Prologue - Epilogue:

Quantum Fluctuations in EFT of S.R. inflation.

- Slow roll = consistent expansion in $1/N$ $N = \#$ of e-folds ~ 50
- Quantum Corrections from flucts $\Rightarrow$ $(H/M_P)^2 \Rightarrow$ EFT valid for $H/M_P \ll 1$ (NOT $\Delta\phi/M_P$) $\Rightarrow$ relief for T/S!
- "Universality" of slow-roll inf:

$$V(\phi) = M^4 N \underbrace{\mathcal{N}\left(\frac{\phi}{\sqrt{N}M_P}\right)}_{\text{"moduli"-type?}} ; \quad M \sim 10^{16}\,\text{GeV}$$

- Non B.D. initial conditions: constraint from renormalization + Back-Reaction of curvature + G.W. $\Rightarrow$ Transfer function $D(k) \lesssim 1/k^2 \Rightarrow$ S.W. multi. Ini. cond $\Longleftrightarrow$ pre-slow roll potential

$\Rightarrow$ CMB QUADRUPOLE SUPP.

Quantum aspects of EFT of SLOW-ROLL INFLATION

DeVega, Sanchez, Boyanovsky

Slow-roll inflation ⇒ a succes. "paradigm" consistent with CMB data.

Simple, yet ROBUST: nearly gaussian, nearly scale invariant, adiabatic, superhorizon perturbations.

Basics:

$$H^2(t) = \frac{\rho(t)}{3M_p^2}$$

single (scalar) field ϕ

$$\mathcal{L} = a^3 \left\{ \frac{\dot{\phi}^2}{2} - \frac{(\nabla\phi)^2}{2a^2} - V(\phi) \right\}$$

$\Phi \sim \langle \phi \rangle =$ homogeneous CONDENSATE only depends on t: $\rho(t) = \frac{\dot{\Phi}^2}{2} + V(\Phi)$

Slow roll: "flat" potential $\Rightarrow$
large number of e-folds, near scale invariance ....
Hierarchy of small dimensionless parameters:

$$\epsilon_V = \frac{M_P^2}{2}\left[\frac{V'(\Phi)}{V(\Phi)}\right]^2 \; ; \quad \eta_V = M_P^2 \frac{V''(\Phi)}{V(\Phi)}$$

$$\xi_V = M_P^4 \frac{V'(\Phi)\,V'''(\Phi)}{V^2(\Phi)} \quad \ldots$$

$$\epsilon_V \sim \eta_V \; ; \quad \xi_V \sim \epsilon_V^2 \quad \ldots$$

CMB data with slow rolling inflaton

$$\Rightarrow V(\Phi) = \underset{\uparrow\ \text{scale}}{M^4}\ \underbrace{v\left(\frac{\Phi}{M_P}\right)}_{\text{dim-less}}$$

$$N[\Phi(t)] = -\frac{1}{M_p^2}\int_{\Phi(t)}^{\Phi_{end}} V(\Phi)\frac{d\Phi}{dV}\, d\Phi$$

$$N \sim 50 = -\frac{1}{M_p^2}\int_{\Phi_{50}}^{\Phi_{end}} V(\Phi)\frac{d\Phi}{dV}\, d\Phi$$

$V(\Phi) + N \Rightarrow$

<u>Define</u>: $\Phi = \sqrt{N}\, M_p\, \chi$ ← dim-less

$$V(\Phi) = N M^4 \omega(\chi)$$

$$\Rightarrow \quad 1 = -\int_{\chi_{50}}^{\chi_{end}} \frac{\omega(\chi)}{\omega'(\chi)}\, d\chi$$

$$\frac{N[\chi]}{N} = -\int_{\chi}^{\chi_{end}} \frac{\omega(\chi)}{\omega'(\chi)}\, d\chi \leq 1$$

$$\Delta\chi = \frac{1}{\sqrt{N}}\frac{\Delta\Phi}{M_p} \Rightarrow \Delta\chi \sim 1 \text{ for}$$

$$\Delta\Phi \sim \sqrt{N}\, M_p \;!!$$

$$\epsilon_V = \frac{1}{2N}\left[\frac{w'(\chi)}{w(\chi)}\right]^2 \; ; \; \eta_V = \frac{1}{N}\frac{w''(\chi)}{w(\chi)}$$

$$\xi_V = \frac{1}{N^2}\frac{w' w'''}{w^2}$$

Slow-roll ≡ SYSTEMATIC EXPANSION IN $1/N$

"Stretched" variables

$$t = \sqrt{N}\,\frac{M_P}{M^2}\,\tau \qquad H = \sqrt{N}\,\frac{M^2}{M_P}\,h$$

$$\ddot{\Phi} + 3H\dot{\Phi} + V'(\Phi) = 0 \longrightarrow \frac{1}{N}\frac{d^2\chi}{d\tau^2} + 3h\frac{d\chi}{d\tau} + w' = 0$$

All "phenomenology" of slow roll infla-tion consistent with $w(\chi) \sim 1$ during SLOW-ROLL

$$|\Delta_k^{(s)}|^2 = \frac{N^2}{12\pi^2}\left(\frac{M}{M_{pl}}\right)^4 \frac{\omega^3}{\omega'^2} \sim 5\times 10^{-5}$$

(WMAP)

$$\Rightarrow \quad M \sim 10^{16}\,\text{GeV} \quad (\omega, \omega' \sim 1)$$

Couplings: $\quad \lambda \Phi^4 \rightarrow \lambda \sim \frac{1}{N}\left(\frac{M}{M_p}\right)^4 \sim 10^{-12}$

"naturally"

EFT description $\Rightarrow \frac{M}{M_p} \ll 1$

Small couplings from "see-saw"

$$\frac{H}{M_p} = \sqrt{N}\left(\frac{M}{M_p}\right)^2 h$$

$h \approx 1$

EFT $\Rightarrow$ $H/M_p \ll 1$

NOT $\frac{\Delta\Phi}{M_p} < 1$: $\quad r \sim \left(\frac{\Delta\Phi}{M_p}\right)^2$ CAN be $\sim \frac{1}{N}$ within EFT!

Quantum fluctuations I:

Inflaton fluctuations: $\phi = \Phi(t) + \underbrace{\varphi(\bar{x}, t)}_{\text{fluctuations.}}$

"zero mode" condensate

$$V(\phi) = V(\Phi) + V'(\Phi)\varphi + \frac{1}{2}V''(\Phi)\varphi^2 + \frac{1}{6}V'''(\Phi)\varphi^3$$

FRW eqn.

$\Downarrow$ Eqn. of motion of inf.

$\Downarrow$ quadratic (Gaussian) flucts.

$\Downarrow$ Non-linear Non-Gauss.

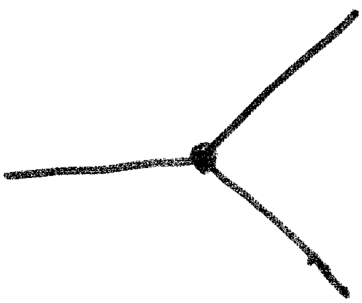
$$\frac{V'''(\Phi)}{2} \equiv g \equiv H \left[\frac{3\eta_V}{2\sqrt{2\epsilon_V}}\right] \frac{H}{M_{pl}} \sim \frac{1}{\sqrt{N}}\left(\frac{M}{M_{pl}}\right)^2$$

dimensions (scale)

$\Downarrow$ Slow roll

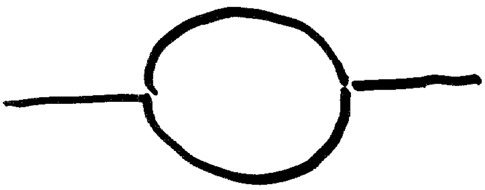
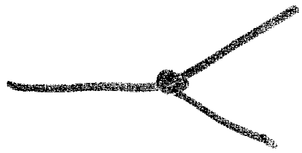
$\Downarrow$ EFT

SELF-INTERACTION $\Rightarrow$ NON-GAUSS.



$\propto \left(\begin{matrix}\text{slow}\\ \text{roll}\end{matrix}\right) \circledast$ EFT

Consequences: Self energy

 $\propto$ $\Big|$  $\Big|^2$

$$\approx \left(\frac{H}{M_{pl}}\right)^2 * (\text{slow roll})$$

$\Rightarrow$ QUANTUM CORRECTIONS TO:

$\underbrace{V_{eff}}_{\text{potential}}$, $\underbrace{\check{\epsilon}_{eff}, \check{\eta}_{eff}}_{\text{slow roll}}$, $\underbrace{\nu_{eff}}_{\text{scaling of superhorizon flucts.}}$

All of $\mathcal{O}\left(\left(\frac{H}{M_{pl}}\right)^2 * \text{slow-roll}\right)$!

EFT: validity $\Longleftrightarrow$ $\frac{H}{M_{pl}} \ll 1$ $\Rightarrow$ small quantum flucts!

Quantum Fluctuations II:

Back reaction from: CURVATURE (R) + TENSOR (GW) + LIGHT SCALARS + LIGHT FERMIONS

LIGHT: $M_{\alpha}^2/H^2 \ll 1$

Q: why only light?

A: Heavy fields integrate out to EFT.

$$\delta V[\Phi] = \underset{\text{curv.}}{\langle T^0_{R0}[\Phi]\rangle} + \underset{\text{G.W.}}{\langle T^0_{h0}[\Phi]\rangle} + \underset{\text{light scal.}}{\langle T^0_{\sigma 0}[\Phi]\rangle}$$

$$+ \underset{\text{fermions}}{\langle T^0_{\psi 0}[\Phi]\rangle}$$

$\langle \ldots\ldots \rangle$ IN "BD vacuum"

$$V_{eff}[\Phi] = V[\Phi] + \delta V[\Phi]$$

$$\delta V[\Phi] = V[\Phi]\left[\frac{H}{M_p}\right]^2 \left[\frac{\text{slow roll}}{\text{slow roll}} + \begin{array}{c}\text{conformal}\\ \text{anomaly}\end{array}\right]$$

$\Downarrow$ EFT!

$\Downarrow$ I.R ~ 1

$\Downarrow$ U.V ~ 20

Quantum Corrections to Physical Observables:

$$\left|\Delta^{(s)}_{eff}\right|^2 = \left|\Delta^{(s)}\right|^2 \left\{ 1 + \frac{2}{3}\left(\frac{H}{4\pi M_P}\right)^2 * \left[1 + \frac{\frac{3}{8} r (n_s - 1) + 2\frac{dn_s}{d\ln k}}{(n_s-1)^2} + \frac{2903}{40} \right] \right\}$$

↑ conf. Anomaly

$$\left|\Delta^{(T)}_{eff}\right|^2 = \left|\Delta^{T}\right|^2 \left\{ 1 - \frac{1}{3}\left(\frac{H}{4\pi M_P}\right)^2 \left[-1 + \frac{1}{8}\frac{r}{n_s - 1} + \frac{2903}{20} \right] \right\}$$

↑ conf. anomaly

$$r_{eff} = r \left\{ 1 - \frac{1}{3}\left(\frac{H}{4\pi M_P}\right)^2 \left[1 + \frac{\frac{3}{8} r (n_s - 1) + \frac{dn_s}{d\ln k}}{(n_s-1)^2} + \frac{8709}{20} \right] \right\}$$

$$V_{eff} = V \left[1 + \frac{1}{3}\left(\frac{H_0}{4\pi M_P}\right)^2 \left(\frac{n_v - 4\epsilon_v}{n_v - 3\epsilon_v} + \frac{3 n_\sigma}{n_\sigma - \epsilon_\sigma} + \mathcal{I} \right) \right]$$

↑ Conf. anomaly $= -145.15$

Quantum Fluctuations III:

Initial Conditions : beyond B. D.

Initial Conditions $\longleftrightarrow$ Back-reactio[n]
of gauge-invariant flucts: CURVATURE
+ G. W.

$$ds^2 = C^2(\eta)\left[d\eta^2 - d\vec{x}^{\,2}\right]$$

$$\Psi_k(\eta) = \frac{1}{C(\eta)}\left[\alpha_k S_\Psi(k,\eta) + \alpha^{+}_{-k} S^{*}_{\Psi}(k,\eta)\right]$$

G.W:

$$h_{ij}(\vec{k},\eta) = \frac{1}{C(\eta) M_p \sqrt{2}} \sum_{\lambda = x,+} \epsilon_{ij}(\lambda,\vec{k})\left[\alpha_k S_h(k,\eta) + h.c.\right]$$

slow-roll: Mode functions

$$\left[\frac{d^2}{d\eta^2} + k^2 - \frac{(\nu^2 - 1/4)}{\eta^2}\right] S(k,\eta) = 0$$

$$S(k,\eta) = A(k)\, g_\nu(k,\eta) + B(k)\, g_\nu^*(k,\eta)$$

$$g_\nu(k;\eta) = \frac{i^{\nu+1/2}}{2}\sqrt{-\pi\eta}\; H_\nu^{(1)}(-k\eta)$$

$$\xrightarrow[|k\eta|\to\infty]{} \frac{e^{-ik\eta}}{\sqrt{2k}} \qquad \text{B.D.}$$

C.C.R $\Rightarrow$ $|A(k)|^2 - |B(k)|^2 = 1$

$|B(k)|^2 = N(k) =$ number of B.D. particles in vacuum state with

NON-BD b.c. B.D $\Rightarrow$ $N_k = 0$

Power spectra: $\dfrac{P(k)}{P^{BD}(k)} = 1 + D(k)$

$D(k) =$ Transfer function for initial condit.

$$= 2\,|B(k)|^2 - 2\,\mathrm{Re}[A(k)\,B^*(k)]$$

Back-reaction: $\rho = \langle 0 | T^0_0 | 0 \rangle$

EFT: RENORMALIZED NOT necessarily renormalizable! : Finite V_{eff} by counter terms, insensitive to U.V. divs.

Short Distance constraints on $N(k)$

$$\Rightarrow \quad N(k) \to N_\mu \left(\frac{\mu}{k}\right)^{4+\delta} \quad \text{as } k \to \infty$$

Small back-reaction: $\langle 0|T_0^0|0\rangle \ll H^2 M_p^2$

$N(k) \lll 1 ! \Rightarrow$

$\Rightarrow$ At large k

$$D(k) \simeq -2\sqrt{N(k)}$$

Both curvature & TENSOR!

Large k- suppression $\Rightarrow$ influence GREATEST in low k! $\Rightarrow$ SACHS–WOLFE plateau.

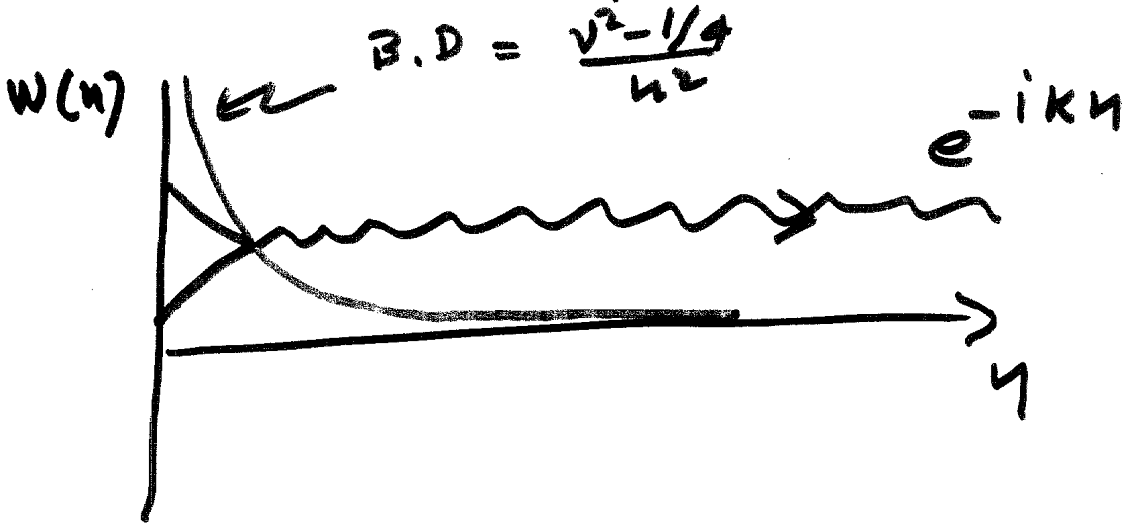
$$C_\ell = C_\ell^{BD} + \Delta C_\ell$$

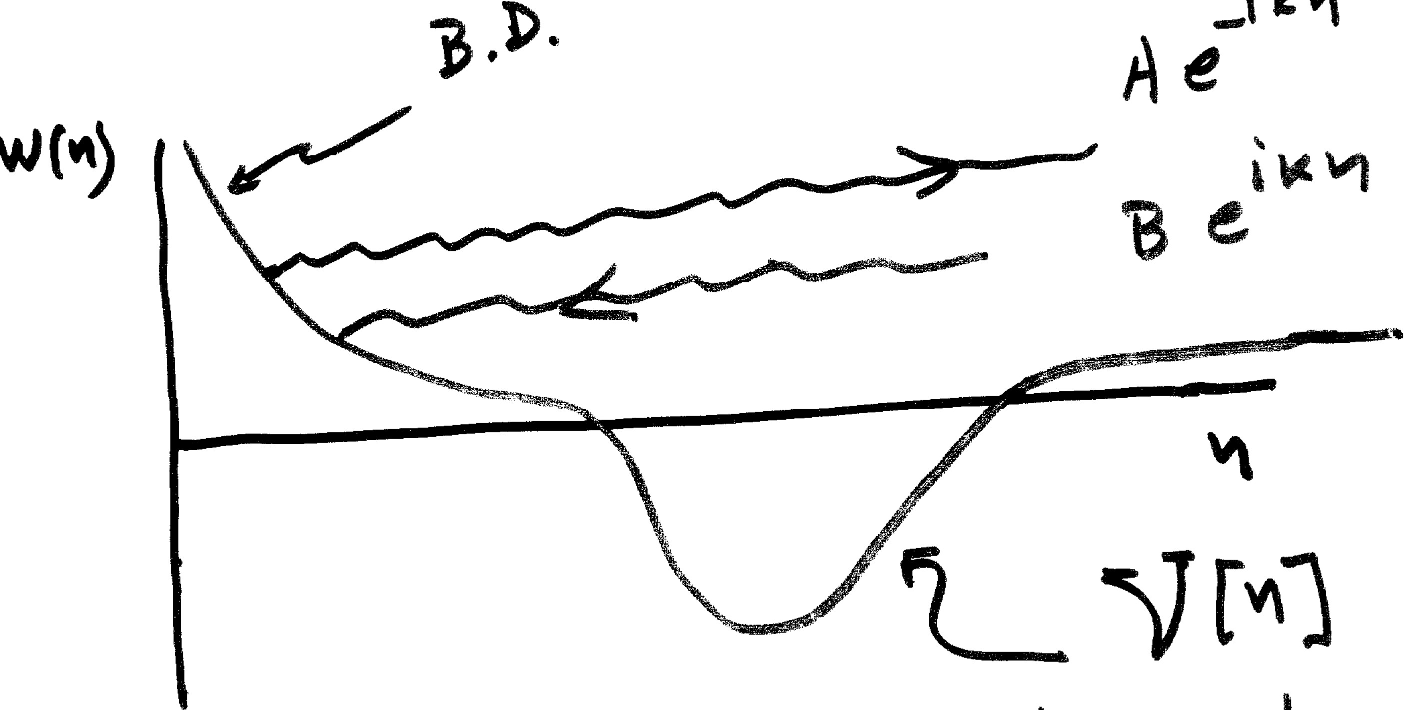
$$\frac{\Delta C_\ell}{C_\ell} \doteq \frac{\int_0^\infty dx\, f_\ell(x)\, D(\mathcal{K}x)}{\int_0^\infty dx\, f_\ell(x)} \qquad \mathcal{K} = \frac{a_0 H_0}{3.3}$$

$f_\ell \sim$ Bessels

Initial conditions from Potentials

$$\left[\frac{d^2}{d\eta^2} + k^2 \underbrace{- \frac{(\nu^2-1/4)}{\eta^2} - \underbrace{V(\eta)}_{0 \text{ for B.D.}}}_{-\text{"potential"} = -W[\eta]}\right] S$$





Ini. Cond $\Longleftrightarrow$ Scatt. by potential.

Potential Scattering $\Longrightarrow D(k)$

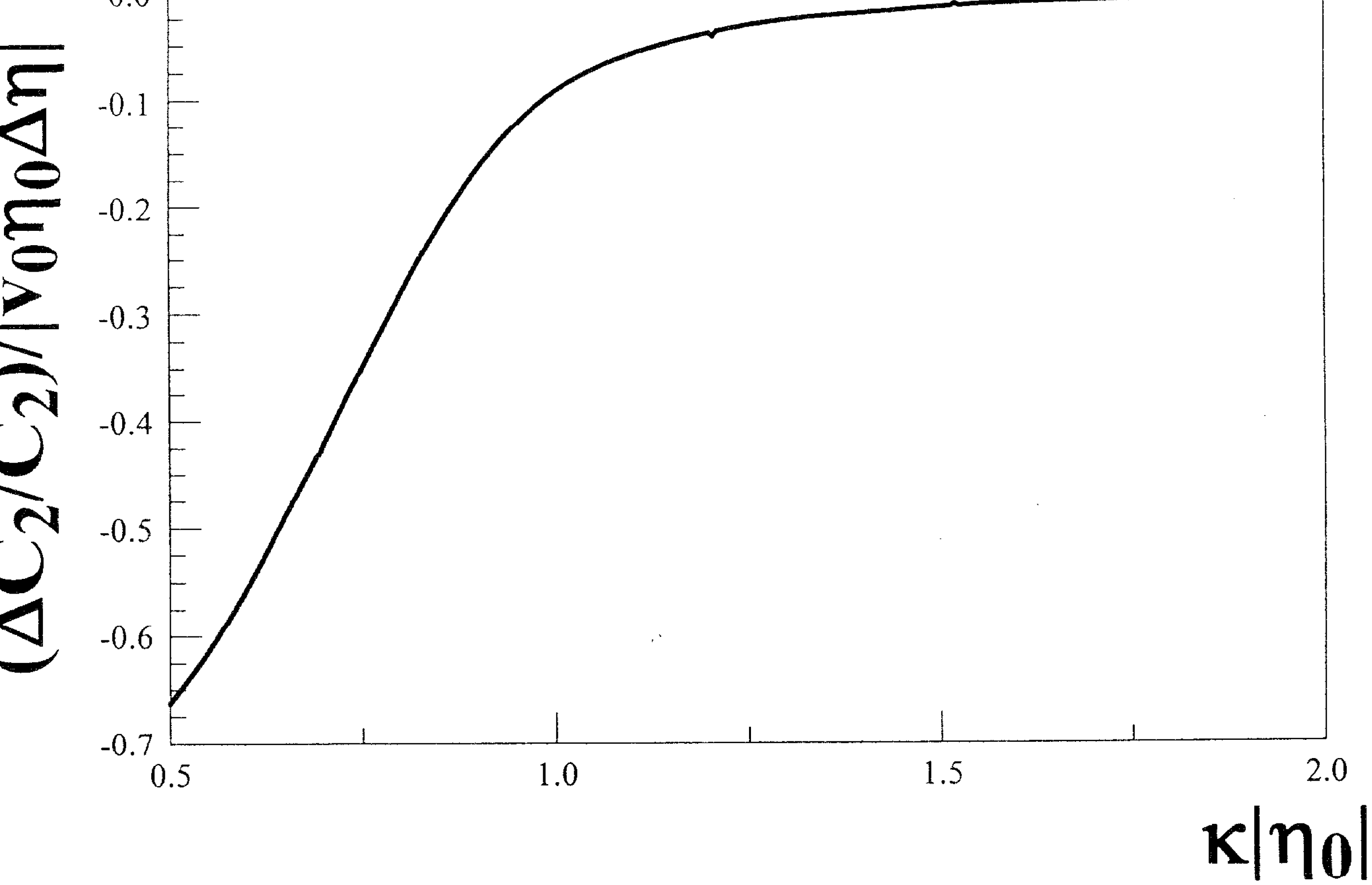
- Narrow ATTRACTIVE potentials localized <u>before</u> last ~ 55 efolds consistent with slow roll $\Longrightarrow$ Quadrupole Suppression

- Height of potential $\sim H^2 \Rightarrow$ CMB QUAD suppression $\sim 10-20\%$.

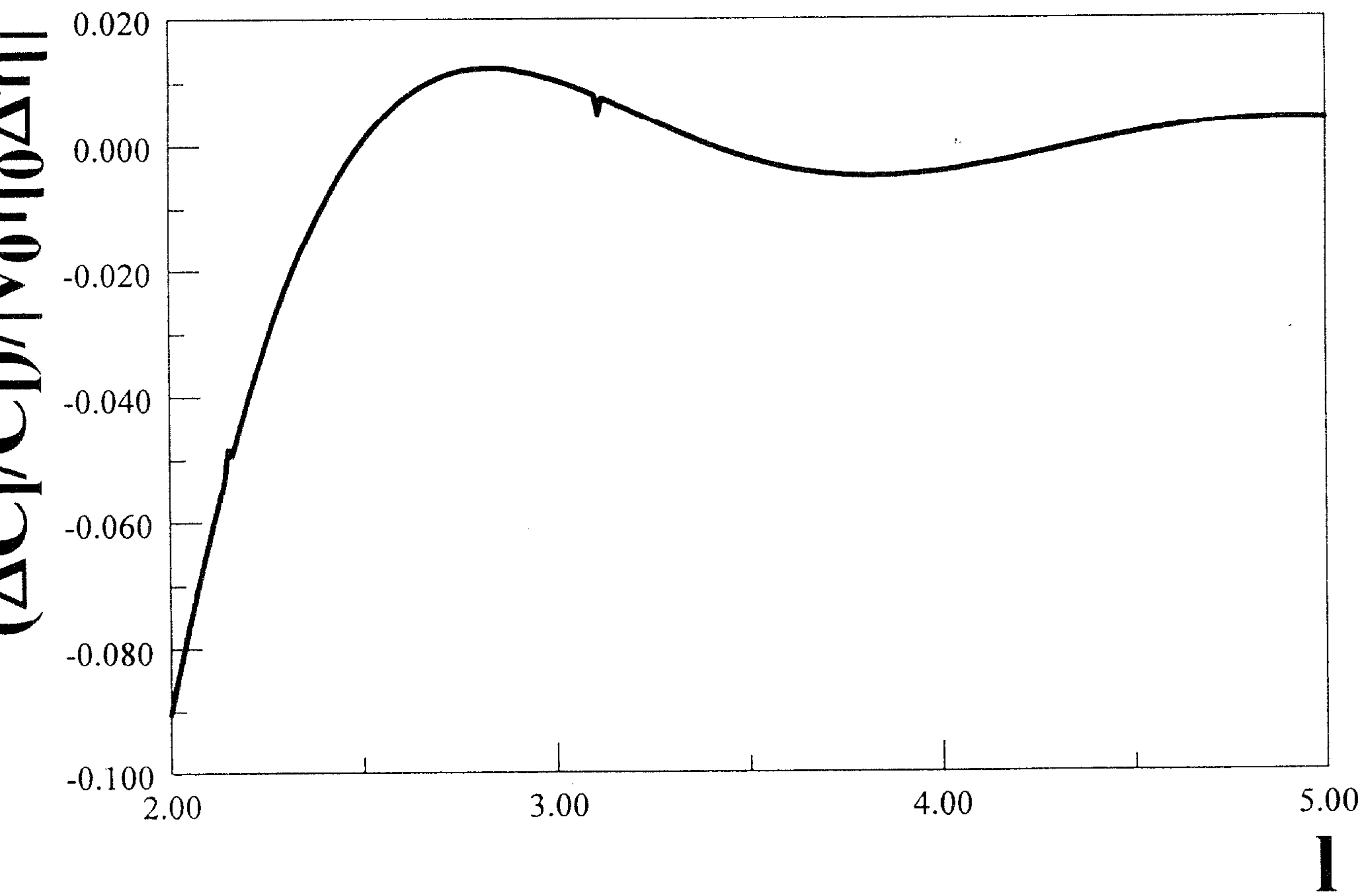
- Narrow + $\mathcal{V}_0 \sim H^2 \Rightarrow N(k) \ll \frac{H^2 M_p^2}{k^4}$ negligible back-reaction !

- $N(k) \lesssim 1/k^4$ at large k!

- Where is $\mathcal{V}(\eta)$ coming from? (TBA..)



0.0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
0.5
1.0
1.5
2.0
$\kappa|\eta_0|$



0.020
0.000
-0.020
-0.040
-0.060
-0.080
-0.100
2.00
3.00
4.00
5.00
l

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