Dark Matter and Neutrino Physics

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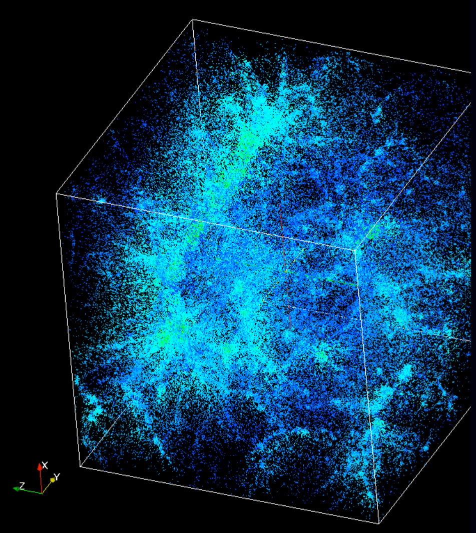
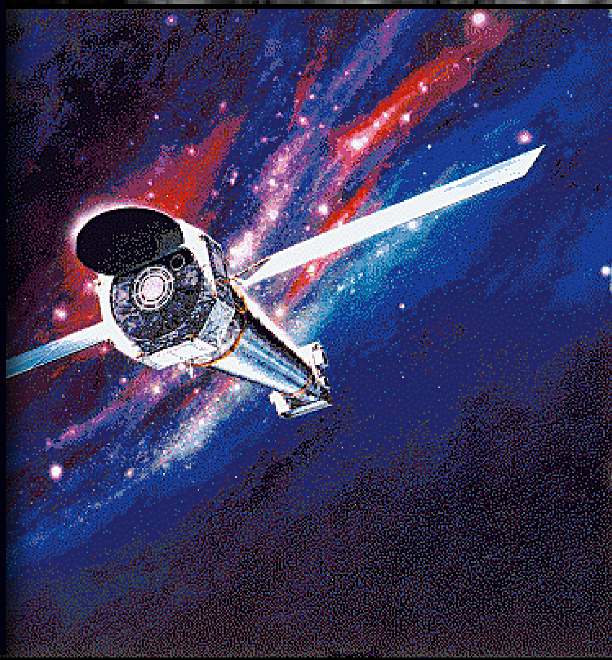




Colloquium Daniel Chalonge
October 26, 2006

The Dark Matter Problem

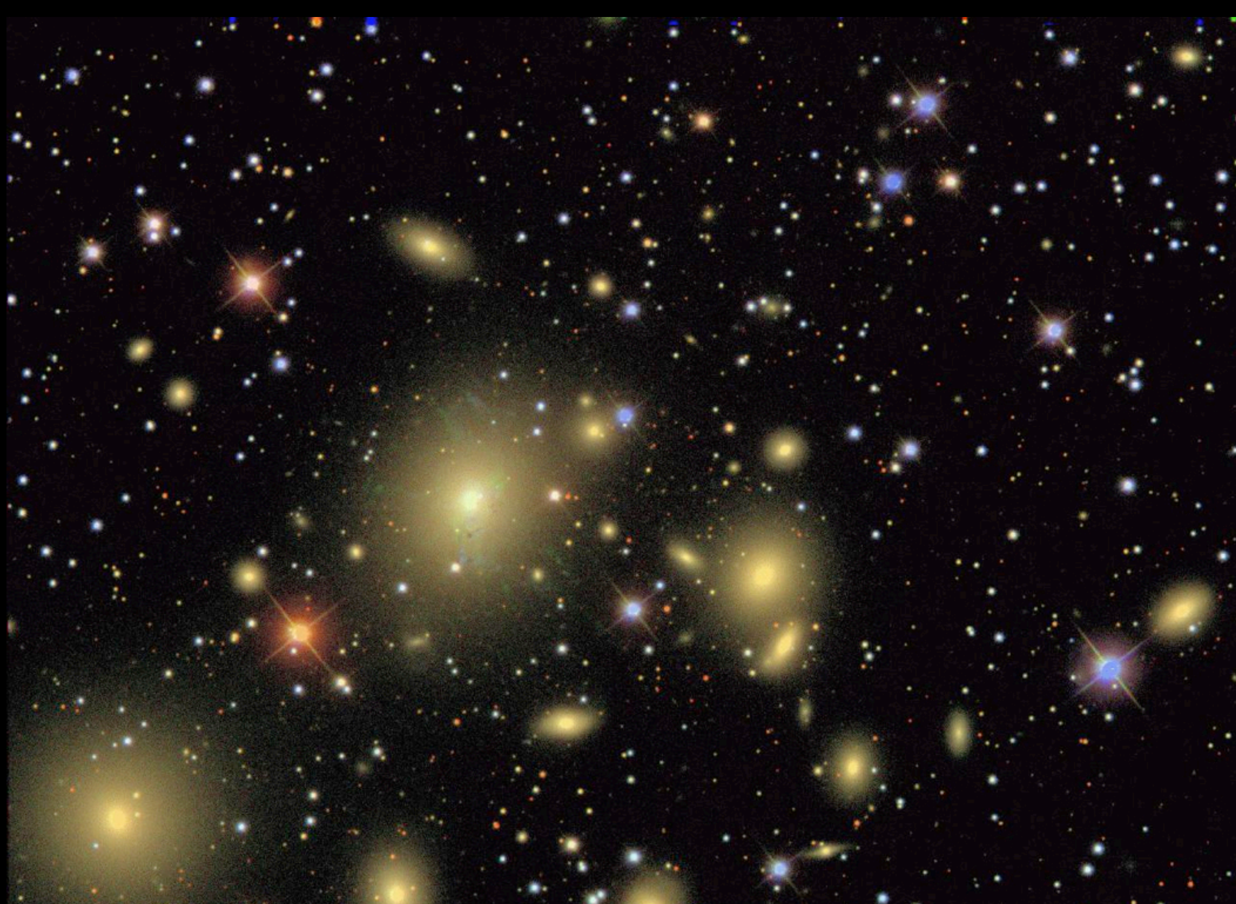
In Terms of a Critical Fraction:

$$\Omega_x \equiv \frac{\rho_x}{\rho_{crit}}$$

First indication: velocity of outer galaxies in a cluster

$$GM(r) = v^2 r$$

Zwicky (1933): the “missing mass”



Counting stars:

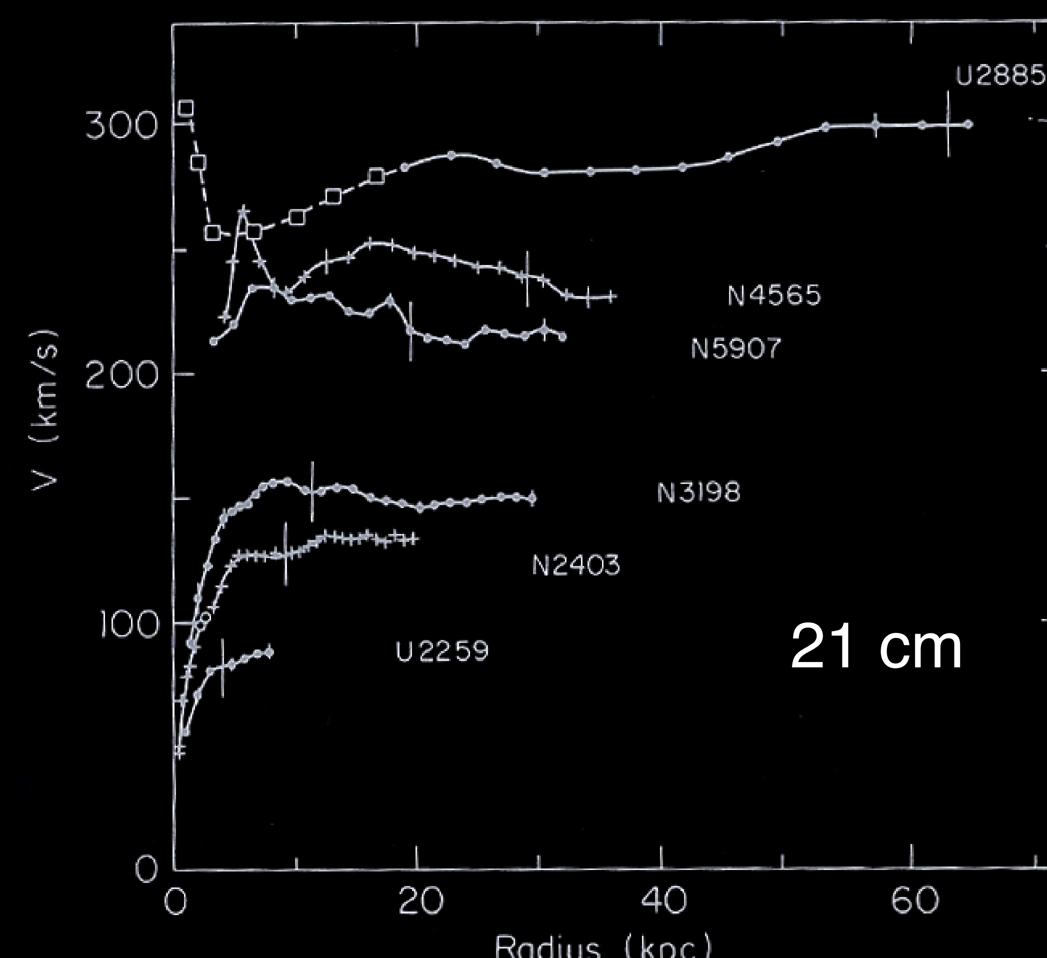
$$\Omega_{LUM} \simeq 0.01 \text{ or less}$$

Rotation curves: stellar, 21 cm:

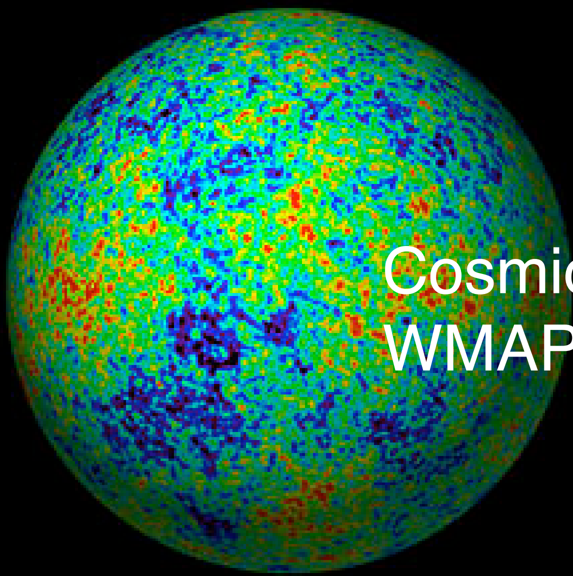
$$\Omega_{Halo} \gtrsim 0.1$$

Virialized galaxy clusters:

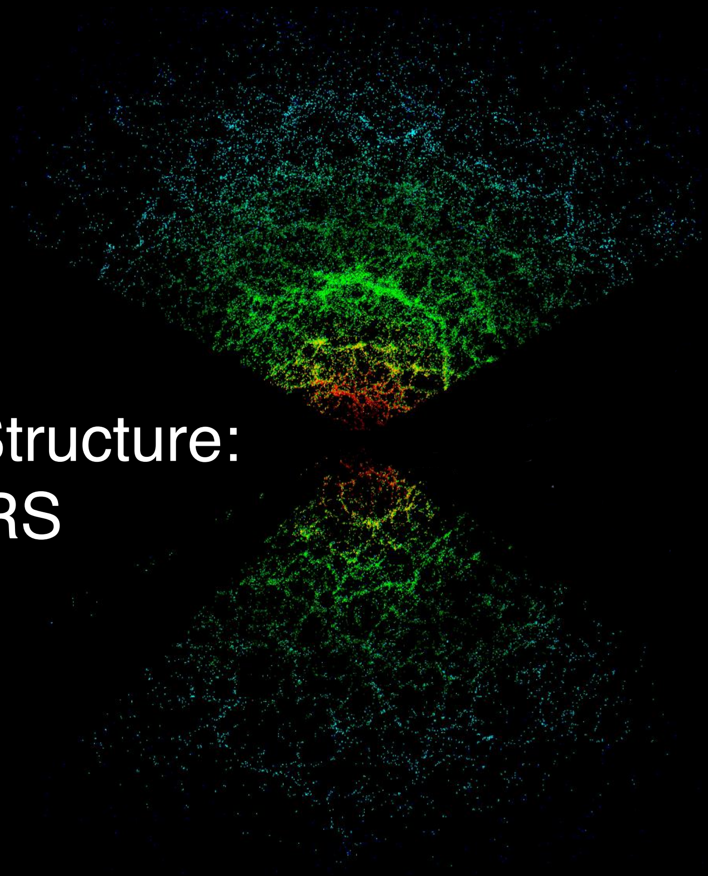
$$\Omega_{clusters} = 0.1 - 0.3$$



Dark Matter Today



Cosmic Microwave Background:
WMAP, ACBAR, CBI, Boomerang



Large Scale Structure:
SDSS, 2dFGRS

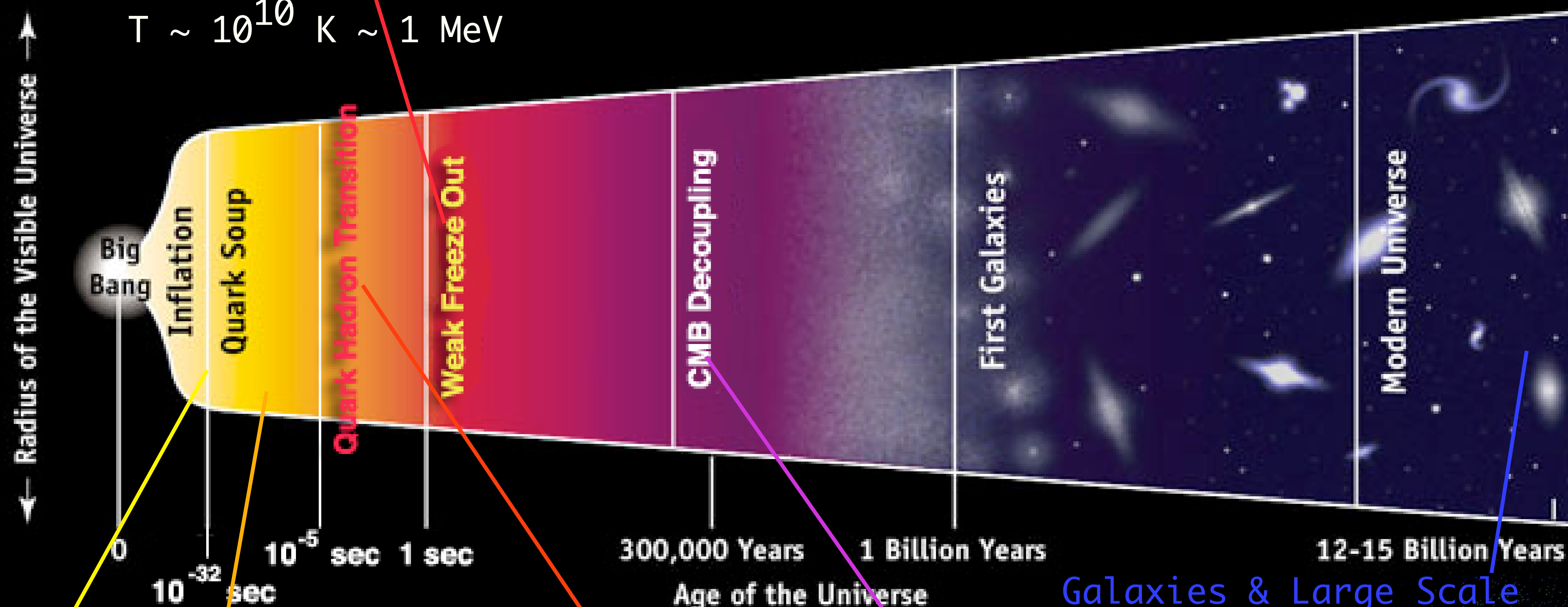
$$\Omega_{\rm DM} = 0.20^{+0.018}_{-0.017}$$

WMAP3 + SDSS LRG 3D P(k)

The Complete History of the Universe

Weak Freeze-Out & Big Bang Nucleosynthesis:

T ~ 10^{10} K ~ 1 MeV



Inflation:

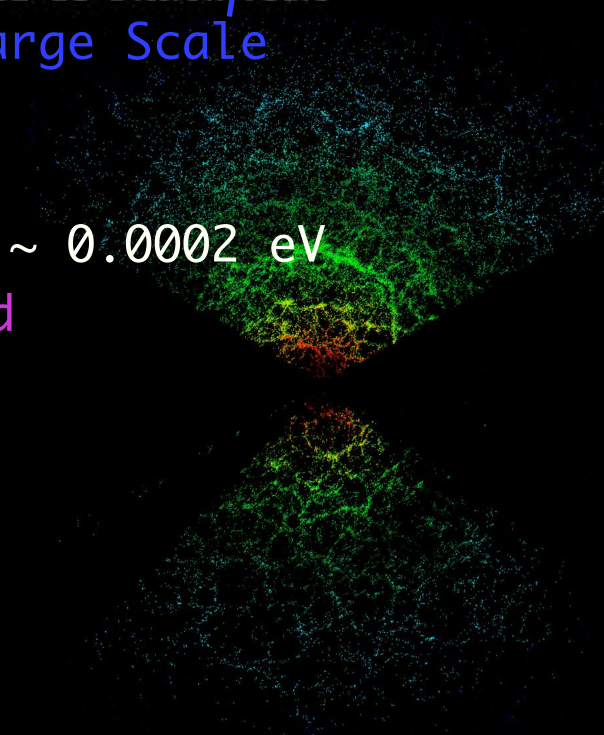
T ~ 10^{29} K ~ 10^{16} GeV

Quark-Hadron transition:

T ~ 10^{12} K ~ 170 MeV

Galaxies & Large Scale Structure Today

T ~ 2.7 K ~ 0.0002 eV

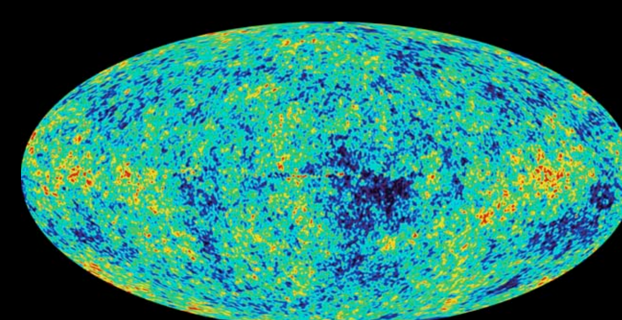


Electroweak Symmetry Breaking:

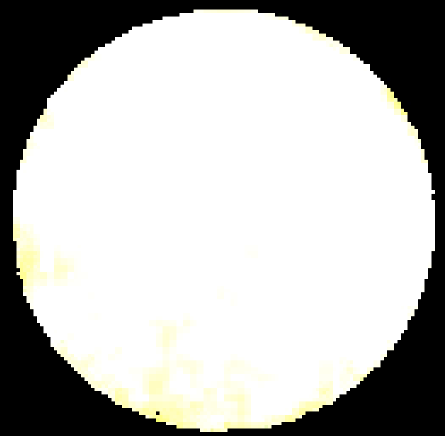
T ~ 10^{15} K ~ 10^2 GeV

Cosmic Microwave Background Decoupling:

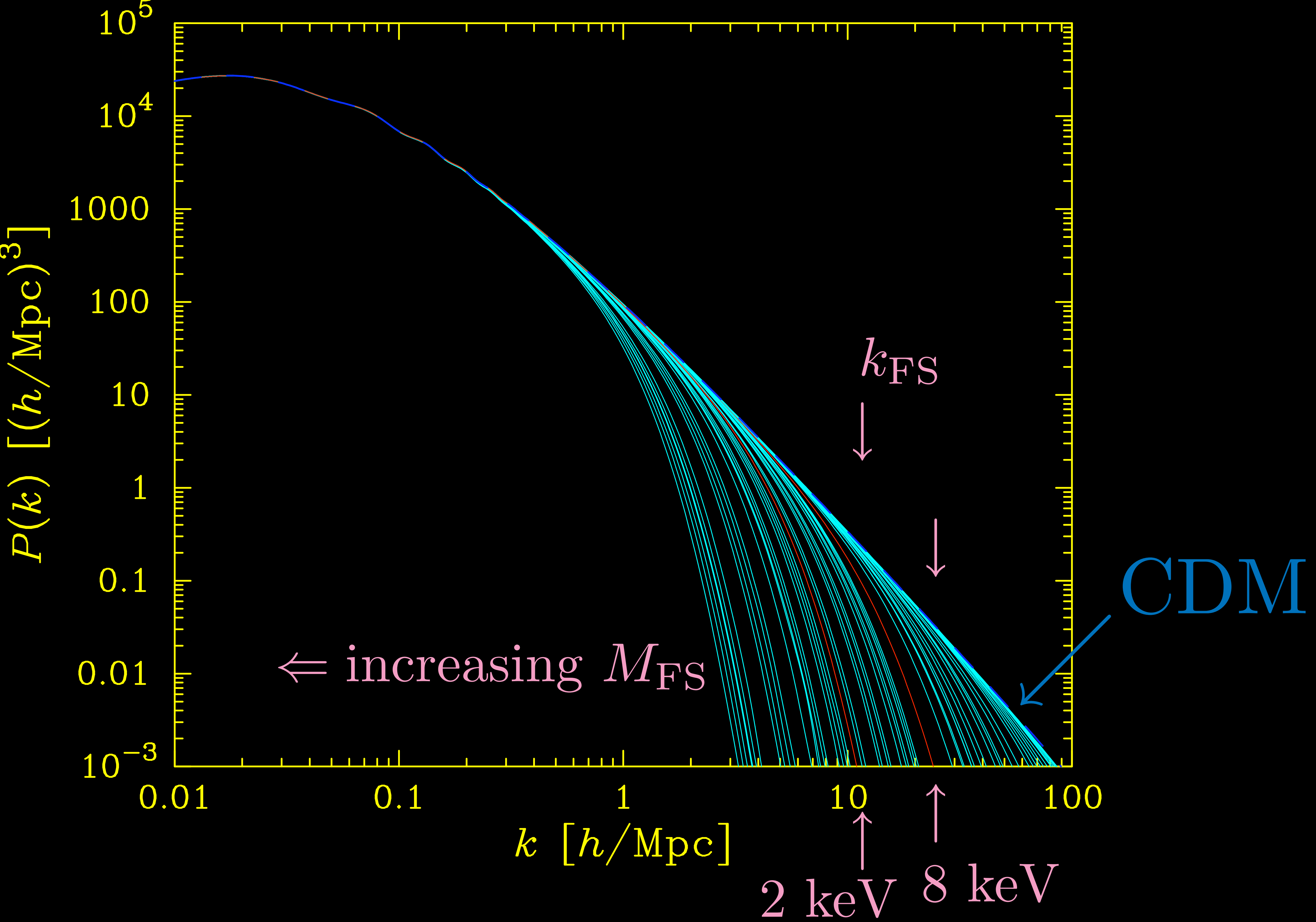
T ~ 3500 K ~ 0.3 eV



CDM Paradigm



The Cosmological Matter Power Spectrum: "Cold" to "Warm" Dark Matter



Problems in Cold Dark Matter?

- Halo Substructure:
  satellite galaxies and sub-halos
  (Klypin et al 1999; Moore et al 1999)
- Halo Cores and Densities:
  (Flores & Primack 1994; Moore 1994)
- Void Galaxy abundances
  (Peebles 2001)
- Angular Momentum Problem
  (Navarro & Benz 1991; Sommer-Larsen & Dolgov 2001)
- Disk Dominated Galaxy Formation
  (Governato et al 2002)

Is the Dark Matter *slightly Warm*?

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Dark matter comes out of the cold

By Jonathan Amos
BBC News science reporter

Astronomers have for the first time put some real numbers on the physical characteristics of dark matter.

The British team used 23 nights of observing time on the VLT

This strange material that dominates the Universe but which is invisible to current telescope technology is one of the great enigmas of modern science.

That it exists is one of the few things on which researchers have been certain.

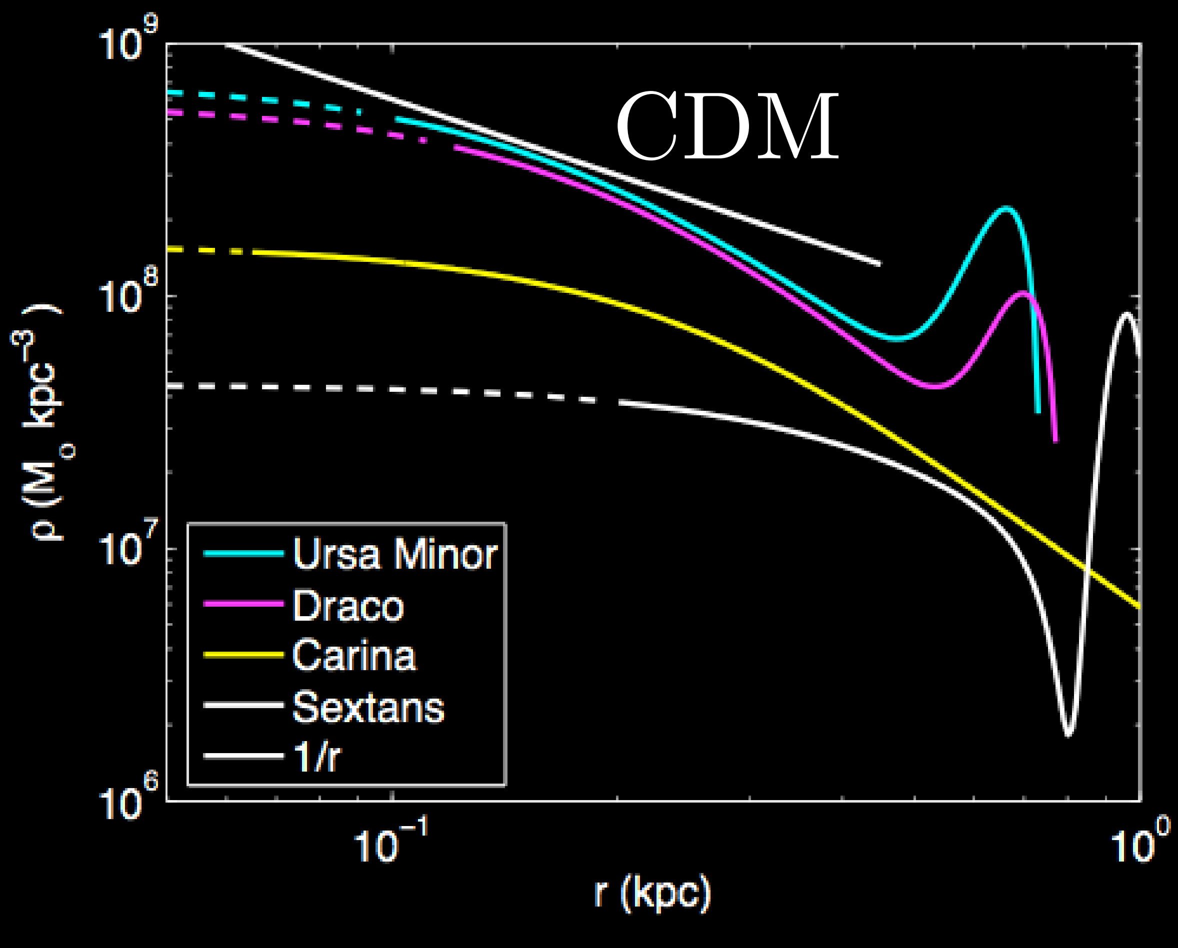
But now an Institute of Astronomy, Cambridge, team has at last been able to place limits on how it is packed in space and measure its "temperature".

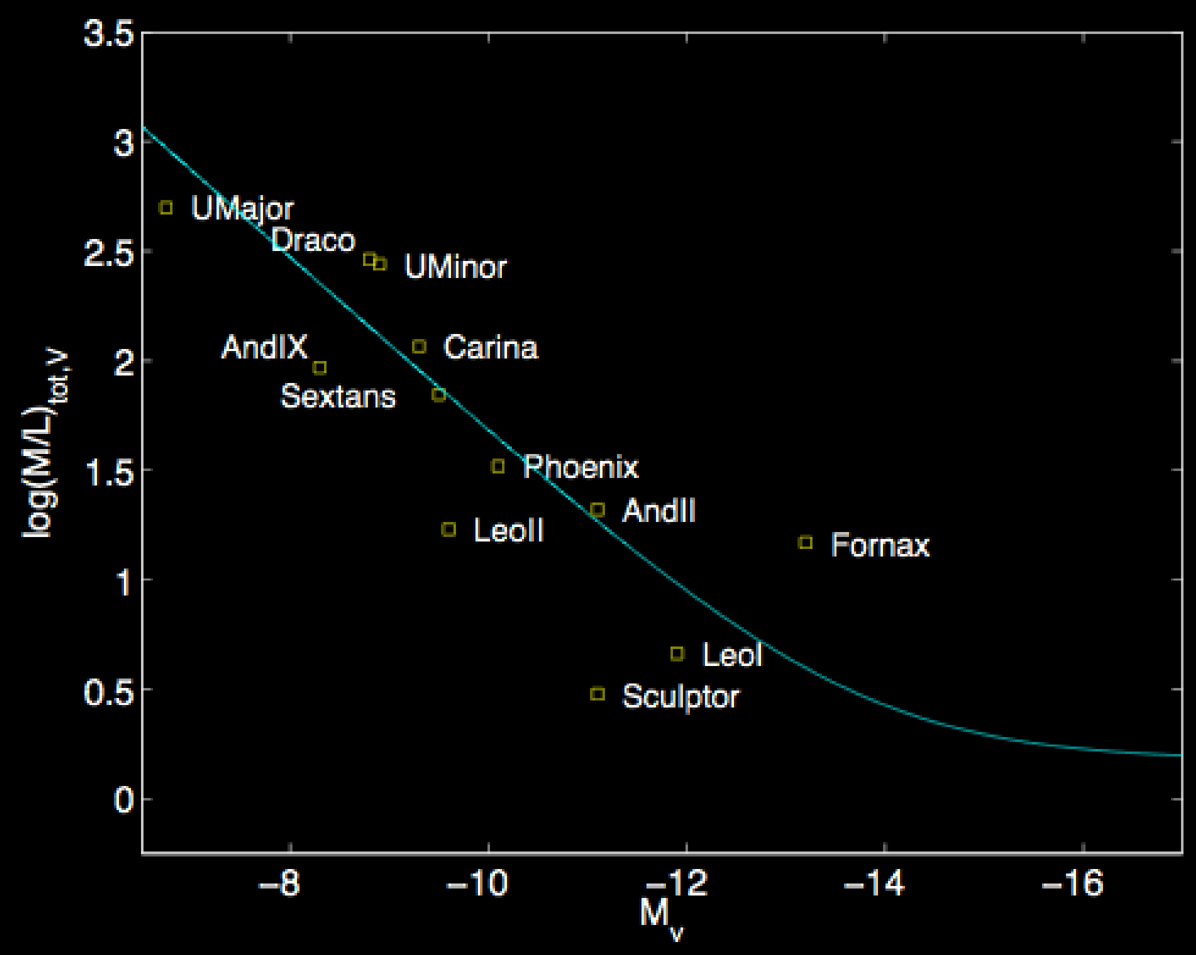
"It's the first clue of what this stuff might be," said Professor Gerry Gilmore. "For the first time ever, we're actually dealing with its physics," he told the BBC News website.

Science understands a great deal about what it terms baryonic matter - the "normal" matter which makes up the stars, planets and people - but it has struggled to comprehend the main material from which the cosmos is constructed.

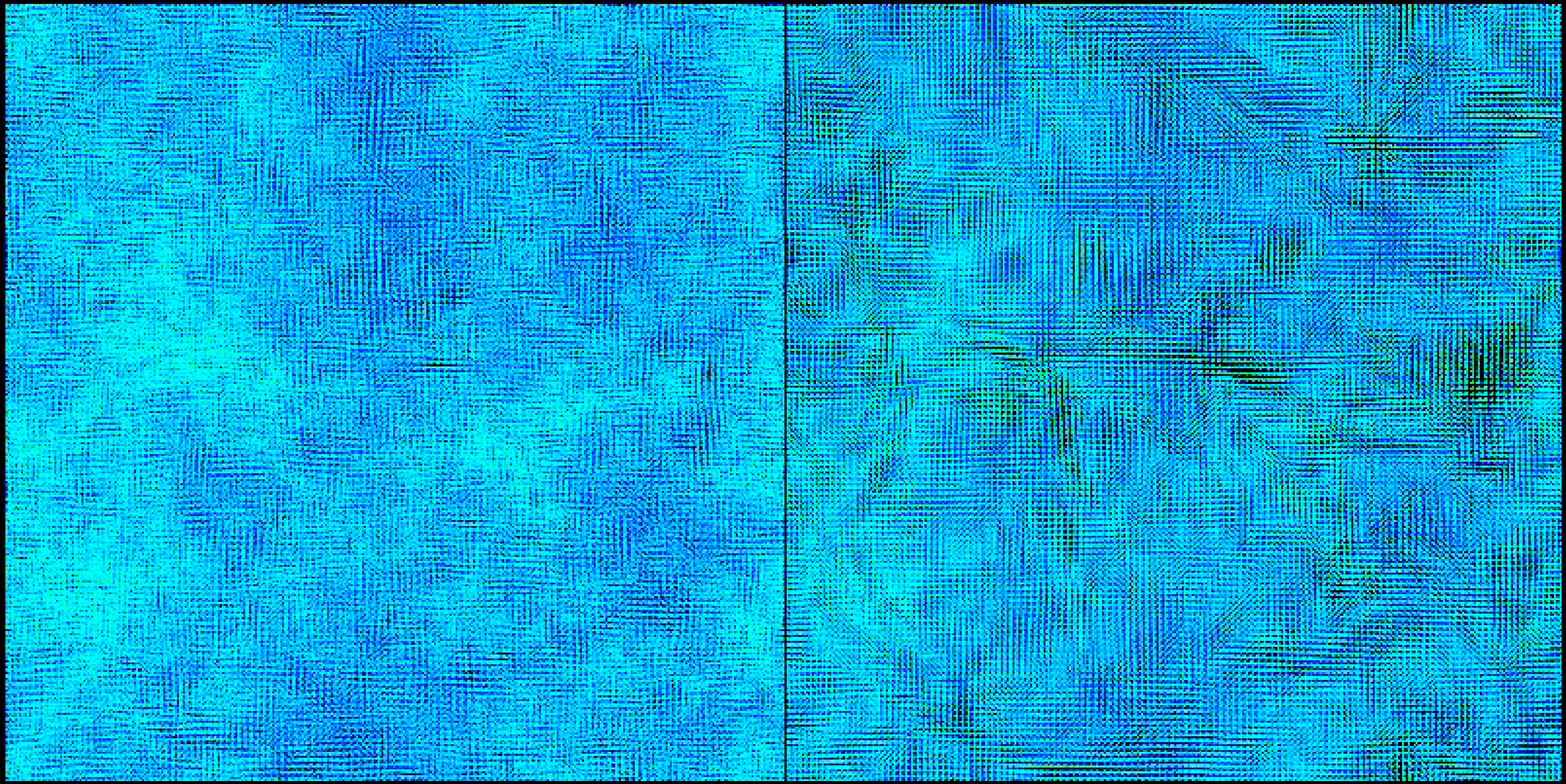
Dwarf Spheroidal Density Profiles from Radial Stellar Velocity Dispersion

- All dwarf spheroidals studied are consistent with NFW and cored profiles, except for UMi, “only consistent with cored profile” [Gilmore et al.,astro-ph/0608528]
- Constant core mass within stellar profile





CDM vs. WDM Structure Formation



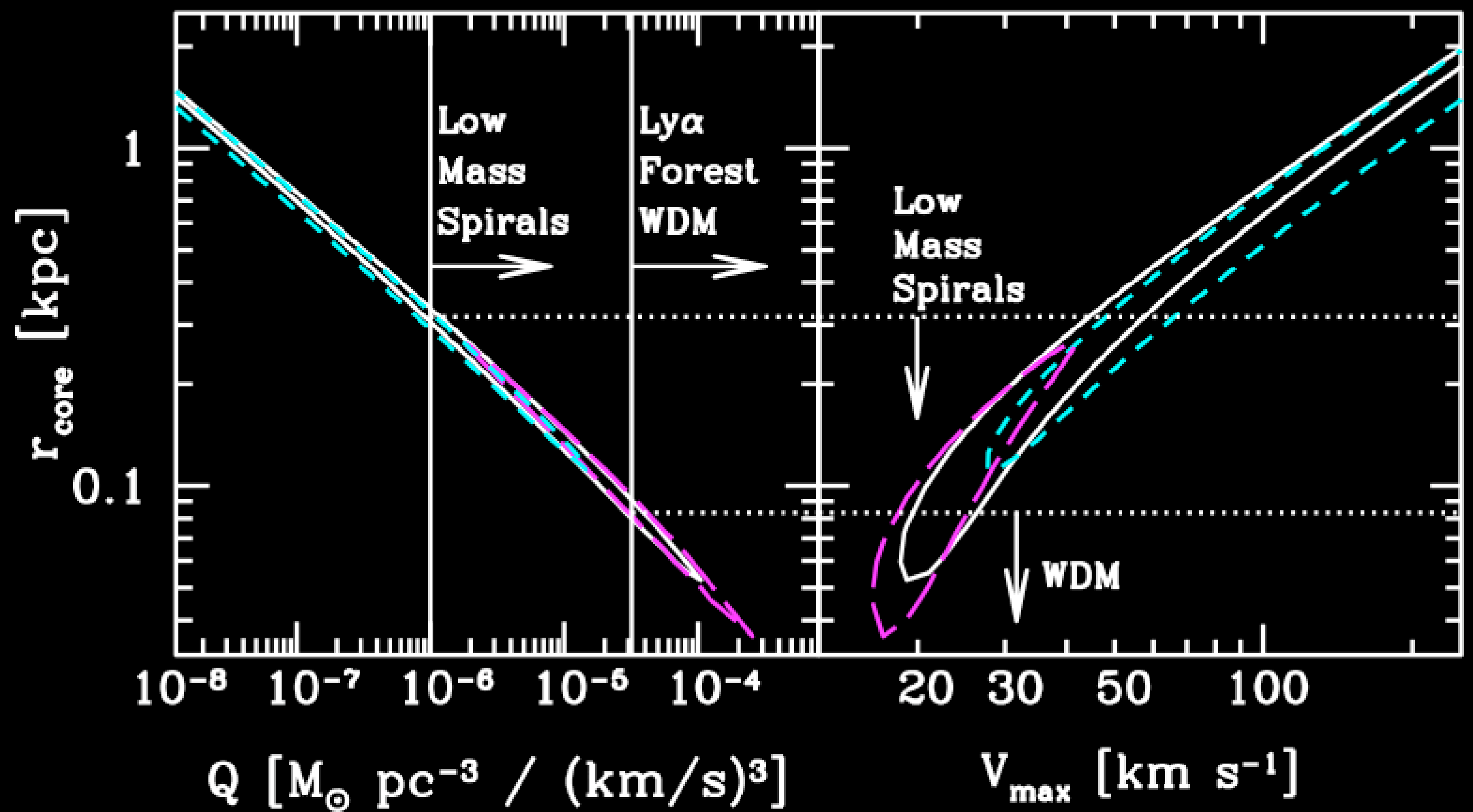
Bode, Ostriker & Turok 2001

Globular Cluster Positions and a Core in Fornax?

- Positions of Globular Clusters away from the center of the Fornax dwarf spheroidal could indicate the presence of a constant-density dark matter core [Goerdt et al 2006; Sanchez-Salcedo et al 2006]

- Cores are in tension or conflict with Lyα Forest constraints [Strigari et al 2006]

phase-space density: $Q \equiv \frac{\rho}{\sigma^3} \propto m_x^4$



The Tremaine-Gunn Bound:
Liouville's Theorem in Statistical Mechanics

phase-space density: $Q \equiv \frac{\rho}{\sigma^3} \propto m_x^4$

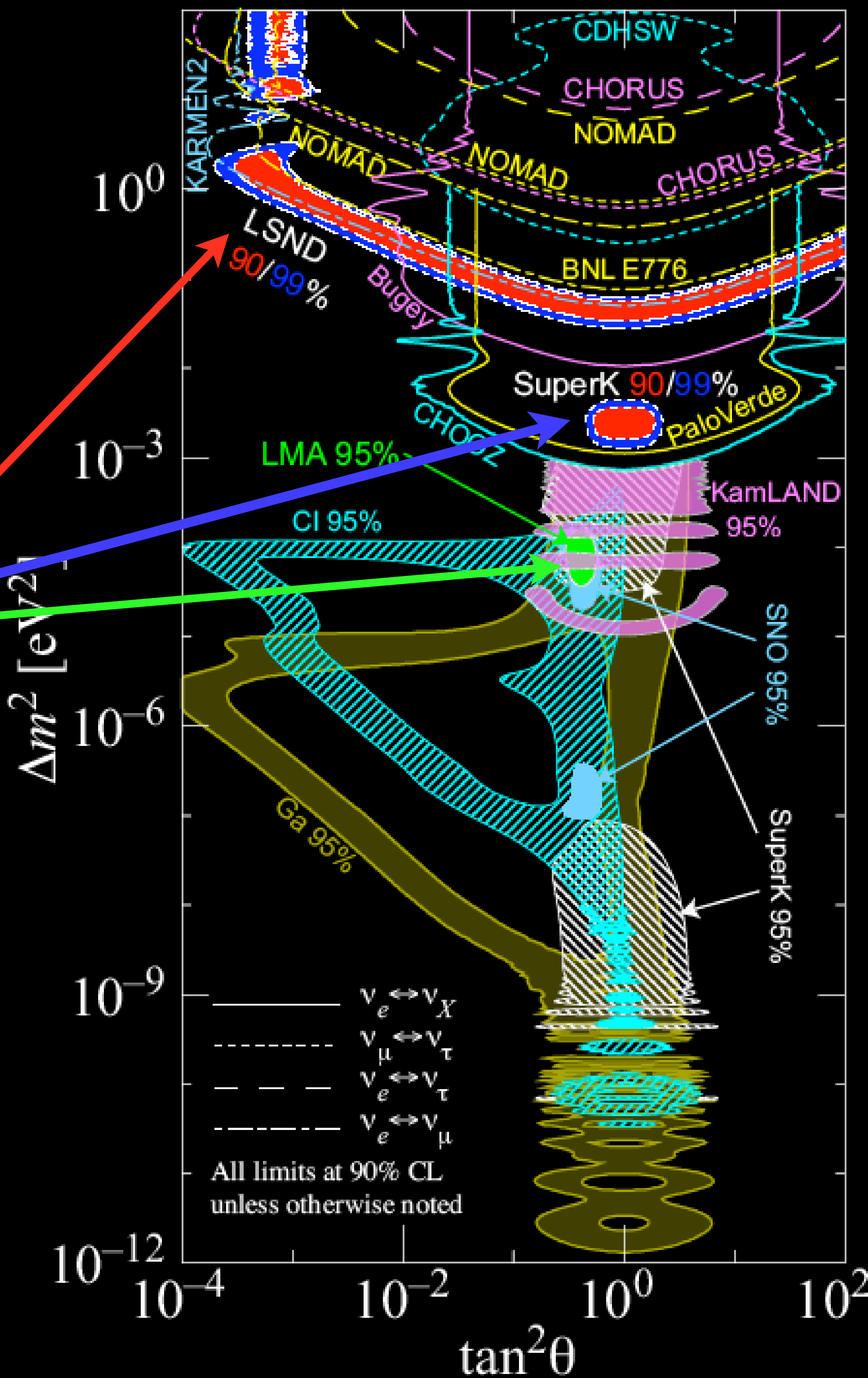
Maxwell statistics: $\rho < 2m_X^4 \left(\frac{\sqrt{2\pi}\sigma}{h_p} \right)^3$

$$\rho = \frac{9\sigma^2}{4\pi G r_c^2}, \text{ King Profile}$$

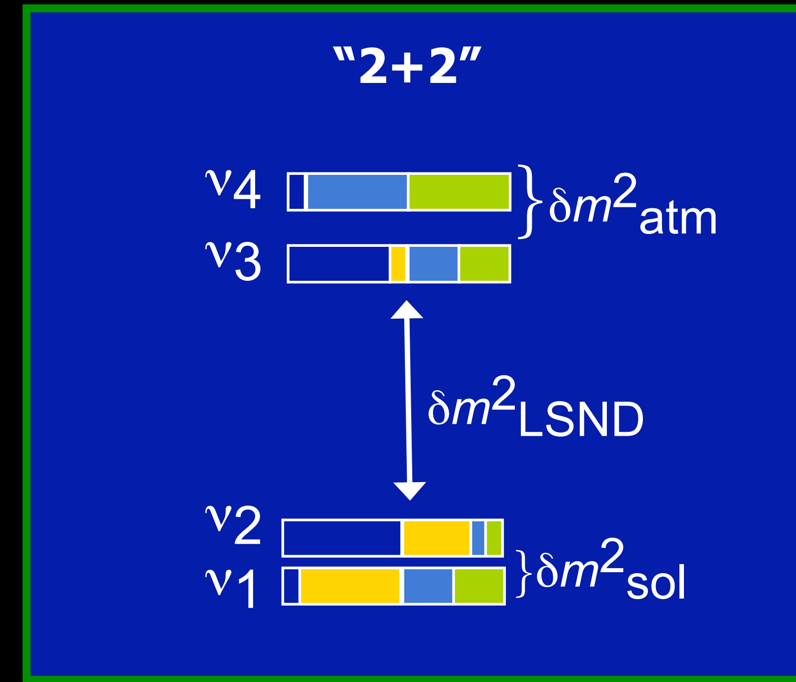
$$\Rightarrow m_X > 0.4 \text{ keV}$$

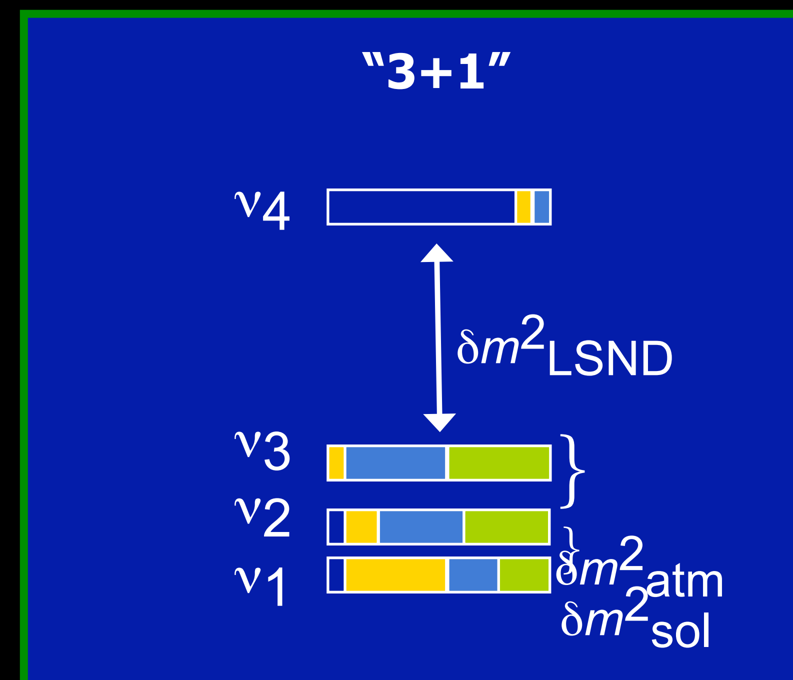
Bode et al (2001)
Tremaine, Gunn (1979)

Experimental Motivation: Too Much of a Good Thing?



Three Mass Differences





PDG, RPP 2004

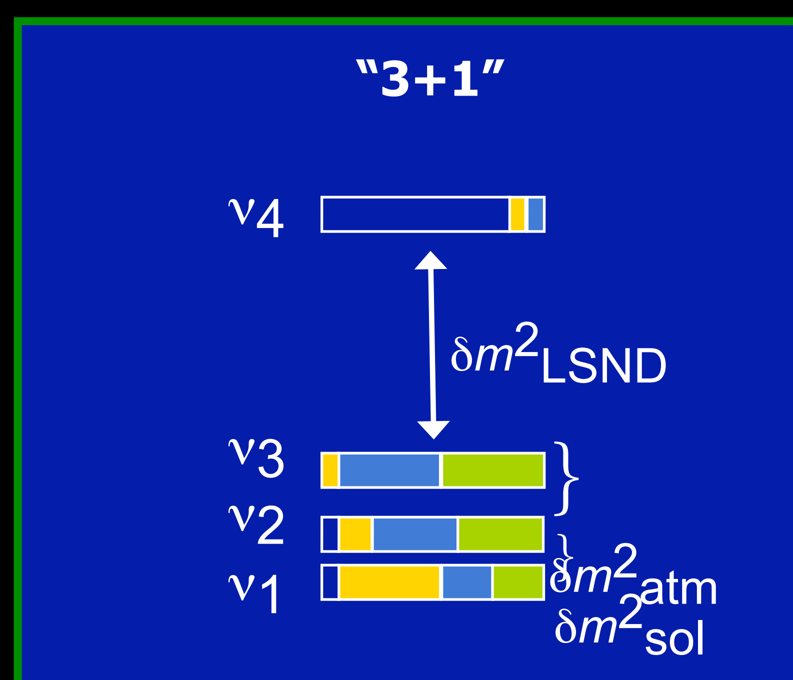
Sterile Neutrino Dark Matter

ν_5

$$\begin{aligned} |\nu_\alpha\rangle &= \cos\theta|\nu_a\rangle + \sin\theta|\nu_b\rangle \\ |\nu_s\rangle &= -\sin\theta|\nu_a\rangle + \cos\theta|\nu_b\rangle \end{aligned}$$

~ 1 keV

$\sin^2 2\theta \sim 10^{-7}$



~ 1 eV

~ 0.01 eV

Sterile Neutrinos

Beyond the Standard Model of Particle Physics

- ν_s Phenomenological Insertion of Majorana & Dirac Mass Terms of Comparable Magnitude (e.g. νMSM)
- ν_s Left-Right Symmetric Models (Pati & Salam 1974; Mohapatra & Pati 1975)
- ν_s Higher Dimensional Operators in String-Inspired models (Langacker 1998)
- ν_s Bulk Fermions in Large Extra Dimensions (ADD; Dvali & Smirnov 2000)
- ν_s Axino in R-parity Violating Minimal Supersymmetric Models (Chun & Kim 1999)
- ν_s Produced non-resonantly in the Early Universe as light WDM (Dodelson & Widrow 1994)
- ν_s Produced Resonantly in the Early Universe as “Cool” Dark Matter (Shi & Fuller, 1999)

The νMSM : a minimalist model

- The Neutrino Minimal Standard Model of Particle Physics [Asaka, Blanchet & Shaposhnikov 2005]
- Add Dirac & Majorana Neutrino Mass Terms to MSM

$$\delta\mathcal{L} = \bar{N}_I i \partial_\mu \gamma^\mu N_I - f^\nu_{I\alpha} \Phi \bar{N}_I L_\alpha - \frac{M_I}{2} \bar{N}^c_I N_I + h.c.$$

- Two heavy sterile neutrinos provide atmospheric & solar mass scales
- Baryogenesis via Leptogenesis
- Light sterile neutrino is the Dark Matter
- More involved models generally involve similar insertions for neutrino mass generation

History: Sterile Neutrinos as Dark Matter

- "Super-weak" neutrinos ($G < G_F$) [Olive & Turner, 1982]: Earlier Decoupling, abundance set by standard dark matter production mechanism of decoupling temperature and degrees of freedom disappearance

- "Sterile" neutrinos [Dodelson & Widrow, 1993]: No SM interactions beyond mass terms, inclusion of finite-temperature modifications to self-energy, lack of thermalization. WDM.

- "Resonant" sterile neutrinos [Shi & Fuller, 1999]: Finite temperature production with non-zero lepton number resonant enhanced production. WDM to CDM. "Cool" Dark Matter.

- "Precision" Sterile Neutrino Dark Matter [Abazajian, Fuller & Patel 2001; KA 2005]: Full momentum-space production description with high-T collision and QCD transition corrections, resonant to non-resonant solutions as a continuum in lepton number.

Neutrinos in the Early Universe

The Two Neutrino Case

Incorporating oscillations, collisions and/or active-active forward scattering necessitates:

$$\rho(\mathbf{p},t) \equiv \langle\psi_\alpha|\widehat{\rho}(\mathbf{p},t)|\psi_\beta\rangle \doteq \begin{pmatrix} \rho_{\alpha\alpha}(\mathbf{p},t) & \rho_{\alpha\beta}(\mathbf{p},t) \\ \rho_{\beta\alpha}(\mathbf{p},t) & \rho_{\beta\beta}(\mathbf{p},t) \end{pmatrix}$$

density-matrix or matrix-of-densities

Evolution of which gives quantities of interest:

$$n_{\nu_\alpha}(p) = \rho_{\alpha\alpha}(p,t)$$

$$n_{\nu_\beta}(p) = \rho_{\beta\beta}(p,t)$$

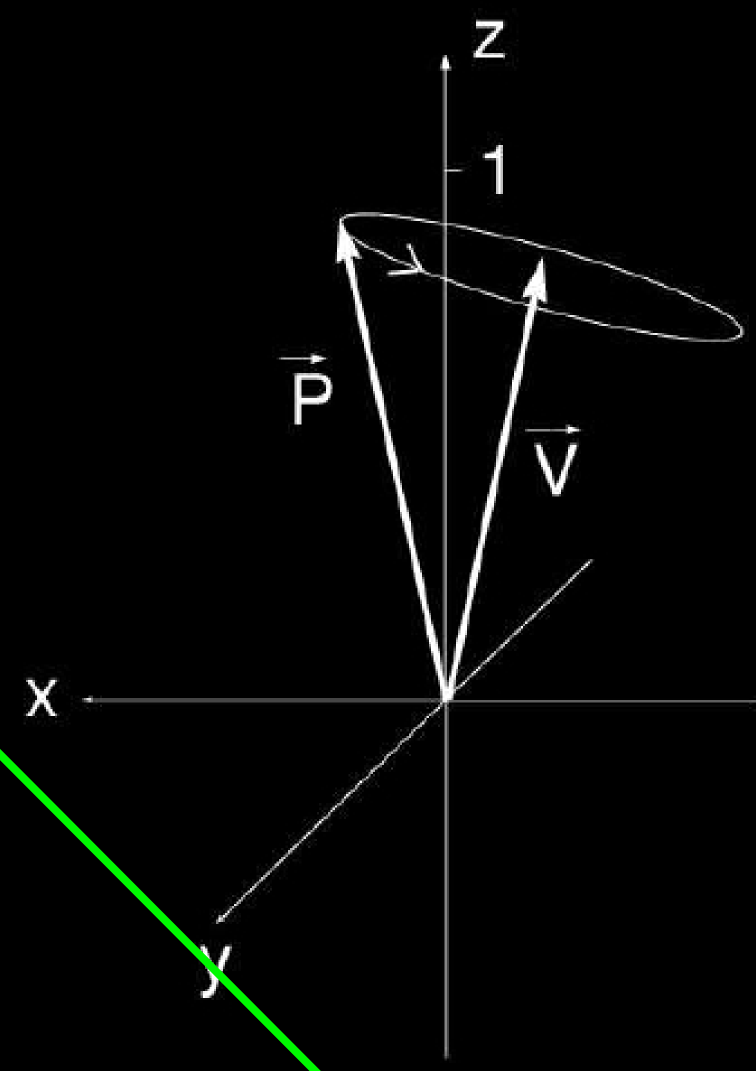
$$\begin{aligned} \rho(p) &= \frac{1}{2}\left[P_0(p) + \mathbf{P}(p)\cdot\sigma\right] \\ &= \frac{1}{2}\begin{pmatrix} P_0(p) + P_z(p) & P_x(p) - iP_y(p) \\ P_x(p) + iP_y(p) & P_0(p) - P_z(p) \end{pmatrix} \end{aligned}$$

Coherent Behavior

$$\partial_t \mathbf{P}(p) = \mathbf{V}(p) \times \mathbf{P}(p)$$

$$\mathbf{V}(p) = \mathbf{\Delta}(p) + \left[V^B(p) + V^T(p)\right]\hat{\mathbf{z}} + \mathbf{V}^S(p)$$

...dipole precession in a magnetic field



Vacuum Oscillation

$e^{+/-}$ / $\mu^{+/-}$ background

Thermal Potential
(finite temperature effects)

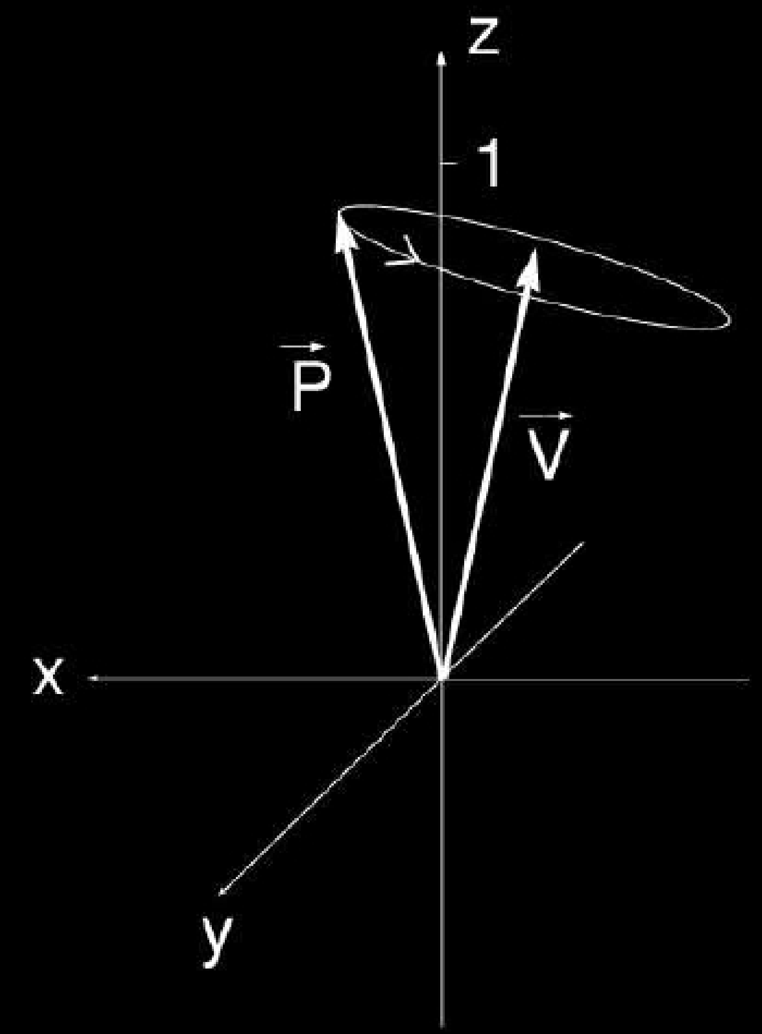
Neutrino Self-Potential

Vacuum Neutrino Oscillations

$$\partial_t \mathbf{P}(p) = \mathbf{V}(p) \times \mathbf{P}(p)$$

$$\mathbf{V}(p) = \boxed{\mathbf{\Delta}(p)} + \left[V^B(p) + V^T(p)\right]\hat{\mathbf{z}} + \mathbf{V}^S(p)$$

$$\mathbf{\Delta}(p) = \frac{\delta m^2}{2p}(\sin 2\theta_0, 0, -\cos 2\theta_0)$$

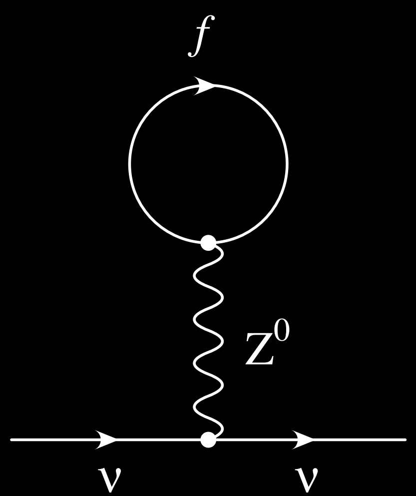


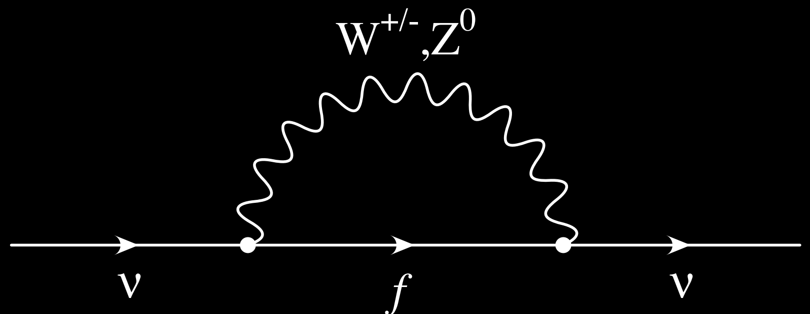
Background and Thermal Potentials

$$\boxed{\mathbf{V}(p) = \mathbf{\Delta}(p) + \left[\boxed{V^B(p)} + \boxed{V^T(p)} \right] \hat{\mathbf{z}} + \mathbf{V}^S(p)}$$

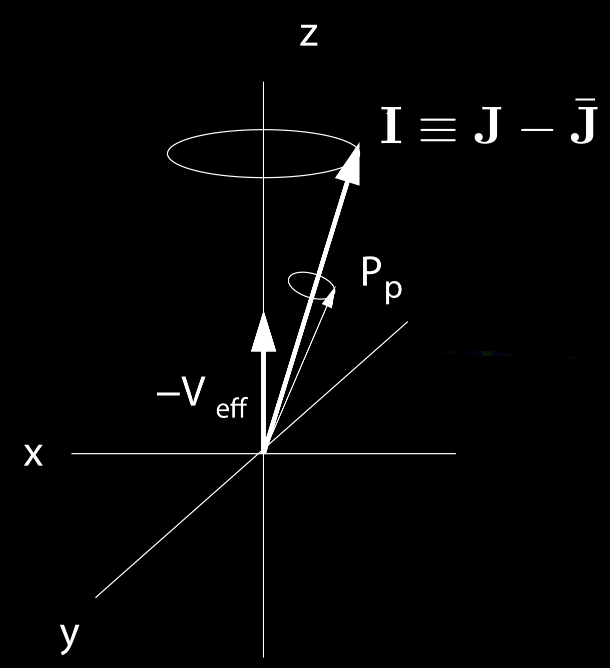
$$V^B(p) = \begin{cases} \sqrt{2} G_F \left[(n_{e^-} - n_{e^+}) - n_n/2 \right] & \text{for } \nu_\alpha \rightleftharpoons \nu_s, \\ \sqrt{2} G_F \left(n_{e^-} - n_{e^+} \right) & \text{for } \nu_e \rightleftharpoons \nu_{\mu,\tau}, \\ 0 & \text{for } \nu_\mu \rightleftharpoons \nu_\tau. \end{cases}$$

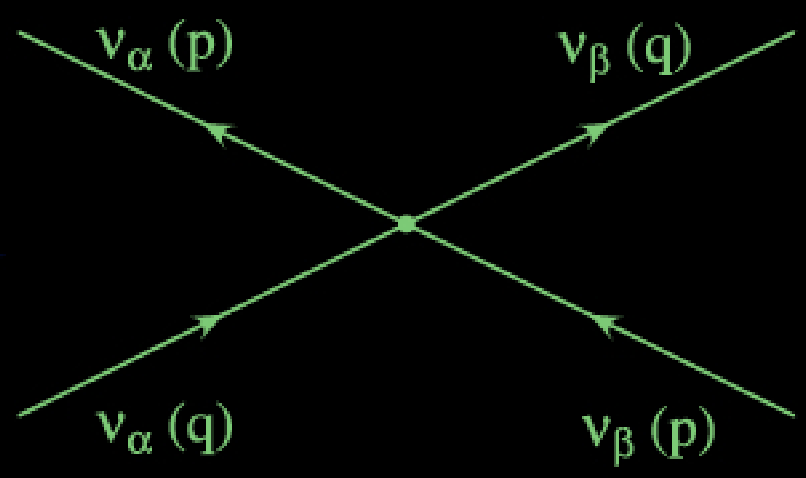
$$\begin{aligned} V^T(p) = & - \frac{8\sqrt{2} G_F p_\nu}{3 m_Z^2} \left(\langle E_{\nu_\alpha} \rangle n_{\nu_\alpha} + \langle E_{\bar{\nu}_\alpha} \rangle n_{\bar{\nu}_\alpha} \right) \\ & - \frac{8\sqrt{2} G_F p_\nu}{3 m_W^2} \left(\langle E_\alpha \rangle n_\alpha + \langle E_{\bar{\alpha}} \rangle n_{\bar{\alpha}} \right), \end{aligned}$$





The Neutrino Self-Potential





$$\mathbf{V}(p) = \mathbf{\Delta}(p) + \left[V^B(p) + V^T(p)\right]\hat{\mathbf{z}} + \boxed{\mathbf{V}^S(p)}$$

$$\text{Sterility} \ \Rightarrow \mathbf{V}^S(p) = V^L(p)\hat{\mathbf{z}}$$

For $\nu_\alpha \rightleftharpoons \nu_\beta$

$$\mathbf{V}^S(p) = 2\sqrt{2}G_F n_{\nu_\alpha}(\mathbf{J} - \bar{\mathbf{J}})$$

where

$$\mathbf{J} \equiv \sum_p \mathbf{P}_p, \qquad \bar{\mathbf{J}} \equiv \sum_p \bar{\mathbf{P}}_p$$

Collisions and Decoherence

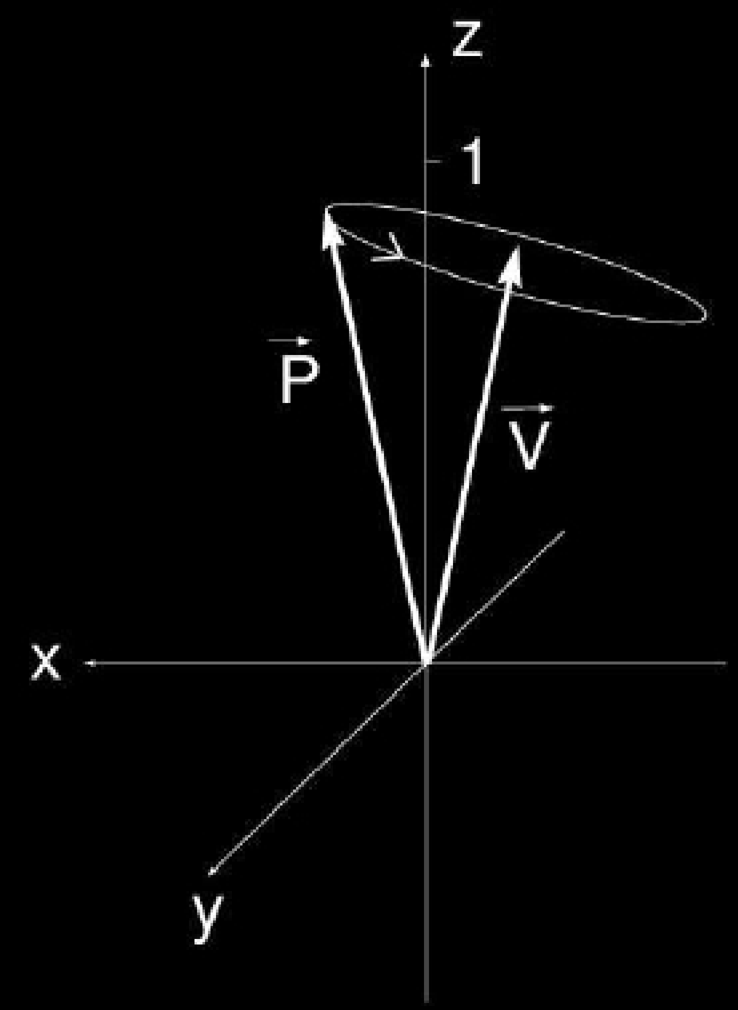
$$\partial_t \mathbf{P}(p) = \mathbf{V}(p) \times \mathbf{P}(p) + \boxed{\mathcal{C}[\mathbf{P}(p)]}$$

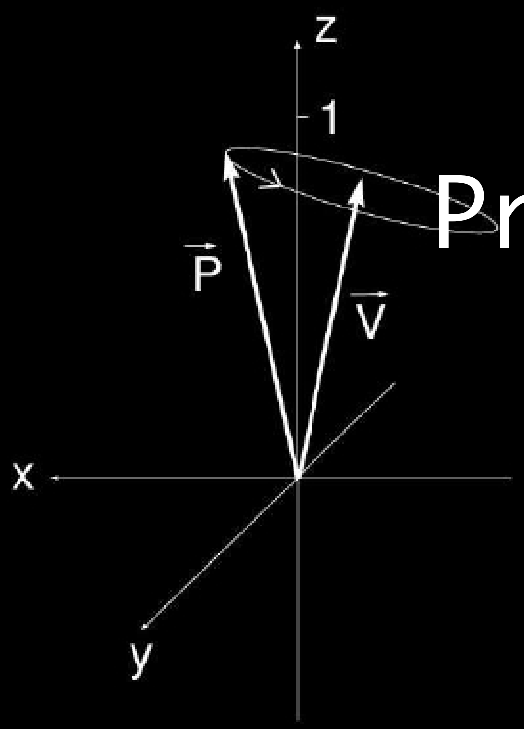
Collision terms:

$$\mathcal{C}[\mathbf{P}(p)] \approx -D(p)\mathbf{P}_T(p) + \int dp' d(p,p')\mathbf{P}_T(p') - \mathbf{C}(p)P_0(p) + \int dp' \mathbf{c}(p,p')P_0(p')$$

where

$$\mathbf{P}_T = P_x(p)\hat{\mathbf{x}} + P_y(p)\hat{\mathbf{y}}$$





Sterile Neutrino Dark Matter Production: Collision Domination

$$\partial_t \mathbf{P}(p) = \mathbf{V}(p) \times \mathbf{P}(p) - D(p)\mathbf{P}_T(p)$$

$$\mathbf{V}(p) = \mathbf{\Delta}(p) + \left[V^T(p) + V^L(p)\right]\hat{\mathbf{z}}$$

$$\Gamma_\alpha(p) = 1.27\, G_{\rm F}^2 p T^4$$

Quasi-Classical Boltzmann Equation:

$$\frac{\partial}{\partial t} f_s(p,t) - H\, p\, \frac{\partial}{\partial p} f_s(p,t) \approx \Gamma(\nu_\alpha \to \nu_s; p, t)\left[f_\alpha(p,t) - f_s(p,t)\right]$$

$$\approx \frac{\Gamma_\alpha(p)}{2}\langle P_m(\nu_\alpha \to \nu_s, t_{\rm in} + \tau)\rangle_\tau \left[f_\alpha(p,t) - f_s(p,t)\right]$$

$$\approx \frac{\Gamma_\alpha(p)}{2}\sin^2 2\theta_m \left[1 + \left(\frac{\Gamma_\alpha(p) l_m}{2}\right)^2\right]^{-1}\left[f_\alpha(p,t) - f_s(p,t)\right]$$

$$\approx \frac{1}{4}\frac{\Gamma_\alpha(p)\Delta^2(p)\sin^2 2\theta}{\Delta^2(p)\sin^2 2\theta + D^2(p) + \left[\Delta(p)\cos 2\theta - V^L - V^T(p)\right]^2}\left[f_\alpha(p,t) - f_s(p,t)\right]$$

Sterile Neutrino Dark Matter Production

$\Gamma_\alpha(p) \sim G_F^2 p T^4 \sim T^5$ $\qquad \Delta^2 \sim p^{-2} \sim T^{-2}$

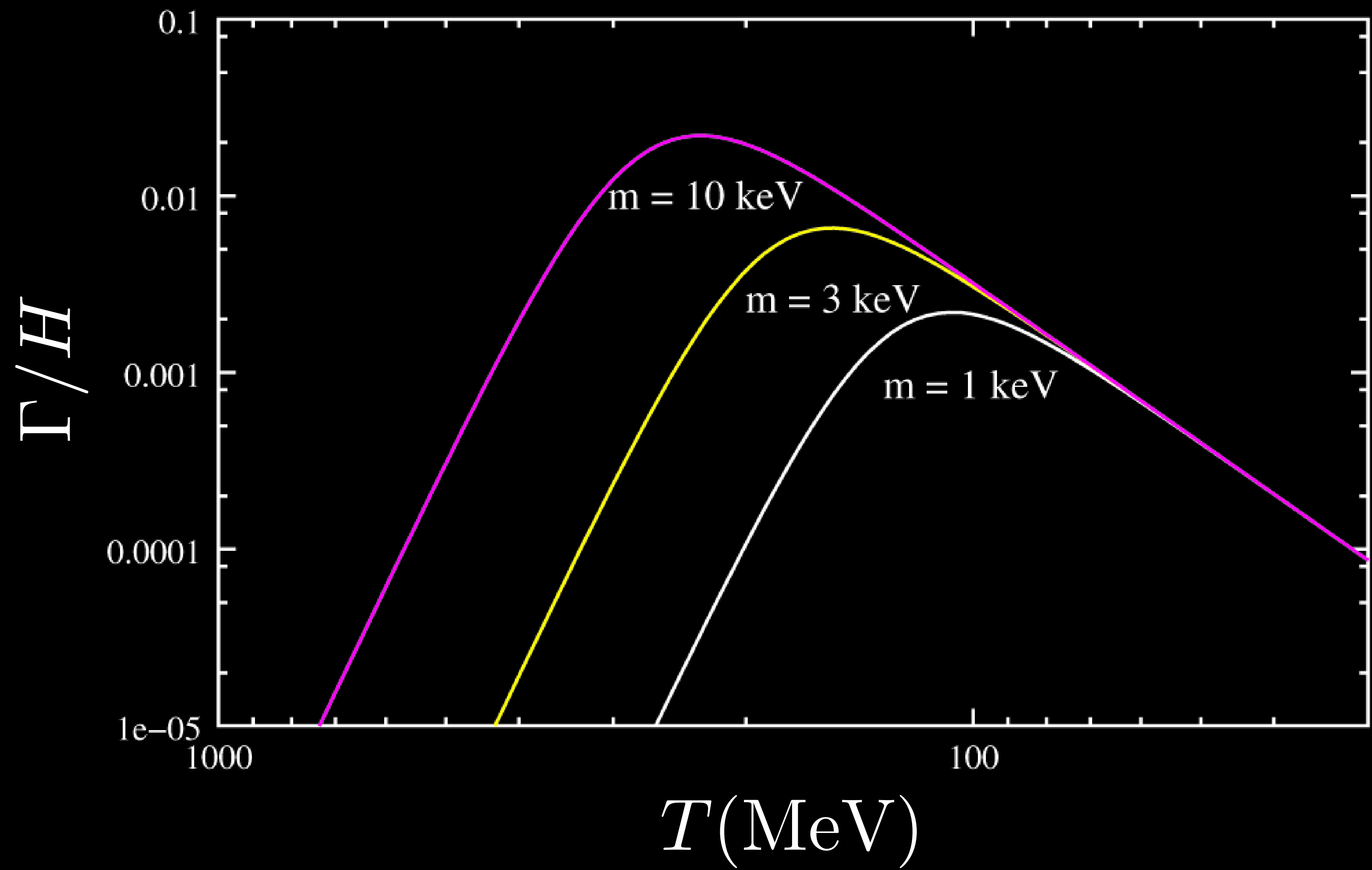
$$\Gamma(\nu_\alpha \to \nu_s) \sim \frac{\Gamma_\alpha(p)\Delta^2(p)\sin^2 2\theta}{\Delta^2(p)\sin^2 2\theta + D^2(p) + [\Delta(p)\cos 2\theta - V^L(p) - V^T(p)]^2}$$

$D(p)^2 \sim T^{10}$ $\qquad [V^T]^2 \sim T^{10}$

$$H^2 = \frac{8\pi}{3} G\rho \sim T^4$$

$$\frac{\Gamma}{H} \sim \begin{cases} T^{-7} & \text{High } T \\ T^3 & \text{Low } T \end{cases}$$

Never in Equilibrium!!



Bulk QCD Thermodynamics

$$\frac{\partial}{\partial t} f_s(p,t) - H\, p\, \frac{\partial}{\partial p} f_s(p,t) = \frac{1}{4} \frac{\Gamma_\alpha(p)\Delta^2(p)\sin^2 2\theta}{\Delta^2(p)\sin^2 2\theta + D^2(p) + [\Delta(p)\cos 2\theta - V^L(p) - V^T(p)]^2} [f_\alpha(p,t) - f_s(p,t)]$$

$$\frac{dT}{dt} = \frac{dr}{dt} \Big/ \frac{dr}{dT} \left[\Rightarrow \frac{dT}{dr} = \left(\frac{4}{3}\rho_* + \rho_s + p_s\right)\left(\frac{d\rho_*}{dT} + \frac{d\rho_s}{dT}\right)^{-1} \right]$$

- Particle mass distribution is nontrivial
- Thermalized presence of hadrons & leptons changes thermal history (time-temperature relation), finite T effects, and scattering rates
- Distorts produced sterile neutrino momentum distributions

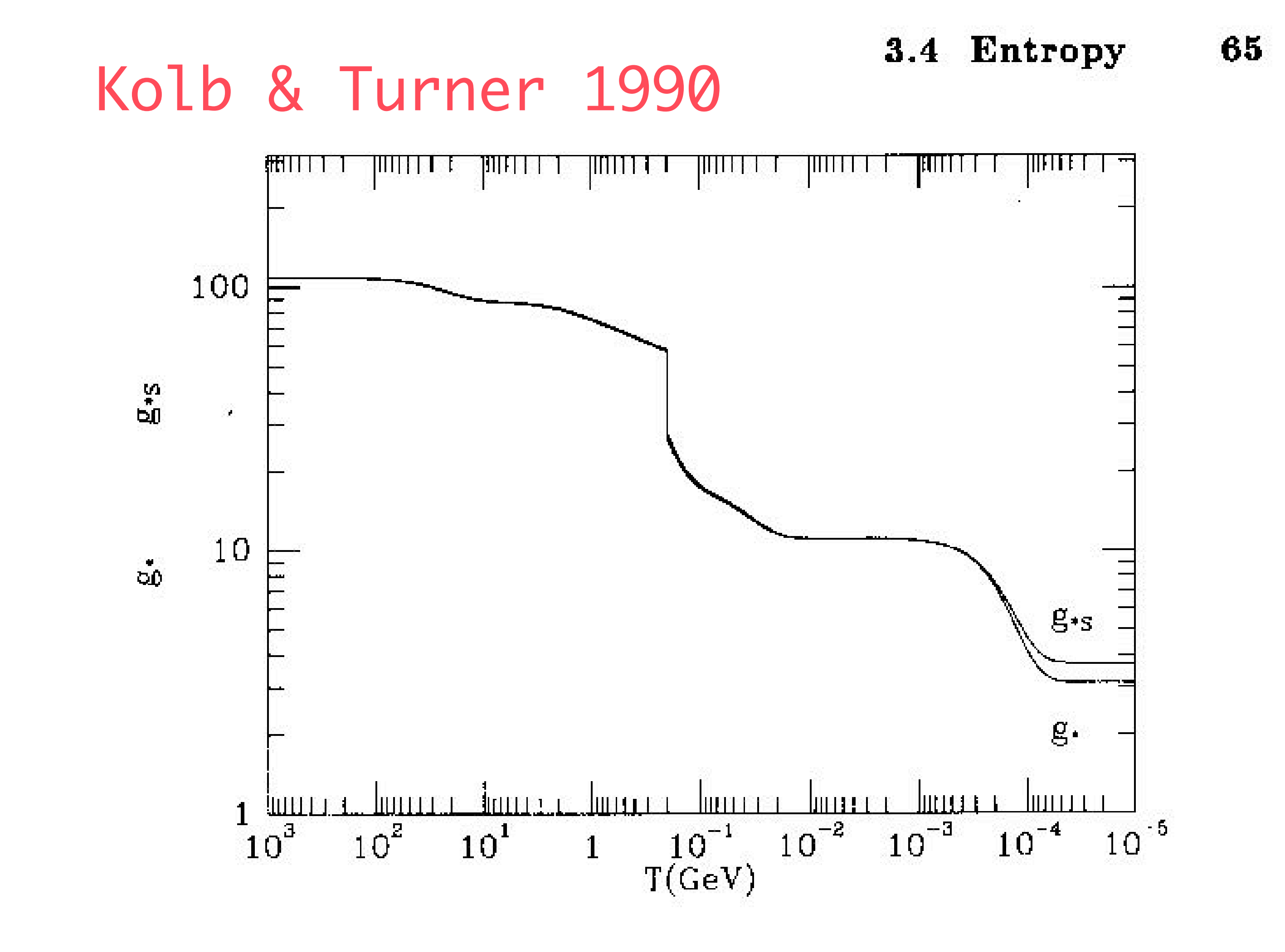
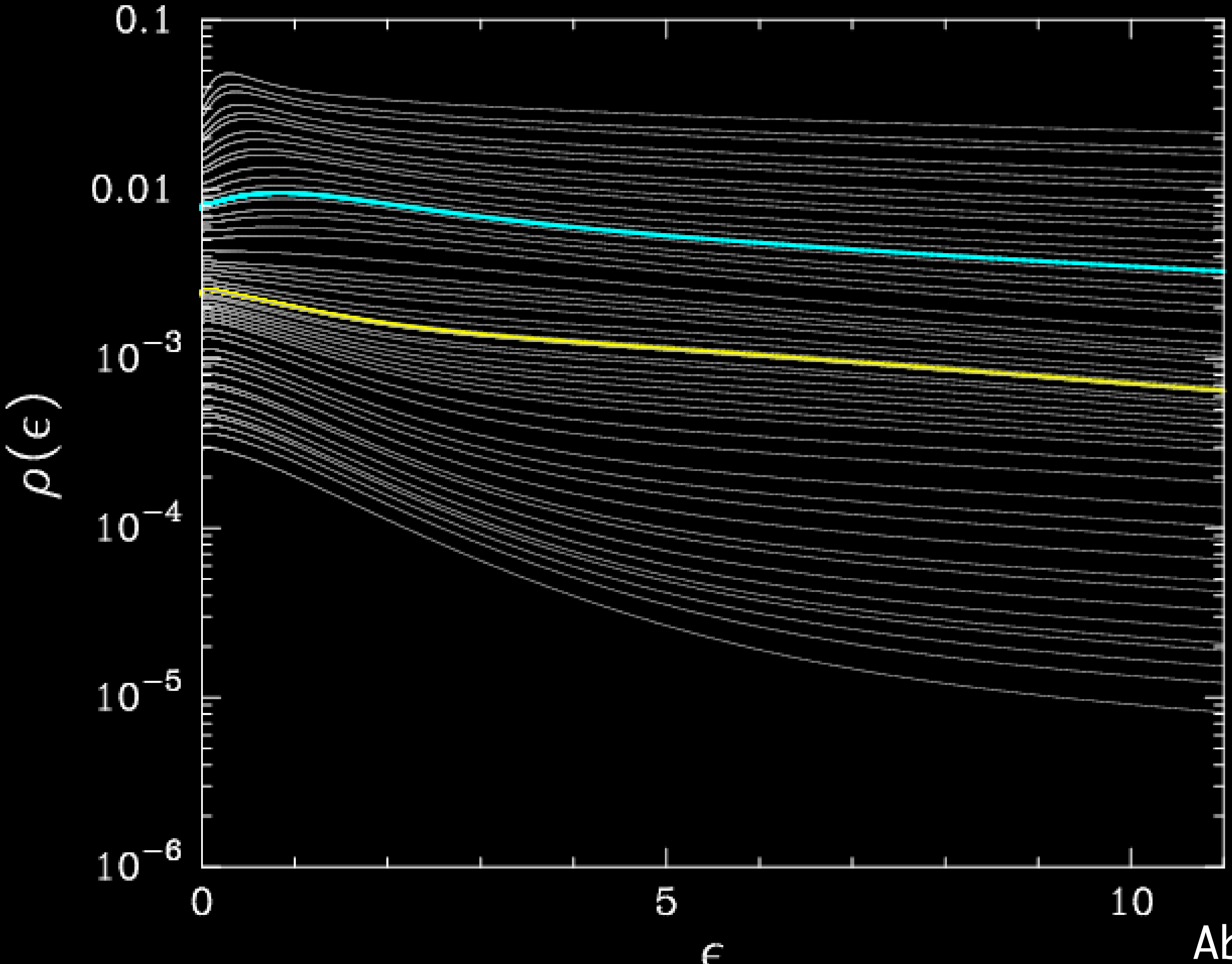


Fig. 3.5: The evolution of $g_*(T)$ as a function of temperature in the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ theory.

Spectral Distribution

$$\rho(\epsilon) = \frac{f_s(\epsilon)}{f_\nu(\epsilon)} \qquad f_\nu(\epsilon) = \frac{1}{e^\epsilon + 1} \qquad \epsilon = p/T$$

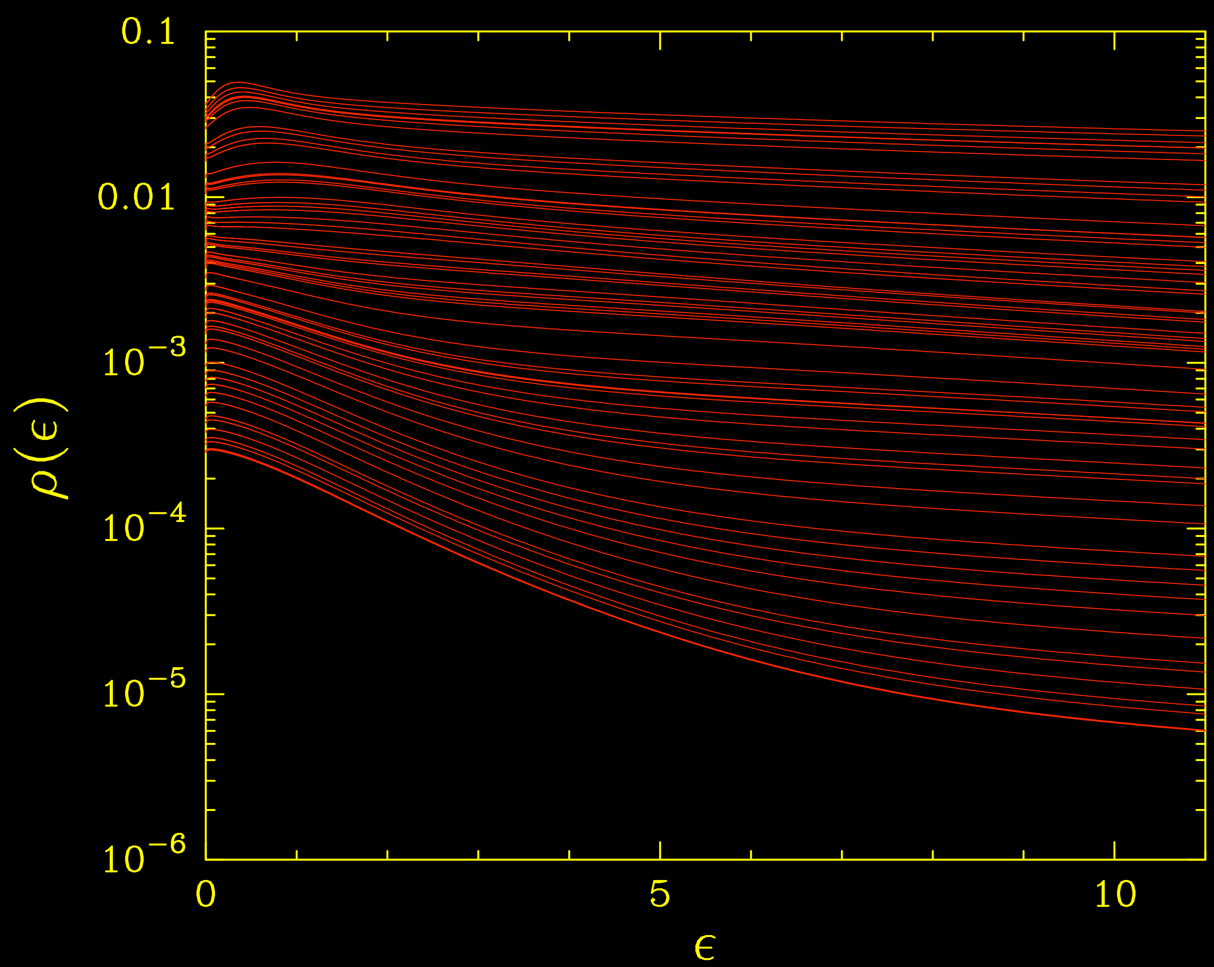


Abazajian 2005

Sterile Neutrino Perturbation Evolution and Structure

$T \sim 10^{11}$ K

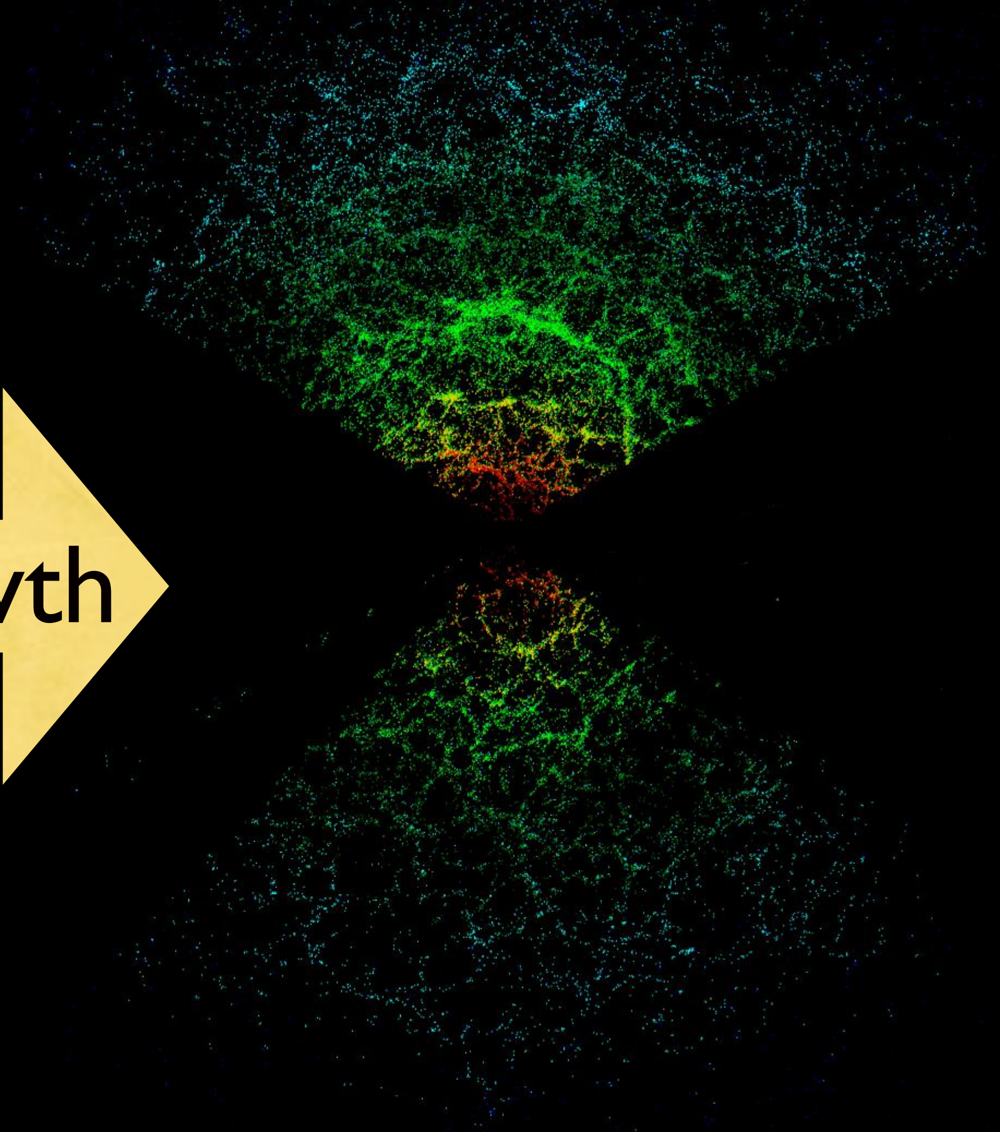
$z \sim 10^{11}$



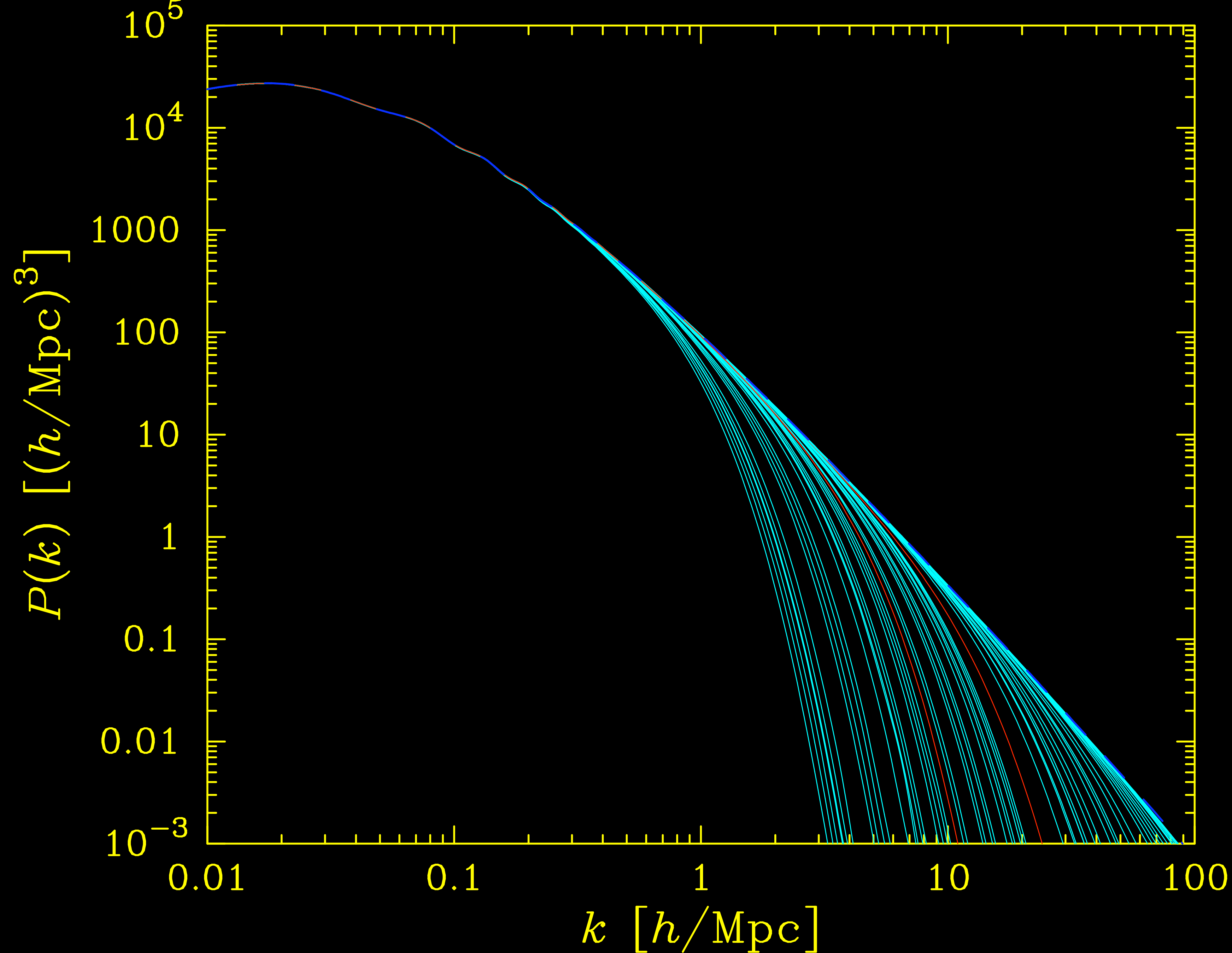
Growth

$T \sim 3\,\mathrm{K}$

$z \sim 0$



The Sterile Neutrino Transfer Function

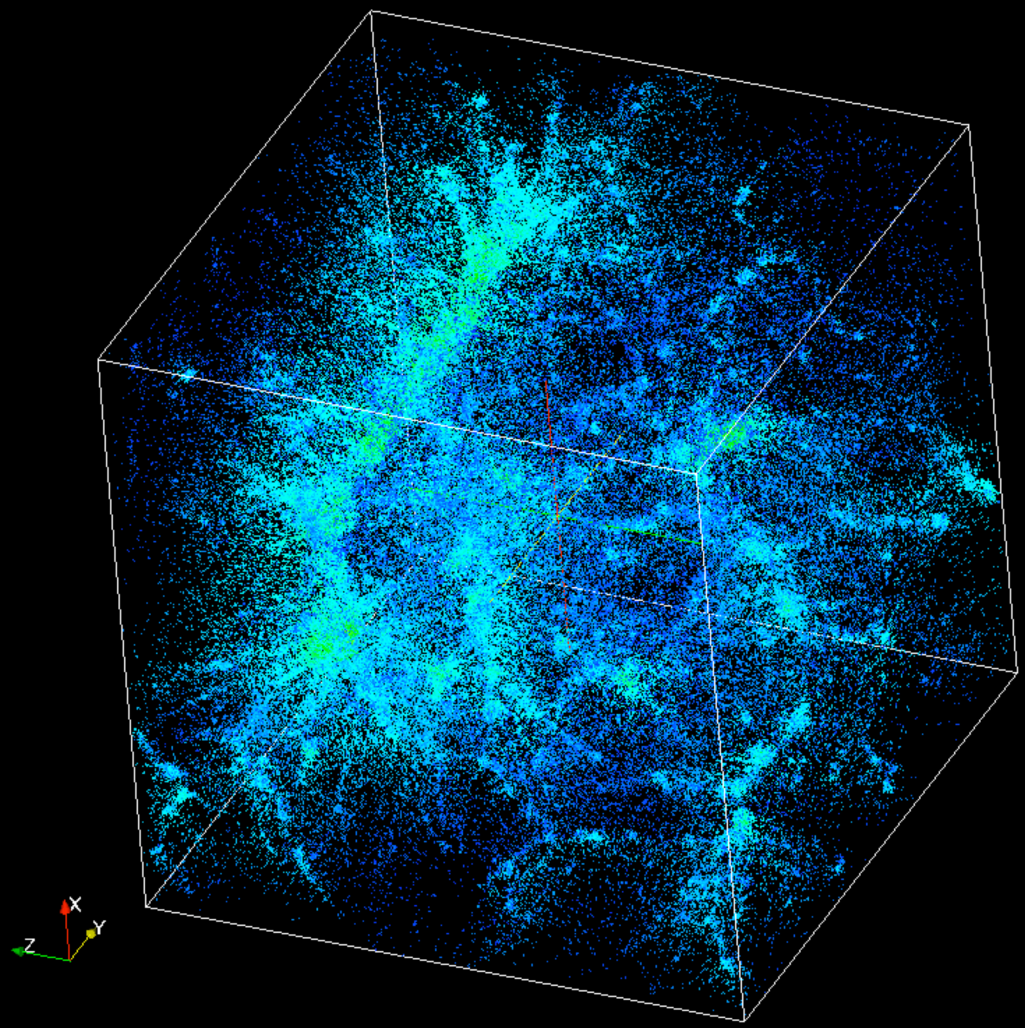


Abazajian (2006)

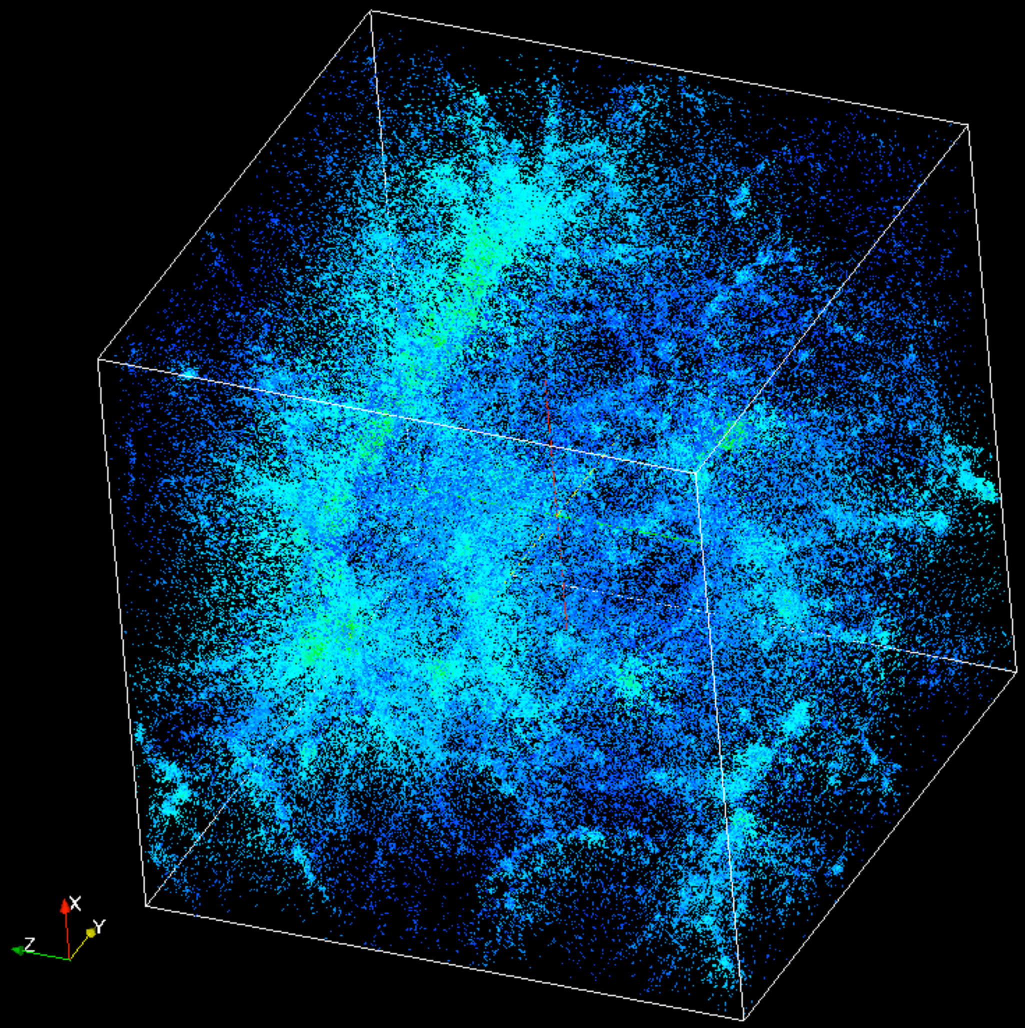
Nonlinear Structure Formation

courtesy Katrin Heitmann

CDM

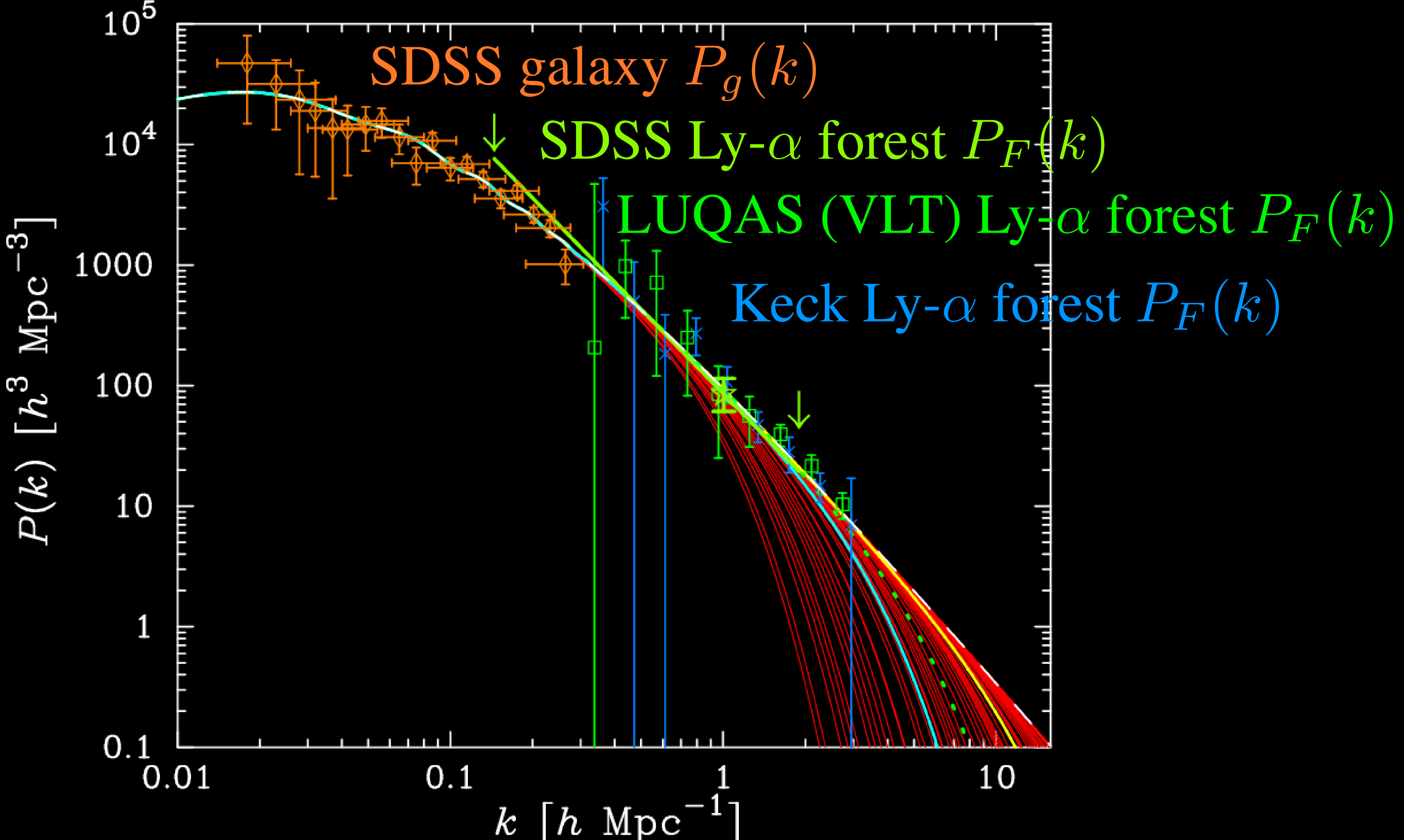


WDM $m_s = 2\ \mathrm{keV}$



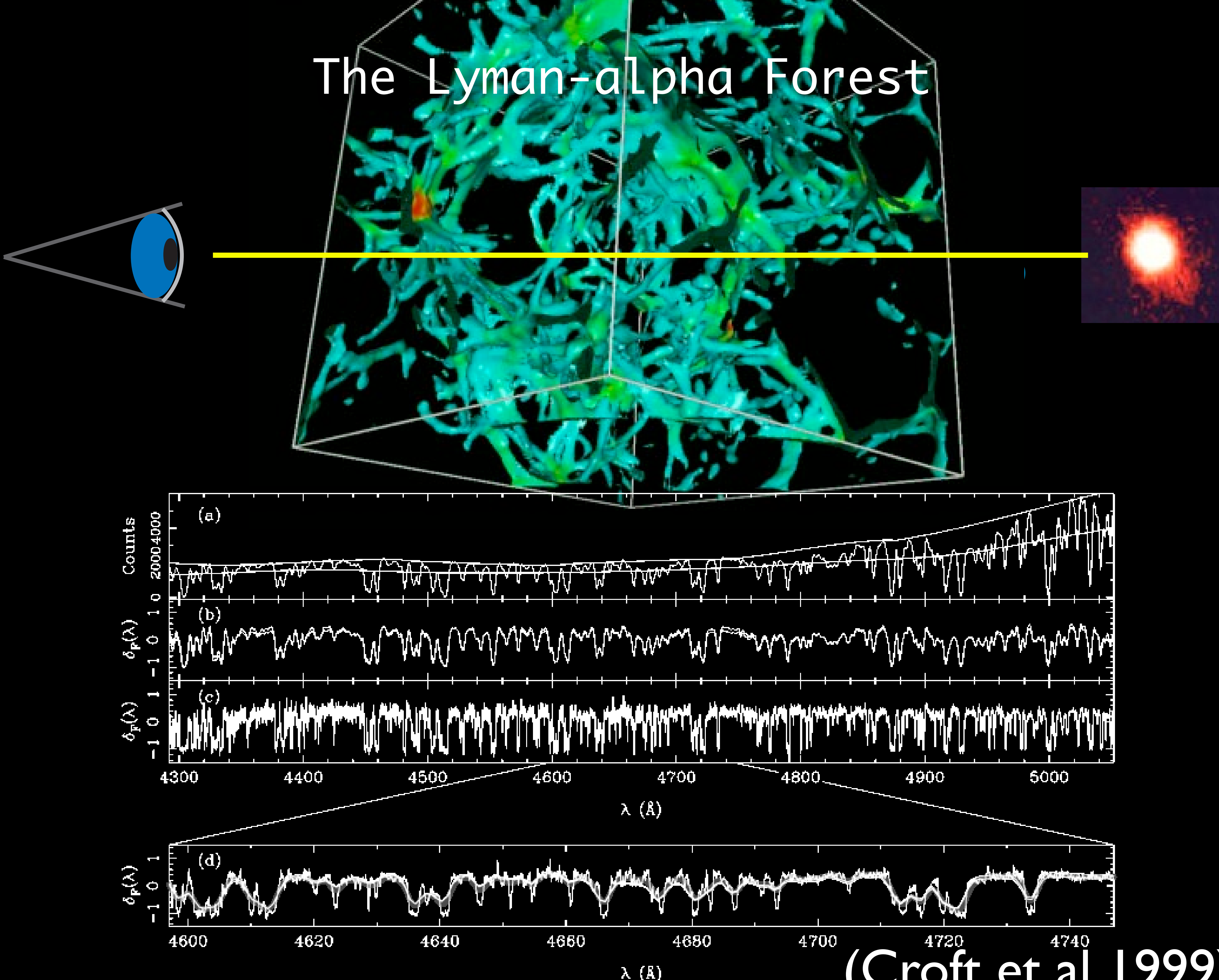
How warm is too warm?

suppression of small scale power



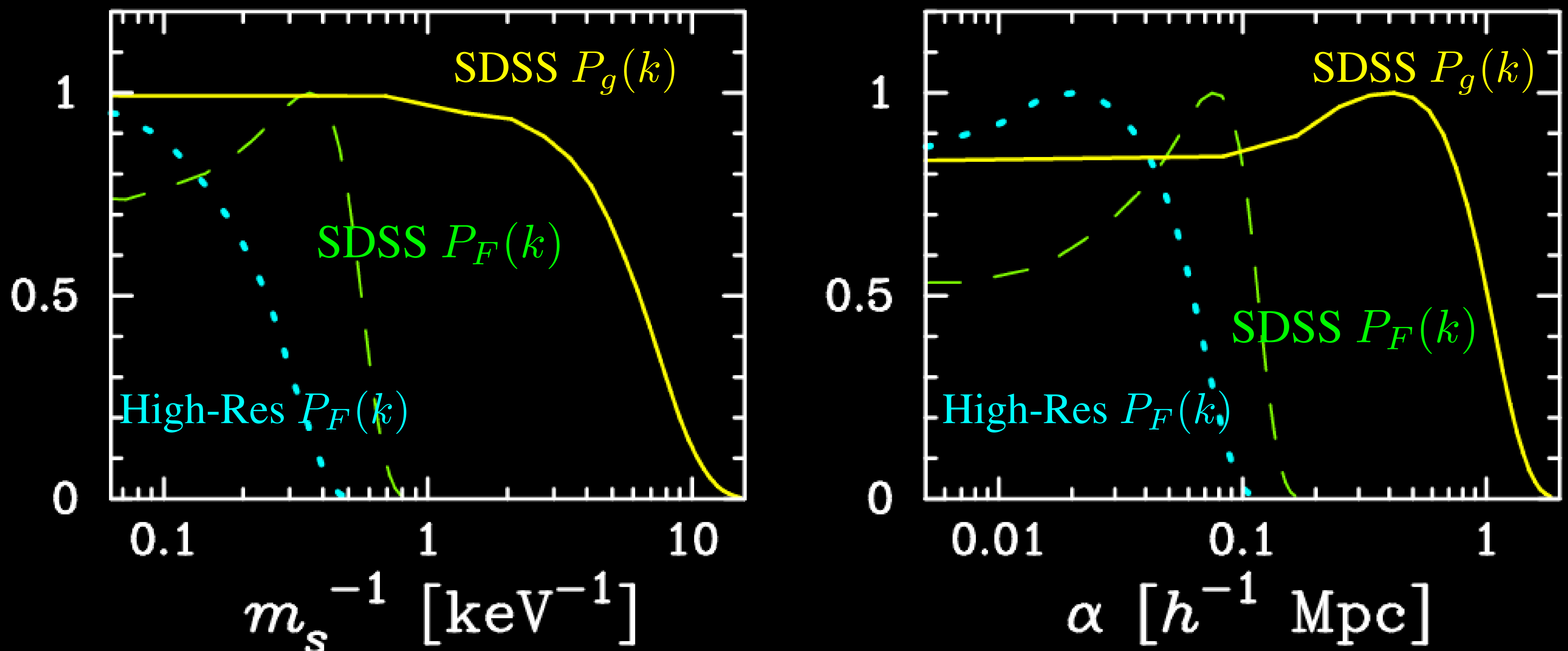
- SDSS 3D P(k) Main Galaxies (Tegmark et al 2003)
- SDSS Lyman-alpha forest (McDonald et al 2005)
- High-Resolution Lyman-alpha forest (Viel, Haehnelt & Springel 2004)
- CMB: WMAP, ACBAR, CBI, VSA, BooMERANG-2K2

The Lyman-alpha Forest



(Croft et al 1999)

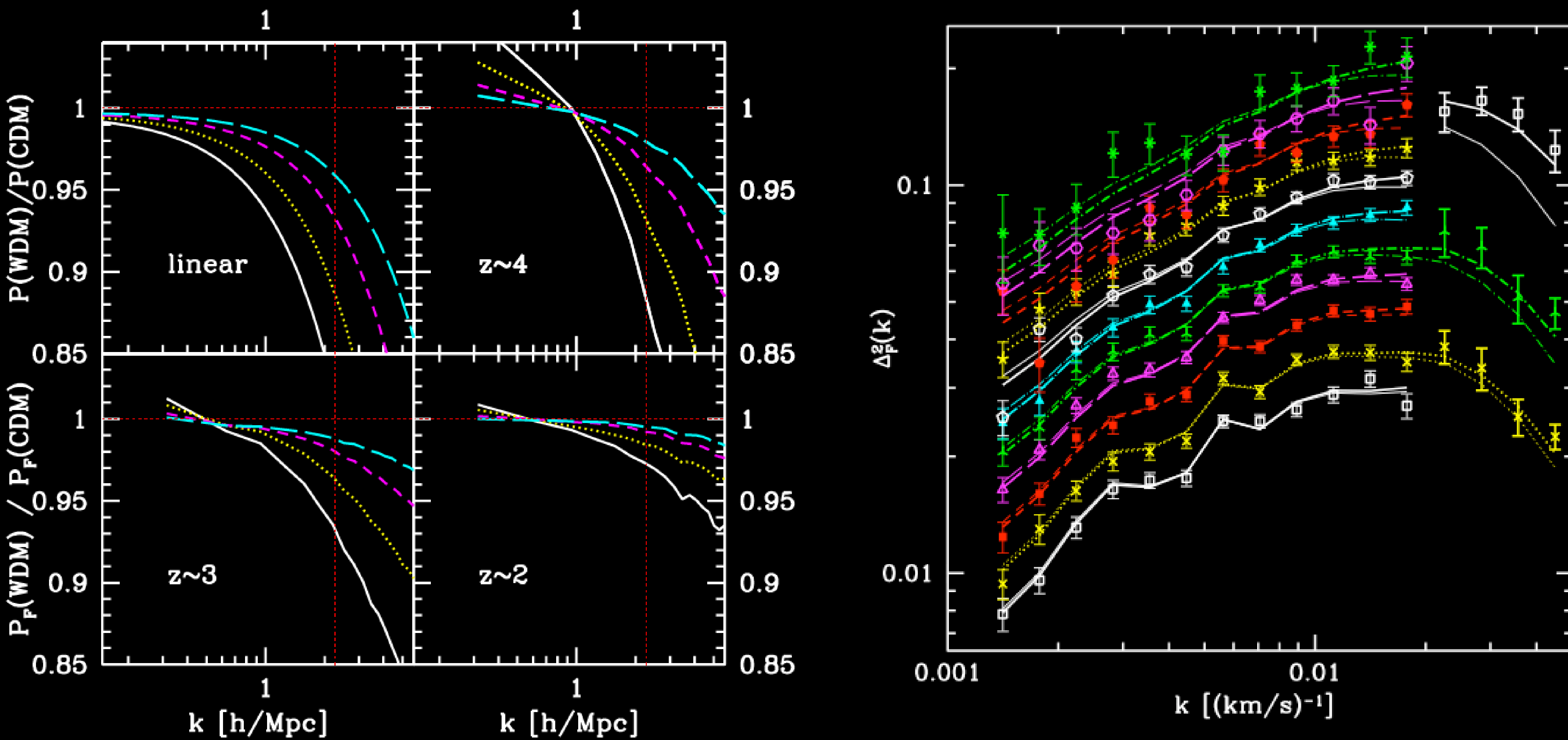
Lower Limits: Particle Mass and Cut-off Scale



SDSS $P_g(k)$: $m_s > 0.11$ keV
SDSS $P_F(k)$: $m_s > 1.7$ keV
High-Res $P_F(k)$: $m_s > 3.0$ keV (?)

(95% CL)

Lyman-alpha Forest Constraints



$m_s > 14$ keV   Seljak et al 2006: WMAP1 + SDSS Pg(k) + Lya + HR

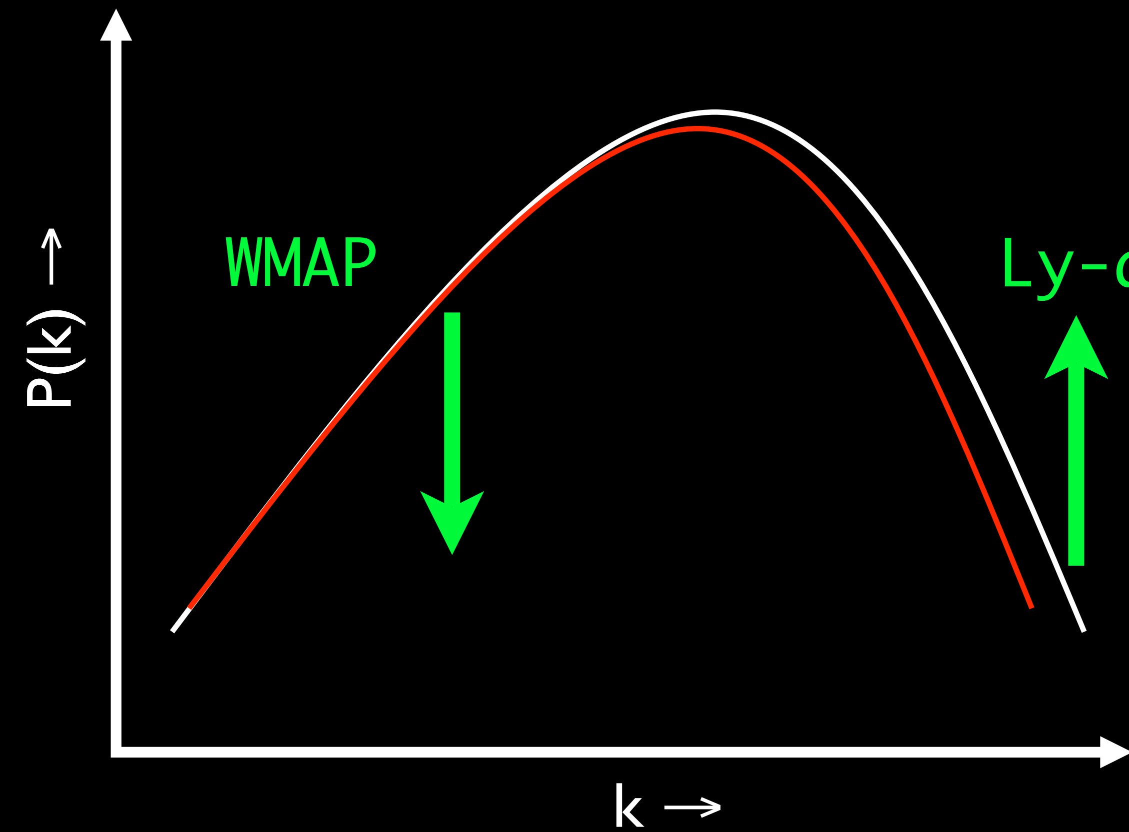
$m_s > 9$ keV   Viel et al 2006: WMAP3 + CMB + 2dFGRS + SDSS Lya

Both depend on the McDonald et al. (2006) SDSS $P_F(k)$ Measurement

SDSS Lyman-alpha Constraints (Seljak et al 2006)

WMAP3:

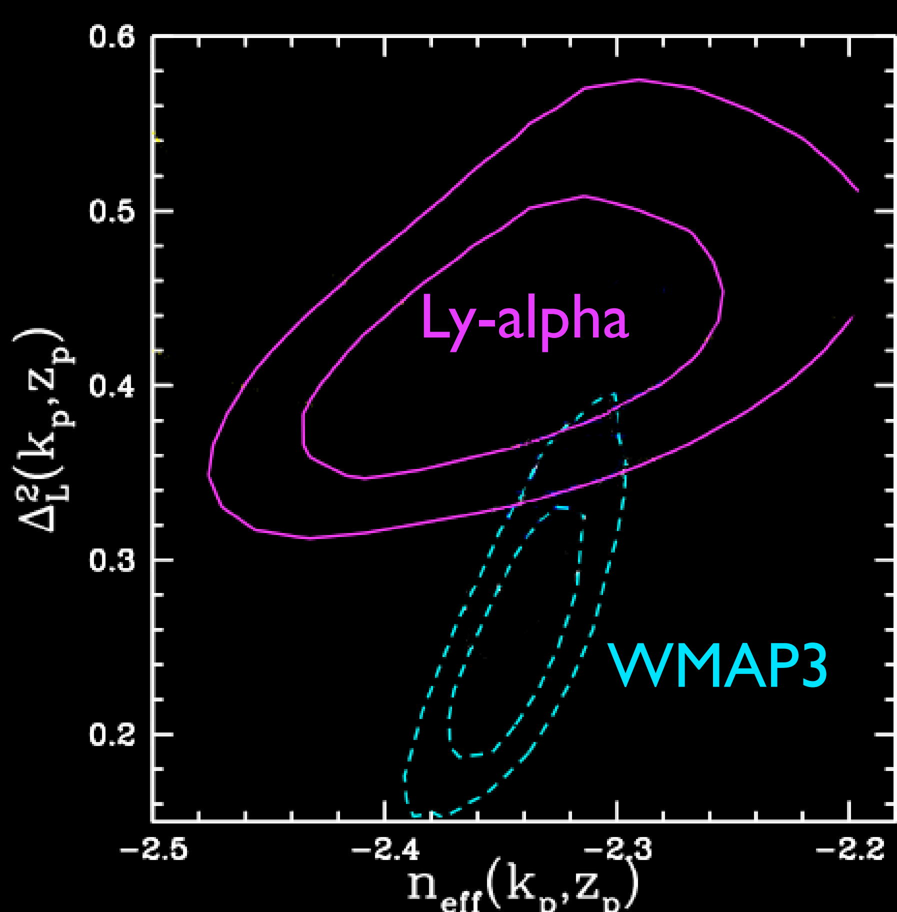
$$\sigma_8 = 0.74^{+0.05}_{-0.04}$$



SDSS Ly-alpha (Seljak et al 2003):

$$\sigma_8 = 0.9^{+0.03}_{-0.03}$$

1% likely!

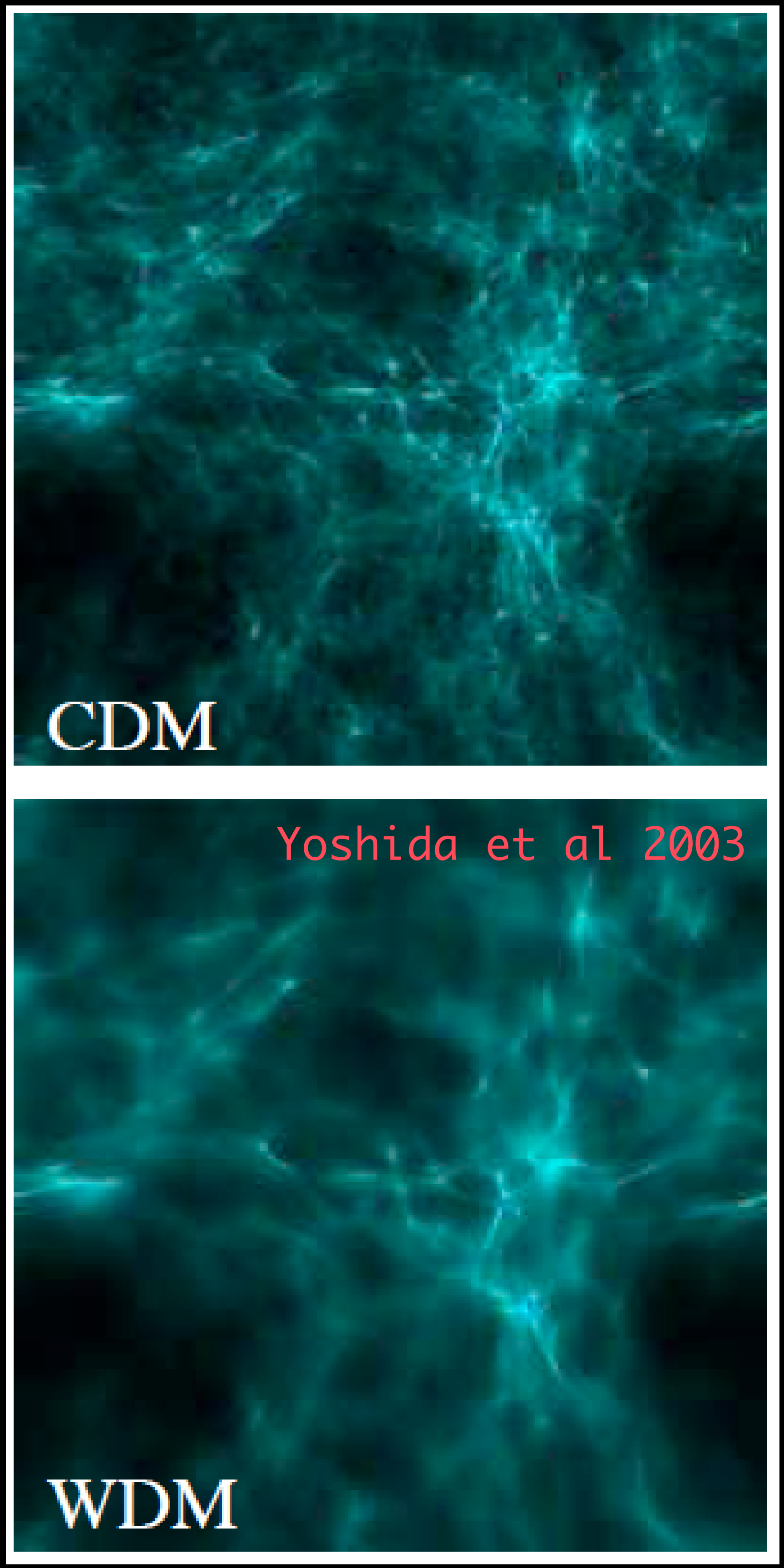


$N_\nu = 5.4^{+0.4}_{-0.6}$ Seljak et al. Lyα

$N_\nu = 3.08^{+0.74}_{-0.68}$ BBN, Cyburt et al. 2004

$N_\nu = 3.04$ (standard model)

Reionization & the "First Stars"

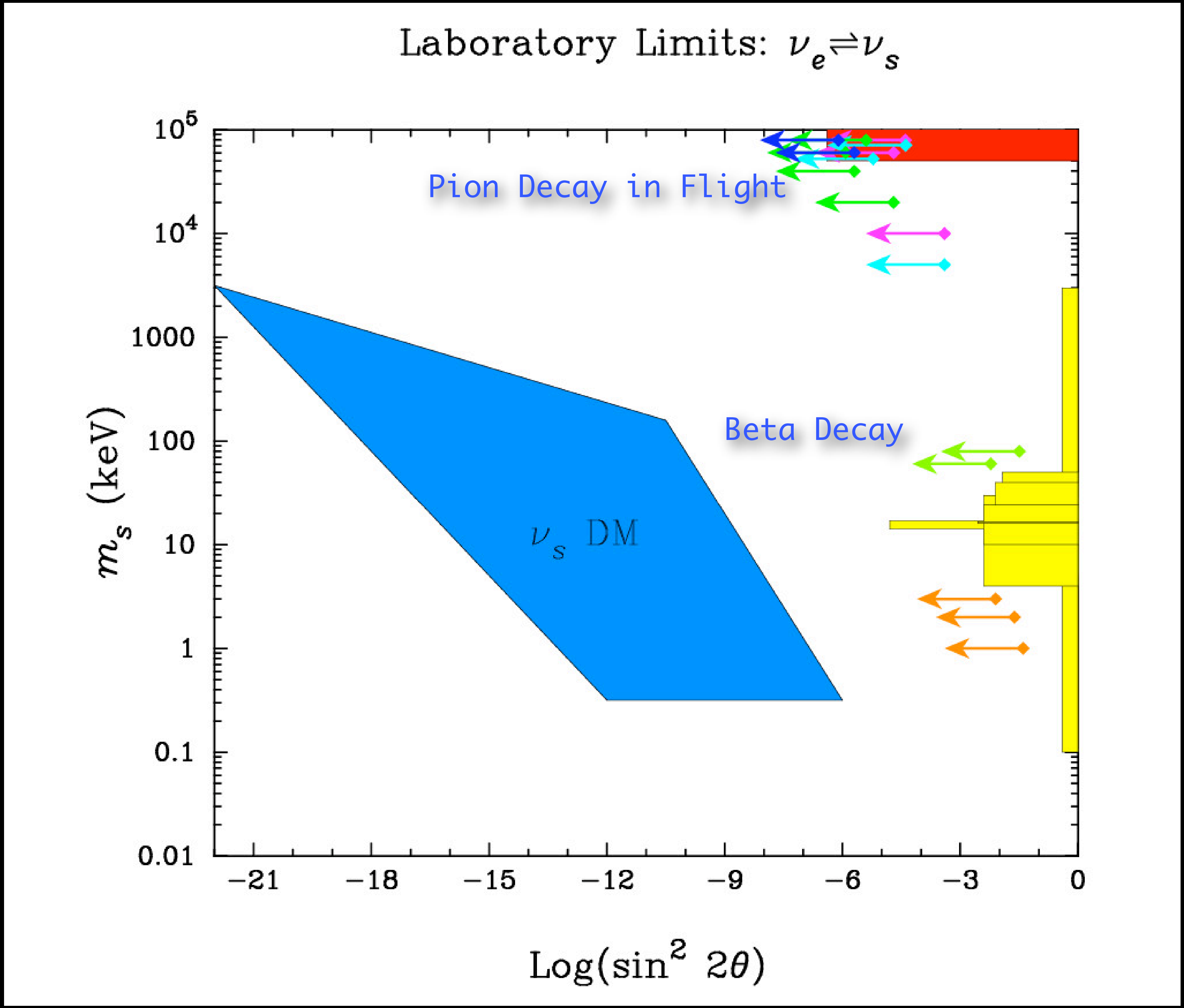


- H_2 formation from ionizing flux of sterile neutrino decay (Biermann & Kusenko 2006)
- Typical initial boxes of 0.186 to 1 Mpc/h
- Delay in halo (and star) formation
  - Sufficient time between initial conditions and halo formation (Heitmann et al '06)
  - Zel'dovich step criterion

    $$\ell \ll \Delta_p/3$$

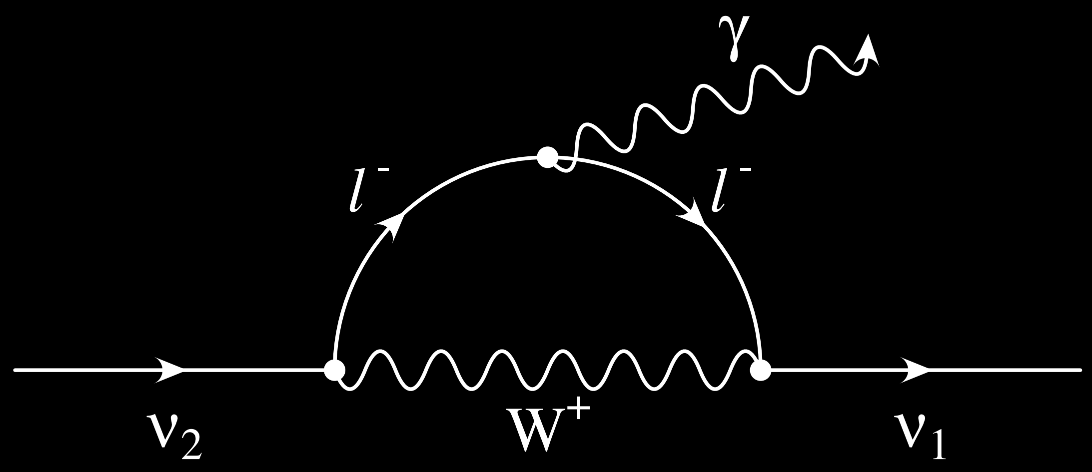
  - Sufficient resolution of large scale fluctuations (Barkana & Loeb '03)

Laboratory Limits



Radiative Decay in the X-ray

Pal & Wolfenstein 1981

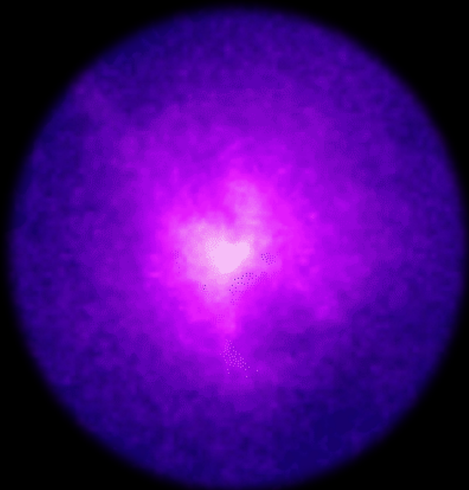


$$\nu_i \longrightarrow \nu_j + \gamma$$

$$E_\gamma = \frac{m_s}{2} \sim 1\ \text{keV}$$

$$\Gamma_\gamma = 6.8 \times 10^{-30}\ \sec^{-1}\ \left(\frac{\sin^2 2\theta}{10^{-7}}\right)\left(\frac{m_s}{1\ \text{keV}}\right)^5$$

Dark Matter Halos as Particle Reservoirs: Detecting Decaying Dark Matter



$\sim 10^{70}$ particles

Background:

- X-ray continuum
- Compact Objects
- Instrumental

Signal:

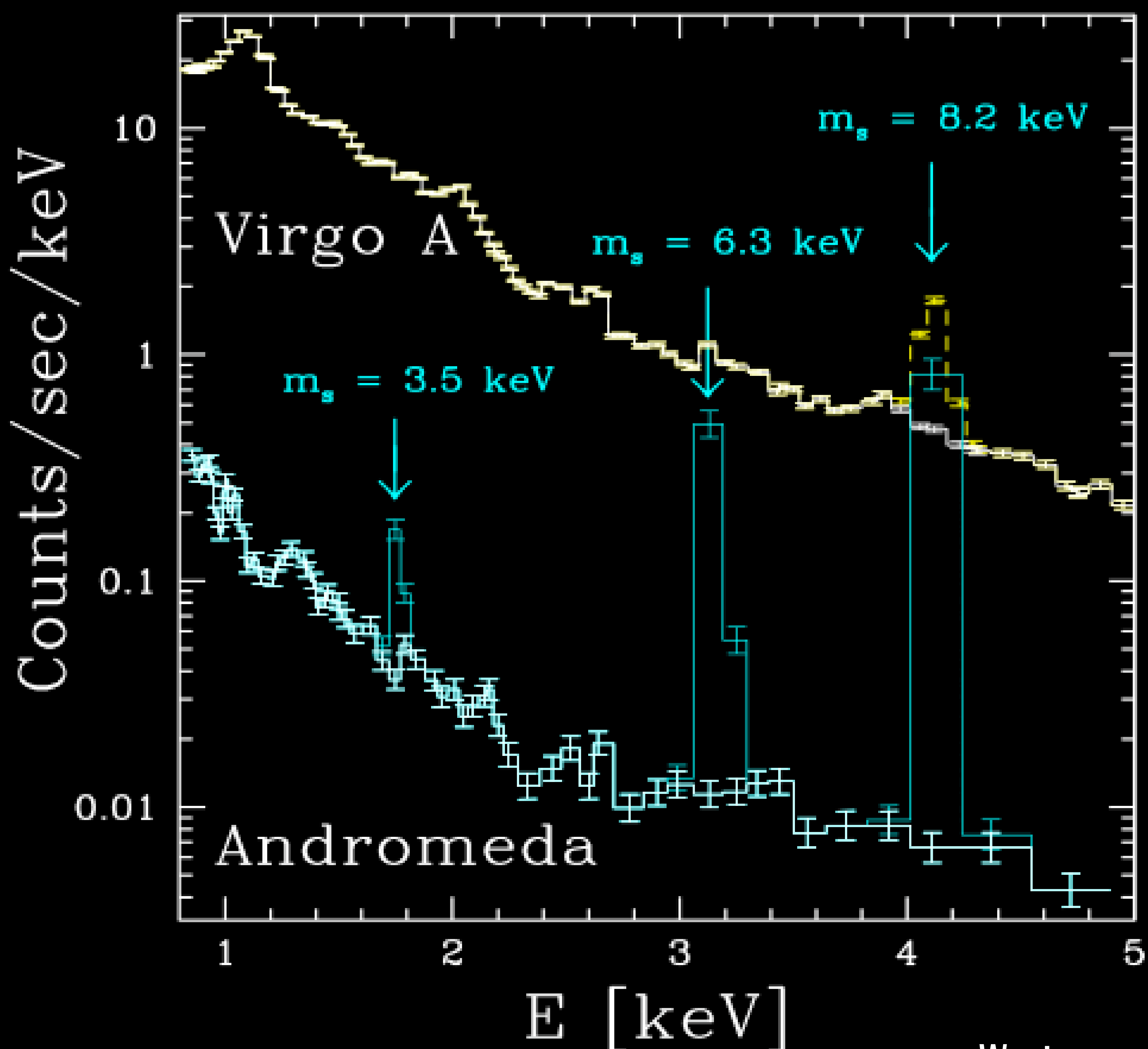
$$F \propto \int d\Omega \;\left[\text{DM density/distance}^2\right] \propto J[\Delta\Omega(\theta)]$$

$$J\left[\Delta\Omega(\theta)\right] = \rho_s \int_0^{2\pi} d\phi \int_0^{\theta} \sin\theta' \left[\int_{x_{\min}(\theta')}^{x_{\max}(\theta')} I[\tilde{r}(x)]\, dx\right] d\theta'$$

$$I_{\rm NFW}\left[\tilde{r}(x)\right] = \frac{1}{\tilde{r}(x)\left[1+\tilde{r}(x)\right]^2}$$

$$I_{\rm BUR}\left[\tilde{r}(x)\right] = \frac{1}{\left[1+\tilde{r}(x)\right]\left[1+\tilde{r}^2(x)\right]}$$

X-ray Limits: Virgo and Andromeda (M31)

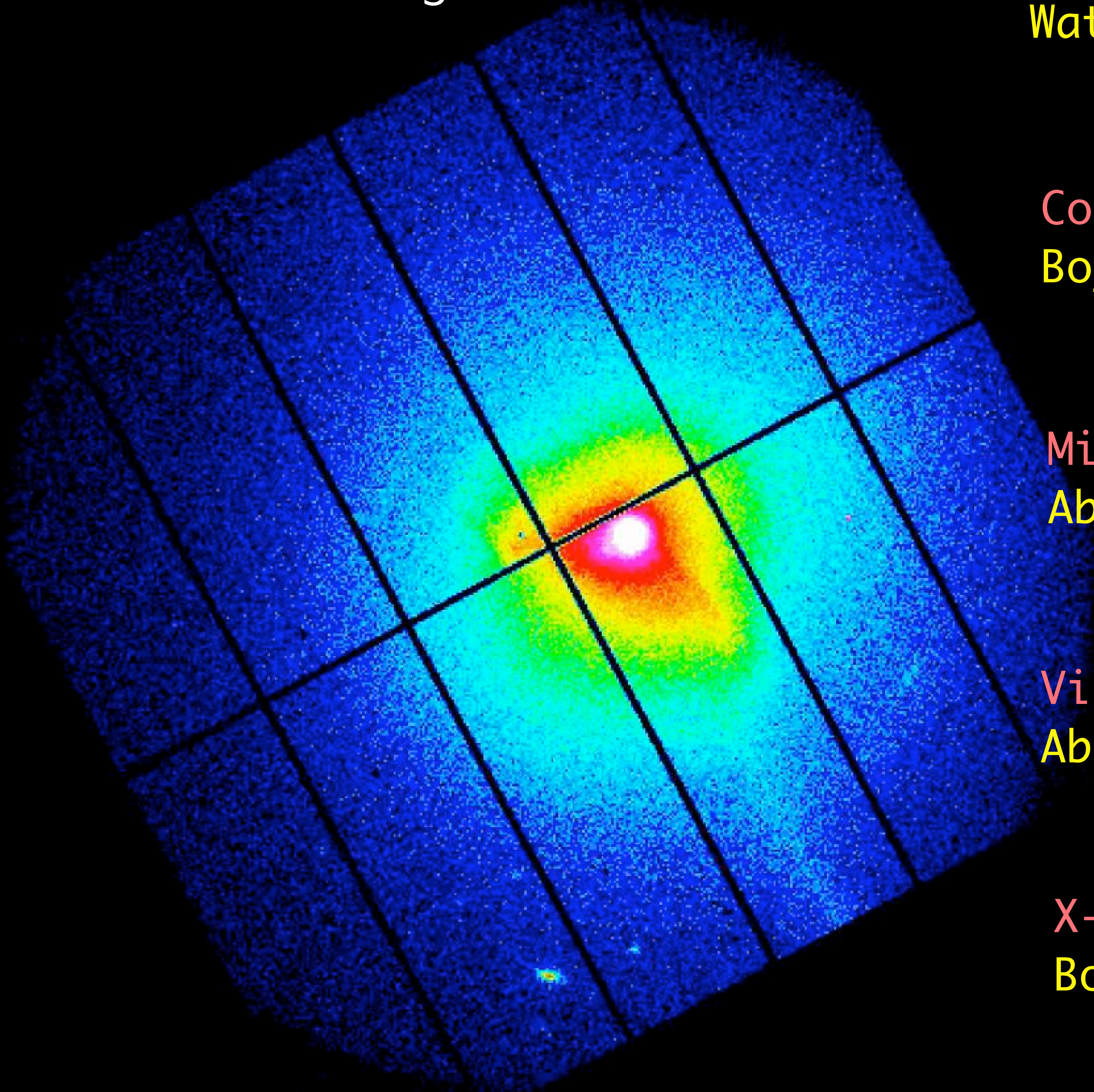


(XMM-Newton)

Watson et al (2006)
Abazajian et al (2001)

X-ray Constraint Summary

XMM Newton: The Virgo Cluster



Andromeda Galaxy:
Watson et al 2006

$$m_s < 3.5 \text{ keV}$$

Coma + Virgo Clusters:
Boyarsky et al 2006

$$m_s < 6.3 \text{ keV}$$

Milky Way in CXB:
Abazajian et al 2006

$$m_s < 6.5 \text{ keV}$$

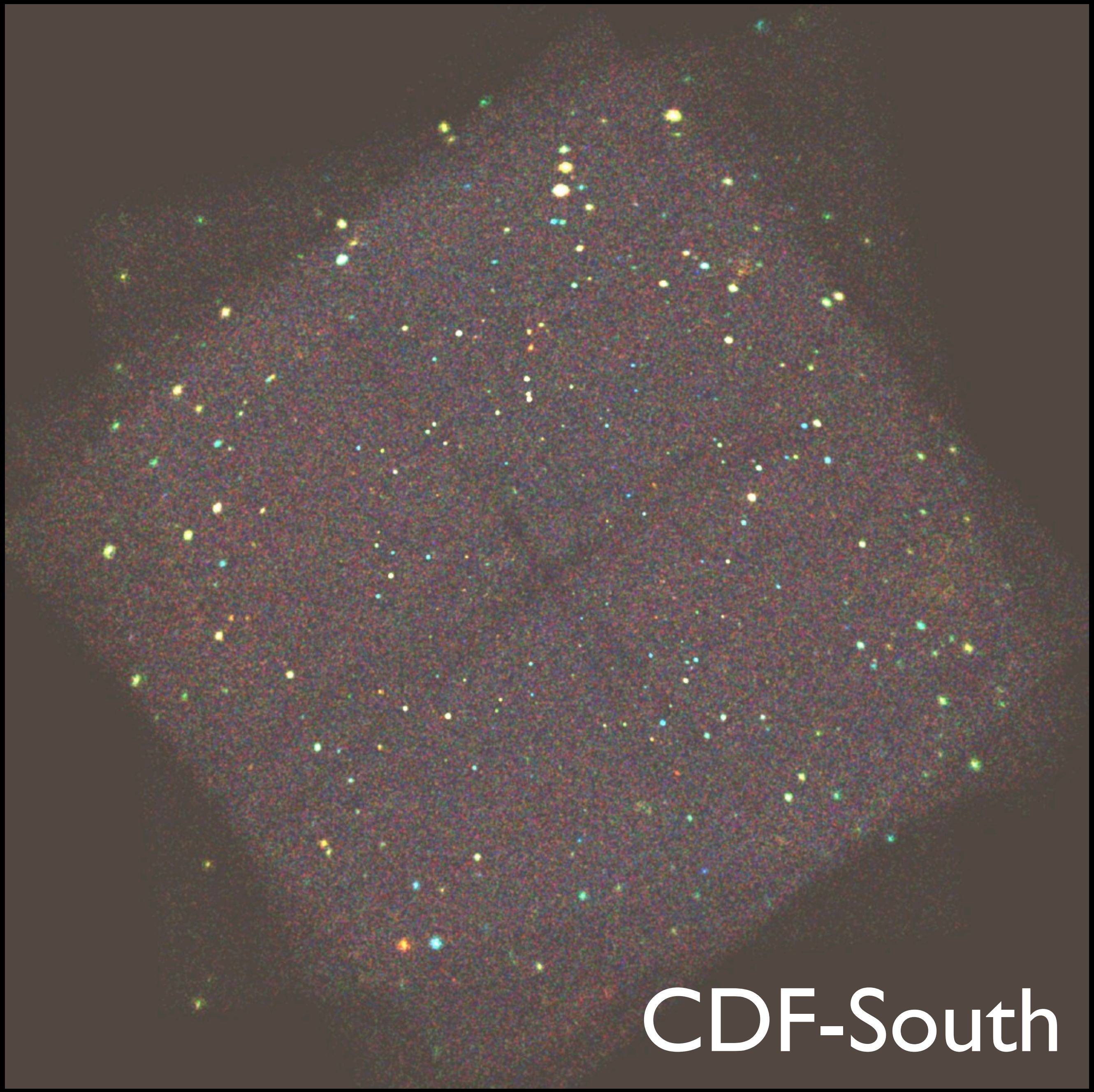
Virgo Cluster:
Abazajian et al 2001

$$m_s < 8.2 \text{ keV}$$

X-Ray Background:
Boyarsky et al 2006

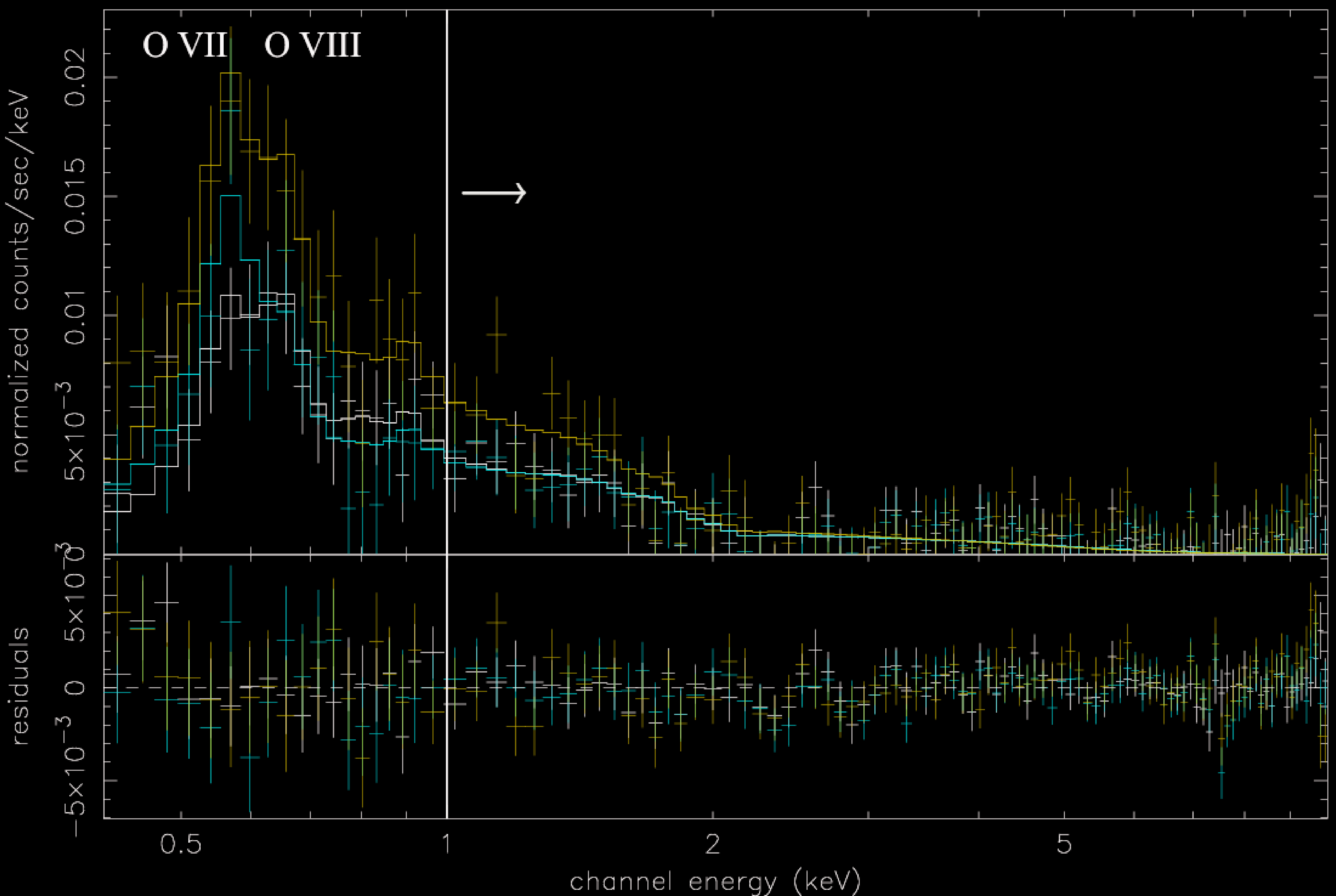
$$m_s < 8.9 \text{ keV}$$

Chandra Deep Field: Milky Way Halo Limits from the Unresesoved X-ray Background


CDF-South

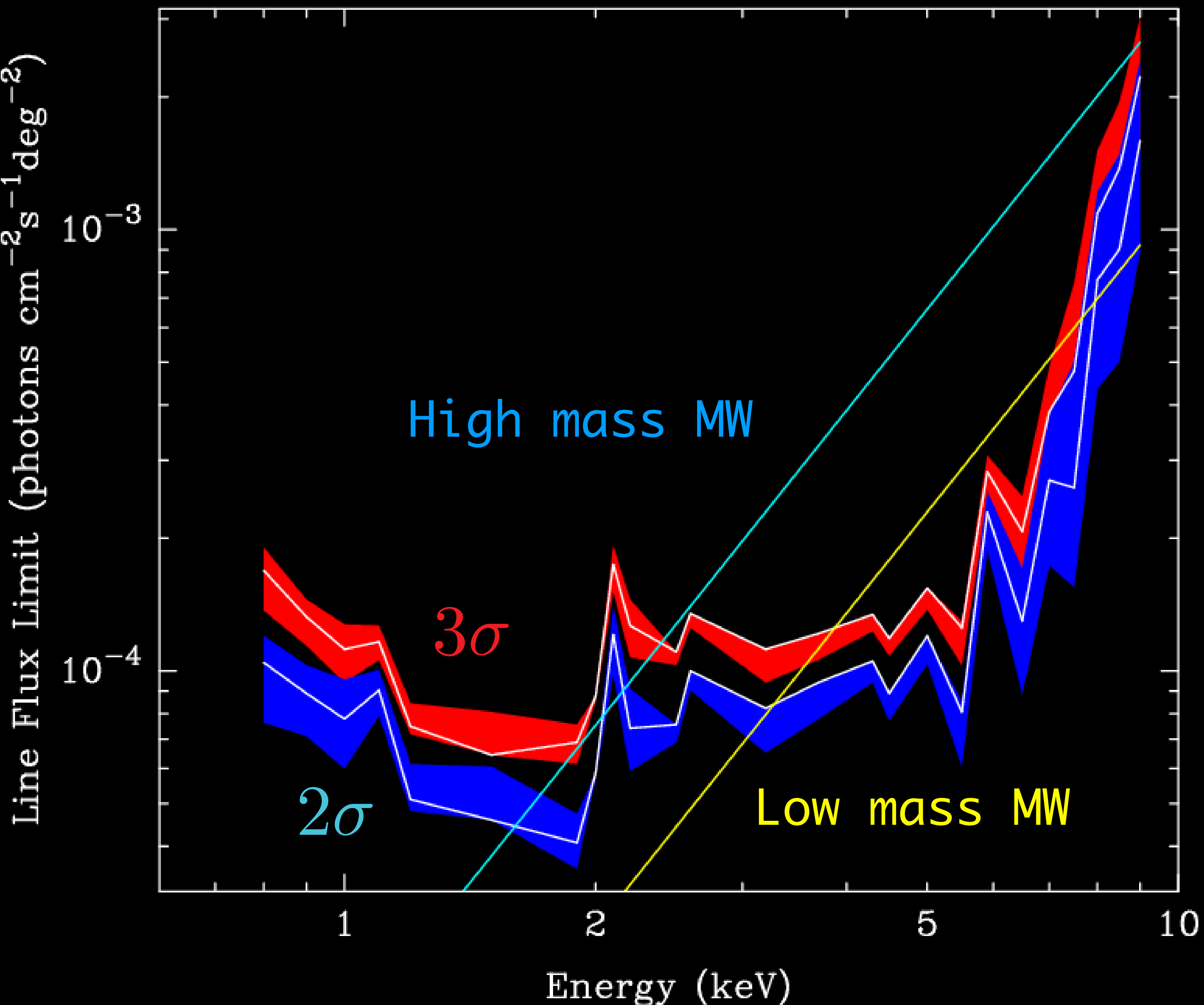

HDF-South

The Unresolved Cosmic X-ray Background



Hickox & Markevitch 2006

Milky Way Line Flux limits from the Chandra Deep Field



High DM mass MW:

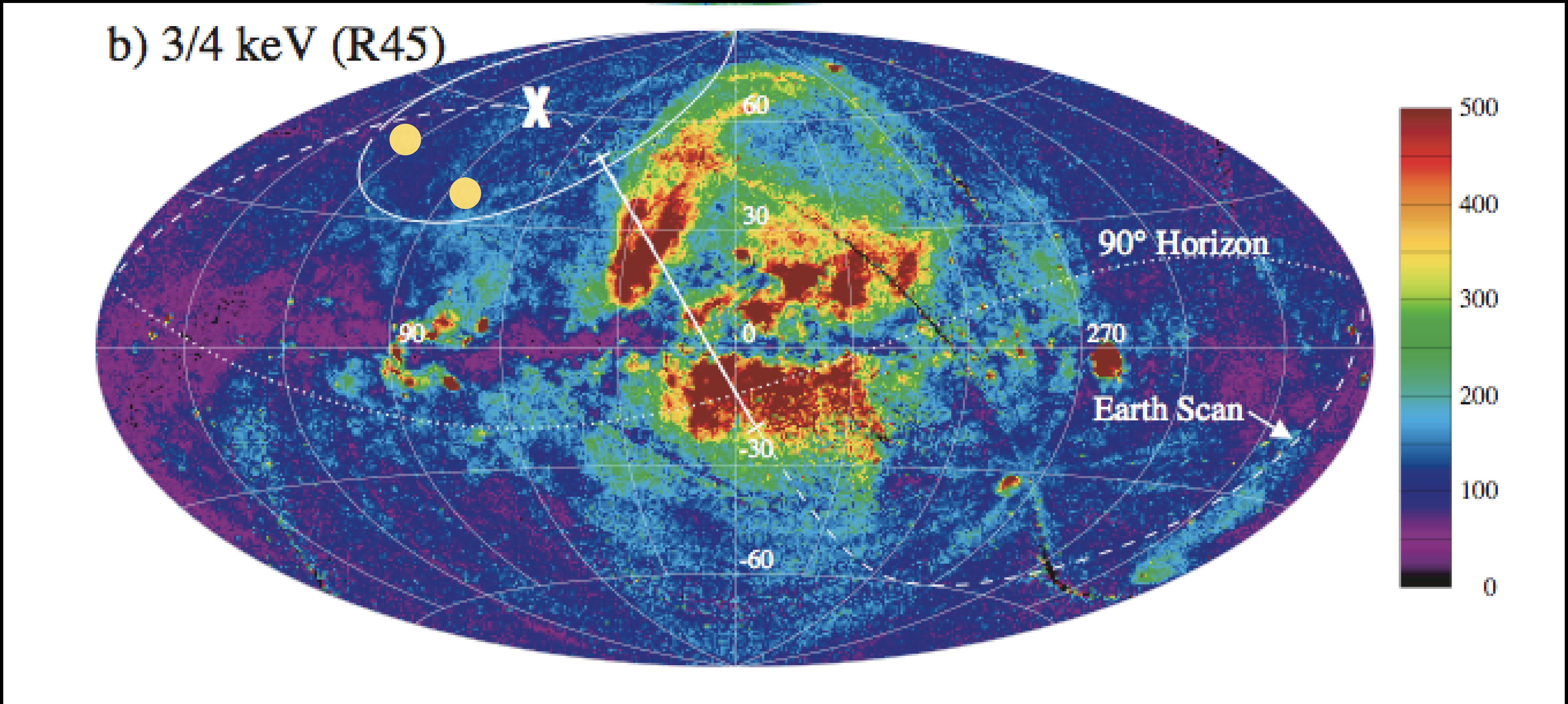
$m_s < 3.4$ keV

Low DM mass MW:

$m_s < 6.5$ keV

Abazajian, Markevitch, Koushiappas & Hickox 2006

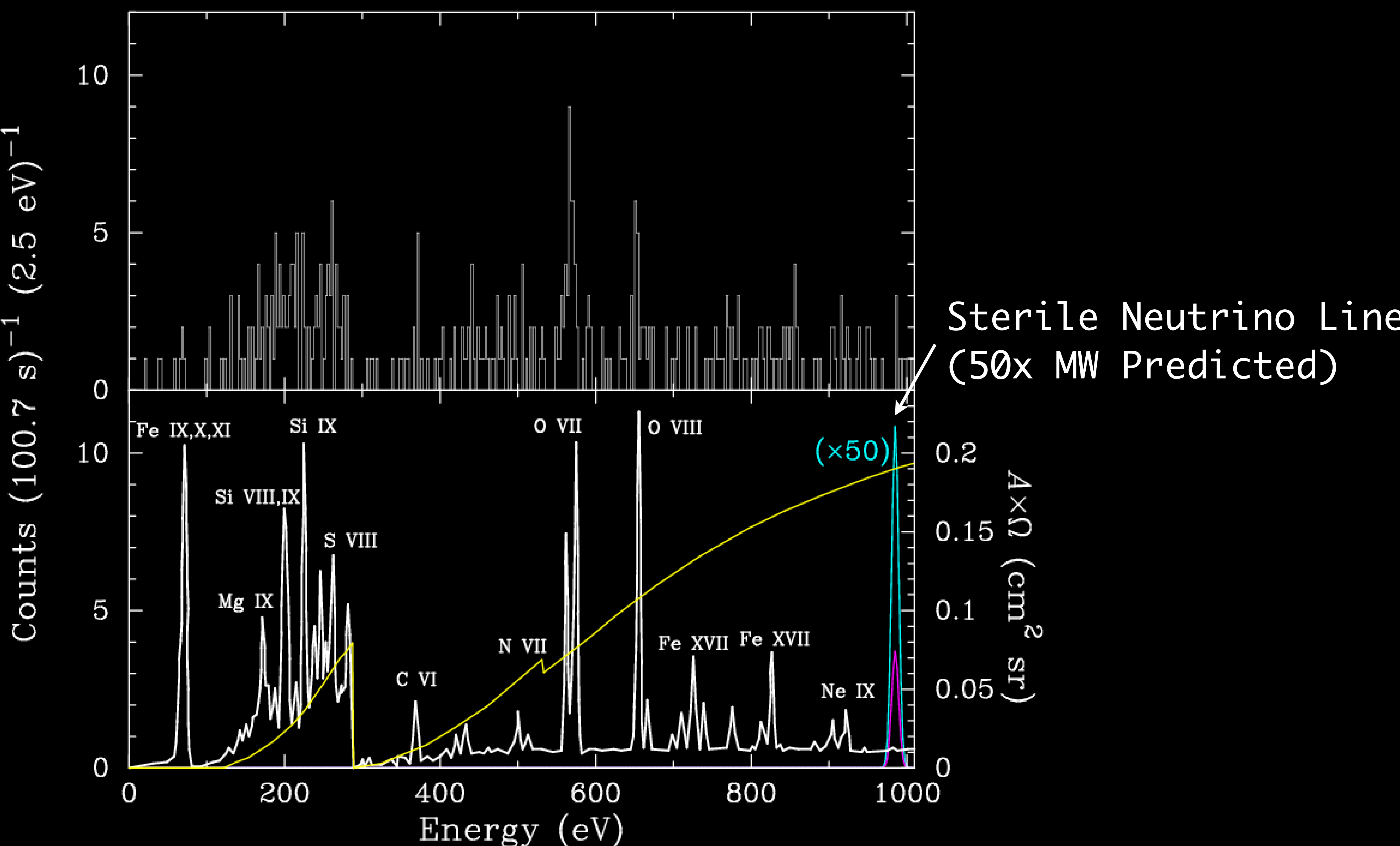
The Soft X-ray Background



100.7 sec X-ray calorimeter exposure on sounding rocket flight from White Sands, NM

Using modern X-ray calorimeters
Resolution ~9 eV
(McCammon et al., 2002)

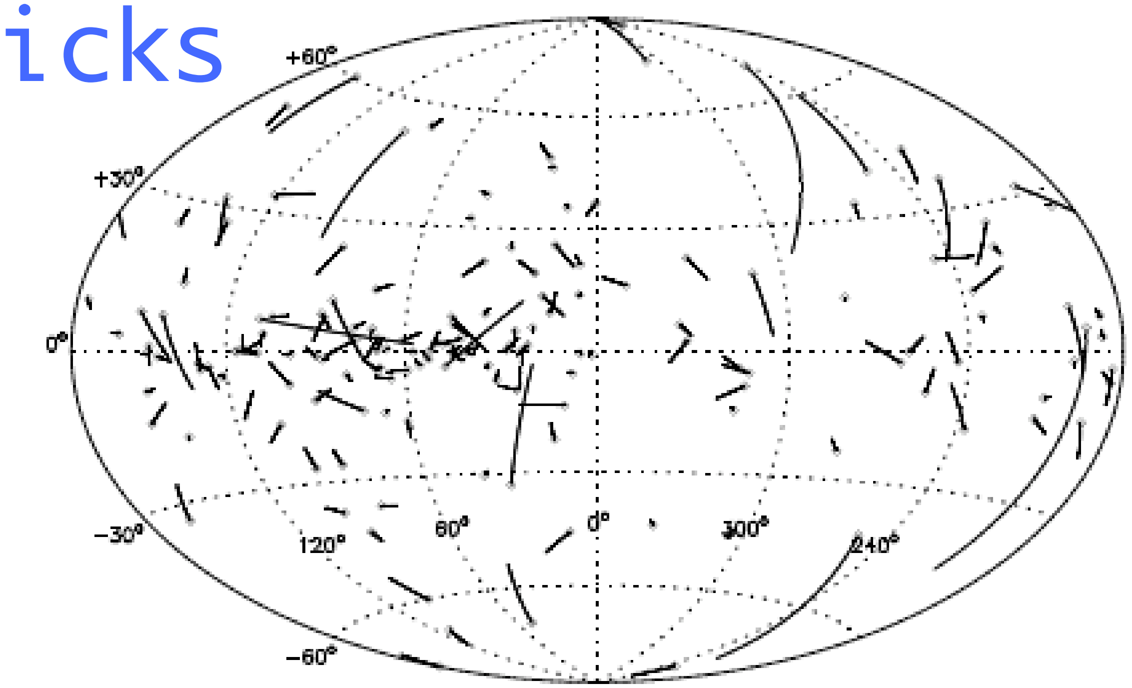
The Soft X-ray Background: Spectrum



(McCammon et al., 2002)

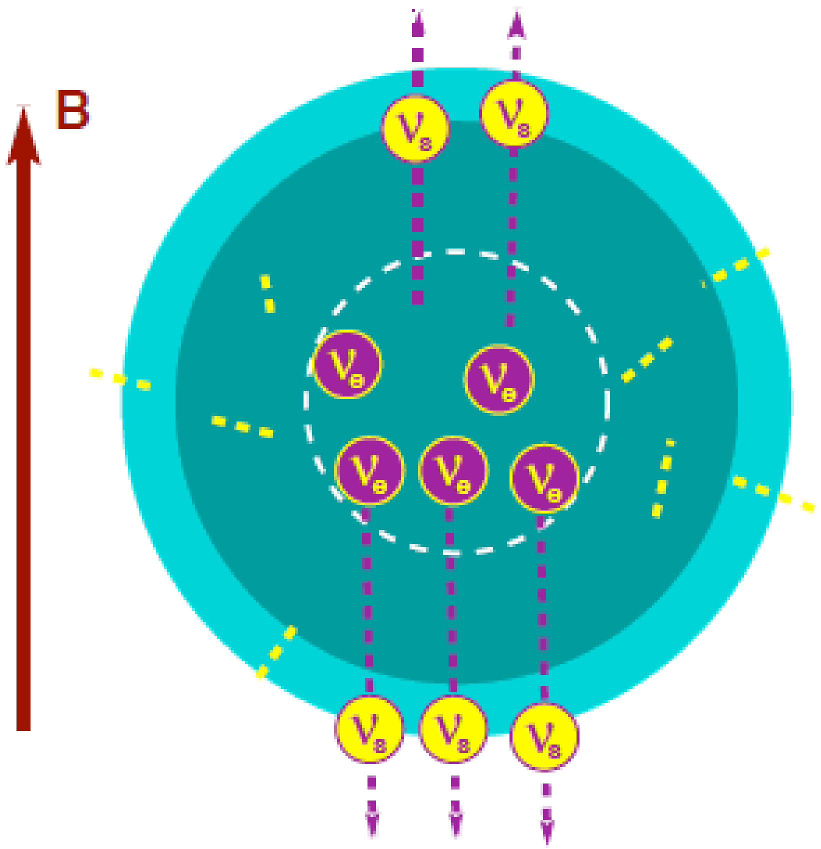
ν_s, Supernovae & Pulsar Kicks

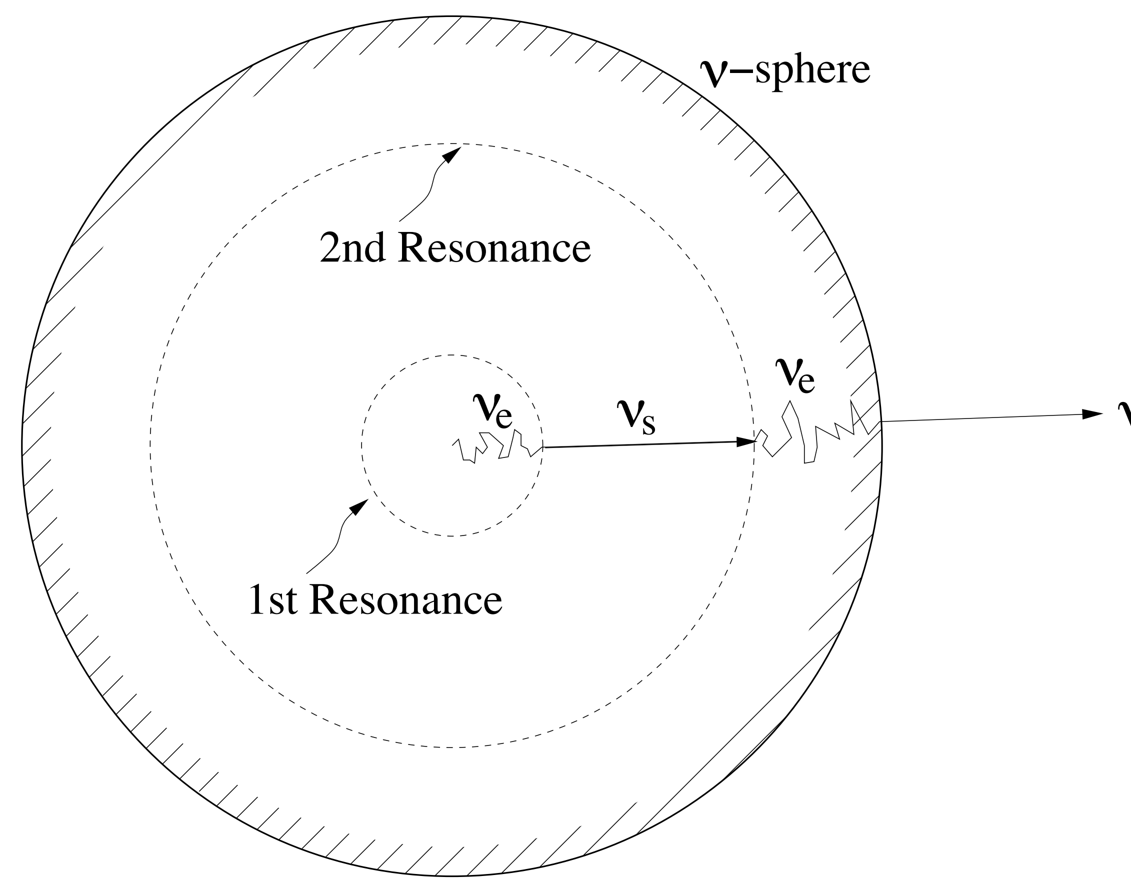
Map of pulsar velocities



Alexander Kusenko (UCLA) *INFO '05*

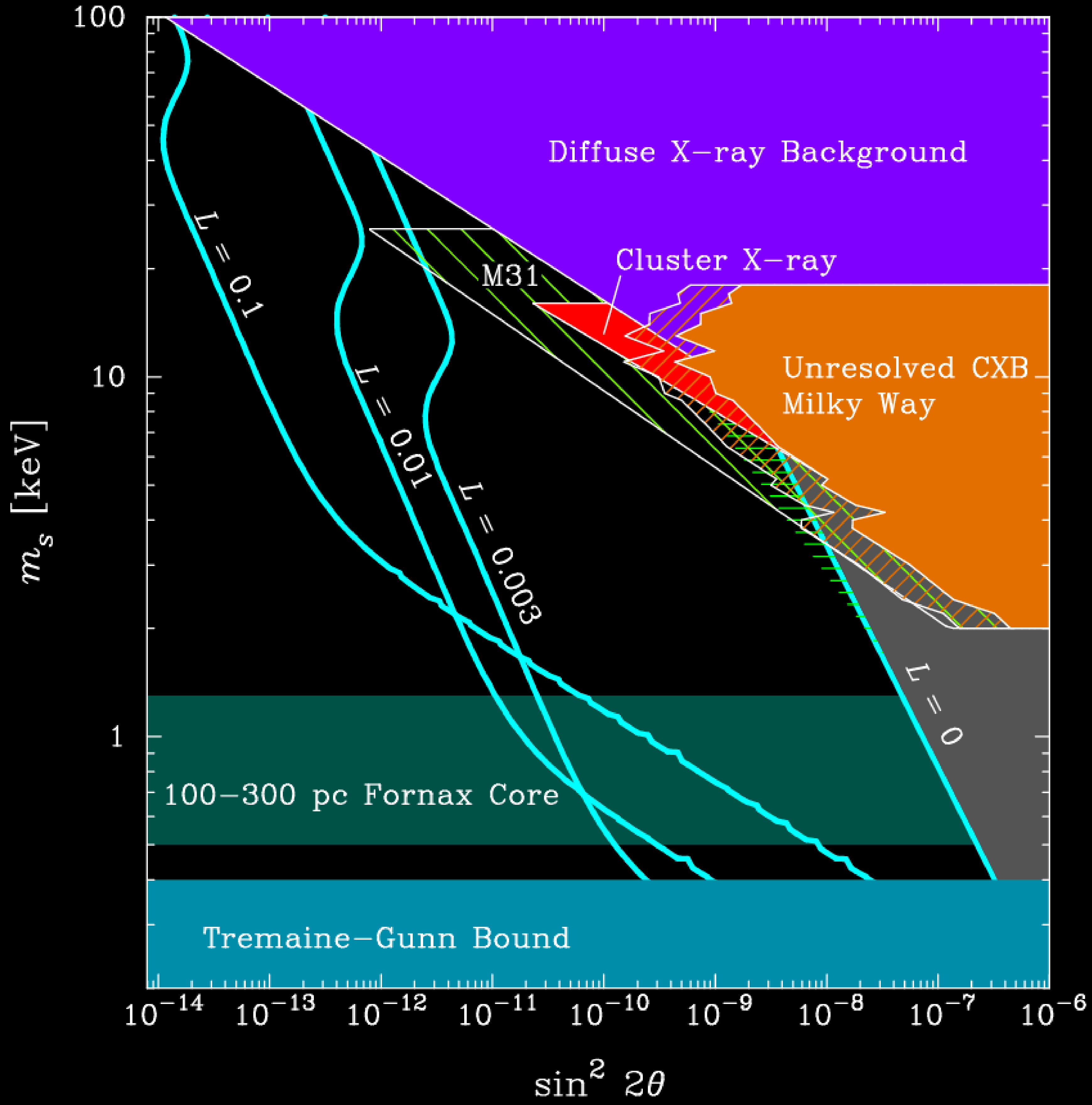
Sterile neutrinos leave the star without scattering. Hence, they give the pulsar a kick.





Segre & Kusenko (1999) Hidaka & Fuller (2006)

Sterile Neutrino Dark Matter Parameter Space



Conclusions

- The mass-generation mechanism for neutrinos may include a dark matter candidate
- Warm Dark Matter may solve several structure formation issues at small scales
- Sterile Neutrino Dark Matter is a natural, minimal candidate
- Evolution of SNDM is solved from production ($T \sim 10^{11}$ K) to onset of nonlinearity at small scales ($z \sim 20$) is known
- Sterile Neutrino Dark Matter is detectable
- Lower limits from the Tremaine-Gunn bound and upper limits from X-ray observations leave a window of $0.5\ \mathrm{keV} < m_s < 3.5\ \mathrm{keV}$