Cosmology with Gravitational Lensing

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Isaac Newton on gravitational lensing (1704)

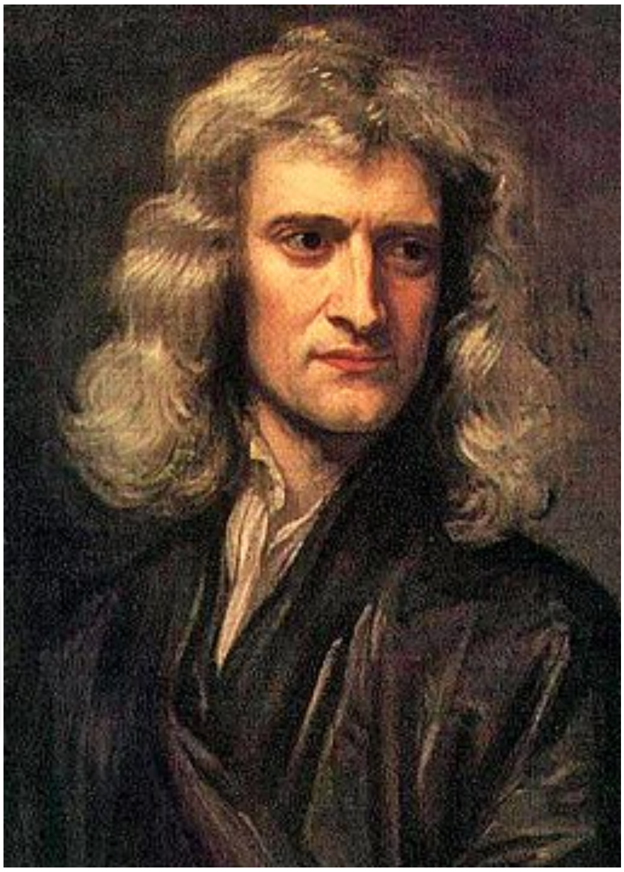
BOOK III.

When I made the foregoing Obſervations, I deſign'd to repeat moſt of them with more care and exactneſs, and to make ſome new ones for determining the manner how the Rays of Light are bent in their paſſage by Bodies, for making the Fringes of Colours with the dark lines between them. But I was then interrupted, and cannot now think of taking theſe things into farther Conſideration. And ſince I have not finiſh'd this part of my Deſign, I ſhall conclude with propoſing only ſome Queries, in order to a farther ſearch to be made by others.

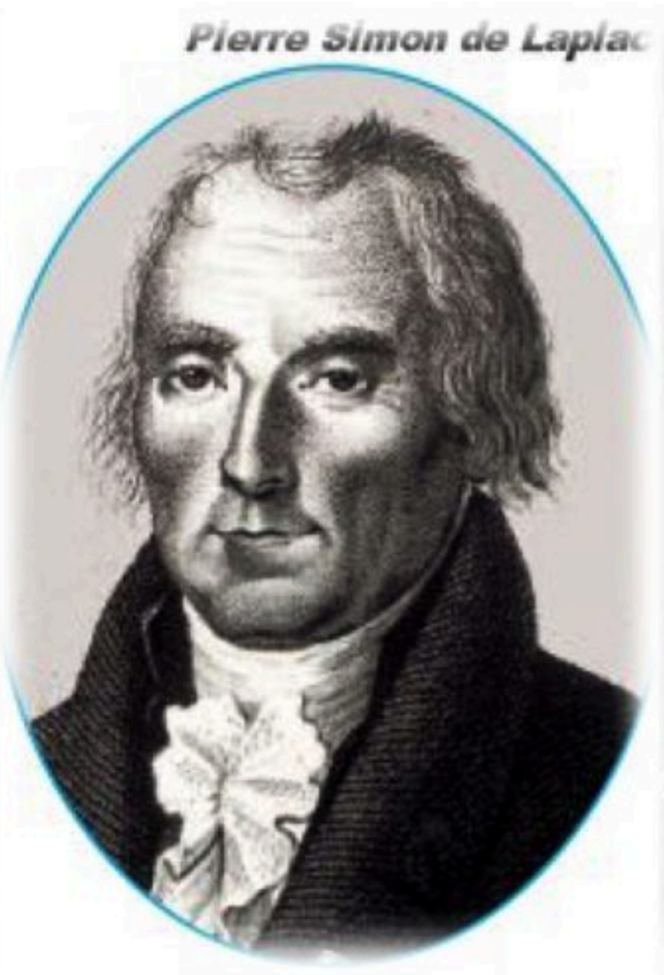
Query 1. Do not Bodies act upon Light at a diſtance, and by their action bend its Rays; and is not this action (*cæteris paribus*) ſtrongeſt at the leaſt diſtance?

Note: Ole Romer had measured speed of light (to 20% precision) in 1676

Newton in confinement (from the plague)



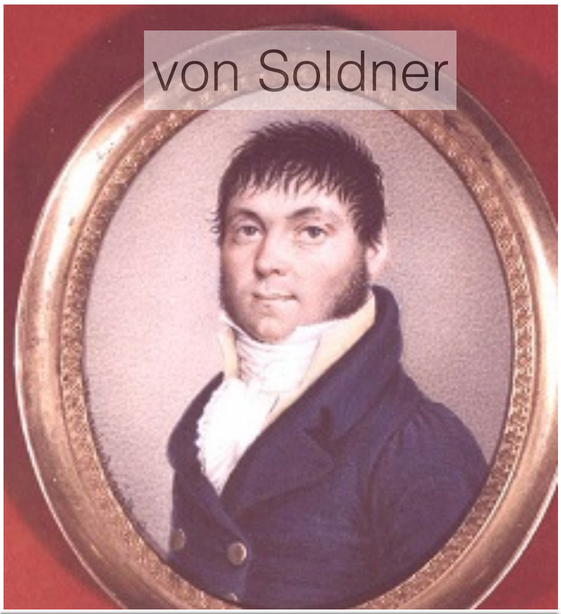
John Mitchell & Laplace -> Black holes ca. 1783

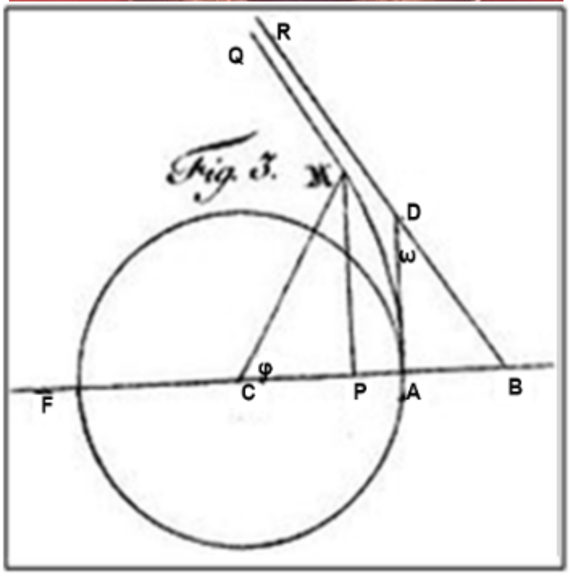


1802: Solar light deflection = 0.84"

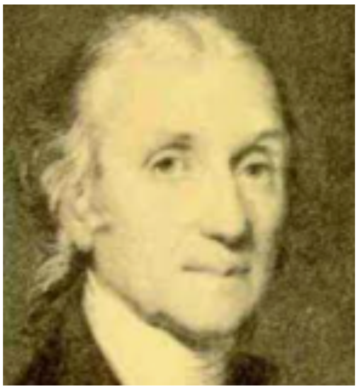
H. Cavendish

missing the famous factor 2 from GR





Newtonian light deflection



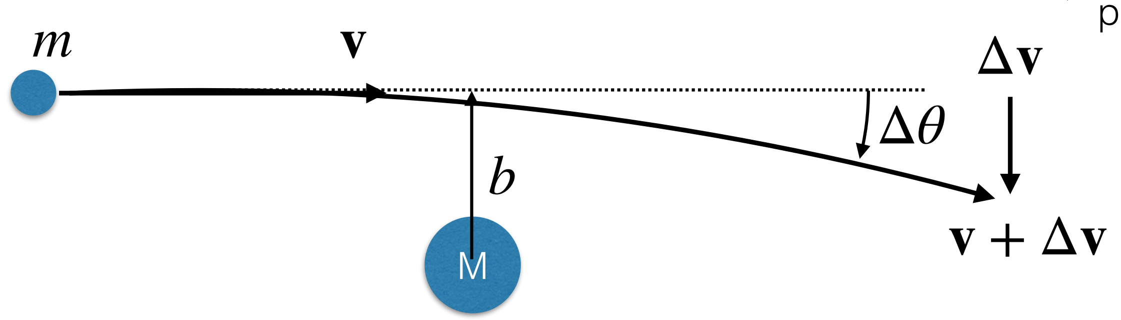
duration of collision

deflection angle

$$\Delta v \sim \boxed{\frac{GM}{b^2}} \times \Delta t \sim \boxed{\frac{GM}{b^2}} \times \frac{b}{v} \quad \Rightarrow \quad \Delta\theta = \frac{\Delta v}{v} = \boxed{\frac{2GM}{b}} \times \frac{1}{v^2}$$

acceleration

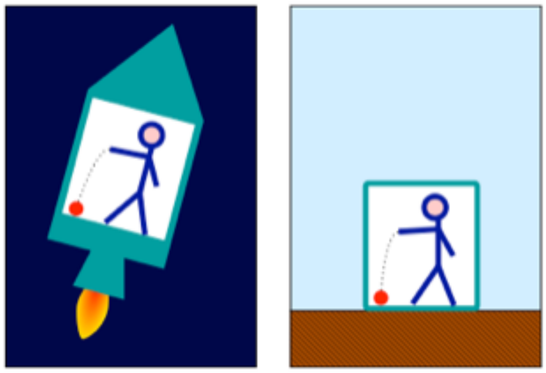
gravitational potential



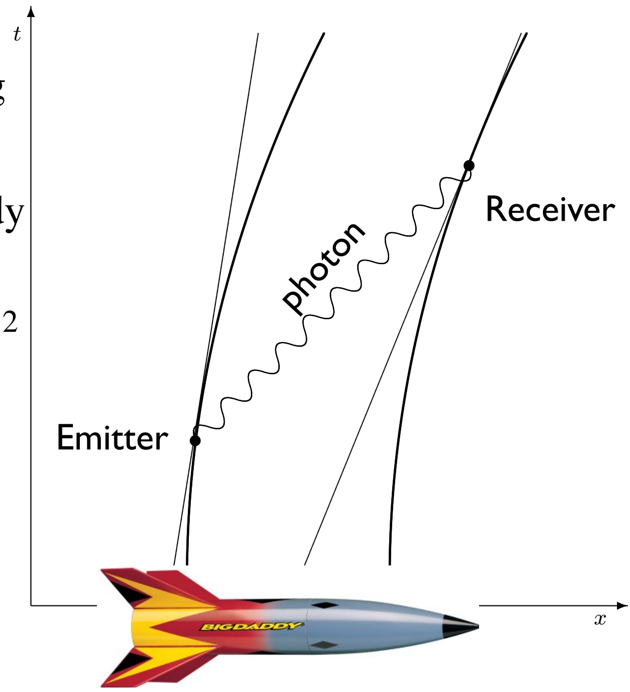
Einstein 1905: special relativity

- prevailing belief <1905: there *is* an absolute reference frame - but the laws of physics (electromagnetism etc.) make it hard to determine
- Einstein: take frame independence of speed of light as *principle;* deny existence of any special frame; define the **spacetime metric:**
    - gives the *proper separation* between two events with $\overrightarrow{dx} = (cdt, \mathbf{dx})$
    - $ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -c^2 dt^2 + |\mathbf{dx}|^2$
- Different observers assign different "*coordinate intervals*" dx^α but agree on squared proper separation ds^2
    - and whether separations are time-like (clocks), space-like (rulers) or null (photon paths)
- "light-cone structure" is an absolute property of space time in SR
    - carries over to "*curved manifolds*" in general relativity (GR)

Einstein ca 1910: happiest thought - the equivalence principle

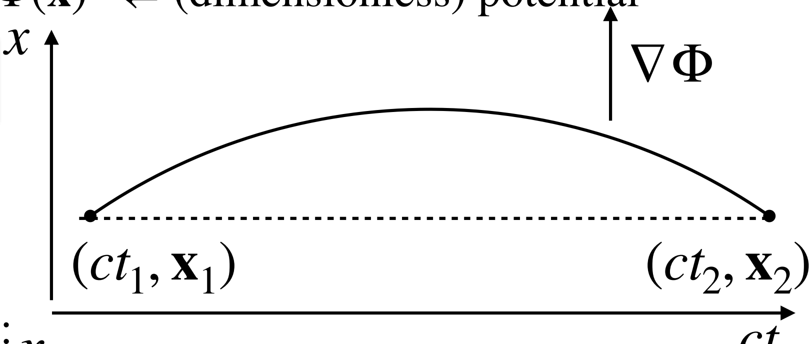


- Reads newspaper article about a tiler falling to his death from a roof
    - going to free-fall "*switches off*" gravity (locally)
- Conversely: physics we see locally sitting on the Earth is the same as if we were in a rocket in empty space being accelerated
    - *gravity and acceleration are equivalent*
- Q: What is the metric of space-time in an accelerating rocket?
    - i.e. SR, but with spatial coordinates tied to the body of the rocket?
- A: (Rindler): $ds^2 = -(1 + 2\mathbf{a}\cdot\mathbf{x}/c^2)c^2 dT^2 + |\mathbf{dx}|^2$
    - so *time is warped in an accelerating frame*
    - time runs faster (slower) at the nose (tail) of the rocket!
    - clocks drift out of synchrony

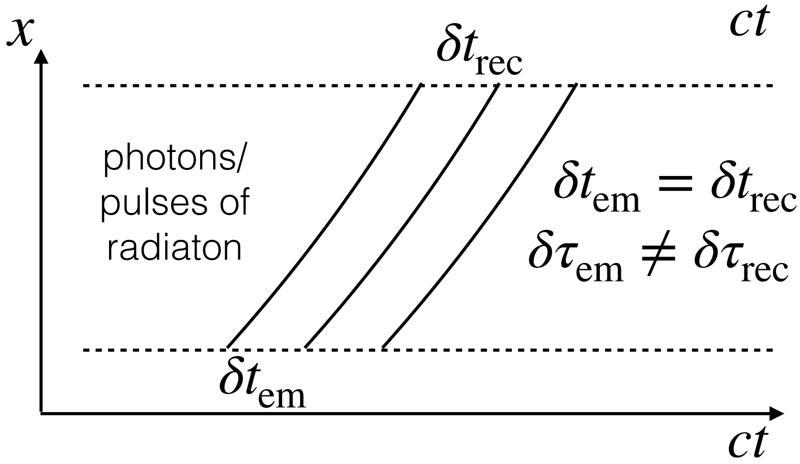


Equivalence -> metric in a gravitational field

- Equivalence: $\mathbf{x}\cdot\mathbf{a}/c^2 \;\Rightarrow\; \mathbf{x}\cdot\nabla\Phi = \Phi(\mathbf{x}) \;\Leftarrow$ (dimensionless) potential
- $\boxed{ds^2 = -(1+2\Phi(\mathbf{x}))c^2dt^2 + |\mathbf{dx}|^2}$

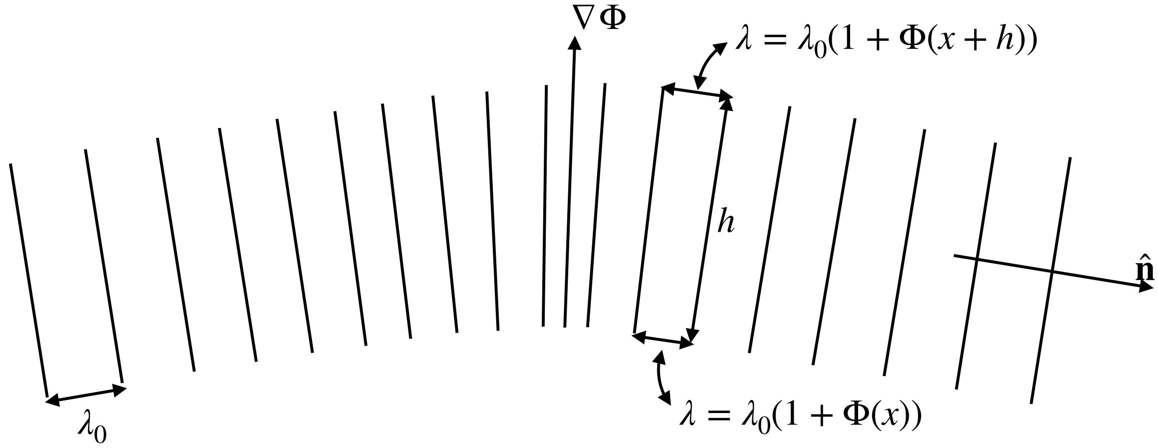


- Explains:
  - parabolic ballistic trajectories
  - "*geodesics*" (maximal proper time):
    - $\int d\tau = \int d\lambda \sqrt{-g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}}$



- Predicts:
  - gravitational redshift
  - light deflection
    - the *same* as Newtonian prediction for a particle moving with speed $v = c$
- Several attempts to measure the light bending by the Sun were unsuccessful (and so failed to prove him wrong!)

Einstein 1910: Light deflection from the equivalence principle



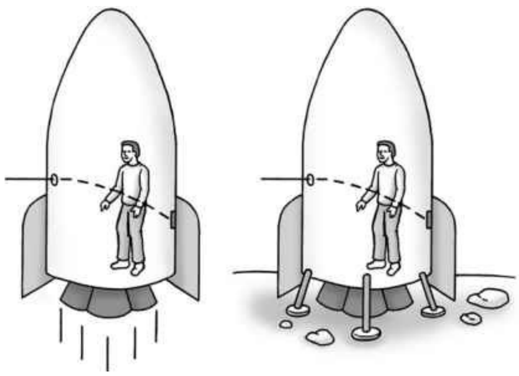
deflection in propagating 1 wavelength

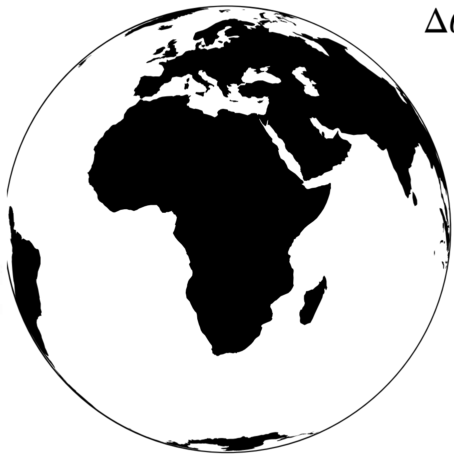
$$\Delta\theta_{\rm def} = \Delta\lambda/h = \lambda_0 \nabla_\perp \Phi$$

gradient perpendicular
to the path

Snell's law:

$$d\hat{\mathbf{n}}/dl = -\nabla_\perp \Phi$$

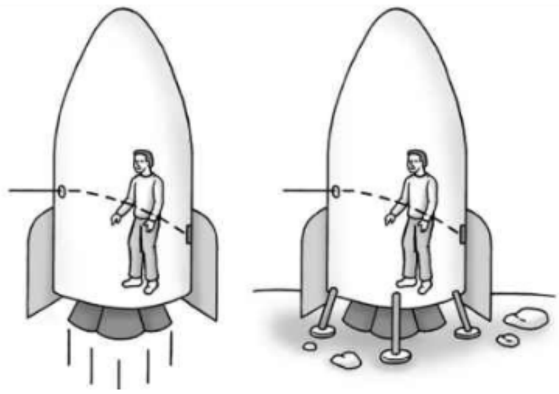




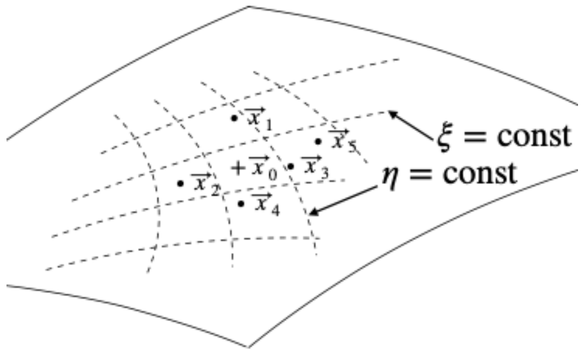
same as Newtonian theory - no extra factor 2

Where does the "extra factor 2" come from?

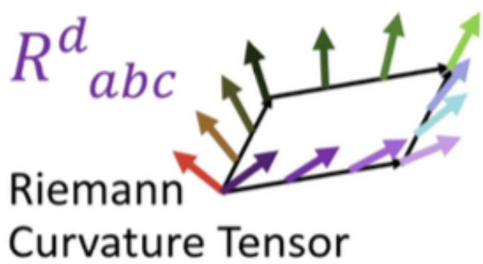
- Q: what's wrong with the EP argument?
- A: nothing - if used for *local light bending*
  - a "flat-space-time" phenomenon
- but it doesn't predict what astronomers see for images of stars seen near the Sun
  - that involves *spatial curvature* also



- The arena of GR is a smooth "*curved manifold*" on which you can lay down curvi-linear coords
- smoothness means you can always find "*locally flat*" coordinates in terms of which *local* physics is just as in SR



- but things like focussing of particles or light depend on the *curvature* of the manifold
- the curvature - a tensor - is encoded in 2nd derivatives of the metric
- and is determined by the matter stress-tensor $T_{\mu\nu}$



Einstein 1910-1915: The gravitational field equations

Newton	Einstein
potential : ϕ	metric : $g_{\mu\nu}(\overrightarrow{x})$
gravity : $\mathbf{g} = -\nabla\phi$	connection : $\Gamma^{\mu}{}_{\nu\beta}$
tide : $\nabla\nabla\phi = \partial^2\phi/\partial x_i \partial x_i$	curvature : $R^{\mu}{}_{\alpha\nu\beta}$
Poisson : $\nabla^2\phi = 4\pi G\rho$	Einstein : $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi\kappa T_{\mu\nu}$
tidal acceleration : $\dfrac{d^2\Delta\mathbf{x}}{dt^2} = -\Delta\mathbf{x}\cdot\nabla\nabla\phi$	geodesic deviation : $\dfrac{d^2\Delta x^{\alpha}}{d\lambda^2} = R^{\alpha}{}_{\beta\mu\nu}\dfrac{dx^{\beta}}{d\lambda}\Delta x^{\mu}\dfrac{dx^{\nu}}{d\lambda}$

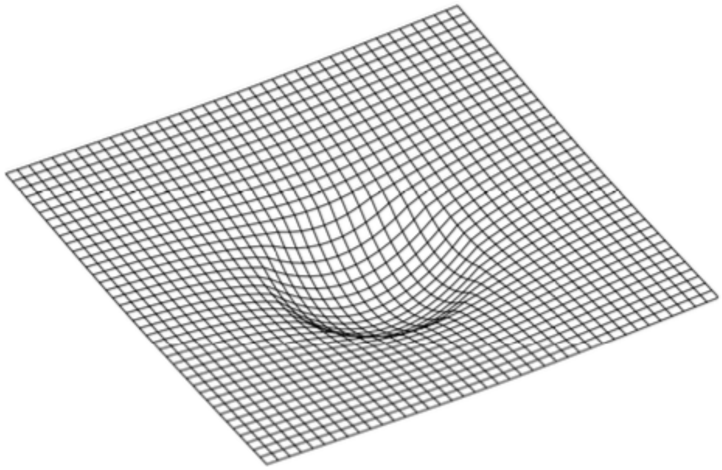
- metric -> connection > curvature generally non-linear. In addition we have coordinate freedom -> complex to solve; hard to interpret solutions
- but for weak fields we can choose coordinates (Lorenz gauge) such that
- $$\boxed{ds^2 = -(1 + 2\Phi)c^2dt^2 + (1 - 2\Phi)\,|\,\mathbf{dx}\,|^2}$$
- where $\nabla^2\Phi = 4\pi G\rho c^2$
- "weak-field" or "Newtonian-limit" metric has warping of *space* as well

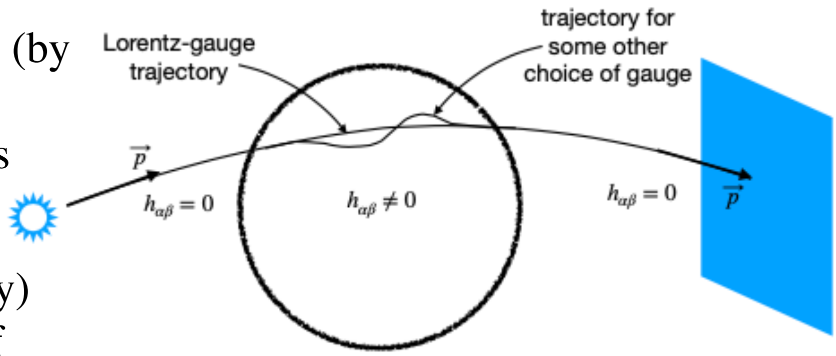
The "refractive index of gravity"

- Weak field metric: $ds^2 = -(1 + 2\Phi)c^2 dt^2 + (1 - 2\Phi)|\mathbf{dx}|^2$ implies that photon trajectories (for which $ds = 0$) have (with $\Phi \ll 1$)
  - *coordinate speed of light* $|\mathbf{dx}|/dt = (1 + 2\Phi(\mathbf{x}))c$
  - so the *effective refractive index* is $n(\mathbf{x}) = 1 - 2\Phi(\mathbf{x})$
- And *Snell's law* is then $d\hat{\mathbf{n}}/d\lambda = -2\nabla_\perp \Phi$
  - so *twice* what Einstein inferred from the EP
- So is the EP invalid?
  - A: No. The EP says how light rays deviate from locally straight lines (geodesics) in the *curved* space $dl^2 = (1 - 2\Phi(\mathbf{x}))|\mathbf{dx}|^2$
    - which is what you would measure with a photographic plate
    - so the extra bending in coordinate space is a *coordinate artefact*
- but, it turns out, this formula does correctly predict the displacement of images that astronomers see

How to understand the extra factor 2 in GR light bending

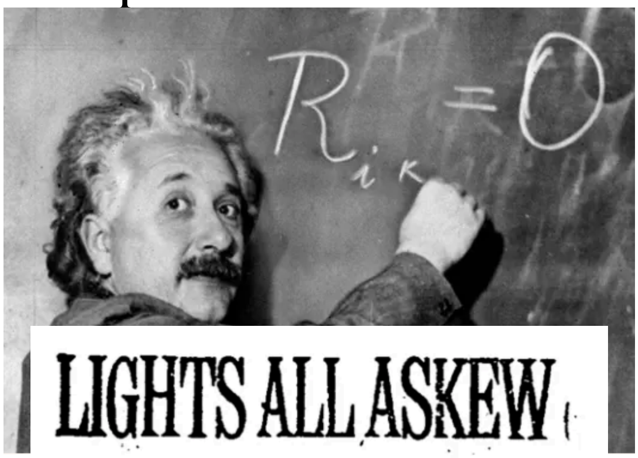
- The equatorial plane through the Sun - a 2D surface - is curved in the same way as the 2-space *embedded* in 3-dimensions shown at the right
- The EP says that physical wavelengths are diminished (gravitational redshift) and that causes local bending (relative to locally straight lines)
- But there is an extra increase of path length for rays that pass close to the Sun because the surface is curved
- And that enhances the global bending (by the famous factor 2) - relative to the coordinate system at $r \rightarrow \infty$, which is spatially flat
- While the coordinate path is (naturally) dependent on the (arbitrary) choice of coordinates - what we measure isn't





Einstein and Eddington's 1919 solar eclipse measurement

- 1911 - rocket thought experiments
  - predicts 0.84" solar bending angle
  - Lenard later accuses AE of plagiarism
- 1912 - Brazilian eclipse experiment
  - failed (to prove him wrong!)
- 1915 - GR paper published (with factor 2)
  - controversy over Hilbert paper
- 1919 - Eddington eclipse trip - success!

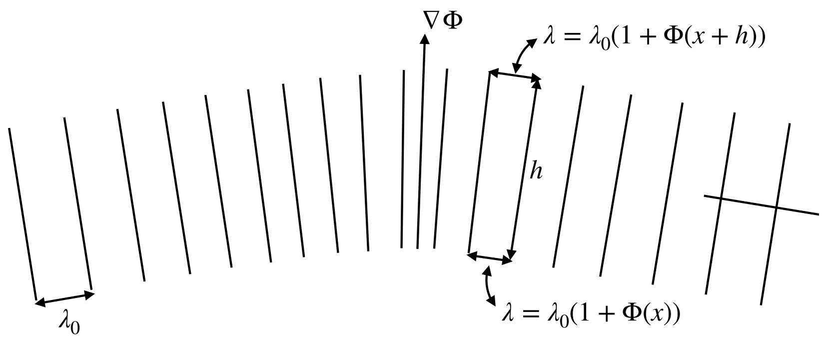


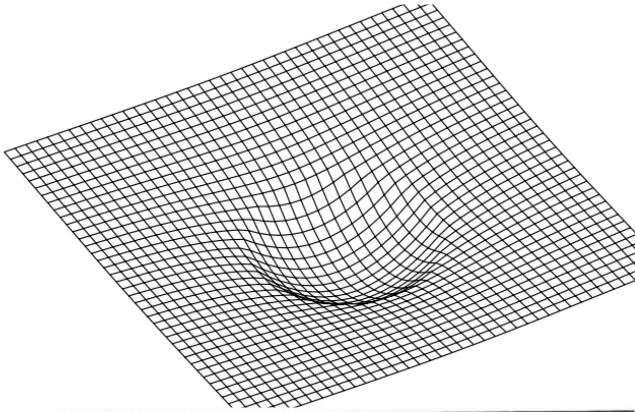
LIGHTS ALL ASKEW IN THE HEAVENS

Men of Science More or Less Agog Over Results of Eclipse Observations.

EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed or Were Calculated to be, but Nobody Need Worry.





deflection in propagating 1 wavelen

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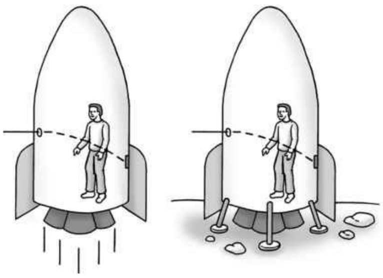
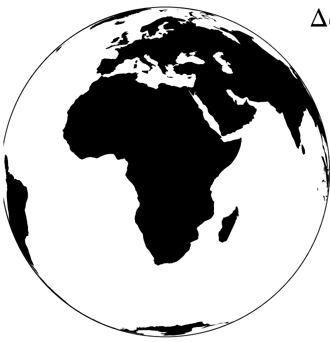
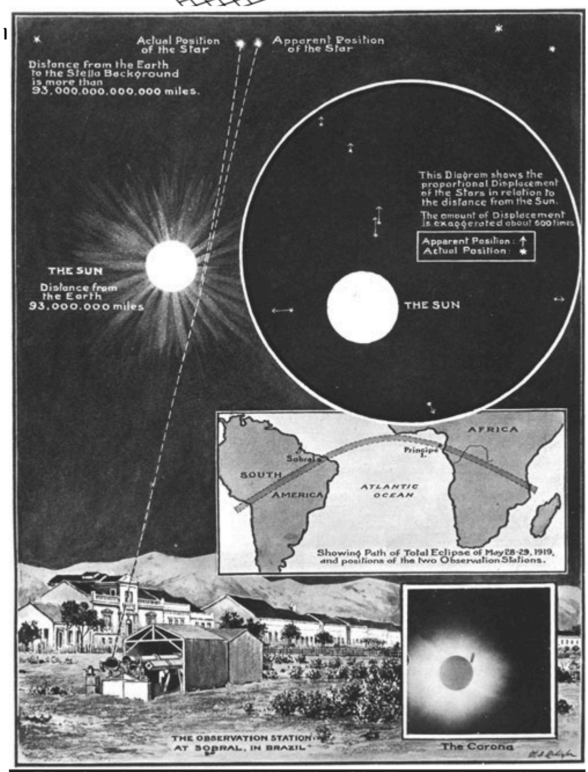




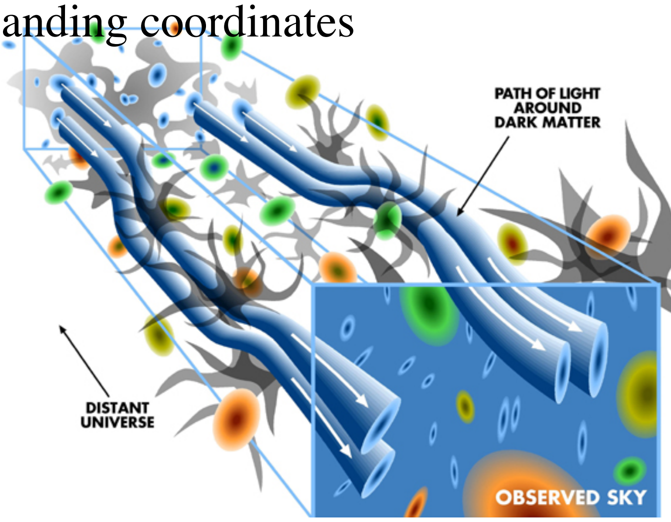


Taken from the 22 November 1919 edition of the Illustrated London News.

- Equivalence principle:
  - light bending as you would measure "in the laboratory" - no factor 2
- Geodesic equation:
  - what astronomers measure "on the sky"

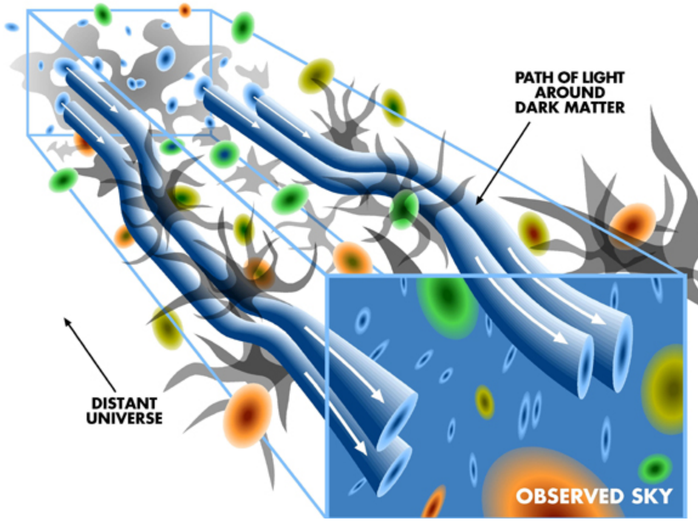
Optical properties of a lumpy *expanding universe*

- Homogeneous universe: metric: $ds^2 = -c^2dt^2 + a^2(t)\,|\mathbf{dx}|^2$
  - scale factor a(t) obeys *Friedmann's eq* $H^2 = (\dot{a}/a)^2 = (8/3)\pi G(\rho + 3P/c^2)$
  - $\mathbf{x}$ is "*comoving/conformal*" coordinate (galaxies have fixed $\mathbf{x}$)
- Lumpiness: $ds^2 = -(1 + 2\Phi(\mathbf{x}))c^2dt^2 + (1 - 2\Phi(\mathbf{x}))a^2(t)\,|\mathbf{dx}|^2$
  - $\Phi(\mathbf{x})$ determined by density *fluctuations* $\delta\rho(\mathbf{x})$ (via Poisson's equation)
  - very good approximation because (peculiar) velocities are slow
  - similar to the weak-field metric in non-expanding coordinates
- Light rays are null paths (ds = 0)
- Same as light rays in "lumpy glass"
  - refractive index $n(\mathbf{x}) = 1 - 2\Phi(\mathbf{x})$
  - n($\mathbf{x}$) = (coordinate speed of light)$^{-1}$
  - Snell's law: Deflection $\theta_{\text{def}} \sim \Phi$



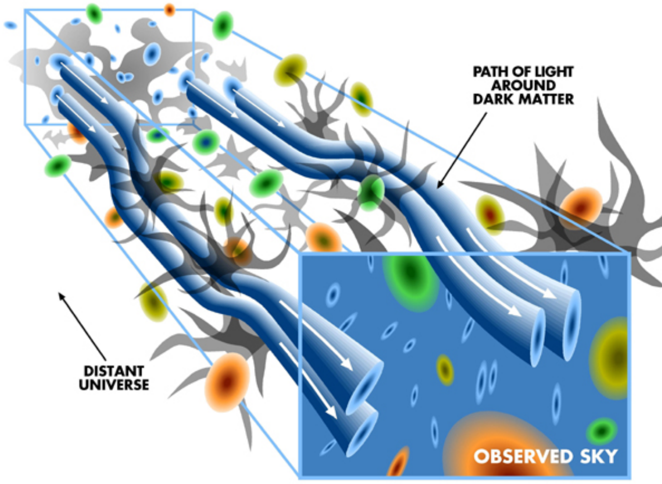
basics of gravitational lensing: Δt, deflection

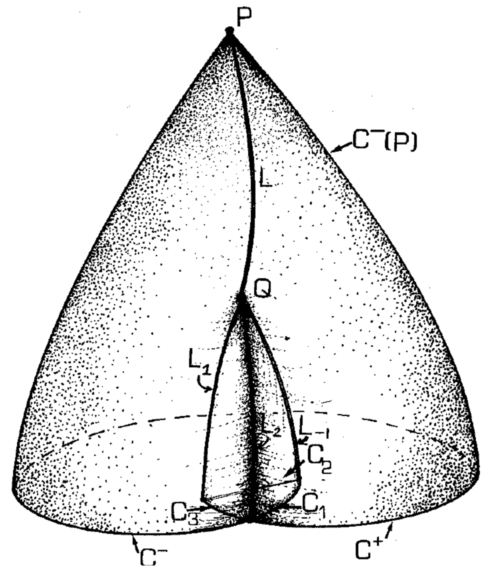
- Gravitational time delay (Shapiro '65): $\Delta t = 2 \int d\lambda \Phi / c$
  - λ = distance: Φ = gravitational field from Δϱ/ϱ
  - measured in "strong lensing" - multiple images of quasars - along with the geometrical time-delay
  - fundamental concept (see Blandford & Narayan '86)
- Light deflection $\theta_1 \sim \int d\lambda \nabla_\perp \Phi \sim G\Delta M / bc^2 \sim (H\lambda/c)^2 \times \Delta\rho/\rho$
  - cumulative deflection is a "random walk'
    - $\theta \sim \sqrt{N}\theta_1 \sim (H\lambda/c)^{3/2} \times \Delta\rho/\rho$
    - $\xi(r) \propto \lambda^{-2} \;\Rightarrow\; \Delta\rho/\rho \propto 1/\lambda$
  - θ dominated by "superclusters" (~30 Mp
    - quite large $\sim 10^{-3}$ radians at high z
    - but hard to observe observable



basics of lensing: Δt, θ_{def} + magnification & shear

- Time delay $\Delta t = 2 \int d\lambda\, \Phi/c$
- Light deflection: $\theta \sim \sqrt{N}\theta_1 \sim (H\lambda/c)^{3/2} \times \Delta\rho/\rho$
  - dominated by large scales ($\lambda \sim 30$ Mpc)
- Weak lensing: the *gradient* of the deflection angle
  - described by a 2x2 image distortion tensor
    - trace: $\kappa \rightarrow$ *magnification* (changes *size* of objects)
    - 2 other components: $\gamma \rightarrow$ *image shear* (changes *shapes*)
    - ~1% at ~ degree scales for sources at z ~ 1 (few % @ z=1000)
    - but grows with decreasing angular scale
  - potentially *very large effects* from small-scale lumpiness



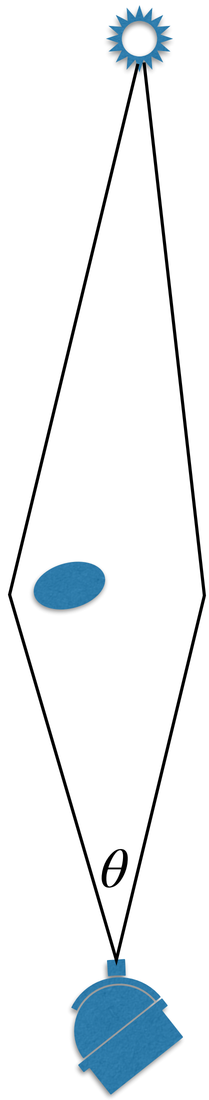


applications of gravitational lensing in cosmology

- Microlensing
    - constraints on e.g. primordial BH DM from MACHO etc
    - μ-lensing at cosmological distances (Gunn & Gott), GRBs etc
- **Strong-lensing**
    - galaxy, cluster masses
    - time delays
- **Weak-lensing - galaxy, cluster + `cosmic'-lensing**
    - Quasar-galaxy associations
    - Image shear and magnification
    - -> DM structure and evolution -> DE
- **Bias in cosmological distances and parameter estimation**

Observable effects 1: deflection and time delays in "strong lenses"

- Fritz Zwicky was the first to point out that galaxies would make effective gravitational lenses and would be able to generate multiple images of background sources
  - particularly for distant sources with an intervening galaxy at roughly half the distance
- In 1979, Walsh, Carswell and Weyman discovered the doubly imaged quasar QSO 0957+561.
- The images of the z = 1.41 QSO have separation of $\theta = 6''$ caused by an intervening lens-galaxy at z = 0.355
- This gives a way to measure the lens-galaxy mass
- In addition, the quasar is time varying, allowing measurement of the light-travel time difference for the two paths which is on the order $\Delta t \sim \theta^2 / H_0$ (Refsdal, 1964).
- This provides a way to measure the age of the universe $t_{\rm U} \simeq 1/H_0$ - but it depends on the assumed profile of the lenses.



Early days of weak lensing

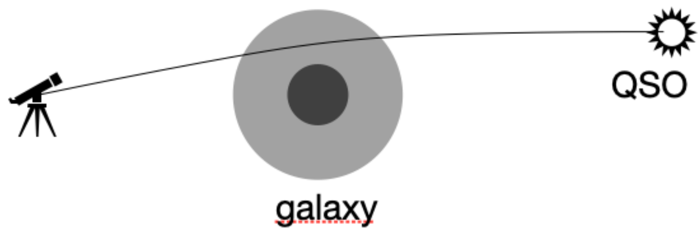
- Jacob B. Zel'dovich's pioneering 1963 paper mentioned the distortion of shapes of galaxies by the tidal shearing of bundles of rays to the source
- but this and subsequent studies focussed on the possibility of bias in mean flux density of distant galaxies
  - without particular reference to what was causing the magnification
- Rachel Webster (1985) proposed that the impact of lensing on the distribution of ellipticities be used as a cosmological probe
- around the mid-late '80s two lensing techniques emerged that were designed to probe the dark matter distribution in and around galaxy haloes
  - note that this was a very lively time for measurement of dark halos using rotation curves and using e.g. relative velocities of pairs of galaxies and using the `cosmic virial theorem' of Davis and Peebles (1983)
- One was "*quasar galaxy associations*" the other was what is now known as "*galaxy-galaxy lensing*"
- Interestingly they gave results that were discrepant with each other

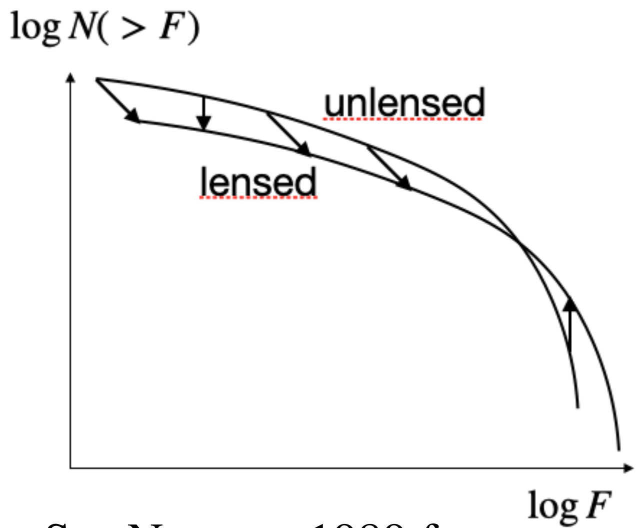
Quasar-galaxy associations

- Quasars had been discovered in the early 60s.
- Their redshifts put them at cosmological distances, and they were interpreted as being powered by accretion onto black holes
- But not all astronomers accepted this, one reason for skepticism being that there were cases where the quasars seemed to be associated with galaxies with much lower redshifts.
- Initially the evidence was questionable and controversial, and people promoting this risked being dismissed as cranks
- But as the samples of e.g. UV-excess quasars from Schmidt telescope surveys grew, the evidence became less anecdotal and statistically stronger.
- As the data improved, the interpretation changed also; rather than being considered to be physical associations - and therefore evidence that the quasar redshifts were non-cosmological - they were interpreted as being sources whose flux-densities were being amplified by the mass in the haloes of the foreground galaxies
- But strangely, the effect was stronger than predicted from kinematic studies

Quasar-galaxy associations

- The first predictions of the effect were by Claude Canizares (1981) and John Peacock (1982)
- A key effect is "*magnification bias*". One aspect of this is that sources that would otherwise be too faint to see could be observed.
- But the other arises from the fact that the way that sources become amplified is by their solid angles becoming larger - *and the same effect dilutes their number density on the sky*
- The net result is an enhancement for bright sources and a diminution for faint ones





- See Narayan 1989 for a particularly clear analysis
- And Benitez et al 2001 for a more recent survey of results
- The large results persist.

Early cosmic-shear results

- The advent of CCD detectors in the 70s radically changed optical astronomy as their linearity and sensitivity were a big advantage
- Initially of very small area, they steadily increased in size and, by the mid 80's were being used - to particular effect by Tyson's group - to do imaging surveys rather than studies of individual objects.
- Valdes, Tyson and Jarvis, in their pioneering study of 1983, measured quadrupole moments M_{ij} of ~45,000 galaxies in 35 fields and computed a mean of the ellipticities $e_1 = \langle M_{xx} - M_{yy} \rangle$ and $e_2 = \langle 2M_{xy} \rangle$ in each field.
- They found a null result.
- This was perhaps not overwhelmingly surprising based on what we now know about the large-scale mass distribution, but one should put this in the context of the time when there were hints of strong inhomogeneity on large scales:
  - one, then relatively recent, discovery was the "Rubin-Ford effect"
  - this was that we have a large (~ 500 km/s) motion with respect to a shell of galaxies at 3500 km/s < cz < 6500 km/s which did *not* agree with our motion with respect to the CMB. I.e. the shell itself had a large peculiar velocity.

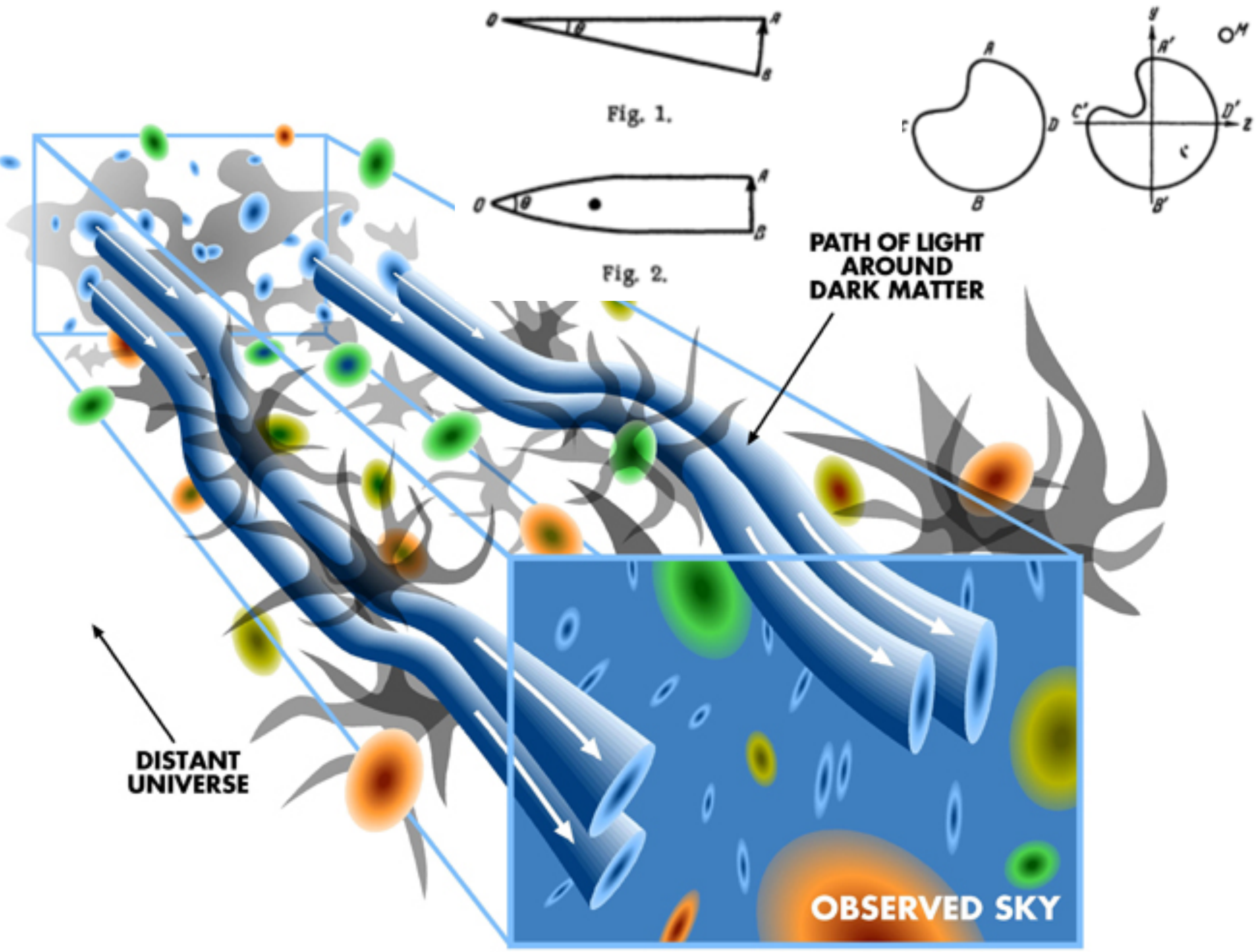
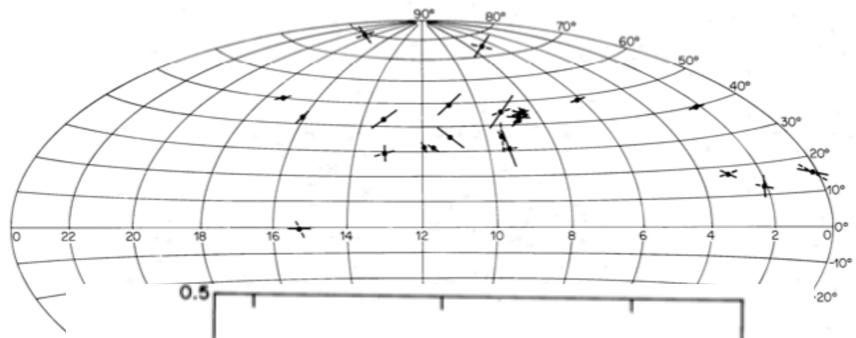


Fig. 1.
Fig. 2.
PATH OF LIGHT
AROUND
DARK MATTER
DISTANT
UNIVERSE
OBSERVED SKY

Early galaxy-galaxy lensing

- Top figure:Valdes et al 1983.
- Tyson, Jarvis, Valdes & Mills (1984) used the same data to measure *galaxy-galaxy lensing*
- Look at bright (foreground) and faint (background) pairs
- The result was a surprisingly weak signal (lower right)
    - barely compatible with kinematic estimates of extended flat rotation curves
    - and very different from what was emerging from quasar-galaxy associations

1983 ALIGNMENT OF FAINT GALAXY IMAGES



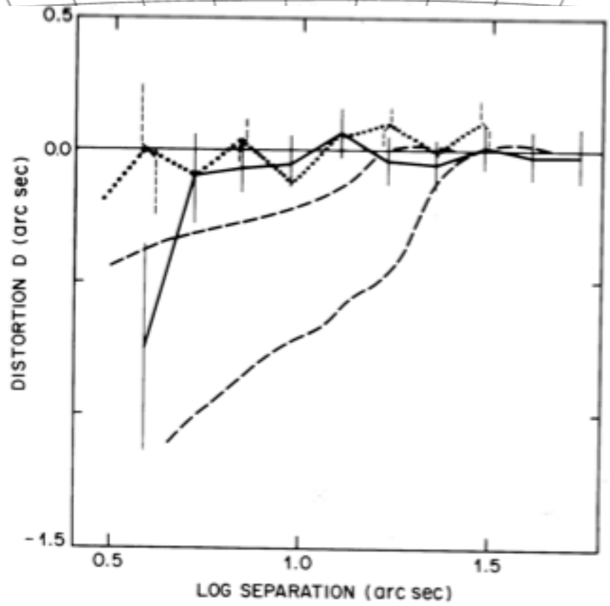


FIG. 1.—The dimensionless gravitational image distortion parameter $\mathscr{D} = \psi(M_r - M_\theta)/(M_r + M_\theta)$ in arc seconds as a function of the radial separation ψ of the foreground-background galaxy pair on the sky in arc seconds (*solid line*). Also shown (*dotted line*) is the result of a control test in which bright stars on the plate were substituted for the foreground galaxy position in the measurement of $\mathscr{D}$. The result is null (within 2 σ) in both cases, 1 σ error bars. Also shown are simulated distortions (*dashed lines*) from galaxies of mass cutoff radius 65 h^{-1} kpc and equivalent circular velocities of 200 and 300 km s^{-1}.

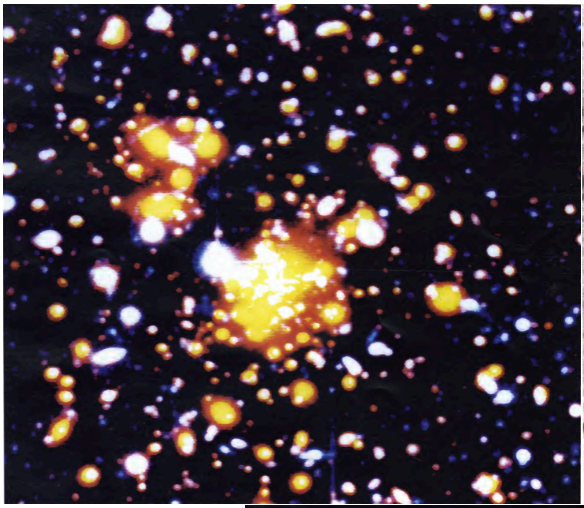
Clusters of galaxies

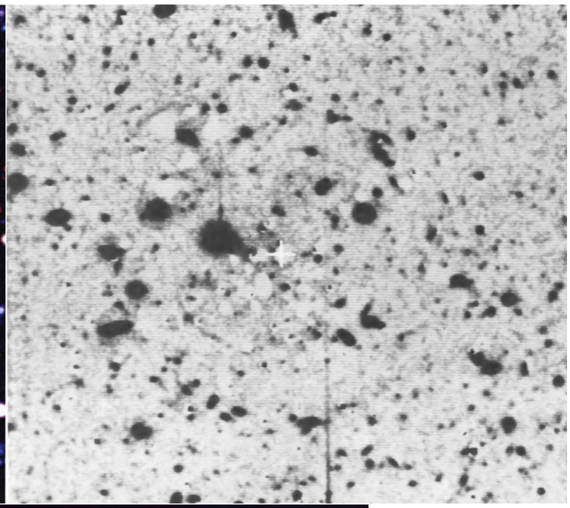
- Largest bound virialised systems $\sim 10^{14}$-$10^{15} M_{sun}$
- Velocity dispersion $\sigma_v \sim 1000$ km/s ($\sim 0.003c$)
- Centres - defined by the brightest galaxy (BCG)
  - Usually very close to peak of light, X-rays, DM

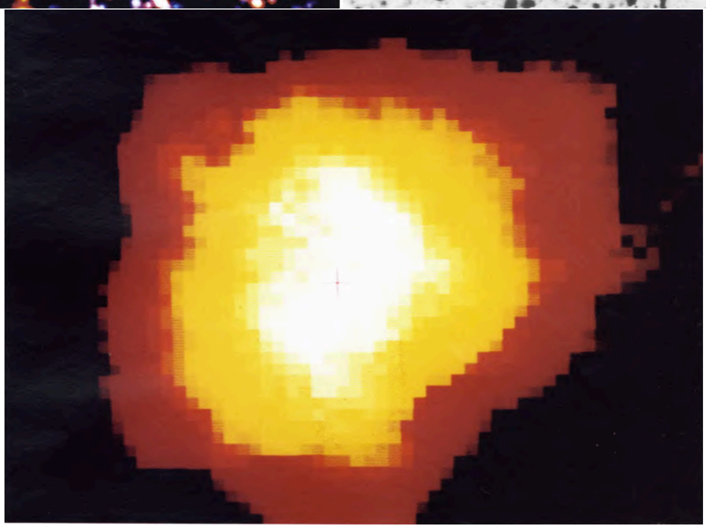
DETECTION OF SYSTEMATIC GRAVITATIONAL LENS GALAXY IMAGE ALIGNMENTS: MAPPING DARK MATTER IN GALAXY CLUSTERS

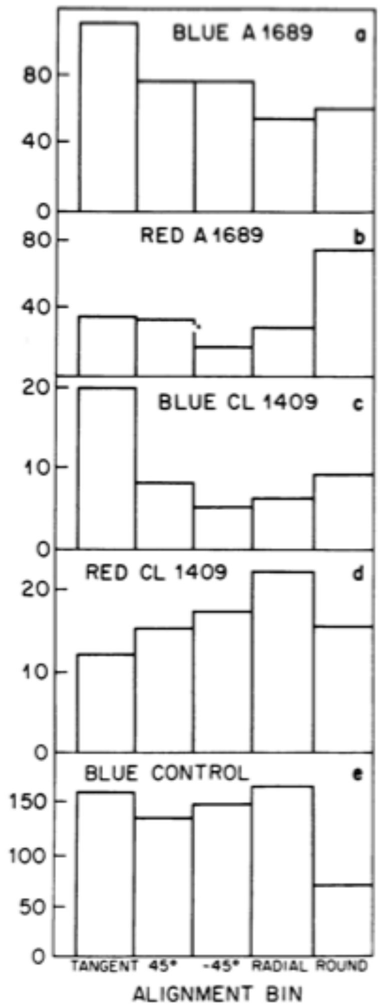
J. A. Tyson,[1,2] F. Valdes,[3] and R. A. Wenk[1]

Received 1989 July 19; accepted 1989 October 16









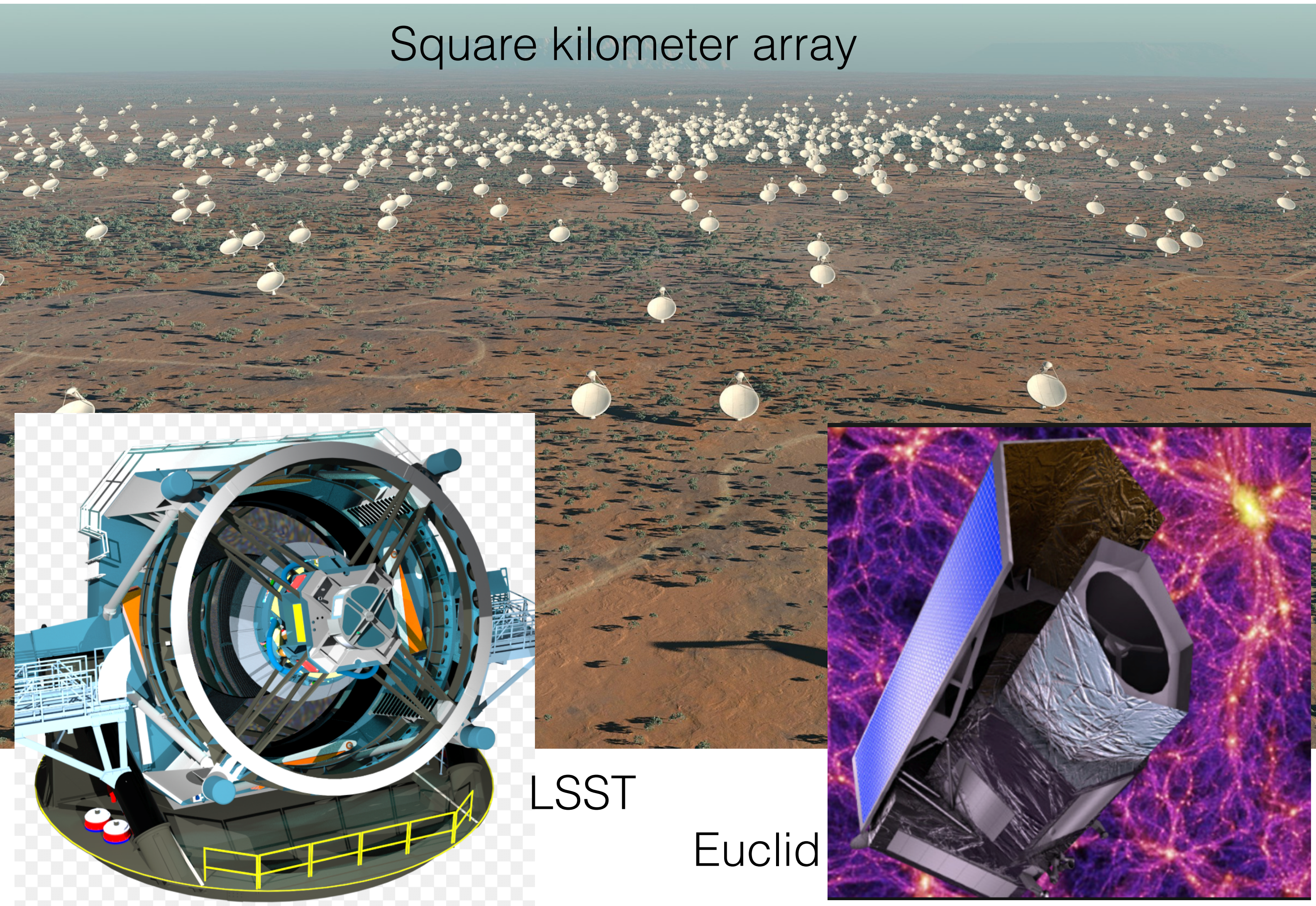
The impact of Tyson, Valdes and Wenk's "mass maps"

- The measurement of the tangential alignment in A1689 (and CL1409) was a revolutionary and exciting event.
  - particularly after the earlier null results on cosmic and GG-lensing
- It was clear that what they were seeing wasn't just coming from the "giant arcs", but was driven by the bulk of the faint galaxies
- Moreover, while the fields were quite small (and the shear correspondingly large), it was evident that, with the number of background galaxies increasing as θ^2, and the signal expected going like $|\gamma| \propto 1/\theta$, the prospects for extending this to larger scales were good
- But what exactly was the colourful DM image actually measuring? The surface density? The potential? Or something else?
- And how could one calibrate the measurement? (it was becoming clear that the null GG-lensing result was due to the seeing diluting the signal)
- The next few years saw intense activity in all of these areas.

Development of weak-lensing

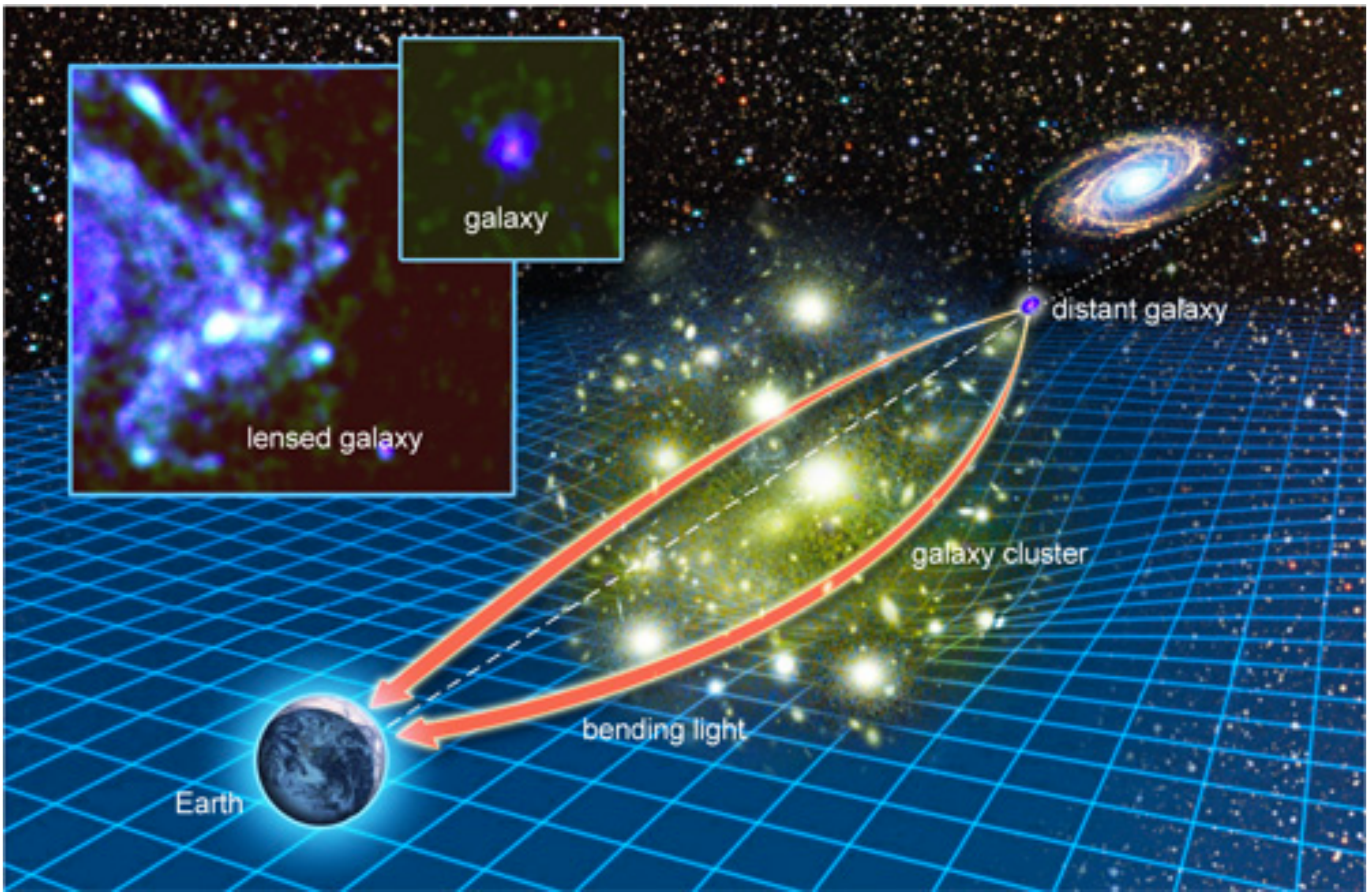
- Pioneering study of Tyson, Valdes & Wenk led the way
- Theory:
  - understanding of what was measured (shear = shape polarisation γ)
  - inversion techniques: $\gamma(\mathbf{r}) \Rightarrow \kappa(\mathbf{r}) = \mathbf{\Sigma}(\mathbf{r})/\mathbf{\Sigma}_{\rm crit}$ $\Leftarrow$ "*mass-maps*"
    - Σ = surface mass density, $\Sigma_{\rm crit} \sim c\rho/H \sim 1 {\rm gm/cm}^2$
  - How to measure power spectrum from observed shear
- Observation:
  - (Total = dark + luminous) projected mass maps
  - Power-spectrum: how much mass $\times$ density fluctuations
  - Constraints on nature of DM (e.g. the "bullet cluster")
- Emphasis shifted from mapping out DM to constraining "dark energy" and testing "modified gravity"
- Became apparent that dedicated facilities were needed => Euclid + LSST

The next decade - a golden age for cosmology



Euclid Science Consortium - 1400 scientists!

DM composition from μ-lensing of individual stars



"Icarus" discovery (Pat Kelly et al)

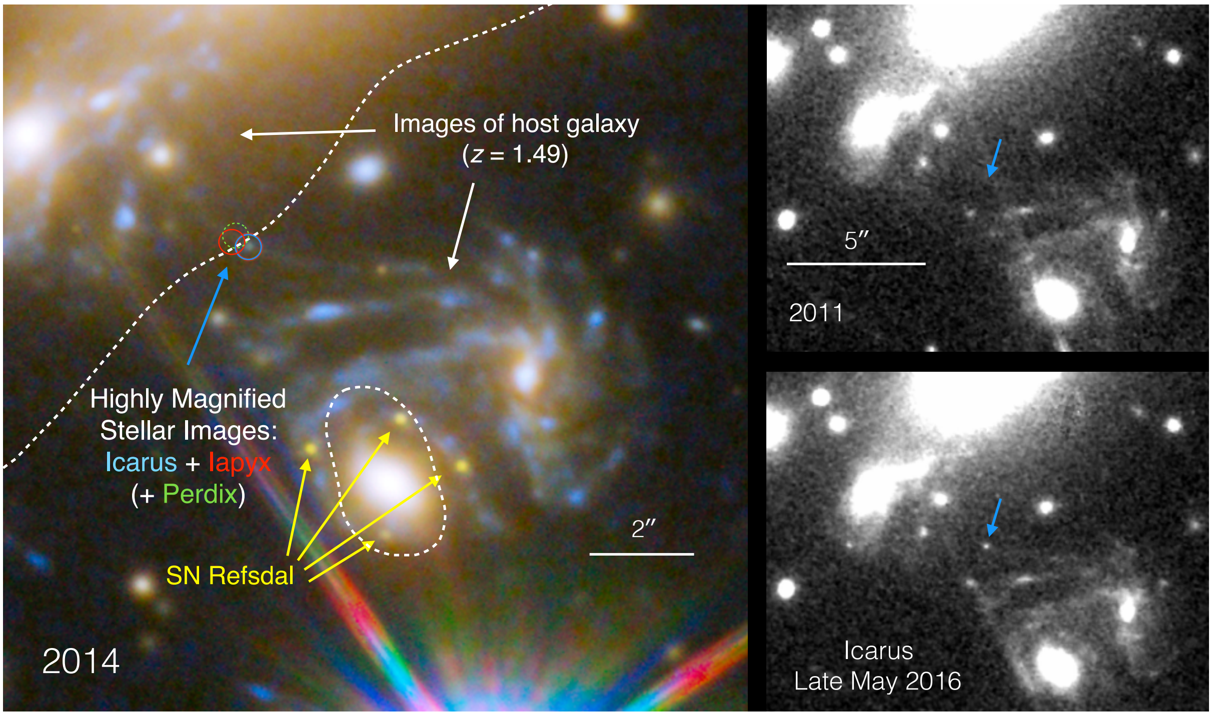


Figure 1: Position of magnified background B-type star coincident with (< 0.1") the MACS J1149 galaxy cluster's critical curve, where magnification rise rapidly. Dashed line in left panel shows location of critical curves from CATS cluster model[14]. The Einstein cross formed from yellow point sources are images of SN Refsdal[3]. Right top panel shows a WFC3 F125W image from 2011, and right bottom shows Icarus near peak brightness in May 2016. Mirrored images of the spiral host galaxy at redshift z = 1.49 lie on opposite sides of the critical curves, and the Bright Cluster Galaxy lies to the upper right. A bright foreground Milky Way star is responsible for the prominent diffraction spikes.

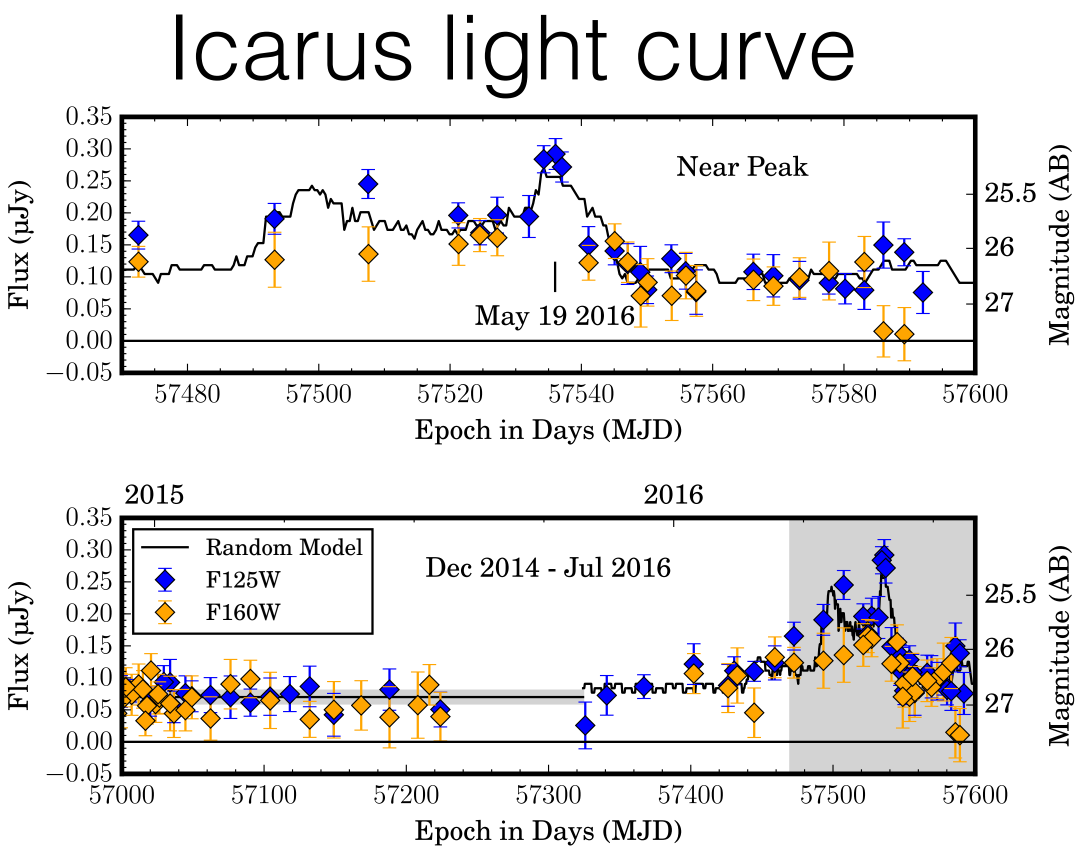
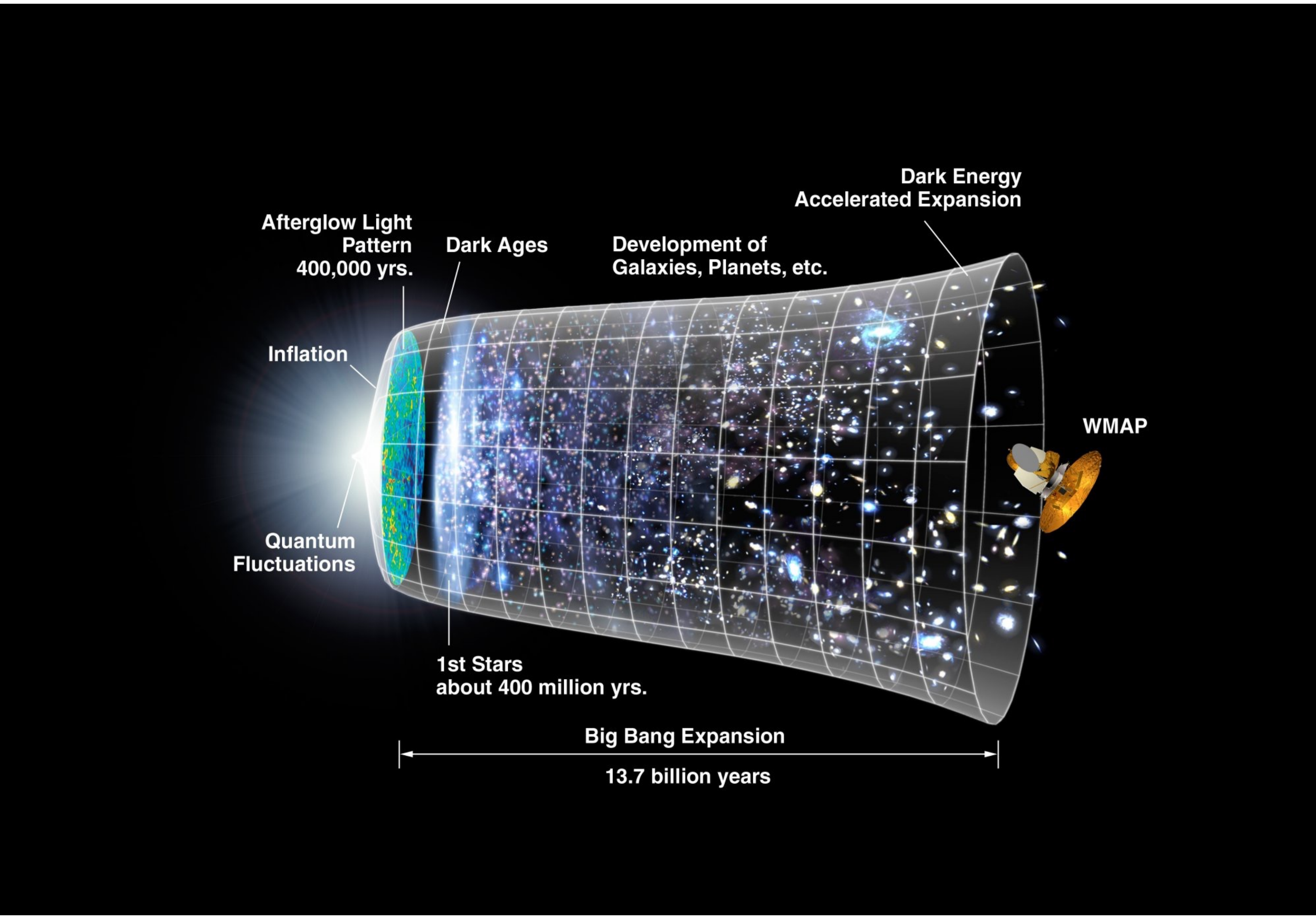


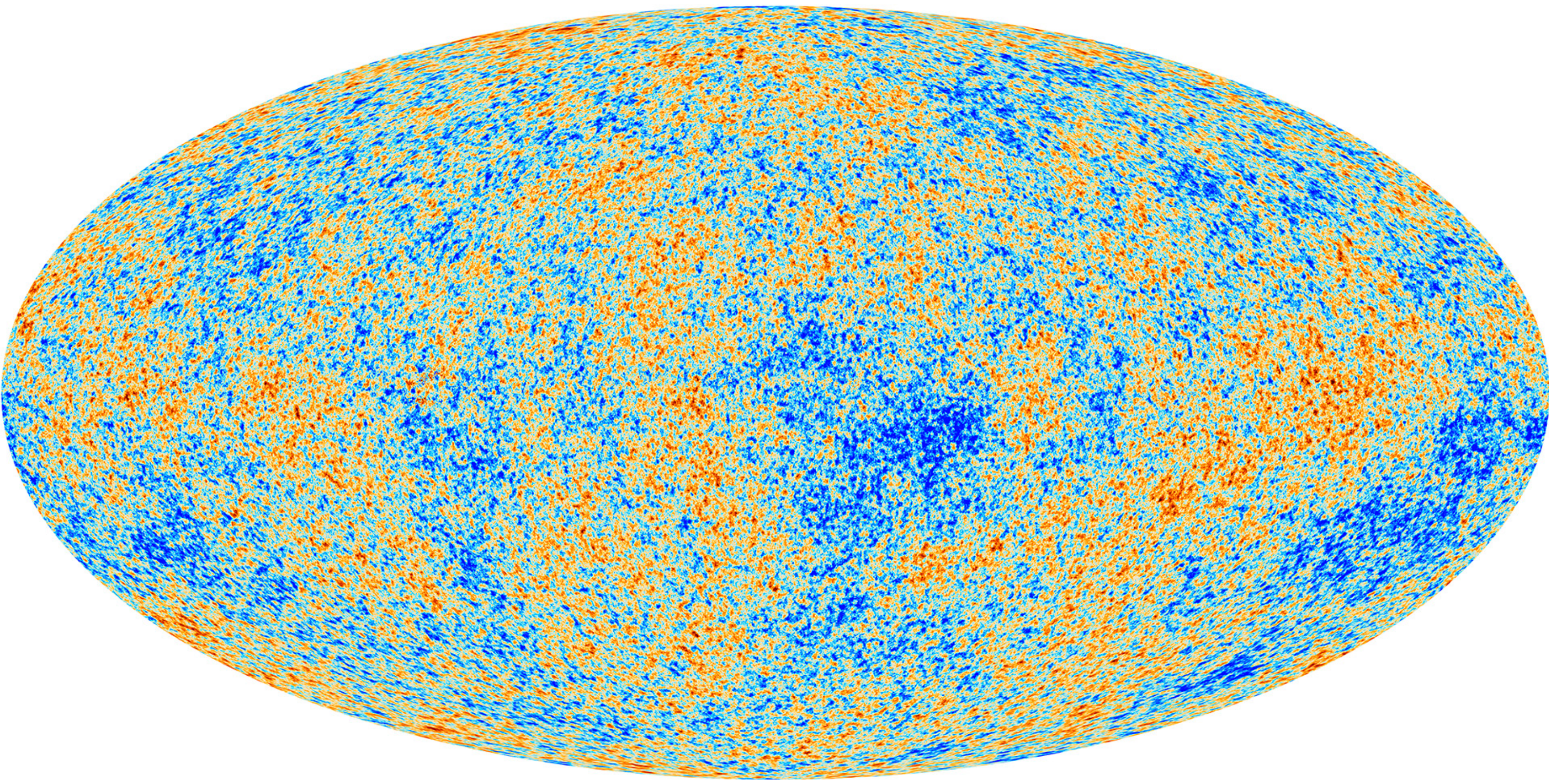
Figure 2: WFC3 F125W and F160W light curve of point source from December 2014 through July 2016 and a simulated light curve of a magnified star near cluster caustic. WFC3 imaging taken on April 29th (MJD 57507) led to the detection of the transient, and we subsequently increased the cadence of visits to 2–3 days. The cluster's potential is responsible for the magnification of ~3,000–50,000× before peak, and we find that the critical curves of stars in the ICL are responsible for the peak with magnification ~10,000–150,000× in May 2016.

The Effect of Gravitational Lensing on Cosmological Parameter Estimation

The scope of modern cosmology



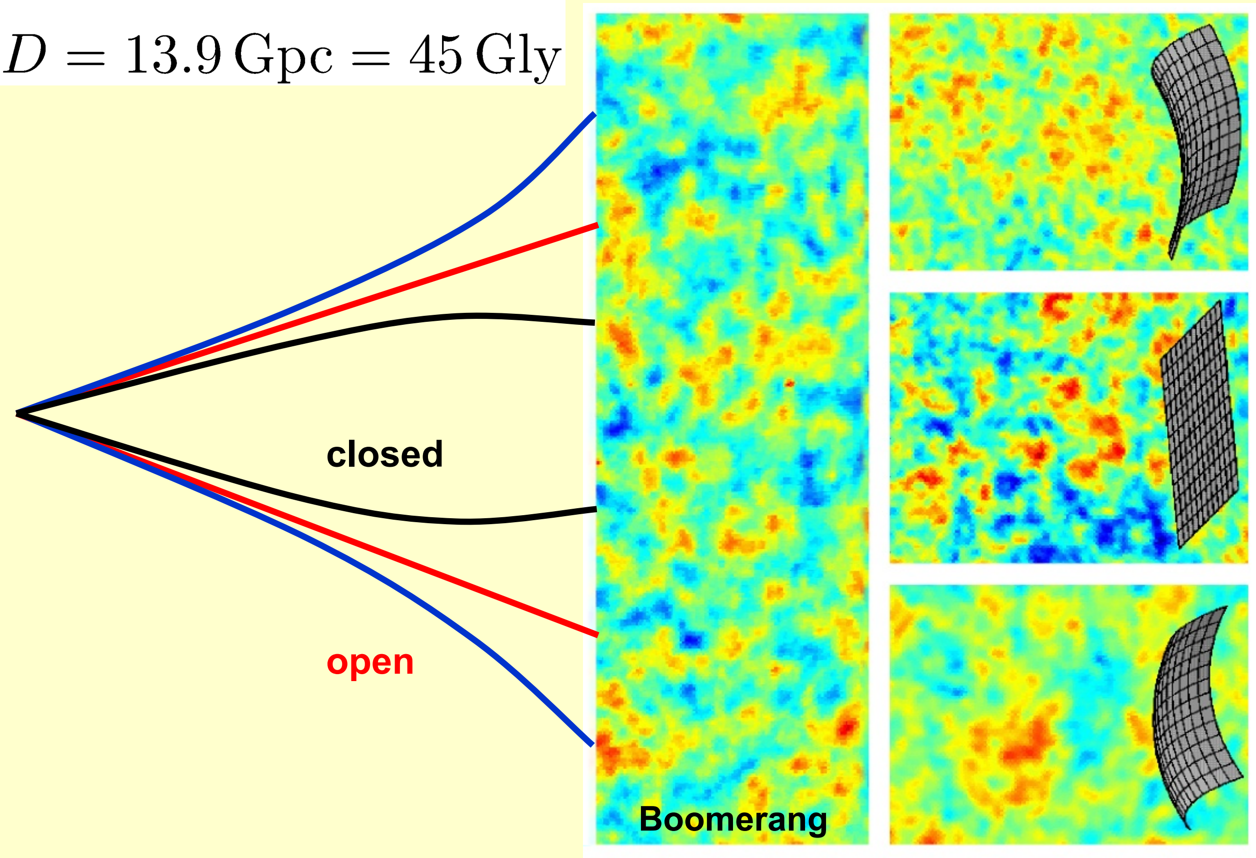
Context: cosmological parameters from the CMB
It is usually assumed that we are looking here at a spherical surface at $z \sim 1100$ with $D = D_0(z=1100)$
But are we?



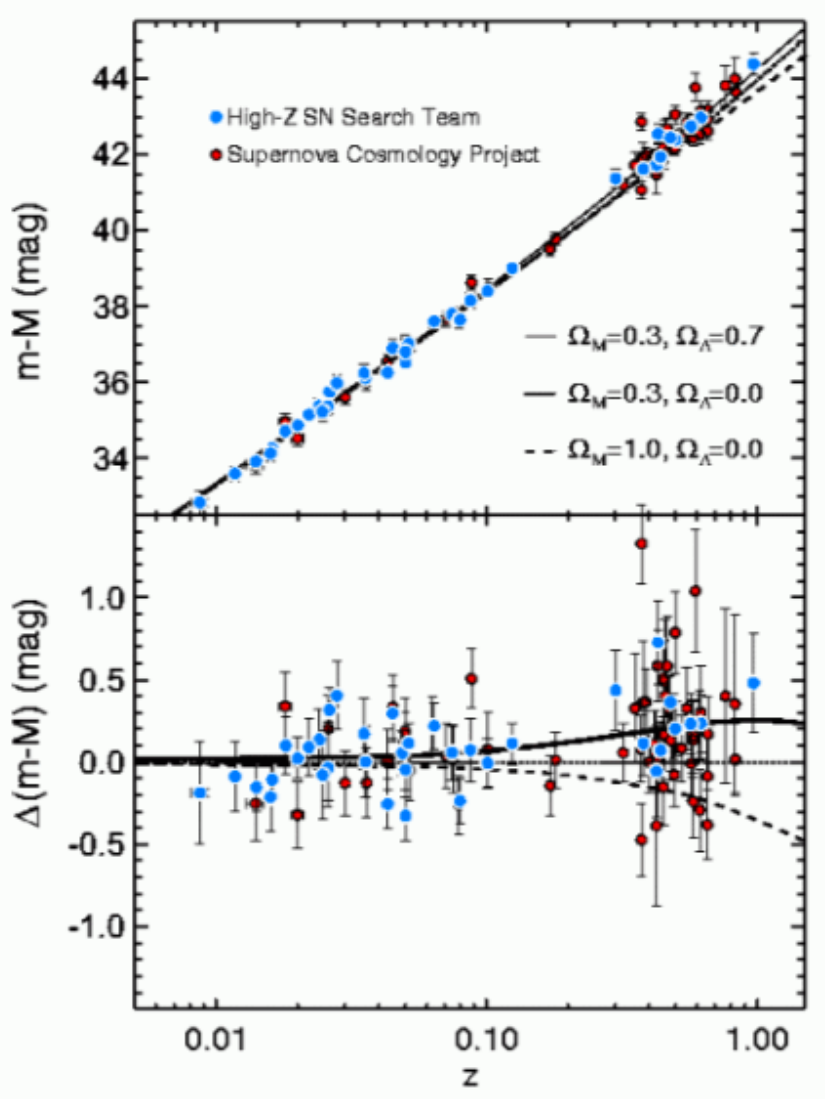
How far away is the CMB?

$$D = \int \frac{c}{H(z)}\, dz$$

$$z = 1080 \Rightarrow D = 13.9\,\mathrm{Gpc} = 45\,\mathrm{Gly}$$



Hubble diagram from SN1a - assumes no flux *bias* from lensing



The problem: is cosmological distance *biased* by structure?

- Distances in cosmology:
  - Local - radar echoes - parallaxes
  - *redshift* (change in size of the Universe)
  - \`*conformal*' or \`*comoving*' distance - appears in metric
  - **angular diameter distance**: angular size: $\theta = d/D_A(z)$
  - **luminosity distance:** flux density: $F = L/(4\pi D_L(z)^2)$
    - apparent distances of "standard candles" or "measuring rods”
    - calculable functions of redshift (absent structure)
- Lensing magnifies or de-magnifies: changes D_A, D_L:
    - they become random functions of direction
- **Q: does structure *bias* angular sizes or flux densities?**
  - **if it does then we will get the wrong cosmological parameters**

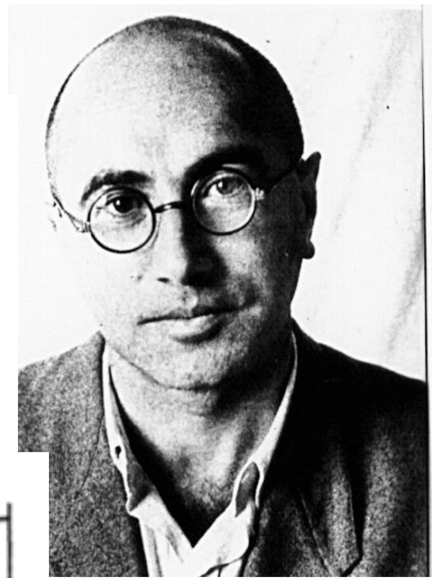
OBSERVATIONS IN A UNIVERSE HOMOGENEOUS IN THE MEAN

Ya. B. Zel'dovich

Translated from Astronomicheskii Zhurnal, Vol. 41, No. 1,
pp. 19-24, January-February, 1964
Original article submitted June 12, 1963

Fig. 1.

Fig. 2.



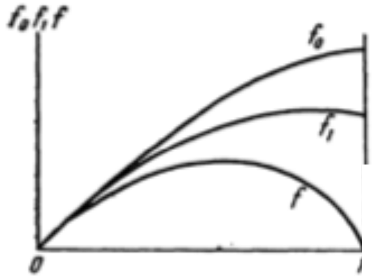
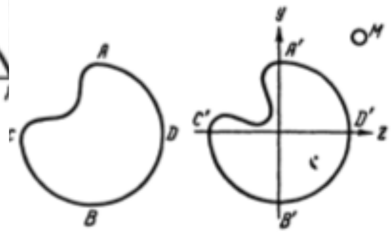


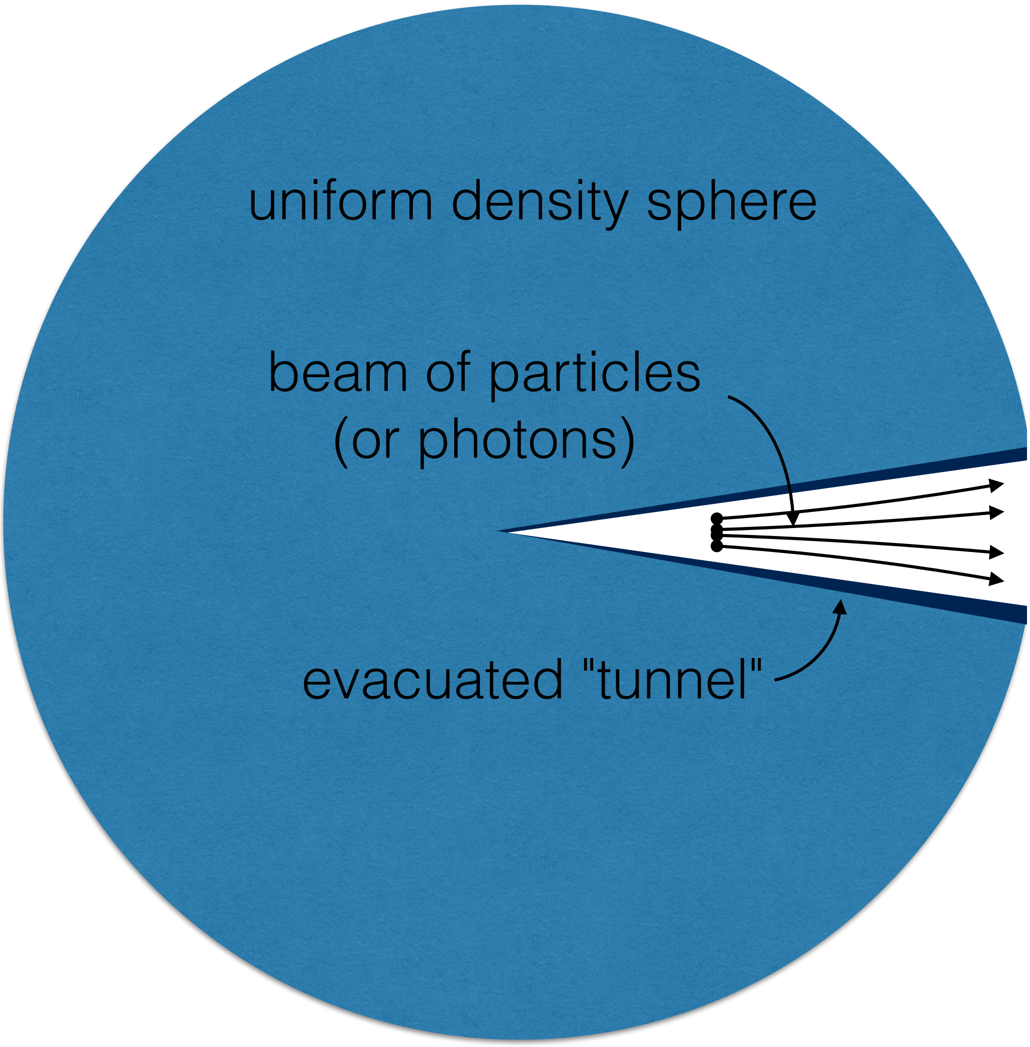
Fig. 6. $f_0 = \Delta\left(1 - \frac{\Delta}{2}\right)$; $f_1 = \frac{2}{5}\,[1 - (1-\Delta)^{5/2}]$; $f = 2(1-\Delta)(1 - \sqrt{1-\Delta})$



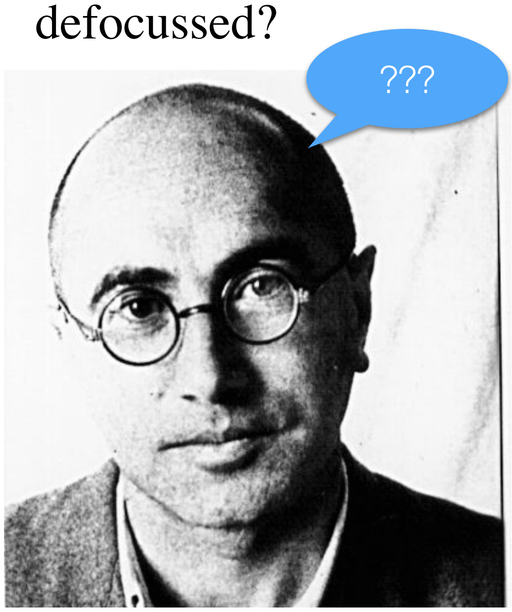
The mass M deforms the observed shape of the object, so that the latter becomes contracted along the axis joining it to M and elongated in the perpendicular direction.

- considers light propagation in *inhomogeneous* cosmologies
- the first known "*cone diagram*"
- angular diameter $D_a(z)$ plots
  - uses $\Delta = z/(1+z)$
- bias in D_a for galaxies seen along underdense lines of sight
- shape distortion from external mass
- FLRW curvature from local light-beam focussing - Raychaudhuri...
  - not $\mathbf{G} = 8\pi G\mathbf{T}$ + symmetry

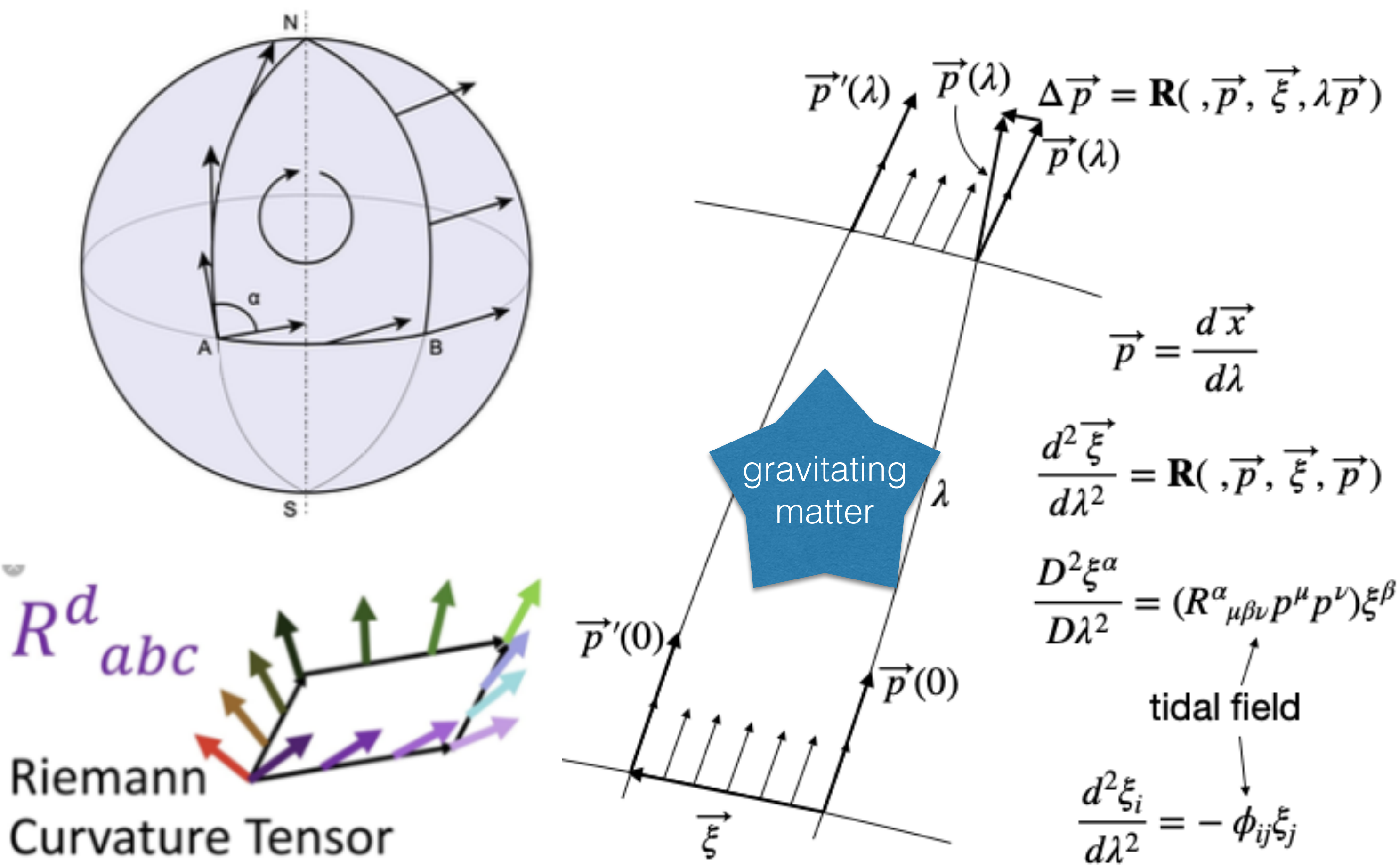
Zel'dovich's 1963 "empty beam" calculation



- Is there a gravitational field in the "tunnel"?
- Would Newton say that a beam of test particles would be defocussed?
- What about a beam of light? Would that get defocussed?

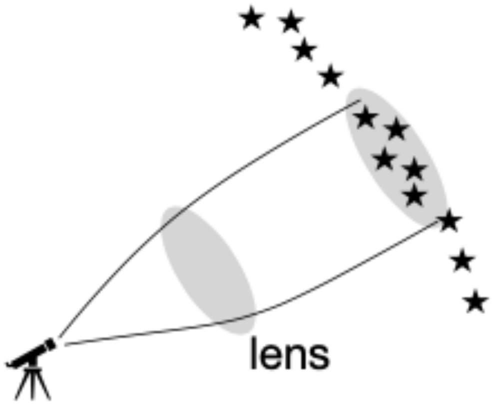


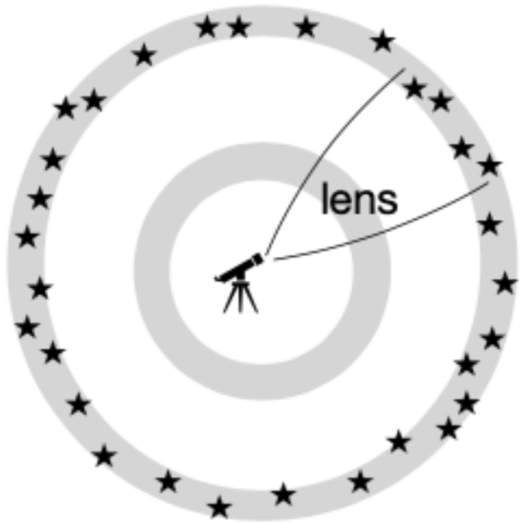
parallel transport -> curvature -> focussing -> Raychaudhuri



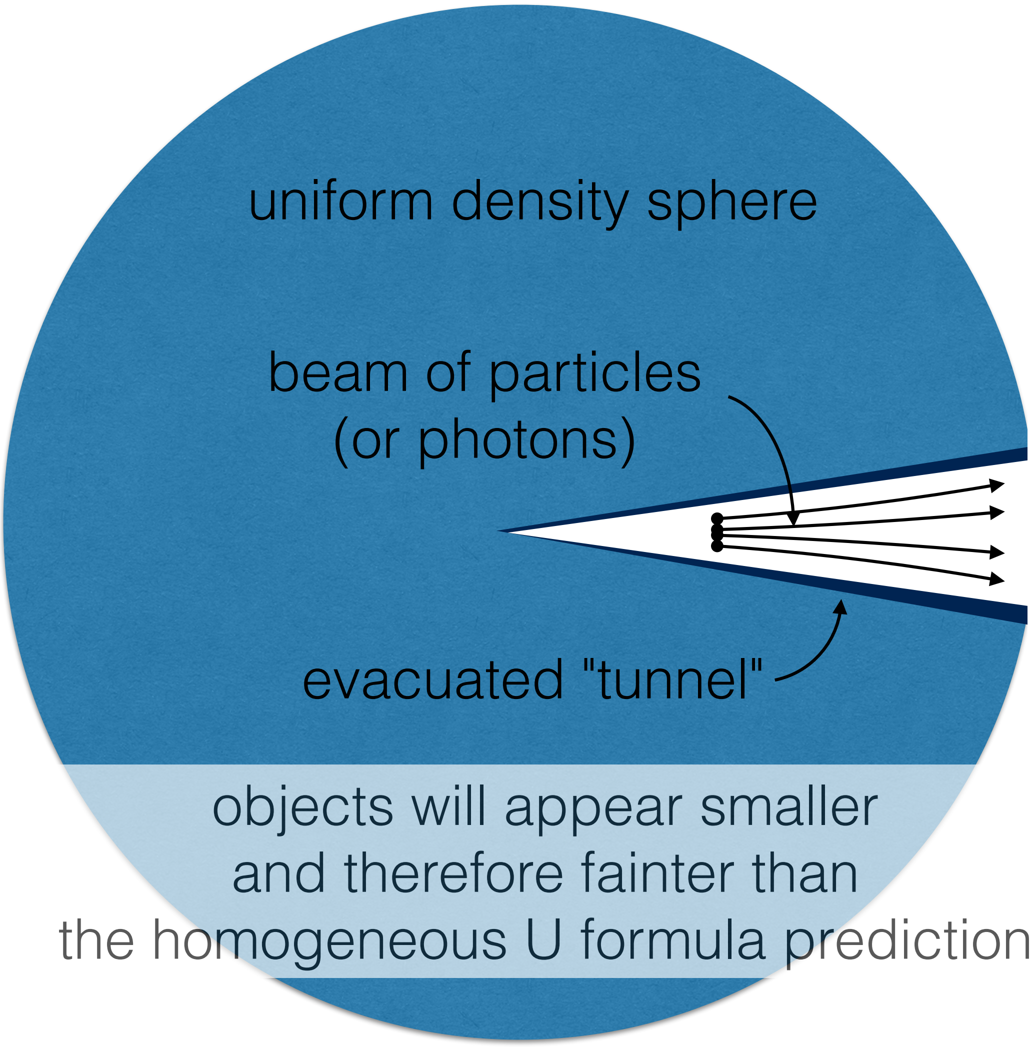
Zel'dovich '63: How Rauchaudhuri => FLRW metric

- With $r = \sqrt{A}$ and affine $d\lambda = -\,ad\tau = a^2 d\chi$ Raychaudhuri's focussing equation is
  - $r'' \equiv \dfrac{d^2 r}{d\lambda^2} = -\dfrac{4\pi G(\rho + P/c^2)}{c^2 a^2} r$
  - comes from GR, but mostly Newton (x2)
- and has solution, for bundle of angle θ at observer,
  - $r = \theta a \sin(\chi)$
  - proof: with $r'/\theta = -\cos(\chi)/a + \sin(\chi)a'$ etc
  - and with Friedmann equation + continuity
    - $a'' = -\dfrac{4\pi G(\rho + P/c^2)}{ac^2} + \dfrac{1}{a^3}$
  - $r''/\theta = \cancel{\dfrac{a'}{a^2}\cos\chi} - \dfrac{1}{a^3}\sin\chi + a''\sin\chi - \cancel{\dfrac{a'}{a^2}\cos\chi}$
- but $ds^2 = -\,c^2 dt^2 + a^2(t)(d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\phi^2))$ implies $r = a\theta\sin\chi$ and hence $D_\mathrm{A} = a\sin\chi$
- so D_A + F-metric can be "derived" from local focussing

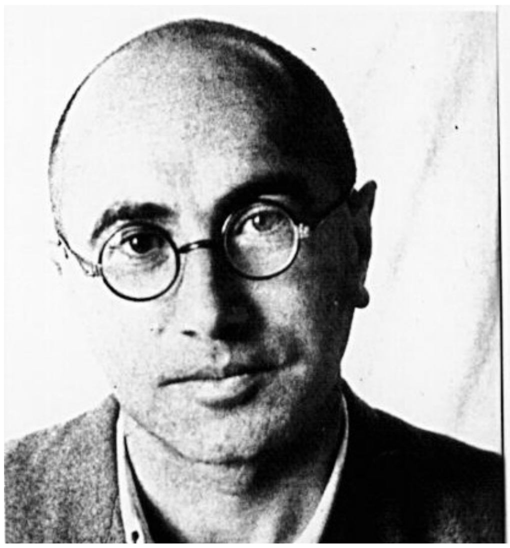




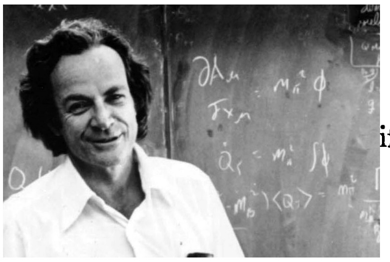
Zel'dovich's 1963 "empty beam" calculation



- Is there a gravitational field in the "tunnel"? **yes**
- Would Newton say that a beam of test particles would be defocussed? **yes**
- What about a beam of light? Would that get defocussed? **no!**



ON THE PROPAGATION OF LIGHT IN INHOMOGENEOUS COSMOLOGIES. I. MEAN EFFECTS



JAMES E. GUNN
ifornia Institute of Technology and Jet Propulsion Laboratory
Received February 23, 1967; revised May 23, 1967

ABSTRACT

The statistical effects of local inhomogeneities on the propagation of light are investigated, and deviations (including rms fluctuations) from the idealized behavior in homogeneous universes are investigated by a perturbation-theoretic approach. The effect discussed by Feynman and recently by Bertotti of the density of the intergalactic medium being systematically lower than the mean mass density is examined, and expressions for the effect valid at all redshifts are derived.

I. INTRODUCTION

In an unpublished colloquium given at the California Institute of Technology in 1964, Feynman discussed the effect on observed angular diameters of distant objects if the intergalactic medium has lower density than the mean mass density, as would be the case if a significant fraction of the total mass were contained in galaxies. It is an obvious extension of the existence of this effect that luminosities will also be affected, though this was apparently not realized at the time. This realization prompted the conviction that the effect of known kinds of deviations of the real Universe from the homogeneous isotropic models (upon which predictions had been based in the past) upon observable quantities like luminosity and angular diameter should be investigated. The author (1967) has recently made such a study for angular diameters; the present work deals primarily with mean statistical effects upon luminosity. A third paper will deal with possible extreme effects one may expect to encounter more rarely. Some of the results discussed here have been discussed independently by Bertotti (1966) and Zel'dovich (1965).

Kantowski '69

CORRECTIONS IN THE LUMINOSITY-REDSHIFT RELATIONS OF THE HOMOGENEOUS FRIEDMANN MODELS

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Received January 22, 1968; revised March 22, 1968

ABSTRACT

In this paper the bolometric luminosity-redshift relations of the Friedmann dust universes ($\Lambda = 0$) are corrected for the presence of inhomogeneities. The "locally" inhomogeneous Swiss-cheese models are used, and it is first shown that the introduction of clumps of matter into Friedmann models does not significantly affect the $R(z)$ or $R(v)$ relations (Friedmann radius versus the redshift or affine parameter) along a null ray. Then, by the use of the optical scalar equations, a linear third-order differential equation is arrived at for the mean cross-sectional area of a light beam as a function of the affine parameter. This differential equation is confirmed by rederiving its small redshift solution from an interesting geometrical point of view. The geometrical argument is then extended to show that "mild" inhomogeneities of a transparent type have no effect on the mean area of a light beam.

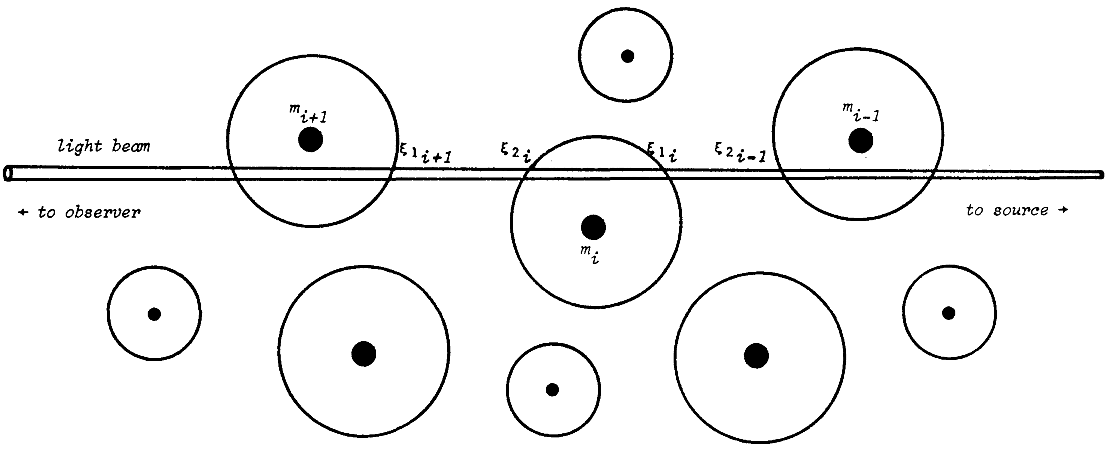

FIG. 1.—Spacelike section of a typical Swiss-cheese universe

Dyer & Roeder '72

THE DISTANCE-REDSHIFT RELATION FOR UNIVERSES WITH NO INTERGALACTIC MEDIUM

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Received 1972 April 19

ABSTRACT

The distance-redshift relation is derived for model universes in which there is negligible intergalactic matter and in which the line of sight to a distant object does not pass close to intervening galaxies. When fitted to observations, this relation yields a higher value of q_0 than does a homogeneous model.

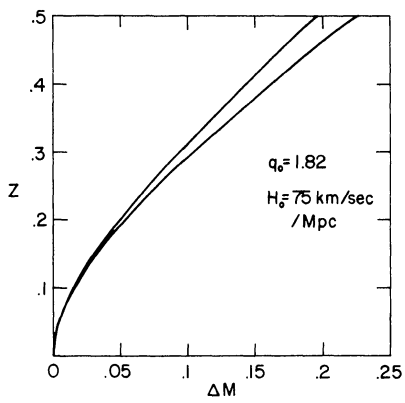
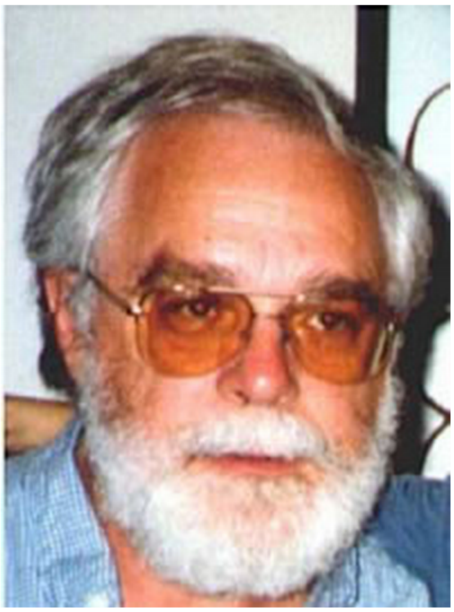


FIG. 1.—The dimming, relative to the homogeneous model, assuming that the beam passes far from any intervening galaxies (*lower curve*) and assuming that the beam passes no closer than 2 kpc to the center of galaxies similar to our own (*upper curve*).



APPARENT LUMINOSITIES IN A LOCALLY INHOMOGENEOUS UNIVERSE

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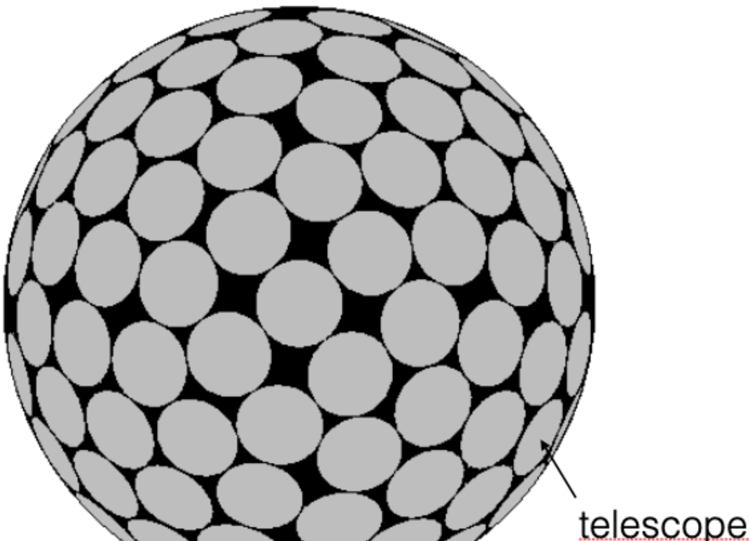
Department of Physics, Harvard University
Received 1976 April 6; revised 1976 May 20

ABSTRACT

Apparent luminosities are considered in a locally inhomogeneous universe, with gravitational deflection by individual clumps of matter taken into account. It is shown that as long as the clump radii are sufficiently small, gravitational deflection by the clumps will produce the same average effect as would be produced if the mass were spread out homogeneously. The conventional formulae for luminosity distance as a function of redshift consequently remain valid, despite the presence of any local inhomogeneities of less than galactic dimensions. For clumps of galactic size, the validity of the conventional formulae depends on the selection procedure used and the redshift of the object studied.



Enter Schneider, Ehlers, Seitz etc... ('80s, '90s)

- Two consistent threads:
  - Lens equation:
    - at least one image is made **brighter**
  - Optical scalar equations (Sachs 1961):
    - from Raychaudhuri
    - -> *focusing theorem* (Seitz+ 1994)
    - Things viewed through 'clumpiness' are further than they appear...
  - opposite to what Zel'dovich, Kantowski, Dyer & Roeder etc concluded
  - and in conflict with Weinberg too...

Seitz, Schneider & Ehlers (1994)

Finally, we have derived an equation for the size of a light beam in a clumpy universe, relative to the size of a beam which is unaffected by the matter inhomogeneities. If we require that this second-order differential equation contains only the contribution by matter clumps as source term, the independent variable is uniquely defined and agrees with the χ-function previously introduced [see SEF, eq. (4.68)] for other reasons. This relative focusing equation immediately yields the result that a light beam cannot be less focused than a reference beam which is unaffected by matter inhomogeneities, prior to the propagation through its first conjugate point. In other words, no source can appear fainter to the observer than in the case that there are no matter inhomogeneities close to the line-of-sight to this source, a result previously demonstrated for the case of one (Schneider 1984) and several (Paper I, Seitz & Schneider 1994) lens planes.

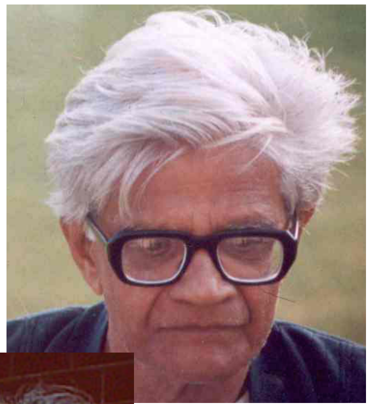
On Seitz, Schneider & Ehlers 94

1992). Taking a somewhat different approach, Seitz, Schneider & Ehlers (1994) have used the optical scalars formalism of Sachs (1961) to show that the square root of the proper area of a narrow bundle of rays $D = \sqrt{A}$ obeys the 'focusing equation':

$$\boxed{\ddot{D}/D = -(R + \Sigma^2).} \qquad (1)$$

Here $\ddot{D}$ is the second derivative of D with respect to affine distance along the bundle; $R = R_{\alpha\beta}k^\alpha k^\beta/2$ is the local Ricci focusing from matter in the beam, which for non-relativistic velocities is just proportional to the matter density; and Σ^2 is the squared rate of shear from the integrated effect of up-beam Weyl focusing – i.e. the tidal field of matter outside the beam. The resulting *focusing theorem* is that the RHS of (1) is non-positive, so that beams are always focused to smaller sizes, at least as compared to empty space-time,

More on the focusing theorem: $\ddot{D}/D = -(R + \Sigma^2)$



- Derived from Sachs '61 "optical scalars"
- from A.K. Raychaudhuri's equation
  - transport of expansion, vorticity and shear
- $R = R_{ab}k^a k^b$ local effect of matter in beam
- Σ^2 is the cumulative effect of matter *outside* the beam
  - Σ being the *rate* of image shearing
- Like cosmological acceleration equation:
  - $d^2a/dt^2 = - 4\pi G(\varrho+3P/c^2)a$
  - so Σ^2 here plays the role of pressure!
- Also recalls Hawking-Ellis singularity theorem
  - both terms are positive => focusing
- e.g. Narlikar (Introduction to Relativity):
  - "*Thus the normal tendency of matter*
  - *is to focus light rays*"

Narlikar on the focusing theorem

The Raychaudhuri equation can be stated in a slightly different form as a *focussing theorem*. In this form it describes the effect of gravity on a bundle of null geodesics spanning a finite cross section. Denoting the cross section by A, we write the equation of the surface spanning the geodesics as $f =$ constant. Define the normal to the cross-sectional surface by $k_i = \partial f/\partial x^i$. Figure 18.3 shows the geometry of the bundle.

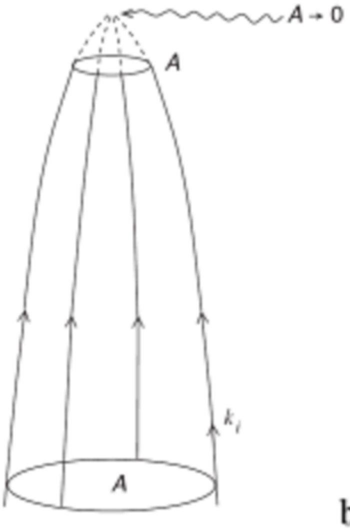


Fig. 18.3. The bundle of geodesics focusses in the future with its cross section A decreasing to zero. This effect was discussed in the context o spacetime singularity by A. K. Raychaudhuri.

Using a calculation similar to that which led to the geodetic deviation equation in Chapter 5, we get the focussing equation as

$$\frac{1}{\sqrt{A}}\frac{d^2\sqrt{A}}{d\lambda^2} = \frac{1}{2}R_{im}k^i k^m - |\sigma|^2, \tag{18.10}$$

Equation (18.10) is similar to the Raychaudhuri equation with $|\sigma|^2$ being the square of the magnitude of shear. With Einstein's equations, we can rewrite (18.10) as

$$\frac{1}{\sqrt{A}}\frac{d^2\sqrt{A}}{d\lambda^2} = -4\pi G\left(T_{im} - \frac{1}{2}g_{im}T\right)k^i k^m - |\sigma|^2. \tag{18.12}$$

For dust we have $T_{im} = \rho u_i u_m$ and this condition is satisfied with the left-hand side equalling $\rho(u_i k^i)^2$. (Remember that k_i is a null vector, so $g_{im}k^i k^m = 0$.) Thus the normal tendency of matter is to focus light rays by gravity.

Kibble & Lieu (2005)

AVERAGE MAGNIFICATION EFFECT OF CLUMPING OF MATTER

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ABSTRACT

The aim of this paper is to reexamine the question of the average magnification in a universe with some inhomogeneously distributed matter. We present an analytic proof, valid under rather general conditions, including clumps of any shape and size and strong lensing, that as long as the clumps are uncorrelated, the average "reciprocal" magnification (in one of several possible senses) is precisely the same as in a homogeneous universe with an equal mean density. From this result, we also show that a similar statement can be made about one definition of the average "direct" magnification. We discuss, in the context of observations of discrete and extended sources, the physical significance of the various different measures of magnification and the circumstances in which they are appropriate.

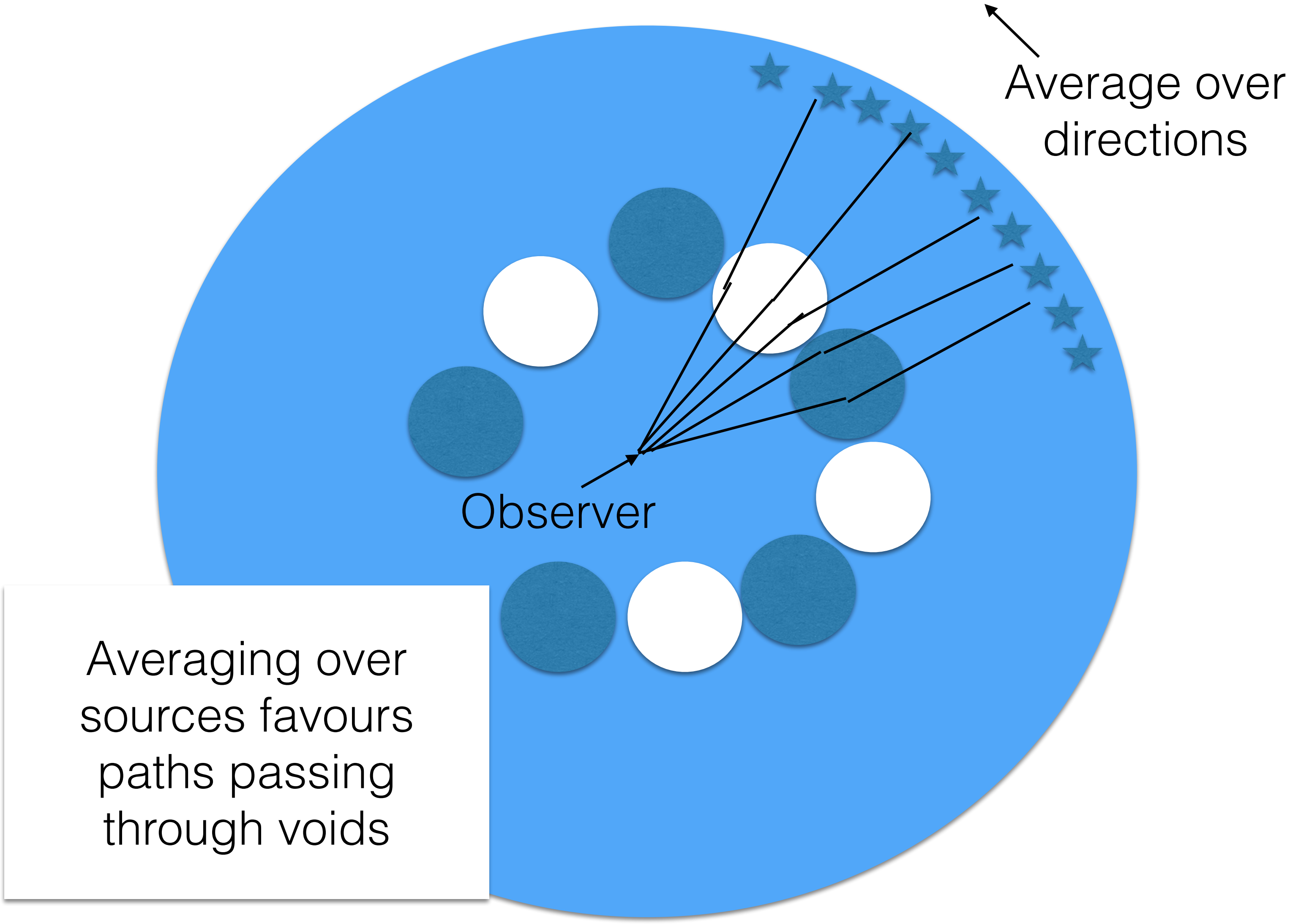
Subject headings: cosmology: miscellaneous — distance scale — galaxies: distances and redshifts — gravitational lensing

Kibble & Lieu 2005

There is another important distinction to be made. We may choose at random one of the sources at redshift *z*, or we may choose a random direction in the sky and look for sources there. These are not the same; the choices are differently weighted. If one part of the sky is more magnified, or at a closer angular-size distance, the corresponding area of the constant-*z* surface will be smaller, so fewer sources are likely to be found there. In other words, choosing a source at random will give on average a smaller magnification or larger angular-size distance.

- Weinberg: $<\mu> = 1$ when averaged over *sources*
- Kibble & Lieu: $<1/\mu> = 1$ when averaged over *directions on the sky*
    - latter is more relevant for CMB observations
        - strictly only valid in weak lensing regime

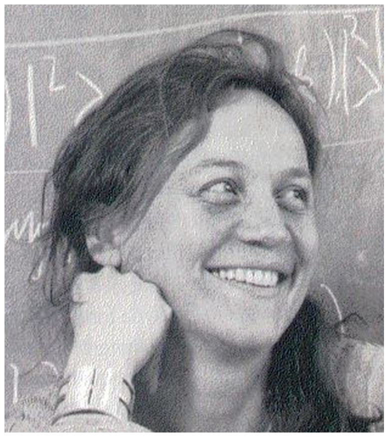
Kibble + Lieu 2005: $\langle\mu\rangle = 1$ vs $\langle 1/\mu\rangle = 1$



Recent developments...

- <u>Backreaction:</u> George Ellis: ”have cosmologists erred in failing to take into account the inherent non-linearity of Einstein's equations?"
  - cosmologists often do <u>linear theory</u> calculations
  - but Einstein's equations (metric <-> matter) are <u>non-linear</u>
  - averaging and non-linearity "do not commute"
  - so is *dark energy* a mirage?
- requires calculations in 2nd order perturbation theory
- now mostly accepted that effects are too small to explain acceleration
- but maybe there are still interesting percent level effects:
  - Recent work: - large ($O(\kappa^2)$) source magnification
  - and similarly large z-surface *area* increase
    - Directly violates Weinberg's assumption
    - in accord with the *focusing theorem* (Seitz, Schneider & Ehlers)



What is the distance to the CMB?
How relativistic corrections remove the tension with local H_0 measurements

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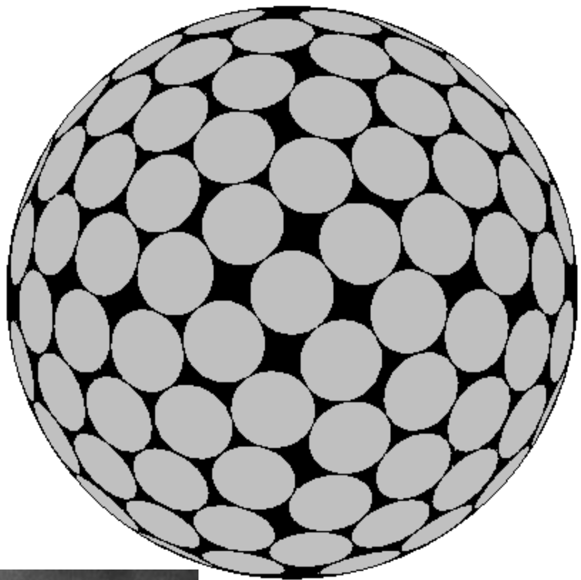
The success of precision cosmology depends not only on accurate observations, but also on the theoretical model – which must be understood to at least the same level of precision. Subtle relativistic effects can lead to biased measurements if they are neglected. One such effect gives a systematic shift in the distance-redshift relation away from its background value, due to the accumulation of all possible lensing events. We estimate the expectation value of this aggregated lensing using second-order perturbations about a concordance background, and show that the distance to last scattering is shifted by several percent. Neglecting this shift leads to significant bias in the background cosmological parameters. We show that this removes the tension between local measurements of H_0 and those measured through the CMB and favours a closed universe.

Is there a flaw in Weinberg's argument?



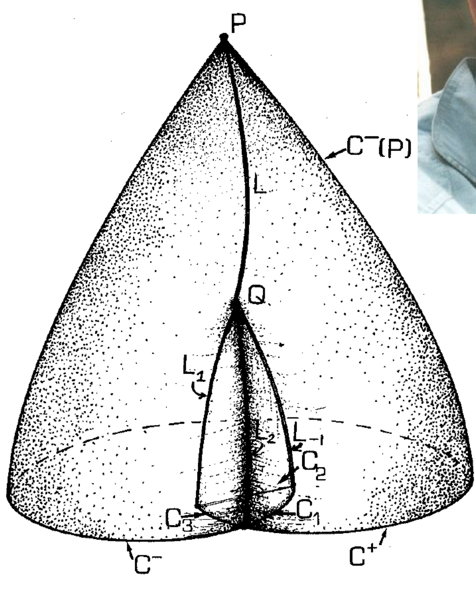


Figure 1: A lens L and resulting caustics on the past light cone $C^-(P)$ (2-dimensional section of the full light cone), showing in particular the cross-over line L_2 and cusp lines L_{-1}, L_1 meeting at the conjugate point Q. The intersection of the past light cone with a surface of constant time defines exterior segments C^-, C^+ of the light cone together with interior segments C_1, C_2, C_3.

Lensing and caustic effects on cosmological distances.

EBD '98

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December 4, 2013

Abstract

We consider the changes which occur in cosmological distances due to the combined effects of some null geodesics passing through low-density regions while others pass through lensing-induced caustics. This combination of effects increases observed areas corresponding to a given solid angle even when averaged over large angular scales, through the additive effect of increases on all scales, but particularly on micro-angular scales; however angular sizes will not be significantly effected on large angular scales (when caustics occur, area distances and angular-diameter distances no longer coincide). We compare our results with other works on lensing, which claim there is no such effect, and explain why the effect will indeed occur in the (realistic) situation where caustics due to lensing are significant. Whether or not the effect is significant for number counts depends on the associated angular scales and on the distribution of inhomogeneities in the universe. It could also possibly affect the spectrum of CBR anisotropies on small angular scales, indeed caustics can induce a non-Gaussian signature into the CMB at small scales and lead to stronger mixing of anisotropies than occurs in weak lensing.

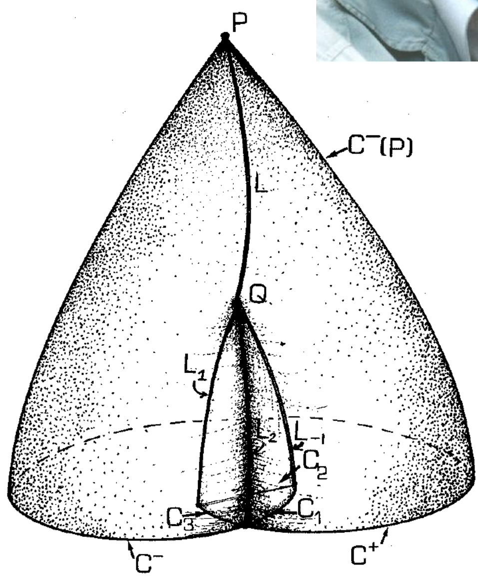
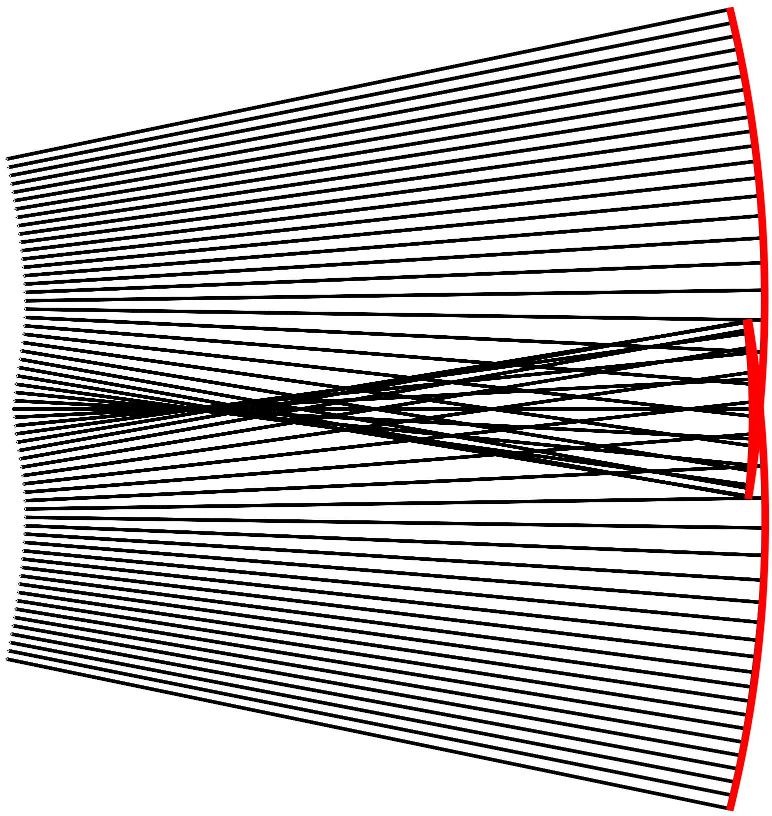


Figure 1: A lens L and resulting caustics on the past light cone $C^{-}(P)$ (2-dimensional section of the full light cone), showing in particular the cross-over line L_2 and cusp lines L_{-1}, L_1 meeting at the conjugate point Q. The intersection of the past light cone with a surface of constant time defines exterior segments C^{-}, C^{+} of the light cone together with interior segments C_1, C_2, C_3.

Ellis, Bassett & Dunsby '98 critique of Weinberg '76

- EDB98 make two points:
- *Weinberg assumes that which is to be proven*
  - true: W76 assumes that the surface of constant z around a source (or observer) is a sphere
- *Small scale strong lensing causes the surface to be folded over on itself so total area greatly enhanced*
  - possibly also true
- *Thus Weinberg's claim is disproved*
  - No: W76 is still valid if multiple images are unresolved



Clarkson et al. 2014

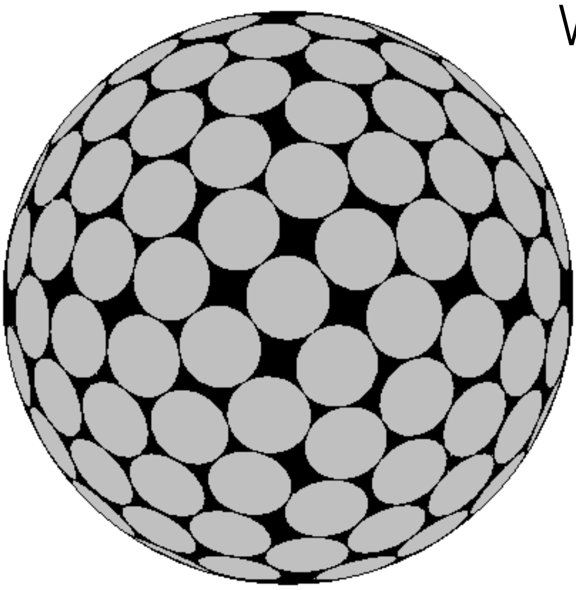
$$\langle \Delta \rangle \simeq \frac{3}{2} \left\langle \left(\frac{\delta d_A}{\chi_s} \right)^2 \right\rangle = \frac{3}{2} \left\langle \kappa^2 \right\rangle \,, \tag{1.5}$$

where κ is the usual linear lensing convergence. This is actually the leading contribution to the expected change to large distances. We prove this remarkably simple and important result in a variety of ways in several appendices. It implies that the total area of a sphere of constant redshift will be larger than in the background. Physically this is because a sphere about us in redshift space is not a sphere in real space — lensing implies that this 'sphere' becomes significantly crumpled in real space, and hence has a larger area. When interpreted

4 Conclusions

We have demonstrated an important overall shift in the distance redshift relation when the aggregate of all lensing events is considered, calculated by averaging over an ensemble of universes. This result is a consequence of flux conservation at second-order in perturbation theory. This is a purely relativistic effect with no Newtonian counterpart — and it is the first quantitative prediction for a significant change to the background cosmology when averaging over structure [21]. The extraordinary amplification of aggregated lensing comes mainly from the integrated lensing of structure on scales in the range 1–100 Mpc, although structure down to 10kpc scales contributes significantly. We have estimated the size of the effect using

Is there really a flaw in Weinberg's argument?



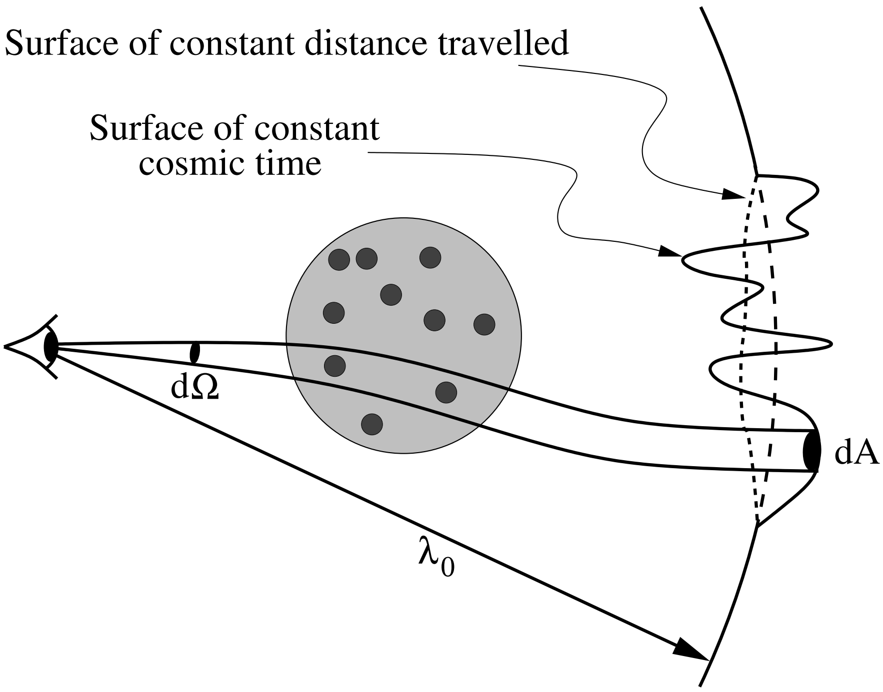
Weinberg assumes surface of constant redshift is a sphere

but if the surface is wrinkled then the area of a surface of constant z may be larger

NK + John Peacock 2016

- Weinberg *assumes* that the area of a surface of constant redshift is unperturbed by lensing by intervening structures
  - same assumption is made by Kibble & Lieu
  - seems reasonable since *static* lenses do not affect redshift
  - and leads to conservation of e.g. source-averaged flux density
    - but not strictly true — breaks down at some level
    - directly challenged by cosmological perturbation theorists
- What *is* the change in the area of the constant-z surface (or cosmic photosphere) caused by structure?
  - Isomorphic to light propagation in lumpy glass
    - What is the area of a wavefront of light (or light from a flash-bulb) propagating out from a point?

KP2016: closing the loophole in Weinberg's argument



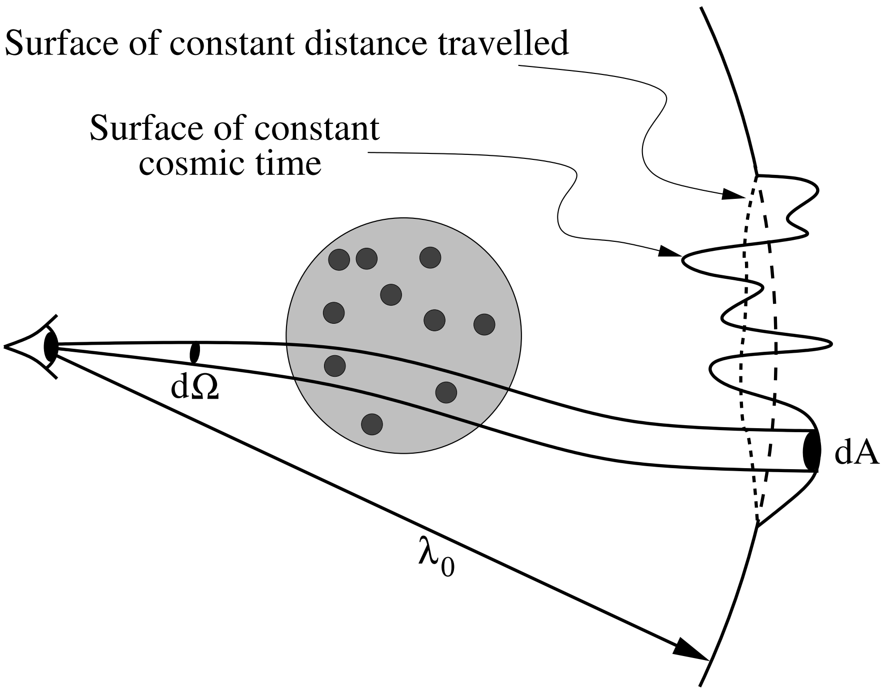
2 effects:

1) wiggly lines don't get as far as straight lines
2) A wrinkly surface has more area than a smooth one

What is the area of a wavy surface?

what property of a wavy surface could the fractional change in surface area depend on?

KP2015: closing the loophole in Weinberg's argument



2 effects:

1) wiggly lines don't get as far as straight lines
2) wrinkly surface has more area than a smooth one

but both effects are ~(bending angle)^2 ~ 10^{-6}

NK + Peacock 2016 - 2nd point

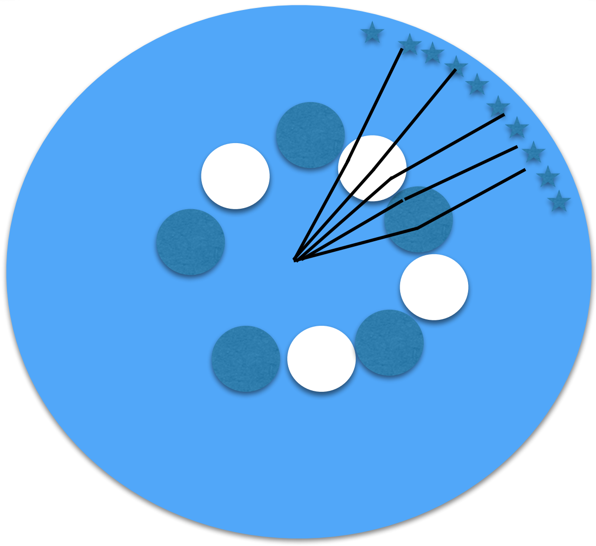
- Perturbation to the *area* is on the order of the <u>mean squared cumulative deflection angle</u>
- This is a one-part-in-a-million effect
  - dominated by large-scale structure
  - see Breont + Fleury 2021 - supports this conclusion
- Relativistic perturbation theory, *focussing theorem* etc. give perturbation to the distance that is on the order of the <u>mean squared convergence</u>
  - much larger
  - dominated by small-scale structure (possibly divergent)
- These effects are real, but are essentially *statistical effects*:
  - E.g. the (source averaged) mean flux magnification μ is unity
    - so $\langle\Delta\mu\rangle_{source} = 0$
  - but μ is a fluctuating quantity
  - so any non-linear function of μ (e.g. $D/D_0 = 1 / \sqrt{\mu}$) will *not* average to unity

KP15: Statistical biases...

- Example: Source averaged distance bias:
  - $D/D_0 = \mu^{-1/2} = (1 + \Delta\mu)^{-1/2} = 1 - \Delta\mu / 2 + 3(\Delta\mu)^2/8 + ...$
  - $\langle D/D_0 \rangle_{\text{source}} = 1 + 3\langle(\Delta\mu)^2\rangle/8 + \ldots = 1 + 3\langle\kappa^2\rangle/8 + \ldots$
- Similarly for source averaged mean inverse magnification
  - $\langle D^2/D_0^2 \rangle_{\text{source}} = 1 + 4\kappa^2 + \ldots$
- *These echo the results from perturbation theory*
- But e.g. the latter is *not* the perturbation to the constant z surface area
  - that would be the average over *directions* rather than over sources
- Similarly, the mean source averaged flux magnification is $<\mu> = 1 + <3\kappa^2 + \gamma^2> + ... = 1 + <4\kappa^2> + ...$
  - but this is the *direction* averaged magnification
- *These come from non-commutativity of averaging and non-linearity*
  - $<f(x)> \,!= f(<x>)$ if x is a fluctuating quantity
  - But are un-related to the non-linearity of Einstein's equations

What about the "*focusing theorem*"?

$$\ddot{D}/D = -(R_{\mu\nu}p^\mu p^\nu + \Sigma^2)$$



- 2 lessons from foregoing:
  - 1) The theorem applies to a bundle of rays fired along a given direction
    - i.e. a *direction* - not *source*-averaged quantity
    - and paths to sources avoid over-densities
    - so care is needed in interpreting this
  - 2) D is a non-linear function of A
    - so, because A is a fluctuation quantity, we automatically expect a statistical bias in D
    - and the size of the effect is $\sim \langle \kappa^2 \rangle$
- So is there a "normal tendency of matter to focus light rays"?
  - as inferred from the averaged focusing theorem
- Not really. It is simply a statistical effect.

Concluding comments....

- The problem of how lensing by cosmic structure affects the mean *distance-redshift relation* (or the mean area of a surface of constant redshift) goes back for at least 50 years
  - Interesting problem....
  - many people played with it...
  - potentially important for "*precision cosmology*" with SN1a and CMB
- A conflict arose in the '80s between Weinberg's flux conservation argument and the contrary indications from the focusing theorem
- This remained unresolved and re-surfaced recently in results of relativistic 2nd order perturbation theory.

Concluding comments continued...

- John Peacock and I believe we have reconciled the conflicts
- We support Weinberg:
  - lensing affects individual source flux densities in a random way
  - but averaged flux density of sources is *almost exactly unperturbed*
- and pay tribute to Kibble and Lieu
  - emphasised the distinction between source and direction averaging
- Our main results:
  - claimed large effect statistical biases.
  - there *is* a bias in the area of constant z or photosphere surfaces - but it is very small ~ 10^{-6}
  - we have tried to clarify how the "*focusing theorem*"does not imply any intrinsic tendency for bundles of rays to be focused as they wend their wiggly way through the lumpy cosmos
- Implication: conventional methods for analysing the CMB & SN1a (mostly) are valid.