Can Trans-Planckian Physics be Seen in the CMB?

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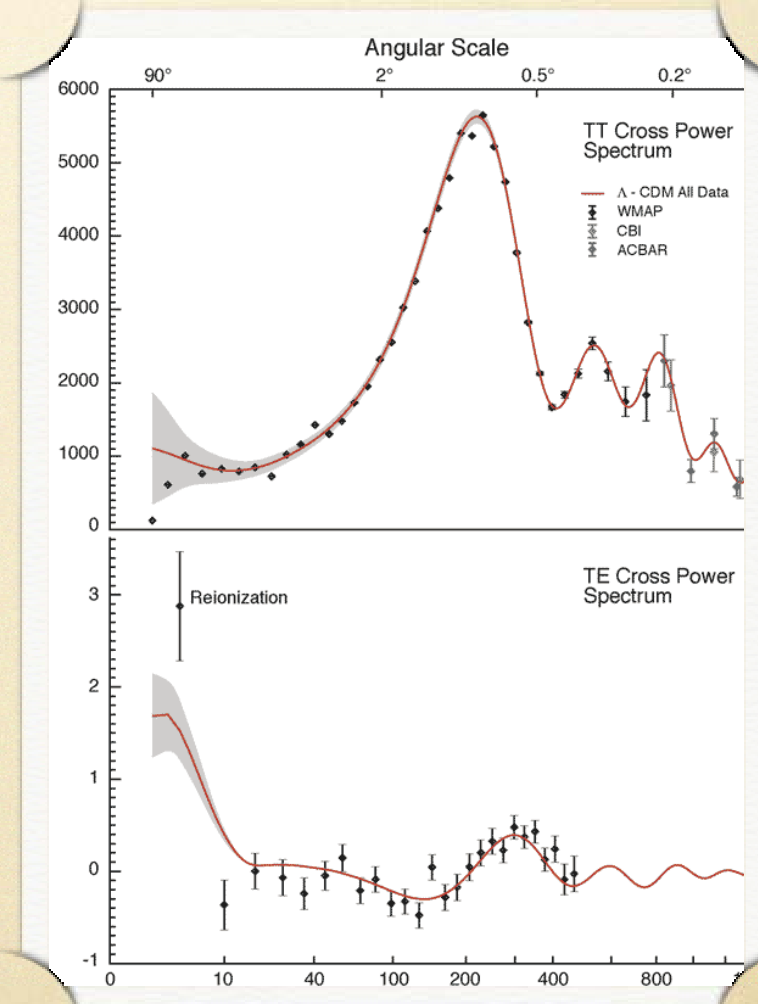
Trans-Planckia

Inflation and Trans-Planckian Physics

- WMAP data are consistent with inflationary paradigm: metric perturbations start as quantum fluctuations which decohere into classical fluctuations.

- Higher resolution probes should give us even better evidence (for or against) inflation as the source of CMB fluctuations.



NASA/WMAP

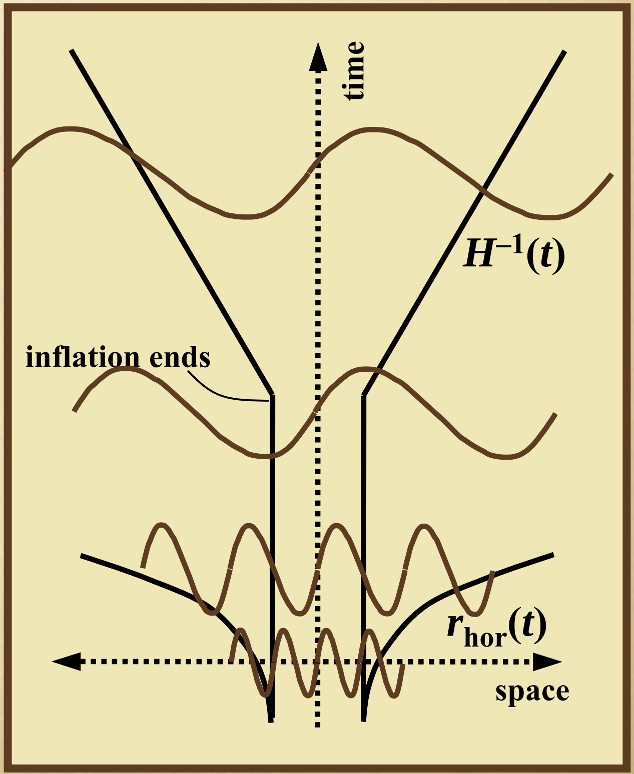
Inflation and Structure Formation

$\phi(t)$ homogeneous part of inflaton

$\chi(\vec{x},t)$ quantum fluctuations of inflaton

$$\begin{aligned} \Phi(\vec{x},t) &= \phi(t) + \chi(\vec{x},t) \\ \langle \chi(\vec{x},t) \rangle &= 0 \\ \langle \chi(\vec{x},t)\chi(\vec{y},t) \rangle &\neq 0 \end{aligned}$$

- quantum fluctuations start with $k > H_{\inf}$
- wavelengths get stretched during inflation
- fluctuations freeze once $k \leq H_{\inf}$
- this translates into curvature perturbations
- when inflation ends, fluct's re-enter the Hubble radius and generate CMB anistropies

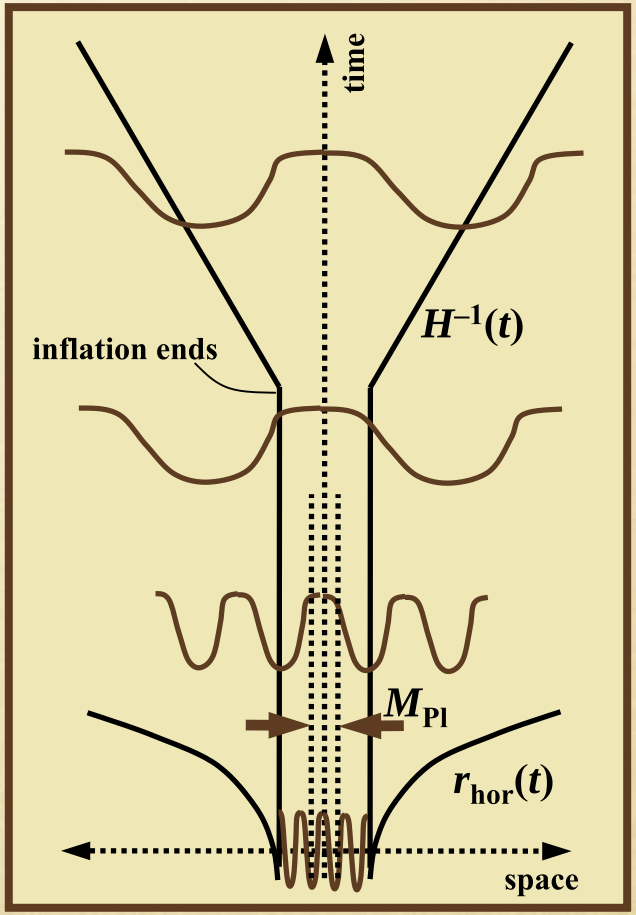


The Trans-Planckian "Problem"

- How big were the fluctuations that contribute to large-scale structure when they were formed?

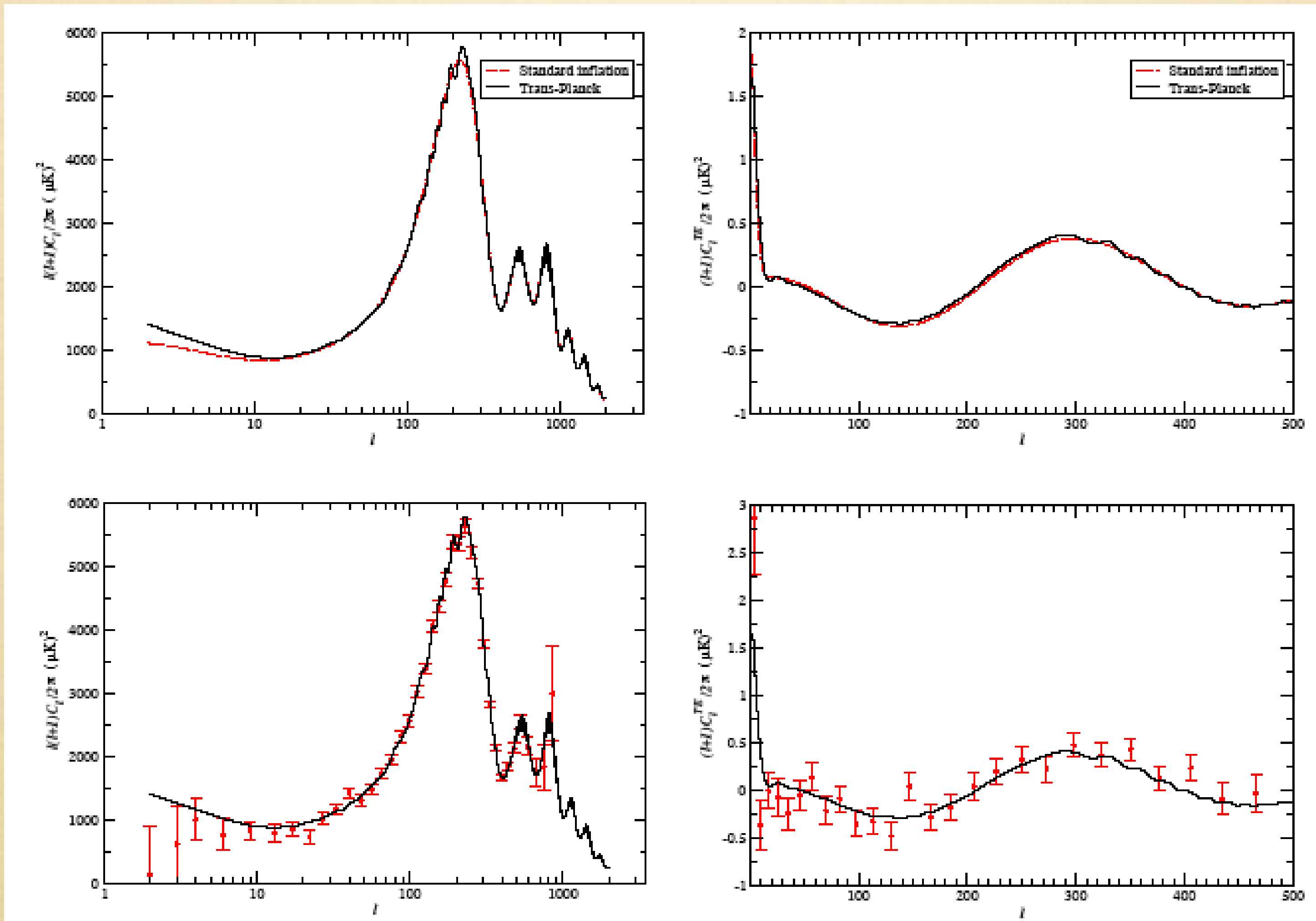
- That depends on how much inflation occurs. 60-70 e-folds solve all the problems, but most models typically have MUCH more

- In fact, in most models (except for very finely-tuned ones) these fluctuations will have started at lengths smaller than l_{Planck}



Does the Physics stretch?

Possible Effects of TP Physics on the CMB?



From Martin and Ringeval: arXiv:astro-ph/0310382

This is both a problem and an opportunity

- If TP physics is infiltrating long-distance observables like the CMB, will we have to know everything to calculate anything?

- On the other hand, we might be able to use the CMB to see physics beyond the Planck scale!

Either way, we need to learn how to calculate these potential corrections consistently

Two approaches:

- Look at specific models (minimal lengths, NC geometry...)

- Try to make generic statements using only symmetries etc.

How big could TP Effects be?

The fiducial measure of the size of TP effects is: $\frac{H_{\mathrm{inf}}^2}{M^2}$

These effects would be too small to measure

Shenker et al: EFT says no bigger than $\frac{H_{\mathrm{inf}}^2}{M^2}$

Model builders: Effects are LARGER! $\frac{H_{\mathrm{inf}}}{M}$

The Initial State

Ground States

How do we define the vacuum?

- In flat space, we can use the isometries of Minkowski space to pick out the vacuum state of the theory. This coincides with the lowest energy state of the Hamiltonian for relativistic FT's

- In FRW spacetimes, this will NOT work! Not enough symmetry to single out only one vacuum

Try: use state that matches to flat space vacuum at short distances, where curvature effects could be neglected.

How do we know that true vacuum remains close to flat vacuum to arbitrarily short distances?

Case Study: De Sitter space

De Sitter space has
an isometry group as large as that
of flat space

Use this to pick out vacuum.

Get an infinite family of choices: the   vacua
α

Only one vacuum matches at short
distances: Bunch-Davies $\alpha \rightarrow -\infty$

BUT: No reason to expect that this is the correct
state!

Effective Field Theories

Effective field theories

- **New physics can have an effect even before we see its cause directly**

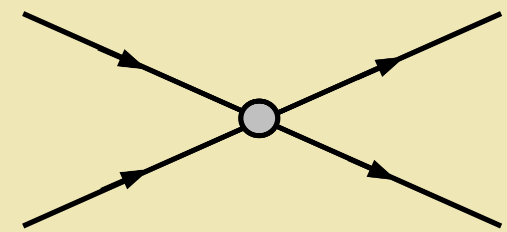
- **An old example: Fermi's theory of ®-decay**

$$L = -\frac{1}{\sqrt{2}}\underbrace{G_F}_{(292\ \mathrm{GeV})^{-2}}[\bar{p}\gamma_\mu n][\bar{e}\gamma^\mu \nu] + \mathrm{h.c.}$$

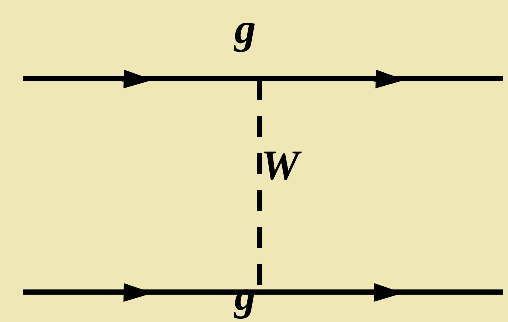
- **The interaction is nonrenormalizable (dim 6)**
  - **but that does not imply that the theory is nonpredictive**

- **At low energies ($E < M_W$), the theory is accurate to order E^2/M_W^2**
  - **we can improve the accuracy by adding higher dimensional operators and using experiments to fix the coefficients**

- **The ratio of the experimentally tested energy and the scale of new physics provides a natural perturbative parameter**



$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{M_W^2}$$



Effective initial states

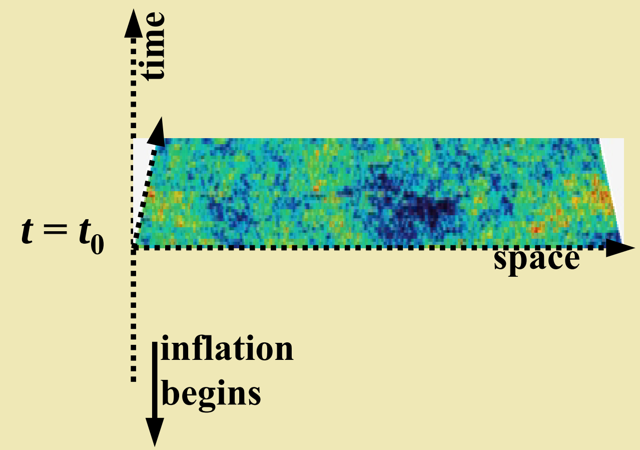
- **This effective theory idea provides a method for estimating the generic trans-Planckian signature in terms of a controlled expansion in H/M**
  - **low scale: inflationary Hubble scale, $H(t)$**
  - **high scale: *any* new physics M**

- **For inflation, the new aspect to the effective theory is that we need to apply it to the *state* in addition to the "bulk" action**

- **Our effective description does not need to make sense to arbitrarily high energies—it just must produce a predictive theory at low energies**

- **Two approaches:**
  - **Lagrangian approach (boundary action)**
    – Schalm, Shiu, van der Schaar, and Greene (2004)
  - **Hamiltonian approach (boundary modes)**
    – Collins and Holman (2005)

Define an initial state at $t = t_0$:



Initial state information:
- **strong physics**
- **excitations**
- **new principles**

$$ds^2 = dt^2 - a^2(t)\, d\vec{x}\cdot d\vec{x}$$

$$S_{t=t_0} = \int d^3\vec{x}\, a_0^3 \Big[z_0 m \varphi^2 + z_1 \varphi\dot{\varphi} + z_2 K\varphi^2 + \frac{z_3}{M} K\varphi\dot{\varphi} + \cdots$$

Calculational Techniques

EFT in Time-Dependent Backgrounds: Formalism

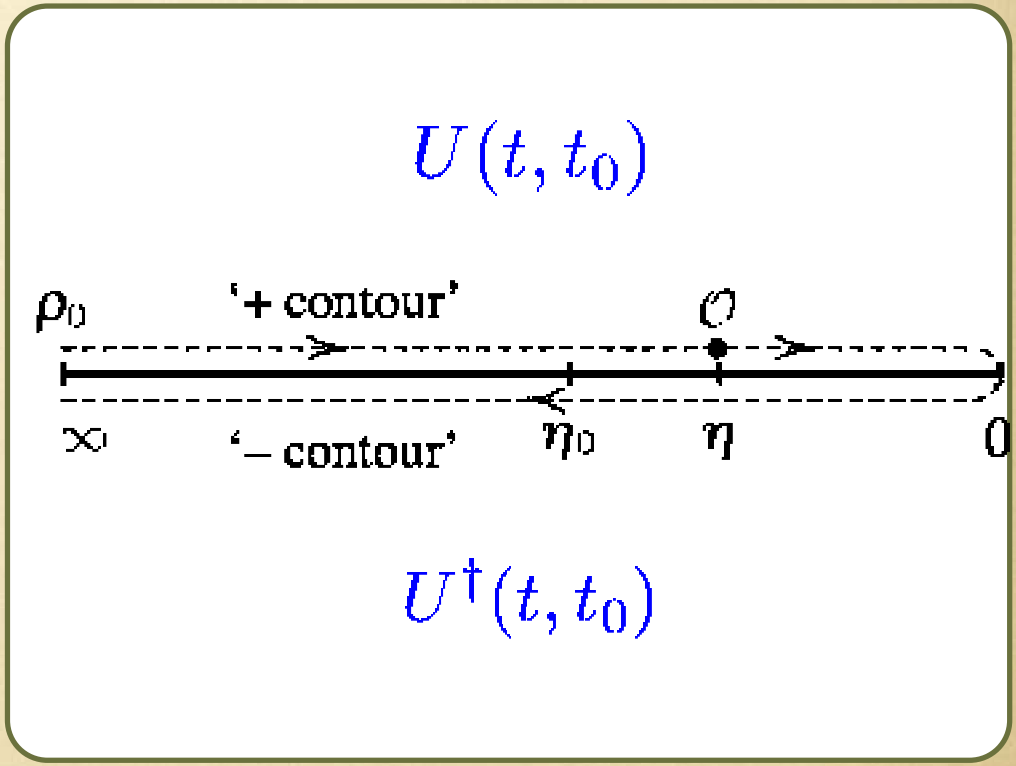
- Std. EFT calculations match observables like S-matrix elements.
- This fails in time-dep backgrounds, since there may be NO asymptotic regions to use for IN/OUT states.
- In particular, QFT in de Sitter space has NO S-matrix formulation!

Schwinger-Keldysh Formalism

- Need a formalism that:
  - Allows us to treat QFT as an initial value problem,
  - Allows us to compute time-dependent observables,
  - Has a well-defined path integral representation, so that a diagrammatic perturbation theory is easy to construct and to calculate in.

Schwinger-Keldysh Formalism (cont'd)

- Schwinger-Keldysh formalism uses the Liouville equation to time-evolve the density matrix ρ
- This involves a CLOSED-TIME CONTOUR since the density matrix evolves with both the time evolution operator and its conjugate.



Schwinger-Keldysh Formalism (cont'd)

We can calculate time-dependent expectation values operators in the interaction picture:

$$\frac{\langle \psi'(t)|\mathcal{O}_I(t)|\psi(t)\rangle}{\langle \psi'(t)|\psi(t)\rangle} = \frac{\langle \psi'(t_0)|T\left(\mathcal{O}_I(t)e^{-i\int_{t_0}^{\infty} dt''\,\left[H_I[\varphi^+]-H_I[\varphi^-]\right]}\right)|\psi(t_0)\rangle}{\langle \psi'(t_0)|T\left(e^{-i\int_{t_0}^{\infty} dt''\,\left[H_I[\varphi^+]-H_I[\varphi^-]\right]}\right)|\psi(t_0)\rangle}$$

We now need contour-Green's functions: there are four of them, depending on which contour the times in the Green's function are located on.

Results

A general initial state

- Let us construct a general propagator, $G_k(t,t')$, that satisfies a simple, linear initial condition,

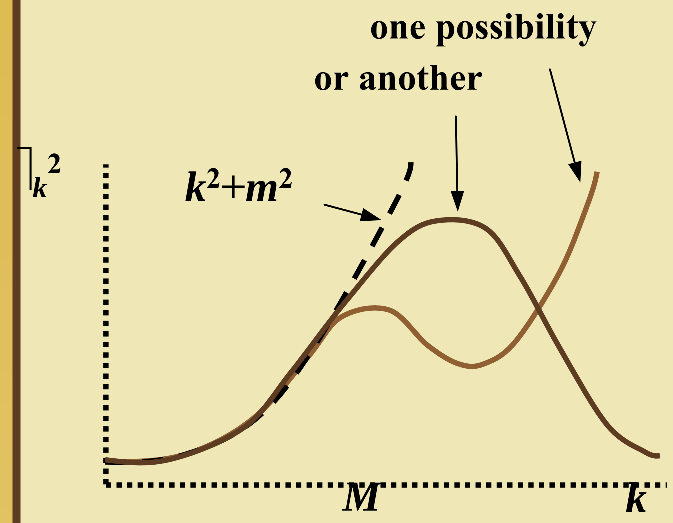
$$\left.\frac{\partial}{\partial t} G_k(t,t')\right|_{t=t_0} = i\,\frac{\omega_k - F_k\,\omega_k^*}{1+F_k}\, G_k(t_0,t')$$

- Here $\rceil_k$ is the vacuum boundary condition
  - The Bunch-Davies vacuum is the no-particle symmetric state *with respect to the free, low-energy action*

- F_k is the "structure function" for the initial state
  - IR aspects are real excitations which are fixed by what we observed
  - but the UV effects are virtual, encoding the mistake we make in extrapolating the free theory states to arbitrarily high energies

- The trans-Planckian signature lies in the latter

Look at a dispersion example again:



Would these examples give very different CMB spectra?

—only the details near *M* are important after renormalization

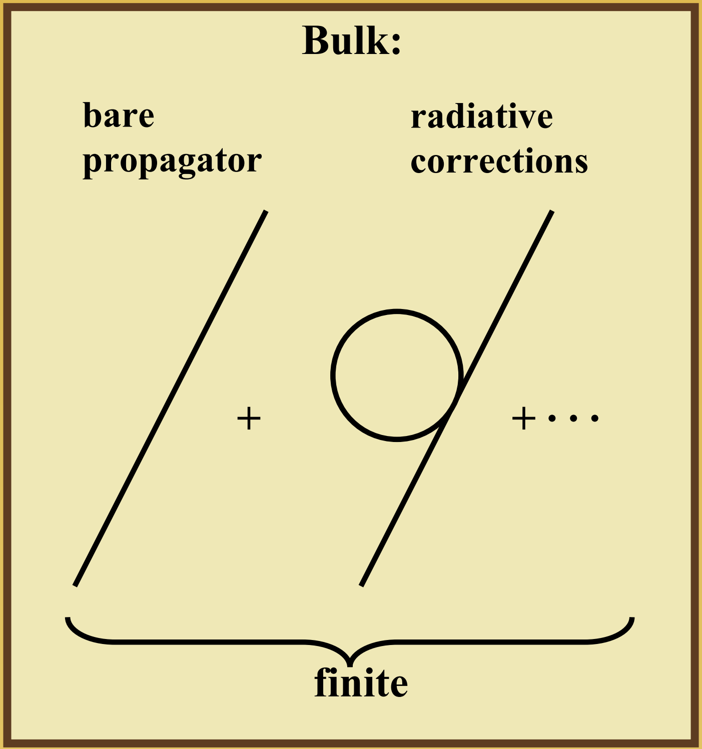
Renormalization

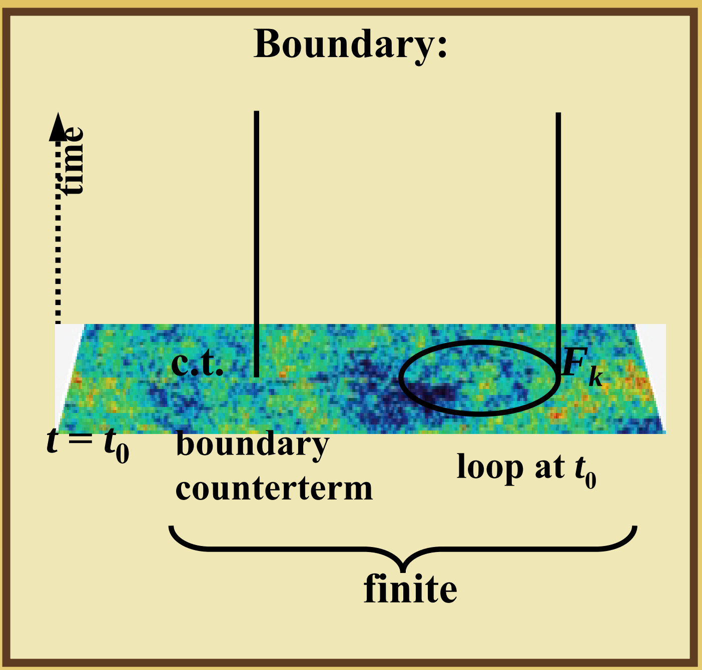
- **The propagator solving the initial condition is**

$$G_k(t,t') = G_k^F(t,t') + F_k G_k^F(t_I,t')$$
$$t_I = 2t_0 - t$$

- **The first term is the usual Feynman propagator for a point source**
  - **In an interacting theory, we encounter the usual bulk divergences which are cancelled by renormalizing the bulk properties of the theory**

- **The second term propagates the initial-state information**
  - **It produces new divergences—but these are all confined to the initial surface, $t = t_0$**
  - **They are cancelled through boundary renormalization**





Boundary renormalization

- hep-th/0501158 & hep-th/0506nnn

- **Expand the boundary structure function in powers of a generalized frequency,**

$$F_k = \sum_{n=0}^{\infty} d_n \frac{H^n(t_0)}{\Omega_k^n(t_0)} + \sum_{n=1}^{\infty} c_n \frac{\Omega_k^n(t_0)}{M^n}$$

- **The first sum (IR): these effects diminish at short distances**
  - **they can produce boundary divergences which are cancelled with *renormalizable* boundary counterterms**

- **The second sum (UV): these effects are unimportant at long distances**
  - **they diverge from vacuum at short scales**
  - **their boundary divergences are cancelled with nonrenormalizable boundary terms**
  - **only the first few are needed for measurements at scales << *M***
  - **boundary Callan-Symanzik equation**

$$\Omega_k(t) \approx \frac{\sqrt{k^2 + m_{\text{eff}}(t)}}{a(t)}$$

Example: $\lfloor\prod^4$ theory to one-loop order

Boundary counterterms:

$$S_{t=t_0} = \int d^3x \sqrt{g_{3d}}\, L_{3d}$$

IR effects: marginal or relevant operators

$$d_0 \to \varphi\partial_t\varphi, \quad K\varphi^2 = 3H\varphi^2$$
$$d_1 \to K\varphi^2$$

UV effects: irrelevant operators

$$c_1 \to \varphi\ddot{\varphi}, \quad \dot{\varphi}^2, \quad K\varphi\dot{\varphi},$$
$$\dot{K}\varphi^2, \quad K^2\varphi^2$$

Experimental outlook

- **The effective initial condition provides a renormalizable, general description of trans-Planckian effects**
  - **leading effect at tree level**
  - **corrections to the Bunch-Davies result scale as H/M**
  - **appear as oscillations about the B-D result with frequency & amplitude correlated**

- **Can we observe a trans-Planckian signal?**
  - **CMB precision for** **(Spergel)**
    - **WMAP (3 year): 10^{-2}**
    - **WMAP (6 year)/PLANCK: 10^{-3}**
    - **Planned Galaxy Surveys: 10^{-4}**
    - **Potential Galaxy Surveys: 10^{-5}**
    - **Ideal bound: 10^{-6}**

- **Future surveys:**
  - **Square kilometer array (2020), 21 cm high redshift gas, cosmic inflation probe, …**

Measuring the power spectrum:

$$\frac{\sigma_P(k)}{P(k)} = \frac{\sqrt{1+n_l/c_l}}{l_{\max}}$$

$l_{\max} \approx 600$ **(6 yr WMAP)**
$l_{\max} \approx 1500$ **(PLANCK)**

from David Spergel's talk at 5th String Cosmology Workshop

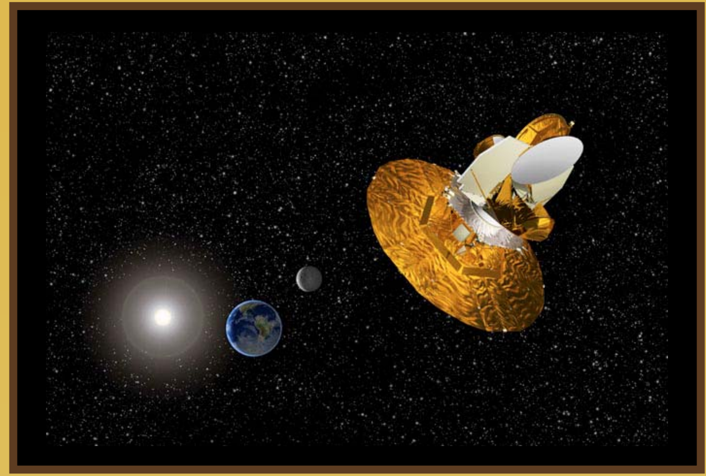
Large scale structure:

$$\frac{\sigma_P(k)}{P(k)} = \frac{\sqrt{1+P(k)V/N_{\text{galaxies}}}}{\sqrt{N_{\text{modes}}}}$$

the same CMB acoustic peaks appear in SDSS data

What we have learned

- **We have developed an effective theory description of an initial state**
  - **it provides a powerful method for finding the leading *generic size* and *shape* of a trans-Planckian signal**
  - **the leading initial state corrections to the Bunch-Davies prediction scale as H/M**
  - **the leading trans-Planckian signature is oscillatory and is specified by 2 parameters (amplitude-frequency/phase)**



… and what comes after

- **Evaluate the detailed form for the power spectrum (include initial state effects carefully)**

  Easther, Kinney & Peiris; Greene, Schalm, Shiu, & van der Schaar; Collins & Holman (in progress)

- **Backreaction questions and naturalness**

  Nitti, Porrati & Rombouts v. Greene, Schalm, Shiu, & van der Schaar; Collins & Holman (in progress)

  - **some controversy still exists**

- **Good experimental outlook**

$$P_{\text{infl}}(k)=\frac{H^2}{4\pi^2}\left[1+O\left(H/M\right)\right]$$

10^{-2} observable now,
10^{-6} observable eventually