Magnetized initial conditions for CMB anisotropies

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“The Magnetized Universe”

M. G. (2004)

- Large-scale magnetic fields (typical length-scales > 1 A.U.) $1 A.U. = 1.49\ 10^{13}\ cm$

- First speculations: early forties (Alfven) late forties (Fermi, Fermi&Chandrasekar) on cosmic ray physics $1\,\mu G = 0.1\ nT = 10^{-26}\ GeV^2$

-Today: magnetic fields measured with various techniques

Zeeman splitting of radio transitions

$$\Delta\nu_Z = \frac{e\overline{B}_{\|}}{2\pi m_e}$$

$$\Delta\nu_{Doppler} \simeq \left(\frac{v_{th}}{c}\right)\nu \gg \Delta\nu_{Zeeman} \simeq \frac{e\overline{B}_{\|}}{2\pi m_e}$$

Synchrotron emission

$$\varepsilon(\nu) = 10^{-23}\, n_{er0}\, L\, \xi(\gamma)\, (6.3\times 10^{18})^{(\gamma-1)/2} (B_{\perp})^{(\gamma+1)/2}\, \nu^{(1-\gamma)/2}\ \ erg\ sec^{-1}\ cm^{-2}\ Hz^{-1}$$

Faraday rotation

$$\Delta\,\phi = \frac{f_e}{2}\left(\frac{\omega_p}{\omega}\right)^2 \omega_B\, \Delta z$$

$$\omega_p = \left(\frac{4\pi n_e e^2}{m_e}\right)^{1/2} \qquad \omega_B = \frac{eB}{mc}$$

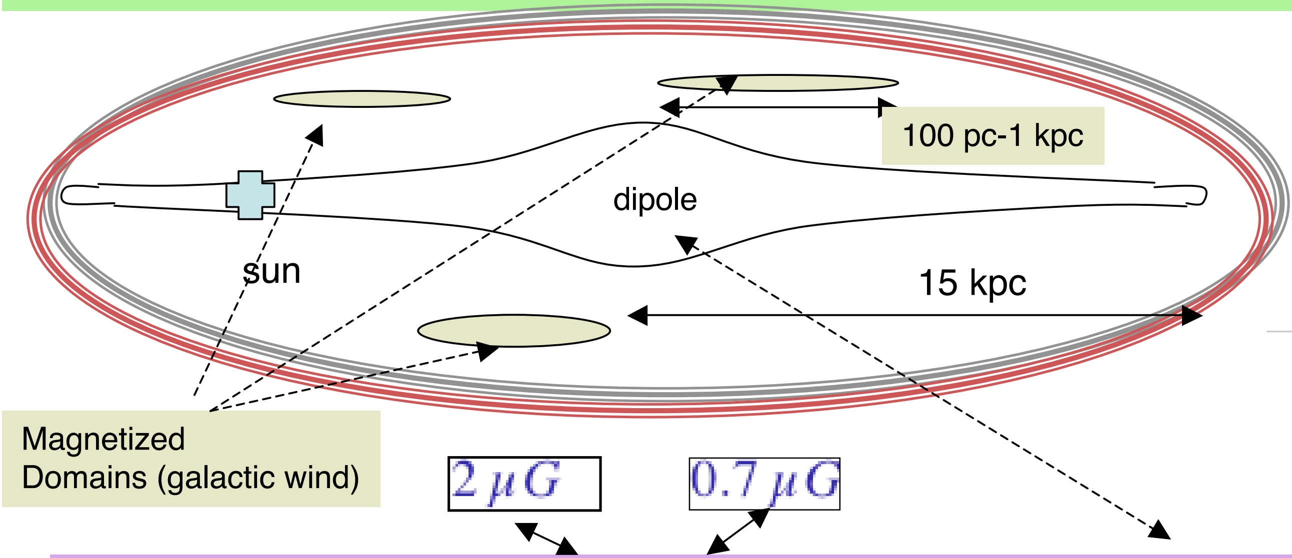
$$\phi = RM\,\lambda^2 + \phi_0$$

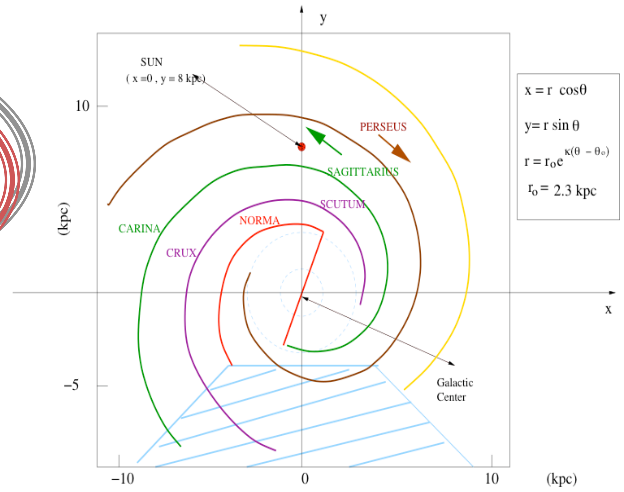
$$RM = \frac{\Delta\phi}{\Delta\lambda^2} = 811.9\int\left(\frac{n_e}{cm^{-3}}\right)\left(\frac{B_{\|}}{\mu G}\right) d\left(\frac{\ell}{kpc}\right)\ \frac{rad}{m^2}$$

$$\langle B_{\|}\rangle = \frac{RM}{DM}$$

$$DM \propto \int n_e d\ell$$

Magnetized galaxies, clusters, and superclusters





Milky way: toroidal field south/north hemisphere; dipoles (center) $10\ rad/m^2$

Local Group: Andromeda, Magellanic Clouds,… $2 - 7\,\mu G$ (elliptical galaxies: shorter scale)

Abell Clusters (like COMA): magnetic fields inside cluster (VLA+ROSAT) [Faraday RM]
Typical RM: $100\ rad/m^2$ $B \sim 0.5\,\mu G = 500\,nG$ $L \sim 50 - 100\,kpc$

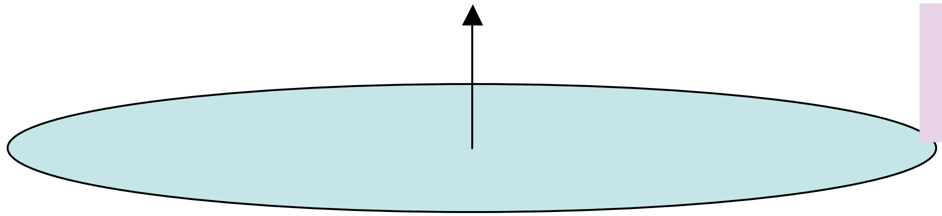
Superclusters: Local Supercluster (Local Group + Virgo Cluster $1.5\,\mu G$
Coma Supercluster (COMA+ Abell 1367) $0.5\,\mu G$?

If true: important for UHECR…

Dynamo and compressional amplification

Galaxy:

$$\lambda_D \simeq \sqrt{\frac{T}{8\pi n_e e^2}}$$



Charged fluid (globally neutral)

Typical rotation period: $P \sim 3 \times 10^8\ yrs$ age $T \sim 10^{10}\ yrs$

Dynamo instability:

$$\alpha = -\frac{\tau_0}{3}\langle \vec{v} \cdot \vec{\nabla} \times \vec{v} \rangle \quad \sim 9.1 \times 10^6\ \frac{cm}{sec}$$

$$\frac{1}{4\pi\sigma} = 10^{25} \frac{cm^2}{sec}$$

$$\boxed{\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B}) + \frac{1}{4\pi\sigma}\nabla^2 \vec{B}}$$

Dynamo term Diffusivity term

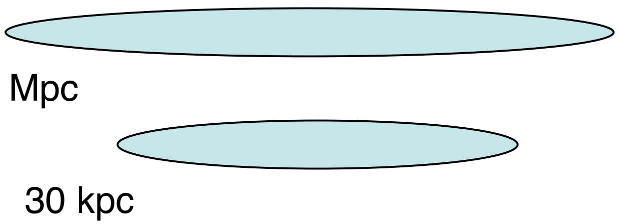
$$\frac{\partial < \vec{B} >}{\partial t} = \alpha \vec{\nabla} \times < \vec{B} > + \frac{1}{\sigma}\nabla^2 < \vec{B} >$$

$$\boxed{1/k \sim L > kpc}$$

Maximal and optimistic amplification: $e^{\Gamma t} \sim e^{T/P} \sim e^N \sim 10^{13}$

$B_i \sim 10^{-19}\ G$ Over L = 30 kpc

Compressional amplification: $B_b = \left(\frac{\rho_b}{\rho_a}\right)^{2/3} B_a$



$$B_i \geq 10^{-23}\ G \quad over \quad L \sim Mpc$$

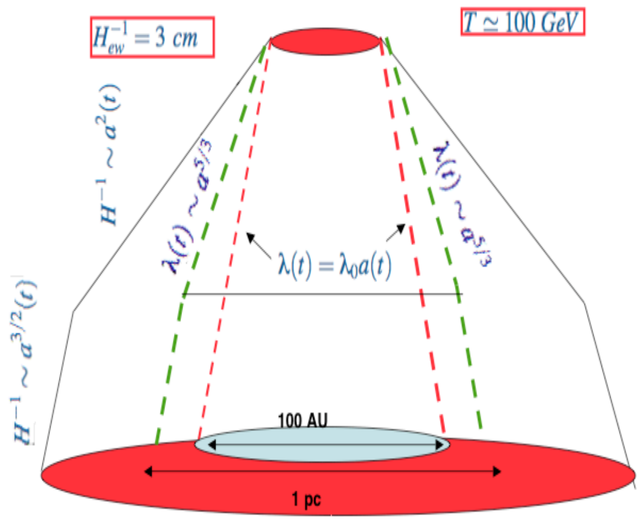
Primordial magnetogenesis

$B_{seed} > 10^{-23} G$ → Too optimistic

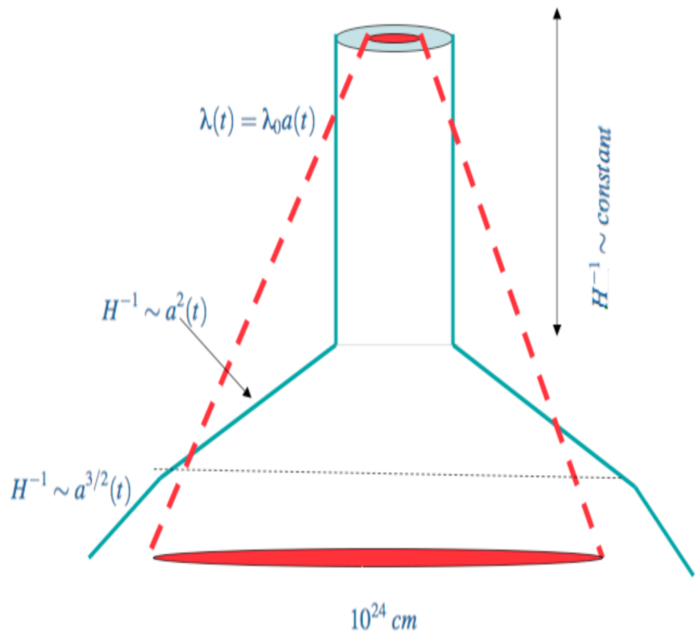
$\boxed{B_{seed} > 10^{-18}\ G}$ → More realistic [flux not exactly conserved, small-scale fields can grow large and swamp dynamo action]

effective e-folds 30->25

CAUSAL mechanisms



``Inflationary'' mechanisms



Plasma physics in FRW space-times

$$ds^2 = a^2(\eta)[d\eta^2 - d\vec{x}^2]$$

Typical scales @ T = 0.3 eV

$$\ell_\gamma \simeq 10^4\,(1+z)^{-2}\,(\Omega_b\,h_0)^{-1}\ Mpc$$

$$\lambda_D = \sqrt{\frac{T_{ei}}{8\pi e^2 n_0}} \simeq 10\left(\frac{n_0}{10^3\ cm^{-3}}\right)^{-1/2}\left(\frac{T_{ei}}{0.3\ eV}\right)^{1/2}\ cm.$$

$$\ell_e \simeq 5.7\times 10^7 cm$$

$$\omega_{pe} \sim MHz \qquad \omega_{pi} \sim kHz$$

$$\omega_{Be} = \frac{eB_0}{m_e c} \simeq 18.08\left(\frac{B_0}{10^{-3}\ G}\right)\ kHz, \qquad \omega_{Bi} = \frac{eB_0}{m_i c} \simeq 9.66\left(\frac{B_0}{10^{-3}\ G}\right)\ Hz$$

If $\omega > \omega_p \qquad \omega > \omega_B$

Two-fluid equations (or even Vlasov-Landau equations)

$\omega > \omega_p \qquad \omega > \omega_B$

Charge continuity

$$n_e' + 3w_e H n_e + (w_e+1)\vec{\nabla}\cdot(n_e\vec{v}_e) = 0,$$
$$n_i' + 3w_i H n_i + (w_i+1)\vec{\nabla}\cdot(n_i\vec{v}_i) = 0,$$

Momentum conservation

$$\rho_e[\vec{v}_e' + H\vec{v}_e + (\vec{v}_e\cdot\vec{\nabla})\vec{v}_e] = -n_e e\left(\vec{E} + \frac{\vec{v}_e}{c}\times\vec{B}\right),$$
$$\rho_i[\vec{v}_i' + H\vec{v}_i + (\vec{v}_i\cdot\vec{\nabla})\vec{v}_i] = n_i e\left(\vec{E} + \frac{\vec{v}_i}{c}\times\vec{B}\right),$$

Maxwell's fields

$$\vec{\nabla}\cdot\vec{E} = 4\pi e(n_i - n_e),$$
$$\vec{\nabla}\cdot\vec{B} = 0, \qquad \vec{\nabla}\times\vec{E} = -\frac{1}{c}\vec{B}'$$
$$\vec{\nabla}\times\vec{B} = \frac{1}{c}\vec{E}' + \frac{4\pi e}{c}(n_i\vec{v}_i - n_e\vec{v}_e)$$

IF $\omega < \omega_p \qquad \omega < \omega_B$ $\longrightarrow$ MHD

$$\vec{J} = e(n_i\vec{v}_i - n_e\vec{v}_e)$$

$$\vec{v} = \frac{m_i\vec{v}_i + m_e\vec{v}_e}{m_i + m_e}$$

MHD description(s)

$$\vec{B} = a^2\vec{b}, \quad \vec{E} = a^2\vec{e}, \quad \vec{J} = a^3\vec{j}, \quad \sigma = \sigma_c a,$$

$$\frac{\partial \vec{B}}{\partial \eta} + \vec{\nabla}\times\vec{E} = 0$$

$$\vec{\nabla}\cdot\vec{E} = 4\pi\rho_q, \qquad \vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{B} = 4\pi\vec{J} + \cancel{\frac{\partial \vec{E}}{\partial \eta}},$$

$$\vec{J} = \sigma(\vec{E} + \vec{v}\times\vec{B}),$$

MHD: effective (one-fluid) plasma description

$$\vec{E} + \vec{v}\times\vec{B} = \underbrace{\frac{1}{\sigma}\vec{J}}_{\text{Resistivity term}} + \cancel{\frac{1}{en_q}(\underbrace{\vec{J}\times\vec{B}}_{\text{Hall term}} - \underbrace{\vec{\nabla}P_e}_{\text{Thermoelectric term}})}.$$

$$\rho_q \simeq 0$$

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\cdot\vec{J} = 0$$

$$\vec{J} = \frac{1}{4\pi}\vec{\nabla}\times\vec{B} \qquad \vec{E} = \frac{1}{4\pi\sigma}\vec{\nabla}\times\vec{B} - \vec{v}\times\vec{B}$$

$$[(\overline{p}+\overline{\rho})\vec{v}]' + \vec{v}\cdot\vec{\nabla}[(\overline{p}+\overline{\rho})\vec{v}] = -\vec{\nabla}\left[\overline{p} + \frac{|\vec{B}|^2}{8\pi}\right] + \frac{[\vec{B}\cdot\vec{\nabla}]\vec{B}}{4\pi} + (\overline{p}+\overline{\rho})\nu\nabla^2\vec{v}$$

Superconducting limit

Resistive limit

(ALFVEN THEOREMS)

$$\frac{d}{dt}\int_\Sigma \vec{B}\cdot d\vec{\Sigma} = 0$$

$$\frac{d}{dt}\int_\Sigma \vec{B}\cdot d\vec{\Sigma} = -\frac{1}{4\pi\sigma}\int_\Sigma \vec{\nabla}\times(\vec{\nabla}\times\vec{B})\cdot d\vec{\Sigma} + O\left(\frac{1}{\sigma^2}\right)$$

$$\frac{d}{dt}\int_V d^3x\vec{A}\cdot\vec{B} = 0$$

$$\frac{d}{dt}\int_V d^3x\vec{A}\cdot\vec{B} = -\frac{1}{4\pi\sigma}\int_V d^3x\vec{B}\cdot\vec{\nabla}\times\vec{B} + O\left(\frac{1}{\sigma^2}\right)$$

Magnetic fields and CMB physics

Uniform magnetic field approximation
[magnetic field along a specific axis].
Simplified estimates
[not so realistic in diverse cases]

FOREGROUNDS & B FIELDS

-- distorsion of the Planckian spectrum
-- shift of the polarization plane of CMB (Faraday rotation)
-- effects on primary anisotropies

Intermediate situation:
uniform magnetic field with
inhomogeneous fluctuations

Fully inhomogeneous
magnetic fields : more
realistic [mathematically
less tractable]

"Zeldovich approximation''

Zeldovich ``approximation" : homogeneous field with (weak) breaking of spatial isotropy

Y. Zeldovich
Sov. Phys. JETP 21
656 (1965)

Magnetic fields weakly breaks spatial isotropy: Bianchi-type I paradigm

(generalizations MG PRD 2000)

$$ds^2 = dt^2 - a^2(t)dx^2 - b^2(t)[dy^2 + dz^2]$$

$$T_x(t) = T_1 \frac{a_1}{a} = T_1 e^{-\int H(t)dt},$$
$$T_y(t) = T_1 \frac{b_1}{b} = T_1 e^{-\int F(t)dt}$$

Electromagnetic radiation propagating along x and y will have a different temperature

$$\boxed{\frac{\Delta T}{T} \sim \int [H(t) - F(t)]\, dt = \frac{1}{2} \int r(t)\, d\log t}$$

$$r(t) = \frac{3[H(t) - F(t)]}{[H(t) + 2F(t)]}$$

Radiation-dominated case

Shear parameter is conserved and proportional to the magnetic energy density

From "Zeldovich" approximation"

$$B_0 \leq 2.23 \times 10^{-9} \ \ Gauss$$

CMB and UNIFORM magnetic fields

If magnetic field is uniform: partial breaking of spatial isotropy. Angular power spectrum DOES depend on m!

Ordinary case

$\langle a_{\ell m} a^*_{\ell' m'} \rangle = C_\ell \delta_{\ell\ell'} \delta_{mm'},$

$$C(\vartheta) = \langle \Delta_I(\hat{n}_2, \tau_0) \Delta_I(\hat{n}_1, \tau_0) \rangle \equiv \frac{1}{4\pi} \sum_\ell (2\ell+1) C_\ell P_\ell(\hat{n}_1 \cdot \hat{n}_2).$$

Uniform field case

$$\langle \Delta_I(\hat{n}_1, \tau_0) \Delta_I(\hat{n}_2, \tau_0) \rangle = \sum_{\ell,\, m} C_\ell(m) Y_{\ell,\, m}(\hat{n}_1) Y^*_{\ell,\, m}(\hat{n}_2)$$

$$+ \sum_{\ell,\, m} D_\ell(m) [Y_{\ell+1,m}(\hat{n}_1) Y^*_{\ell-1,m}(\hat{n}_2) + Y_{\ell-1,m}(\hat{n}_1) Y^*_{\ell+1,m}(\hat{n}_2)]$$

$B_0 \leq 2 \times 10^{-9}\ Gauss$

$a_{\ell+1,m}$ $\quad a_{\ell-1,m}$ $\quad$ correlation

$\vec{B} = \vec{B}_0 + \delta\vec{B}, \quad |\delta\vec{B}| < |\vec{B}_0|$

G. Chen, et al APJ (2004)

Faraday rotation by a UNIFORM magnetic field

Kosowsky& Loeb ApJ (97)
MG PRD (97), MG(PRD,2005)

From two-fluid description:

$$\Delta\phi = f_e \frac{e}{2m_e}\left(\frac{\omega_p}{\omega}\right)^2 (\vec{B}\cdot\hat{z})\delta z$$

$$\langle(\Delta\phi)^2\rangle^{1/2} \simeq 1.6^0 \left(\frac{B}{B_c}\right)\left(\frac{\omega_M}{\omega}\right)^2$$

$$B_c \sim 10^{-3}\ G$$

$$\omega_F = \frac{d\phi}{d\eta} = \frac{e^3\, n_e\, x_e \vec{B}\cdot\vec{q}}{8\pi^2\, m_e^2\, \nu^2}\frac{a}{a_0}$$

Axial symmetry around k, e.g. B II k (!)

$$\Delta_Q' + (ik\mu + \tau')\Delta_Q - 2\omega_F\Delta_U = \frac{\tau'}{2}[1 - P_2(\mu)]S_Q$$
$$\Delta_U' + (ik\mu + \tau')\Delta_U + 2\omega_F\Delta_Q = 0$$

$$\tau' = x_e\, n_e\, \sigma_T \frac{a}{a_0}$$

$$S_Q = \Delta_{I,2} + \Delta_{Q,0} + \Delta_{Q,2}$$

Visibility function

$$(\Delta_Q \pm i\Delta_U) = \frac{3}{4}(1-\mu^2)\int_0^{\eta_0} d\eta e^{-ik\mu\Delta\eta} K(\eta)\, S_Q(\eta)\, e^{\mp 2i\omega_F\Delta\eta}$$

E-modes are ROTATED into B-modes !

$$(\Delta_Q \pm i\Delta_U)(\hat{n}) = \sum_{\ell m} a_{\pm 2,\ell m}\ {}_{\pm 2}Y_{\ell m}(\hat{n})$$

$$a_{E,\ell m} = -\frac{1}{2}(a_{2,\ell m} + a_{-2,\ell m})$$
$$a_{B,\ell m} = \frac{i}{2}(a_{2,\ell m} - a_{-2,\ell m}).$$

$$E(\hat{n}) = \sum_{lm} a_{E,\ell m} Y_{lm}(\hat{n}), \qquad B(\hat{n}) = \sum_{lm} a_{B,\ell m} Y_{lm}(\hat{n}).$$

Limits from Faraday rotation

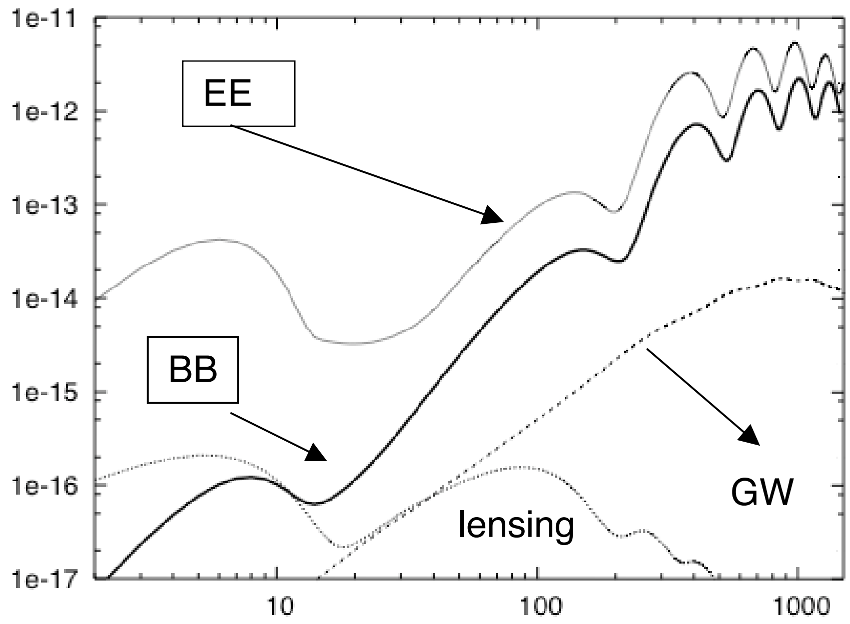
TB correlations smaller than initial TE correlations by a factor $\omega_F < 1$ $\rightarrow TB \simeq \omega_F\, TE$

TE and EE correlations are a factor ω_F^2 smaller than their values in the absence of magnetic field

From WMAP TE correlations $B_0 < 10^{-8}\ Gauss, \quad @\ 30\ GHz$

Other B-modes: GW and weak lensing (frequency independent)

$$\frac{\ell(\ell+1)C_\ell^{BB}}{\omega_F^2}$$



Scoccola et al (PRD 2004)

$r \simeq 10^{-2}$

Yet other B-modes can be obtained from magnetized birefringence (PNGB)

Dispersion relations in a cold plasma
With magnetic field and PNGB [MG (PRD) 2004]

Topological classification of inhomogeneous magnetic fields

MG PRD (1998)

No matter how inhomogeneous the field is, ALFVEN theorems hold!

$$\vec{F}_B = \vec{J}\times\vec{B} \simeq \frac{1}{4\pi}(\vec{\nabla}\times\vec{B})\times\vec{B}$$

Lorentz force in MHD

$$\vec{B}\cdot\vec{\nabla}\times\vec{B}$$

Magnetic gyrotropy

$$\langle[(\vec{\nabla}\times\vec{B})\times\vec{B}]^2\rangle = 0$$

$$\langle[\vec{B}\cdot\vec{\nabla}\times\vec{B}]^2\rangle \neq 0$$

Maximally helical field
(also approximately FORCE FREE)

$$\langle[(\vec{\nabla}\times\vec{B})\times\vec{B}]^2\rangle \neq 0$$

$$\langle[\vec{B}\cdot\vec{\nabla}\times\vec{B}]^2\rangle = 0$$

Minimally helical
(magnetic helicity is approximately ZERO)

$$\langle B_i^*(\vec{p})B_j(\vec{k})\rangle = i\,(2\pi)^3\,\delta^{(3)}(\vec{k}-\vec{p})\frac{k_\ell}{k}\,\varepsilon_{ij\ell}\,P_G(k)$$

$$P_G(k) = \tilde{A}k^m$$

Hypermagnetic fields
[m. shaposhnikov, M.G PRL]

Pogosian et al [PRD]

$$\langle B_i^*(\vec{p})B_j(\vec{k})\rangle = (2\pi)^3\delta^{(3)}(\vec{k}-\vec{p})P_{ij}P_B(k)$$

$$P_B(k) = A\,k^n$$

$$P_{ij}(k) = \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)$$

$$\boxed{\delta\tau_{\mu\nu} = \delta\tau_{\mu\nu}^{(S)} + \delta\tau_{\mu\nu}^{(V)} + \delta\tau_{\mu\nu}^{(T)}}$$

Decompose the electromagnetic energy-momentum
Tensor with respect to spatial rotations

Metric Fluctuations

$$\delta\tau_{\mu\nu} = \delta\tau^{(S)}_{\mu\nu} + \delta\tau^{(V)}_{\mu\nu} + \delta\tau^{(T)}_{\mu\nu}$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu}(\eta) + \boxed{\delta g_{\mu\nu}(\vec{x},\eta)}$$

Overall ten degrees of freedom.

$$\boxed{\delta g_{\mu\nu} = \delta g^{(S)}_{\mu\nu} + \delta g^{(V)}_{\mu\nu} + \delta g^{(T)}_{\mu\nu}}$$

Four scalars

Two divergenceless vectors (four d.f.)

Rank-two tensor in 3-dimensions (divergenceless and traceless)

$$\begin{matrix} 2\phi & -\partial_i B \\ -\partial_i B & 2(\psi\delta_{ij} - \partial_i\partial_j E) \end{matrix}$$

4

$$\begin{matrix} 0 & -Q_i \\ -Q_i & \partial_i W_j + \partial_j W_i \end{matrix}$$

4

$$\partial_i W^i = \partial_i Q^i = 0$$

$$\begin{matrix} 0 & 0 \\ 0 & -h_{ij} \end{matrix}$$

2

$$\partial_i h^i_j = h^i_i = 0$$

To fix completely coordinate system: four conditions. Gauge two scalars (E=0, B=0, for instance) and one vector (for instance W).

$$\Phi = \phi + \frac{1}{a}[(B - E')a]'$$

$$\Psi = \psi - \frac{a'}{a}(B - E')$$

2 gauge-invariant scalars

$$Q_i + W_i'$$

One gauge-invariant vector

Magnetized tensor and vector modes

TENSORS (gauge-invariant)

$$\left(\frac{\Delta T}{T}\right)_t = -\frac{1}{2}\int_{\eta_i}^{\eta_f} h'_{ij} n^i n^j d\eta.$$

Tensor Sachs-Wolfe

$$h_i^{j\prime\prime} + 2H h_i^{j\prime} - \nabla^2 h_i^j = -16\pi G a^2 \delta_t \tau_i^j$$

VECTORS

$$Q_i \to \tilde{Q}_i = Q_i - \zeta_i'$$

$$W_i \to \tilde{W}_i = W_i + \zeta_i$$

Choose gauge

$$\tilde{W}_i = 0$$

$$\nabla^2 \vec{Q} = -16\pi G a^2 (p+\rho)\vec{V} + \frac{16\pi G}{\sigma a^2}\vec{F}_B(\vec{x})$$

From (0i) component of perturbed Einstein equations

$$\nabla^2(\vec{Q}' + 2H\vec{Q}) = \frac{16\pi G}{a^2}\vec{F}_B(\vec{x})$$

From (ij) equation

$$\vec{V}' + \left[4H + \frac{p' + \rho'}{p+\rho}\right]\vec{V} + \frac{\vec{F}_B(\vec{x})}{a^4(p+\rho)} = 0$$

From covariant conservation equation

$$\vec{F}_B = \frac{1}{4\pi}(\vec{\nabla}\times\vec{B})\times\vec{B}$$

$$\vec{V}(\vec{x},\eta) = -\frac{1}{a^4(\eta)(p+\rho)}\int \vec{F}_B(\vec{x},\eta)d\eta - \frac{\vec{C}(\vec{x})}{16\pi G a^4(\eta)(p+\rho)}$$

$$\nabla^2\vec{Q} = \frac{16\pi G}{a^2(\eta)}\int \vec{F}_B(\vec{x},\eta)d\eta + \frac{\vec{C}(\vec{x})}{a^2(\eta)}$$

Decaying Mode

Vector Sachs-Wolfe

$$\left(\frac{\Delta T}{T}\right)_v = [-\vec{V}\cdot\vec{n}]_{\eta_i}^{\eta_f} + \frac{1}{2}\int_{\eta_i}^{\eta_f}(\partial_i Q_j + \partial_j Q_i)n^i n^j d\eta.$$

Convolutions

M. Gasperini, M. G. G. Veneziano, PRD,PRL 1995: conjecture similar constraint for fully inhomogeneous fields with ``flat” energy spectrum. $P(k) = A\, k^n$ $\langle B_i(\vec{k}) B_j(\vec{p}) \rangle = P_{ij}(k) P(k) \delta^{(3)}(\vec{k} - \vec{p})$

Magnetic field smoothed over a comoving scale L ~ 1 Mpc $B_L \lesssim 10^{-9}\ Gauss$ n ~ -3

$$\langle B_i(\mathbf{k}) B_j^*(\mathbf{k}') \rangle = \frac{(2\pi)^{n+8}}{2} \frac{B_L^2}{\Gamma\left(\frac{n+3}{2}\right)} P_{ij} \frac{k^n}{k_L^{n+3}} \delta(\mathbf{k} - \mathbf{k}'), \qquad k < k_D,$$

Correlator of Lorentz Force (convolution)

Consider, for instance, VECTOR modes in the absence of magnetic gyrotropy.

$$\langle F_i^{(B)}(\mathbf{k}) F_j^{(B)*}(\mathbf{k}') \rangle \equiv P_{ij} |F^{(B)}(k)|^2 \delta(\mathbf{k} - \mathbf{k}')$$

$$k^3 |F^{(B)}(k)|^2 \simeq \frac{1}{8\pi(2n+3)} \left[\frac{(2\pi)^{n+5} B_L^2}{2\Gamma\left(\frac{n+3}{2}\right) \rho_\gamma} \right]^2 \left[\left(\frac{k_D}{k_L}\right)^{2n+3} \left(\frac{k}{k_L}\right)^3 + \frac{n}{n+3} \left(\frac{k}{k_L}\right)^{2n+3} \right],$$

If n > -3/2 dissipation scale dominates. If n <-3/2 large-scale properties of the field dominate.

$$L_D \sim 10^{-3} \left(\frac{\overline{B}}{10^{-9}\ Gauss} \right) h_0^{-1/2}\ Mpc$$

Smaller than Silk scale $k_D \simeq 10 - 20\, Mpc^{-1}$

Spectra of vectors and tensors

For vectors TT, BB, TE correlations have been estimated (both analytically and numerically)
BB correlation larger than EE correlation, furthermore for large multipoles:

$$C_\ell^{BB} \propto \ell^2, \qquad 100 < \ell < 500$$

For tensors, BB correlation slightly smaller than EE correlation, furthermore, for large multipoles

$$C_\ell^{BB} \propto \ell, \qquad 100 < \ell < 500$$

Most interesting case n ~ -3 (large-scale effect of magnetic field is dominant).

Mack et al (PRD 2002) -> (Lewis PRD 2004).

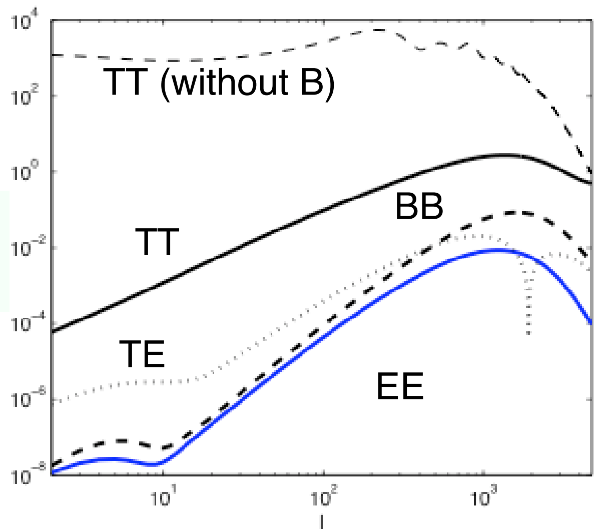
$$B_L \leq 7 \times 10^{-9} \;\; Gauss \qquad n = -2.9$$

$$B_L \leq 10^{-10} \;\; Gauss \qquad n = 0$$

$$B_L \leq 2 \times 10^{-13} \;\; Gauss \qquad n = 2$$

n=-2.9

$$\frac{\ell(\ell+1)}{2\pi\,\mu K^2} C_\ell$$



Magnetized scalar modes

Most complicated… M.G. PRD (2004)

(conformally Newtonian gauge)

$$\nabla^2\psi - 3H(H\phi+\psi') = 4\pi G a^2[\delta T_0^0 + \delta\tau_0^0]$$

Hamiltonian constraint

$$-\partial^i(H\phi+\psi') = 4\pi G a^2(\delta T_0^i + \delta\tau_0^i),$$

Momentum constraint

$$\left[\psi'' + H(2\psi'+\phi') + (2H'+H^2)\phi + \frac{1}{2}\nabla^2(\phi-\psi)\right]\delta_i^j - \frac{1}{2}\partial_i\partial^j(\phi-\psi) = -4\pi G a^2[\delta T_i^j + \delta\tau_i^j],$$

$$\delta\tau_i^j = \frac{1}{4\pi a^4}\left[E_i E^j + B_i B^j - \frac{1}{2}(\vec{B}^2+\vec{E}^2)\delta_i^j\right],$$

What plays the role here is the SCALAR part of the Lorentz force $\vec{\nabla}\cdot\vec{F}_B = \vec{\nabla}\cdot[(\vec{\nabla}\times\vec{B})\times\vec{B}]$

And the ENERGY density $\vec{B}^2$

For instance:

$$-3H(H\phi+\psi') - k^2\psi = \frac{3}{2}H^2[(R_\nu\delta_\nu + (1-R_\nu)\delta_\gamma) + \Omega_B(k) + \Omega_b\delta_b + \Omega_c\delta_c],$$

$$\Omega_B(k,\eta) = \frac{\rho_B}{\rho} = \frac{1}{8\pi\rho a^4}\int d^3p\, B_i(|\vec{p}-\vec{k}|)B^i(p).$$

$$\boxed{\theta = \partial_i v^i}$$

$$\theta_b' = -H\theta_b - c_s^2\nabla^2\delta_b - \nabla^2\phi + \frac{4}{3}\frac{\Omega_\gamma}{\Omega_b}a n_e x_e \sigma_T(\theta_\gamma-\theta_b) + \frac{\vec{\nabla}\cdot[\vec{J}\times\vec{B}]}{a^4\rho_b}$$

Magnetized adiabatic and non-adiabatic modes

M.G. PRD 2004

- Solve everything consistently for PHOTONS + NEUTRINOS + BARYONS +CDM (boring but doable)

-Magnetic fields modify qualitatively and quantitatively the nature of the behaviour of the various modes

- MOST GENERAL solution includes FIVE MODES

ONE Magnetized adiabatic mode

$$\varsigma = \frac{T^3}{n_{cdm}}$$

$$S = \frac{\delta\varsigma}{\varsigma} = \frac{3}{4}\delta_r - \delta_{cdm} = 0$$

$$\delta_\gamma \simeq \delta_\nu \simeq \frac{4}{3}\delta_{cdm} \simeq \frac{4}{3}\delta_b$$

FOUR Magnetized non-adiabatic modes

Baryon isocurvature mode
CDM isocurvature mode
Neutrino isocurvature velocity mode
Neutrino isocurvature density mode

[Some of isocurvature modes are singular on the longitudinal gauge: go to synchronous gauge]

Remarks:

-Since neutrinos free stream (unlike photons) we treat them through an ``improved” fluid system where the quadrupole and octupole moments of the neutrino phase space distribution are dynamical.

-The five magnetized solution define the correct initial conditions to be imposed on the lowest multipoles of the Boltzmann hierachies.

Example/1: explicit expression for adiabatic mode

$$\bar{\delta}_b = \bar{\delta}_c = -\frac{3}{2}\phi_0 - \frac{(525+188R_\nu+16R_\nu^2)}{60\,(25+2R_\nu)}\phi_0 k^2\eta^2,$$

$$\bar{\delta}_\gamma = \bar{\delta}_\nu = -2\phi_0 - \frac{(525+188R_\nu+16R_\nu^2)}{45\,(25+2R_\nu)}\phi_0 k^2\eta^2,$$

$$\bar{\phi} = \phi_0 - \frac{(75+14\,R_\nu - 8\,R_\nu^2)}{90\,(25+2R_\nu)}\phi_0 k^2\eta^2,$$

$$\bar{\psi} = \left(1+\frac{2}{5}R_\nu\right)\phi_0 - \frac{(75+79R_\nu+8R_\nu^2)}{90\,(25+2R_\nu)}\phi_0 k^2\eta^2,$$

$$\bar{\theta}_\nu = \frac{\phi_0}{2}k^2\eta - \frac{(65+16R_\nu)}{36\,(25+2R_\nu)}\phi_0 k^4\eta^3,$$

$$\bar{\theta}_b = \frac{\phi_0}{2}k^2\eta - \frac{(75+14\,R_\nu-8\,R_\nu^2)}{360\,(25+2R_\nu)}\phi_0 k^4\eta^3,$$

$$\bar{\theta}_c = \frac{\phi_0}{2}k^2\eta - \frac{(75+14\,R_\nu-8\,R_\nu^2)}{360\,(25+2R_\nu)}\phi_0 k^4\eta^3,$$

$$\bar{\theta}_\gamma = \frac{\phi_0}{2}k^2\eta - \frac{(25+8R_\nu)}{20\,(25+2R_\nu)}\phi_0 k^2\eta^2,$$

$$\bar{\sigma}_\nu = \frac{\phi_0}{15}k^2\eta^2 - \frac{(65+16R_\nu)}{540\,(25+2R_\nu)}\phi_0 k^4\eta^4,$$

$$\delta_{b,c} = \bar{\delta}_{b,c} - \frac{3}{4}\Omega_B - \left[\frac{69-61R-8R_\nu^2}{60\,(25+2R_\nu)}\right]\Omega_B k^2\eta^2,$$

$$\delta_\gamma = \bar{\delta}_\gamma - \Omega_B + \left[\frac{237+152R_\nu+16R_\nu^2}{90\,(25+2R_\nu)}\right]\Omega_B k^2\eta^2,$$

$$\delta_\nu = \bar{\delta}_\nu - \Omega_B - \left[\frac{375-207R_\nu-152R_\nu^2-16R_\nu^3}{90R_\nu\,(25+2R_\nu)}\right]\Omega_B k^2\eta^2,$$

$$\phi = \bar{\phi} - \left[\frac{6-8R_\nu+2R_\nu^2}{45\,(25+2R_\nu)}\Omega_B\right]k^2\eta^2,$$

$$\psi = \bar{\psi} - \left[\frac{69-61R_\nu-8R_\nu^2}{180\,(25+2R_\nu)}\Omega_B\right]k^2\eta^2,$$

$$\theta_\gamma = \bar{\theta}_\gamma - \frac{\Omega_B}{4}k^2\eta + \left[\frac{(7+8R_\nu)}{40\,(25+2R)}\Omega_B\right]k^4\eta^3,$$

$$\theta_\nu = \bar{\theta}_\nu - \frac{\Omega_B}{4}\frac{R_\gamma}{R_\nu}k^2\eta - \left[\frac{45-29R_\nu-16R_\nu^2}{72R_\nu\,(25+2R_\nu)}\Omega_B\right]k^4\eta^3,$$

$$\theta_b = \theta_c = \bar{\theta}_b - \left[\frac{6-8R_\nu+2R_\nu^2}{180\,(25+2R_\nu)}\Omega_B\right]k^4\eta^3,$$

- Difference of two longitudinal fluctuations determined by magneticv energy density

-Adiabatiticity condition enforced to lowest order in $k\eta$ but it is violated to next order, e.g. $|k\eta|^2$

-This is the result in the force free case; Lorentz force can be included but the expression is more cumbersome

-Corrections to adiabaticity are of order $\Omega_B(k)k^2\eta^2$ [small outside the horizon]

Three regimes emerge naturally: quasi-adiabatic regime $\Omega_B(k) \leq \phi_0(k)$ $\Omega_B(k) \ll \phi_0(k)$

isocurvature regime $\Omega_B(k) > \phi_0(k)$ Adiabatic regime

Example/2: magnetized baryon (CDM) isocurvature mode

MG PRD (2004)

$$h \simeq (-4\overline{\Omega}_b\eta + 6\overline{\Omega}_b\eta^2),$$
$$\xi \simeq \frac{2}{3}\overline{\Omega}_b\eta - \overline{\Omega}_b\eta^2,$$

$$\Omega_{b,c} = \overline{\Omega}_{b,c}\frac{a(\eta)}{a(\eta)+1},$$
$$\Omega_B = \overline{\Omega}_B\frac{1}{a(\eta)+1},$$

$$\delta_\gamma \simeq \left(-\frac{8}{3}\overline{\Omega}_b\eta + 4\overline{\Omega}_b\eta^2\right), -\overline{\Omega}_B(1-\eta+\eta^3),$$
$$\delta_b = (1 - 2\overline{\Omega}_b + 3\overline{\Omega}_b\eta^2),$$
$$\delta_\nu \simeq \left(-\frac{8}{3}\overline{\Omega}_b\eta + 4\overline{\Omega}_b\eta^2\right),$$
$$\delta_c \simeq 2\overline{\Omega}_b + 3\overline{\Omega}_b\eta^2,$$
$$\theta_\gamma \simeq -\frac{1}{3}\overline{\Omega}_b k^2\eta^2 - \frac{k^2}{16}\overline{\Omega}_B(4\eta - 2\eta^2 + \eta^4),$$
$$\theta_\nu \simeq -\frac{1}{3}\overline{\Omega}_b k^2\eta^2 - \frac{k^2}{16}\overline{\Omega}_B(4\eta - 2\eta^2 + \eta^4)\frac{R_\gamma}{R_\nu},$$
$$\theta_c = 0,$$
$$\sigma_\nu \simeq -\frac{2}{3}\frac{\overline{\Omega}_b}{2R_\nu + 15}k^2\eta^3 - \frac{\overline{\Omega}_B}{4R_\nu}(1-\eta+\eta^3),$$

Recall that modes based on anisotropic stresses (like neutrino velocity and density modes are divergent in the longitudinal gauge but completely finite in the synchronous gauge.

Modifications of tight coupling approximation

Scalar spectra?

Surprisingly enough: scalar modes less studied!

Presumption: scalar modes do not give stronger constraints since radiation pressure prevents the induced density fluctuations from growing effectively before recombination and the compressional modes are erased up to the Silk scale.

No systematic attempt has been made to constrain various scalar modes

Abelian-Higgs model

MG &
M. Shaposhnikov
(PRD 2000)

$$S = \int d^4x\sqrt{-g}\left[(D_\mu\phi)^* D^\mu\phi - m^2\phi^*\phi - \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta}\right] \qquad D_\mu = \partial_\mu - ieA_\mu$$

$$S = \int d^3x d\eta\left[\eta^{\mu\nu}D_\mu\Phi^* D_\nu\Phi + \left(\frac{a''}{a} - m^2a^2\right)\Phi^*\Phi - \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta}\right], \qquad \Phi = a\phi$$

Evolution of Higgs field IS NOT conformally invariant

$$\boxed{\Phi'' - \nabla^2\Phi - \frac{a''}{a}\Phi + m^2a^2\Phi = 0.}$$

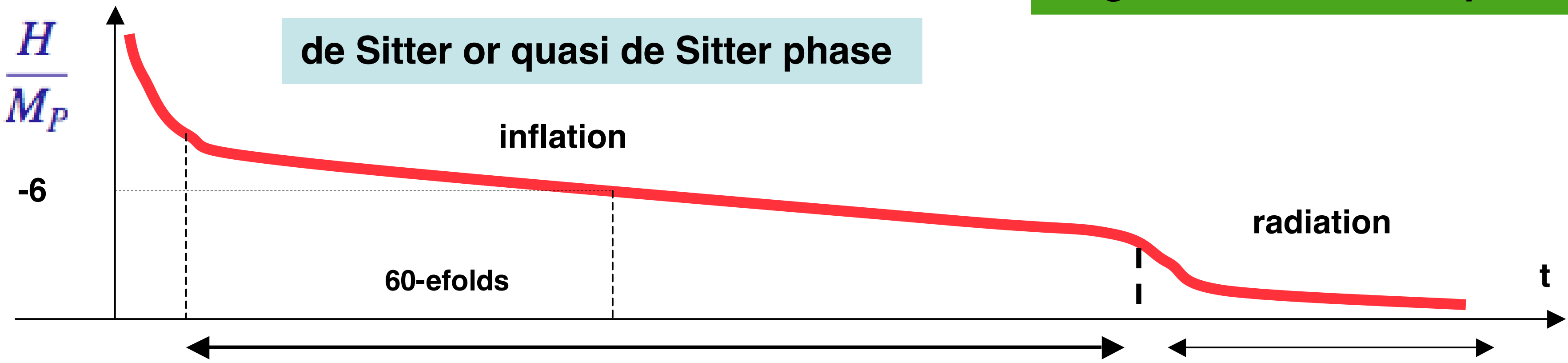
Evolution of GAUGE field IS conformally invariant

Charge and current density fluctuations are produced

$$J^\mu = e\left[\Phi_2\partial^\mu\Phi_1 - \Phi_1\partial^\mu\Phi_2\right]$$

Since : $\langle J_\mu(x)J_\nu(y)\rangle \neq 0$ **@large scales**

Magnetic fields can be produced



Charge density fluctuations and electric fields dissipated at finite conductivity(Vlasov Landau approach).

Induced magnetic field: $B_L \sim 10^{-46}\ Gauss$ $\quad n \sim 1$ $\quad r_L \sim 10^{-80}$

String Cosmology

Dynamical extra-dimensions

$$ds^2 = a^2(\eta)[d\eta^2 - d\vec{x}^2] - b^2(\eta)\gamma_{ab}dy^a dy^b$$

Conformal invariance naturally broken

$$\partial_\mu\left[b^n(\eta)\cancel{a^4(\eta)}\frac{\eta^{\alpha\mu}}{\cancel{a^2(\eta)}}\frac{\eta^{\beta\nu}}{\cancel{a^2(\eta)}}F_{\alpha\beta}\right] = 0$$

Volume of Internal dimensions

Constraints from BBN. Not present variation (Webbet al. PRL)

Compactification of 6 internal dimensions around a torus: T_6

$$S_{eff} = -\int d^4x\sqrt{-G}\,e^{-\varphi}\left[\frac{1}{2\lambda_s^2}\left(R + G^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi - \frac{1}{12}H_{\mu\nu\alpha}H^{\mu\nu\alpha}\right) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \ldots..\right]$$

$$\varphi = \Phi - \log V_6$$

Four-dimensional dilaton field

$$H^{\mu\nu\alpha} = e^{\varphi}\frac{\varepsilon^{\mu\nu\alpha\rho}}{\sqrt{-g}}\partial_\rho\sigma$$

Pseudo-scalar field

(density fluctuations)

$$\ell_P^2 = e^{\varphi}\lambda_s^2$$

Electromagnetic fluctuations

ELLIPSES are very important!

= other Abelian fields +
Corrections in powers of e^{φ}

Corrections in powers of $\lambda_s^2\partial^2$

Needs to be massive (Newton law)
May be also very massive (> 10 TeV)
Because of BBN considerations.

Evolution of the gauge coupling?

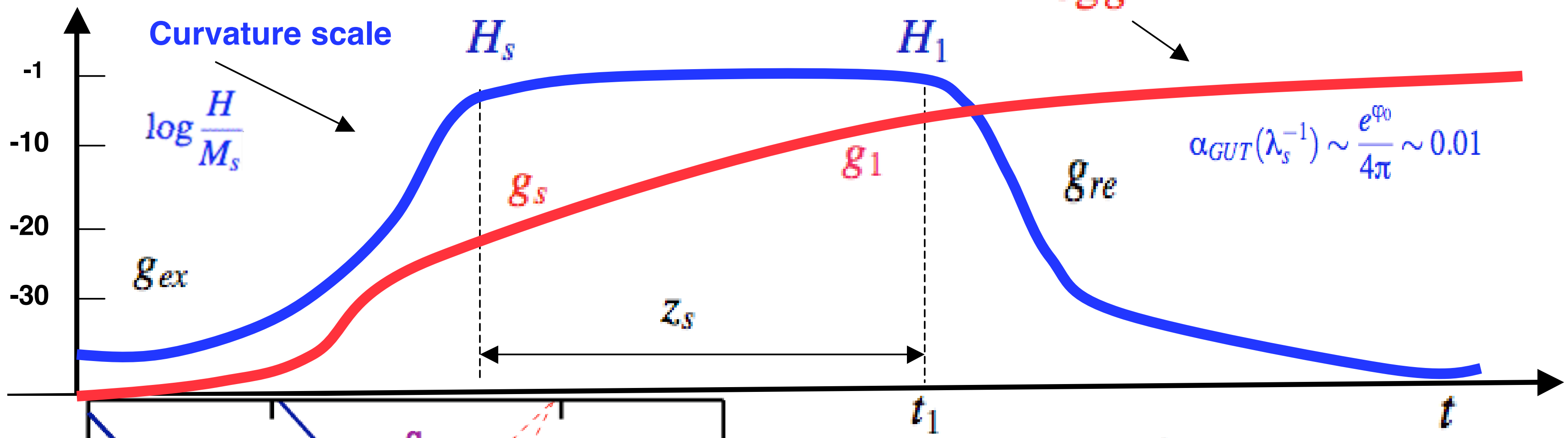
Different spectra of gauge fields depending on the model

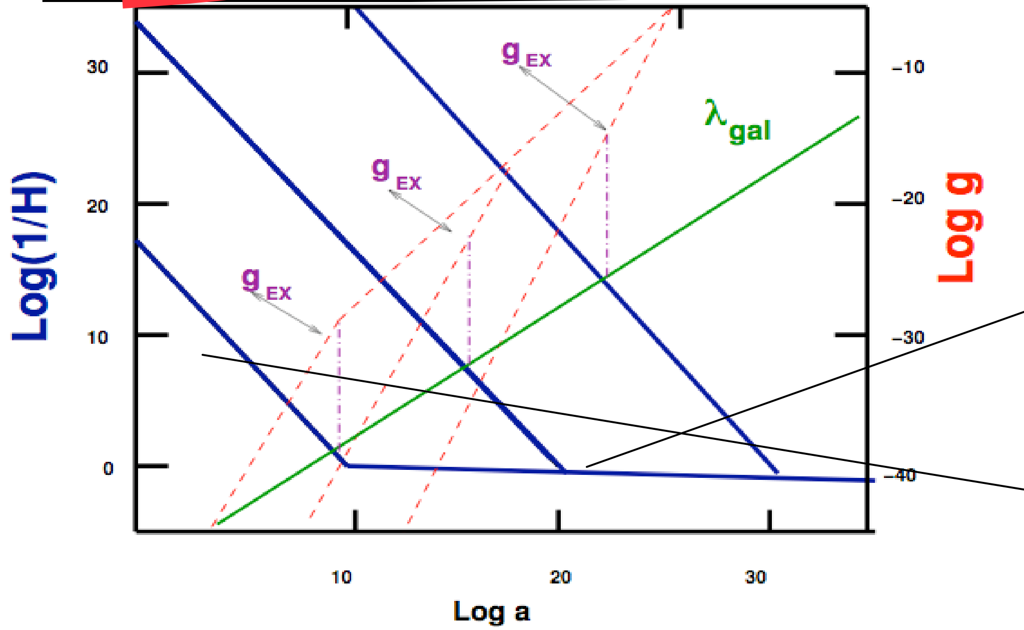
Dilaton evolves and breaks conformal invariance.

$$A_i'' + \left[k^2 - g(g^{-1})''\right] A_i = 0$$

$$g = e^{\varphi/2}$$

$$\log g$$





$$\left(\frac{g_{re}}{g_{ex}}\right)^2$$ **amplification**

$$B_L \sim 10^{-10} - 10^{-11} \quad Gauss$$

$$-3 \leq n \leq -2.5$$

$$B_L \sim 10^{-24} - 10^{-26} \quad Gauss$$

$$n \simeq 1 - \sqrt{3}$$

M. Gasperini, M. G., G. Veneziano PRL (95)

GW & hypermagnetic fields (tensor modes)

$$\Omega_{GW} = \frac{\rho_{GW}}{\rho_{critical}}$$

$T > T_c$

If hypermagnetic background Is present-> GW

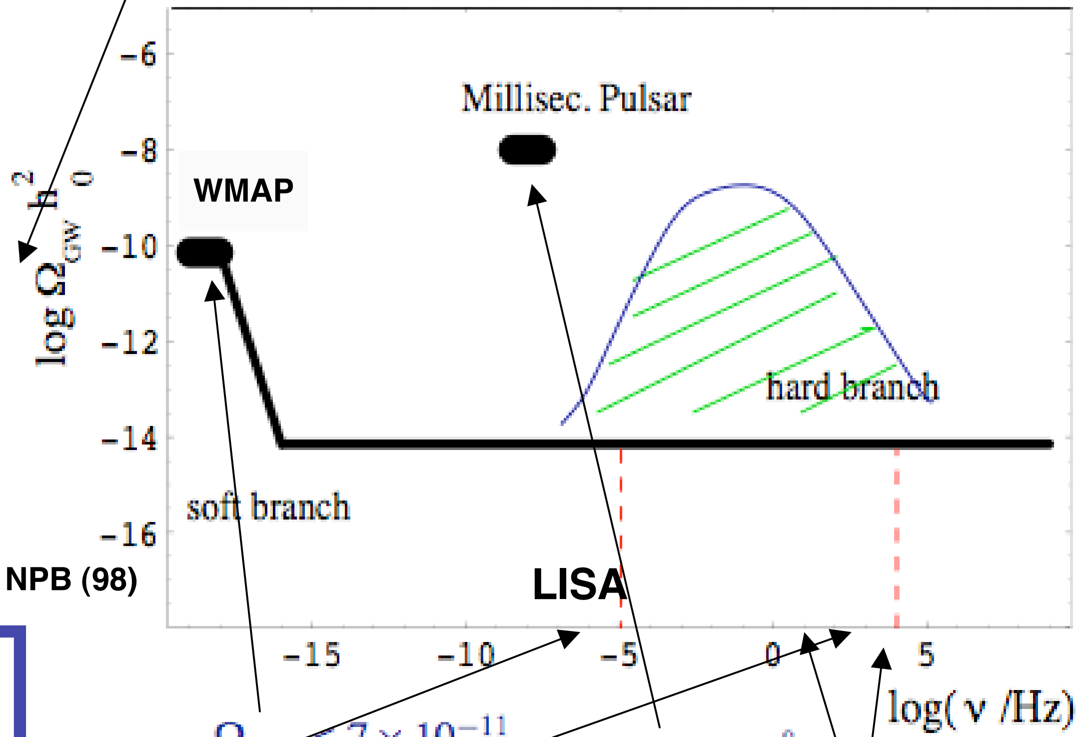
TT component of hypermagnetic Energy-momentum tensor May affect the TT component Of the fluctuations of the geometry

$\boxed{h_0^2 \Omega_{GW} \sim 10^{-7} - 10^{-8}}$

$\boxed{\vec{H}_Y / T_c \geq 0.3}$

signal

Kajantie et al NPB (98)



$\Omega_{GW} < 7 \times 10^{-11}$ $\Omega_{GW} < 10^{-8}$

$$\nu_{ew} \sim 2.01 \times 10^{-7} \left(\frac{N_{eff}}{100}\right)^{1/6} \left(\frac{T}{GeV}\right) Hz$$

$$\nu_{\sigma} \sim 10^8 \nu_{ew}$$

VIRGO/LIGO

BBN: $\int_{\nu_n}^{\nu_1} \Omega_{GW} \frac{d\nu}{\nu} < 0.2 \times 10^{-5}$

$T \sim 100 GeV$ $\nu_{ew} \sim 10^{-5}\, Hz$ $\nu_{\sigma} \sim 10^3\, Hz$

M.G. PRD [2000]

$\Omega_{GW} < 10^{-15}$ **inflation**

Concluding remarks

Magnetic field exist over different length-scales.

Compressional amplification + dynamo

Clusters

Magnetic field fluctuations over larger scales (>> Mpc).

CMB tools applied to large-scale magnetic fields

Uniform //vs// inhomogeneous field calculations

From analysis of Faraday rotation : weak constraint

B modes induced

GW (over much smaller frequencies)

Scalar modes

Foregrounds…

Large-scale structure….

NON- gaussiantity….

Next time.