On Cosmic Acceleration without Dark Energy

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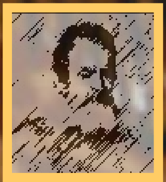
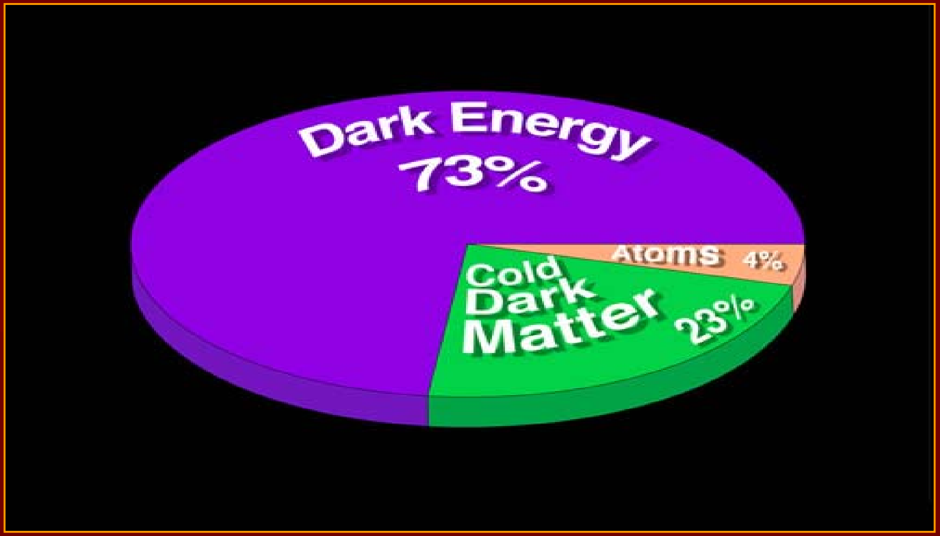


based on:

- E.W. Kolb, S. Matarrese, A. Notari & A. Riotto, 2005, PRD **71**, 023524
- E. Barausse, S. Matarrese & A. Riotto, 2005, PRD 71, 063537
- E.W. Kolb, S. Matarrese, A. Notari & A. Riotto, hep-th/0503117
- E.W. Kolb, S. Matarrese & A. Riotto, astro-ph/0506534

The cosmic budget



- *Only about 4% of the cosmic energy budget is in the form of ordinary "baryonic" matter, out of which only a small fraction shines in the galaxies (quite likely most of the baryon reside in filaments forming the Warm-Hot Intergalactic Medium (WHIM), a sort of cosmic web connecting the galaxies and clusters of galaxies).*

- *About 23% of the cosmic budget is made of Dark Matter, a collisionless component whose presence we only perceive gravitationally. The most likely candidates are hypothetical particles like neutralinos, axions, etc….*

- *About 73% of the energy content of our Universe is in the form of some exotic component, called Dark Energy, or "Quintessence", which causes a large-scale cosmic repulsion among celestial objects, thereby mimicking a sort of anti-gravity effect. The simplest dark energy candidate is the Cosmological Constant Λ.*

What is dark energy made of ?

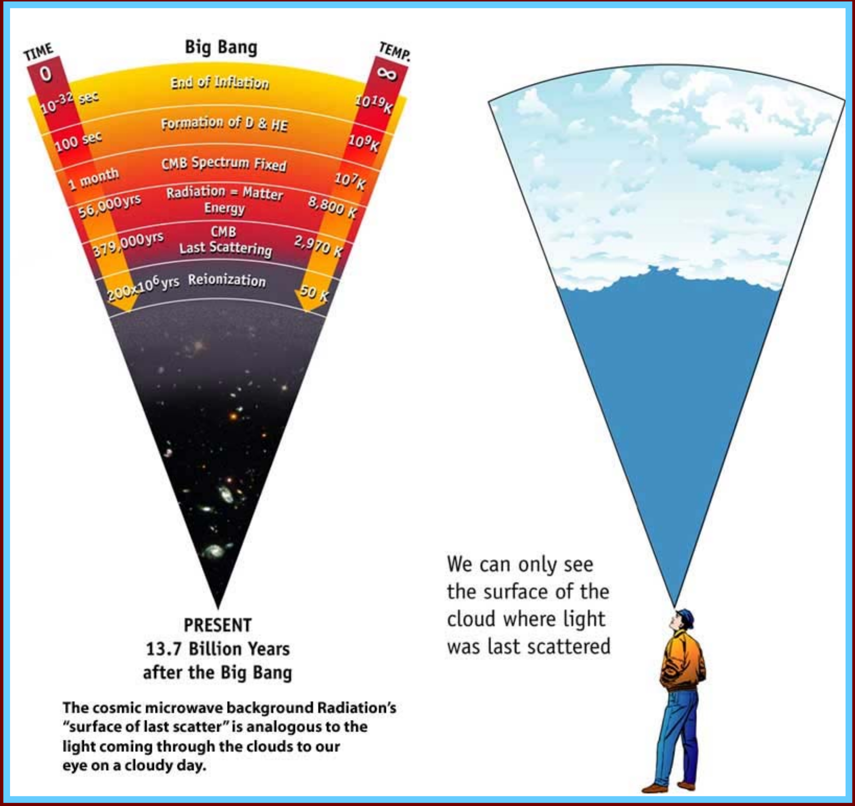
Observations triggered a lot of theoretical activity:

- ✓ Cosmological constant
- ✓ Quintessence
- ✓ String Effects
- ✓ Quantum Gravity effects
- ✓ Modification of GR
- ✓ Back-reaction

An alternative to Dark Energy: back-reaction of inhomogeneities

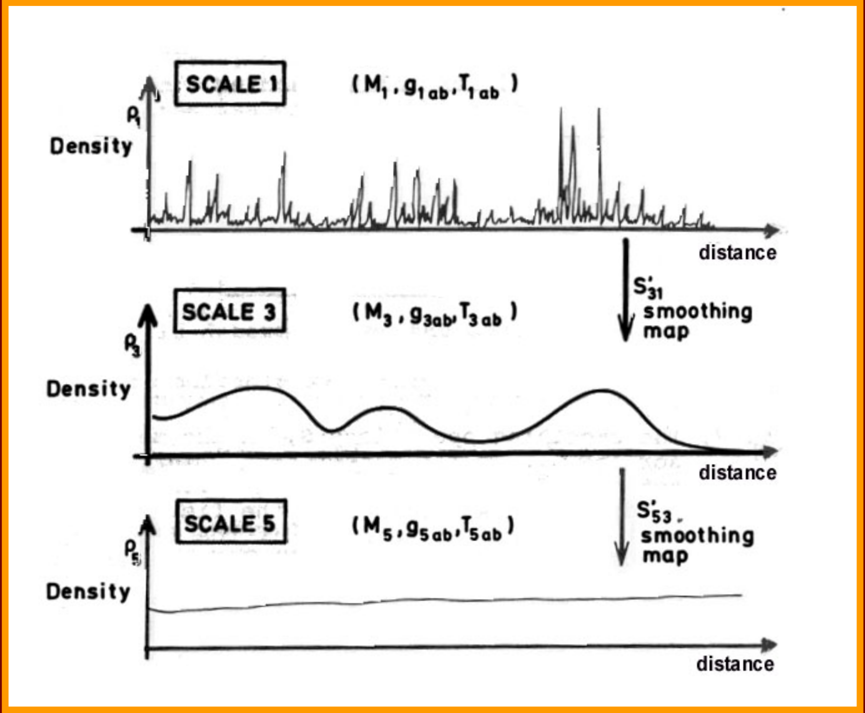
"Observational Cosmology"

- ☞ The standard cosmological model is based on both observational evidence (e.g. the quasi-perfect isotropy of the CMB) and on a priori philosophical assumptions: the Copernican Principle, according to which all comoving cosmic observers at a given cosmic time see identical properties around them. An alternative approach, called "Observational Cosmology" was proposed by Kristian & Sachs (1966), following earlier ideas by McCrea (1935). The idea is that of building our cosmological model solely on the basis of observations within our past-light-cone, without any a priori symmetry assumptions. Schucking (1964) was a proponent of this approach at a Galileo Commemoration in Padova. But the most important contribution was given by Ellis in 1983, with his talk at the International GR Conference in Padova.



Smoothing and back-reaction

- Ellis realizes that "smoothing" of space-time irregularities plays a central role in any observational approach. He however realizes that smoothing necessarily modifies the structure of Einstein's equations (smoothing and evolution do not commute), leading to an extra "back-reaction" term in their RHS. He also states "there is no reason why the effective stress-energy tensor [i.e. that including back-reaction] should obey the usual energy conditions" [$P>-\rho/3$, Hawking & Ellis 1973], even when the original one does. I.e. smoothing may lead to the avoidance of singularities. But it also implies that back-reaction may lead to accelerated expansion starting from a standard fluid with positive or zero pressure.



from: Ellis (1984)

GR dynamics of an inhomogeneous Universe

- ✓ *Consider Einstein's equations for a fluid of pressureless and irrotational matter:*

$$G^{\mu}{}_{\nu} = 8\pi G\, \rho\, u^{\mu} u_{\nu}$$

- ✓ *Describe the system in the synchronous and comoving gauge assuming no global symmetry whatsoever*

$$ds^2 = - dt^2 + h_{ij}(\mathbf{x},t)\, dx^i\, dx^j$$

- ✓ *Given the fluid four-velocity* $u^{\mu} = (1,0,0,0)$ *define, by covariant differentiation, the volume expansion* Θ *describing the expansion or contraction of fluid elements while its trace-free part, the shear* $\sigma^i{}_j$*, describing the distortion of fluid elements by the tidal interaction with the surrounding matter:*

$$\Theta^i{}_j = u^i{}_{;j} = \frac{1}{2} h^{ik} \dot{h}_{kj} = \sigma^i{}_j + \frac{1}{3}\Theta\,\delta^i{}_j$$

Einstein's equations

$$\frac{2}{3}\Theta^2 - 2\sigma^2 + R = 16\pi G\,\rho$$

energy constraint (00)

$$\sigma^i_{j|i} - \frac{2}{3}\Theta_{,j} = 0$$

momentum constraint (0i)

$$\dot{\Theta} + \Theta^2 + R = 12\pi G\,\rho$$

expansion evolution equation (i=j)

$$\dot{\sigma}^i_j + \Theta\sigma^i_j + R^i_j - \frac{1}{3}R\delta^i_j = 0$$

shear evolution equation (i≠j)

$$\dot{\Theta} + \frac{1}{3}\Theta^2 + 2\sigma^2 + 4\pi G\,\rho = 0$$

Raychaudhuri equation

$$\dot{\rho} = -\Theta\rho$$

mass conservation

Background: Friedmann equations

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho$$

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

$$\dot{\rho} = -3\frac{\dot{a}}{a}\rho$$

Homogeneous and isotropic form taken by Einstein equations for pure matter with zero spatial curvature (Einstein-de Sitter model).

__Solution:__

$a(t) \sim t^{2/3}$,

$\rho = 1/(6\pi G t^2)$,

$q=1/2$

Dealing with inhomogeneities: smoothing

Smoothing over a finite volume

see: Ellis 1983; Carfora & Marzuoli 1984; Buchert & Ehlers 1997; Buchert 2000, 2001

$$a_D(t) = \left(\frac{V_D}{V_{D_0}}\right)^{1/3} , \quad V_D = \int_D \sqrt{h}\, d^3x \,,$$

$$a_D(t) = a(t)\, e^{-\Psi_\ell(t)+\Psi_{\ell 0}}$$

Coarse-graining: averaging over a comoving domain D comparable with our present-day Hubble volume

$$\Psi_\ell(\mathbf{x}_{\rm obs}, t) \equiv \ln a - \frac{1}{3}\ln\left(\int_D \sqrt{h}\, d^3x\right) + \text{const.}$$

Ψ_l has a residual x-dependence labeling the specific Hubble-size patch around a given cosmic observer

$$H_D = \frac{\dot a_D}{a_D} = \frac{1}{3}\langle\Theta\rangle_D = -\dot\Psi_\ell + \frac{2}{3t}$$

$$\langle\Theta\rangle^{\cdot}_D = \langle\dot\Theta\rangle_D + \langle\Theta^2\rangle_D - \langle\Theta\rangle^2_D \geq \langle\dot\Theta\rangle_D$$

The non-commutation of averaging and evolution comes from the time-dependence of the coarse-graining volume element (via the 3-metric determinant)

Can spatial smoothing be removed from the definition of background cosmological parameters?

- In a inhomogeneous Universe "cosmological parameters like the rate of expansion or the mass density are to be considered as volume-averaged quantities, and only these can be compared with cosmological observations" (Ellis & Buchert gr-qc/0506106).

- Nonetheless, as some authors (e.g. Flanaghan 2005) have recently proposed to define cosmological parameters like H_0 and q_0 by angular averaging only, let's look at this issue in more detail.

- As soon as the back-reaction of perturbations on the Hubble rate is considered (e.g. by a second-order perturbations calculation) **huge** Newtonian corrections appear, spoiling the FRW background input value of H: these are the same terms leading to a harmless contribution to back-reaction if spatially averaged over a large volume.

$$\frac{\overline{\langle\delta\theta\rangle}}{3H} = -\frac{25}{24}a^2H^2\int_0^\infty \frac{dk}{k^3}\Delta^2(k,a)$$

$$+\frac{25}{4}a^2H^2\int_0^\infty \frac{dk}{k^3}\Delta^2(k,a)W^2(kR) - \frac{5}{6}\int_0^\infty \frac{dk}{k}\Delta^2(k,a)W^2(kR)$$

Newtonian term: it becomes very large (because of unfiltered small-scale effects) if spatial smoothing is removed, i.e. if R→ 0 (see Kolb et al. 2005a)

The scale-factor of our Hubble patch

$$a_D(t) \propto a_{FRW}(t)\exp(-\Psi_\bullet(t))$$

$$\Psi_\bullet(t) = -\frac{1}{3}\ln\langle(1+\delta_{FRW})^{-1}\rangle_{in} + const$$

Acceleration in our local Hubble patch is possible if the <u>mean rarefaction factor</u> (w.r.t. an underlying FRW model) $<(1+\delta_{FRW})^{-1}>_{in}$ *grows fast enough to overshoot the FRW background evolution* *($<.>_{in}$ indicates averages over the initial, i.e. post-inflationary volume)*

Ψ_I is by construction a super-Hubble perturbation: sub-Hubble Fourier modes of $(1+\delta_{FRW})^{-1}$ *are filtered out. Nonetheless, the evolution of our super-Hubble mode is fed by the non-linear evolution of sub-Hubble (i.e. observable) perturbations.*

Effective Friedmann equations

see: Buchert (2000, 2001)

$$\frac{\ddot{a}_D}{a_D} = -\frac{4}{3}\pi G\left(\rho_{\text{eff}} + 3P_{\text{eff}}\right)$$

$$H_D^2 = \frac{8\pi G}{3}\rho_{\text{eff}} \,,$$

$$\rho_{\text{eff}} = \langle\rho\rangle_D - \frac{Q_D}{16\pi G} - \frac{\langle R\rangle_D}{16\pi G} \,,$$

$$P_{\text{eff}} = -\frac{Q_D}{16\pi G} + \frac{\langle R\rangle_D}{48\pi G} \,,$$

mean curvature

$$\dot{\rho}_{\text{eff}} = -3H_D\left(\rho_{\text{eff}} + P_{\text{eff}}\right) \,.$$

kinematical back-reaction:

$$\langle\rho\rangle^{\cdot}_D = -3H_D\langle\rho\rangle_D$$

$$\boxed{Q_D = \frac{2}{3}\left(\langle\Theta^2\rangle_D - \langle\Theta\rangle_D^2\right) - 2\langle\sigma^2\rangle_D}$$

Back-reaction and averaging

$$\left(a_D^6 Q_D\right)^{\cdot} + a_D^4 \left(a_D^2 \langle R \rangle_D\right)^{\cdot} = 0 \,.$$

Integrability Condition: only exists in GR (no Newtonian analogue)

$$h_{ij} = a^2(t) e^{-2\Psi_\ell(t)} \widetilde{h}_{ij}(\mathbf{x}, t) \quad \begin{cases} \Theta = 3\frac{\dot{a}}{a} + \widetilde{\Theta} - 3\dot{\Psi}_\ell \\ \sigma^i{}_j = \widetilde{\sigma}^i{}_j, \end{cases}$$

Consider Ψ_l as a space-time dependent conformal rescaling.

$$Q_D = \frac{2}{3}\langle \widetilde{\Theta}^2 \rangle_D - 2\langle \widetilde{\sigma}^2 \rangle_D$$

Q_D is only contributed by sub-Hubble fluctuations (but feels super-Hubble modes via time-evolution of the background)

$$\langle R \rangle_D = a^{-2} e^{2\Psi_\ell} \left\langle \widetilde{R} + 4\widetilde{\nabla}^2 \Psi_\ell - 2\widetilde{\nabla}^k \Psi_\ell \widetilde{\nabla}_k \Psi_\ell \right\rangle_D$$

$\langle R \rangle_D$ gets contributions both from super-Hubble and sub-Hubble modes

Can irrotational dust undergo acceleration?

- ✓ *According to the Raychaudhuri equation for irrotational dust each fluid element can only undergo decelerated (q>0) or free (q = 0) expansion → the strong energy condition is satisfied*

$$q = -\frac{3\dot{\Theta} + \Theta^2}{\Theta^2} = \frac{6\sigma^2 + 12\pi G\rho}{\Theta^2} \geq 0$$

- ✓ *However, coarse-graining over a finite volume D makes acceleration (q < 0) possible by the time-dependence of the averaging volume (via the metric determinant) → the strong energy condition can be violated*

$$q_D = -\frac{3\dot{\Theta}_D + \Theta_D{}^2}{\Theta_D{}^2} = \frac{-3Q_D + 12\pi G\langle\rho\rangle_D}{\Theta_D{}^2} \quad \text{can have both signs}$$

$$\text{as} \quad Q_D \equiv \frac{2}{3}\left\langle(\Theta - \langle\Theta\rangle)^2\right\rangle_D - 2\left\langle\sigma^2\right\rangle_D \quad \text{can be positive}$$

The back-reaction equation of state

$$\left(a_D^6 Q_D\right)^{\cdot} + a_D^4\left(a_D^2\langle R\rangle_D\right)^{\cdot} = 0\,.$$

<u>Integrability Condition</u>: only exists in GR (no Newtonian analogue)

$$\rho_{\mathrm{eff}} = \langle\rho\rangle_D - \frac{Q_D}{16\pi G} - \frac{\langle R\rangle_D}{16\pi G}\,,$$

$$P_{\mathrm{eff}} = -\frac{Q_D}{16\pi G} + \frac{\langle R\rangle_D}{48\pi G}\,,$$

$$Q_D \propto a_D^{-6}$$

- ✓ Stiff-matter-like solution (negligible)

$$\langle R\rangle_D \propto a_D^{-2}$$

- ✓ Standard curvature term: only possibility if only super-Hubble modes are present

$$Q_D = -\frac{1}{3}\langle R\rangle_D$$

- ✓ Effective cosmological constant:

$$\boxed{\langle R\rangle_D = -3\,\Lambda_{\mathrm{eff}}}$$

A heuristic argument for acceleration

The local expansion rate can be written as

$$H_D = \frac{2}{3t} + \frac{1}{3}\frac{\langle (1+\delta_{\rm FRW})^{-1}\,\theta\rangle_{D_{\rm in}}}{\langle (1+\delta_{\rm FRW})^{-1}\rangle_{D_{\rm in}}}$$

θ = *peculiar volume expansion factor*: it is positive for underdense fluid patches. In order for the kinematical back-reaction Q_D to be positive and large what really matters is that non-linear structures have formed in the Universe, so that a large variance of Θ arises (Θ is narrowly peaked around its FRW value as long as perturbations stay linear).

Hence H_D is expected to be enhanced w.r.t. its FRW value by the back-reaction of inhomogeneities, eventually leading to acceleration

Solving Einstein's equations for the coarse-grained Universe description

The problem can be approached by different, and to some extent complementary, techniques:

- ✓ Use approximate solution of Einstein's equations to fit our local scale-factor by appropriate smoothing.
- ✓ Use approximate solution of Einstein's equations to compute the back-reaction equation of state. Next solve the effective Friedmann equations to find our local scale-factor.
- ✓ Study the behavior of perturbations within our past light-cone to assess the form of the "back-reaction equation of state" directly from observations.

General property of back-reaction

- ✓ One can integrate the Q_D $\langle R \rangle_D$ relation obtaining

$$\langle R \rangle_D = -Q_D - \frac{6\kappa_D}{a_D^2} + \frac{4}{a_D^2}\int_0^{a_D} da\, a Q_D(a)$$

where κ_D is a generally time-dependent integration constant

- ✓ Replacement in the first Friedmann equation leads to:

tiny if computed over a region $D \sim 1/H_0$, by inflationary initial conditions

$$H_D^2 = \frac{8\pi G}{3}\langle \rho \rangle_D - \frac{\kappa_D}{a_D^2} + \frac{2}{3a_D^2}\int_0^{a_D} da\, a Q_D(a)$$

where Q_D is not a free parameter but it should be computed consistently from the non-linear dynamics of perturbations. Note once again that a constant and positive Q_D would mimic a cosmological constant term

Inconsistency of the Newtonian approach to back-reaction

- ✓ Back-reaction is a genuinely GR problem and the connection between kinematical back-reaction and mean curvature (yielding the possibility of acceleration) has NO NEWTONIAN ANALOG.

- ✓ No matter how good the Newtonian approximation is in describing matter clustering in the Universe, it completely fails if applied to study back-reaction.

$$\boxed{Q_D^{\mathrm{Newtonian}} = \langle \nabla \cdot [\mathbf{u}\,(\nabla \cdot \mathbf{u}) - (\mathbf{u} \cdot \nabla)\,\mathbf{u}] \rangle_D}$$

- ✓ Indeed, Ehlers & Buchert (1996) have shown EXACTLY that in Newtonian theory Q_D is a total divergence term, which by Gauss theorem can be transformed into a tiny surface term. Many authors (e.g. Siegel & Fry 2005) have used various approximations to recover this result and, based on it, reached the erroneous conclusion that back-reaction is negligible.

The effect of (pure) super-Hubble perturbation modes

$$\left(a_D^6 Q_D\right)^{\cdot} + a_D^4 \left(a_D^2 \langle R \rangle_D\right)^{\cdot} = 0\,.$$

- ✓ Let's take the extreme (and unrealistic) situation in which there are only super-Hubble modes. In such a case the kinematical back-reaction identically vanishes and the only consistent solution of the integrability condition is a standard curvature term $\langle R \rangle_D$ ✪ $1/a_D^2$. The same result can be obtained by a renormalization group resummation of a gradient expansion

$$Q_D = \frac{2}{3}\langle \widetilde{\Theta}^2 \rangle_D - 2\langle \widetilde{\sigma}^2 \rangle_D$$

$$\langle R \rangle_D \;=\; a^{-2} e^{2\Psi_\ell} \left\langle \widetilde{R} + 4\widetilde{\nabla}^2 \Psi_\ell - 2\widetilde{\nabla}^k \Psi_\ell \widetilde{\nabla}_k \Psi_\ell \right\rangle_D$$

hence pure super-horizon modes *<u>cannot</u>* lead to the observed accelerated expansion of the Universe. They can only produce a curvature term which, for inflationary initial conditions is bound to be tiny today

The effect of observable, i.e. sub-Hubble, perturbation modes

- Dealing with the back-reaction of sub-Hubble modes is far more complex, as a reliable evaluation of the effect can only be obtained by a non-Newtonian and non-perturbative approach to the non-linear dynamics of perturbations.

- We used two alternative approaches:

  a) a higher-order gradient expansion in the comoving and synchronous gauge

  b) a non-perturbative approach in the weak-field limit of the Poisson gauge

Solving Einstein's equations

A non-perturbative solution of Einstein equations is obtained by a gradient-expansion (Lifshits & Khalathnikov 1970). It contains terms of any perturbative order with a given number of spatial gradients. At lowest order it coincides with the "separate Universe" approximation (Salopek & Bond 1991). Higher order terms describe the Universe at higher and higher resolution. Initial conditions ("seeds") from single-field slow-roll inflation. The range of validity at order n (i.e. with 2n gradients) is $(k/aH)^{2n}\varphi^{n+1} < 1$ ➒ *scales down to a few Mpc and even below (depending on the specific term under consideration); see Salopek et al. 1993.*

$$ds^2 = -dt^2 + a^2(t)\gamma_{ij}dx^i dx^j$$

$$\gamma_{ij} = e^{-2\Psi}\left(\delta_{ij} + \chi_{ij}\right)$$

traceless perturbation

$$\psi = \frac{5}{3}\varphi + \frac{a}{18}\left(\frac{2}{H_0}\right)^2 e^{10\varphi/3}\left[\nabla^2\varphi - \frac{5}{6}(\nabla\varphi)^2\right]$$

... plus higher-derivative terms

Matarrese, Pillepich & Riotto, 2005 in prep.

$\varphi \sim 10^{-5}$ = peculiar gravitational potential (related to linear density contrast δ by cosmological Poisson equation, $\nabla^2\varphi \sim \delta$)

How back-reaction gets big

The general rule goes as follows:

- *Newtonian terms like $\nabla^2\varphi/H_0^2$, which would be the largest ones by themselves, add up to give a pure total derivative contribution to Q_D so their space average always yields a tiny surface term (10^{-5}).*
- *Post-Newtonian terms, like $(\nabla\varphi)^2/H_0^2$ are small but cannot lead to a total derivatives.*
- *Therefore a combination of the two can be as large as required.*

The averaging volume window function becomes ineffective when ensemble expectation values of products of φ are considered.

$$\langle \nabla\varphi\nabla\varphi\rangle_R = \frac{1}{(2\pi)^3}\int d^3k W(kR)\int d^3p\, \mathbf{p}(\mathbf{p}+\mathbf{k})\varphi(\mathbf{p})\varphi(-\mathbf{p}-\mathbf{k})$$

$$ensemble \quad mean \rightarrow \delta^{(3)}(\mathbf{p}-\mathbf{p}+\mathbf{k}) \Rightarrow W(kR) \rightarrow W(0)=1 \rightarrow \langle \nabla\varphi\nabla\varphi\rangle_R = \langle \nabla\varphi\nabla\varphi\rangle_{R=0}$$

The small-scale behavior of products of $\nabla\varphi$ and $\nabla^2\varphi$ yields the back-reaction: terms like $(\nabla^2\varphi)^2(\nabla\varphi)^2$ (in proper units) are sizeable and lead to an effective dark-energy contribution.

The appearance of instabilities

- ✓ The calculation of back-reaction from sub-Hubble perturbation modes can be also performed using the weak-field limit of GR in the Poisson gauge (see Seljak & Hui 1996; Siegel & Fry 2005). However, it is crucial that quantities are expressed in terms of the fluid proper time; this involves both a kinematical back-reaction term (once again because of non-commutativity of averaging and time evolution) and a lapse function N. Accounting for these contributions shows that post-Newtonian terms appear also in this approach and become as big as the FRW background at recent times, thus leading to an instability of the underlying background (flat matter-dominated FRW) model

$$\boxed{Q_D \sim c^2|R_D| \sim H^2\langle\delta^2\,(v/c)^2\rangle_D \sim \Lambda_{eff}}$$

- ✓ This instability can have been the origin of the present phase of accelerated expansion in our Hubble patch.

- ✓ Note that by no means our findings rely on the existence of extra non-Newtonian terms affecting the dynamics of LSS. The perturbations which create the instability are just the familiar Newtonian ones that lead to LSS formation. It is only in the back-reaction effects that they combine to produce a non-Newtonian expression.

The state of the controversy

where we still disagree

We conclude that cosmological models based on GR with irrotational initial conditions and perturbations only at and above the Hubble scale and only matter fields that conform to the SEC cannot explain the accelerating expansion. This paper does not exclude the possibility of using backreaction from sub-Hubble perturbations to explain the accelerating expansion. The latter possibility is difficult to investigate as it involves complicated non-linear physics; the perturbative calculations [33], which account for the nonlinear evolution of density perturbations, but not for strong field GR effects, suggest that the sub-Hubble backreaction is small [37]. Nevertheless, only a full non-perturbative analysis would be definitive since there are rare regions of the universe such as black holes that cannot be described as a perturbation of an FRW spacetime.

Hirata and Seljak (2005)

we don't believe such highly non-linear structures are relevant

Conclusions

- ✓ *Averaging inhomogenous cosmology leads to effective Friedmann equations with extra (back-reaction) source terms, showing that acceleration is possible even if the Universe content is pure dust:*

  - ✓ *if super-Hubble perturbations only are considered, back-reaction only amounts to a tiny curvature term*

  - ✓ *when sub-Hubble perturbations are considered, their non-linear dynamics leads to an instability, driven by a post-Newtonian term, that mimics the effect of Dark Energy*

<u>If our findings are confirmed</u>:

- ✓ *Backreaction could be the reason why the observable Universe is presently undergoing a phase of accelerated expansion, thereby suggesting a natural solution of the coincidence problem*

- ✓ *There would be no need for Dark Energy at a fundamental level: matter and gravity would be the only players!*