New advancements in AdS/CFT in lower dimensions

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1. Introduction

String Theory: Most consistent theory unifying Quantum Gravity and the Standard Model of particle interactions

In the last 20 years: Rich mathematical structure with far reaching applications

Picture changing paradigm:

AdS/CFT or gauge/gravity duality

(Maldacena'97)

Gauge theory
strongly (weakly)
interacting
(without gravity)

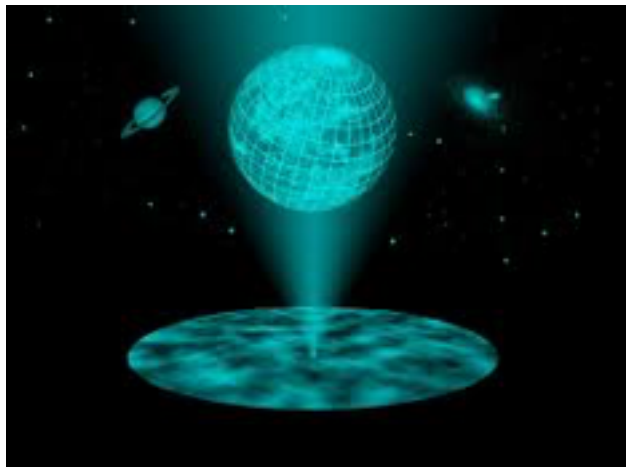
$\longleftrightarrow$

String Theory
weakly (strongly)
interacting
(with gravity)

Used to gain information about strongly coupled gauge systems (quark-gluon plasma, condensed matter systems, neutron stars).

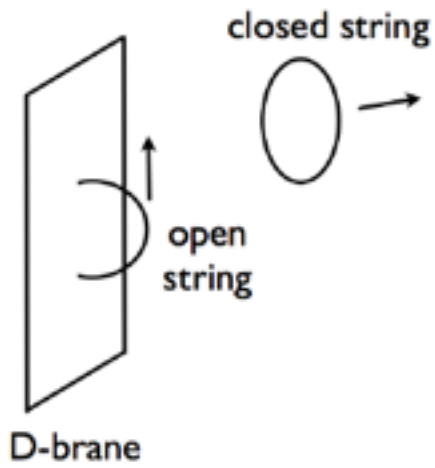
Possibility to address conceptual issues about quantum gravity (concrete realization of quantum gravity and spacetime as emergent phenomena).

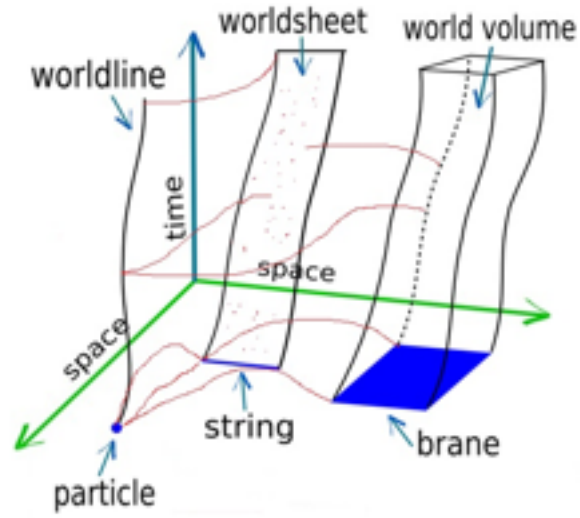
Concrete implementation of the Holographic principle, which underlies our current theoretical understanding of Black Holes



D-branes

String theory is more than a theory of strings





D-branes are BPS objects, satisfying $q = T$:

$$T_{F1} = \frac{1}{l_s^2}\,; \qquad T_{Dp} = \frac{1}{l_s^{p+1}}\frac{1}{g_{st}}\,; \qquad T_{NS5} = \frac{1}{l_s^6}\frac{1}{g_{st}^2}$$

They occur as classical solutions of the low energy sugras

D-branes are dynamical due to the open strings attached

Their dynamics can be studied using open string perturbation theory

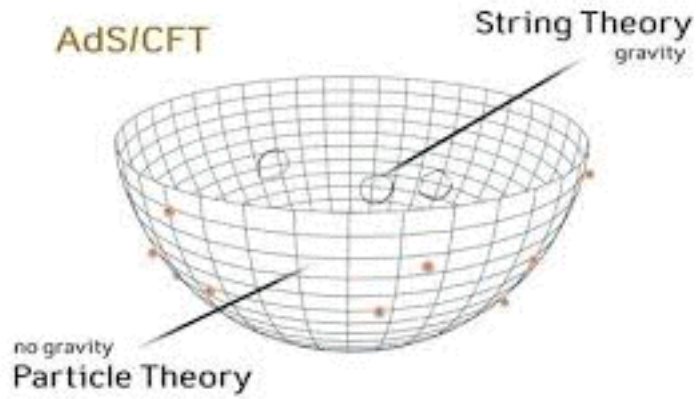
This represents an unexpected window into the dynamics of non-perturbative objects!

Their dual description as end-points of open strings and as sols to sugras have brought about the most important developments in String Theory in the last 20 years:

- AdS/CFT correspondence
- Black Hole thermodynamics

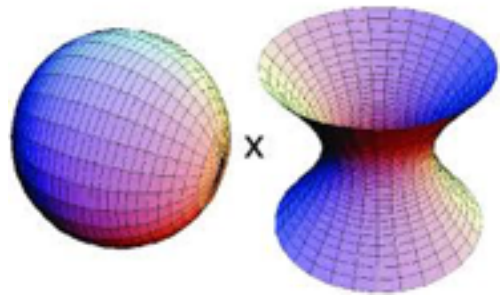
The AdS/CFT correspondence

Maldacena'97:



Equivalence between Type IIB string theory on $AdS_5 \times S^5$ and 4 dim $\mathcal{N} = 4$ SYM

AdS_5 : anti-de Sitter space in 5 dim (maximally symmetric solution of Einstein's eqs. with a negative cosmological constant)



Based on the study of a stack of N coincident D3-branes and its two dual descriptions:

- As a system of open strings
- As solution of the eqs. of motion of Type IIB sugra

As a system of open strings:

4d $\mathcal{N} = 4$ SYM theory. This is a conformal field theory.

As a solution to Type IIB sugra:

$AdS_5 \times S^5$ is the spacetime that arises around N D3-branes

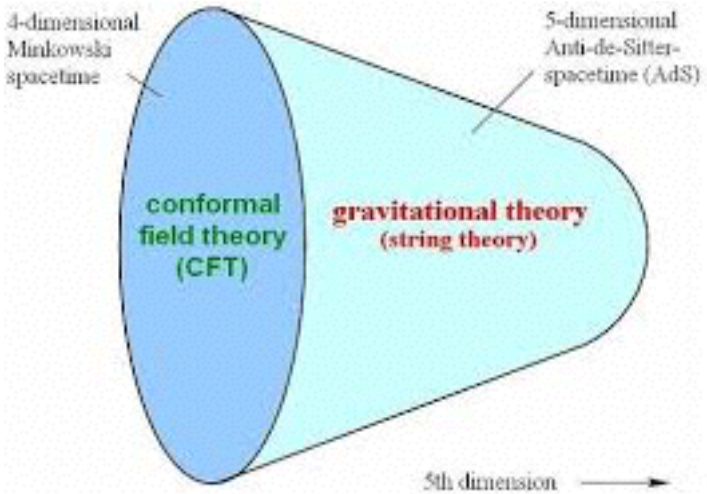
Type IIB sugra is dual to $\mathcal{N} = 4$ SYM in the planar limit and strong 't Hooft coupling $\rightarrow$ weak/strong duality

Beyond the sugra limit, the correspondence states that Type IIB superstring theory is dual to $\mathcal{N} = 4$ SYM

10d closed string theory dual to a 4d gauge theory!

In a suitable coordinate system $ds^2_{AdS_5} = dr^2 + e^{2r} ds^2_{\mathbb{R}^{1,3}}$

The boundary of AdS_5 is flat space. There is where the field theory lives:



$\rightarrow$ Explicit realization of the holographic principle

The AdS/CFT correspondence has been extended to other systems of branes:

For example: From D1-D5 branes:

Type IIB string theory on $AdS_3 \times S^3 \times CY_2$ dual to the CFT living in the D1-D5 system: 2d CFT with (4,4) susy

This example has been used to test the correspondence:

- 2d CFTs are inherent to String Theory. They are infinite dim and therefore much more tractable than higher dim ones
- The D1-D5 system is amenable for standard worldsheet methods

It is also the best scenario for the study of Black Holes

AdS/CFT and Black Holes

- Holography:

Black Holes: Thermodynamical systems with temperature and entropy

$$S = \frac{A}{4G}$$ (Bekenstein, Hawking 70's)

Proportional to the area of the horizon!

Holographic principle: The degrees of freedom of quantum gravity in a region can be encoded on the boundary of that region ('t Hooft'93, Susskind'94)

A consistent picture is reached if gravity in d dim is equivalent to a local field theory in d-1 dim, as predicted by AdS/CFT

- Microscopical description of entropy:

Intersecting D-brane configuration describing a black hole as supergravity solution (BPS):

$$S_{\text{closed}} \sim A$$

As the end points of open strings the D-branes are described by a field theory, and one can compute the number of states:

$$S_{\text{open}} \sim \ln \Omega$$

For the D1-D5 system:

$$S_{\text{closed}} = 2\pi\sqrt{Q_m(\frac{1}{2}Q_e^2+1)} \quad , \quad S_{\text{open}} = 2\pi\sqrt{\frac{1}{2}Q_m Q_e^2+1}$$

Perfect agreement when $Q_e >> 1$! (Strominger, Vafa'96)

2. Our work

5d extremal Black Holes have $AdS_3 \times S^2$ near horizon geometries

These have associated 2d (0,4) dual CFTs

Our goal: Find explicit classes of $AdS_3 \times S^2$ solutions to massive IIA sugra, and construct the 2d dual CFTs

Extend the duality to $AdS_2 \times S^2$ (near horizon geometries of 4d extremal Black Holes), and construct the dual super quantum mechanics

Explicit scenarios for the microscopical description of BH

Based on 1908.09851, 1909.10510, 2011.00005, 2011.13932, 2021.04682 and the review article "New advancements in AdS/CFT in lower dimensions", published in Universe 7 (2021) 7, 250, with Anayeli Ramirez

3. AdS_3 solutions to massive IIA with (0,4) susy

Explicit construction of the Killing spinors that realise the bosonic subalgebra $SO(2,2) \oplus SU(2)$ of $AdS_3 \times S^2$:

$AdS_3 \times S^2 \times M_4 \times I$ solutions to massive IIA, with

- $M_4 = CY_2$: class I
- $M_4 =$ Kähler : class II

Concentrate in class I:

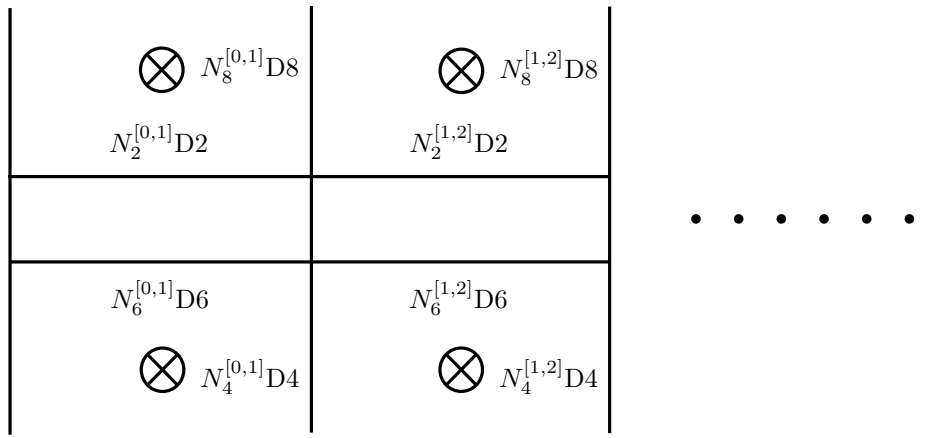
$$ds^2 = \frac{u}{\sqrt{h_4 h_8}}\left(ds^2_{AdS_3} + \frac{h_8 h_4}{4h_8 h_4 + u'^2} ds^2_{S^2}\right) + \sqrt{\frac{h_4}{h_8}} ds^2_{CY_2} + \frac{\sqrt{h_4 h_8}}{u} d\rho^2$$

u, h_8 : Linear functions in ρ ; h_4 : Function of ρ and CY_2

D2 and D6 branes are stretched between NS5-branes. They play the role of colour branes

D4 and D8 are perpendicular, and play the role of flavour branes

Hanany-Witten brane set-up:



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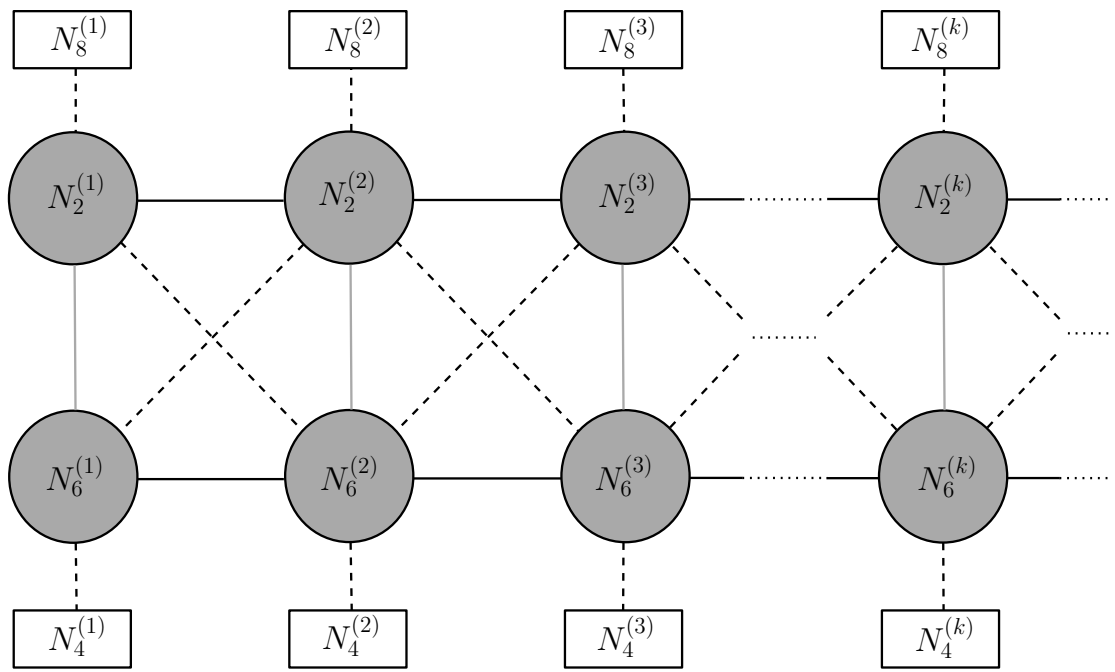
4. The 2d dual CFT

Main features of the brane set-up:

D2 and (wrapped) D6 colour branes

D4 and (wrapped) D8 flavour branes



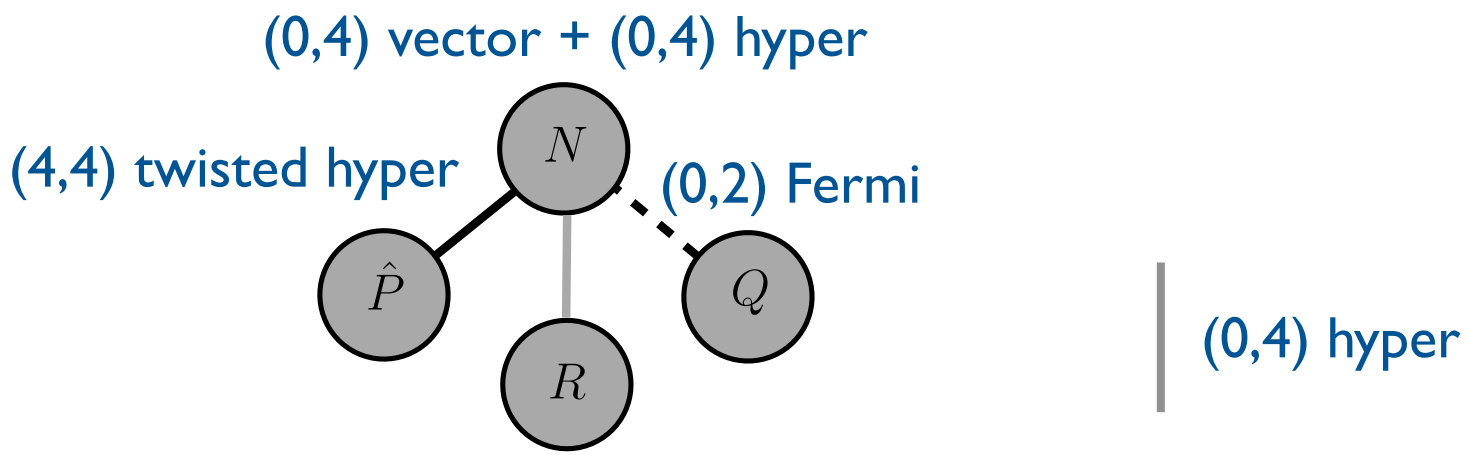
We can build 2d (0,4) quivers with (0,4) vector multiplets, hyper multiplets, twisted hypermultiplets and (0,2) Fermi multiplets

Tong'15

These come from the quantisation of the open strings with ends attached to the different stacks of branes

The resulting gauge theories can be anomalous, since left and right moving fermions need not be paired together

Non-anomalous quivers are obtained by assembling the building block:



This building block is non-anomalous if $2R = Q$

This must be satisfied at each node of our quiver

This is indeed satisfied with the quantised charges associated to the solutions!

We can then compute the central charge of the CFT in the IR, using that the (0,4) superconformal algebra relates the central charge with the R-symmetry anomaly (= level of the superconformal R-symmetry)[1]

$$c = 3k = 6(n_{hyp} - n_{vec})$$ (Putrov, Song, Yan'15)

[1] The energy momentum tensor and the R-symmetry current sit in the same multiplet

The holographic central charge, in turn, is computed from

$$c_{hol} = \frac{3R}{2G_3} = \frac{3}{\pi}\int d\rho\, h_4 h_8$$ (Brown, Henneaux'86)

Both expressions should agree in the holographic limit.

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Both expressions should agree in the holographic limit.

Taking $h_4(\rho) = \alpha_k + \frac{\beta_k}{2\pi}(\rho - 2\pi k)$ and

$h_8(\rho) = \mu_k + \frac{\nu_k}{2\pi}(\rho - 2\pi k)$ for $\rho \in [2\pi k, 2\pi(k+1)]$

we get

$$c = 6\sum_{k=1}^{P} \alpha_k \mu_k$$ to leading order from both expressions!

(Couzens, Y.L., Petri, Vandoren, 21)

In Couzens, Y.L., Petri, Vandoren, 21 we interpreted these solutions as describing chains of black strings stacked on top of each other along an interval

5. New AdS_2 solutions in Type II

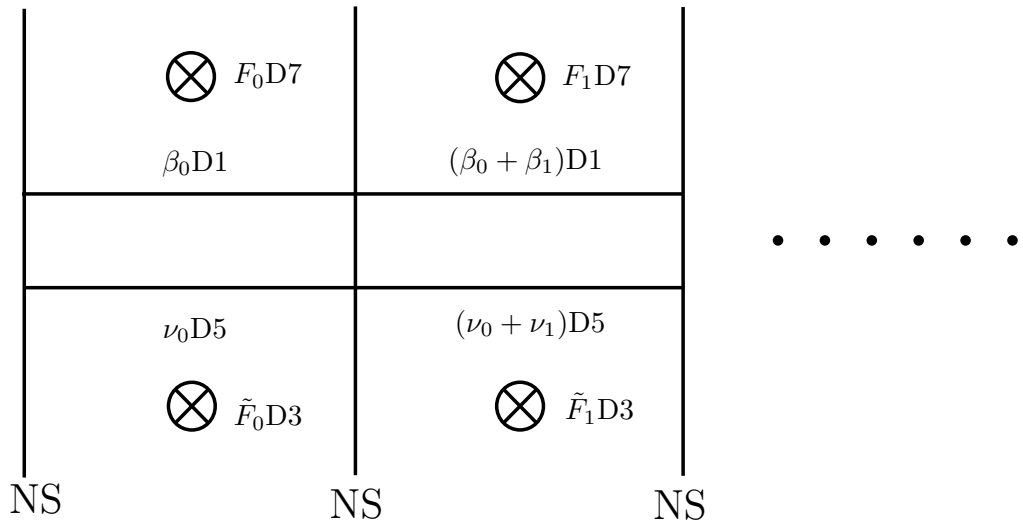
One class in Type IIB is constructed by T-dualising the $AdS_3 \times S^2 \times \mathrm{CY}_2 \times I$ solutions to massive IIA

This gives a family of $AdS_2 \times S^2 \times \mathrm{CY}_2 \times S^1 \times I$ solutions with 4 supersymmetries.

These solutions extend the class of $AdS_2 \times S^2 \times \mathrm{CY}_2 \times \Sigma_2$ solutions of Type IIB by Chiodaroli, Gutperle, Krym'09; Chiodaroli, D'Hoker, Gutperle'09, for Σ_2 an annulus, to include D7-branes

The dual SCQM can be understood as a compactification of the 2d CFT dual to the AdS_3 sols

From the geometry, one constructs the Hanany-Witten brane set-ups:



From them we can construct quiver gauge theories built out of 1d N=4 matter multiplets

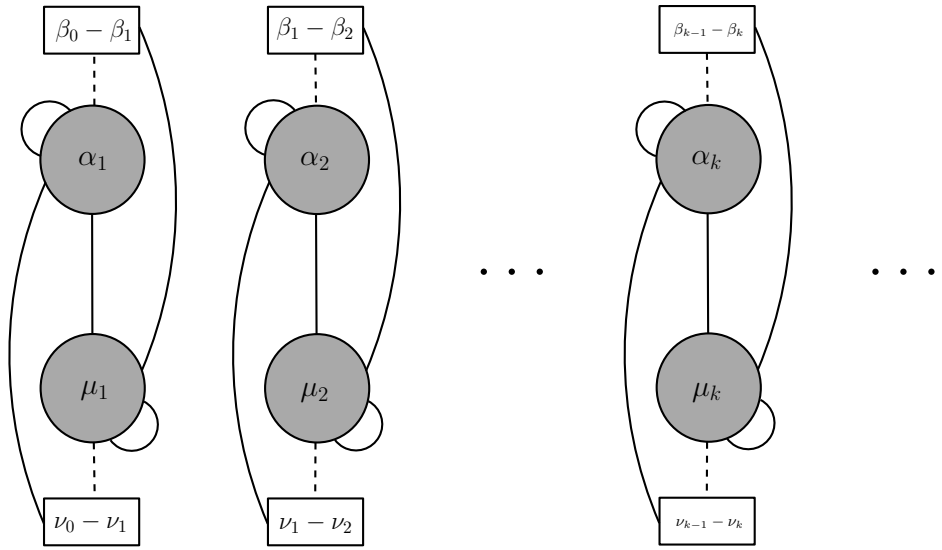
Using $c = 6(n_{hyp} - n_{vec})$ for the conformal quantum mechanics we find perfect agreement with the holographic result.

A class of solutions to massive IIA is obtained by analytically continuing the $AdS_3 \times S^2 \times \mathrm{CY}_2 \times I$ solutions.

The associated brane set-up is

	t	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
$D0$	$\times$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$D4$	$\times$	$\times$	$\times$	$\times$	$\times$	$-$	$-$	$-$	$-$	$-$
$D4'$	$\times$	$-$	$-$	$-$	$-$	$-$	$\times$	$\times$	$\times$	$\times$
$D8$	$\times$	$\times$	$\times$	$\times$	$\times$	$-$	$\times$	$\times$	$\times$	$\times$
$F1$	$\times$	$-$	$-$	$-$	$-$	$\times$	$-$	$-$	$-$	$-$

From it we can construct the quivers



The formula $c = 6(n_{hyp} - n_{vec})$, agrees, once more, with the holographic central charge!

6. Conclusions

- AdS_3 solutions to massive IIA with (0,4) susy classified
- 2d dual CFTs identified as IR fixed points of 2d QFTs built out of (0,4) multiplets
- Duality checked with the computation of the central charge
- Interpreted as chains of black strings
- New classes of AdS_2 solutions in IIB and massive IIA, extending previous classifications
- Interpretable as chains of black holes?

These solutions are ready for a careful study from a Black Hole perspective!

THANKS!