INFLATION AT THE GUT SCALE CONFRONTED TO WMAP AND THE QUADRUPOLE SUPPRESSION

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X^{th} COLLOQUIUM COSMOLOGIE

OBSERVATOIRE DE PARIS

CONCORDANCE MODEL:

STANDARD COSMOLOGY TODAY

ΛCDM — COLD DARK MATTER $+ \Lambda > 0$

EXPLAINS THE OBSERVATIONS:

THREE YEARS OF WMAP DATA

SMALL SCALE CMB DATA (SMALL ANGLE / HIGH ℓ)

LIGHT ELEMENTS ABUNDANCES

LARGE SCALE STRUCTURE (LSS) OBSERVATIONS

SUPERNOVA LUMINOSITY/DISTANCE RELATIONSHIP

LYMAN α FOREST OBSERVATIONS

GRAVITATIONAL LENSING OBSERVATIONS

H_0 (HUBBLE CONSTANT) MEASUREMENTS (HST)

PROPERTIES OF CLUSTER OF GALAXIES

⋮

CONCORDANCE (standard) MODEL OF THE UNIVERSE

THE UNIVERSE STARTS BY AN

$\boxed{\text{INFLATIONARY ERA}}$

$$ds^2 = dt^2 - a^2(t)\, d\vec{x}^2$$

INFLATION = ACCELERATED EXPANSION $\frac{d^2a}{dt^2} > 0$

DURING THE INFLATIONARY ERA THE UNIVERSE EXPANDED BY AL LEAST SIXTY EFOLDS $e^{60} \simeq 10^{26}$.

INFLATION LASTS $\simeq 10^{-34}$ sec.

INFLATION ENERGY SCALE $\sim 10^{16}$ GeV
GRAND UNIFICATION SCALE

DURING INFLATION THE MATTER CAN BE DESCRIBED BY A SCALAR FIELD: THE INFLATON

LAGRANGEAN:

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

WHAT IS THE INFLATON?

IT IS AN EFFECTIVE FIELD.

IT CAN DESCRIBE A FERMION-ANTIFERMION PAIR: $\langle \bar{\psi} \psi \rangle \sim \phi$, ψ = GUT FERMION, ϕ = INFLATON

SUCH CONDENSATE CAN DOMINATE THE EXPECTATION VALUE OF THE HAMILTONIAN ($\sim T_{00}$) AND THEREFORE GOVERN THE COSMOLOGICAL EXPANSION.

(RECALL THAT $\langle \psi \rangle = 0$)

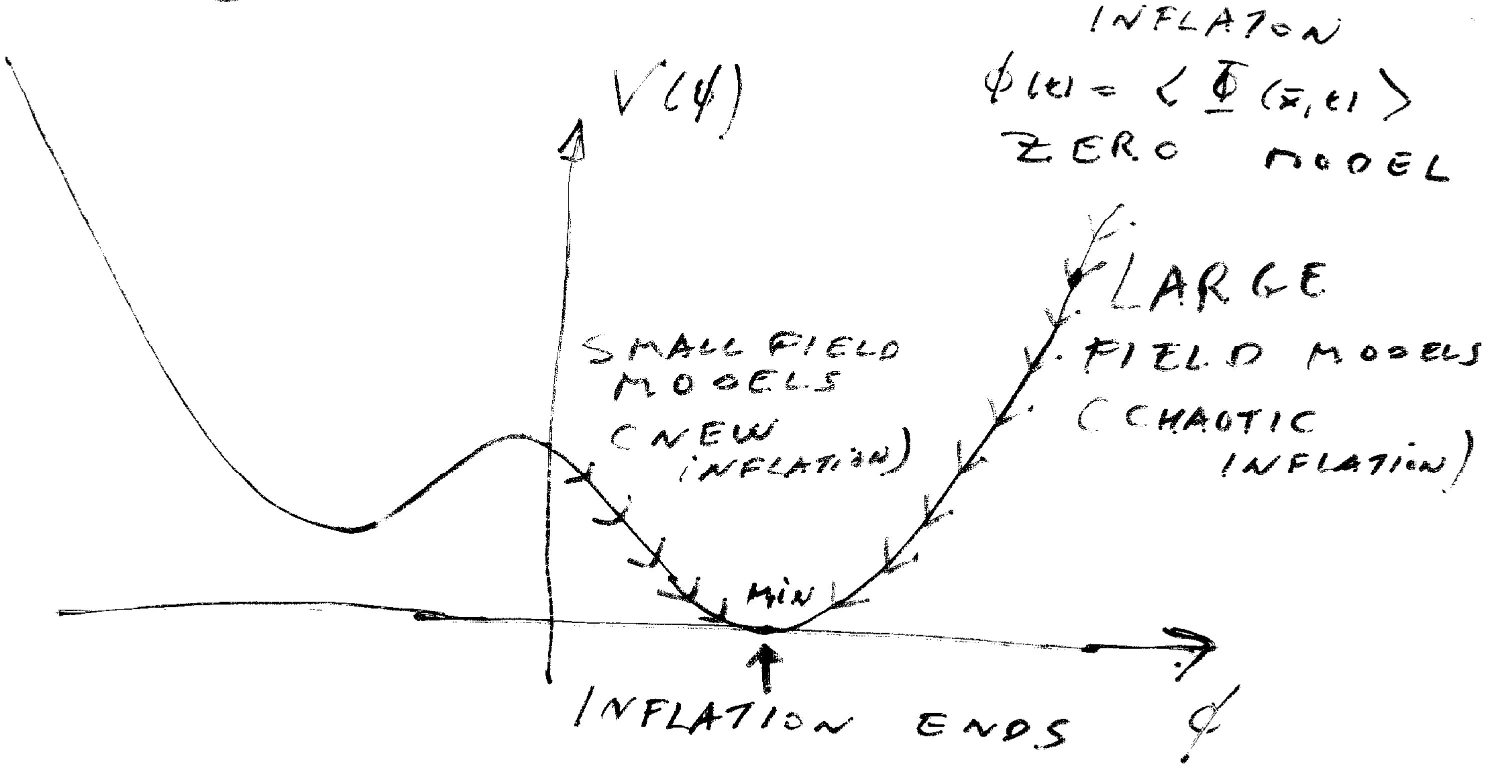
RELEVANT EFFECTIVE THEORIES IN PHYSICS:

1) GINSBURG-LANDAU THEORY OF SUPERCONDUCTIVITY. IT IS AN EFFECTIVE THEORY FOR COOPER PAIRS IN THE MICROSCOPIC BCS THEORY OF SUPERCONDUCTIVITY

2) THE $O(4)$ SIGMA MODEL FOR PIONS, THE σ and PHOTONS AT ENERGIES $\lesssim 1$ GeV. MICROSCOPIC THEORY IS QCD: quarks and gluons

$\pi \simeq \langle q \bar{q} \rangle$, $\sigma \simeq \langle q \bar{q} \rangle$

SLOW ROLL INFLATON MODELS



$V(\text{MIN}) = V'(\text{MIN}) = 0$ $\Big\{$ INFLATION ENDS AFTER A FINITE NUMBER OF EFOLDS

SLOW-ROLL + WMAP+LSS DATA IMPLY FOR THE INFLATON POTENTIAL

$$V(\phi) = N M^4 \, w\left(\frac{\phi}{\sqrt{N} M_{Pl}}\right)$$

$M_{Pl} = 2.4 \; 10^{18}$ Ge[V] Planck MASS $= \frac{1}{\sqrt{8\pi G}}$

$$\chi \equiv \frac{\phi}{\sqrt{N} M_{Pl}}$$ slow inflaton field

M = INFLATION ENERGY SCALE

N = NUMBER OF EFOLDS SINCE HORIZON EXIT TILL END OF INFLATION $\simeq 50$

$w(\chi)$, χ ARE OF ORDER ONE

$M \simeq 10^{-3} M_{Pl}$

M IS FIXED BY THE AMPLITUDE OF CMB ANISOTROPY

WE USE A POLYNOMIAL $w(x)$.

A QUARTIC ORDER IS RENORMALIZABLE.

HIGHER ORDER POLYNOMIALS ARE ACCEPTABLE SINCE INFLATION IS AN EFFECTIVE THEORY

+ LARGE FIELD INFLATION, − SMALL FIELD INFLATION

$$w = w_0 \pm \frac{x^2}{2} + G_3 x^3 + G_4 x^4 .$$

NORMALIZATION of w

$G_3, G_4 \sim O(1)$

$$V = N M^4 w\left(\frac{\Phi}{\sqrt{N} M_{Pl}}\right) = V_0 \pm \frac{m^2}{2}\Phi^2 + g\,\phi^3 + \lambda\,\phi^4$$

see-saw structure

the inflaton is very light

$$m = \frac{M^2}{M_{Pl}} , \quad g = \frac{m}{\sqrt{N}} \underbrace{\left(\frac{M}{M_{Pl}}\right)^2}_{\sim 10^{-6}} G_3 , \quad V_0 = N M^4 w_0$$

$m = \frac{M}{M_{Pl}} \times M \simeq 10^{-3} M \ll M.$

$$\lambda = \underbrace{\left(\frac{M}{M_{Pl}}\right)^4}_{\sim 10^{-12}} \frac{G_4}{N}$$

NO FINE TUNING: SMALL COUPLINGS ARISE NATURALLY AS RATIO OF THE ENERGY SCALES: INFLATION (GUT) AND PLANCK

FOR A GENERAL POTENTIAL:

$$V(\Phi) = \sum_n c_n \Phi^n , \quad c_n = \frac{m^2}{(N M_{Pl}^2)^{\frac{n}{2}-1}} \, G_n \qquad G_n \sim O(1)$$

$$w(x) = \sum_n G_n x^n$$

DEFINE <u>DIMENSIONLESS</u> AND <u>SLOW</u> VARIABLES

$$\chi = \frac{\Phi}{\sqrt{N}\, M_{Pl}} \quad , \quad \tau = \frac{m\, t}{\sqrt{N}} \quad , \quad h(\tau) = \frac{H(t)}{\sqrt{N}\, m}$$

slow inflaton $\left(m = \frac{M^2}{M_{Pl}} \right)$ — slow time — slow Hubble

$$w(\chi) = \frac{V(\Phi)}{N M^4}$$

slow & dimension-less inflaton POTENTIAL

$\boxed{N \sim 50}$ $\boxed{M = \text{ENERGY INFLATION SCALE}}$

EVOLUTION EQUATIONS:

$$\begin{cases} 3 h \dfrac{d\chi}{d\tau} + w'(\chi) = -\dfrac{1}{N} \dfrac{d^2\chi}{d\tau^2} \\[2ex] h^2 = \dfrac{1}{3} w(\chi) + \dfrac{1}{6N} \left(\dfrac{d\chi}{d\tau} \right)^2 \end{cases}$$

CORRECTIONS TO SLOW-ROLL

HIGHER ORDERS IN SLOW ROLL ARE OBTAINED <u>SYSTEMATICALLY</u> BY EXPANDING IN 1/N:

$$\chi(\tau) = \chi_{sr}(\tau) + \frac{1}{N} \chi^{(1)} + \frac{1}{N^2} \chi^{(2)} + \dots$$

BOYANOVSKY, H.J. DE VEGA, N. SANCHEZ

Phys. Rev. <u>D73</u>, 023008 (2006)

PRIMORDIAL POWER SPECTRUM

$$P(k) = |\Delta_k^{(s)}|^2 \, k^{n_s - 1} \longleftarrow \text{Adiabatic Scalar Perturbations}$$

TO ORDER $\frac{1}{N}$ $\equiv$ FIRST ORDER SLOW ROLL

$$n_s - 1 = -3M_{Pl}^2 \left[\frac{V'(\phi)}{V(\Phi)}\right]^2 + 2M_{Pl}^2 \frac{V''(\phi)}{V(\phi)} = -\frac{3}{N}\left[\frac{w'(\chi)}{w(\chi)}\right]^2 + \frac{2}{N}\frac{w''}{w(\chi)}$$

Φ, χ = inflaton field at horizon exit.

RATIO OF TENSOR TO SCALAR PERTURBATIONS: $r = \left|\frac{\Delta_k^{(T)}}{\Delta_k^{(s)}}\right|^2 = 8M_{Pl}^2\left[\frac{V'(\phi)}{V}\right]^2 = \frac{8}{N}\left[\frac{w'(\chi)}{w(\chi)}\right]$

$$|\Delta_k^{(s)}|^2 = \frac{1}{12\pi^2 M_{Pl}^6}\frac{V^3(\phi)}{[V'(\phi)]^2} = \frac{N^2}{12\pi^2}\left(\frac{M}{M_{Pl}}\right)^4 \frac{w^3(\chi)}{[w'(\chi)]^2}$$

FOR ALL SLOW ROLL INFLATION MODELS $\quad |\Delta_k^{(s)}| \simeq \frac{N}{2\pi\sqrt{3}}\left(\frac{M}{M_{Pl}}\right)^2, \quad N \simeq 50$

THE 2003 WMAP RESULT $|\Delta_k^{(s)}| = (0.467 \pm 0.022) 10^{-}$

$$\Rightarrow \left(\frac{M}{M_{Pl}}\right)^2 \simeq 1.02 \times 10^{-5} \Rightarrow M = 0.77 \times 10^{16}\ \text{GeV}$$

$$\boxed{\text{THE INFLATION ENERGY SCALE IS THE GRAND UNIFICATION ENERGY SCALE!}}$$

Trimomial Inflaton Potential: ~~small~~ and large field inflation (new & chaotic).

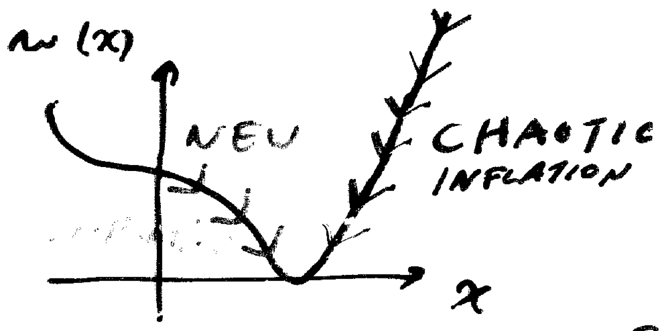
$$w(x) = w_0 \pm \frac{x^2}{2} + G_3 x^3 + G_4 x^4$$

$$w(x_{min}) = w'(x_{min}) = 0$$

$$h = \frac{G_3}{\sqrt{G_4}}$$ ASYMMETRY PARAMETER

$w_0, G_3, G_4 = O(1)$

G_4 = quartic coupling



FOR THIS FAMILY OF MODELS

RED

$n_s < 1$ TILTED , $\frac{dn_s}{d\ln k} < 0$

new: $r \leq \frac{8}{N} = 0.16$ (For $G_3 = G_4 = 0$, x^2 model)

chaotic $\frac{8}{N} \leq r \leq \frac{16}{N} = 0.32$ ($G_4 \to \infty$ limit) x^4 model

GENERAL PROPERTIES:

r grows with the asymmetry h for fixed G_4

n_s increases and then decreases with G_4 for fixed h. There is a maximum value for n_s but no lower bound.

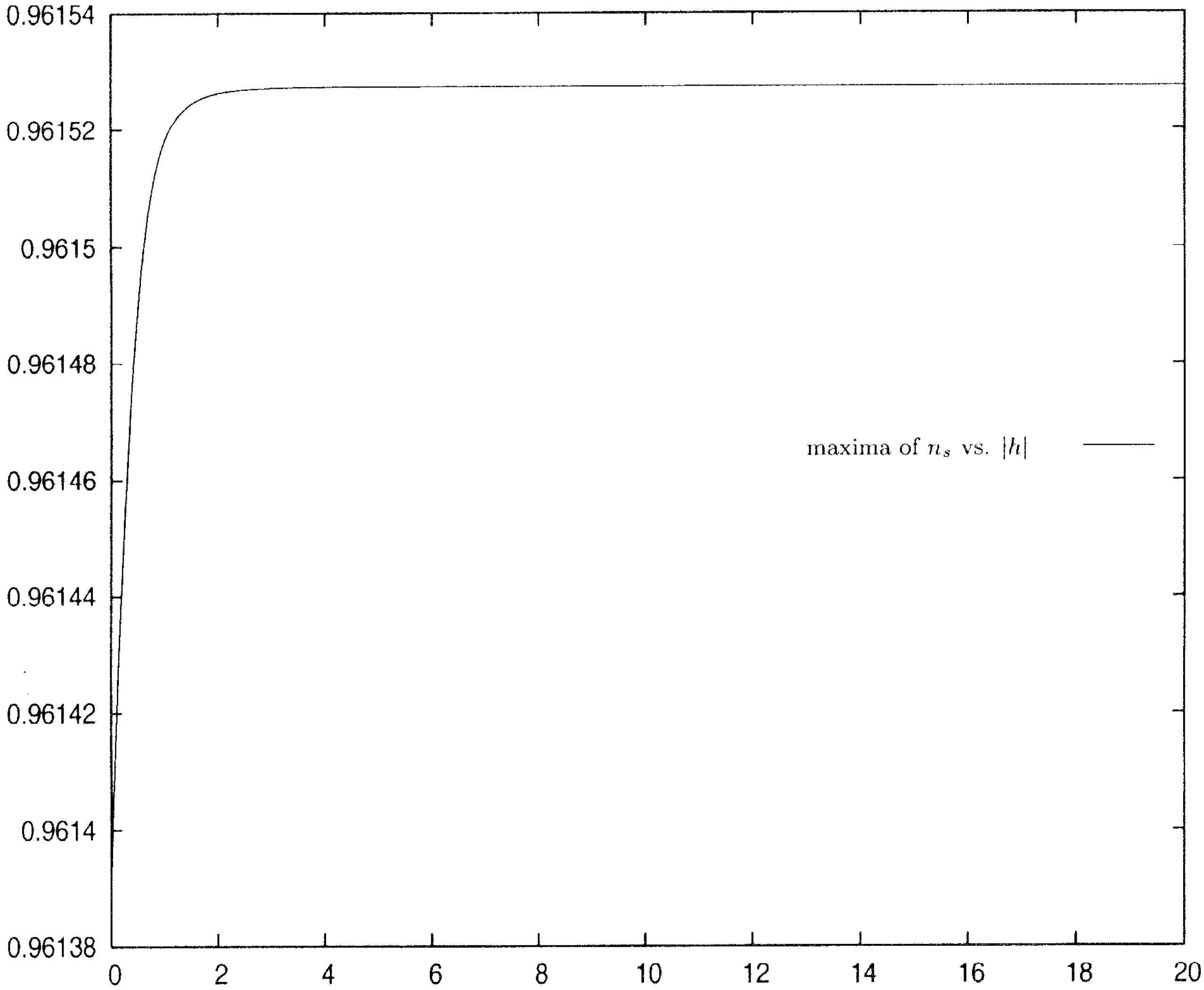


FIG. 6: New Inflation. Maxima of n_s plotted vs. the asymmetry of the potential $|h|$. The limiting value for large $|h|$ is $n_s^{maximum} = 0.961528\ldots$

n_s has for each asymmetry $|h|$ a maximum value varying the quartic coupling

No lower bound on n_s.

Since the observed value of ns is smaller than 0.962... this $\overbrace{\text{model}}^{\text{trinomial}}$ is acceptable

n_s is a two-valued function of A_s

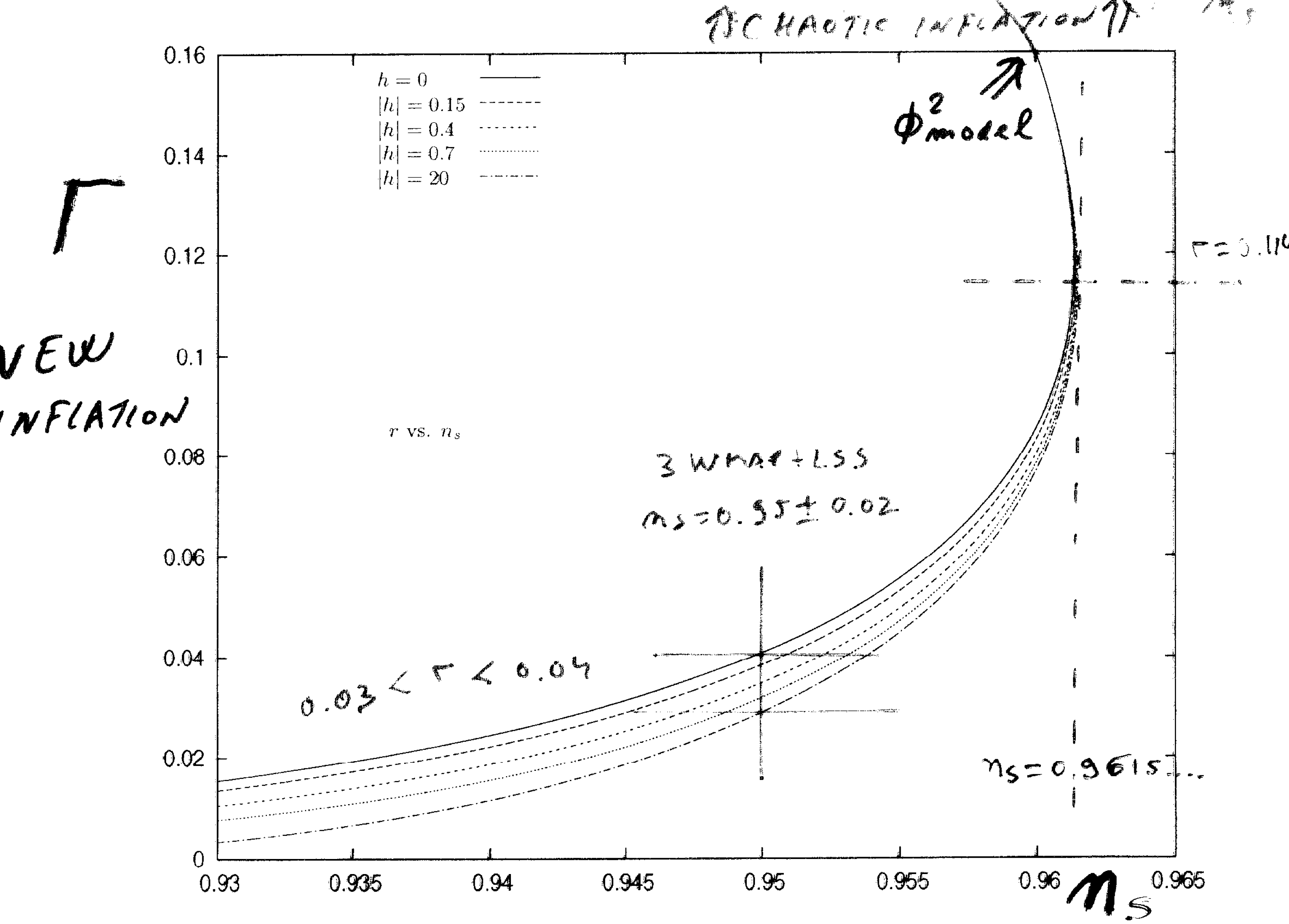


FIG. 4: New Inflation. r as a function of n_s for the asymmetry of the potential $|h| = 0, 0.15, 0.4, 0.7$ and 20. For a given n_s, r monotonically and slowly decreases with increasing $|h|$. $r = r(n_s)$ is not too sensitive to h. The maximun value of n_s is $n_s^{maximum} = 0.961528\ldots$ and the corresponding r is $r_{max} = 0.114769\ldots$. The maximun value of r is $r_{abs\ max} = 0.16$ and corresponds to the quadratic potential setting $y = 0$ in eq.(3.2). For $n_s = 0.95$ (the three years WMAP value), we find $0.03 < r < 0.04$.

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astro-ph/0604136
Single Field Inflation models allowed and ruled out by the three years WMAP data

Phys. Rev. D74, 063519 (2006)

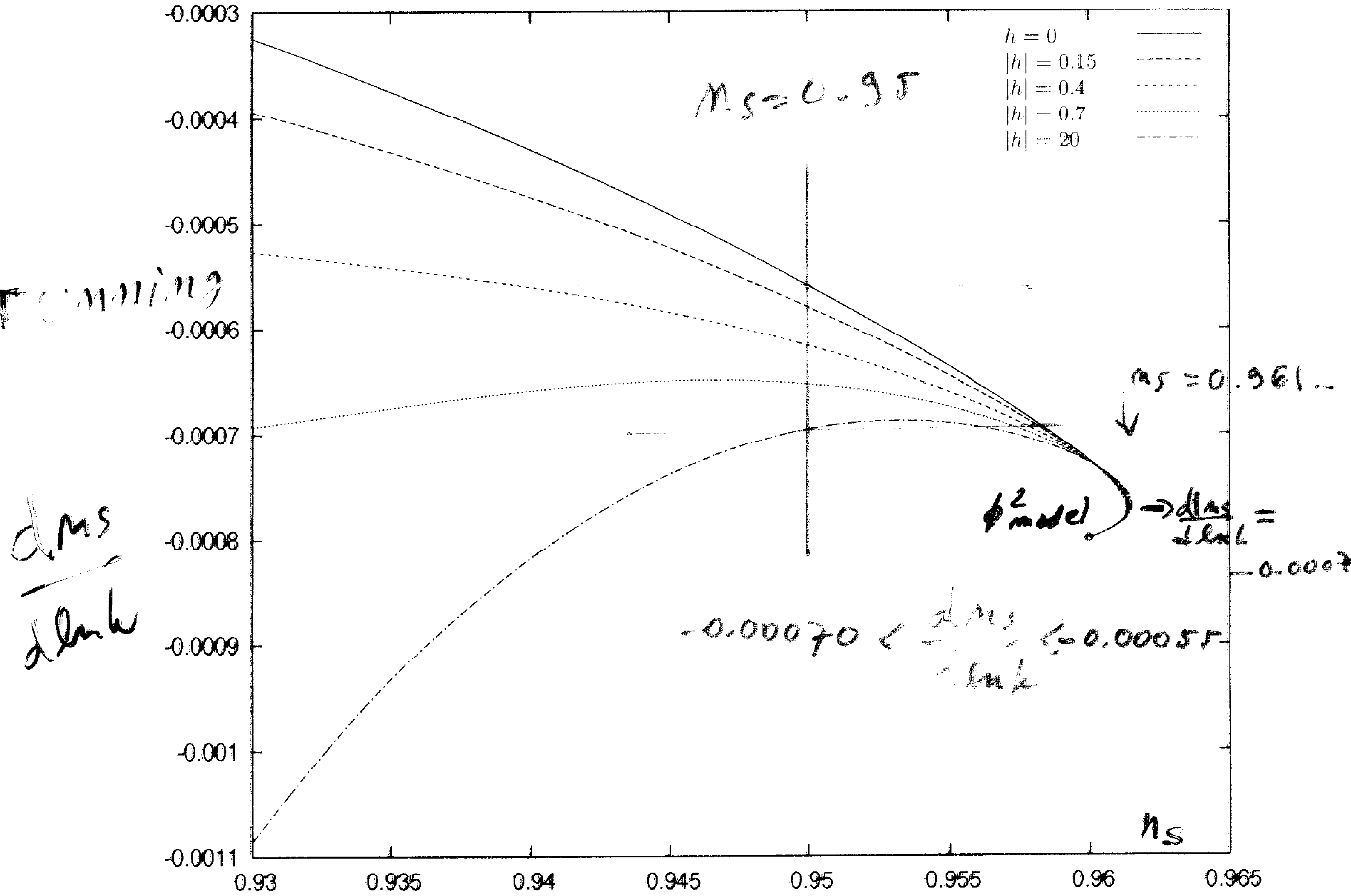


FIG. 5: New Inflation. The running $dn_s/d\ln k$ as a function of n_s for the asymmetry of the potential $|h| = 0, 0.15, 0.4, 0.7$ and 20. The running turns out to be always **negative** in new inflation. For $n_s < 0.96$, the running $dn_s/d\ln k$ decreases with increasing $|h|$. The opposite happens for $n_s > 0.96$. In the last case the dependence on h is weak. We find $dn_s/d\ln k = -0.00077\ldots$ at the branch point $n_s = 0.961\ldots$ for all $|h|$. The point $n_s = 1 - \frac{2}{N} = 0.96$, $\frac{dn_s}{d\ln k} = -\frac{2}{N^2} = -0.0008$ is reached for all values of h and corresponds to the monomial potential eq.(5.2). For $n_s = 0.95$ (the three years WMAP value), we find $-0.00070 < dn_s/d\ln k < -0.00055$.

$\frac{dn_s}{d\ln k}$ is a two valued function of n_s

WMAP 3 yrs.
A. G. Sánchez et al.
MNRAS, 366, 189 (2006)

$\Big\}$ red tilted $n_s = 0.95 \pm 0.02$

FROM THIS VALUE New inflation trinomial potential

predicts $0.03 < r < 0.04$

$$-0.0007 < \frac{dn_s}{d\ln k} < -0.00055$$

chaotic inflation with trinomial potential

predicts $r = 0.27$

3yr WMAP + SDSS says that $r < 0.28$ (95% CL)

THIS $\boxed{\text{DISFAVOURS}}$ CHAOTIC INFLATION

WE FIND A SIMPLE RELATION BETWEEN THE INFLATION ENERGY SCALE M, n_s and r

$$10^4 \left(\frac{M}{M_{Pl}}\right)^2 = 1.27 \sqrt{r(1-n_s)}$$ SMALL AND LARGE FIELD INFLATION

$$10^4 \left(\frac{M}{M_{Pl}}\right)^2 = 1.27 \sqrt{r(n_s-1+\frac{3}{8}r)}$$ HYBRID INFLATION.

$$1.27 = 5\pi\sqrt{3}\ 10^3 \left|\delta^{(S)}_{k\,ad}\right| \ ; \ \left|\delta^{(S)}_{k\,ad}\right| = 0.467\ 10^{-4}$$ WMAP

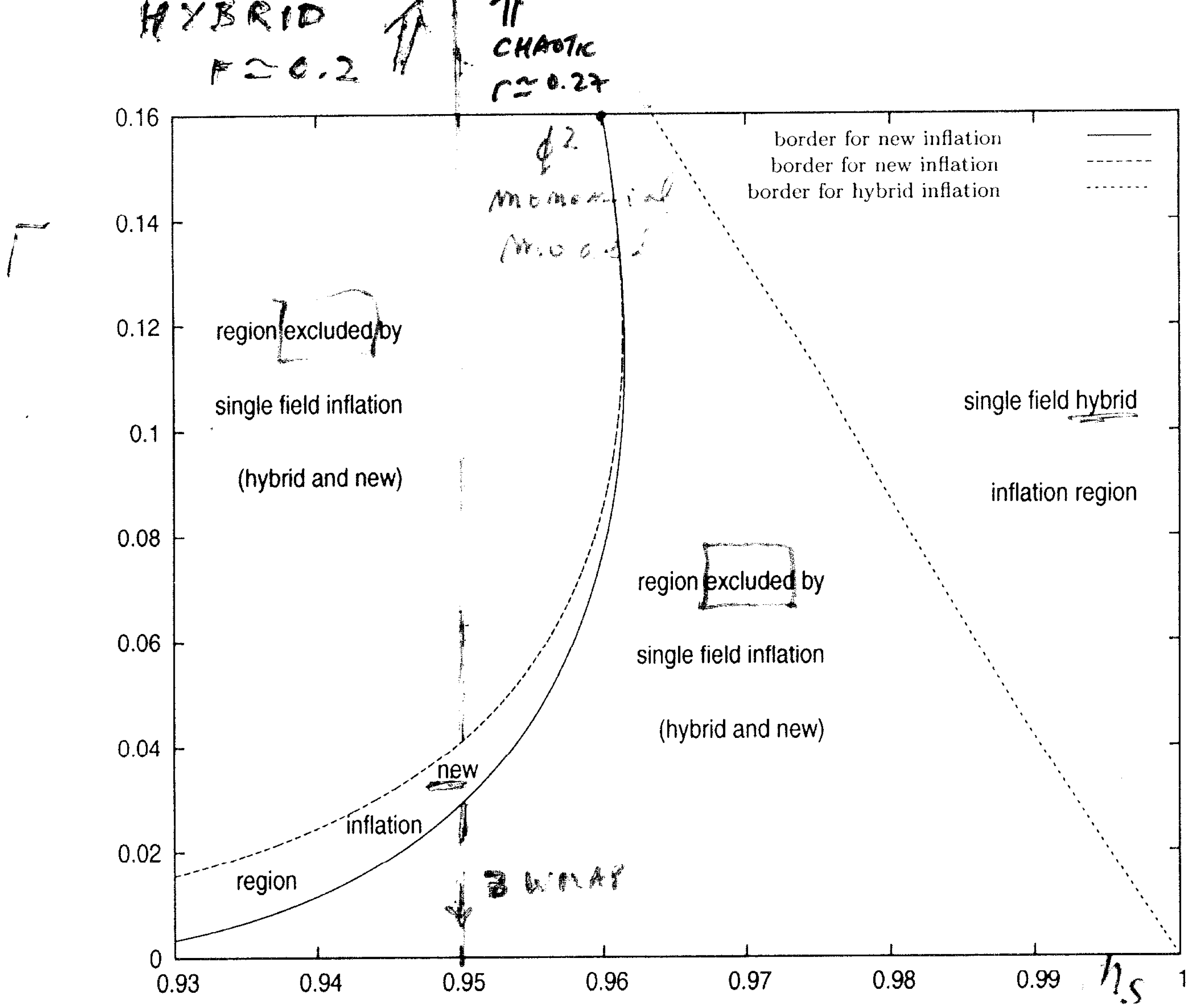


FIG. 24: Regions described in the (r, n_s)-plane by new and single field hybrid inflation for $n_s < 1$. The hybrid inflation border corresponds to $\mu^2 = 1.7\,\Lambda$. For $n_s > 1$, all values of (r, n_s) can be described by hybrid inflation (at least for $r < 0.2$, $n_s < 1.15$). The excluded region cannot be described by single field inflation (neither hybrid inflation, nor new inflation). Two or more fields inflation could describe such regions.

THE SLOW ROLL REGIME IS AN **ATTRACTOR**.

STARTING WITH $\dot{\Phi}^2 \sim V(\phi)$

KINETIC ENERGY $\sim$ POTENTIAL ENERGY

THE INFLATON FIELD ALWAYS REACHES THE SLOW ROLL REGIME $\dot{\Phi}^2 \ll V(\phi)$ AFTER **A FEW EFOLDS**. (TYPICALLY ONE EFOLD)

SLOW ROLL HAS A **LARGE** BASIN OF ATTRACTION.

DEFINE:

using the Friedmann equation

$$y^2 \equiv \frac{\dot{\Phi}^2}{2 M_{Pl}^2 H^2} = 3\left[1 - \frac{V(\phi)}{3 M_{Pl}^2 H^2}\right] = \epsilon_V$$

$$= \frac{\dot{x}^2}{2 h^2 N} = 3\left[1 - \frac{w(x)}{3 h^2}\right]$$

SINCE $V \geq 0$, $0 \leq y^2 \leq 3$ IN GENERAL

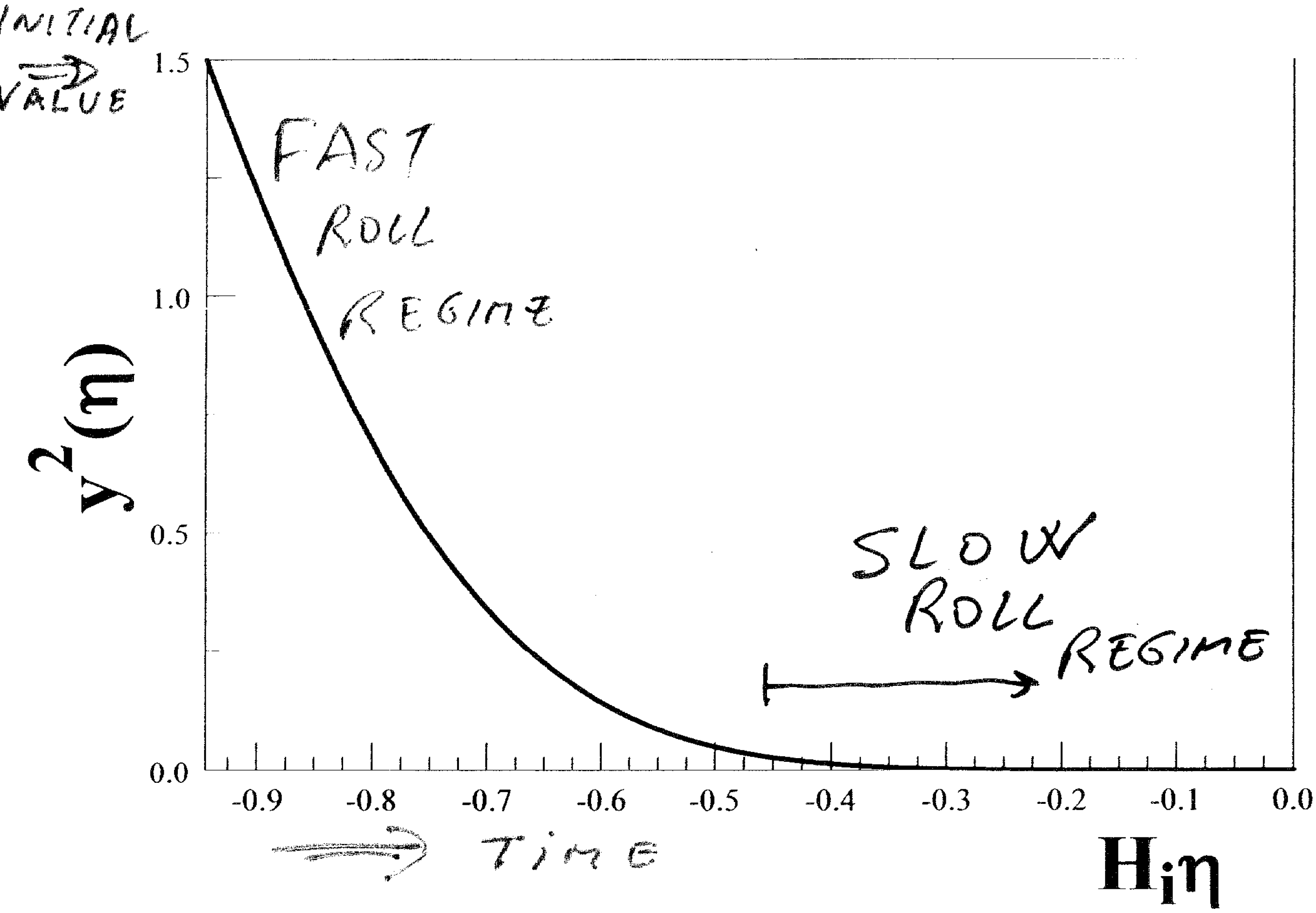
IN SLOW ROLL: $y^2 \sim O\left(\frac{1}{N}\right) \ll 1$

$$N \sim 50$$

$y^2 = O(1) =$ **FAST ROLL**

$0 < y^2 < 3$

NEW INFLATION



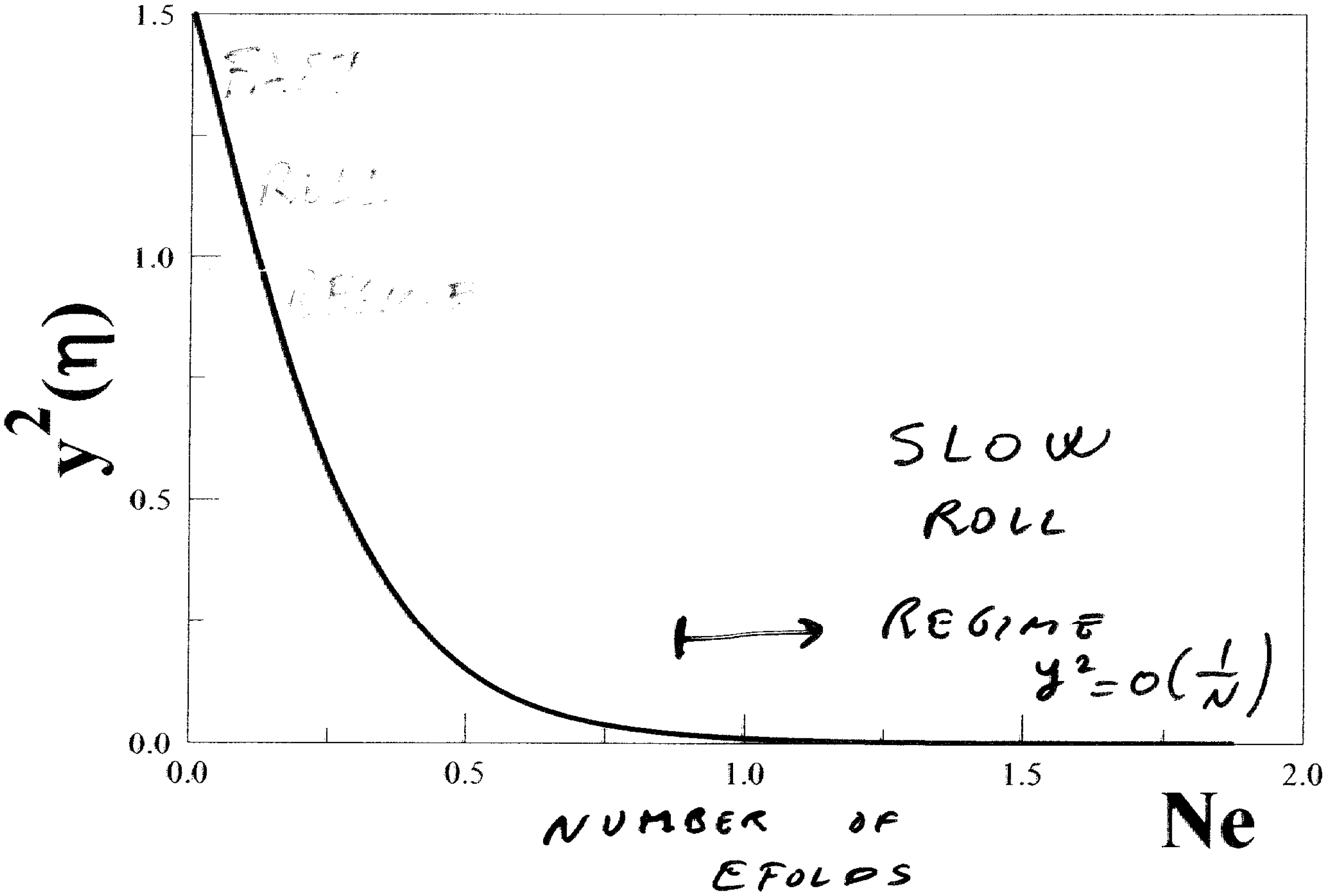
$$y^2 \equiv \frac{\dot{\Phi}^2}{2M_{Pl}^2 H^2} = 3\left[1 - \frac{V(\Phi)}{3M_{Pl}^2 H^2}\right] = \frac{\dot{\chi}^2}{2N h^2}$$

$0 \leq y^2 \leq 3$, $y^2 = \epsilon_v = O\left(\frac{1}{N}\right)$ in slow roll

NOTICE THAT $\frac{\ddot{a}}{a} = H^2(1-y^2)$, FOR $y^2 > 1$ IT IS NOT INFLATION, BUT DESACCELERATED EXPANSION.

EXTREME FAST ROLL $y^2 \gg 1$, $V(\Phi)$ NEGLIGEABLE
extreme analytic solution

$H = \frac{1}{3t}$, $a = a_0 t^{1/3}$, $\Phi = -\sqrt{\frac{2}{3}} M_{Pl} \log(m t)$



$N \simeq 50$

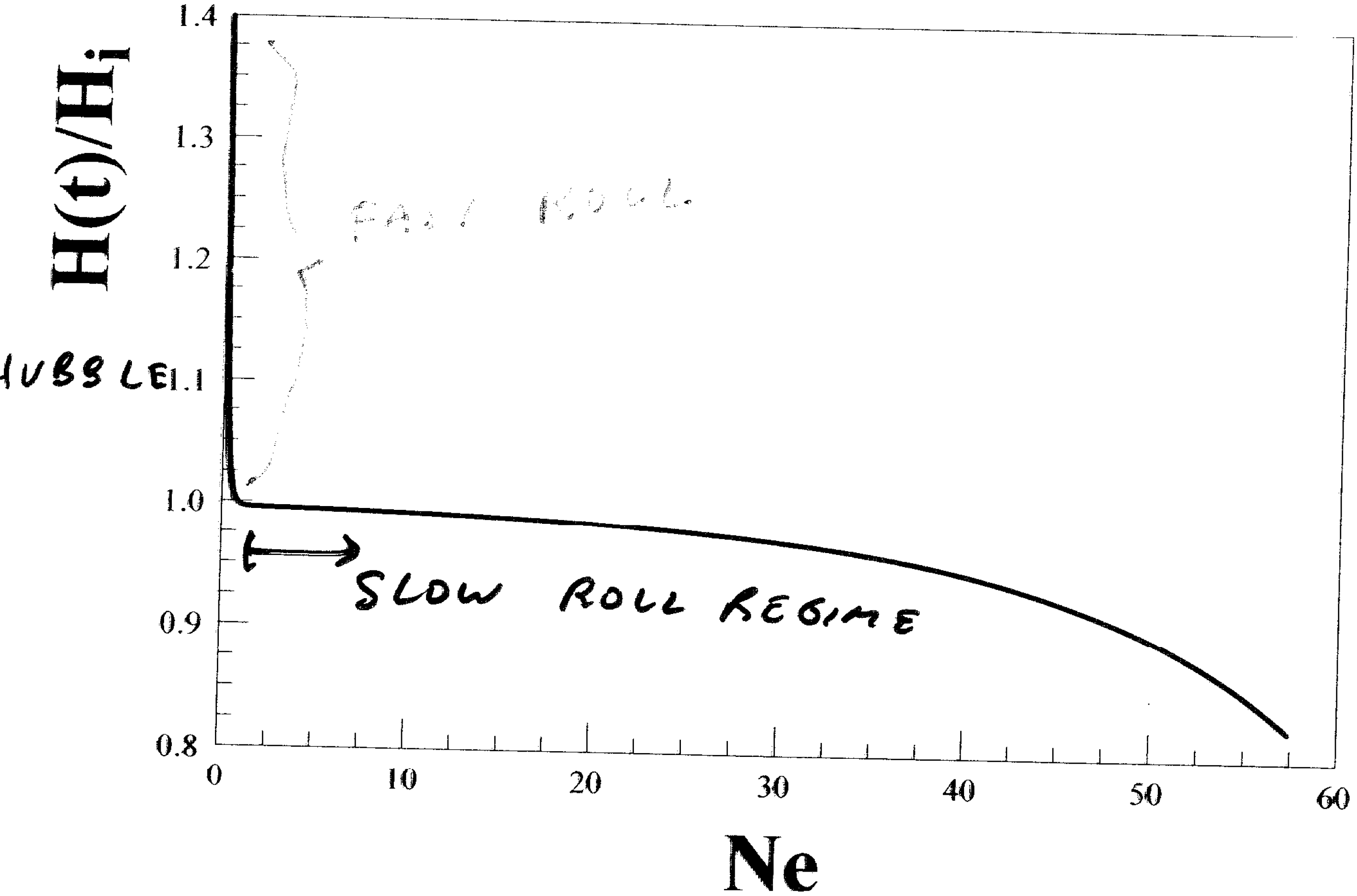
$\Rightarrow$ INFLATION LAST ~ 60 efolds

NEW INFLATION (SMALL FIELD)

$$V(\phi) = \frac{\lambda}{4}\left(\phi^2 - \frac{m^4}{\lambda M_{Pl}^2}\right)^2 , \quad \chi = \frac{\phi}{\sqrt{N}\, M_{Pl}}$$

$$w(\chi) = \frac{G}{4}\left(\chi^2 - \frac{1}{G}\right)^2 , \quad V(\phi) = N M^4 w(\chi)$$

$G = 0.4$ IN THE PLOT



FAST ROLL TAKES PLACE

DURING THE FIRST EFOLD