M. Giovannini, PRD 73, 101302 (2006);
PRD 74, 063002 (2006);
CQG 23, 4991 (2006)

Cosmic magnetic fields and scalar CMB anisotropies

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Observatoire de Paris, October 2006

A Magnetized Universe?

- Large-scale magnetic fields (typical length-scales > 1 A.U.) $1 A.U. = 1.49\ 10^{13}\ cm$
- First speculations: early forties (Alfven) late forties (Fermi, Fermi & Chandrasekar) on cosmic ray physics $1\ \mu G = 0.1\ nT = 10^{-26}\ GeV^2$

-Today: magnetic fields measured with various techniques

Zeeman splitting of radio transitions

$$\Delta\nu_Z = \frac{e\bar{B}_{\|}}{2\pi m_e}$$

$$\Delta\nu_{Doppler} \simeq \left(\frac{v_{th}}{c}\right)\nu \gg \Delta\nu_{Zeeman} \simeq \frac{e\bar{B}_{\|}}{2\pi m_e}$$

Synchrotron emission

$$\varepsilon(\nu) = 10^{-23}\, n_{er0}\, L\, \xi(\gamma)\, (6.3\times 10^{18})^{(\gamma-1)/2} (B_{\perp})^{(\gamma+1)/2}\, \nu^{(1-\gamma)/2}\ \ erg\ sec^{-1}\ cm^{-2}\ Hz^{-1}$$

Faraday rotation

$$\Delta\,\phi = \frac{f_e}{2}\left(\frac{\omega_p}{\omega}\right)^2 \omega_B\, \Delta z$$

$$\omega_p = \left(\frac{4\pi n_e e^2}{m_e}\right)^{1/2} \qquad \omega_B = \frac{eB}{mc}$$

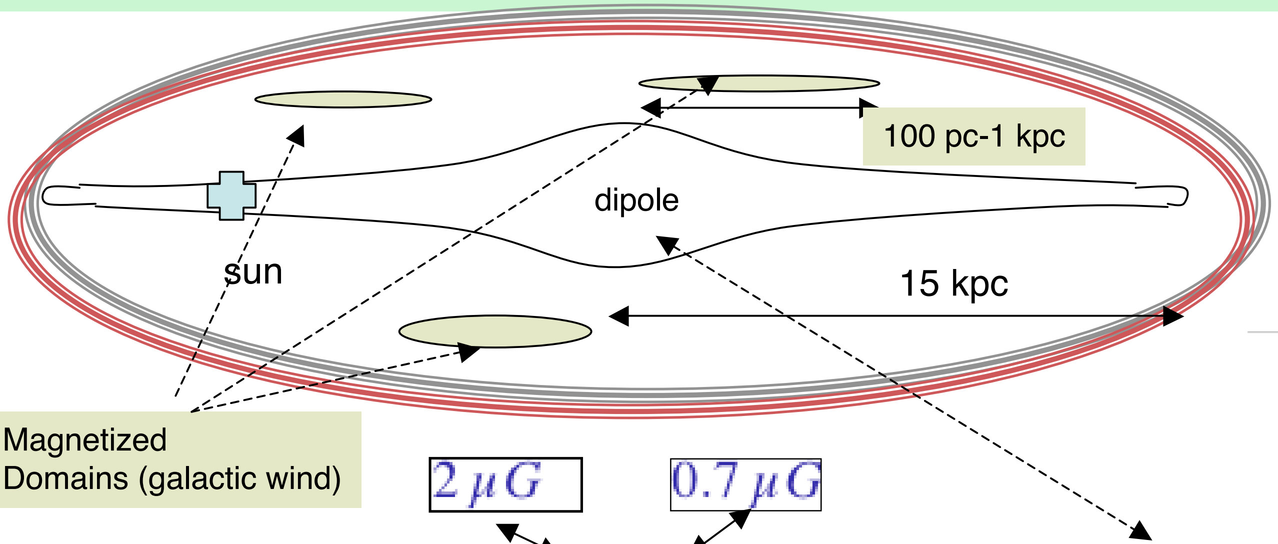
$$\phi = RM\,\lambda^2 + \phi_0$$

$$RM = \frac{\Delta\phi}{\Delta\lambda^2} = 811.9\int\left(\frac{n_e}{cm^{-3}}\right)\left(\frac{B_{\|}}{\mu G}\right) d\left(\frac{\ell}{kpc}\right)\ \frac{rad}{m^2}$$

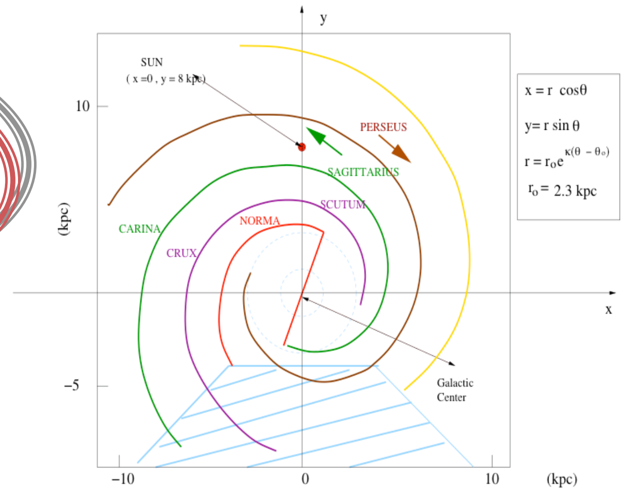
$$DM \propto \int n_e d\ell$$

$$\langle B_{\|}\rangle = \frac{RM}{DM}$$

Magnetized galaxies, clusters, and superclusters



SGP survey
Parkes



Magnetized
Domains (galactic wind)

$2\,\mu G$ $0.7\,\mu G$

Milky way: toroidal field south/north hemisphere; dipoles (center) $10\ rad/m^2$

Local Group: Andromeda, Magellanic Clouds,… $2 - 7\,\mu G$ (elliptical galaxies: shorter scale)

Abell Clusters (like COMA): magnetic fields inside cluster (VLA+ROSAT) [Faraday RM]
Typical RM: $100\ rad/m^2$ $B \sim 0.5\,\mu G = 500\,nG$ $L \sim 50 - 100\ kpc$

Superclusters: Local Supercluster (Local Group + Virgo Cluster $1.5\,\mu G$
Coma Supercluster (COMA+ Abell 1367) $0.5\,\mu G$?

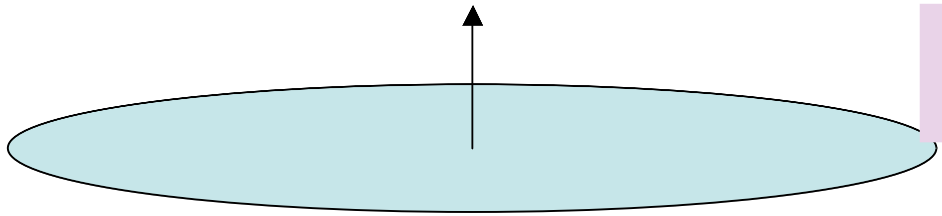
If true: important for UHECR… Hercules / Perseus-Pisces $B_L \simeq 0.5\mu G$ $L \simeq 500\ kpc$ Kronberg (2006)

$n_e \simeq 10^{-6}\ cm^{-3}$ GRG

Dynamo and compressional amplification

Galaxy:

$$\lambda_D \simeq \sqrt{\frac{T}{8\pi n_e e^2}}$$



Charged fluid (globally neutral)

Typical rotation period: $P \sim 3 \times 10^8\ yrs$ age $T \sim 10^{10}\ yrs$

Dynamo instability:

$$\alpha = -\frac{\tau_0}{3}\langle \vec{v} \cdot \vec{\nabla} \times \vec{v} \rangle$$

$$\sim 9.1 \times 10^6\ \frac{cm}{sec}$$

$$\frac{1}{4\pi\sigma} = 10^{25}\frac{cm^2}{sec}$$

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B}) + \frac{1}{4\pi\sigma}\nabla^2 \vec{B}$$

$$\frac{\partial < \vec{B} >}{\partial t} = \alpha \vec{\nabla} \times < \vec{B} > + \frac{1}{\sigma}\nabla^2 < \vec{B} >$$

$$1/k \sim L > kpc$$

Dynamo term Diffusivity term

Maximal and optimistic amplification: $e^{\Gamma t} \sim e^{T/P} \sim e^N \sim 10^{13}$

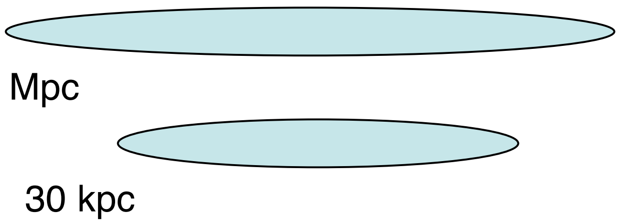
$B_i \sim 10^{-19}\ G$ Over L = 30 kpc

Compressional amplification:

$$B_b = \left(\frac{\rho_b}{\rho_a}\right)^{2/3} B_a$$

Clash: dynamo versus helicity conservation.
Brandenburg & Subramanian

$$\frac{d}{dt}\int_V d^3x \vec{A} \cdot \vec{B} = -\frac{1}{4\pi\sigma}\int_V d^3x \vec{B} \cdot \vec{\nabla} \times \vec{B} + O\left(\frac{1}{\sigma^2}\right)$$



$$B_i \geq 10^{-23}\ G \quad over \quad L \sim Mpc$$

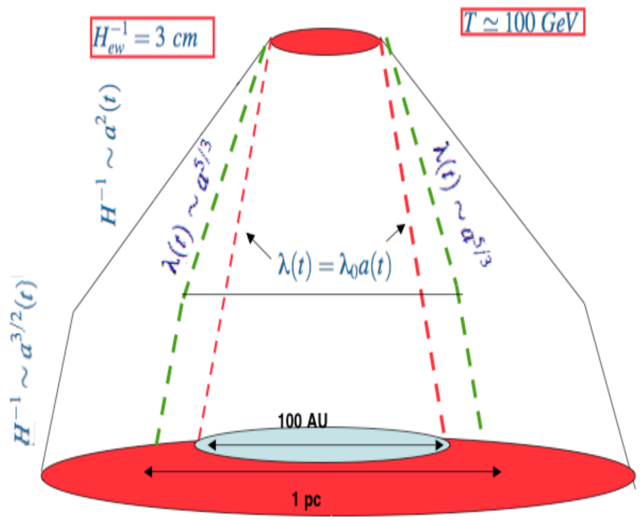
Primordial magnetogenesis

$B_{seed} > 10^{-23} G$ → Too optimistic

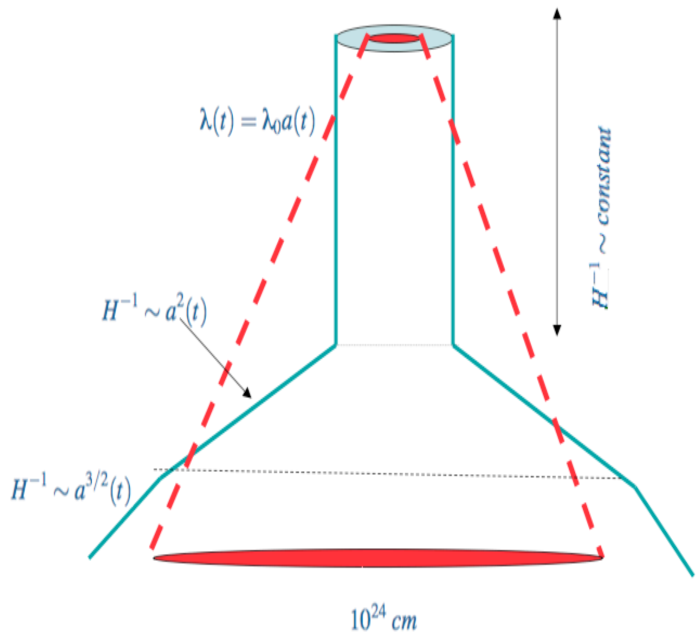
$B_{seed} > 10^{-18}\ G$ → More realistic [flux not exactly conserved, small-scale fields can grow large and swamp dynamo action]

effective e-folds 30->25

CAUSAL mechanisms



``Inflationary'' mechanisms



Magnetic fields and CMB physics

LET US SUPPOSE....

Uniform magnetic field approximation
[magnetic field along a specific axis].
Simplified estimates
[not so realistic in diverse cases]

FOREGROUNDS & B FIELDS

-- distorsion of the Planckian spectrum
-- shift of the polarization plane of CMB (Faraday rotation)
-- effects on primary anisotropies

Intermediate situation:
uniform magnetic field with
inhomogeneous fluctuations

Fully inhomogeneous
magnetic fields : more
realistic [mathematically
less tractable]

Plasma physics in FRW space-times

$$ds^2 = a^2(\tau)[d\tau^2 - d\vec{x}^2]$$

Kinetic (Vlasov-Landau) description

↓

Two-fluid plasma description

↓

One-fluid plasma description (MHD)

$$\lambda_D = \sqrt{\frac{T}{8\pi n_e e^2}}$$

$$\omega_p = \sqrt{\frac{4\pi n_e e^2}{m_e}}$$

Zeldovich approximation (1965)

Zeldovich ``approximation" : homogeneous field with (weak) breaking of spatial isotropy

Y. Zeldovich
Sov. Phys. JETP 21
656 (1965)

Magnetic fields weakly breaks spatial isotropy: Bianchi-type I paradigm

(generalizations MG PRD 2000)

$$ds^2 = dt^2 - a^2(t)dx^2 - b^2(t)[dy^2 + dz^2]$$

Electromagnetic radiation propagating along x and y will have a different temperature

$$T_x(t) = T_1 \frac{a_1}{a} = T_1 e^{-\int H(t)dt},$$

$$T_y(t) = T_1 \frac{b_1}{b} = T_1 e^{-\int F(t)dt}$$

→

$$\frac{\Delta T}{T} \sim \int [H(t) - F(t)]\, dt = \frac{1}{2} \int r(t)\, d\log t$$

$$r(t) = \frac{3[H(t) - F(t)]}{[H(t) + 2F(t)]}$$

Radiation-dominated case

Shear parameter is conserved and proportional to the magnetic energy density

From "Zeldovich" approximation"

$$\frac{B_0^2}{\rho_\gamma} \leq 10^{-6} \quad \rightarrow \quad B_0 \leq 2.23 \times 10^{-9} \; Gauss$$

More accurate estimates based on modified angular power spectrum lead to quantitatively similar estimates.

G. Chen, et al APJ (2004)

Faraday rotation by a UNIFORM magnetic field

From two-fluid description:

Kosowsky& Loeb ApJ (97)
MG PRD (97), MG(PRD,2005)

$$\Delta\phi = f_e \frac{e}{2m_e}\left(\frac{\omega_p}{\omega}\right)^2 (\vec{B}\cdot\hat{z})\delta z$$

$$\langle(\Delta\phi)^2\rangle^{1/2} \simeq 1.6^0 \left(\frac{B}{B_c}\right)\left(\frac{\omega_M}{\omega}\right)^2$$

$$B_c \sim 10^{-3}\ G$$

$$\omega_F = \frac{d\phi}{d\eta} = \frac{e^3\, n_e\, x_e \vec{B}\cdot\vec{q}}{8\pi^2\, m_e^2\, \nu^2}\frac{a}{a_0}$$

Axial symmetry around k, e.g. B II k (!)

$$\Delta_Q' + (ik\mu + \tau')\Delta_Q - 2\omega_F\Delta_U = \frac{\tau'}{2}[1 - P_2(\mu)]S_Q$$
$$\Delta_U' + (ik\mu + \tau')\Delta_U + 2\omega_F\Delta_Q = 0$$

$$\tau' = x_e\, n_e\, \sigma_T \frac{a}{a_0}$$

$$S_Q = \Delta_{I,2} + \Delta_{Q,0} + \Delta_{Q,2}$$

Visibility function

$$B_0 < 10^{-8}\ Gauss, \quad @\ \ 30\ GHz$$

$$(\Delta_Q \pm i\Delta_U) = \frac{3}{4}(1-\mu^2)\int_0^{\eta_0} d\eta e^{-ik\mu\Delta\eta} K(\eta)\, S_Q(\eta)\, e^{\mp 2i\omega_F\Delta\eta}$$

From WMAP TE correlations

E-modes are ROTATED into B-modes !

$$(\Delta_Q \pm i\Delta_U)(\hat{n}) = \sum_{\ell m} a_{\pm 2,\ell m}\ {}_{\pm 2}Y_{\ell m}(\hat{n})$$

$$a_{E,\ell m} = -\frac{1}{2}(a_{2,\ell m} + a_{-2,\ell m})$$
$$a_{B,\ell m} = \frac{i}{2}(a_{2,\ell m} - a_{-2,\ell m}).$$

$$E(\hat{n}) = \sum_{lm} a_{E,\ell m} Y_{lm}(\hat{n}), \qquad B(\hat{n}) = \sum_{lm} a_{B,\ell m} Y_{lm}(\hat{n}).$$

Fully inhomogeneous magnetic fields

$$\langle B_i(\vec{k},\tau)B^j(\vec{p},\tau)\rangle = \frac{2\pi^2}{k^3}P_i^j(k)\delta^{(3)}(\vec{k}+\vec{p})$$

$$B_i(\vec{k},\tau) = a^2(\tau)\mathcal{B}_i(\vec{k},\tau)$$

Spectral index

$$P_i^j(k) = P_B(k)\left(\delta_{ij} - \frac{k_ik_j}{k^2}\right) + iQ_B(k)\varepsilon_{ij\ell}\frac{k^\ell}{k}, \qquad P_B(k) = A_B\left(\frac{k}{k_p}\right)^{\varepsilon}, \qquad Q_B(k) = \tilde{A}_B k^{\tilde{\varepsilon}}$$

MHD approach:

$$\vec{J} = \frac{1}{4\pi}\vec{\nabla}\times\vec{B}, \qquad \vec{E} = \frac{\vec{\nabla}\times\vec{B}}{4\pi\sigma} - \vec{v}\times\vec{B}$$

$$\frac{1}{a^4}\vec{\nabla}\cdot[\vec{J}\times\vec{B}] = \frac{1}{4\pi a^4}\vec{\nabla}\cdot[(\vec{\nabla}\times\vec{B})\times\vec{B}]$$

Divergence of “Lorentz Force”

$$\delta_s\mathcal{T}_0^0 = \delta_s\rho_B, \qquad \delta_s\mathcal{T}_i^j = -\delta_s p_B\delta_i^j + \tilde{\Pi}_i^j$$

Not all independent !

$$\delta_s\rho_B = \frac{|B^2(\vec{x},\tau)|^2}{8\pi\, a^4(\tau)}, \qquad \delta_s p_B = \frac{\delta\rho_B}{3}$$

$$\tilde{\Pi}_i^j = \frac{1}{4\pi\, a^4(\tau)}\left[B_iB^j - \frac{B^2}{3}\delta_i^j\right]$$

Magnetic energy density and pressure

Anisotropic stress

Inhomogeneities in FRW models

$$\delta \mathcal{T}_{\mu\nu} = \delta \mathcal{T}^{(S)}_{\mu\nu} + \delta \mathcal{T}^{(V)}_{\mu\nu} + \delta \mathcal{T}^{(T)}_{\mu\nu}$$

$$g_{\mu\nu} \to g_{\mu\nu}(\tau) + \delta g_{\mu\nu}(\vec{x},\tau) \Longrightarrow \delta g_{\mu\nu}(\vec{x},\tau) = \delta_s g_{\mu\nu}(\vec{x},\tau) + \delta_v g_{\mu\nu}(\vec{x},\tau) + \delta_t g_{\mu\nu}(\vec{x},\tau)$$

10 degrees of freedom

$$ds^2 = a^2(\tau)[d\tau^2 - d\vec{x}^2]$$

$$\delta_s g_{\mu\nu}(\vec{x},\tau) = a^2(\tau)\begin{pmatrix} 2\phi & -\partial_i B \\ -\partial_i B & 2(\psi\delta_{ij} - \partial_i\partial_j E) \end{pmatrix}$$

→ **4 d. f.**

$$\delta_v g_{\mu\nu}(\vec{x},\tau) = a^2(\tau)\begin{pmatrix} 0 & -Q_i \\ -Q_i & \partial_i W_j + \partial_j W_i) \end{pmatrix}$$

→ **4 d. f.**

$$\partial_i Q^i = 0, \qquad \partial_i W^i = 0$$

$$\delta_t g_{\mu\nu}(\vec{x},\tau) = a^2(\tau)\begin{pmatrix} 0 & 0 \\ 0 & -h_{ij} \end{pmatrix}$$

→ **2 d. f.**

$$\partial_i h^i_j = 0, \qquad h^i_i = 0$$

1) Vector modes (easier)

2) Tensor modes (gauge-invariant)

3) Scalar modes (most complicated)

→ 4) CMB anisotropies induced by scalar modes

5) CMB polarization

Also covariant approach (C. Tsagas and collaborators)

Magnetized curvature perturbations

Choose a gauge (for instance conformally Newtonian)

$$\mathcal{H} = \frac{a'}{a}$$

$$\zeta = -\psi - \frac{\delta\rho_t + \delta\rho_B}{\rho_t'}\mathcal{H}$$

Hamiltonian constraint

Density contrast on uniform curvature hypersurfaces

$$\zeta = \mathcal{R} + \frac{\nabla^2\psi}{12\pi G a^2 (p_t + \rho_t)}$$

$$\mathcal{R} = -\psi - \frac{\mathcal{H}(\mathcal{H}\phi + \psi')}{\mathcal{H}^2 - \mathcal{H}'}$$

$$\zeta(k,\tau) \simeq \mathcal{R}(k,\tau) + \mathcal{O}(|k\tau|^2)$$

Curvature fluctuations on comoving orthogonal hypersurfaces

$\rho_t = \rho_\gamma + \rho_\nu + \rho_b + \rho_c$ $+\Lambda$ (does not fluctuate)

Photons

neutrinos

Baryons

CDM

$$\delta\rho_t = \delta\rho_\gamma + \delta\rho_\nu + \delta\rho_b + \delta\rho_c$$

Evolution equations

Photons and baryons : tightly coupled at early times

$$\theta_\gamma \simeq \theta_b = \theta_{\gamma b}$$

$$\theta'_{\gamma b} + \frac{\mathcal{H} R_b}{(1+R_b)}\theta_{\gamma b} + \frac{\nabla^2\delta_\gamma}{4(1+R_b)} + \nabla^2\phi = \frac{3}{4}\frac{\vec{\nabla}\cdot[\vec{J}\times\vec{B}]}{a^4\rho_\gamma(1+R_b)}$$

$$\delta_\gamma = \frac{\delta\rho_\gamma}{\rho_\gamma} \qquad \theta_{\gamma b} = \partial_i v^i_{\gamma b}$$

$$R_b = \left(\frac{698}{z+1}\right)\left(\frac{\omega_b}{0.023}\right)\left(\frac{\omega_\gamma}{2.47\times 10^{-5}}\right)^{-1}$$

Neutrinos : collisionless below 1 MeV

$$\theta'_\nu + \frac{1}{4}\nabla^2\delta_\nu + \nabla^2\phi = \nabla^2\sigma_\nu, \qquad \delta'_\nu = 4\psi' - \frac{4}{3}\theta_\nu, \qquad \sigma'_\nu = \frac{4}{15}$$

$$\delta_\nu = \frac{\delta\rho_\nu}{\rho_\nu} \qquad \theta_\nu = \partial_i v^i_\nu$$

CDM : only coupled through metric fluctuations

$$\theta'_c + \mathcal{H}\theta_c + \nabla^2\phi = 0, \qquad \delta'_c = 3\psi' - \theta_c, \qquad \delta_c = \frac{\delta\rho_c}{\rho_c}$$

$$\theta_c = \partial_i v^i_c$$

+ PERTURBED EINSTEIN EQUATIONS COUPLING PLASMA & MAGNETIC FIELDS

Anisotropic stress: important aspect (neutrinos + magnetic fields)

$$\nabla^4(\phi-\psi) = 12\pi G a^2[(p_\nu+\rho_\nu)\nabla^2\sigma_\nu + (p_\gamma+\rho_\gamma)\nabla^2\sigma_B]$$

$$\nabla^2\sigma_B = \frac{3}{16\pi a^4\rho_\gamma}\vec{\nabla}\cdot[(\vec{\nabla}\times\vec{B})\times\vec{B}] + \frac{\nabla^2\Omega_B}{4}$$

$$\Omega_B(\vec{x}) = \frac{\delta\rho_B(\tau,\vec{x})}{\rho_\gamma(\tau)}$$

Magnetized adiabatic and non-adiabatic modes

- Solve everything consistently for PHOTONS + NEUTRINOS + BARYONS +CDM (boring but doable)

-Magnetic fields modify qualitatively and quantitatively the nature of the behaviour of the various modes

M.G. PRD 2004

- MOST GENERAL solution includes FIVE MODES

ONE Magnetized adiabatic mode

$$\varsigma = \frac{T^3}{n_c}$$

$$\mathcal{S} = \frac{\delta\varsigma}{\varsigma} = \frac{3}{4}\delta_r - \delta_c$$

$$\delta_\gamma \simeq \delta_\nu = \frac{4}{3}\delta_c$$

Verified for adiabatic mode
If $k\tau \ll 1$

FOUR Magnetized non-adiabatic modes

Baryon isocurvature mode
CDM isocurvature mode
Neutrino isocurvature velocity mode
Neutrino isocurvature density mode

[Some of isocurvature modes are singular on the longitudinal gauge: go to synchronous gauge]

Remarks:

-Since neutrinos free stream (unlike photons) we treat them through an ``improved” fluid system where the quadrupole and octupole moments of the neutrino phase space distribution are dynamical.

-The five magnetized solution define the correct initial conditions to be imposed on the lowest multipoles of the Boltzmann hierachies.

Magnetized adiabatic mode

$$|k\tau| \ll 1$$

$$\delta_\gamma = \delta_\nu = -2\phi_i - R_\gamma \Omega_B, \qquad \delta_b = \delta_c = -\frac{3}{2}\phi_i - \frac{3}{4} R_\gamma \Omega_B$$

Density contrasts

$$\theta_{\gamma b} = \frac{k^2\tau}{4}[2\phi_i + R_\nu \Omega_B - 4\sigma_B], \qquad \theta_c = \frac{k^2\tau}{2}\phi_i, \qquad \theta_\nu = \frac{k^2\tau}{2}\left[\phi_i - \frac{R_\gamma \Omega_B}{2}\right] + k^2\tau \frac{R_\gamma}{R_\nu}\sigma_B$$

Peculiar velocities

$$\psi_i = \phi_i\left(1 + \frac{2}{5}R_\nu\right) + \frac{R_\gamma}{5}(4\sigma_B - R_\nu \Omega_B), \qquad \sigma_\nu = -\frac{R_\gamma}{R_\nu}\sigma_B + \frac{k^2\tau^2}{6R_\nu}(\psi_i - \phi_i).$$

Metric variables & quadrupole moment of neutrino phase space distribution

Notation

$$R_\gamma = 1 - R_\nu, \qquad R_\nu = \frac{r}{1+r}, \qquad r = \frac{7}{8}N_\nu \left(\frac{4}{11}\right)^{4/3} \equiv 0.681\left(\frac{N_\nu}{3}\right)$$

Angular power spectrum

$$k_L = 1\ \mathrm{Mpc}^{-1}, \qquad k_p = 0.002\ \mathrm{Mpc}^{-1}$$

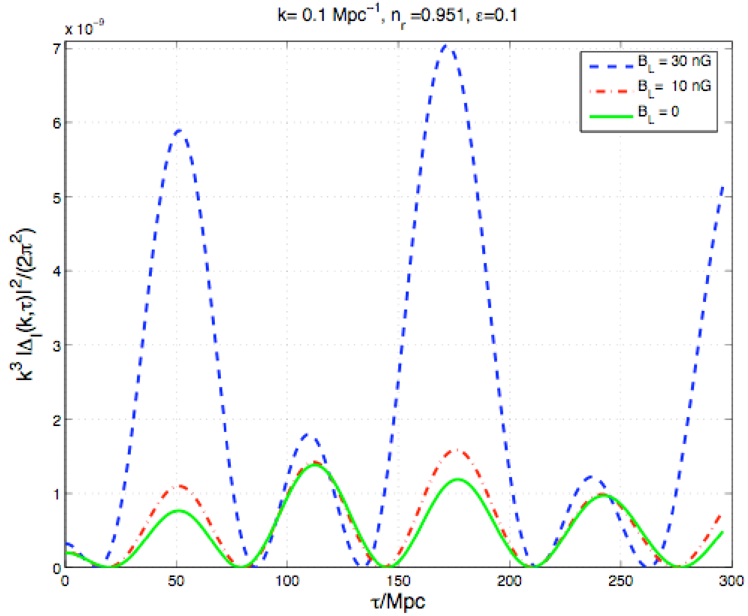
$$C_\ell = \Bigg[\frac{A_\zeta}{25} Z_1(n_r,\ell) + \frac{9}{100} R_\gamma^2 \overline{\Omega}_{BL}^2 Z_2(\varepsilon,\ell) - \frac{4}{25}\sqrt{A_\zeta A_S} Z_1(n_{rs},\ell)\cos\gamma_{rs}$$
$$+ \frac{4}{25} A_S Z_1(n_s,\ell) - \frac{3}{25}\sqrt{A_\zeta} R_\gamma \overline{\Omega}_{BL} Z_3(n_r,\varepsilon,\ell)\cos\gamma_{br}$$
$$+ \frac{6}{25}\sqrt{A_S} R_\gamma \overline{\Omega}_{BL} Z_3(n_s,\varepsilon,\ell)\cos\gamma_{bs}\Bigg],$$

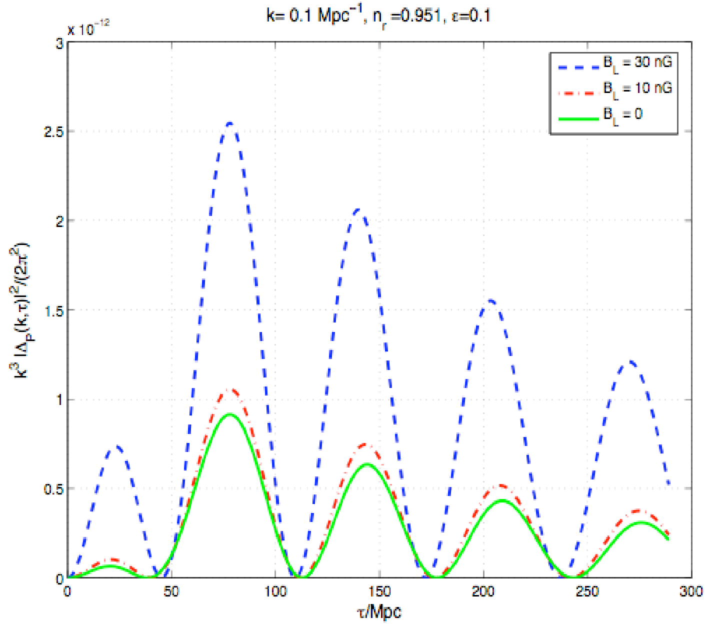
$$Z_1(n,\ell) = \frac{\pi^2}{4}\left(\frac{k_0}{k_p}\right)^{n-1} 2^n \frac{\Gamma(3-n)\Gamma\left(\ell+\frac{n-1}{2}\right)}{\Gamma^2\left(2-\frac{n}{2}\right)\Gamma\left(\ell+\frac{5}{2}-\frac{n}{2}\right)},$$

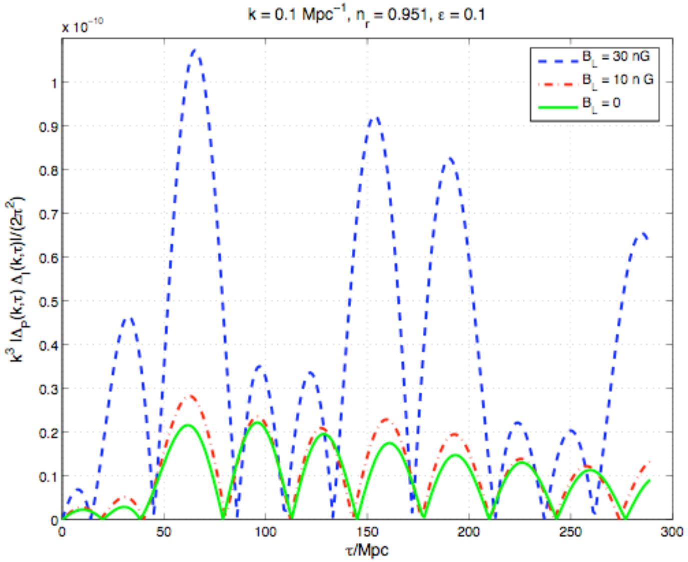
$$Z_2(\varepsilon,\ell) = \frac{\pi^2}{2} 2^{2\varepsilon} F(\varepsilon)\left(\frac{k_0}{k_L}\right)^{2\varepsilon} \frac{\Gamma(2-2\varepsilon)\Gamma(\ell+\varepsilon)}{\Gamma^2\left(\frac{3}{2}-\varepsilon\right)\Gamma(\ell+2-\varepsilon)},$$

$$Z_3(n,\varepsilon,\ell) = \frac{\pi^2}{4} 2^{\varepsilon} 2^{\frac{n+1}{2}} \sqrt{F(\varepsilon)}\left(\frac{k_0}{k_L}\right)^{\varepsilon}\left(\frac{k_0}{k_p}\right)^{\frac{n+1}{2}} \frac{\Gamma\left(\frac{5}{2}-\varepsilon-\frac{n}{2}\right)\Gamma\left(\ell+\frac{\varepsilon}{2}+\frac{n}{4}-\frac{1}{4}\right)}{\Gamma^2\left(\frac{7}{4}-\frac{\varepsilon}{2}-\frac{n}{4}\right)\Gamma\left(\frac{9}{4}+\ell-\frac{\varepsilon}{2}-\frac{n}{4}\right)},$$

Numerical solutions: magnetized adiabatic mode/1







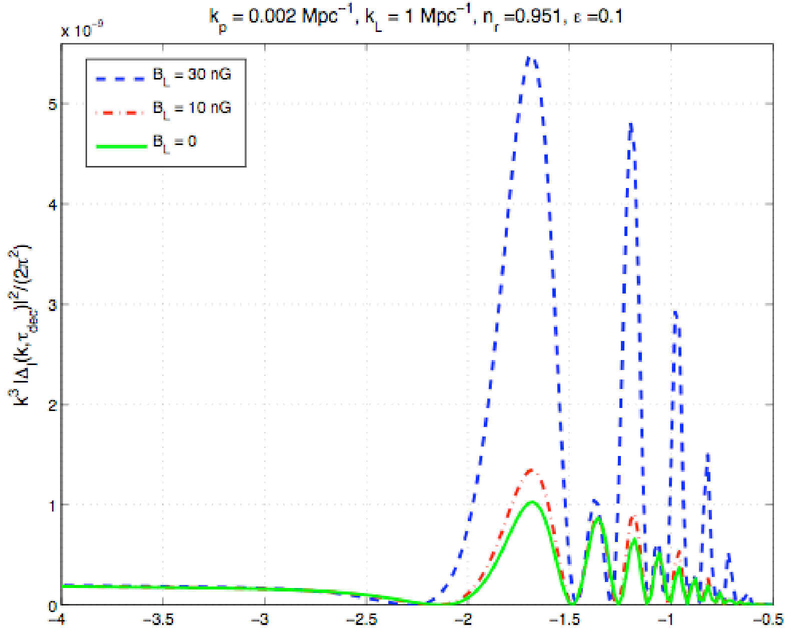
B_L Magnetic field smoothed over typical scale L

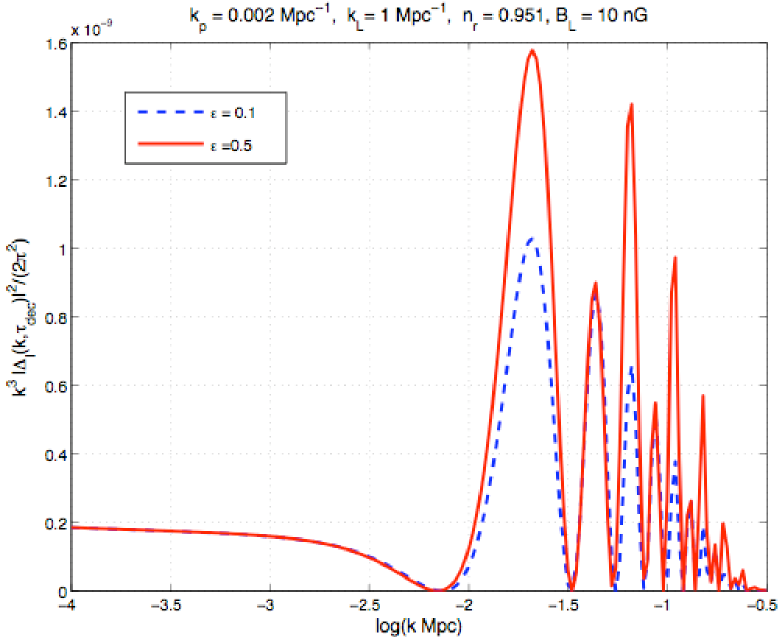
ε Spectral index

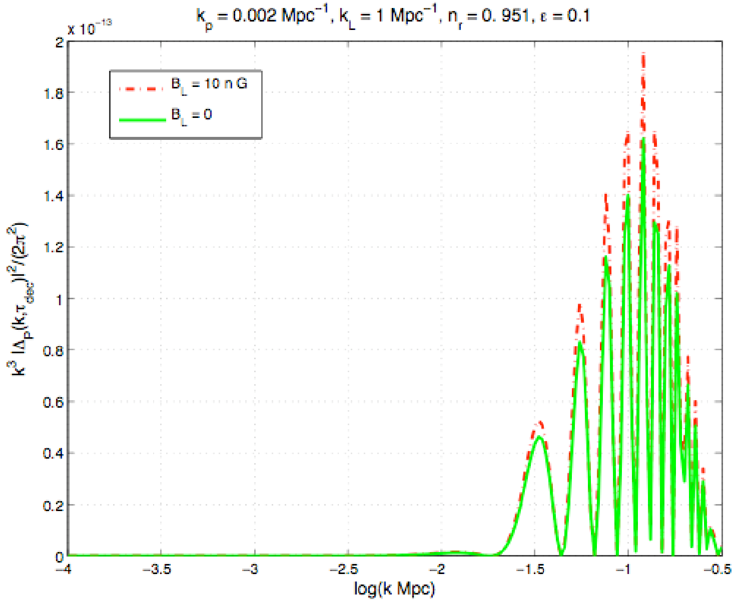
$\omega_\gamma = 2.47 \times 10^{-5}$ $\omega_c = 0.111$

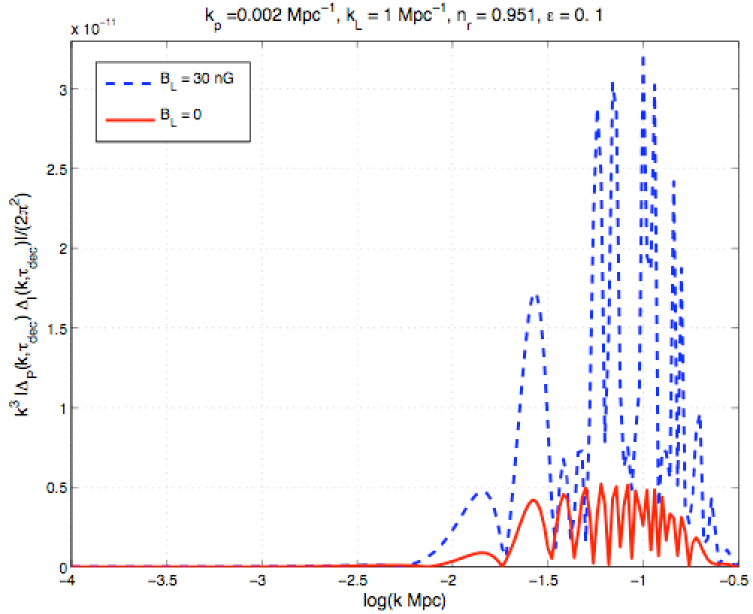
$\omega_b = 0.023$ $h = 0.73$ $N_\nu = 3$

Numerical solutions: magnetized adiabatic mode/2

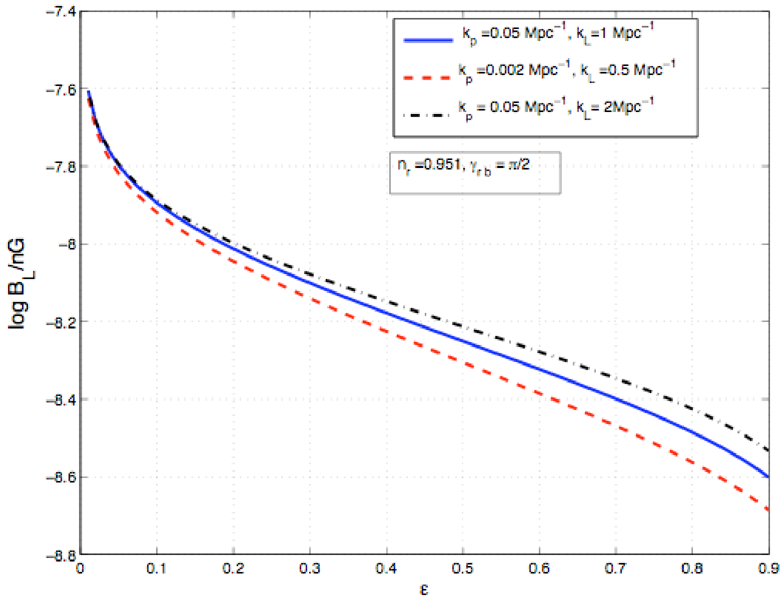


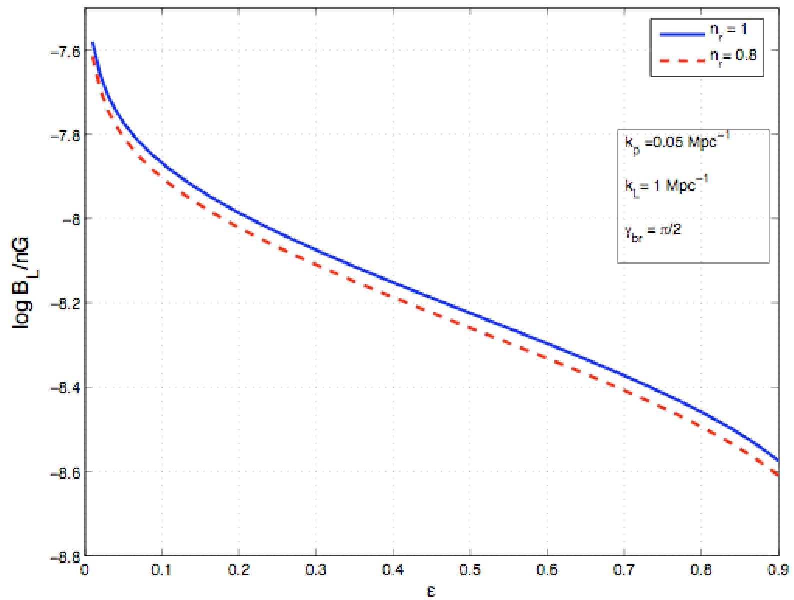


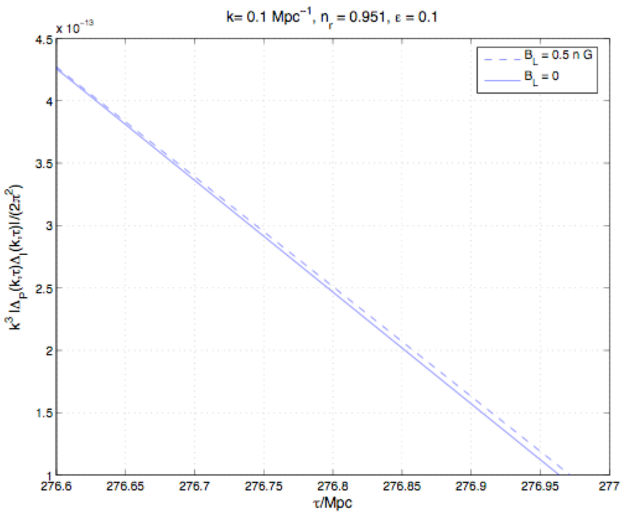


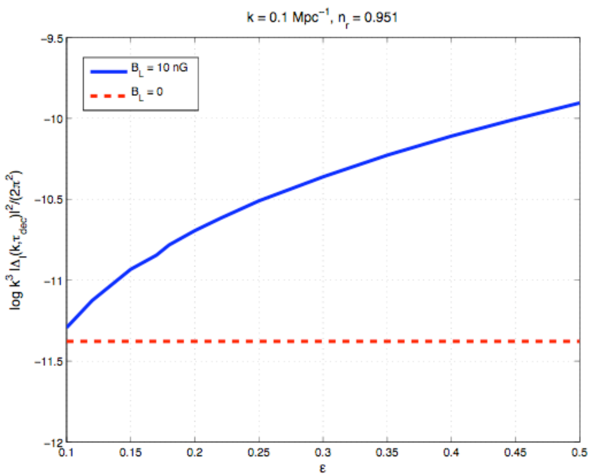


Constraints









Final remarks

Large scale magnetic fields (LSMF) exist in nature

Their origin is still debatable

Primordial or not primordial?

CMB as a tool to scrutinize the origin of LSMF

Redundant set of informations : Faraday effect, Primary anisotropies, vector and tensor modes...

New formalism to describe magnetized (scalar) CMB anisotropies.

LSMF can be as large as 0.1 nG over Mpc