Möbius Chain Boundary Conditions: A New Topological Resource for Quantum Error Cancellation

José Luis Mac Loughlin[1] and Norma G. Sanchez[2]

[1]*School of Arts and Sciences, Museum House and Quantum Laboratory, La Plata City 1900, Provincia de Buenos Aires, Argentina*

[2]*International School and Institute of Astro-Physics Daniel Chalonge - Hector de Vega, CNRS, INSU-Institut National des Sciences de l'Univers, Sorbonne Université, 75014 Paris, France. Norma.Sanchez@orange.fr, https://chalonge-devega.fr*

(Dated: July 12, 2026)

We formulate a Möbius-chain mechanism for passive (topological, geometric) cancellation of coherent quantum errors. This work is the second step in a broader research program on **Möbius Quantum Computation (MQC)**: **(I)** In Paper I, we introduced MQC (Möbius Quantum Computation) as a topological boundary framework for quantum information processing, where **non-orientable boundary conditions** act as active ingredients in the effective quantum evolution of the quantum states and information processing. **(II)** Here we specialize such framework to one concrete consequence: the suppression of coherent errors by Möbius chain boundary conditions. **(III)** In contrast with conventional quantum error correction, which relies on redundant logical encoding, syndrome extraction, and active recovery operations, the present approach uses **a global topological and non-orientable identification**. **(IV)** A one-dimensional quantum chain is endowed with a Möbius boundary condition such that a quantum amplitude returning after one complete traversal is compared with a twisted copy of itself. **(V)** We show that coherent phase errors accumulated along the ordinary branch can act **against their topologically reflected contribution** and they cancellate themselves. **(VI)** The resulting mechanism is geometric, boundary-driven, and passive. **(VII)** We formulate the boundary condition, derive the basic cancellation relation, discuss residual errors due to imperfect symmetry, and compare the proposal with dynamical decoupling, decoherence-free subspaces, stabilizer codes, and topological quantum computation. **(VIII)** The present results establish Möbius boundary conditions as a promising and efficient geometric resource that may complement conventional quantum error mitigation and fault-tolerant architectures. **Keywords: Quantum Computation; Möbius Chain; Geometric Error Cancellation; Topological Boundary Conditions; Quantum Information Processing; Coherent Errors.**

I. INTRODUCTION

Quantum computation is limited not only by stochastic decoherence but also by coherent and systematic errors. Such errors include phase drifts, calibration imperfections, systematic over-rotations, residual couplings, and slowly varying control errors. Because coherent errors can add constructively over many operations, they may be especially damaging in long quantum circuits.

Standard quantum error correction suppresses errors by encoding logical qubits into larger Hilbert spaces, measuring stabilizers, and applying active recovery operations. This strategy is essential for fault-tolerant architectures, but it is resource intensive. It is therefore useful to investigate complementary passive mechanisms capable of suppressing specific classes of errors before, or in parallel with, active correction.

The present paper is part of a research program on Möbius quantum computation. In Paper I [1], Möbius quantum computation was proposed as a topological boundary framework for quantum information processing, based on the idea that non-orientable boundary conditions can act as active constraints on quantum evolution rather than as merely geometrical illustrations.

The Möbius strip, owing to its single-sided non-orientable structure, provides a simple mathematical setting in which global boundary identification, geometric phase, and quantum-state evolution can be connected.

This paper here develops the next step of that program. Instead of presenting the general framework again, we focus on one concrete mechanism: coherent error cancellation in a Möbius quantum chain.

The essential idea of this paper is that a quantum state propagating on a closed chain with a Möbius boundary identification does not return to itself trivially. Instead, the return amplitude is related to the initial amplitude by a twist. This global constraint can force opposite coherent phase contributions to interfere destructively. The cancellation is not produced by measurement or redundancy, but by the global topology of the boundary condition itself.

The goal of this paper is intentionally focused:

(i) We formulate a minimal Möbius-chain model,

(ii) derive the cancellation of symmetric coherent phase perturbations,

(iii) identify the residual error for imperfect symmetry, and

(iv) discuss the physical meaning of Möbius boundary conditions as a topological resource for passive quantum error suppression.

The present article should therefore be viewed as the second contribution of a broader MQC (Möbius Quantum Computation) research programme we recently started in [1].

In Paper I [1] we introduced the general quantum topological computing framework, whereas the present work investigates one specific physical consequence, namely passive (geometric/topological) coherent error cancellation induced by Möbius boundary conditions.

Further developments, including topological gate implementations and quantum algorithms based on Möbius topology, will be investigated in future work.

II. MÖBIUS BOUNDARY CONDITIONS AS A QUANTUM RESOURCE

The Möbius strip is the simplest non-orientable surface with a single boundary component. Its standard identification may be represented as

$$(x,0) \equiv (L-x,1), \tag{1}$$

where $x \in [0,L]$ labels the coordinate along the strip.

The important physical point is not the literal construction of a mechanical strip, but the existence of an effective boundary rule that identifies the end of a quantum trajectory with a reversed or twisted copy of its beginning.

In quantum information language, such a rule may be represented by a Möbius boundary operator $\mathcal{B}_M$. Acting on a state $|\psi\rangle$, this operator imposes a global constraint on the admissible evolution,

$$|\psi(L)\rangle = e^{i\phi_M}\mathcal{T}|\psi(0)\rangle, \tag{2}$$

where ϕ_M is a twist phase and $\mathcal{T}$ is an internal twist operation.

(i) Depending on the physical realization, $\mathcal{T}$ may act on spin, pseudospin, qubit orientation, path degree of freedom, or another internal two-level structure.

(ii) Such boundary condition is the basic resource considered here.

(iii) It enriches the usual Hilbert-space evolution by imposing a non-orientable global identification.

(iv) In this sense, the Möbius boundary condition is not an additional local interaction. It is a topological rule selecting how amplitudes are compared after a complete traversal.

III. MÖBIUS QUANTUM CHAIN

Consider a one-dimensional quantum chain of length L, parametrized by

$$x \in [0,L]. \tag{3}$$

For an ordinary periodic chain, the wavefunction satisfies

$$\psi(L) = \psi(0). \tag{4}$$

For a Möbius quantum chain, we impose instead

$$\psi(L) = e^{i\phi_M}\mathcal{T}\psi(0). \tag{5}$$

The simplest idealization is obtained when the twist acts as a sign or orientation inversion,

$$\mathcal{T}^2 = 1, \qquad \mathcal{T}\psi = \pm\psi. \tag{6}$$

A full traversal of the chain therefore combines dynamical propagation with a topological return rule. The state does not simply come back to the same local orientation. It comes back as a twisted copy of itself.

The effective propagation around the chain may be written schematically as

$$U_M = \mathcal{B}_M U_0, \tag{7}$$

where U_0 denotes the ordinary dynamical evolution and $\mathcal{B}_M$ encodes the Möbius boundary identification.

In regimes where the topological contribution can be represented by a global phase, one may write

$$U_M = e^{i\gamma_M} U_0, \tag{8}$$

where γ_M denotes a geometric or topological phase associated with the Möbius traversal.

In the present paper, however, we emphasize not only the phase itself but the cancellation of coherent error contributions induced by the twisted return.

IV. COHERENT PHASE ERROR ON AN ORDINARY CHAIN

Let a coherent phase error be described by a slowly varying perturbation $\epsilon(x)$, so that the wavefunction acquires the local phase

$$\psi(x) \longrightarrow e^{i\epsilon(x)}\psi(x). \tag{9}$$

On an ordinary chain, the accumulated coherent phase after one traversal is

$$\Delta_{\rm ord} = \int_0^L \epsilon(x)\,dx. \tag{10}$$

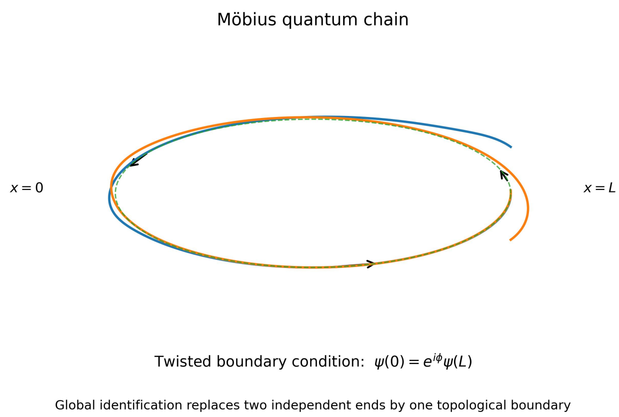

FIG. 1. Möbius quantum chain with a twisted boundary condition. The two ends of the chain are not identified trivially, as in an ordinary periodic chain, but through a Möbius rule $\psi(L) = e^{i\phi_M}\mathcal{T}\psi(0)$. This global identification acts as a topological boundary resource.

If the same systematic perturbation is repeated over many cycles, the accumulated phase may grow coherently,

$$\Delta_{\mathrm{ord}}^{(N)} = N\Delta_{\mathrm{ord}}. \tag{11}$$

This constructive accumulation is one of the reasons coherent errors can be more dangerous than purely random errors.

Random errors may partially average out, while systematic coherent errors can reinforce themselves.

On the contrary, as we show here, the presence of a Möbius strip topology (Möbius boundary conditions) naturally produces in a clean (geometric) annhilation of one error by each other, instead of the error construction which operates in the trivial topology.

V. MÖBIUS ERROR CANCELLATION

The Möbius boundary condition changes the global phase balance. Because the state returns through a twisted identification, the second contribution to the accumulated phase is effectively compared with a topologically reversed branch. We write the total Möbius phase mismatch as

$$\Delta_M = \frac{1}{2}\left[\int_0^L \epsilon(x)\,dx - \int_0^L \epsilon_M(x)\,dx\right], \tag{12}$$

where $\epsilon_M(x)$ denotes the error field seen on the twisted branch after the Möbius identification.

For a symmetric coherent perturbation,

$$\epsilon_M(x) = \epsilon(x), \tag{13}$$

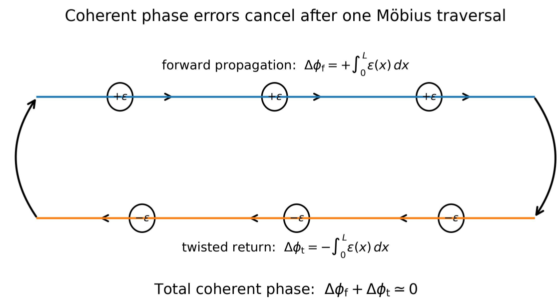

FIG. 2. Coherent phase-error cancellation in a Möbius quantum chain. A phase accumulated along the ordinary branch is compared with the contribution accumulated along the twisted return branch. For symmetric coherent perturbations, the two contributions cancel in the global Möbius boundary condition.

Eq. (12) gives

$$\Delta_M = 0. \tag{14}$$

Thus the coherent error cancels at the level of the global boundary condition. The cancellation is not local. It is a result of comparing the forward branch with its topologically related Möbius return, as a consequence of the non-global orientation of the Möbius manifold.

More generally, if the perturbation is not perfectly symmetric, we may write

$$\epsilon_M(x) = \epsilon(x) + \delta\epsilon(x), \tag{15}$$

and the residual Möbius phase becomes

$$\Delta_M = -\frac{1}{2}\int_0^L \delta\epsilon(x)\,dx. \tag{16}$$

The remaining error is therefore controlled by the asymmetry of the perturbation rather than by its full magnitude. This is the key suppression mechanism.

VI. BOUNDARY CANCELLATION AND QUANTUM GATES

Let U_g be a quantum gate acting locally or sequentially on the chain, and let U_M denote the global Möbius propagation operator. A circuit layer may be represented schematically as

$$U_{\mathrm{layer}} = U_M U_g. \tag{17}$$

The Möbius operation is not a replacement for the logical gate. Rather, it acts as a topological boundary filter that modifies the way coherent errors accumulate.

If a gate suffers from a coherent over-rotation,

$$U_g \longrightarrow e^{-i(\theta+\delta\theta)H}, \tag{18}$$

then, an ordinary repeated circuit accumulates the systematic shift $\delta\theta$.

On the contrary, in a Möbius chain, the topological return can pair the over-rotation with a reversed or twisted (with opposite sign) contribution. The leading residual error is then determined by the mismatch between the two branches,

$$\delta\theta_{\text{eff}} \sim \frac{1}{2}\left(\delta\theta - \delta\theta_M\right). \tag{19}$$

For $\delta\theta_M = \delta\theta$, the leading coherent contribution does cancel.

In realistic implementations, the cancellation will be approximate, and its quality will depend on the degree to which the Möbius branch reproduces the same systematic error with the exact required topological sign or orientation relation. However, and in any case, the topological Möbius mechanism produces an efficient coherent error mitigation effect.

VII. COMPARISON WITH OTHER ERROR-SUPPRESSION MECHANISMS

The Möbius mechanism is distinct from standard stabilizer quantum error correction. It does not require syndrome extraction, measurement feedback, or redundant logical encoding.

It is also distinct from dynamical decoupling, where time-dependent pulse sequences average unwanted interactions. On the contrary, in our topological framework here the error suppression arises from a static global boundary condition.

Our paper here is related in spirit to decoherence-free subspaces, because both approaches use structure to reduce sensitivity to selected perturbations. However, decoherence-free subspaces rely on symmetry in the system-environment coupling and do encode information in protected subspaces.

The Möbius mechanism instead uses a non-orientable boundary identification to compare two topologically related error contributions.

This approach is also related to geometric and topological approaches, but it differs from topological quantum computation based on anyons or braiding.

In the present work, topology enters through boundary identification and global phase cancellation. The Möbius chain may therefore be viewed as a boundary-engineered passive error-suppression layer.

VIII. PHYSICAL INTERPRETATION AND POSSIBLE IMPLEMENTATIONS

The cancellation mechanism can be summarized as follows:

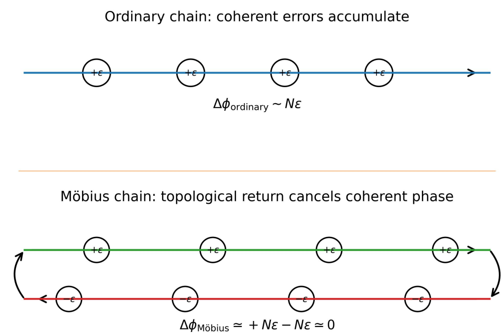

FIG. 3. Comparison between an ordinary chain and a Möbius chain. In an ordinary chain, coherent errors can accumulate constructively. In a Möbius chain, the twisted return branch can cancel the leading coherent contribution, leaving only the asymmetry between the two branches.

(i) In an ordinary (topologically trivial) closed chain, a coherent error accumulated over a full loop returns with the same orientation and can reinforce itself.

(ii) In a Möbius chain instead, the returning amplitude is identified with a twisted copy of the initial amplitude. This twist reverses or compares the contribution of symmetric coherent perturbations. The error is therefore converted from an accumulated global phase into a difference between two topologically related branches of different sign.

(iii) This mechanism is expected to be most effective for slowly varying, systematic, and spatially correlated errors. It is not expected to cancel arbitrary local stochastic noise.

(iv) Therefore, Möbius error cancellation should be regarded as complementary to, not a replacement for, conventional quantum error correction.

(v) Possible physical settings include photonic waveguide arrays with engineered path topology, superconducting circuits with synthetic boundary conditions, cold atoms in programmable optical potentials, trapped-ion chains with controlled phase-space loops, and topological materials supporting non-trivial boundary modes.

(vi) In each case, the essential requirement is not the literal construction of a physical Möbius strip, but the implementation of an effective non-orientable boundary identification in the quantum dynamics.

IX. CONCLUSIONS

We have formulated a Möbius boundary mechanism for coherent quantum error cancellation. This paper extends the Möbius quantum computation program introduced in Paper I by focusing on a concrete application: passive error suppression in a Möbius quantum chain.

A chain with a twisted Möbius identification can force symmetric coherent phase errors to cancel after a complete traversal. The leading residual error is controlled by the asymmetry between the two ordinary and twisted branches.

The central result is the cancellation relation

$$\Delta_M = 0 \tag{20}$$

for symmetric coherent perturbations.

This result shows that global topology can act as a passive error-suppression resource.

This framework can be applied in numerical simulations, circuit realizations, and experimental implementations in superconducting qubits, photonic circuits, trapped ions, cold atoms, and synthetic quantum matter platforms.

The present analysis establishes the theoretical feasibility of the coherent error cancellation induced by M"obius boundary conditions.

The approach here was intentionally conceptual to illustrate the essential principle and does not yet include hardware-dependent noise models, finite-temperature environments, or device-specific imperfections.

These aspects will require dedicated numerical simulations and experimental implementations.

Consequently, the present work should be regarded as a proof of principle demonstrating a new topological mechanism for passive quantum error suppression.

Beyond its immediate application to coherent error suppression, the Möbius framework suggests that non-orientable topological boundary conditions may constitute a broader design principle for quantum information processing.

This perspective opens new directions connecting topology, quantum dynamics, and quantum computation.

[1] J.L. Mac Loughlin, N. G. Sanchez, Möbius Quantum Computation: A New Topological Boundary framework for Quantum Information Processing, Paper I, Researchgate DOI: 10.13140/RG.2.2.33458.36806 (July 2026). https://chalonge-devega.fr/Mobius_Topology_Quantum_Computation.pdf

[2] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000).

[3] D. Deutsch, Quantum theory, the Church–Turing principle and the universal quantum computer, Proc. R. Soc. Lond. A **400**, 97 (1985).

[4] P. W. Shor, Scheme for reducing decoherence in quantum computer memory, Phys. Rev. A **52**, R2493 (1995).

[5] A. M. Steane, Error correcting codes in quantum theory, Phys. Rev. Lett. **77**, 793 (1996).

[6] D. Gottesman, Stabilizer codes and quantum error correction, Ph.D. thesis, California Institute of Technology (1997).

[7] D. A. Lidar and K. B. Whaley, Decoherence-free subspaces and subsystems, in *Irreversible Quantum Dynamics*, edited by F. Benatti and R. Floreanini (Springer, Berlin, 2003).

[8] L. Viola and S. Lloyd, Dynamical suppression of decoherence in two-state quantum systems, Phys. Rev. A **58**, 2733 (1998).

[9] A. Y. Kitaev, Fault-tolerant quantum computation by anyons, Ann. Phys. **303**, 2 (2003).

[10] C. Nayak, S. H. Simon, A. Stern, M. Freedman, and S. Das Sarma, Non-Abelian anyons and topological quantum computation, Rev. Mod. Phys. **80**, 1083 (2008).

[11] B. Simon, Holonomy, the quantum adiabatic theorem, and Berry's phase, Phys. Rev. Lett. **51**, 2167 (1983).

[12] M. V. Berry, Quantal phase factors accompanying adiabatic changes, Proc. R. Soc. Lond. A **392**, 45 (1984).

[13] Y. Aharonov and J. Anandan, Phase change during a cyclic quantum evolution, Phys. Rev. Lett. **58**, 1593 (1987).

[14] P. Zanardi and M. Rasetti, Holonomic quantum computation, Phys. Lett. A **264**, 94 (1999).

[15] B. M. Terhal, Quantum error correction for quantum memories, Rev. Mod. Phys. **87**, 307 (2015).

[16] J. Preskill, Quantum computing in the NISQ era and beyond, Quantum **2**, 79 (2018).

[17] M. Z. Hasan and C. L. Kane, Colloquium: Topological insulators, Rev. Mod. Phys. **82**, 3045 (2010).

[18] X.-L. Qi and S.-C. Zhang, Topological insulators and superconductors, Rev. Mod. Phys. **83**, 1057 (2011).

[19] J. E. Moore, The birth of topological insulators, Nature **464**, 194 (2010).

[20] Z.-Y. Xue and C.-Y. Ding, Recent Advances on Nonadiabatic Geometric Quantum Computation, arXiv quant-ph 2511.07119 (2025).