = p_c/T_d$

$$\langle \vec{V}^2(t) \rangle = \langle \frac{\vec{P}_f^2(t)}{m^2} \rangle = \frac{\int \frac{d^3 P_f}{(2\pi)^3} \frac{\vec{P}_f^2}{m^2} f_d[a(t)\; P_f]}{\int \frac{d^3 P_f}{(2\pi)^3} f_d[a(t)\; P_f]} = \left[\frac{T_d}{m\; a(t)}\right]^2 \frac{\int_0^\infty y^4 f_d(y) dy}{\int_0^\infty y^2 f_d(y) dy} \; .$$

The formula for the Mass of the Dark Matter particles

Energy Density: $\rho_{DM}(t) = g \int \frac{d^3 P_f}{(2\pi)^3} \sqrt{m^2 + P_f^2}\; f_d[a(t)\, P_f]$

g : $\#$ of internal degrees of freedom of the DM particle, $1 \le g \le 4$. For $z \lesssim 30 \;\Rightarrow$ DM particles are non-relativistic:

$$\rho_{DM}(t) = m\; g\; \frac{T_d^3}{a^3(t)} \int_0^\infty y^2\; f_d(y)\; \frac{dy}{2\pi^2}\; .$$

Using entropy conservation: $T_d = \left(\frac{2}{g_d}\right)^{\frac{1}{3}} T_\gamma\; (1 + z_d)$,

$g_d =$ effective $\#$ of UR degrees of freedom at decoupling,

$T_\gamma = 0.2348$ meV , $1\ \mathrm{meV} = 10^{-3}$ eV.

Today $\Omega_{DM} = \rho_{DM}(0)/\rho_c = 0.105/h^2$ and we obtain for the mass of the DM particle:

$$m = 6.986\ \mathrm{eV}\; \frac{g_d}{g \displaystyle\int_0^\infty y^2\; f_d(y)\; dy}$$. Goal: determine m and g_d

Phase-space density invariant under universe expansion

Using again entropy conservation to replace T_d yields for the one-dimensional velocity dispersion,

$$\sigma_{DM}(z) = \sqrt{\frac{1}{3}\,\langle \vec{V}^2\rangle(z)} = \frac{2^{\frac{1}{3}}}{\sqrt{3}}\,\frac{1+z}{g_d^{\frac{1}{3}}}\,\frac{T_\gamma}{m}\,\sqrt{\frac{\int_0^\infty y^4\,F_d(y)\,dy}{\int_0^\infty y^2\,F_d(y)\,dy}} =$$

$$= 0.05124\,\frac{1+z}{g_d^{\frac{1}{3}}}\,\frac{\mathrm{keV}}{m}\,\left[\frac{\int_0^\infty y^4\,F_d(y)\,dy}{\int_0^\infty y^2\,F_d(y)\,dy}\right]^{\frac{1}{2}}\,\frac{\mathrm{km}}{\mathrm{s}}.$$

Phase-space density: $\mathcal{D} \equiv \dfrac{n(t)}{\langle \vec{P}^2_{phys}(t)\rangle^{\frac{3}{2}}} \overset{\mathrm{non-rel}}{=} \dfrac{\rho_{DM}}{3\sqrt{3}\,m^4\,\sigma^3_{DM}}$

$\mathcal{D}$ is computed theoretically from freezed-out distributions:

$$\mathcal{D} = \frac{g}{2\,\pi^2}\,\frac{\left[\int_0^\infty y^2 F_d(y)dy\right]^{\frac{5}{2}}}{\left[\int_0^\infty y^4 F_d(y)dy\right]^{\frac{3}{2}}}$$

Theorem: The phase-space density $\mathcal{D}$ can only decrease under self-gravity interactions (gravitational clustering) [Lynden-Bell, Tremaine, Henon, 1986].

Mass Estimates of DM particles

Our previous formulas yield for relics decoupling UR at LTE:

$$m = \left(\frac{Z}{g}\right)^{\frac{1}{4}} \ \mathrm{keV} \begin{cases} 0.568 \\ 0.484 \end{cases}, \ g_d = g^{\frac{3}{4}} \ Z^{\frac{1}{4}} \begin{cases} 155 & \text{Fermions} \\ 180 & \text{Bosons} \end{cases}.$$

Since $g = 1 - 4$, we see that $g_d > 100 \Rightarrow T_d > 100$ GeV.
$1 < Z^{\frac{1}{4}} < 5.6$ for $1 < Z < 1000$.
Example: for DM Majorana fermions $(g = 2)$ $m \simeq 0.85$ keV.

Sterile neutrinos ν as DM decoupling out of LTE and UR.
ν is a singlet Majorana fermion with a Majorana mass m_ν coupled with a Yukawa-type coupling $Y \sim 10^{-8}$ to a real scalar field χ. χ is more strongly coupled to the particles in the Standard Model. [Chikashige,Mohapatra,Peccei (1981), Gelmini,Roncadelli (1981), Schechter, Valle (1982), Shaposhnikov, Tkachev (2006), Boyanovsky (2008)]

Relics decoupling non-relativistic

$$F_d^{NR}(p_c) = \frac{2^{\frac52}\,\pi^{\frac72}}{45}\; g_d\, Y_\infty\; \left(\frac{T_d}{m}\right)^{\frac32}\; e^{-\frac{p_c^2}{2\,m\,T_d}} = \frac{2^{\frac52}\,\pi^{\frac72}}{45}\; \frac{g_d\, Y_\infty}{x^{\frac32}}\; e^{-\frac{y^2}{2\,x}}$$

$Y(t) = n(t)/s(t)$, $n(t)$ number of DM particles per unit volume, $s(t)$ entropy per unit volume, $x \equiv m/T_d$, $T_d < m$.

$Y_\infty = \frac{1}{\pi}\sqrt{\frac{45}{8}}\;\frac{1}{\sqrt{g_d}\;T_d\;\sigma_0\;M_{Pl}}$ late time limit of Boltzmann.

σ_0: thermally averaged total annihilation cross-section times the velocity.

From our general equations for m and g_d:

$m = \frac{45}{4\,\pi^2}\;\frac{\Omega_{DM}\;\rho_c}{g\;T_\gamma^3\;Y_\infty} = \frac{0.748}{g\;Y_\infty}$ eV and $m^{\frac52}\;T_d^{\frac32} = \frac{45}{2\,\pi^2}\;\frac{1}{g\;g_d\;Y_\infty}\;Z\;\frac{\rho_s}{\sigma_s^3}$

Finally:

$$\sqrt{m\;T_d} = 1.47\;\left(\frac{Z}{g_d}\right)^{\frac13}\,\mathrm{keV}. \quad m = 3.67\;\mathrm{keV}\;Z^{\frac13}\;\frac{g_d^{\frac{1}{12}}}{\sqrt{g}}\;\sqrt{\frac{\sigma_0}{\mathrm{pb}}}$$

We used ρ_{DM} today and the decrease of the phase space density by a factor Z. 1 pb $= 10^{-36}\;\mathrm{cm}^2 = 0.257\;/(10^5\;\mathrm{GeV}^2)$.

Relics decoupling non-relativistic 2

Allowed ranges for m and T_d.

$m > T_d > b$ eV where $b > 1$ or $b \gg 1$ for DM decoupling in the RD era

$$\left(\frac{Z}{g_d}\right)^{\frac{1}{3}} 1.47 \text{ keV} < m < \frac{2.16}{b} \text{ MeV } \left(\frac{Z}{g_d}\right)^{\frac{2}{3}}$$

$g_d \simeq 3 \quad \text{for} \quad 1 \text{ eV} < T_d < 100 \text{ keV}$ and $1 < Z < 10^3$

$1.02 \text{ keV} < m < \frac{104}{b} \text{ MeV} \quad , \quad T_d < 10.2$ keV.

D. Boyanovsky, H. J. de Vega, N. Sanchez,
Phys. Rev. D 77, 043518 (2008), arXiv:0710.5180.
H. J. de Vega, N. G. Sanchez, arXiv:0901.0922.

Only using ρ_{DM} today (ignoring the phase space density information) gives:

$$\sigma_0 = 0.16 \text{ pbarn} \frac{g}{\sqrt{g_d}} \frac{m}{T_d}$$

http://pdg.lbl.gov

Dark Energy

$76 \pm 5\%$ of the present energy of the Universe is Dark !
Current observed value:
$\rho_\Lambda = \Omega_\Lambda \, \rho_c = (2.39 \text{ meV})^4 \quad , \quad 1 \text{ meV} = 10^{-3} \text{ eV}.$
Equation of state $p_\Lambda = -\rho_\Lambda$ within observational errors.
Quantum zero point energy. Renormalized value is finite.
Bosons (fermions) give positive (negative) contributions.
Mass of the lightest particles ~ 1 meV is in the right scale.
Spontaneous symmetry breaking of continuous symmetries produces massless scalars as Goldstone bosons. A small symmetry breaking provide light scalars: axions,majorons...
Observational Axion window $10^{-3} \text{ meV} \lesssim M_{\text{axion}} \lesssim 10 \text{ meV}$.
Dark energy can be a cosmological zero point effect. (As the Casimir effect in Minkowski with non-trivial boundaries).
We need to learn the physics of light particles (< 1 MeV), also to understand dark matter !!

Summary and Conclusions

- We formulate inflation as an effective field theory in the Ginsburg-Landau spirit with energy scale $M \sim M_{GUT} \sim 10^{16}$ GeV $\ll M_{Pl}$. Inflaton mass small: $m \sim H/\sqrt{N} \sim M^2/M_{Pl} \ll M$. Infrared regime !!
- For all slow-roll models $n_s - 1$ and r are $1/N, \; N \sim 60$.
- MCMC analysis of WMAP+LSS data plus this theory input indicates a spontaneously broken inflaton potential: $w(\chi) = \frac{y}{32}\left(\chi^2 - \frac{8}{y}\right)^2 \; , \; y \simeq 1.26$.
- Lower Bounds: $r > 0.023$ (95% CL) , $r > 0.046$ (68% CL). The most probable values are $r \simeq 0.051$($\Leftarrow$ measurable !!) $n_s \simeq 0.964$.
- Model independent analysis of dark matter points to a particle mass at the keV scale. T_d may be > 100 GeV. DM is cold.

Summary and Conclusions 2

- CMB quadrupole suppression may be explained by the effect of fast-roll inflation provided the today's horizon size modes exited by the end of fast-roll inflation.
- Quantum (loop) corrections in the effective theory are of the order $(H/M_{Pl})^2 \sim 10^{-9}$. Same order of magnitude as loop graviton corrections.

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Future Perspectives

The Golden Age of Cosmology and Astrophysics continues.

A wealth of data from WMAP (7 yr), Planck, Atacama Cosmology Tel and further experiments are coming.

Galaxy and Star formation. DM properties from astronomical observations. Better bounds on DM cross-sections.

DM in planets and the earth. Flyby and Pioneer anomalies?

The Dark Ages...Reionisation...the 21cm line...

Nature of Dark Energy? 76% of the energy of the universe.

Nature of Dark Matter? 83% of the matter in the universe.

Light DM particles are strongly favoured $m_{DM} \sim$ keV.

Sterile neutrinos? Some unknown light particle ??

Need to learn about the physics of light particles (< 1 MeV).

END

THANK YOU FOR YOUR ATTENTION