GAUGE INVARIANT CURVATURE PERTURBATION

scalar curvature

Newtonian potential, Hubble, inflaton fluctuation

$$R(\vec{x},t) = -\psi(\vec{x},t) - \frac{H(t)}{\dot{\Phi}(t)}\,\phi(\vec{x},t)$$

inflaton ev

THESE FLUCTUATIONS AROUND THE FRW GEOMETRY ARE RESPONSIBLE OF THE CMB ANISOTROPIES AND THE LSS.

GAUGE INVARIANT POTENTIAL:

$$u(\vec{x},t) = -z(t)\,R(\vec{x},t)$$

$$z(t) \equiv a(t)\,\dot{\Phi}(t)/H(t)$$

IN FOURIER SPACE:

$$\tilde{u}(\vec{k},t) = \alpha_R(k)\,S_R(k,\eta) + \alpha_R^{+}(k)\,S_R^{*}(k,\eta)$$

creation/annihilation operators

The mode functions obey:

$$\left[\frac{d^2}{d\eta^2} + k^2 - \frac{1}{z}\frac{d^2 z}{d\eta^2}\right] S_R(k,\eta) = 0$$

$$\begin{cases} \eta = \text{conformal time} \\ t = \text{cosmic time}, \quad d\eta = dt/a \end{cases} \qquad ds^2 = a^2(\eta)(d\eta^2 - d\vec{x}^2) = dt^2 - a^2 d\vec{x}^2$$

For Tensor Fluctuations:

$$\left[\frac{d^2}{d\eta^2} + k^2 - \frac{1}{a}\frac{d^2 a}{d\eta^2}\right] S_T(k,\eta) = 0$$

THE MODE FUNCTIONS OF FLUCTUATIONS OBEY A SCHRÖDINGER-LIKE EQUATION WITH POTENTIAL

$$W(\eta) = \left\{ \begin{array}{ll} \frac{1}{z}\frac{d^2 z}{d\eta^2} & \text{SCALAR} \\ \frac{1}{a}\frac{d^2 a}{d\eta^2} & \text{TENSOR} \end{array} \right\} = \frac{\nu^2 - 1/4}{\eta^2} + \mathcal{V}(\eta)$$

$W(\eta)$ DURING SLOW ROLL

LIKE A CENTRIFUGAL BARRIER

$\mathcal{V}(\eta) = 0$ DURING <u>SLOW-ROLL</u>

$\mathcal{V}(\eta) \neq 0$ DURING <u>FAST ROLL</u>

$$\nu_R = \frac{3}{2} + 3\epsilon_v - \eta_v + O\left(\frac{1}{N^2}\right), \quad \nu_T = \frac{3}{2} + \epsilon_v + O\left(\frac{1}{N^2}\right)$$

$$\epsilon_v = \frac{M_{pl}^2}{2}\left[\frac{V'(\phi)}{V(\phi)}\right]^2 = O\left(\frac{1}{N}\right), \quad \eta_v = M_{pl}^2 \frac{V''(\phi)}{V(\phi)} = O\left(\frac{1}{N}\right)$$

DURING SLOW ROLL:

$$S(k,\eta) = A(k)\, g_\nu(k,\eta) + B(k)\, f_\nu(k,\eta)$$

$$g_\nu(k,\eta) = \frac{i^{\nu+\frac{1}{2}}}{2}\sqrt{-\pi\eta}\, H_\nu^{(1)}(-k\eta), \quad g_{\frac{3}{2}}(k,\eta) = \frac{e^{-ik\eta}}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right)$$

Hankel functions

$$f_\nu(k,\eta) = g_\nu(k,\eta)^*,$$

BUNCH DAVIES INITIAL CONDITIONS:

$$S(k,\eta) = \frac{e^{-ik\eta}}{\sqrt{2k}}$$

FOR $\mathcal{V}(\eta) = 0$ THIS IMPLIES $\left\{ \begin{array}{l} A(k) = 1 \\ B(k) = 0 \end{array} \right.$

A NON-ZERO $\mathcal{V}(\eta)$ PRODUCES:

$$A(k) = 1 + i \int_{\eta_0}^{0} g_\nu^*(k,\eta)\, \mathcal{V}(\eta)\, S(k,\eta)\, d\eta$$

($0 \to$ END OF INFLATION)

$$B(k) = -i \int_{\eta_0}^{0} g_\nu(k,\eta)\, \mathcal{V}(\eta)\, S(k,\eta)\, d\eta$$

($\eta_0 \to$ BEGINING OF INFLATION)

Analogous to a one-dimensional scattering problem in the $(-\eta)$-axis.

POWER SPECTRUM (SCALAR AND TENSOR)

$$P(k) = \frac{k^3}{2\pi^2} \langle 0 | \left| \frac{S(k,\eta)}{a} \right|^2 | 0 \rangle_{\eta \to 0^-} = P^{sr}(k) \left[1 + D(k) \right]$$

EFFECT OF $\mathcal{V}(\eta)$

$$P_S^{sr}(k) = A_S^2 \left(\frac{k}{k_0} \right)^{n_S - 1}, \quad P_T^{sr}(k) = A_T^2 \left(\frac{k}{k_0} \right)^{n_T}$$

STANDARD SLOW-ROLL POWER SPECTRA

TRANSFER FUNCTION $D(k)$:

$$D(k) = 2|B(k)|^2 - 2\,\mathrm{Re}\left[A(k)\, B(k)^*\, i^{2\nu - 3} \right]$$

FOR $\mathcal{V} = 0$, $B = 0$, $A = 1 \Rightarrow D(k) = 0$

FAST ROLL MODIFIES THE POWER SPECTRUM BY THE FACTOR

$1 + D(k)$ and $1 + D_T(k)$ resp.

CHANGE IN THE C_ℓ DUE TO $\nu(\eta)$

$$C_\ell = C_\ell^{sr} + \Delta C_\ell \; ; \quad \frac{\Delta C_\ell}{C_\ell} = \frac{\int_0^\infty D(\mathcal{K} x)\, f_\ell(x)\, dx}{\int_0^\infty f_\ell(x)\, dx}$$

$$\mathcal{K} = \frac{a_0 H_0}{3.3}$$

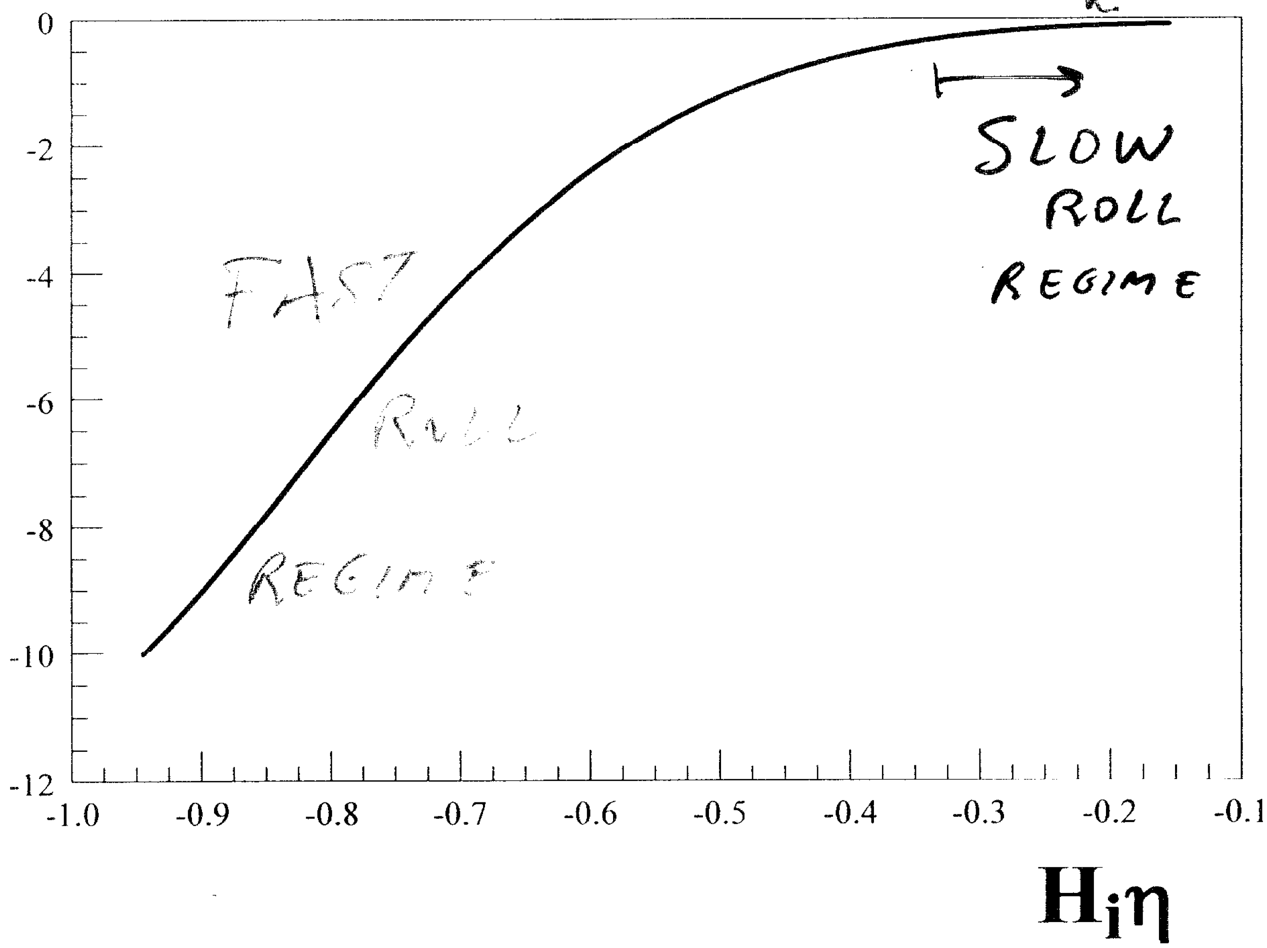
$$f_\ell(x) = \underset{x^{n_T - 1}}{x^{n_s - 2}} \left[j_\ell(x) \right]^2$$

j_ℓ = spherical Bessel function

POTENTIAL FELT BY THE

SCALAR FLUCTUATIONS

$V_R(\eta) \ll H^2$

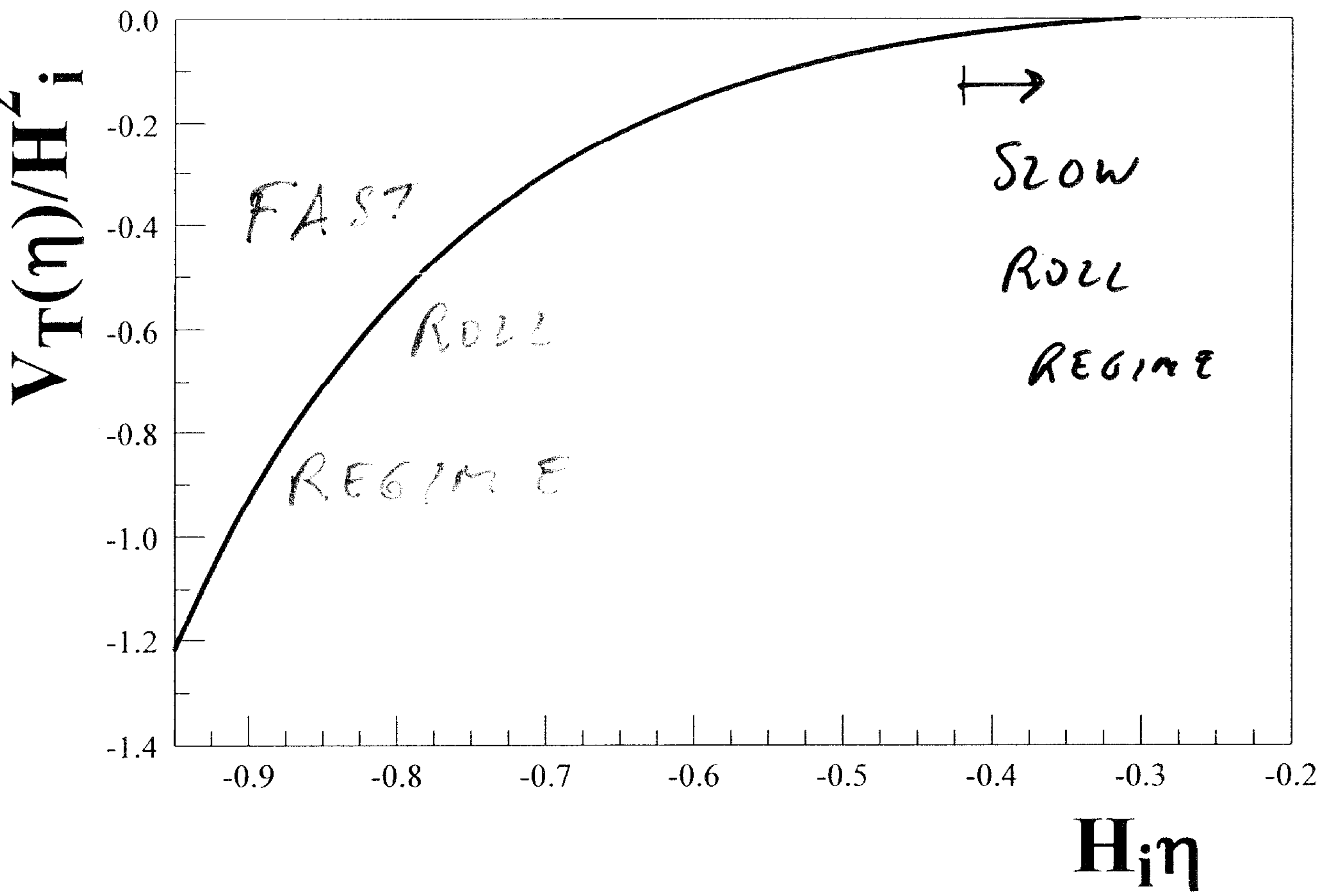


$\mathcal{V}_R(\eta)$ IS ATTRACTIVE:

$\mathcal{V}_R < 0$

POTENTIAL FELT BY THE

TENSOR FLUCTUATIONS $V_T(\eta) \ll H^2$



$\mathcal{V}_T(\eta)$ IS ATTRACTIVE TOO!

H_i = Hubble during slowroll inflation

$H_i \sim 4m$

HOWEVER, $\mathcal{V}_T \sim \frac{1}{10} \mathcal{V}_R$

$\mathcal{V}$ MUCH SMALLER FOR TENSOR PERTURBATIONS

BORN APPROXIMATION FOR THE ΔC_ℓ

SINCE $\mathcal{V}(\eta)$ IS QUITE SMALL WE CAN USE BORN APPROXIMATION:

mode functions $\quad S_\nu(k,\eta) \simeq g_\nu(k,\eta)\quad$ Hankel functions $\eta/=0$ modes

(We checked against exact solutions).

The transfer function results:

$$(k) = \frac{1}{k}\int_{-\eta_0}^{0} d\eta\, \mathcal{V}(\eta)\left[\sin(2k\eta)\left(1-\frac{1}{k^2\eta^2}\right)+\frac{2}{k\eta}\cos(k\eta)\right]\left[1+O\left(\frac{1}{k\eta}\right)\right]$$

and then,

$$\frac{\Delta C_2}{C_2} = \frac{\int_0^\infty D(\varkappa x) f_2(x)\,dx}{\int_0^\infty f_2(x)\,dx} = \frac{1}{\varkappa}\int_{-\eta_0}^{0} d\eta\, \mathcal{V}(\eta)\, \Psi(\varkappa x)\,.$$

$\Psi(x) > 0$ for $\eta < 0$

ATTRACTIVE $\mathcal{V}(\eta) < 0 \Rightarrow \boxed{\Delta C_2 < 0}$

QUADRUPOLE SUPPRESION

IN GENERAL $\quad \boxed{\Delta C_\ell \sim \frac{1}{\ell^2}}$

$\Delta C_\ell < 0$

$\Psi(x)$ IS AN ODD FUNCTION : $\Psi(-x) = -\Psi(x)$

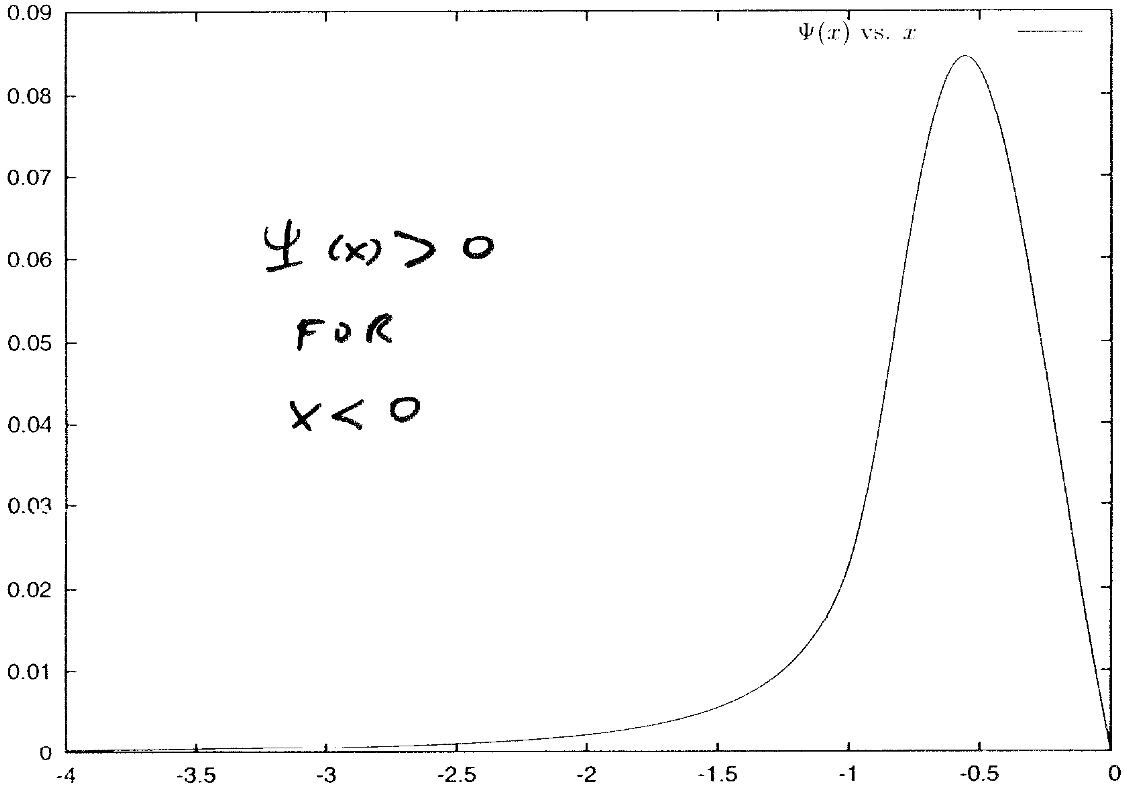


FIG. 1: The odd function $\Psi(x)$ vs. x for negative x [see eq.(5.20)]. This function convoluted with the potential $\mathcal{V}(\eta)$ yields the change on the quadrupole $\frac{\Delta C_2}{C_2}$ [see eq.(5.19)].

$$\Psi(x) \equiv 3\int_0^\infty \frac{dy}{y}\left[j_2(y)\right]^2\left[\left(y^2-\frac{1}{x^2}\right)\sin(2xy)+\frac{2y}{x}\cos(2yx)\right]$$

$$= \frac{1}{105x^2}\left[p(x)(1-x)^3\log\left|1-\frac{1}{x}\right| - (x\to -x)\right] + \frac{2}{105x} - \frac{13x}{126} +$$

$$+ \frac{22x^3}{105} - \frac{2x^5}{21}.$$

where $p(x) = 10x^6+30x^5+33x^4+19x^3+9x^2+3x+1$

$$\Psi(x) \underset{x\to 0}{=} -\frac{x}{6} + O(x^3), \quad \Psi(x) \underset{x\to\infty}{=} -\frac{1}{60x^3} + O\left(\frac{1}{x^5}\right)$$

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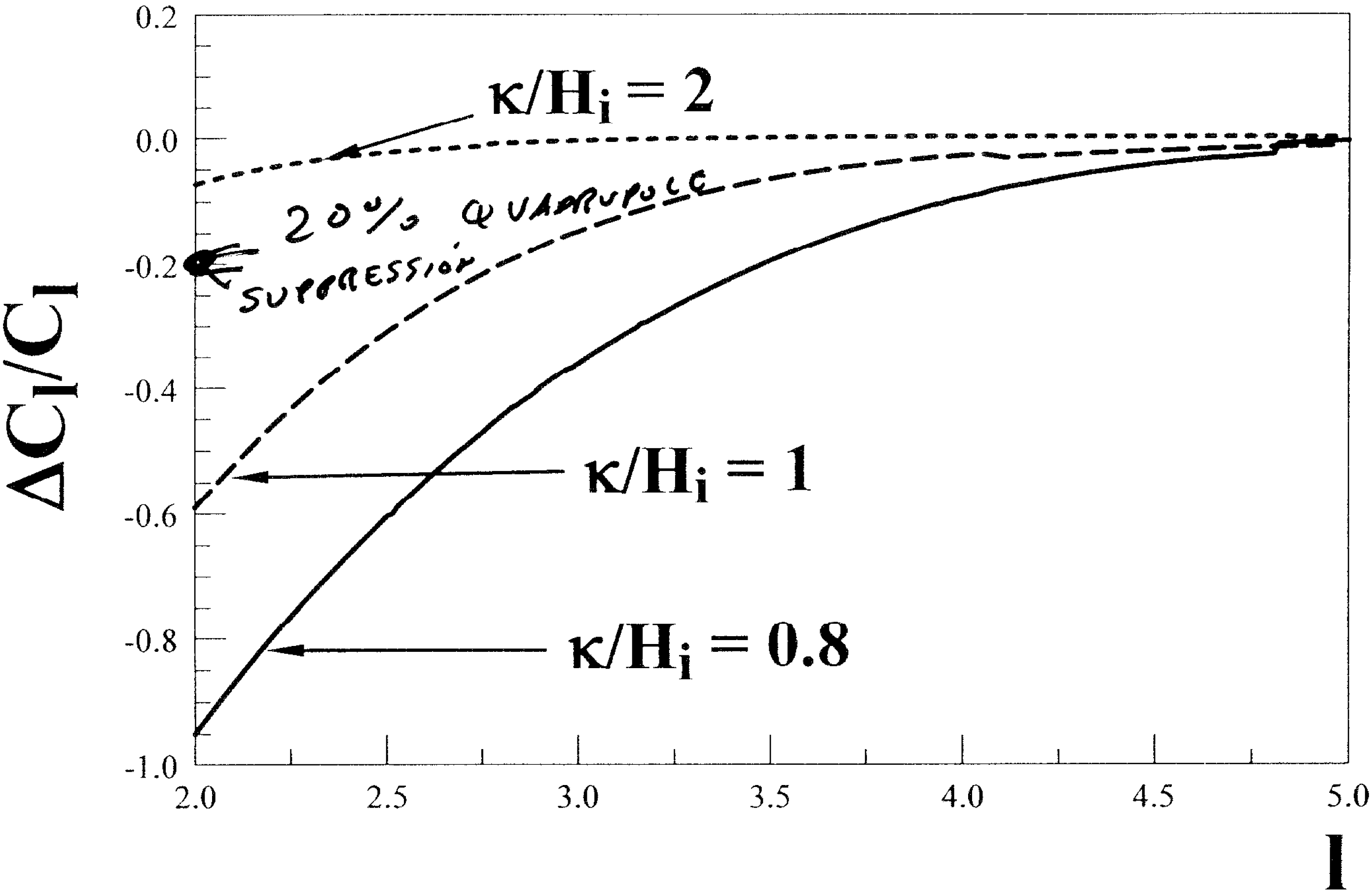
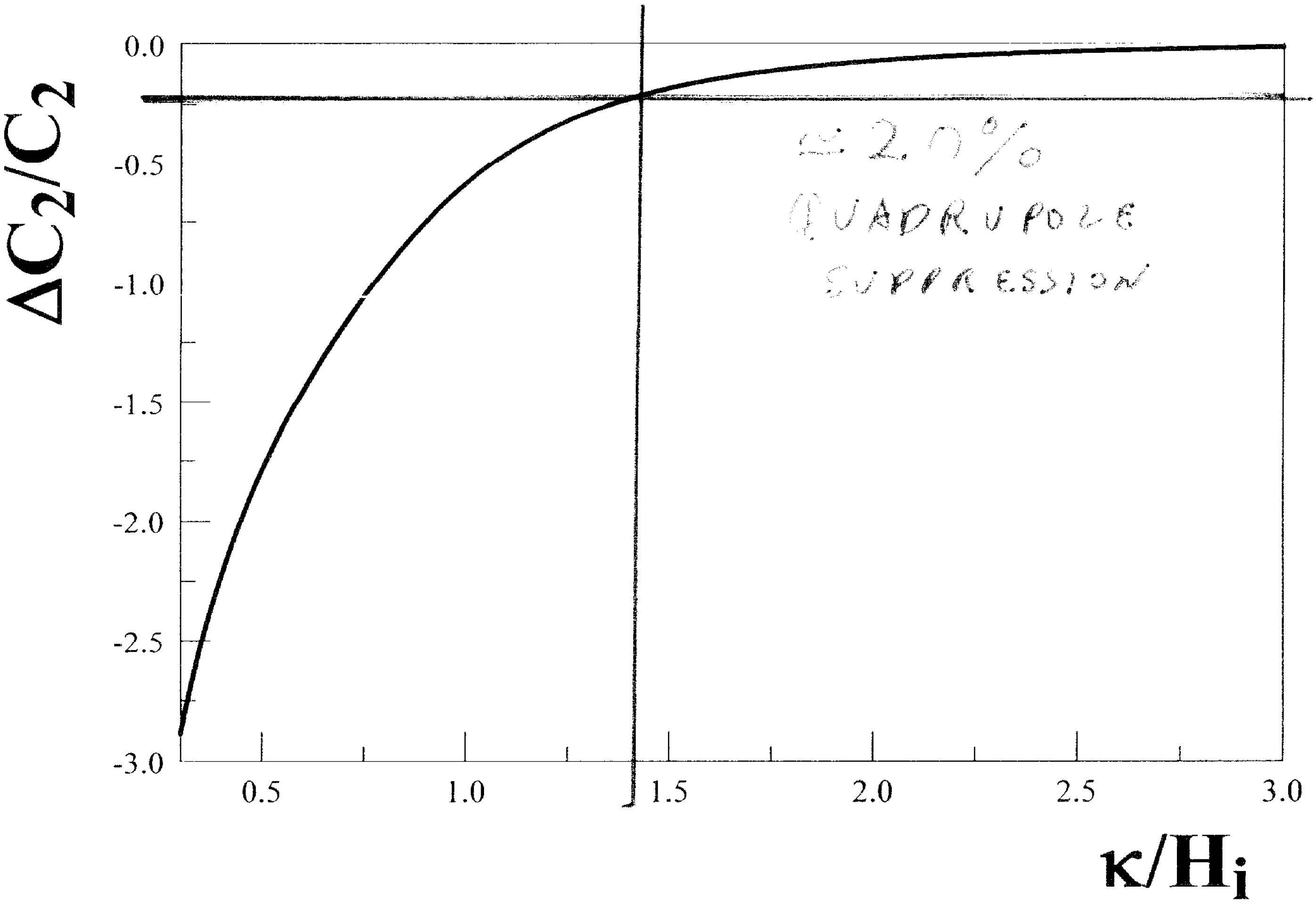


$$\frac{\Delta C_\ell}{C_\ell} \sim \frac{1}{\ell^2}$$

FOR $-\frac{\Delta C_2}{C_2} \sim 0.2$, $\frac{\Delta C_\ell}{C_\ell}$ FOR $\ell > 2$

CANNOT BE SEEN WITH THE PRESENT DATA



$\mathcal{K} = \frac{a_0 H_0}{3.3}$ $\qquad$ H_i = Hubble during slow roll

20% QUADRUPOLE SUPPRESSION FOR $\frac{\mathcal{K}}{H_i} \simeq 1.4$

$$k_Q \simeq a_0 H_0 = a_{sr} H_i \Rightarrow \frac{\mathcal{K}}{H_i} = \frac{a_{sr}}{3.3} = 1.4$$

$$\Rightarrow a_{sr} \simeq 4.6 \simeq e^{1.5}$$

THE QUADRUPOLE MODES SHOULD EXIT THE HORIZON 1.5 efolds AFTER FAST ROLL STARTS (AND 0.5 efold AFTER SLOW ROLL BEGINS)

QUARTIC COUPLING: $\lambda = \frac{G_4}{N} \left(\frac{M}{M_{Pl}} \right)^4$

$$N = \log \frac{a(\text{END INFLATION})}{a(\text{HOR EXIT})}$$

λ RUNS LIKE IN FOUR DIMENSIONAL RENORMALIZATION GROUP IN FLAT EUCLIDEAN SPACE.

CONJECTURE: INFLATION IS HOVERING NEAR A TRIVIAL GAUSSIAN INFRARED POINT.

SLOW ROLL INFLATION MAY THUS ENJOY UNIVERSALITY PROPERTIES

D. BOYANOVSKY, H. J. DE VEGA, N. G. SANCHEZ
astro-ph/0507595; Phys. Rev. D73, 023008 (2006).
CLARIFYING INFLATION MODELS:
SLOW-ROLL AS AN EXPANSION IN 1/Nefolds
AND NO FINE TUNING.

THE SCALE OF INFLATION

Grand Unification Idea (GUT)

) RENORMALIZATION GROUP RUNNING OF ELECTROMAGNETIC, WEAK AND STRONG COUPLINGS SHOWS THAT THEY MEET AT $M_1 \sim 10^{16}$ GeV

) NEUTRINO MASSES (from ν oscillations) ARE EXPLAINED BY THE SEE-SAW MECHANISM

$$\boxed{\Delta m_\nu \sim \frac{M_{Fermi}^2}{M_2} \quad \sim 0.05\,eV}$$

$M_{Fermi} \sim 250$ GeV (weak interactions)

$M_2 \sim 10^{16}$ GeV

) WE SAW THAT THE INFLATON POTENTIAL IS

$$\underbrace{m^2 M_{pe}^2}_{M_3^4} \, v\left(\frac{\phi}{M_{pe}}\right)$$

$m \sim 10^{13}$ GeV

$M_{pe} \simeq 10^{19}$ GeV

HENCE, $M_3 \simeq 10^{16}$ GeV $\qquad m = \frac{M_3^2}{M_{pe}} =$ AGAIN SEE-SAW TYPE

CONCLUSION: $M_1 = M_2 = M_3 = M_{GUT} = 10^{16}$ GeV

MOREOVER, MODULI POTENTIALS ARE

$(m_{SUSY})^4 \, v(\phi/m_{pe}) \Longrightarrow$ $\boxed{m_{SUSY} = M_{GUT}}$

THE GEOMETRY IS DE SITTER $e^{Ht} = -\frac{1}{H\eta}$

$ds^2 = dt^2 - e^{2Ht} d\vec{x}^2$

$$ds^2 = \frac{1}{(H\eta)^2}\left[d\eta^2 - d\vec{x}^2\right] \Leftarrow \text{SCALE INVARIANT!}$$

DURING $N \sim 60$ efolds $\qquad \eta$ = CONFORMAL TIME

$\Rightarrow$ FLUCTUATIONS SPECTRUM SCALE INVARIANT TOO UP TO $\frac{1}{N}$ corrections

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INFLATON POTENTIAL

$$V(\phi) = N M^4 w\left(\frac{\Phi}{\sqrt{N} M_{PL}}\right), \quad w(x) = O(1),\ x = O(1)$$

MODULI POTENTIALS:

$$m_{SUSY}^4\, v\left(\frac{\phi}{M_{PL}}\right)$$

THEY ressemble inflaton potentials if

$$m_{SUSY} \simeq M = 10^{15}\ \text{GeV}$$

FIRST OBSERVATION OF SUSY IN NATURE ?

THE GEOMETRY IS DE SITTER $e^{Ht} = -\frac{1}{H\eta}$

$ds^2 = dt^2 - e^{2Ht} d\vec{x}^2$

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THEY Ressemble inflaton potentials if

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FIRST OBSERVATION OF SUSY IN NATURE ?

F. A. QUESTIONS ABOUT INFLATION

The INFLATON POTENTIAL

Q: WHERE DID THIS FUNCTION COME FROM?

A: IT IS AN EFFECTIVE POTENTIAL DESCRIBING SCALAR CONDENSATE (À LA GINSBURG-LANDAU) IN A GUT THEORY.

Q: WHY IS THE POTENTIAL SO FLAT?

A: BECAUSE IT IS A FUNCTION OF $\dfrac{\Phi}{\sqrt{N}\, M_{Pl}}$

$N \equiv$ number of efolds ~ 50.

Q: WHY DID THE FIELD START AT THE CHOSEN $\phi(0)$?

A: $\phi(0)$ IS CHOSEN IN EACH MODEL SUCH THAT THE INITIAL ENERGY IS AT THE GUT SCALE.

Q: HOW DO WE CONVERT THE FIELD ENERGY COMPLETELY INTO PARTICLES?

IT IS A NONPERTURBATIVE OUT OF EQUILIBRIUM THAT HAS BEEN EXPLICITLY COMPUTED.

SUMMARY

INFLATION : AN EFFECTIVE THEORY

THE INFLATON FIELD : A SCALAR CONDENSATE DOMINATING THE ENERGY OF THE UNIVERSE

ONLY SCALAR CONDENSATES ARE ISOTROPIC

FUNDAMENTAL THEORY : GRAND UNIFIED GAUGE THEORY SU(5), O(10), ... ?

GINSBURG-LANDAU APPROACH TO INFLATON MODELS

INFLATON MODEL TO GUT
AS
SIGMA MODEL TO QCD.

STRUCTURE OF THE INFLATON POTENTIAL:

$$V(\Phi) = N M^4 w\left(\frac{\Phi}{\sqrt{N} M_{Pl}}\right)$$

$w(MIN) =$
$= w'(MIN) = 0$

M = inflation scale $= 3 \cdot 10^{16}$ GeV (WMAP) = GUT SCALE

N = number of efolds ~ 50

GETS RID OF FINE TUNING

$w(\chi)$ and ALL ITS PARAMETERS ARE $O(1)$

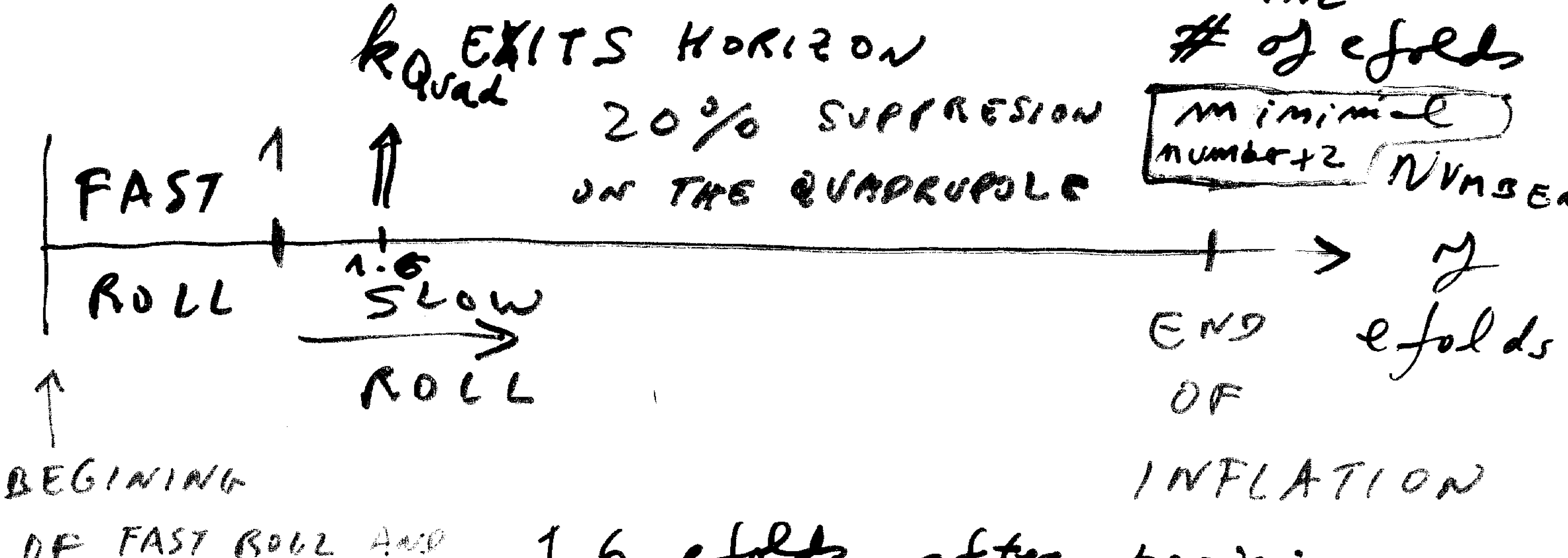
SLOW-ROLL BUILT-IN. HIGHER ORDERS IN SLOW-ROLL AS A SYSTEMATIC $1/N$ EXPANSION.

BIG CHANGE IN Φ $\rightarrow$ SMALL CHANGE OF $V(\Phi)$

FOR **ALL** SLOW ROLL EFFECTIVE MODELS OF INFLATION THERE IS GENERICALLY A **FAST ROLL** STAGE FOLLOWED BY SLOW ROLL.

AN **ATTRACTIVE** POTENTIAL $\mathcal{V}(\eta)$ IS GENERATED ON THE SCALAR AND TENSOR PERTURBATIONS DURING **FAST ROLL**.

THE k MODES EXITING THE HORIZON VERY SOON AFTER THE BEGINING OF SLOW ROLL GET **PARTIALLY** SUPPRESED BY $\mathcal{V}(\eta)$



1.6 efolds after begining

MAIN REFERENCES

D. Boyanovsky, H. J. de Vega, N. G. Sánchez, Clarifying inflation models: slow roll as an expansion in 1/Nefolds. Phys. Rev. D73, 023008 (200

D. Cirigliano, H. J. de Vega, N. G. Sánchez
Clarifying inflation models: The precise inflationary model from effective field theory and the WMAP data. Phys. Rev. D71, 103518 (2005).

H. J. de Vega, N. G. Sánchez, Single Field Inflation Models Allowed and Ruled Out by the three years WMAP data, Phys. Rev. D74, 06351 (2006)

D. Boyanovsky, H. J. de Vega, N. G. Sánchez
CMB Quadrupole suppression
I. Initial Conditions of Inflationary Perturbations astro-ph/0607508.
II. The Early Fast Roll Stage astro-ph/0607487.

PHASE TRANSITIONS IN THE EARLY AND THE PRESENT UNIVERSE, hep-ph/0602002, ANN. REV. PART. & NUCL. PHYS. (2006). D. BOYANOVSKY, H. J. DE VEGA, D. J. SCHWARZ