Inflationary Gravitational Waves, CMB Polarization, and Direct Detection

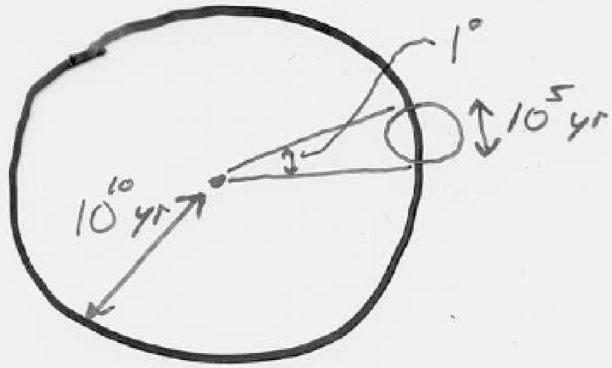
Marc Kamionkowski

(Caltech)

Paris, 1 July 2005

First, brief review of inflation….

2. Why is the Universe so smooth?



$\sim$ 40,000 causally-disconnected regions of early U have same T to $\sim 10^{-5}$.

Also, at BBN,

^4He, D, ^7Li
= nonlinear functions of ρ_b

$\Rightarrow$ Would get different abundances if $\exists$ small-scale fluctuations

Fluctuations must be small in early U on all scales.

$\Rightarrow$ perturbations $\exists$, but are small and nearly scale-invariant

Inflation: Basic Idea

As Universe expands, ~~the~~ Hubble length H^{-1} increases. During inflation,

$$\frac{d}{dt}\left[\frac{H^{-1}}{a}\right] < 0,$$

comoving Hubble length decreases with time, and objects/info/curvature exit horizon leaving (classically) smooth, empty Universe.

$$H = \frac{\dot{a}}{a} \Rightarrow \frac{d}{dt}\left[\frac{H^{-1}}{a}\right] = \frac{d}{dt}\left[\frac{1}{\dot{a}}\right] = \frac{-\ddot{a}}{\dot{a}^2}$$

$$\Rightarrow \ddot{a} > 0 \quad \text{for inflation}$$

2nd Friedmann eqn: $\frac{\ddot{a}}{a} = \frac{-4\pi G}{3}(\rho + 3p)$

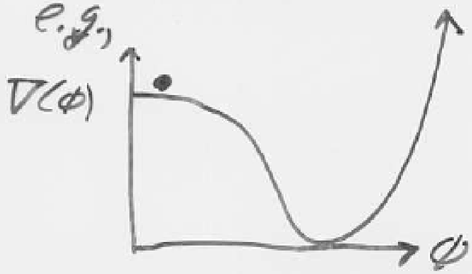
$$\Rightarrow \text{need} \quad w = \frac{p}{\rho} < \frac{-1}{3}$$

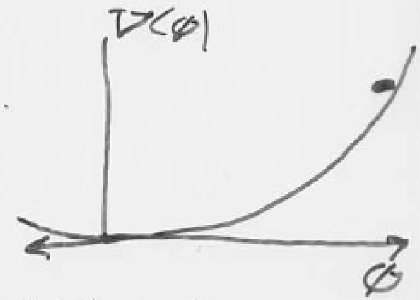
(~ dark energy)

Basic Mechanism

Postulate scalar field $\phi(\vec{x}, t)$
with potential-energy density $V(\phi)$:

e.g.,





If field homogeneous (why?), then

energy density: $\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$

pressure: $p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$,

so if $\dot{\phi}^2 < 2V(\phi) \Rightarrow$ inflation.

For flat $V(\phi)$, $\dot{\phi}^2 \propto a^{-6}$, $V(\phi) \propto a^0$;

so solution $\rightarrow$ inflation if $V(\phi)$ sufficiently flat.

Equations of motion:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3m_{pl}^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{dV}{d\phi}$$

$$\left(\text{i.e., } \Box\phi = -\frac{dV}{d\phi} \text{ in } ds^2 = dt^2 - a^2(t)\, d\vec{x}^2\right)$$

Slow-roll approximation:

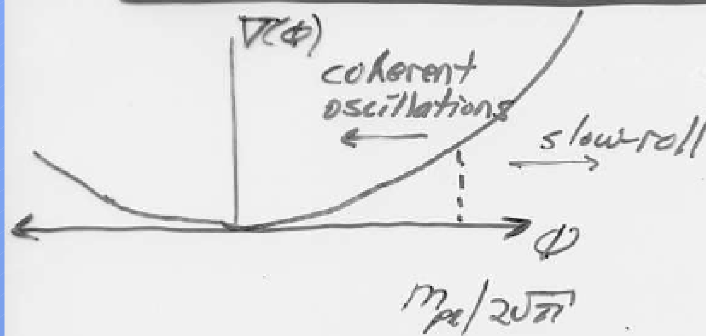
$$H^2 \simeq \frac{8\pi V}{3m_{pl}^2} \qquad 3H\dot{\phi} \simeq -V'(\phi)$$

if $\epsilon(\phi) \equiv \frac{m_{pl}^2}{16\pi}\left(\frac{V'}{V}\right)^2 \ll 1$

$$\eta(\phi) \equiv \left|\frac{m_{pl}^2}{8\pi}\frac{V''}{V}\right| \ll 1$$

(and require $\dot{\phi}$ sufficiently small at onset).

Simple Example: Chaotic Inflation



$$V(\phi)=\frac{1}{2}m^2\phi^2 \qquad \frac{V'}{V}=\frac{2}{\phi} \qquad \frac{V''}{V}=\frac{1}{2\phi^2}$$

slow-roll: $(\epsilon \lesssim 1)$ for $\phi \gtrsim \frac{1}{2\sqrt{\pi}}m_{pl}$

and $\eta \lesssim 1$ for $\phi \gtrsim \frac{m_{pl}}{4\sqrt{\pi}}$

For $\phi \lesssim \frac{m_{pl}}{2\sqrt{\pi}}$, field oscillates coherently:

$$\ddot{\phi}+3\cancel{H\dot{\phi}}+m^2\phi=0$$

when $H \ll m$, $\phi(t) \propto e^{\pm imt-\frac{3}{2}Ht}$,

so $\bar{\rho}(t) \propto \langle \dot{\phi}^2 \rangle \propto e^{-3Ht}$

i.e, gas of zero-momentum ϕ particles

<u>Reheating</u>: ϕ particles decay to SM particles which make up primordial plasma

Inflaton perturbations:

$$\ddot{\phi} + 3H\dot{\phi} + \nabla^2\phi + \frac{dV}{d\phi} = 0$$

Let $\phi(\vec{x},t) = \phi(t) + \delta\phi(\vec{x},t)$. Then

$$(\delta\ddot{\phi})_{\vec{k}} + 3H(\delta\dot{\phi})_{\vec{k}} + \left(\frac{k}{a}\right)^2 \delta\phi_{\vec{k}} + \frac{1}{2}m^2\delta\phi_{\vec{k}} = 0$$

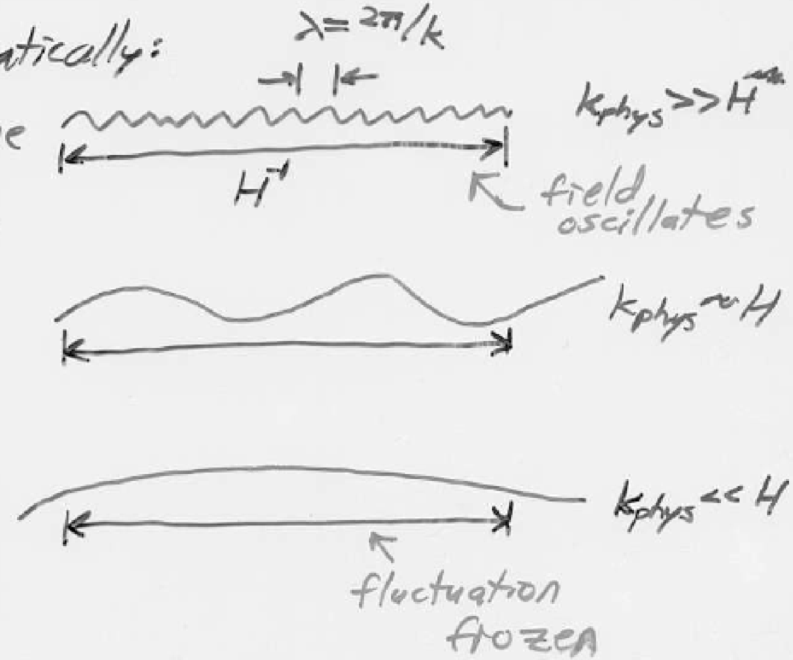
where $m^2 = V''$ and $(\delta\phi) \ll \phi$

I.e., to lowest order in $\delta\phi$, each $\vec{k}$ mode evolves independently

Schematically:

early time
$\frac{k}{a} \gg H$
$\lambda \ll H^{-1}$

late time
$\frac{k}{a} \ll H$
$\lambda \gg H^{-1}$



Slow-roll: $\eta \propto \frac{V''}{V} << 1 \Rightarrow m^2 << H^2$, so

$$(\delta\ddot{\phi}_{\vec{k}}) + 3H(\delta\dot{\phi}_{\vec{k}}) + (k/a)^2 \delta\phi_{\vec{k}} = 0,$$

or, writing $\delta\phi_{\vec{k}} = w_k(t)\, a_{\vec{k}} + w_k^*(t)\, a_{\vec{k}}^\dagger$,

$$\ddot{w}_k + 3H\dot{w}_k + (k/a)^2 w_{\vec{k}} = 0$$

which, for H=const, has soln:

$$w_k(t) = L^{-3/2} \frac{H}{(2k)^{3/2}} \left(i + \frac{k}{aH}\right) e^{ik/aH}$$

$$\stackrel{\text{early}}{=} \left(\frac{1}{aL}\right)^{3/2} \sqrt{\frac{1}{2E_{\vec{k}}}}\, e^{-iE_{\vec{k}}t}$$ flat-space result

(with $E_{\vec{k}} = k/a$),

which has, at early times, $\langle |\delta\phi_{\vec{k}}|^2 \rangle = |w_k|^2$.

At time t_* shortly after horizon exit,

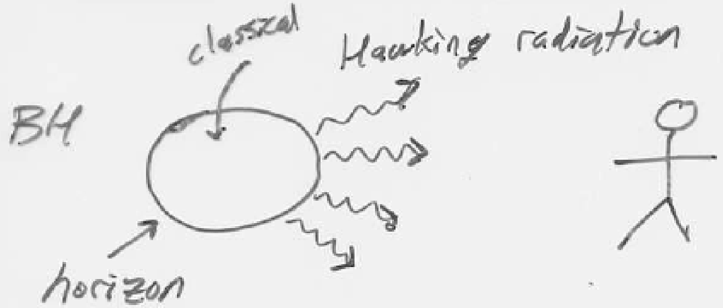
$$\langle |\delta\phi_{\vec{k}}|^2 \rangle = \frac{H^2(t_*)}{2L^3k^3},$$

and classical spectrum of frozen ϕ fluctuations emerges with power spectrum,

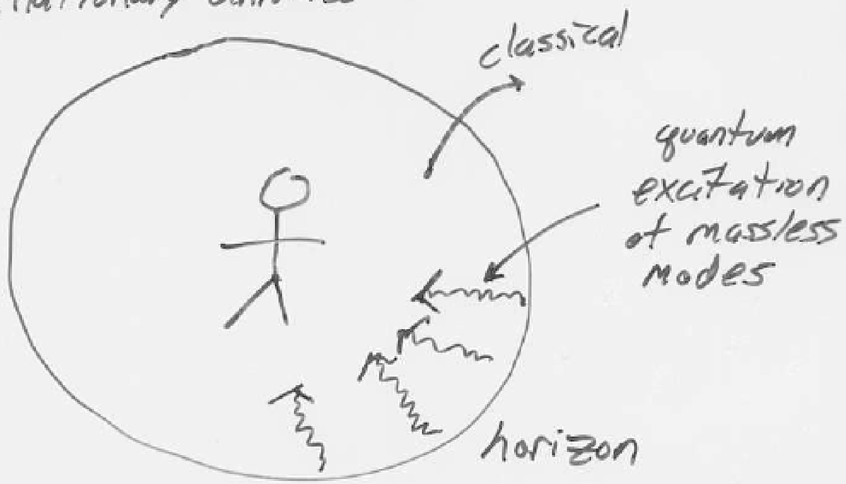
and Gaussian!

$$P_\phi(k, t_*) = \left(\frac{H}{2\pi}\right)^2 \Bigg|_{k=aH}$$

Black-hole analogy



Inflationary Universe



Evolution of perturbations from horizon exit to horizon re-entry to today is fully classical, but requires some nasty G.R. Result for power spectrum of curvature $\mathcal{R}^*$ (total density) is

$$P_{\mathcal{R}}(k) = \frac{8}{3\pi^2} \frac{V}{m_{pl}^2 \epsilon} \quad \propto \frac{V^3}{V'^2}$$

with V, ϵ evaluated at $k = aH$

COBE: $\left[P_{\mathcal{R}}(k)\right]^{1/2} \simeq 5 \times 10^{-5}$ at $k = a_0 H_0$

$$\Rightarrow \frac{V^{3/2}\, 16\sqrt{2}\, \pi^{3/2}}{V' m_{pl}^3} = 5.2 \times 10^{-4}$$

(assuming no GW contribution).

or $\frac{V^{1/4}}{\epsilon^{1/4}} = 6.6 \times 10^{16}$ GeV

$$\Rightarrow V^{1/4} \lesssim 6 \times 10^{16} \text{ GeV}$$

* $\mathcal{R} \to \Phi$ = Newtonian potential on subhorizon scales

The (scalar or density-perturbation) spectral index.

Matter power spectrum $P(k) \propto k^{n_s}$

related to Φ through Poisson,

$$\nabla^2 \Phi = 4\pi G \bar{\rho}\, \delta \qquad \delta \equiv \frac{\delta\rho}{\rho}$$

So $P_{\mathcal{R}} \propto k^{n_s - \cancel{4}\,1}$

$$\Rightarrow \quad n_s - \cancel{4}\,1 \equiv \frac{d \ln P_{\mathcal{R}}}{d \ln k}$$

Using $d\ln k = \frac{dk}{k} = \frac{H\,da}{Ha} = \frac{da}{a} = \frac{\dot{a}}{a} dt = H\,dt$

and $dt = -(3H/V')\,d\phi$,

$$\frac{d}{d\ln k} = \frac{-m_{pl}^2}{8\pi} \frac{V'}{V} \frac{d}{d\phi}$$

$$\Rightarrow \boxed{n_s = 1 - 6\epsilon + 2\eta}$$

WMAP: $n_s = 0.99 \pm 0.04$

$$n_s \begin{cases} > 1 & \text{"blue" spectrum} \\ \simeq 1 & \text{Peebles-Harrison-Zeldovich} \\ < 1 & \end{cases}$$

"Running" of the spectral index

$$\boxed{\begin{aligned} \frac{dn}{d\ln k} &= -16\,\epsilon\eta + 24\,\epsilon^2 + 2\xi^2 + \text{HOT} \\ \xi^2 &\equiv \frac{m_{pl}^4}{64\pi^2}\frac{V'V'''}{V^2} \end{aligned}}$$

Adiabatic vs. Isocurvature:

If ϕ = inflaton, and $\delta\phi \to \delta\rho$, then decay of inflaton is same everywhere, so $\delta_x \equiv \delta\rho_x/\rho_x = \delta\rho/\rho$

for x = baryons, DM, ν's, γ's

i.e., no "entropy" perturbations

$\Rightarrow$ adiabatic

Isocurvature: Suppose DM comes from decay of non-inflaton χ (a "spectator") that attains fluctuations $\delta\chi$ during inflation. Then may have $\frac{\delta\rho_b}{\rho_b} \neq \frac{\delta\rho_\gamma}{\rho_\gamma} \neq \frac{\delta\rho_{dm}}{\rho_{dm}}$

Gravitational Waves:

Tensor perturbations h_{ij} to metric,

$$ds^2 = -dt^2 + a^2(t)\,d\vec{x}^2 + a^2 h_{ij}\,dx^i dx^j$$

satisfy KG eqn:

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} + (k/a)^2 h_{ij} = 0$$

i.e., propagating massless modes = gravitons.
Get excited QM during inflation.
2 Polarization states $(+, \times)$ get
power spectra,

$$P_+(k) = P_\times(k) = \frac{1}{2}P_{GW}(k) \propto \left(\frac{H}{2\pi}\right)^2.$$

Multiplicative coeff obtained by expanding
Einstein-Hilbert action,

$$S = \frac{1}{16\pi G}\int \sqrt{-g}\,R\,d^4x$$

to quadratic order in $h_+, h_\times$. Result:

$$\Rightarrow \quad P_{GW}(k) = \frac{m_{pl}^2}{4\pi}\left(\frac{H}{2\pi}\right)^2\Bigg|_{k=aH}$$

primordial!!

GW spectral index:

$$\boxed{n_{grav} = \frac{d\ln P_{GW}(k)}{d\ln k} = -2\epsilon}$$

Processed spectrum:

For $\epsilon=0$, all h same as enter horizon. Then, h decays by

$$\rho_{GW} \propto \frac{1}{a^4} \propto \dot{h}^2 \propto (k/a)^2 h^2$$

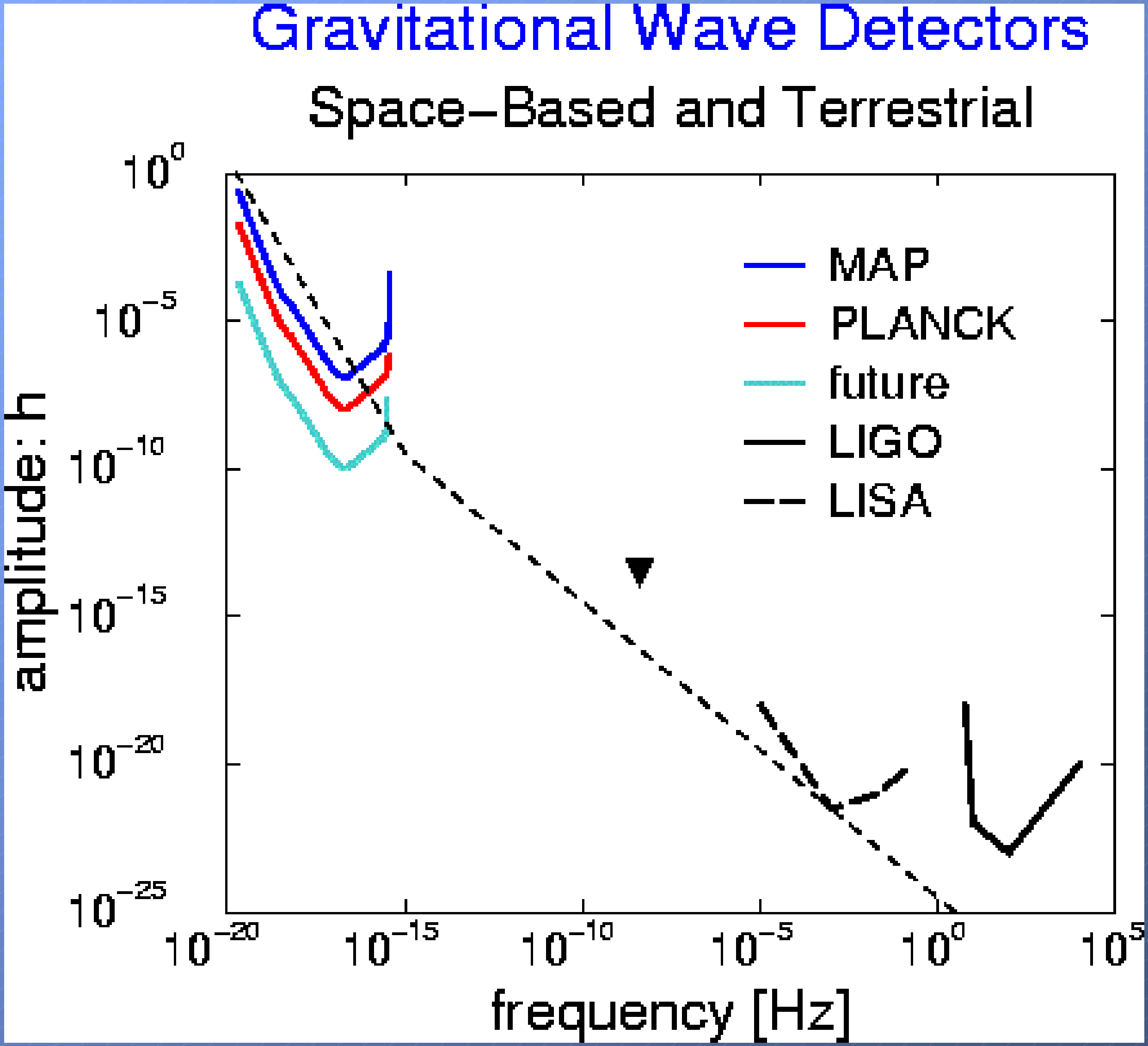
$$\Rightarrow h \propto a^{-1}(t) \text{ after horizon crossing.}$$

For modes $k > k_{eq}$ that enter horizon during RD, $aH \propto a^{-1}$, so $a \propto k^{-1}$ at horizon crossing, and so

$$h_k \propto k^{-1} \text{ today for } k > k_{eq}.$$

(similar arguments for $k < k_{eq}$ leading to steeper low-k spectrum).

Important: $h_k \propto \sqrt{V}$



Caldwell, MK, Wadley

Summary of Inflationary Observables

(1) Density-perturbation amplitude

$$\frac{\delta\rho}{\rho} \propto \frac{V^{3/2}}{V'}$$

(2) Spectral index for $\delta\rho/\rho$:

$$n_s = 1 - 6\epsilon + 2\eta$$

$$\epsilon \propto \left(\frac{V'}{V}\right)^2 \qquad \eta \propto \frac{V''}{V}$$

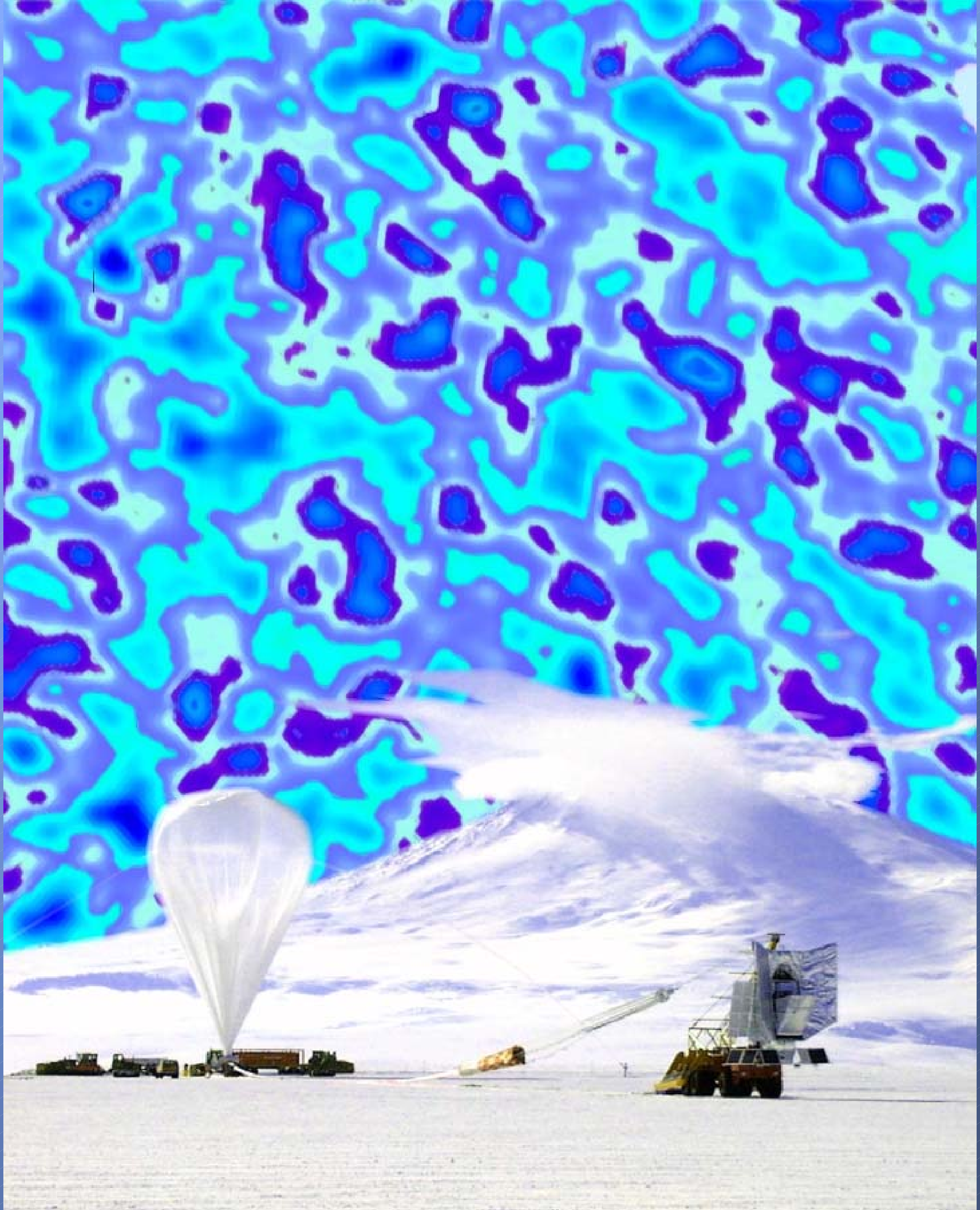
(3) Running of spectral index
(higher order in $\frac{d}{d\phi}$ of $V(\phi)$)

(4) GW amplitude $\propto V$

(5) GW spectral index $n_{grav} \propto \epsilon$

(6) Non-gaussianity:

Bispectrum $\propto \epsilon$



What is the geometry of the Universe?

$$\Omega \equiv \frac{\rho}{\rho_c} \equiv \frac{\rho}{3H_0^2/(8\pi G)}$$

- Flat (Euclidean) $\Omega = 1$
- Closed (bound) $\Omega > 1$
- Open (unbound) $\Omega < 1$

Cosmological geometry: The shape of spacetime

General relativity: Matter warps spacetime

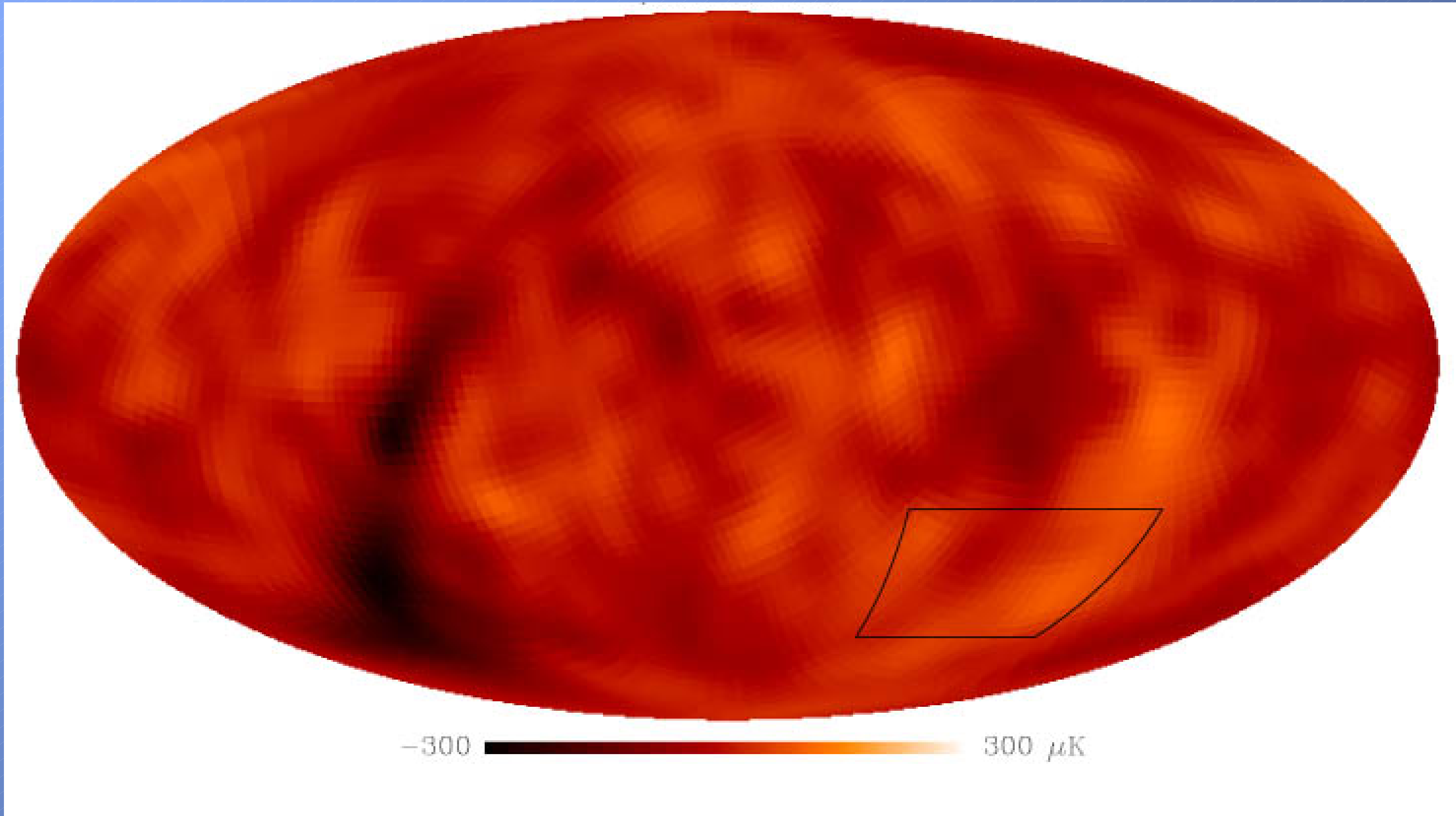
QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

"Open"
(Less matter)

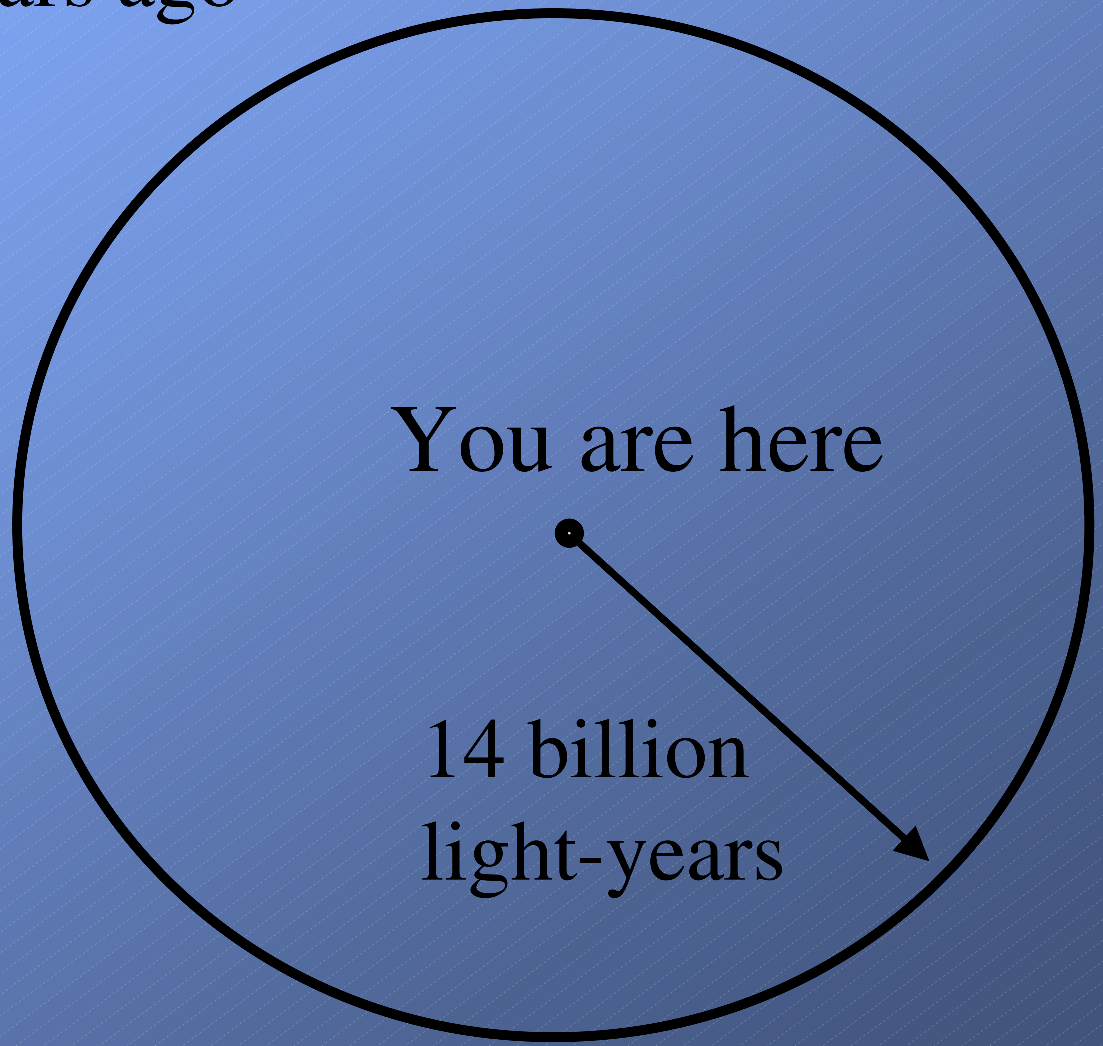
"Flat"
(critical density)

"Closed"
(more matter)

NASA COBE map of CMB temperature (1991-1994)



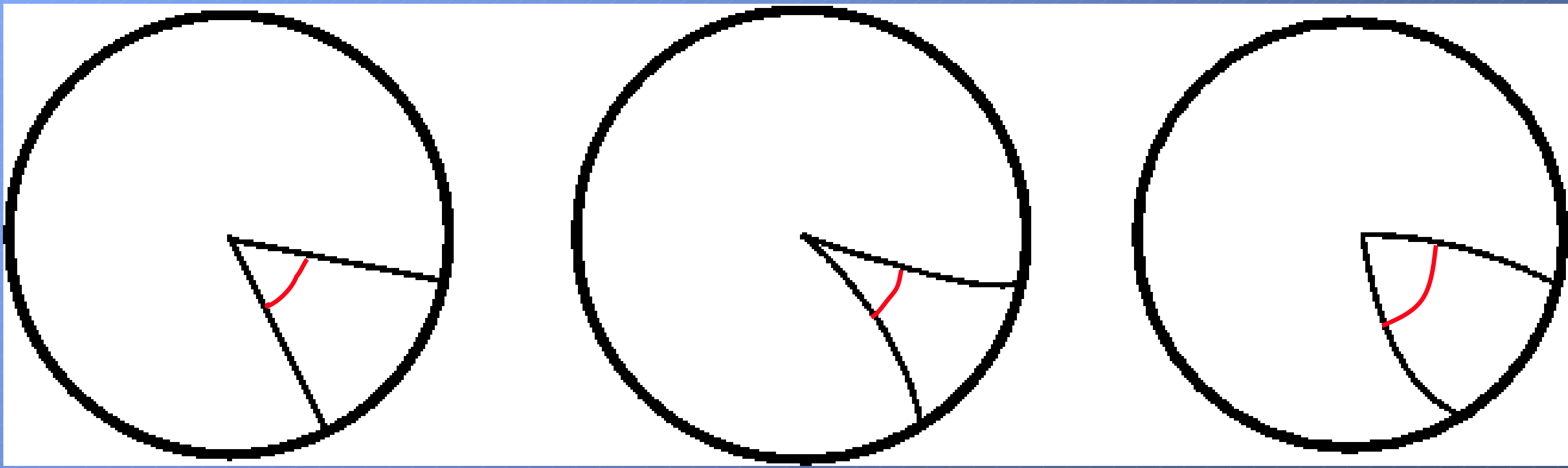
CMB that we see originates from edge of observable Universe as it was ~400,000 years after the big bang, ~14 billion years ago



MK, Spergel, & Sugiyama 1994

The Geometry of the Universe

Warped spacetime acts as lens:



“flat” “open” “closed”

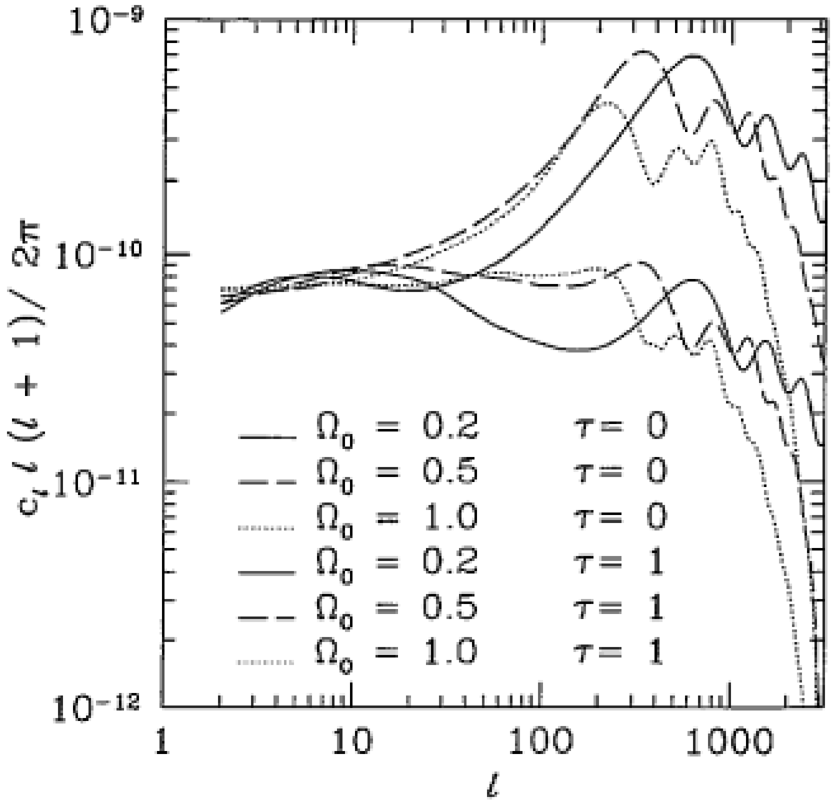
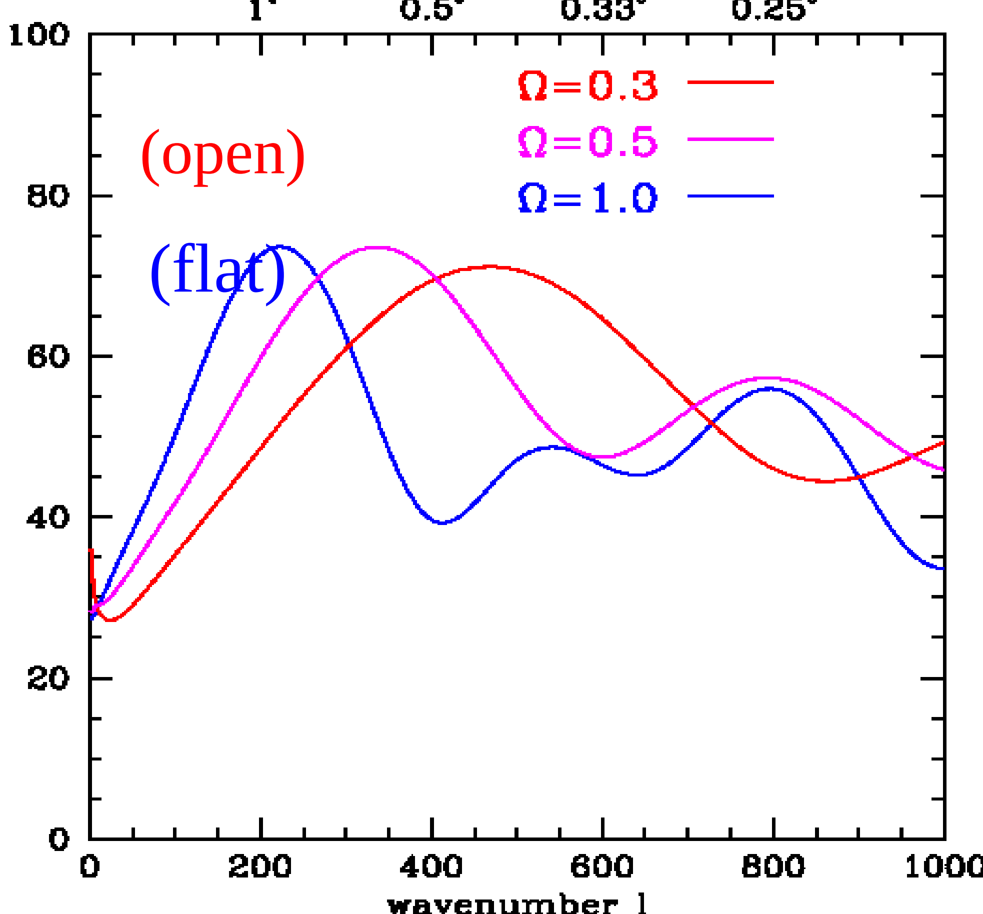


FIG. 2.—*COBE*-normalized CMB spectrum as a function of multipole moment l for several values of Ω and for optical depths $\tau = 0$ (no reionization) and $\tau = 1$. Here we have taken $\Omega_b = 0.06$ and $h = 0.5$. Throughout the paper, $\Lambda = 0$.



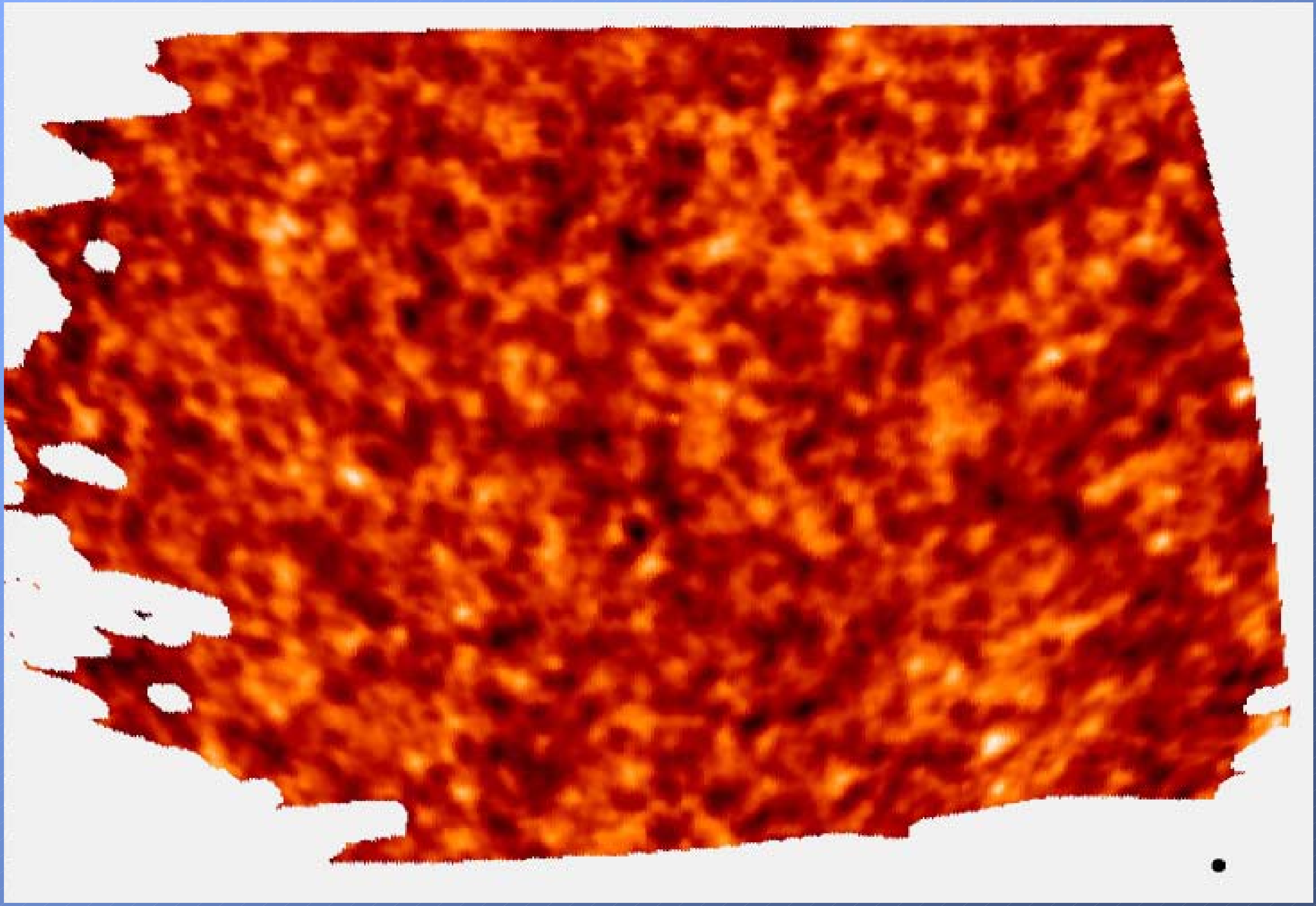
CMB determination

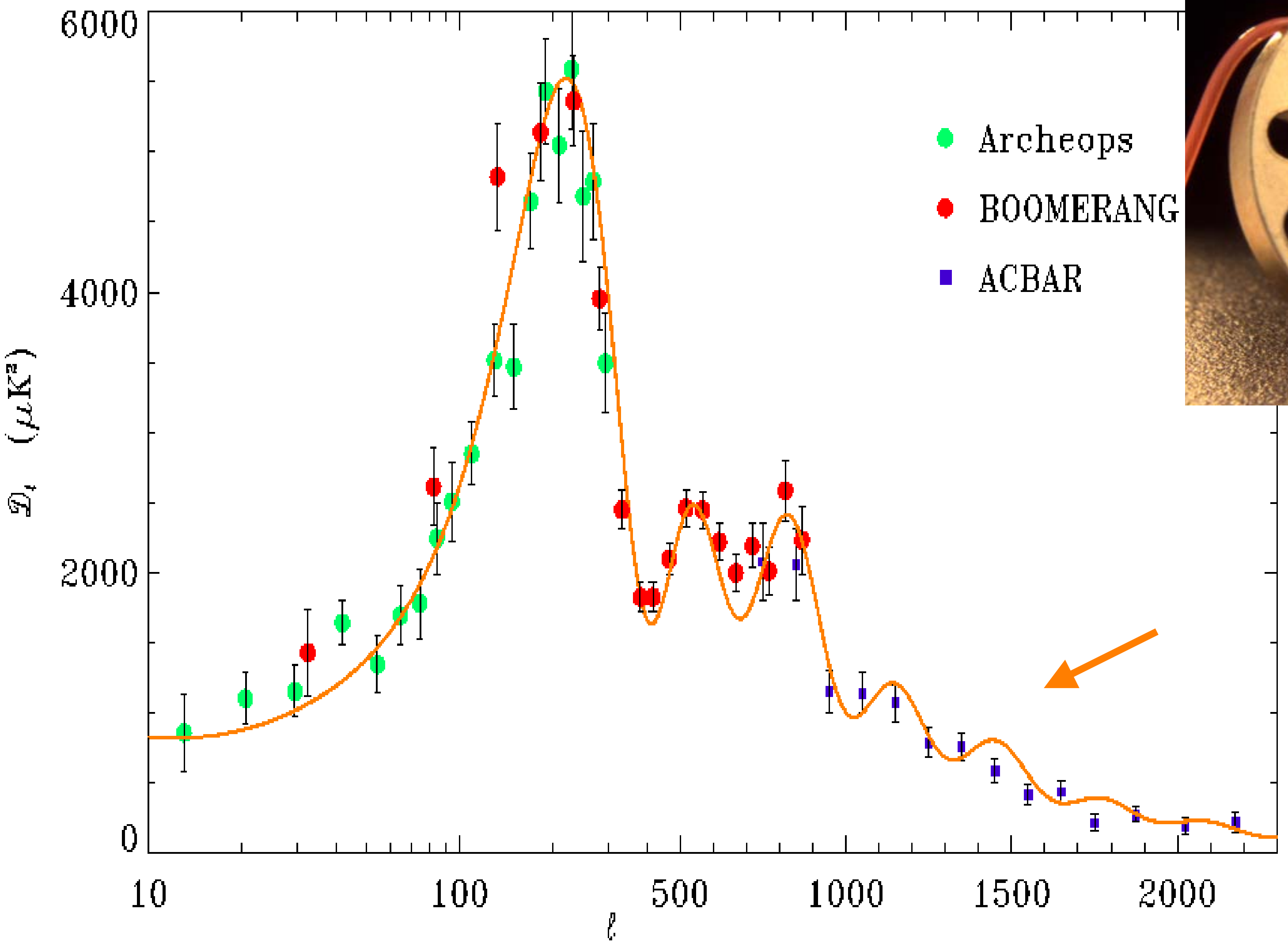
of the geometry

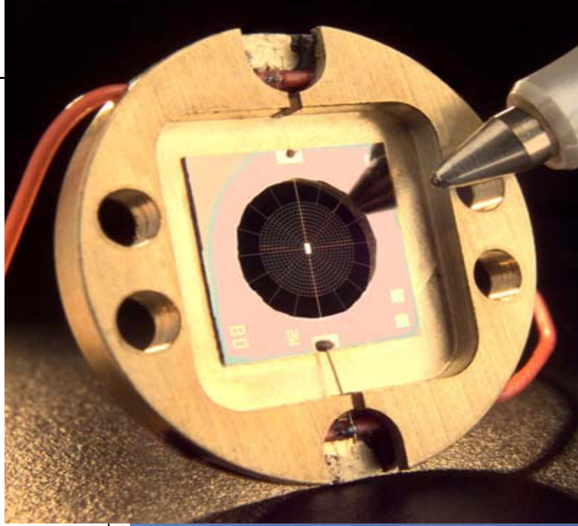
(MK, Spergel,

and Sugiyama, 1994)

BOOMERanG map of CMB (2000)







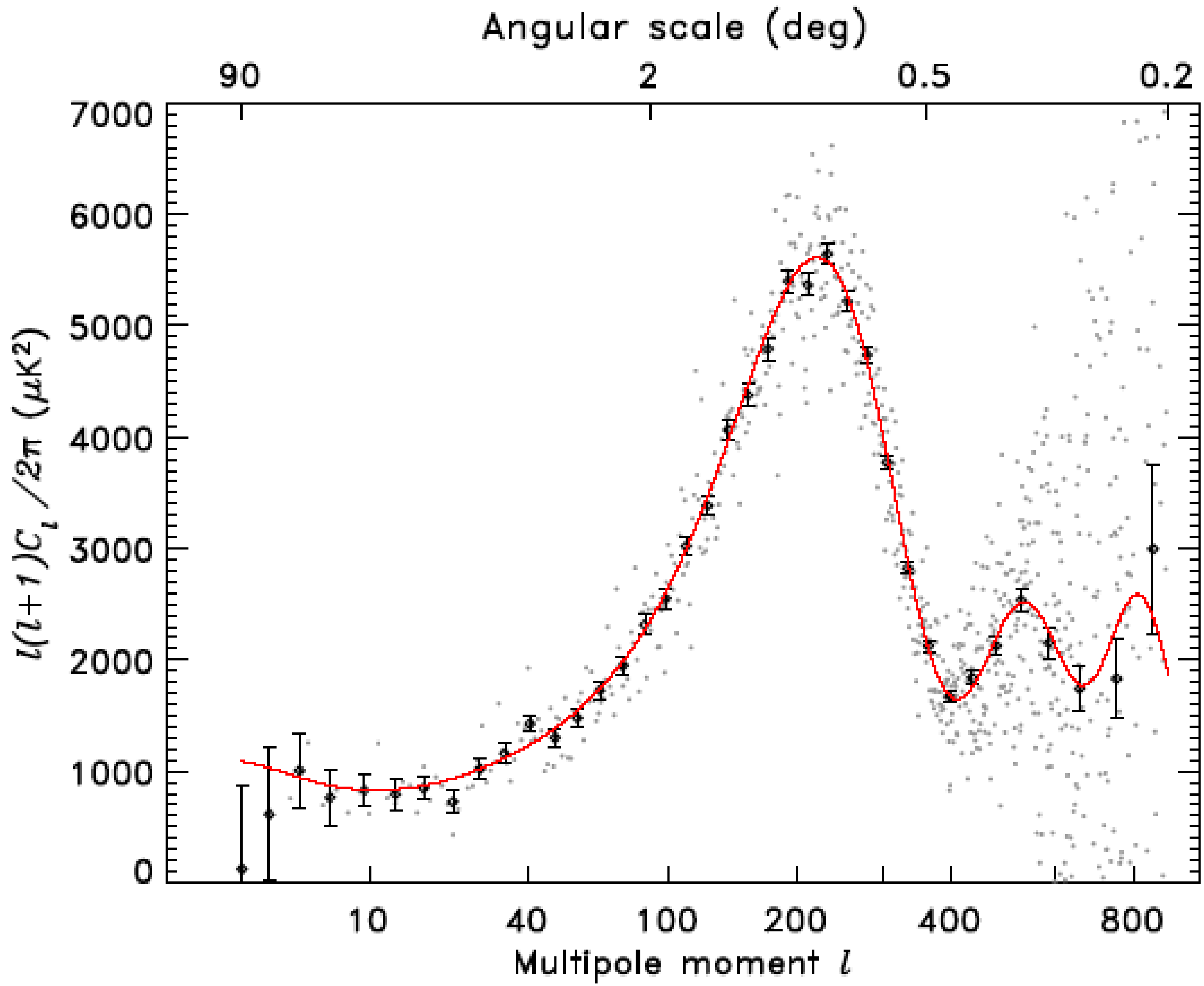
Pre-WMAP bolometer-based measurements: BOOMERanG, ARCHEOPS, ACBAR

BOOMERanG (2002)

TABLE 5

RESULTS OF PARAMETER EXTRA

Priors	Ω_{tot}	n_s	$\Omega_b h^2$	$\Omega_{\text{CDM}} h^2$	Ω
Weak only	1.02 ± 0.06	$0.96^{+0.10}_{-0.09}$	$0.022^{+0.004}_{-0.003}$	0.13 ± 0.05	(0.51
LSS	$1.02^{+0.04}_{-0.05}$	$0.97^{+0.10}_{-0.08}$	$0.022^{+0.004}_{-0.003}$	$0.13^{+0.03}_{-0.02}$	0.55
SN1a	$1.02^{+0.07}_{-0.05}$	$0.99^{+0.11}_{-0.10}$	0.023 ± 0.004	0.10 ± 0.04	0.73
LSS and SN1a	$0.99^{+0.03}_{-0.04}$	$1.03^{+0.10}_{-0.09}$	$0.023^{+0.003}_{-0.003}$	$0.14^{+0.03}_{-0.02}$	0.65
$h = 0.71 \pm 0.08$	$0.98^{+0.04}_{-0.05}$	$0.97^{+0.10}_{-0.09}$	$0.022^{+0.004}_{-0.003}$	$0.14^{+0.05}_{-0.04}$	0.62
Flat	(1.00)	$0.95^{+0.09}_{-0.08}$	0.021 ± 0.003	0.13 ± 0.04	(0.57
Flat and LSS	(1.00)	$0.98^{+0.10}_{-0.07}$	0.021 ± 0.003	0.13 ± 0.01	0.62
Flat and SN1a	(1.00)	$0.98^{+0.11}_{-0.09}$	0.022 ± 0.003	$0.12^{+0.01}_{-0.02}$	0.68
Flat, LSS and SN1a...	(1.00)	$1.03^{+0.10}_{-0.09}$	0.023 ± 0.003	0.13 ± 0.01	0.66



Now even more precise from WMAP

Where did large scale structure (e.g., galaxies, clusters, larger-scale clustering) come from?

explosions

Late-time phase transitions

Superconducting cosmic strings

Cosmic strings

textures

Global monopoles

Soft phase transitions

Isocurvature CDM perturbations

Seed models

Primordial adiabatic perturbations

Isocurvature baryon perturbations

Rolling scalar fields

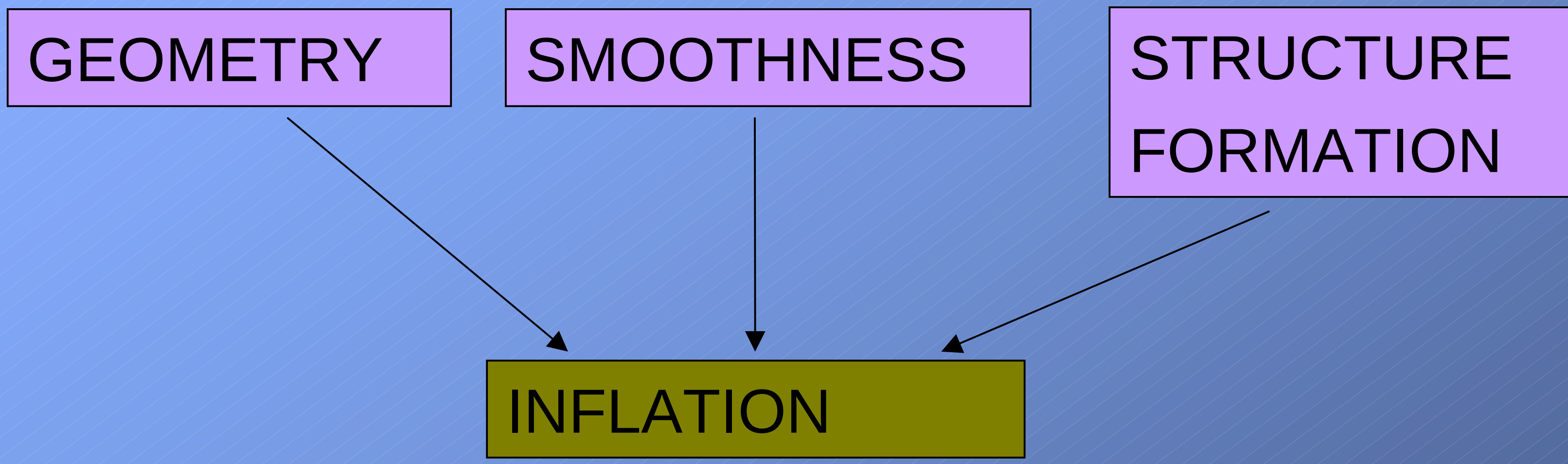
Loitering universe

Pre-CMB

Where did large-scale structure (e.g., galaxies, clusters, larger-scale clustering) come from?

Post CMB:

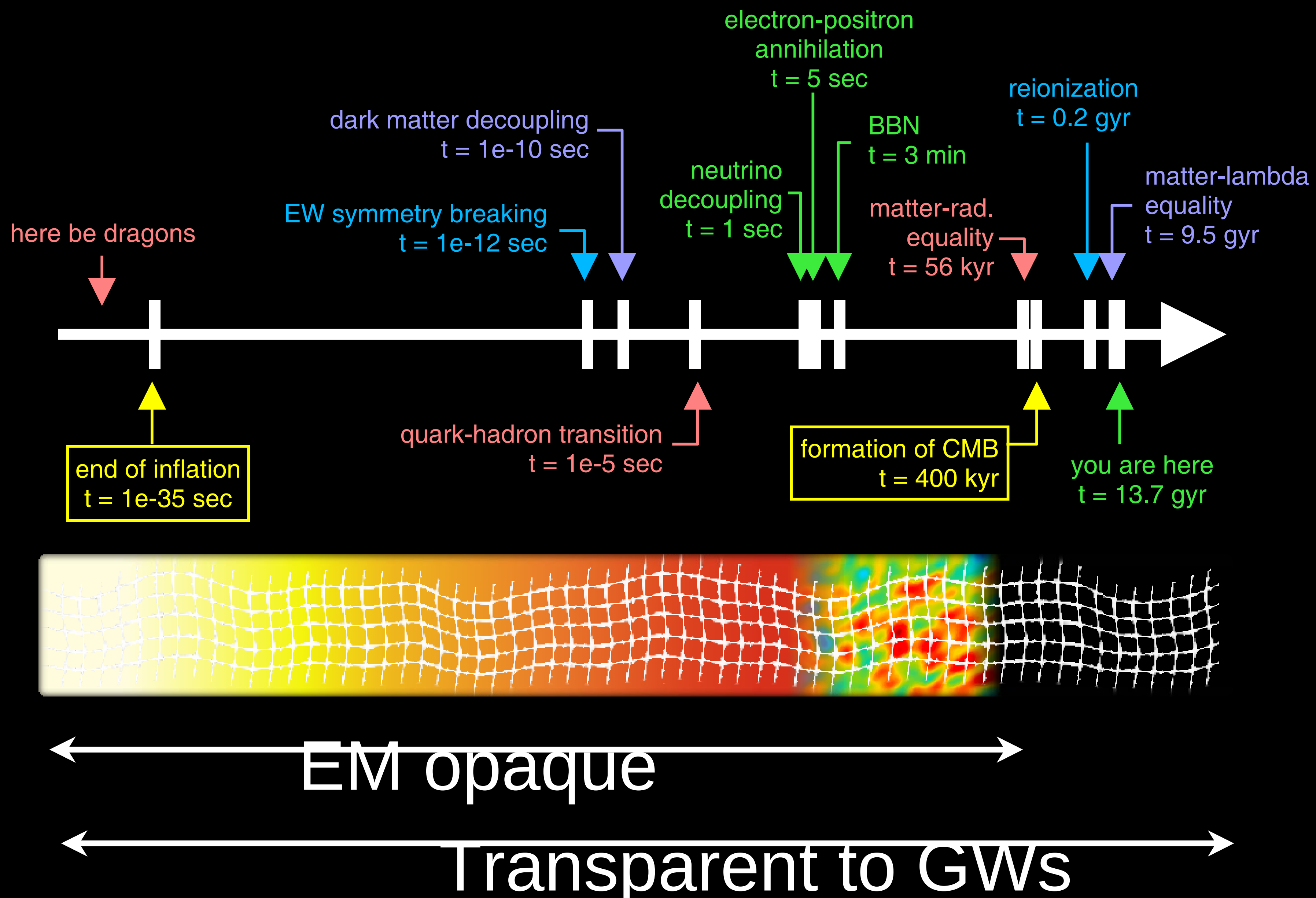
gravitational infall from nearly scale-invariant spectrum of primordial adiabatic perturbations

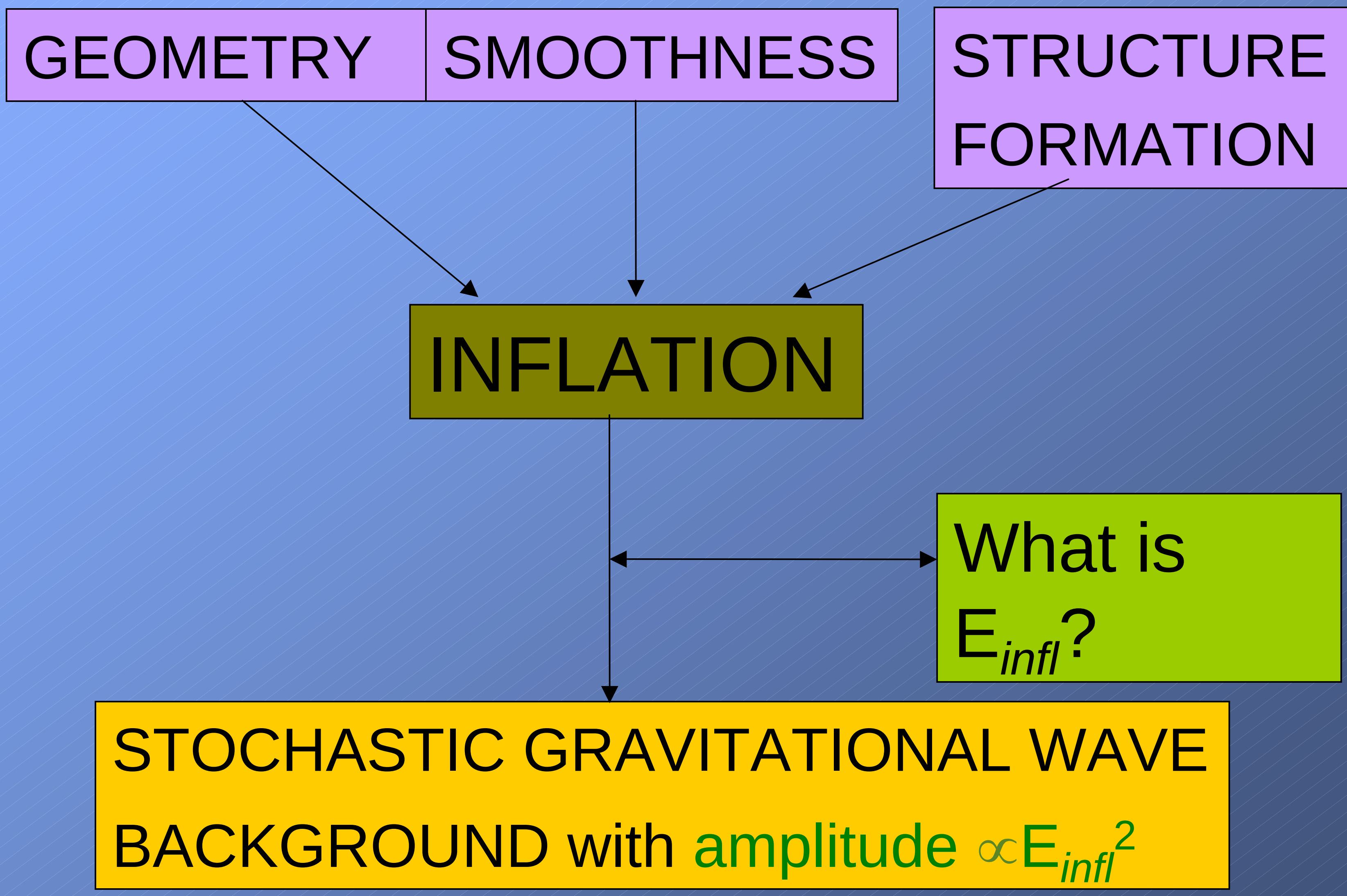

GEOMETRY
SMOOTHNESS
STRUCTURE FORMATION
INFLATION

WHAT NEXT???

History of the universe (to scale!)

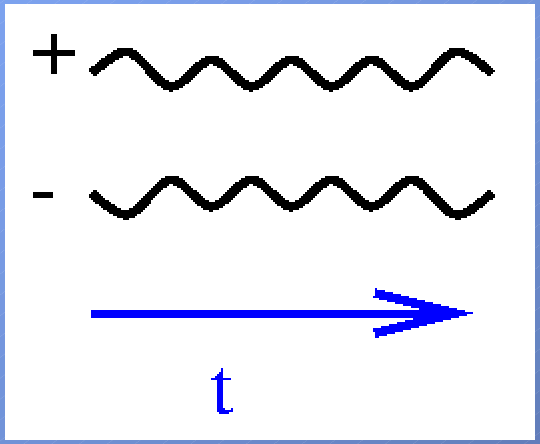
(from H. C. Chiang)




GEOMETRY
SMOOTHNESS
STRUCTURE FORMATION
INFLATION
What is E_{infl}?
STOCHASTIC GRAVITATIONAL WAVE BACKGROUND with amplitude $\propto E_{infl}^2$

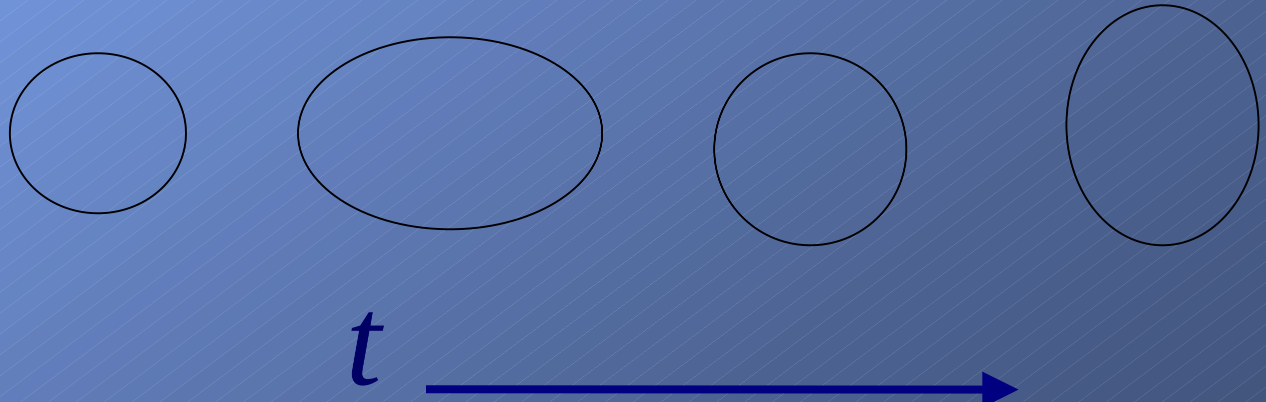
Detection of electromagnetic wave:

look for oscillations of test charges

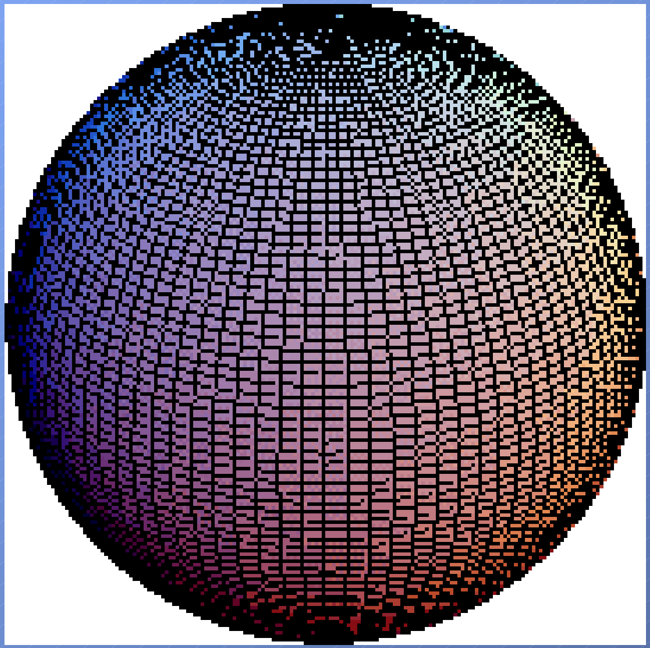


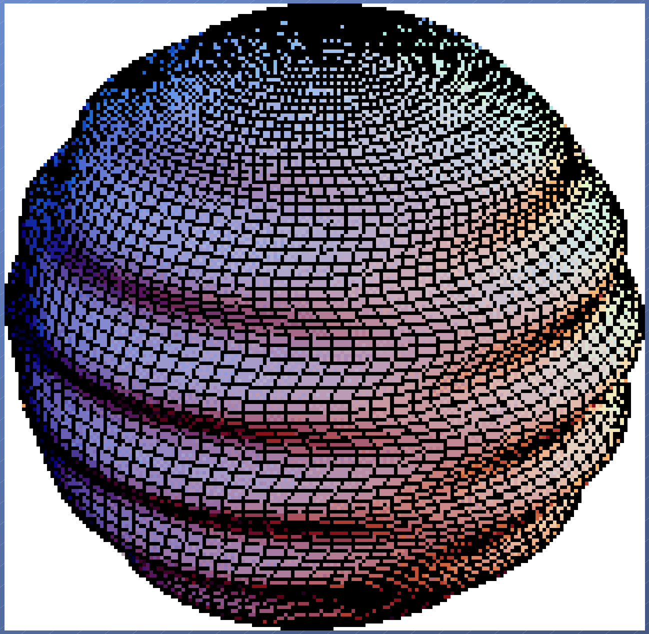
Detection of gravitational wave:

look for quadrupole oscillations of a ring of test masses



Detection of ultra-long-wavelength GWs from inflation: use plasma at CMB surface of last scatter as sphere of test masses.





(from H. C. Chiang)

Polarization in the CMB

Polarization induced by Thomson scattering

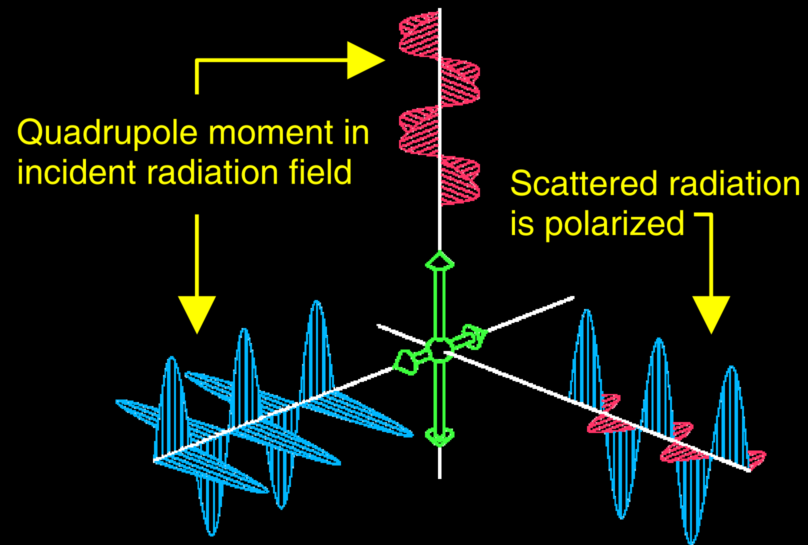


Image: M. Hedman

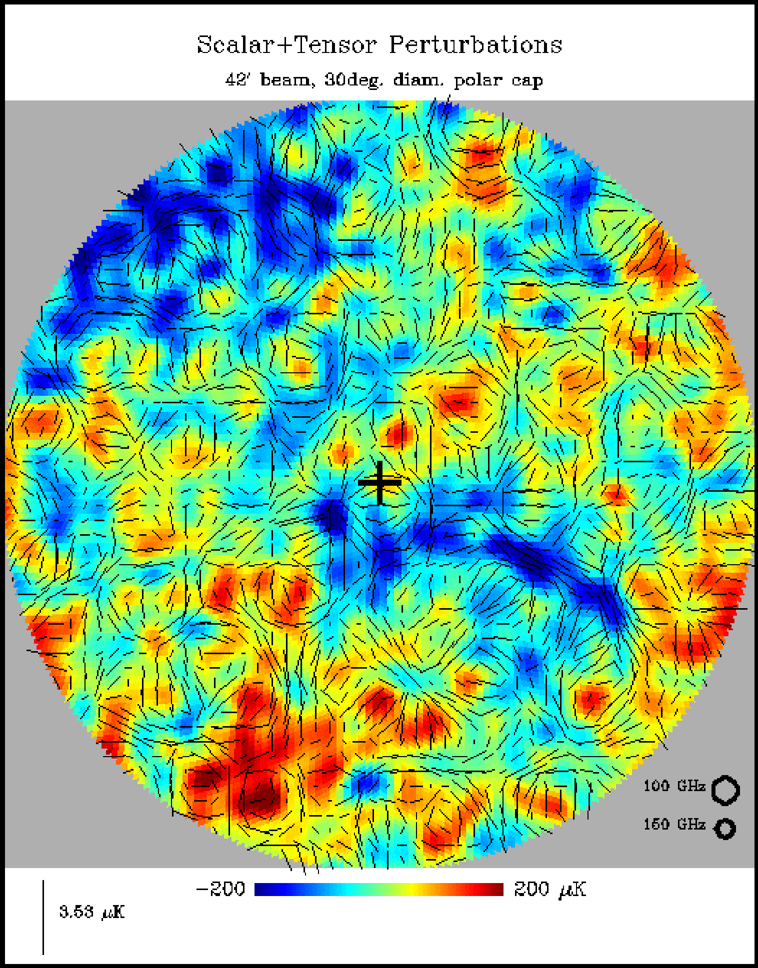


Image: E. Hivon

Detection of gravitational waves with CMB polarization

(MK, Kosowsky, Stebbins, 1996; Seljak & Zaldarriaga 1996)

Temperature map: $T(\hat{n})$

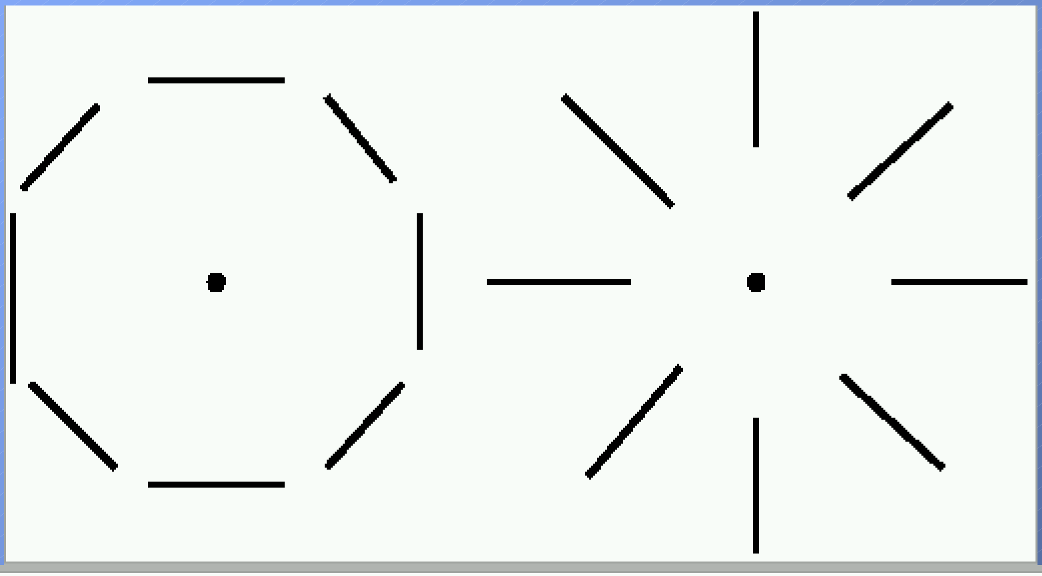
Polarization Map: $\vec{P}(\hat{n}) = \vec{\nabla} A + \vec{\nabla} \times \vec{B}$

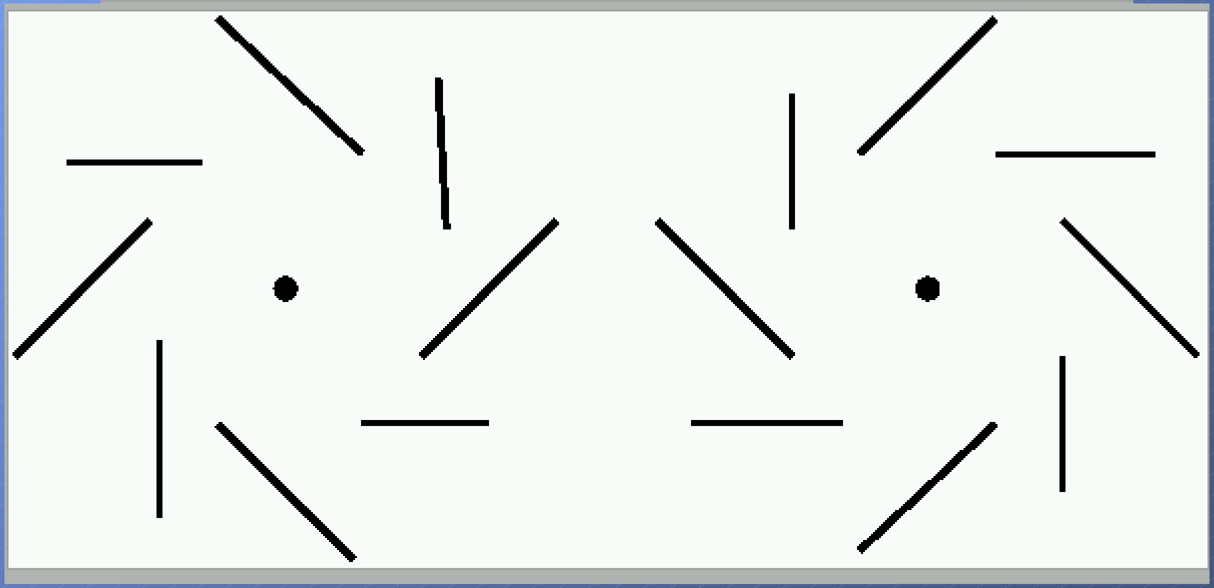
Density perturbations have no handedness"
so they can*not* produce a polarization with a curl

Gravitational waves do have a handedness, so they can (and do) produce a curl

➡ Model-independent probe of gravitational waves!

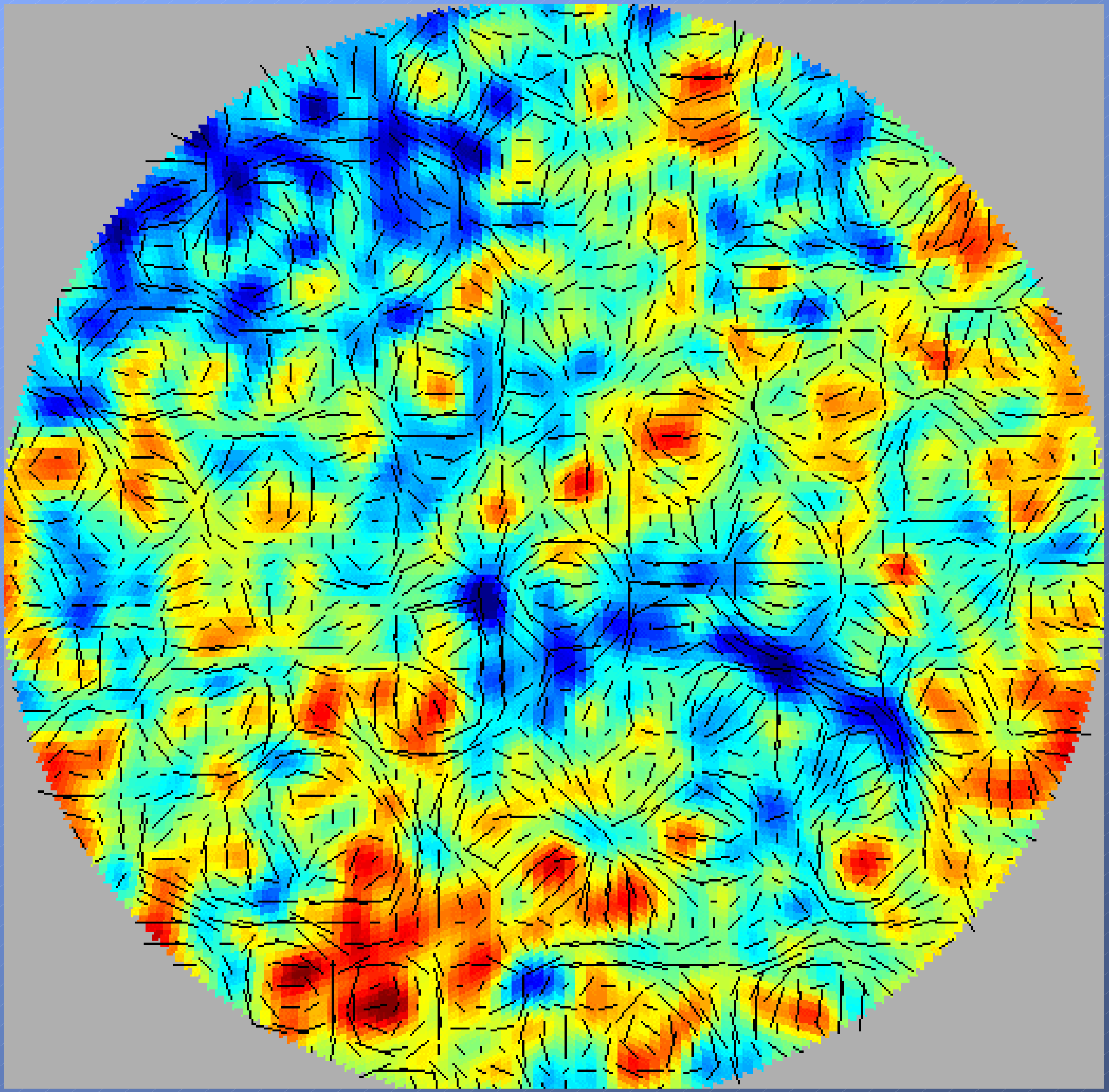
“Curl-free” polarization patterns



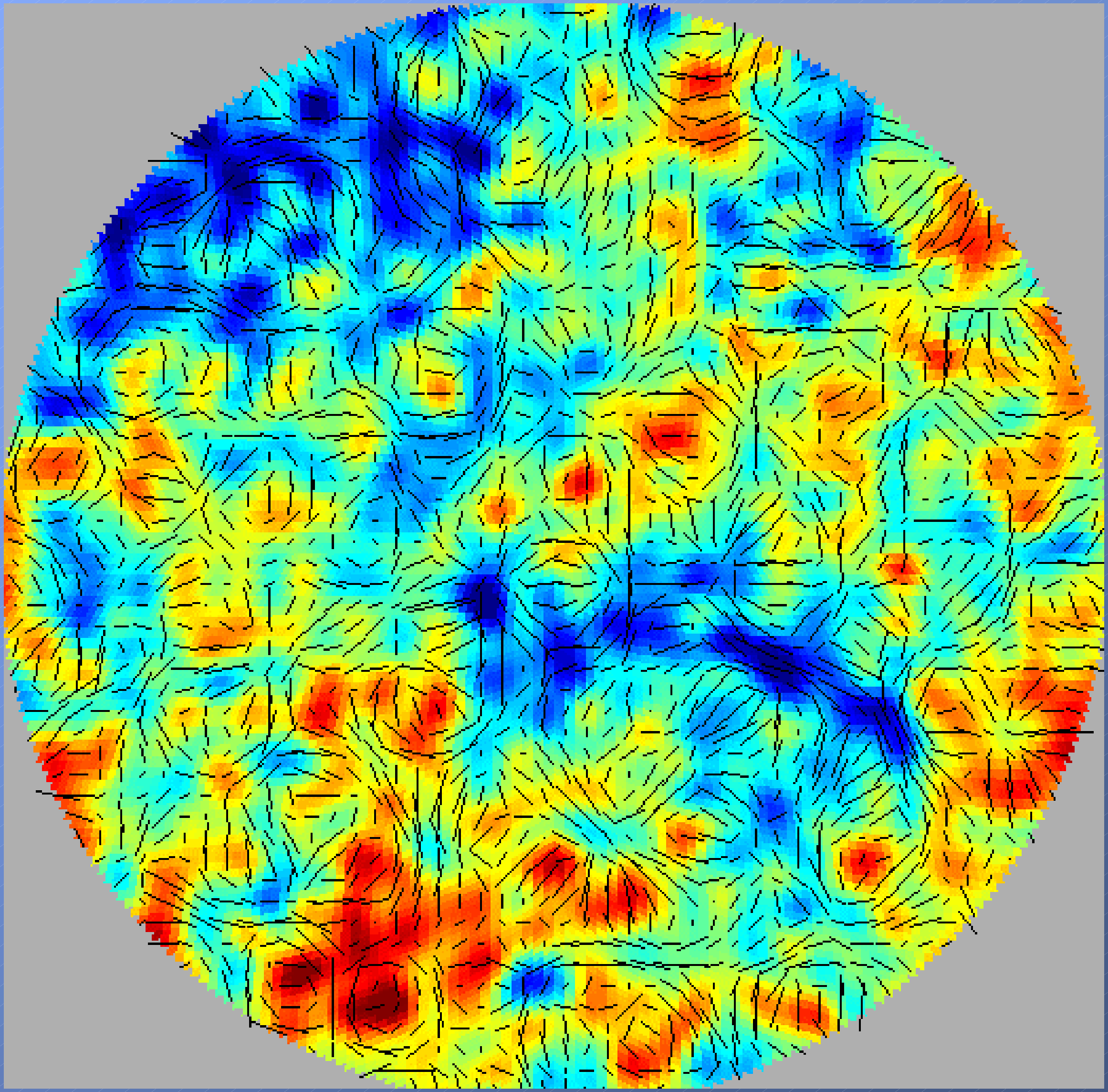


“curl” patterns

No Gravity Waves

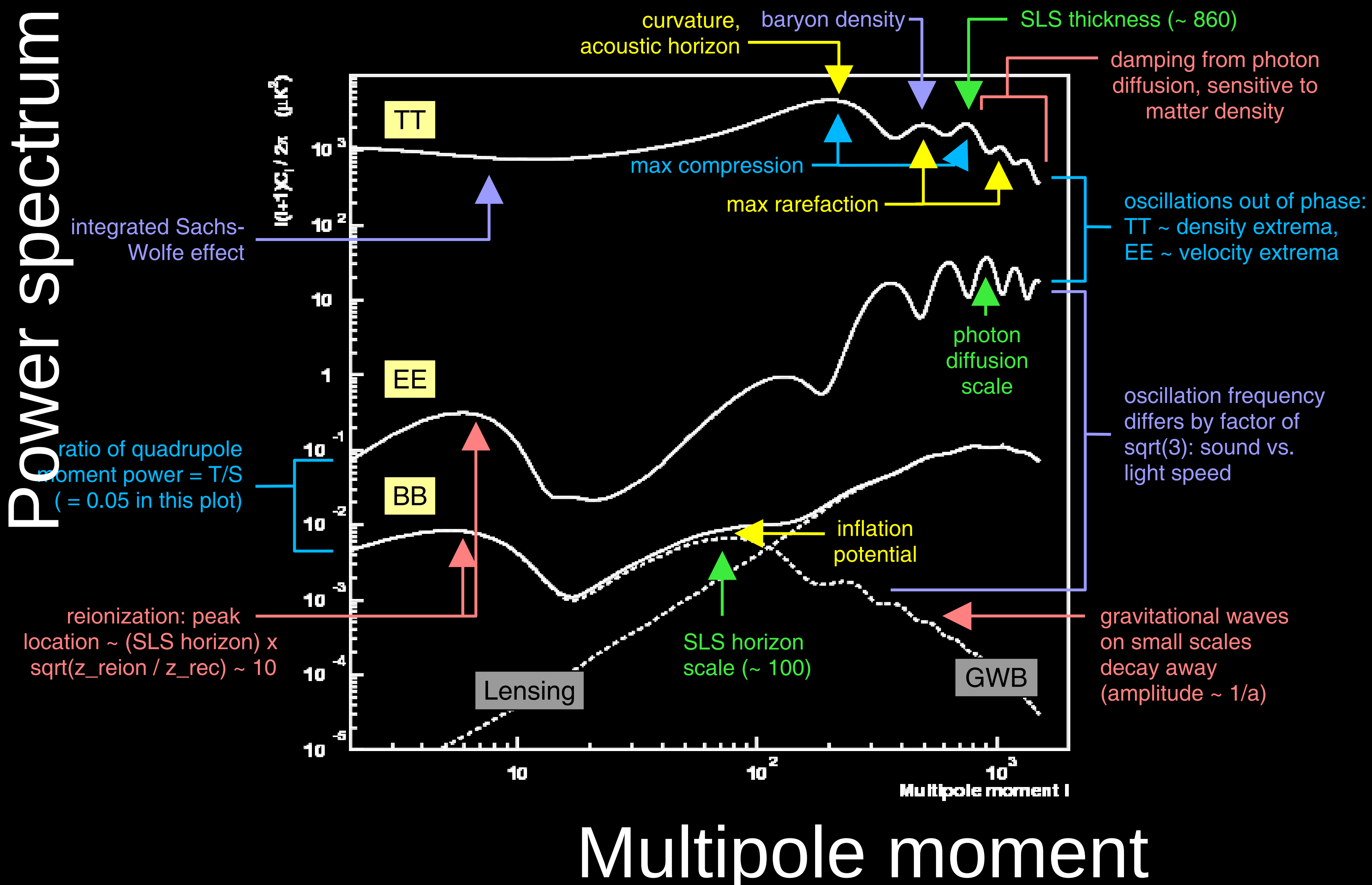


Gravity Waves

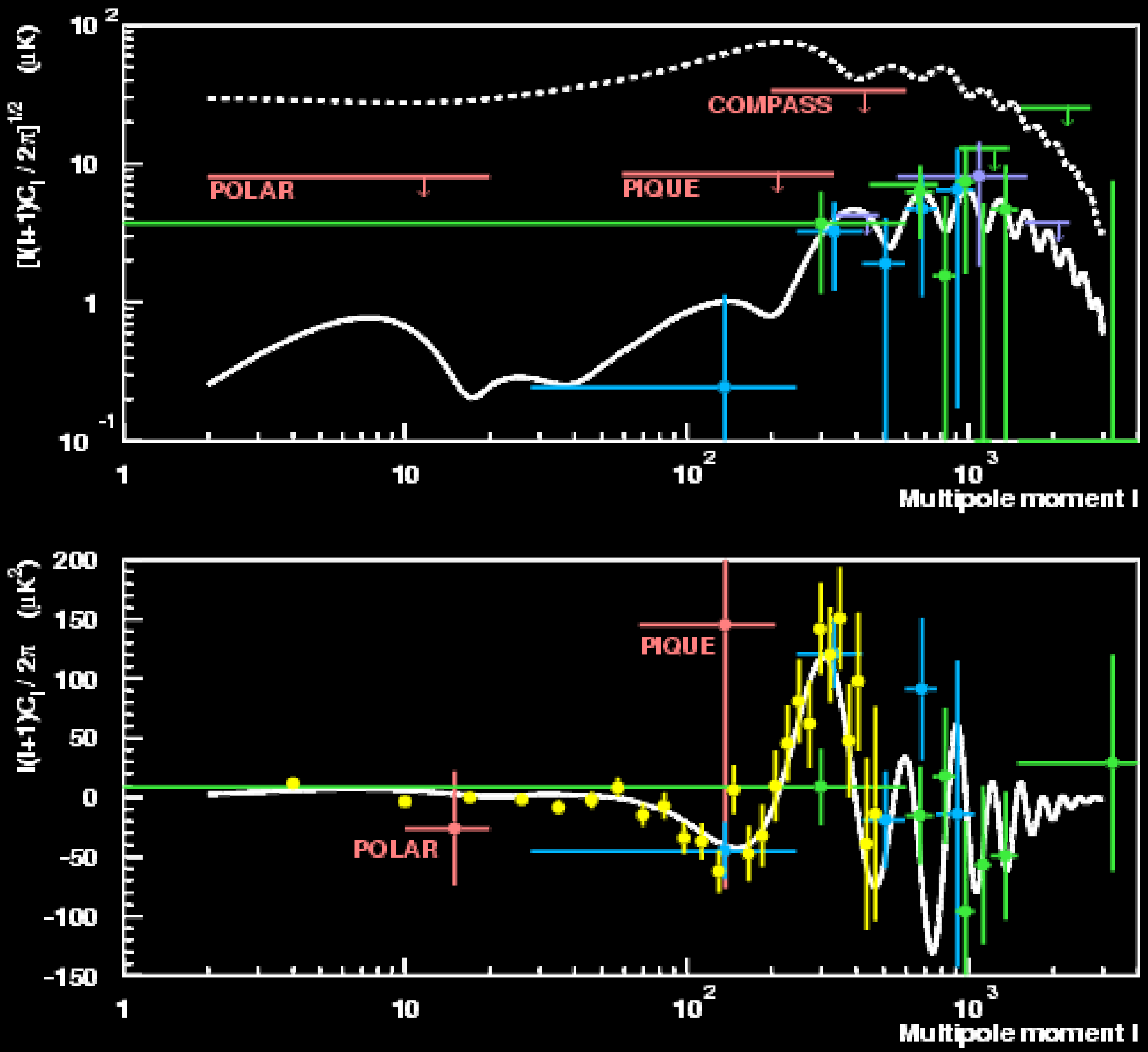


Power spectrum anatomy 101

(from H. C. Chiang)



Which experiments have measured what



Temperature spectrum

Beaten to death by numerous experiments

EE and TE polarization

POLAR, PIQUE, COMPASS
WMAP
CBI
DASI
CAPMAP

BB polarization

Upper limits ~ 2 μK^2 (from DASI, CBI, B2K)

T/S < 0.9 from WMAP

Recall, GW amplitude is $\propto E_{\mathrm{infl}}^2$

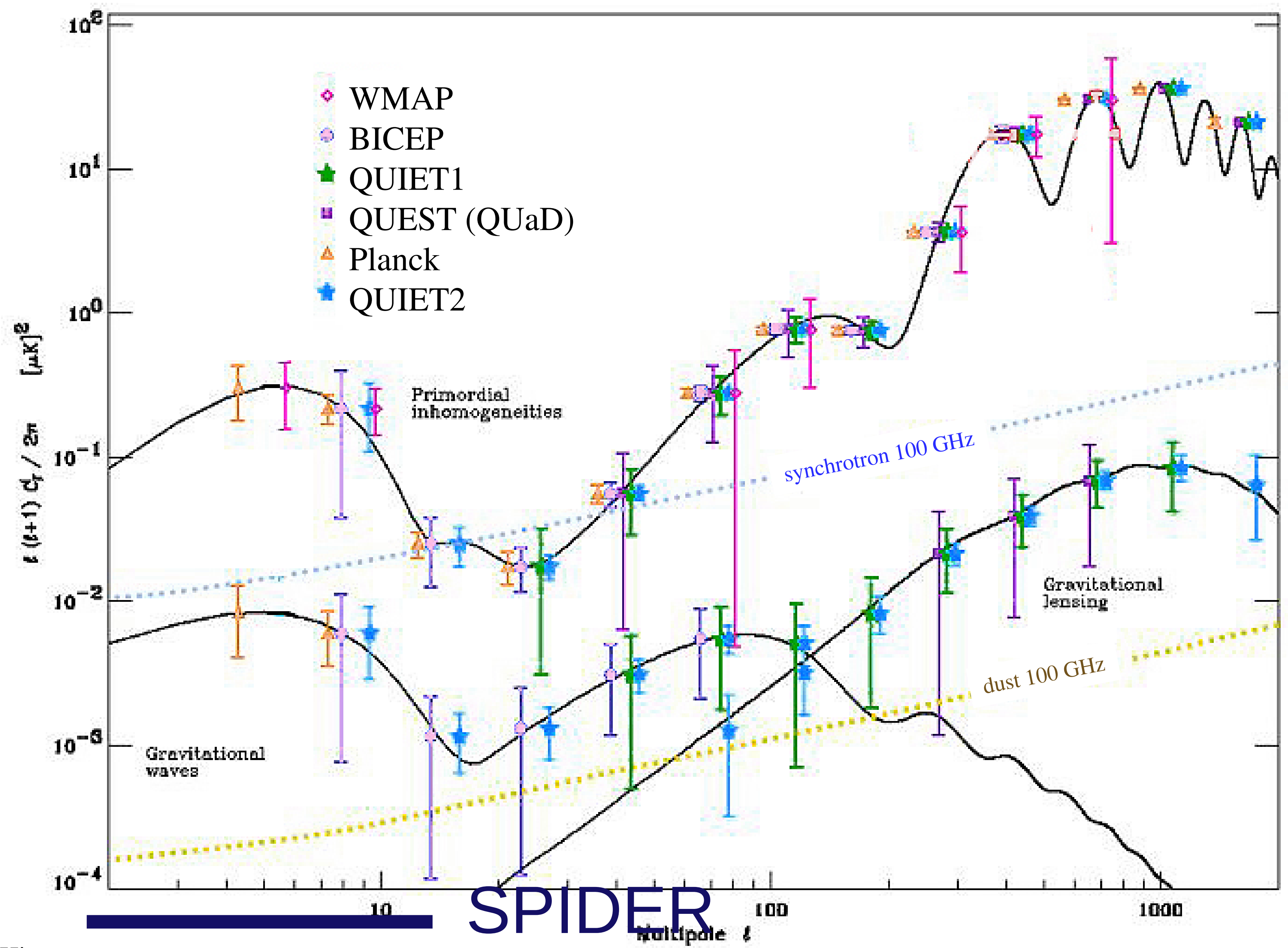
GWs $\Rightarrow$ ΔT

And from COBE, $E_{\mathrm{infl}} < 3\times10^{16}$ GeV

GWs $\Rightarrow$ unique polarization pattern. Is it detectable?

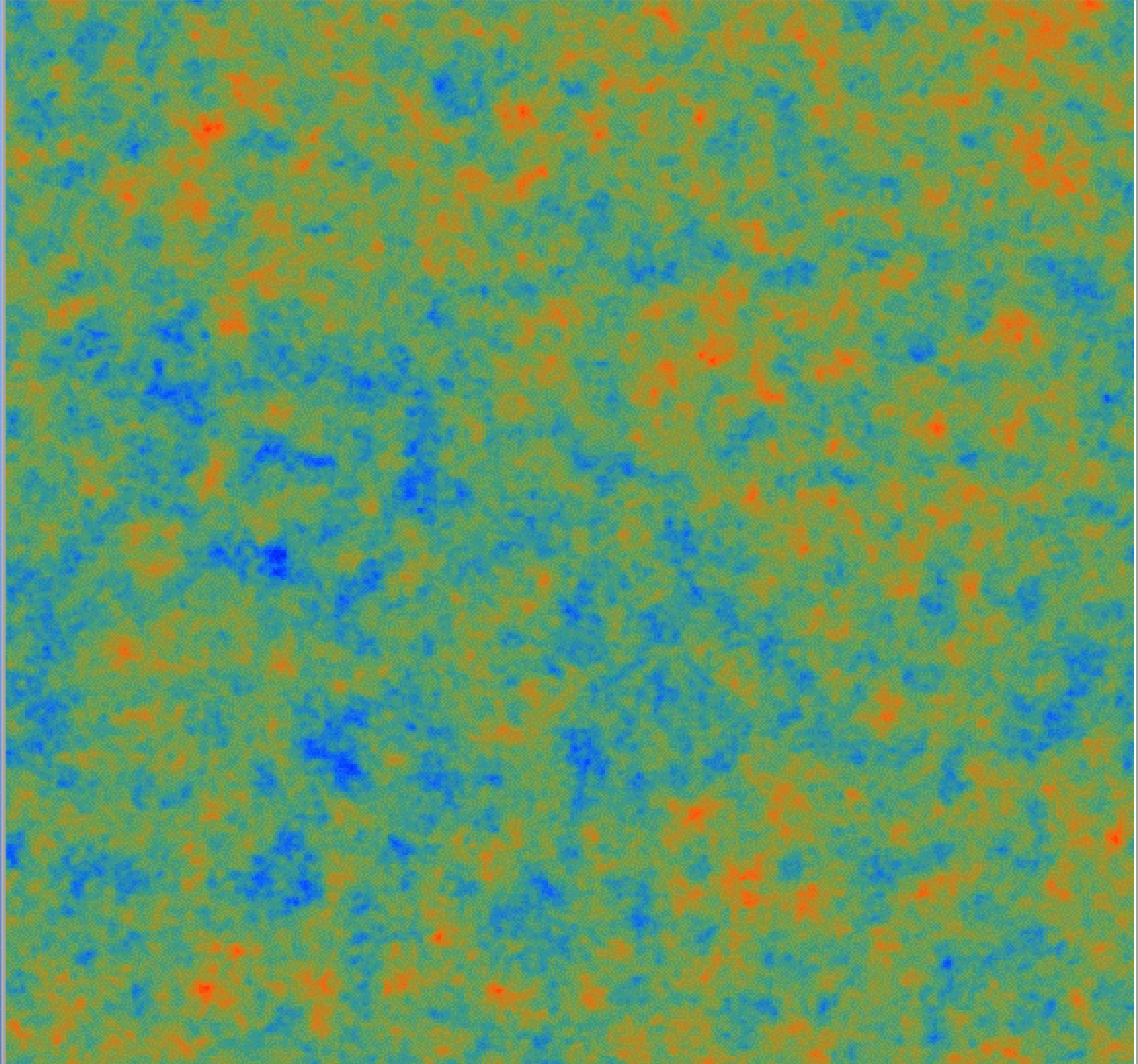
If $E << 10^{15}$ GeV (e.g., if inflation from PQSB), then polarization far too small to ever be detected.

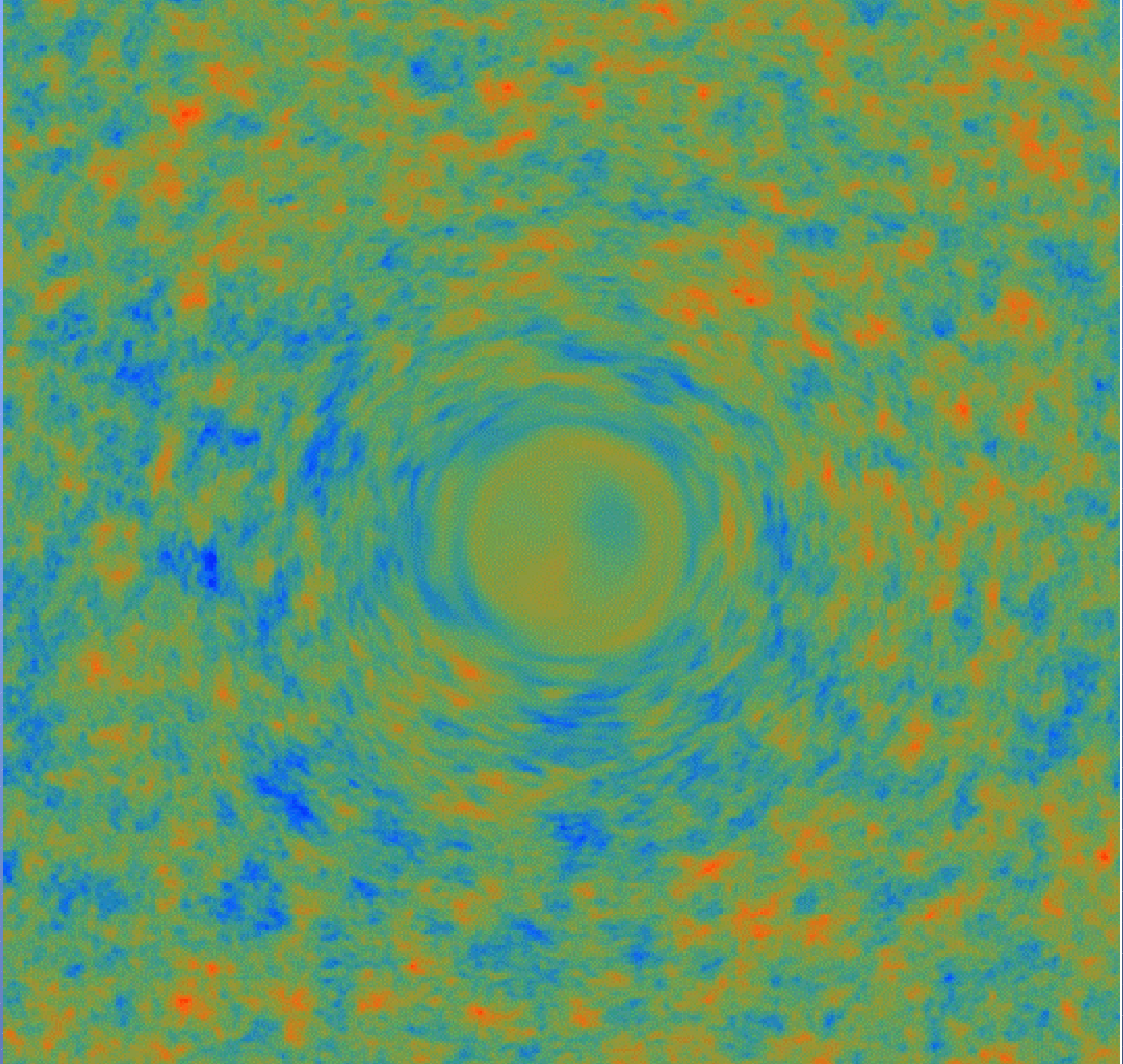
But, if $E \sim 10^{15\text{-}16}$ GeV (i.e., if inflation has something to do with GUTs), then polarization signal is conceivably detectable by Planck or realistic post-Planck experiment!!!


WMAP
BICEP
QUIET1
QUEST (QUaD)
Planck
QUIET2
Primordial inhomogeneities
synchrotron 100 GHz
Gravitational lensing
dust 100 GHz
Gravitational waves
$\ell(\ell+1)\,C_\ell\,/\,2\pi\ \ [\mu K]^2$
Multipole ℓ
10
100
1000
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
SPIDER
Hivon

(Kesden, Cooray, MK 2002;
Knox, Song 2002)

To go beyond Planck, will require high resolution temperature and polarization maps to disentangle cosmic shear contribution to curl component from that due to inflationary gravitational waves.





Lensing shifts position on sky:

$$\begin{pmatrix} T \\ Q \\ U \end{pmatrix}_{obs.} (\vec{\theta}) = \begin{pmatrix} T \\ Q \\ U \end{pmatrix}_{ls} (\vec{\theta} + \delta\vec{\theta}) \simeq \begin{pmatrix} T \\ Q \\ U \end{pmatrix}_{ls} (\vec{\theta}) + \delta\vec{\theta} \cdot \nabla \begin{pmatrix} T \\ Q \\ U \end{pmatrix}_{ls} (\vec{\theta}),$$

Where the projected grav potential is

$$\varphi(\hat{n}) = -2 \int_0^{r_{ls}} dr \frac{d_A(r_{ls}, r)}{d_A(r,0) d_A(r_{ls},0)} \Phi(r, \hat{n}r)$$

Define the Fourier-mode angles

$$\frac{l_x^2 - l_y^2}{l_x^2 + l_y^2} = \cos 2\phi_{\vec{l}}, \qquad \frac{2 l_x l_y}{l_x^2 + l_y^2} = \sin 2\phi_{\vec{l}},$$

The grad/curl (E/B) Fourier modes are:

$$\begin{pmatrix} G \\ C \end{pmatrix}(\vec{l}) = \frac{1}{2}\begin{pmatrix} \cos 2\phi_{\vec{l}} & \sin 2\phi_{\vec{l}} \\ \sin 2\phi_{\vec{l}} & -\cos 2\phi_{\vec{l}} \end{pmatrix}\begin{pmatrix} Q \\ U \end{pmatrix}(\vec{l}).$$

Or inverted…

$$Q(\vec{l}) = 2G(\vec{l})\cos 2\phi_{\vec{l}}, \qquad U(\vec{l}) = -2G(\vec{l})\sin 2\phi_{\vec{l}},$$

So lensing induces change to Q…

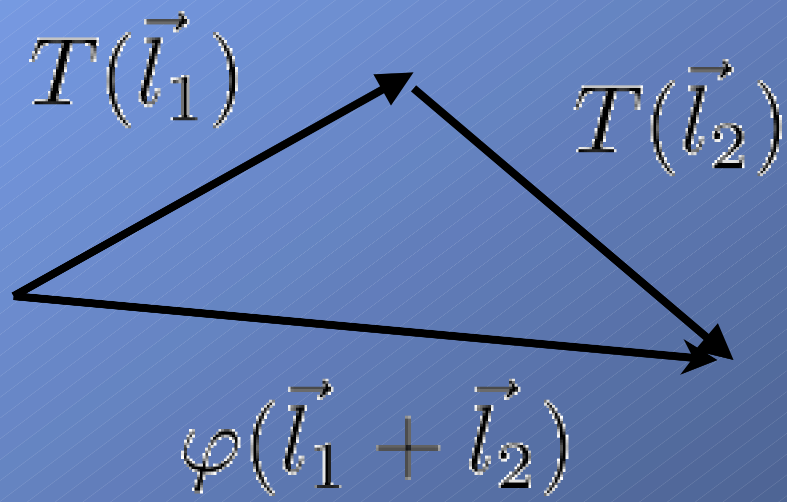
$$\delta Q(\vec{\theta}) = \nabla Q \cdot \nabla \varphi = \int \frac{d^2\vec{l}}{(2\pi)^2} e^{-i\vec{l}\cdot\vec{\theta}} (\nabla Q \cdot \nabla \varphi)_{\vec{l}},$$

and analogously for U

and in Fourier space…

$$\delta Q(\vec{l}) \equiv (\nabla Q \cdot \nabla \varphi)_{\vec{l}} = 2 \int \frac{d^2 \vec{l}_1}{(2\pi)^2} [\vec{l}_1 \cdot (\vec{l} - \vec{l}_1)] G(\vec{l}_1) \varphi(\vec{l} - \vec{l}_1) \cos 2\phi_{\vec{l}_1},$$

and similarly for U

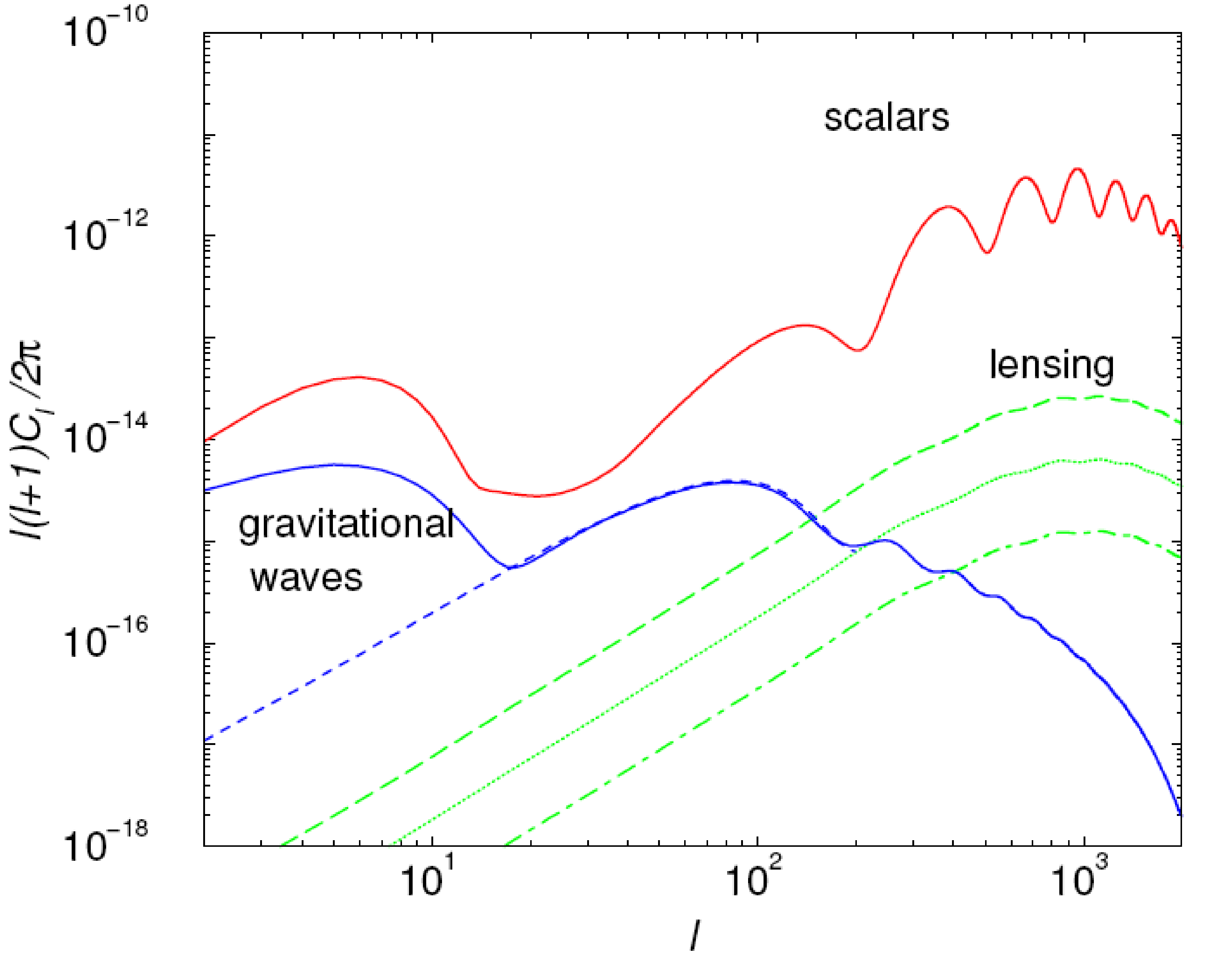


and so lensing induces a curl

$$
\begin{aligned}
C(\vec{l}) &= \frac{1}{2}[\sin 2\phi_{\vec{l}} Q(\vec{l}) - \cos 2\phi_{\vec{l}} U(\vec{l})] \\
&= \int \frac{d^2\vec{l_1}}{(2\pi)^2}[\vec{l_1}\cdot(\vec{l}-\vec{l_1})]G(\vec{l_1})\varphi(\vec{l}-\vec{l_1})\sin 2(\phi_{\vec{l}}-\phi_{\vec{l_1}}) \\
&= \int \frac{d^2\vec{l_1}}{(2\pi)^2}[\vec{l_1}\cdot(\vec{l}-\vec{l_1})]G(\vec{l_1})\varphi(\vec{l}-\vec{l_1})\sin 2\phi_{\vec{l_1}}\,.
\end{aligned}
$$

even if there was no primordial curl, with power spectrum

$$
C_l^{\rm CC} = \int \frac{d^2\vec{l_1}}{(2\pi)^2}[\vec{l_1}\cdot(\vec{l}-\vec{l_1})]^2 \sin^2 2\phi_{\vec{l_1}}\, C^{\varphi\varphi}_{|\vec{l}-\vec{l_1}|} C^{\rm GG}_{l_1}
$$



10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-18}
10^{1}
10^{2}
10^{3}
l
$l(l+1)C_l/2\pi$
scalars
lensing
gravitational
waves

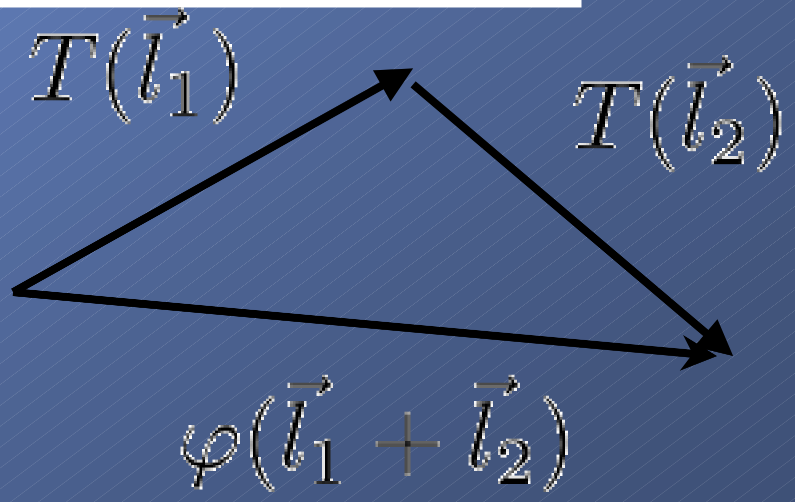
How can we correct for it? T also lensed. In absence of lensing,

$$\left\langle T(\vec{l})T(\vec{l}')\right\rangle = 0 \quad \text{for} \quad \vec{l} \neq \vec{l}'.$$

but with lensing,

$$\left\langle T(\vec{l})T(\vec{l}')\right\rangle = f(\vec{l},\vec{l}')\varphi(\vec{L}) \qquad \vec{l} \neq \vec{l}',$$

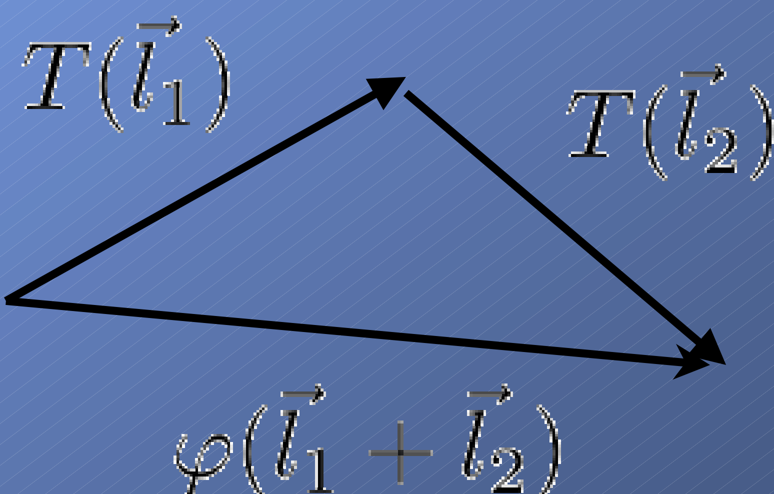
$$f(\vec{l},\vec{l}') = C_l^{\mathrm{TT}}(\vec{L}\cdot\vec{l}) + C_l^{\mathrm{TT}}(\vec{L}\cdot\vec{l}').$$

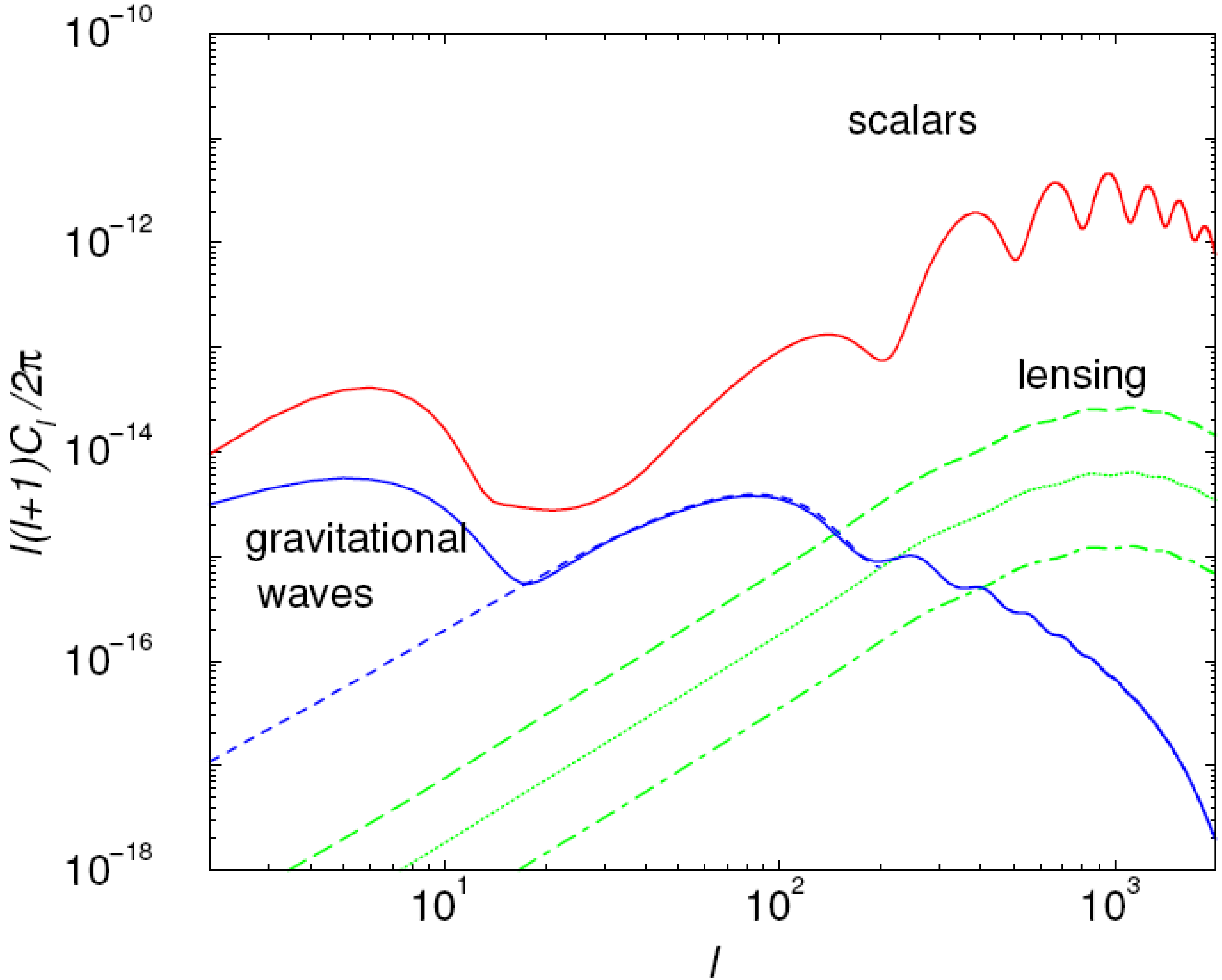


We can therefore reconstruct the deflection angle….

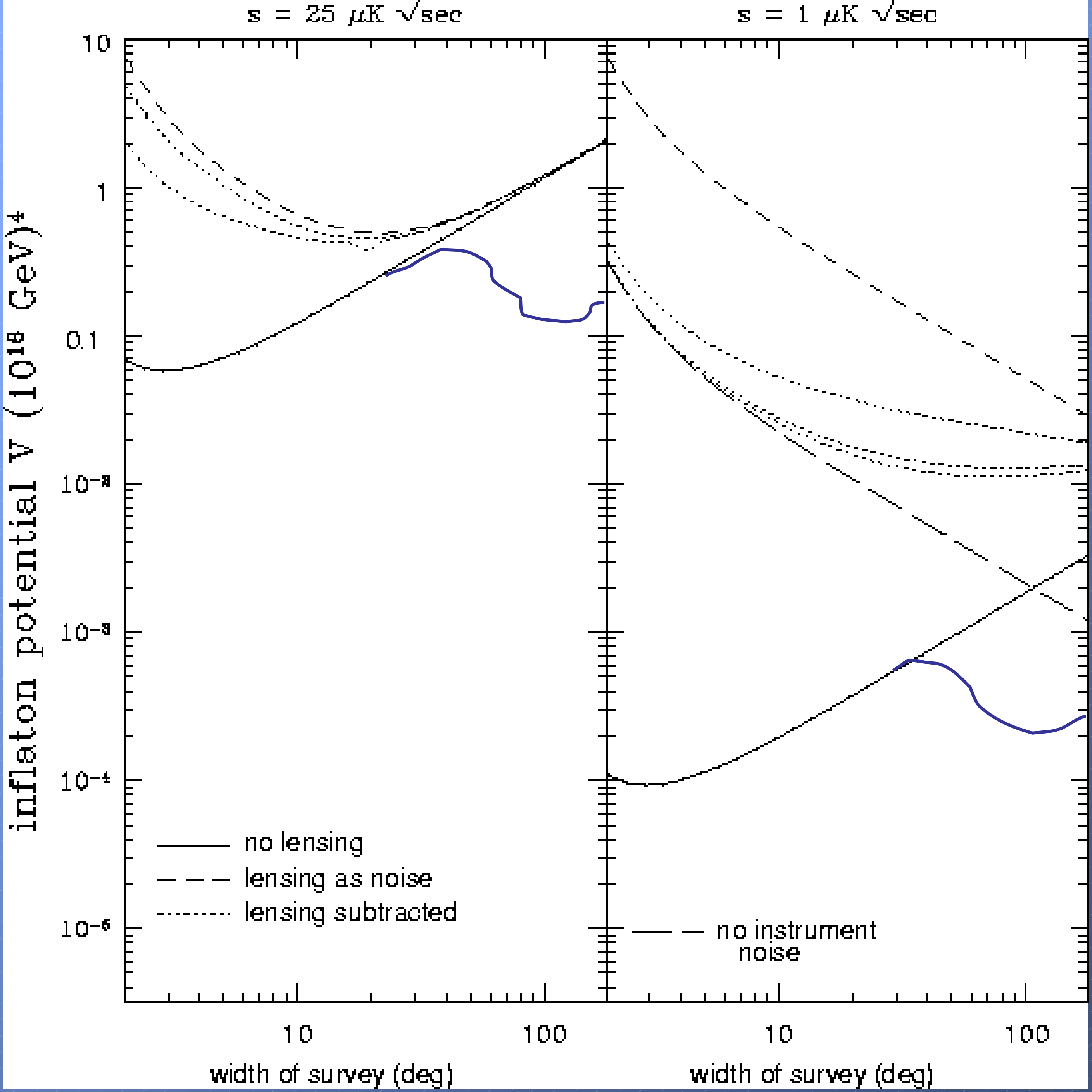
$$\delta\vec{\theta}(\vec{L}) = \frac{i\vec{L}A(L)}{L^2}\int \frac{d^2\vec{l_1}}{(2\pi)^2} T(\vec{l_1})T(\vec{l_2})F(\vec{l_1},\vec{l_2}),$$

$$F(\vec{l_1},\vec{l_2}) \equiv \frac{f(\vec{l_1},\vec{l_2})}{2C_{l_1}^{\mathrm{TT,t}}C_{l_2}^{\mathrm{TT,t}}}, \qquad A(L) = L^2\left[\int \frac{d^2\vec{l_1}}{(2\pi)^2} f(\vec{l_1},\vec{l_2})F(\vec{l_1},\vec{l_2})\right]^{-1}$$





scalars
lensing
gravitational
waves
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-18}
10^{1}
10^{2}
10^{3}
l
$l(l+1)C_l/2\pi$



s = 25 μK √sec
s = 1 μK √sec
inflaton potential V (10^16 GeV)^4
10
1
0.1
10^-2
10^-3
10^-4
10^-5
10
100
width of survey (deg)
no lensing
lensing as noise
lensing subtracted
no instrument noise

Another possibility to correct for cosmic shear

(Sigurdson, Cooray 2005)

- Use 21-cm probes of hydrogen distribution to map mass distribution between here and z=1100

Brief aside: Probes of parity violation in CMB

(Lue, Wang, MK 1999)

Might new physics responsible for inflation be parity violating?
TC and TG correlations in CMB are parity violating.
Can be driven, e.g., by terms of form $\phi R\tilde{R}$ during inflation or $\phi F\tilde{F}$ since recombination

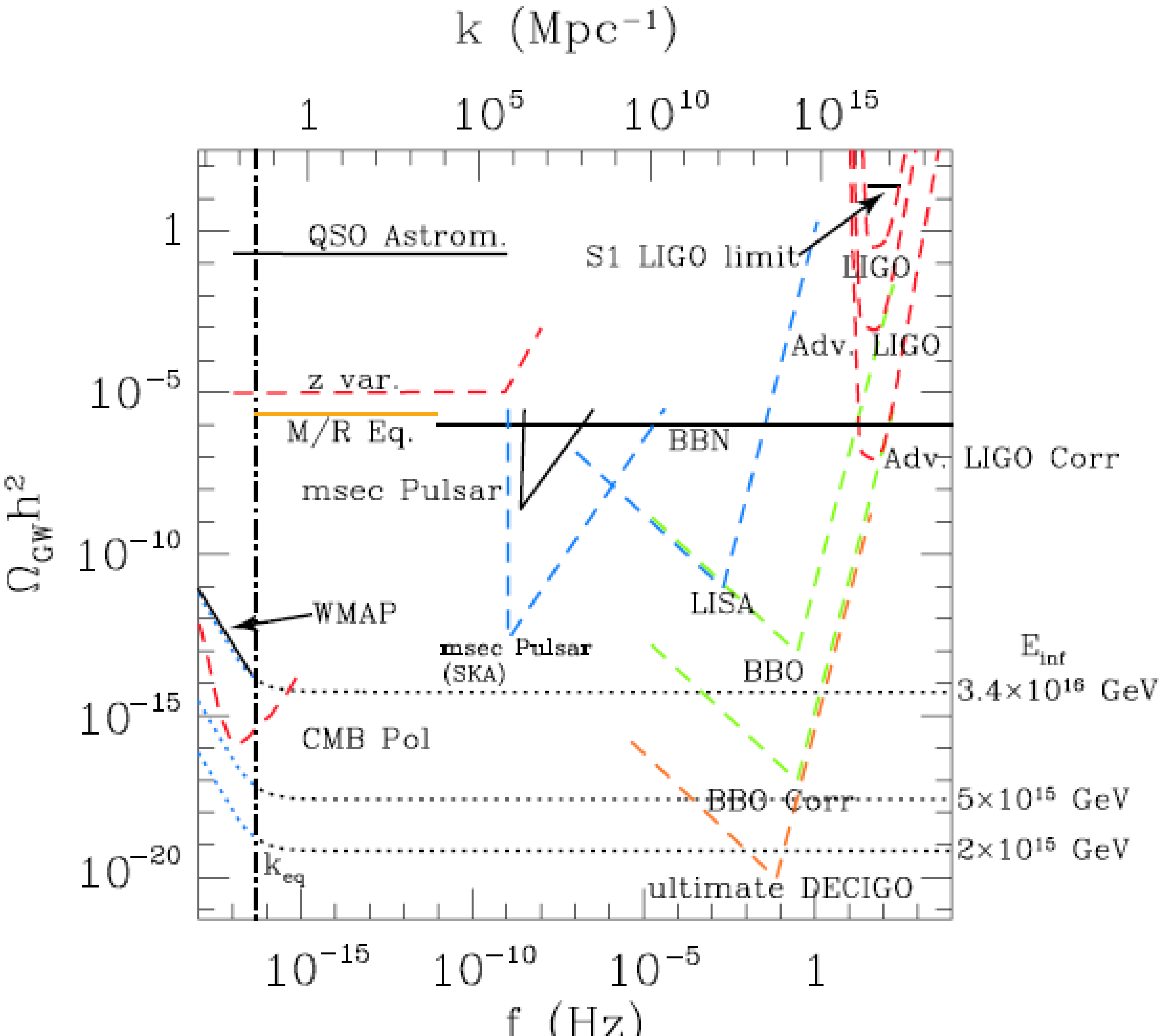
Direct Detection of Inflationary Gravitational Waves?

(T. L. Smith, MK, Cooray, astro-ph/0506422)

Mission concept studies:

•NASA: Big-Bang Observer (BBO)
•Japan: Deci-Hertz Gravitational-Wave Observatory (DECIGO)

seek to detect directly inflationary gravitational-wave background at ~0.1-Hz frequencies


k (Mpc⁻¹)
1
10^5
10^10
10^15
1
10^−5
10^−10
10^−15
10^−20
$\Omega_{GW} h^2$
QSO Astrom.
S1 LIGO limit
LIGO
Adv. LIGO
z var.
M/R Eq.
BBN
Adv. LIGO Corr
msec Pulsar
LISA
WMAP
msec Pulsar
(SKA)
BBO
E_{inf}
3.4×10^{16} GeV
CMB Pol
BBO Corr
5×10^{15} GeV
2×10^{15} GeV
k_{eq}
ultimate DECIGO
10^−15
10^−10
10^−5
1
f (Hz)

Survey some "toy" models for inflation:

"power-law"

$$V(\phi) = V_0 e^{-p\phi/m_{\rm Pl}}$$

"chaotic"

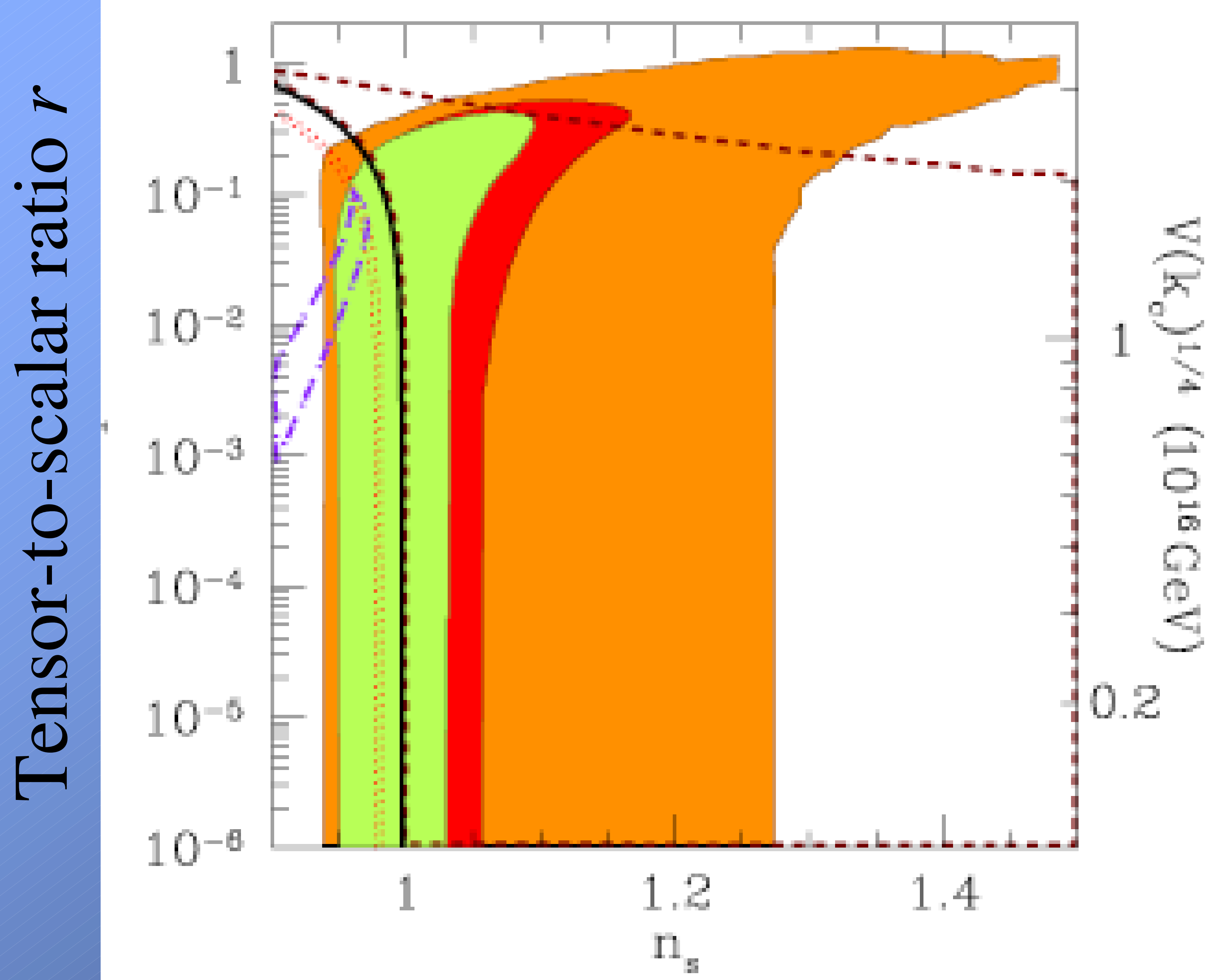
$$V(\phi) = V_0(\phi/M_{\rm Pl})^\alpha$$

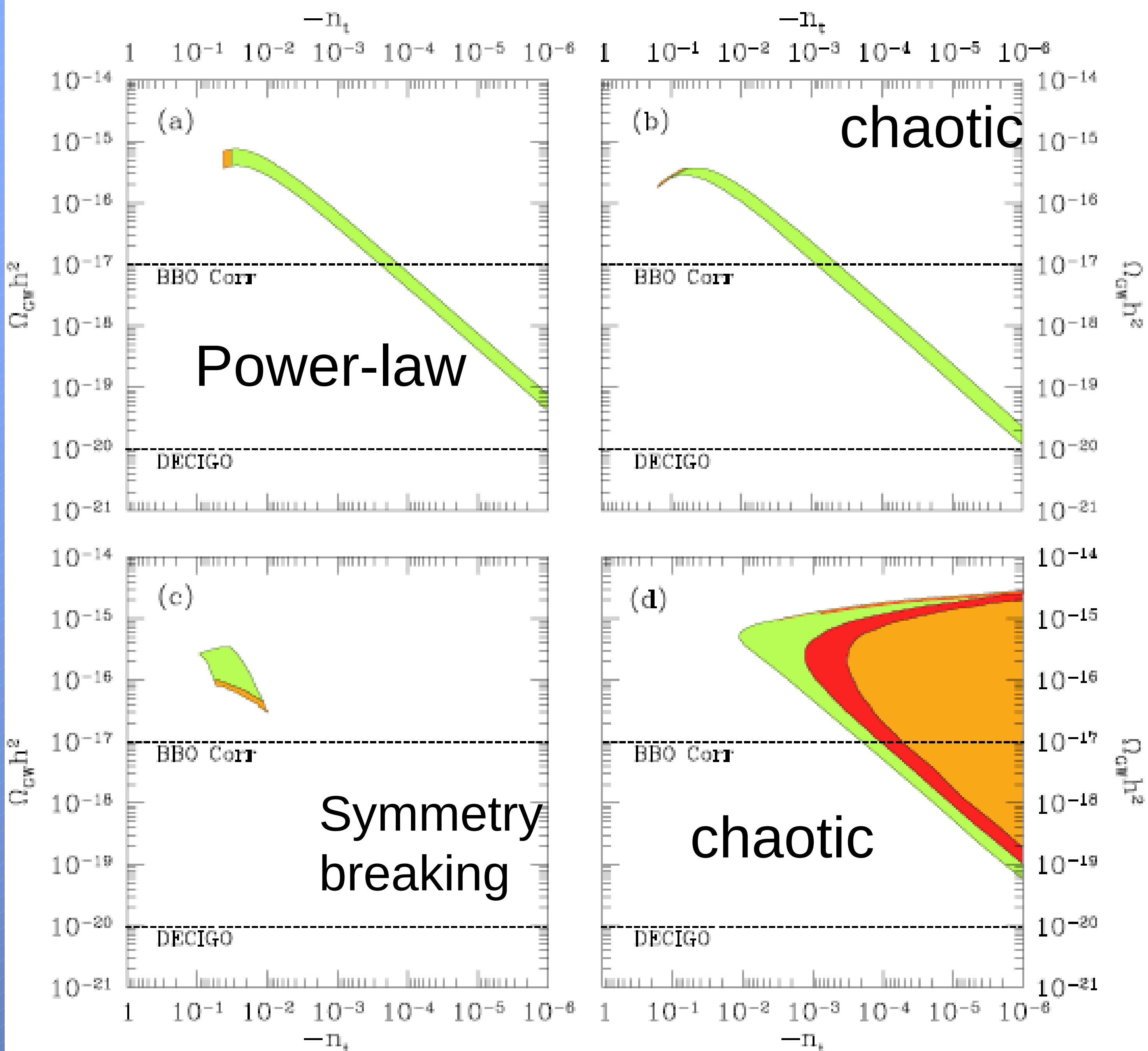
"hybrid"

$$V(\phi) = V_0\left[1 + \left(\frac{\phi}{\mu}\right)^2\right]^2$$

"symmetry-breaking"

$$V(\phi) = V_0\left[1 - \left(\frac{\phi}{\nu}\right)^2\right]^2$$


Tensor-to-scalar ratio r
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
1
1.2
1.4
n_s
$V(k_o)^{1/4}$ (10^{16}GeV)
1
0.2



(a)
Power-law
(b)
chaotic
(c)
Symmetry
breaking
(d)
chaotic
BBO Corr
DECIGO
$-n_t$
$\Omega_{GW}h^2$

Other possibilities?

- Phantom energy ($w<-1$) driven inflation (Baldi, Finelli, Matarrese 2005)
- Pre-big-bang, cyclic, and ekpyrotic models

may produce “blue” GW spectra: larger direct signal, without increasing CMB signal

Conclusions

- IGWB is detectable in many inflation models
- IGWB probably not directly detectable if does not show up in CMB polarization
- Large lever arm between CMB and BBO/DECIGO scales provides unique probe of inflationary models