

# Why is the WMAP quadrupole power small ?

$$\Delta T^2 = C_2^3 / \pi \Rightarrow 1250 (\mu K)^2$$

$$\text{COBE} + \text{WMAP} \Rightarrow 256 (\mu K)^2$$

Sig .  $\approx 5\%$

-0.02

+0.02

Pavel Naselsky

Niels Bohr Institute , Denmark

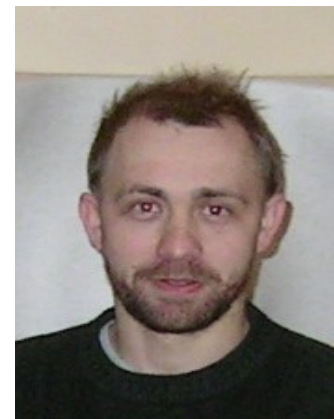
# Collaborators within DK-*Planck* community



**Igor D. Novikov ( NBI)**



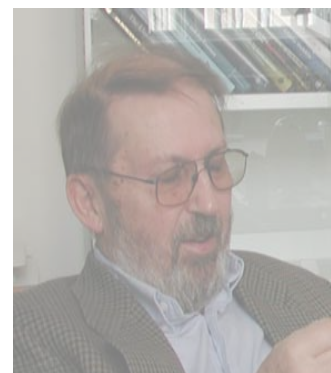
**Andrei Doroshkevich  
(ASC FIRAN)**



**Oleg V. Verhodonov  
(SAO RAN)**



**Lung-Yih Chiang (NBI)**



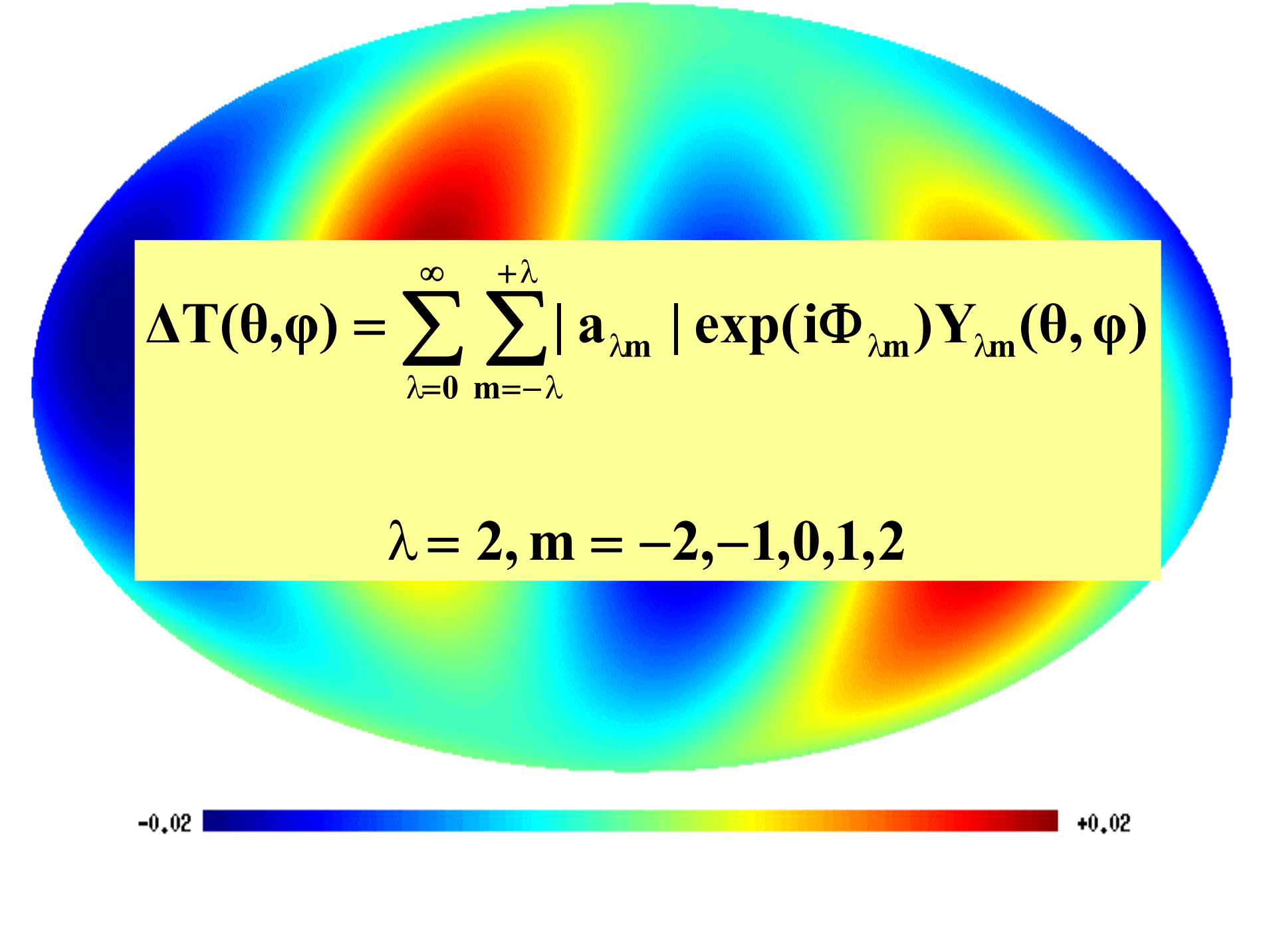
**Per Rex Christensen (NBI)**

AND with thanks to



"What really interests me is whether God had any choice when he created the World"

A. Einstein



$$\Delta T(\theta, \varphi) = \sum_{\lambda=0}^{\infty} \sum_{m=-\lambda}^{+\lambda} |a_{\lambda m}| \exp(i\Phi_{\lambda m}) Y_{\lambda m}(\theta, \varphi)$$

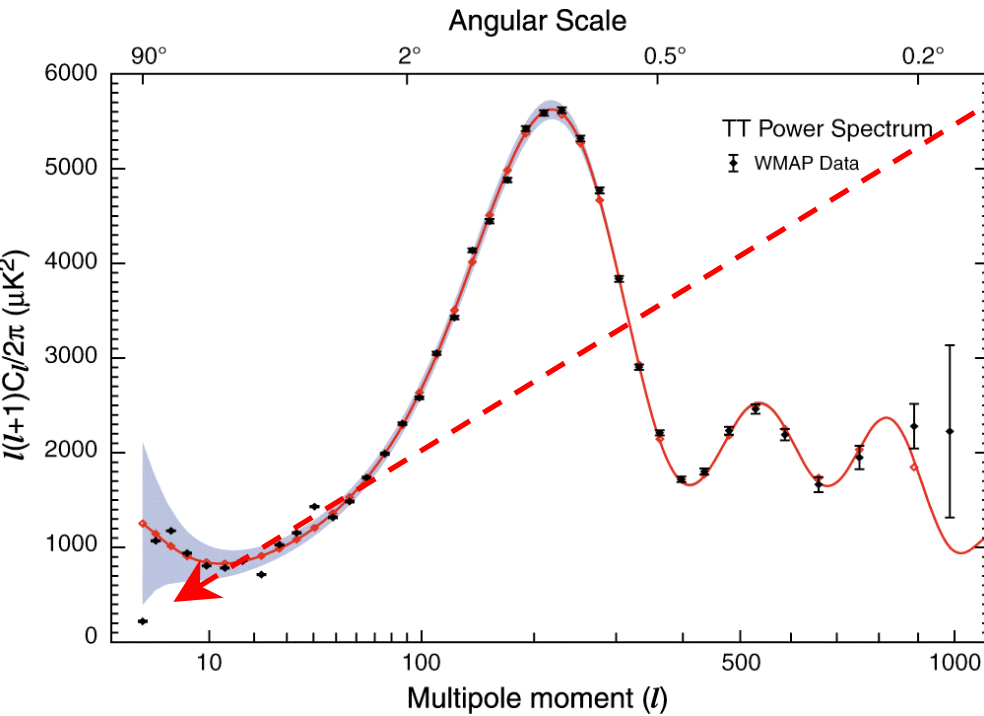
$$\lambda = 2, m = -2, -1, 0, 1, 2$$

-0,02



+0,02

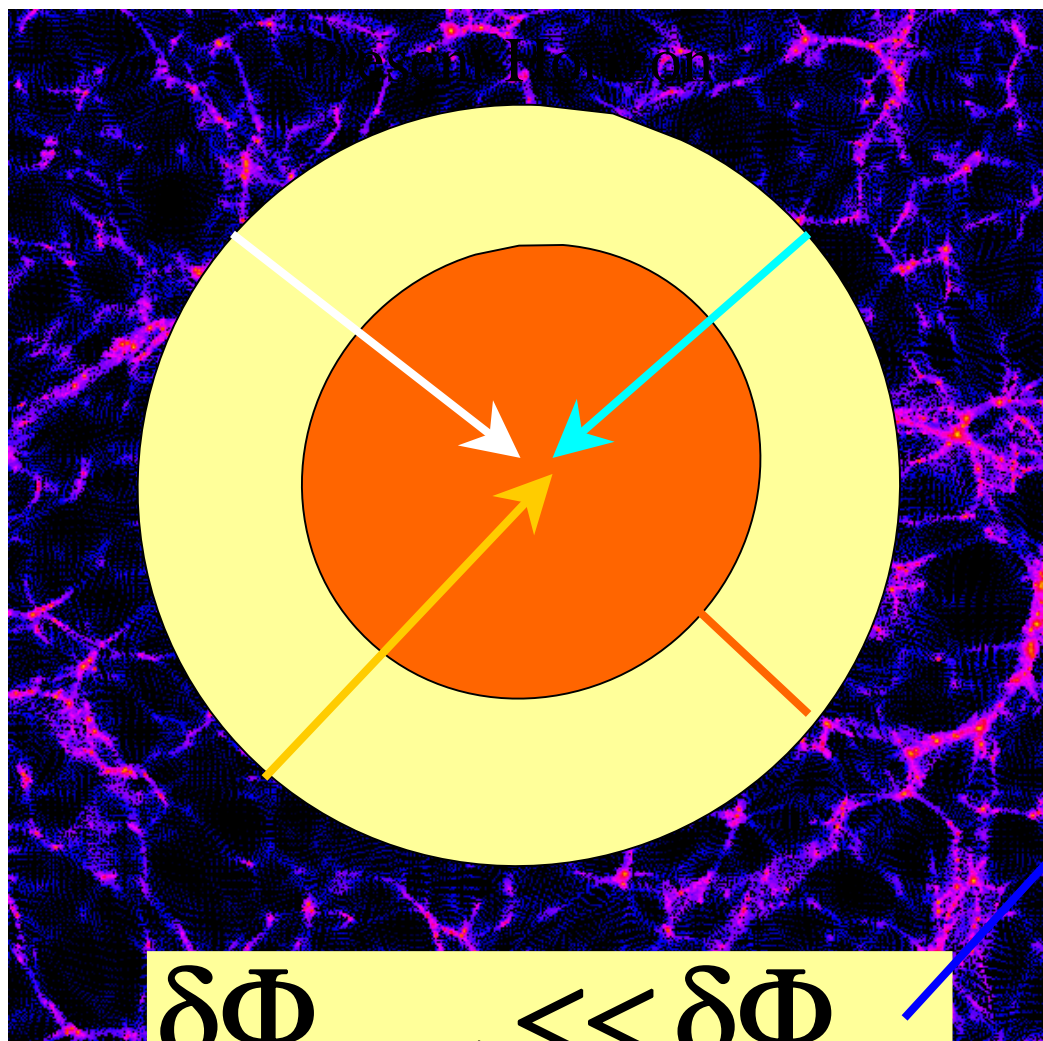
# WHY Quadrupole ?



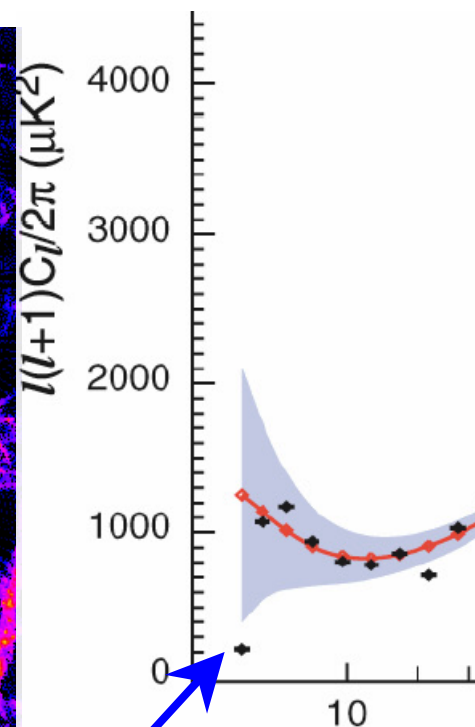
**1. Direct test on dark energy and  
Its interaction with dark matter**



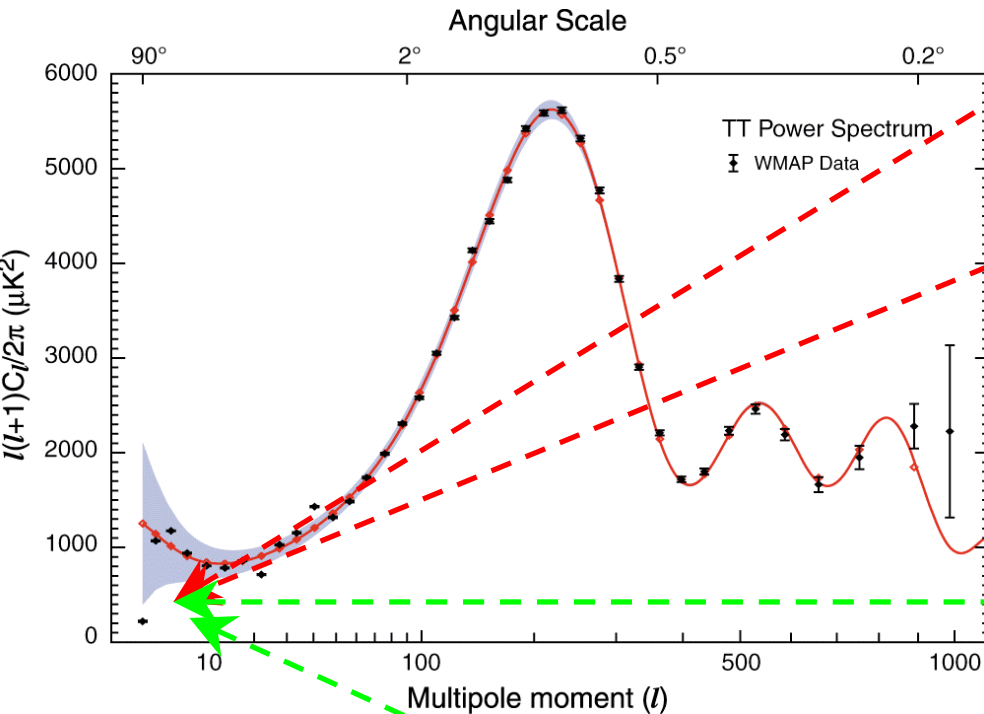
$$\delta\Phi_{\text{out}} > \delta\Phi_{\text{in}}$$



$$\delta\Phi_{\text{out}} \ll \delta\Phi_{\text{in}}$$



# WHY Quadrupole ?



1. Direct test on dark energy and its interaction with dark matter
2. Direct test on the expansion law between recombination and reionization
3. Test on the early stage of inflation



**BSI (peculiarities of  $V$ )**

## 4. Non-trivial topology

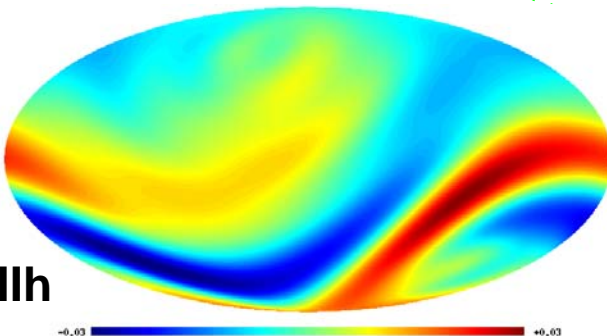
The end of the horn is infinite in length, but incredibly narrow

If you travelled out of the flared end of the horn, you would find yourself travelling back in on the opposite side

TOO DISTANT TO OBSERVE

OBSERVABLE UNIVERSE

## 5. Direct test on statistical isotropy



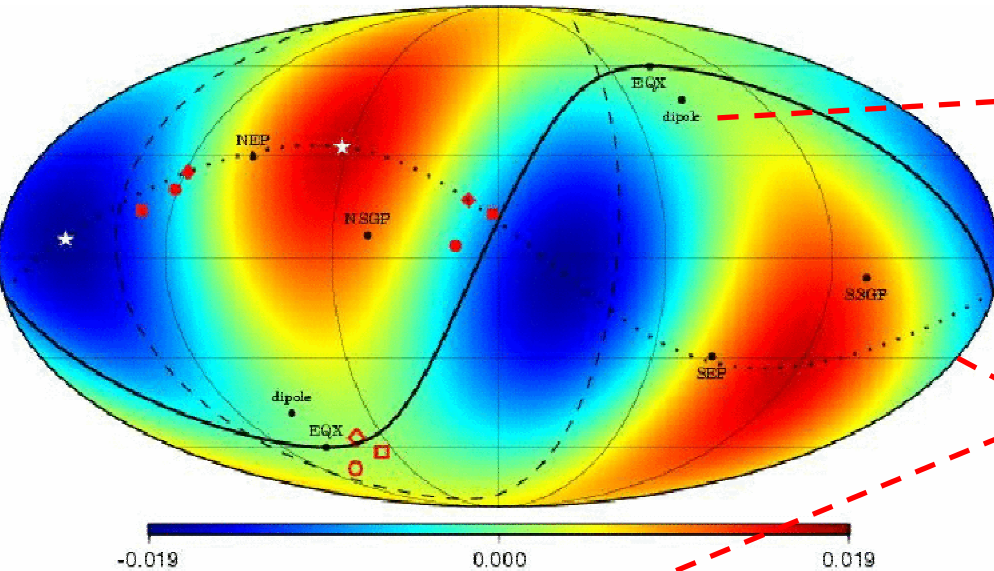
**Bianchi VIIh**



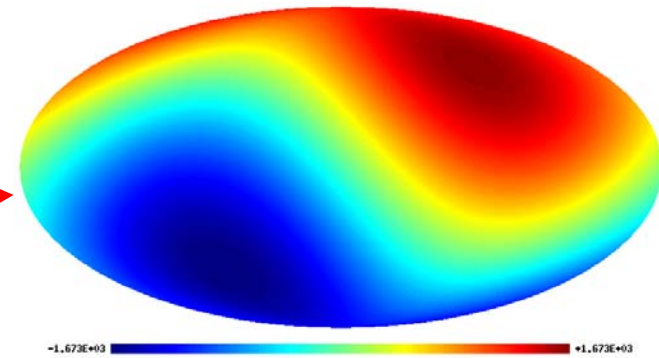
# WHY Quadrupole ?

## Correlations with Galactic structures

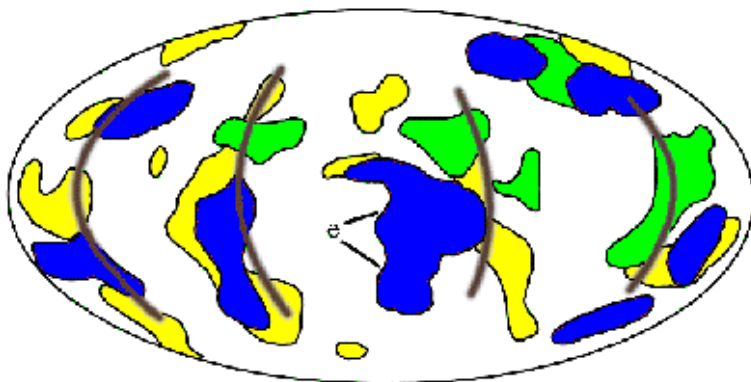
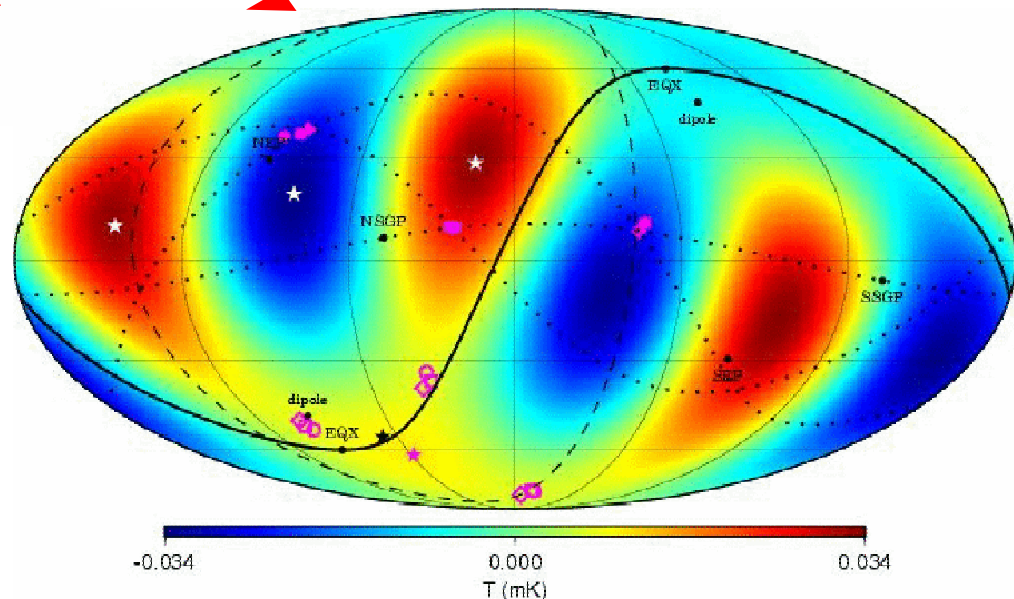
(multipole vector analysis, Copi et al, 2004, 2006)



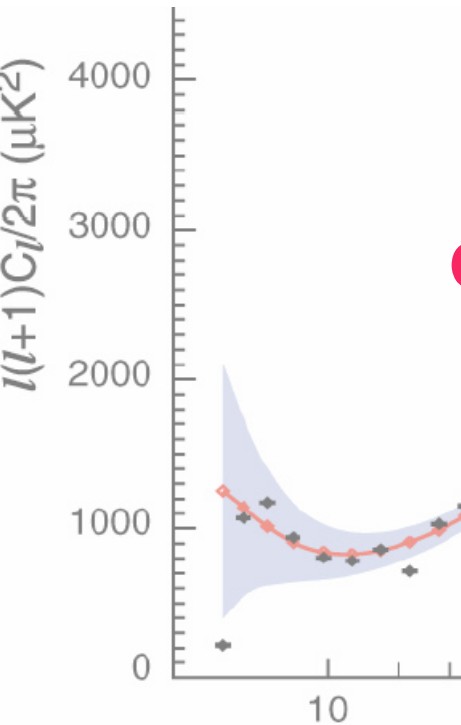
dipole



Q-Oct correlation



# Non-Gaussianity and (or) statistical anisotropy ?



**OK ! The power of the quadrupole is small.**

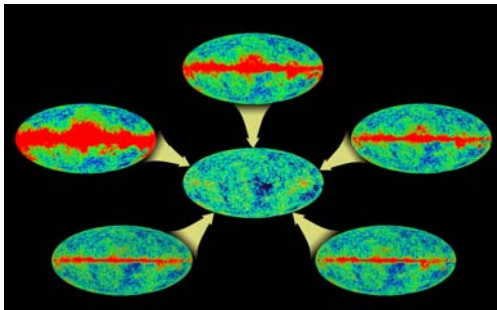
**1. Is the quadrupole some peculiar realization of the Gaussian random process ? YES !**

**a) Quantum (Gaussian) fluctuations are still OK!**

**b) Standard model of inflation is OK !**

**c) Methods of the CMB-Foreground separation are OK !**

**Minimal Variance Method**

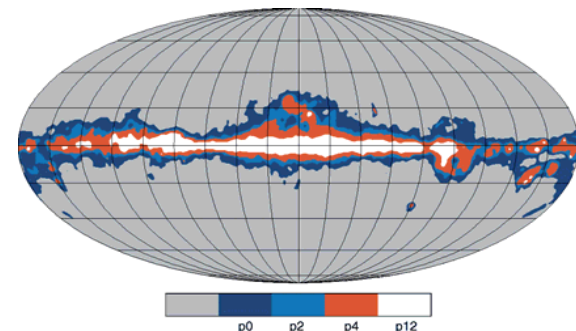


$$\Delta T(\theta_p, \varphi_p) = \sum_{j=1}^N \omega_j T_j(\theta_p, \varphi_p)$$

$$\text{Var}(\Delta T(\theta_p, \varphi_p)) \Rightarrow \min(\omega_j)$$

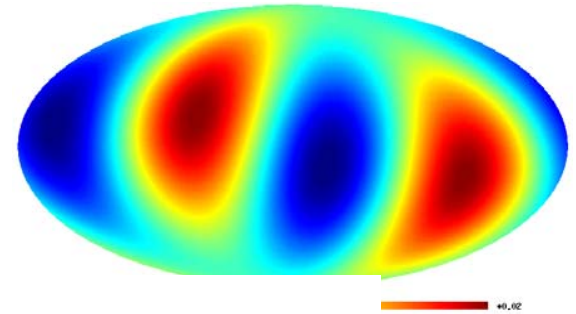
$$\sum_{j=1}^N \omega_j = 1$$

**MASK**



OK ! The power of the quadrupole is small.

Is the WMAP quadrupole non-Gaussian ? Yes !



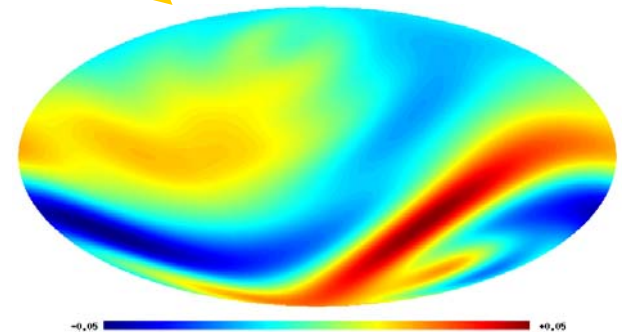
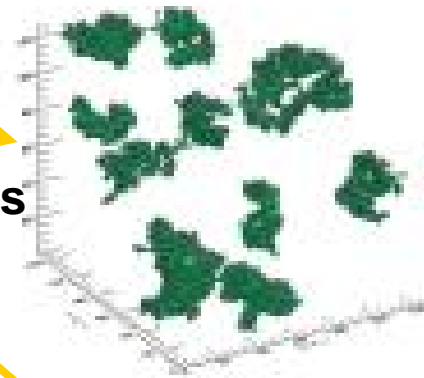
## Link with new physics

Order of quantum fluctuations at Super Large Scales  
and chaos at small scales ?

Bianchi VIIh type statistical anisotropy ?

MVM is no longer available

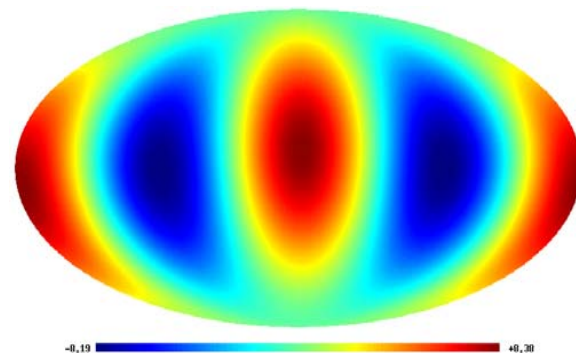
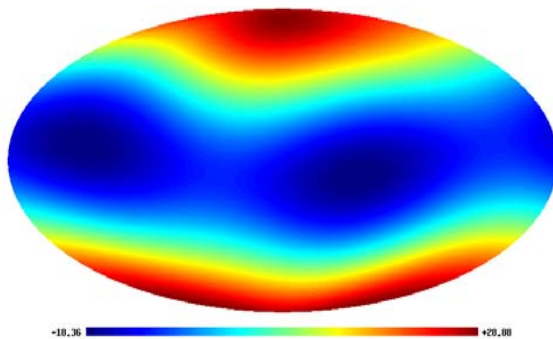
Galactic masks are not available



# HOW we may test statistical properties of quadrupole ?

## Problems

- Statistically not well established (the only 5 numbers !)
- The only one realization on the radio sky
- Very high level of uncertainties of the power spectrum (“cosmic variance”)



# Quadrupole components

$$a(2,m) = \left\{ \begin{array}{c} a(2,0) \\ a(2,1) \\ a(2,-1) \\ a(2,2) \\ a(2,-2) \end{array} \right\}$$

For Gaussian random field  $P[a(2,m)]$  uniformly distributed over  $m$  !

# Amplitudes and Phases

$$\mathbf{a}_{lm} = |\mathbf{a}_{l,m}| \exp(i\Phi_{l,m})$$

$$C_\lambda = \frac{1}{2\lambda+1} \sum_m |\mathbf{a}_{\lambda m}|^2$$

$$C_2 = \frac{1}{5} |\mathbf{a}_{20}|^2 + \frac{2}{5} |\mathbf{a}_{21}|^2 + \frac{2}{5} |\mathbf{a}_{22}|^2$$

“Naive” expectations

$$\frac{1}{5} |\mathbf{a}_{20}|^2 \sim \frac{2}{5} |\mathbf{a}_{21}|^2 \sim \frac{2}{5} |\mathbf{a}_{22}|^2$$

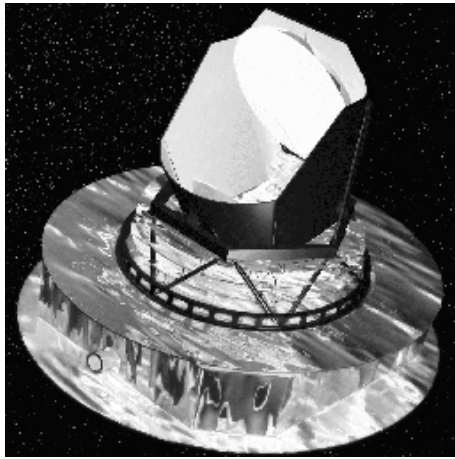
The WMAP reality

$$|\mathbf{a}_{20}| = 0.0115\text{mK};$$

$$|\mathbf{a}_{21}| = 0.00486\text{mK} \sim 0.2 |\mathbf{a}_{22}|$$

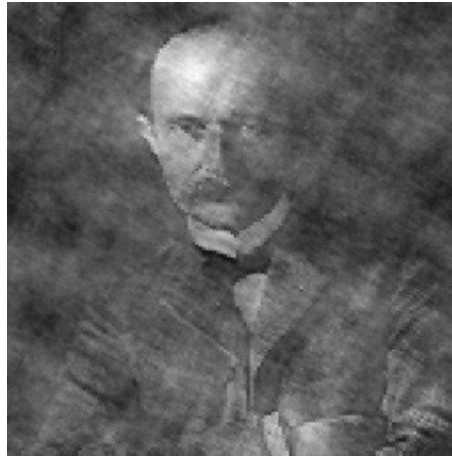
$$|\mathbf{a}_{22}| = 0.0236\text{mK} \sim 2 |\mathbf{a}_{20}|$$





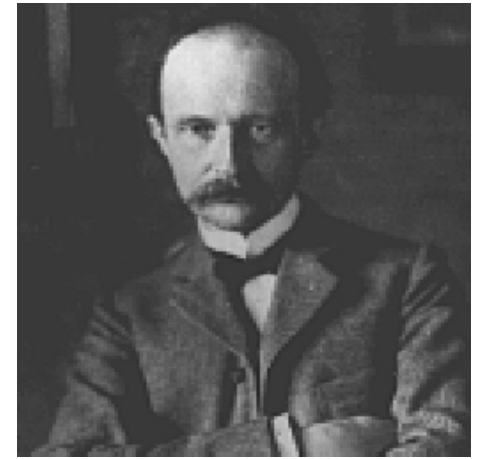
Planck satellite

$$|\delta k| \exp(i\Phi_k)$$



transformed Planck

$$\text{FT}^{-1}[|\delta k| \exp(i\Phi_k)]$$



Max Planck

$$|\delta k| \exp(i\Phi_k)$$

Planck satellite and transformed Planck have the same power spectrum (same  $|\delta k|$ ), they have different “faces” due to different phases:

It is phase  $\Phi_k$  that keep Max’s face, not amplitude  $|\delta k|$  !!

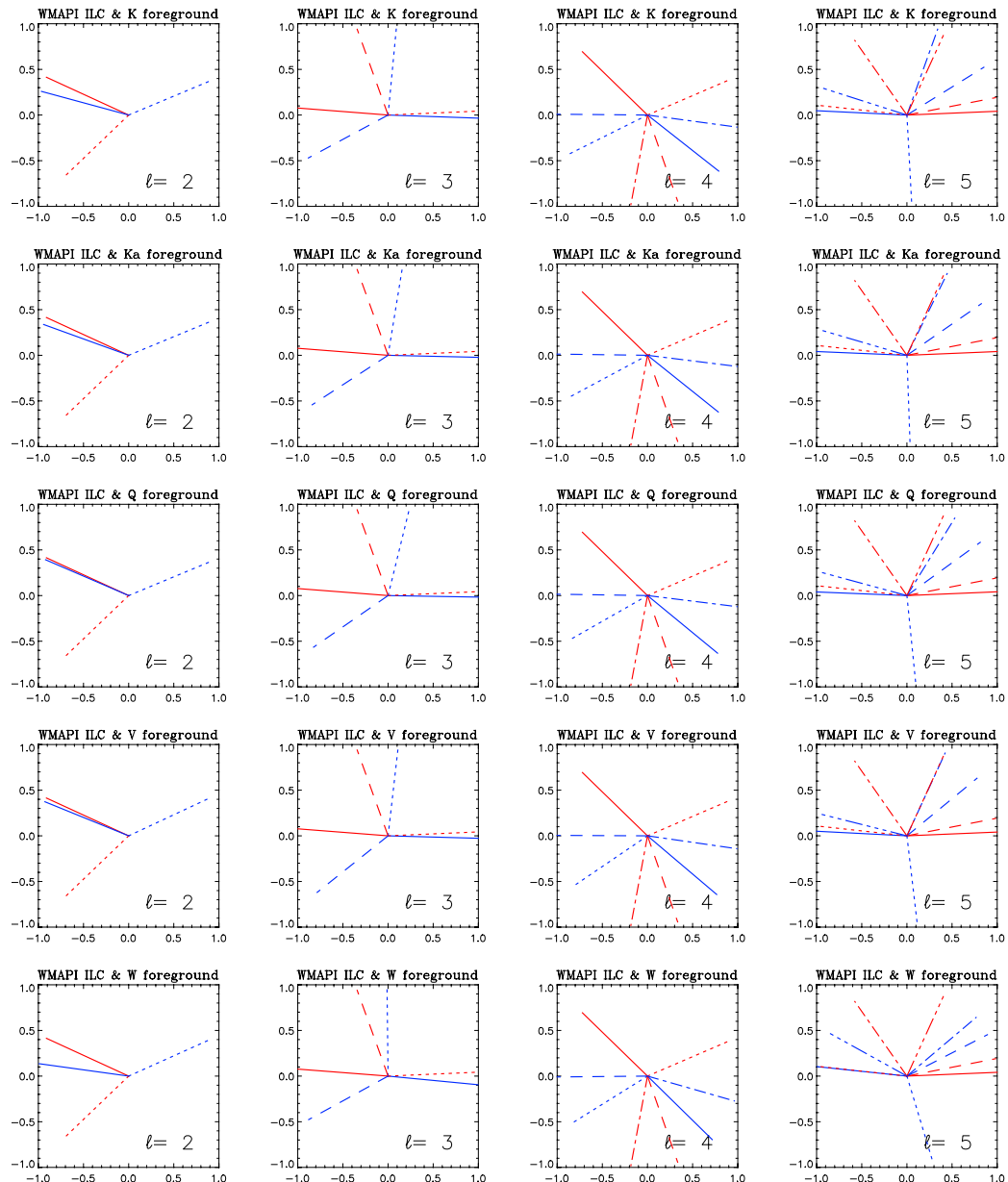
- 1. If CMB is Gaussian, the phases should be non-correlated and uniformly distributed at  $[0, 2\pi]$  for any cosmological models ( $\Omega_{\text{tot}}, \Omega_{\text{b}}, \Omega_{\Lambda}, \dots$ )**
- 2. If the topology of the Universe is non-trivial, the phases of spherical harmonics decomposition should have some auto-correlations.**
- 3. If foregrounds and CMB are separated correctly, their phases should cross-correlate as minimal as possible.**

# Analysis of CMB-Foreground coupling through phases

ILC (I)

For  $m=1$

$$\xi_{L1} - \Phi_{L1} \Rightarrow (0, \pi)$$



# Analysis of CMB-Foreground coupling through phases

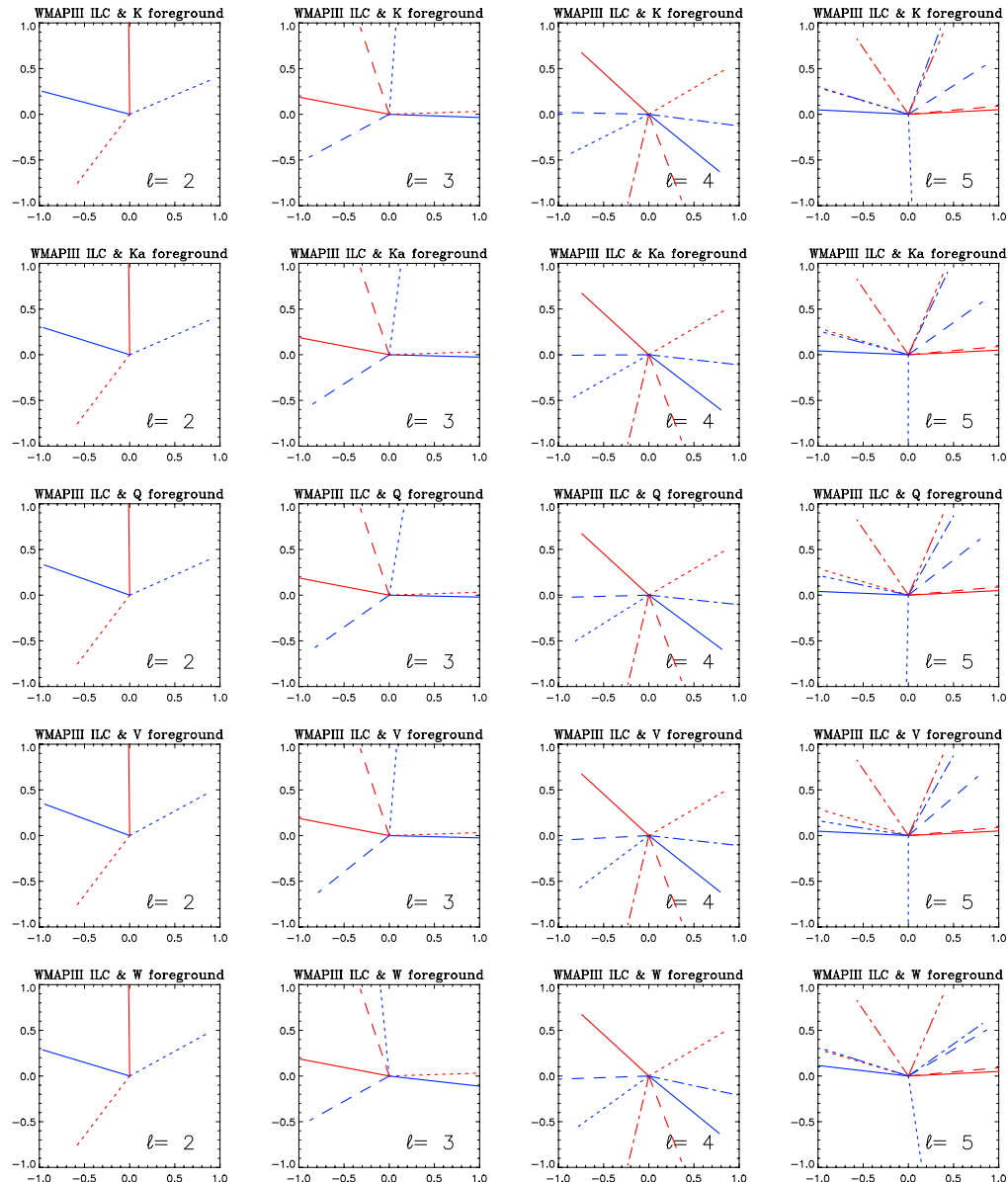
ILC (III)

For  $l+m=\text{even}$

$$\xi_{lm} - \Phi_{lm} \Rightarrow (0, \pi)$$

For  $l+m=\text{odd}$

$$\xi_{lm} - \Phi_{lm} \Rightarrow (\pi/2, 3\pi/2)$$



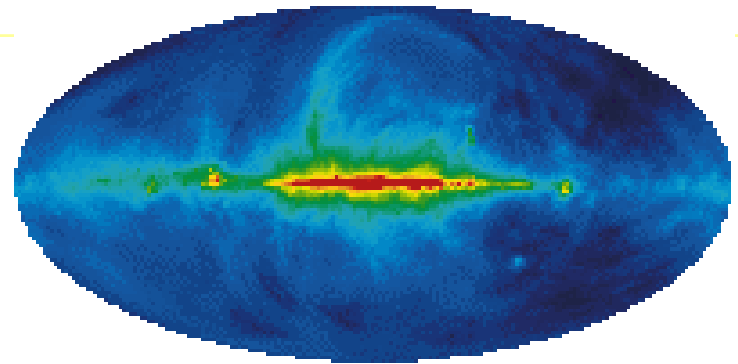
# Link to the MVM. Statistical independency

$$C_\lambda = \frac{1}{2\lambda+1} \sum_{m=-l}^l |c_{l,m}| |F_{l,m}| \cos(\xi_{l,m} - \Phi_{l,m}) \Rightarrow (l=2)$$

$$\frac{1}{5} |c_{20}| |F_{20}| \cos(\xi_{20} - \Phi_{20}) + \frac{2}{5} |c_{21}| |F_{21}| \cos(\xi_{21} - \Phi_{21}) + \frac{2}{5} |c_{22}| |F_{22}| \cos(\xi_{22} - \Phi_{22}) = 0$$

$0, \pi$

$$|F_{20}| \geq |F_{22}| \gg |F_{21}|$$



$$\cos(\xi_{22} - \Phi_{22}) \sim \frac{|\mathbf{c}_{20}| |\mathbf{F}_{20}|}{|\mathbf{c}_{22}| |\mathbf{F}_{22}|} (\pm 1) \Rightarrow$$

$$|\mathbf{c}_{20}| \sim \frac{|\mathbf{F}_{22}| |\mathbf{c}_{22}|}{|\mathbf{F}_{20}|} < |\mathbf{c}_{22}|$$

$$\xi_{22} - \Phi_{22} = (0, \pi) + \beta,$$

$$\beta \ll \pi / 2$$

$$\cos(\xi_{21} - \Phi_{21}) \sim \frac{1}{2} \beta^2 \frac{|\mathbf{F}_{22}| |\mathbf{c}_{22}|}{|\mathbf{F}_{21}| |\mathbf{c}_{21}|} \Rightarrow$$

$$\xi_{21} - \Phi_{21} \sim \pi / 2 + \frac{1}{2} \beta^2 \frac{|\mathbf{F}_{22}| |\mathbf{c}_{22}|}{|\mathbf{F}_{21}| |\mathbf{c}_{21}|}$$



## Statistical independency (predictions)

$$|c_{21}| \ll |c_{22}|, |c_{20}| \quad \xi_{22} - \Phi_{22} \Rightarrow (0, \pi) \quad \xi_{21} - \Phi_{21} \sim \pi/2$$

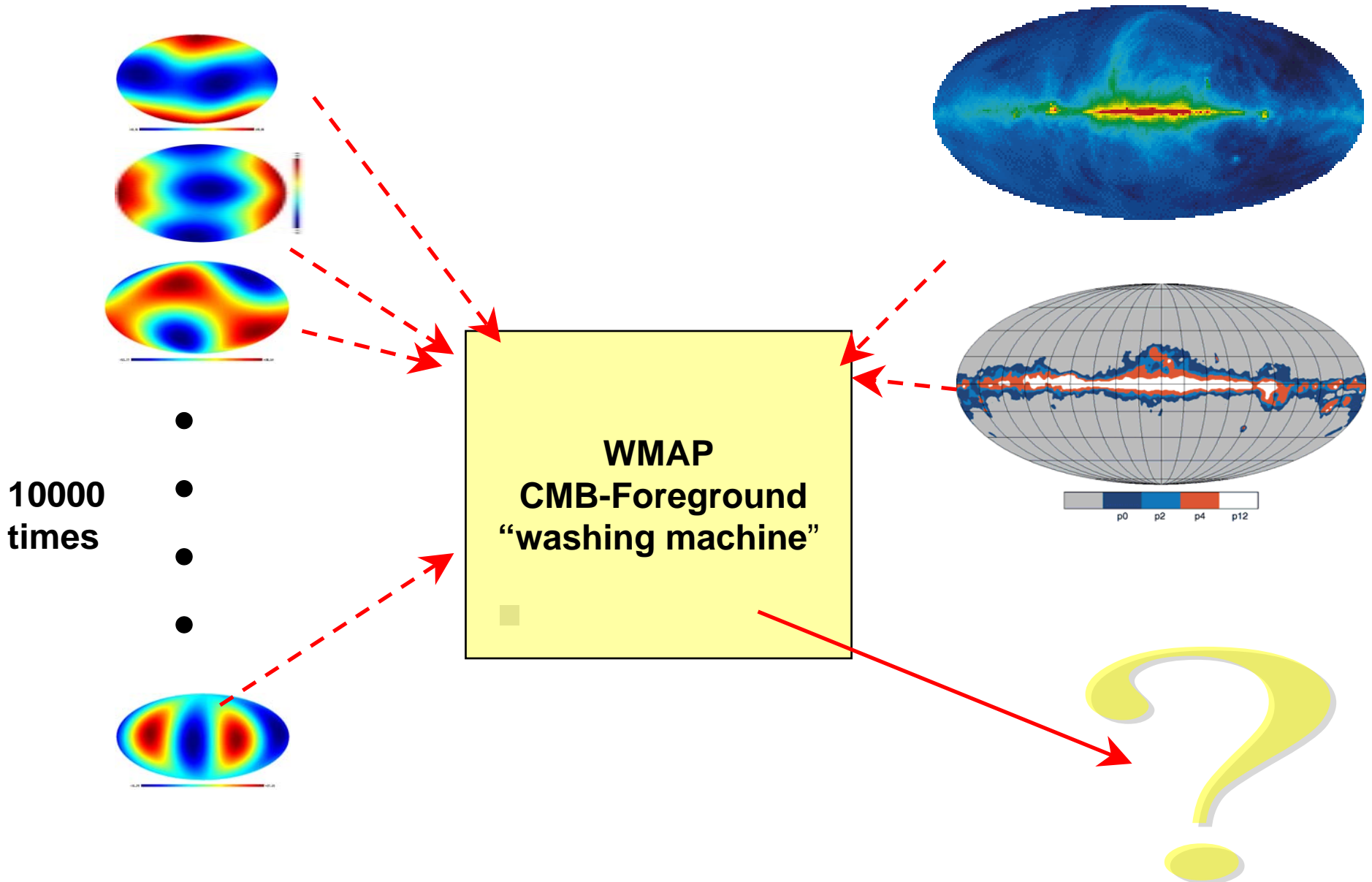
## WMAP results

$$|c_{20}| = 0.0115 \text{mK};$$

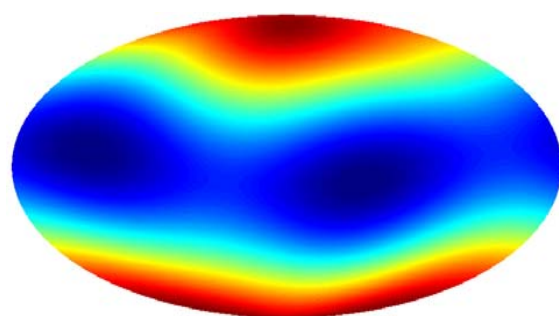
$$|c_{21}| = 0.00486 \text{mK} \sim 0.2 |a_{22}|$$

$$|c_{22}| = 0.0236 \text{mK} \sim 2 |a_{20}|$$

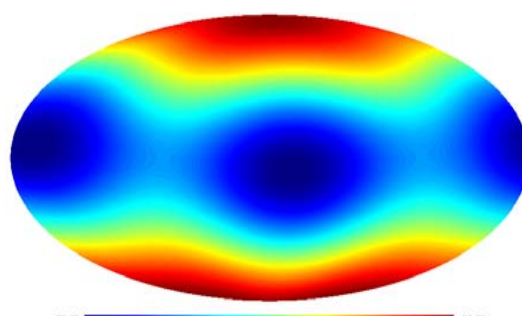
# WMAP simulations



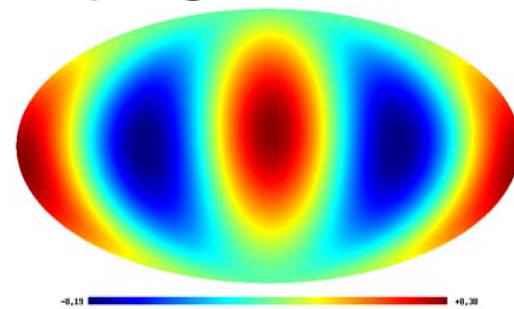
# 10000 realizations of the CMB



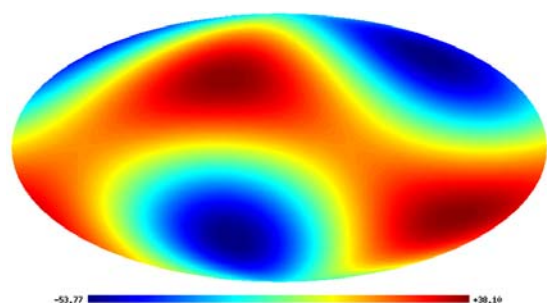
**N1 input map**



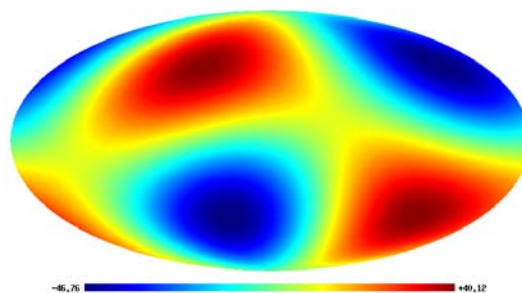
**N1 output map**



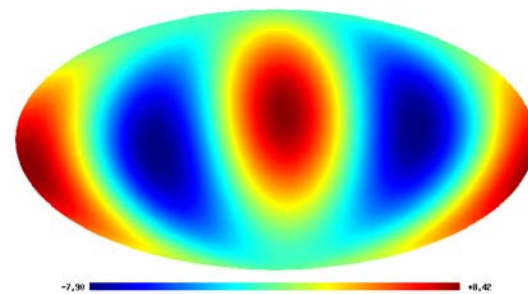
**N1 in-output map**



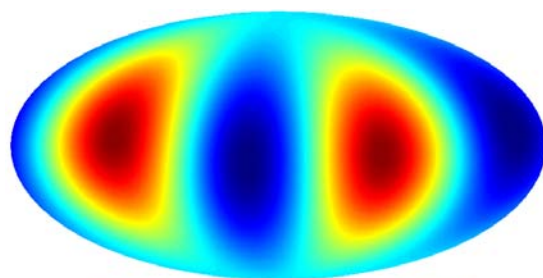
**N4 input map**



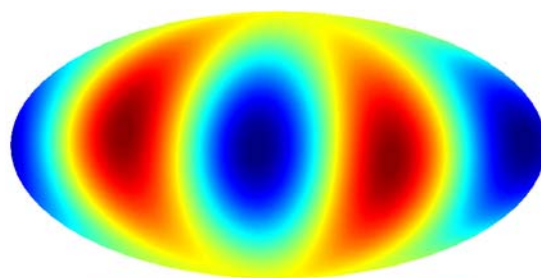
**N4 output map**



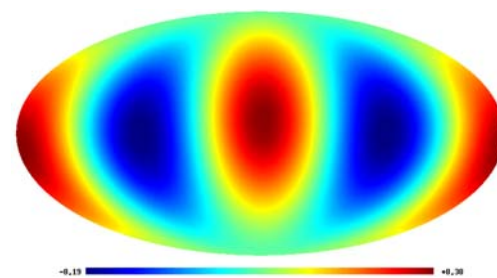
**N4 in-output map**



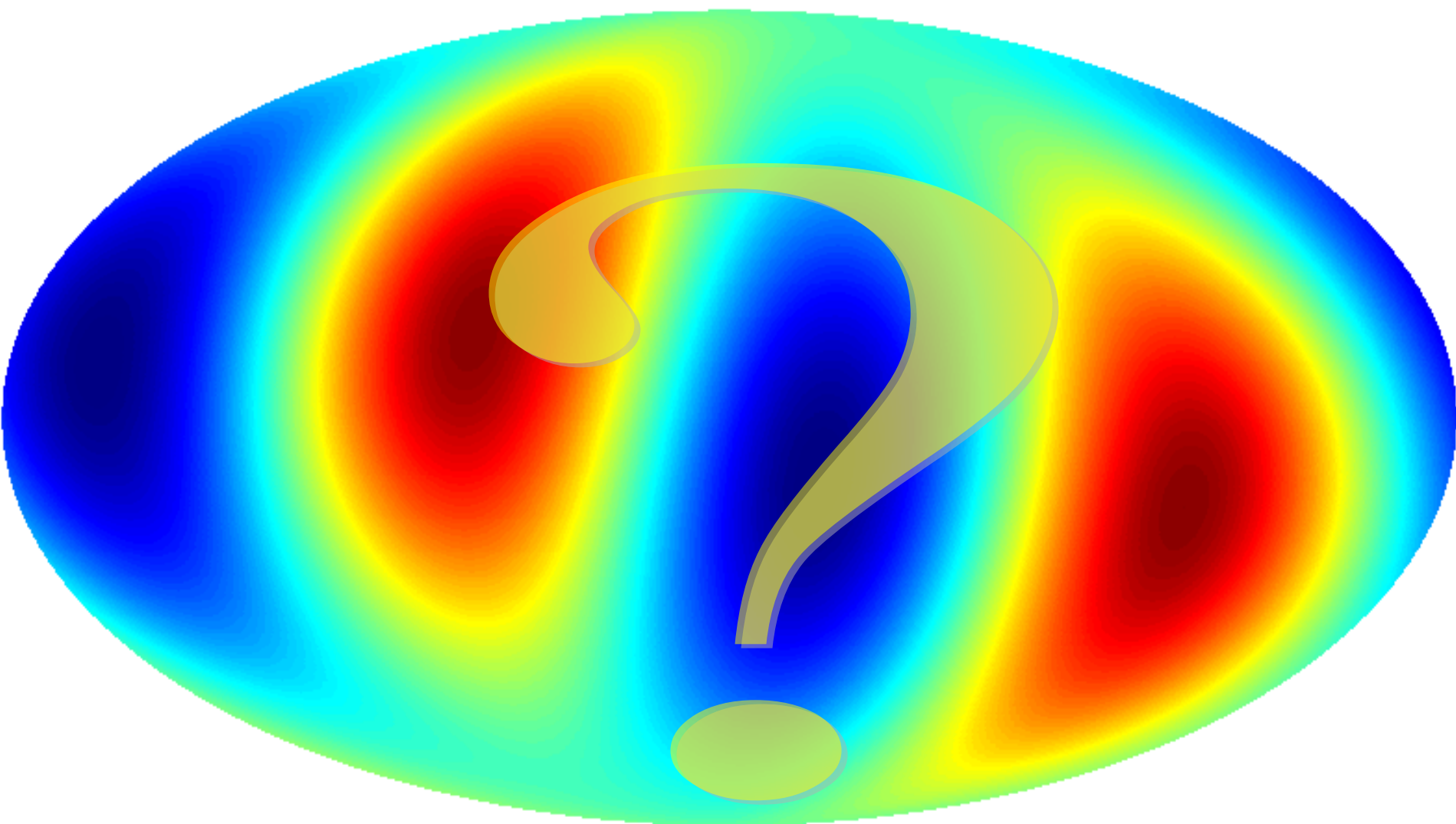
**N8 input map**



**N8 output map**



**N8 in-output map**



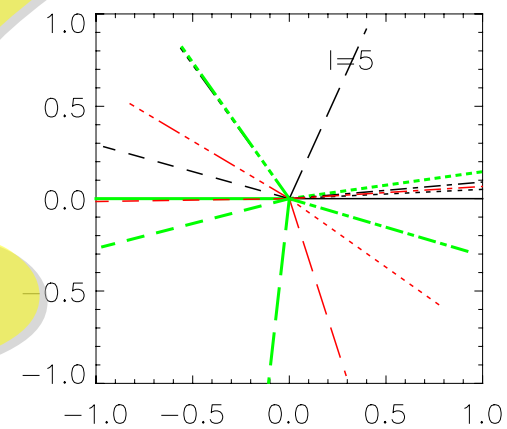
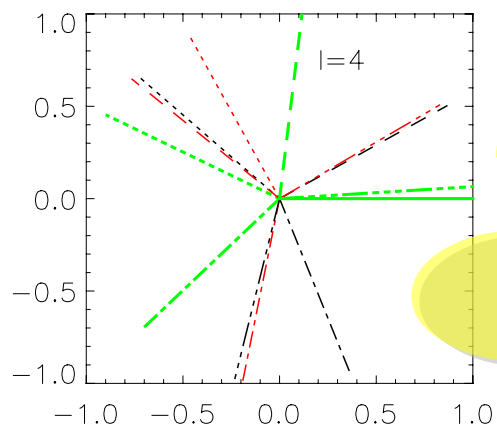
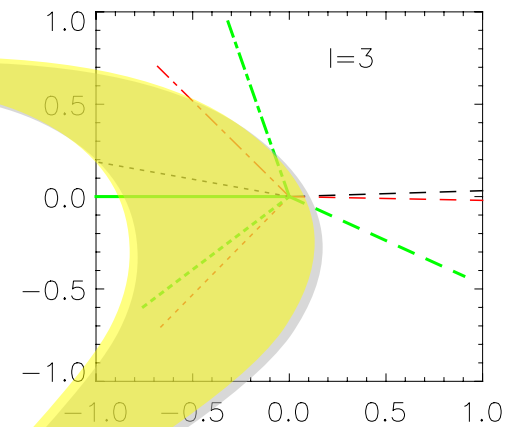
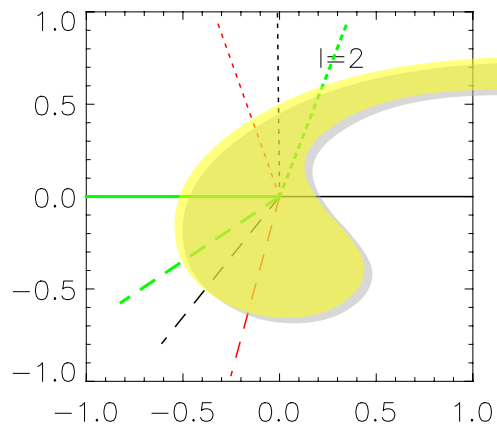
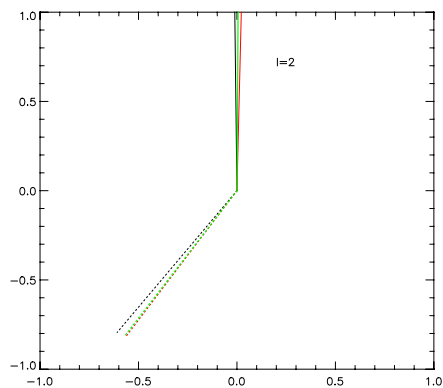
-0.02

+0.02

< 70%

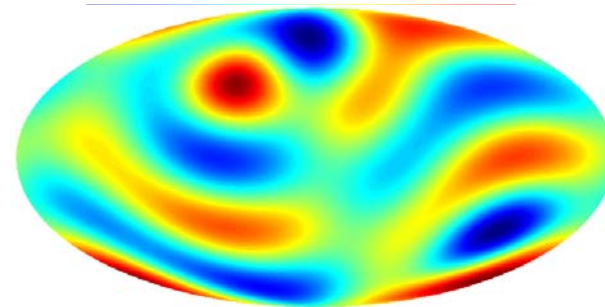
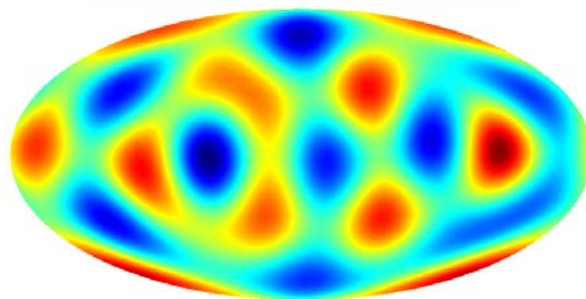
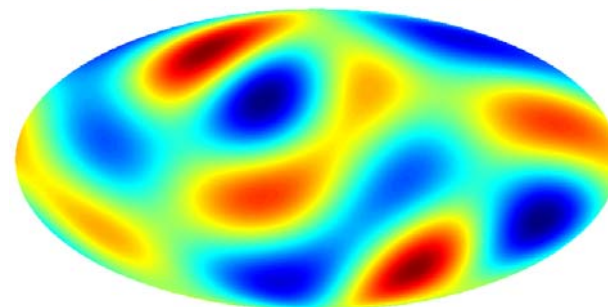
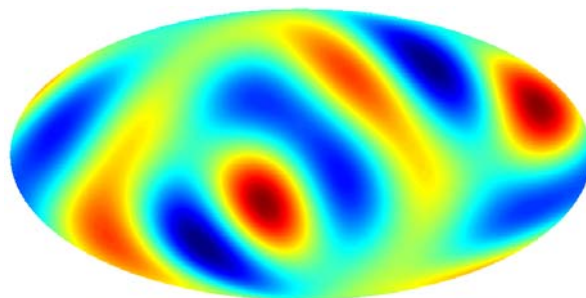
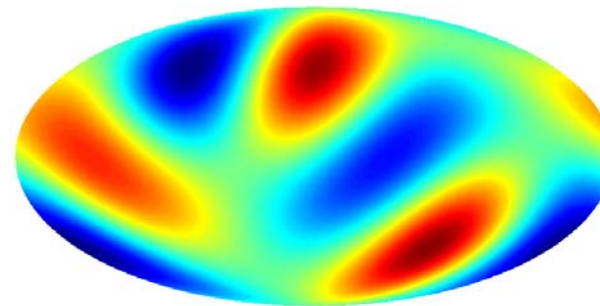
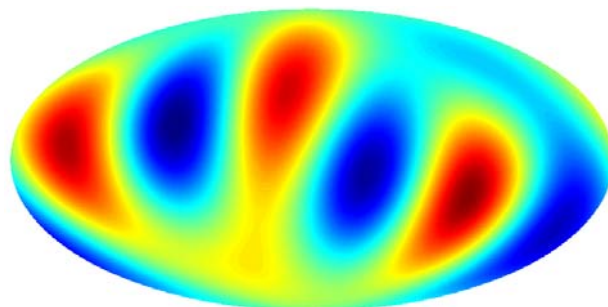
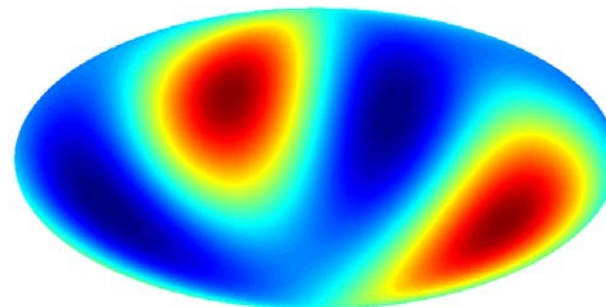
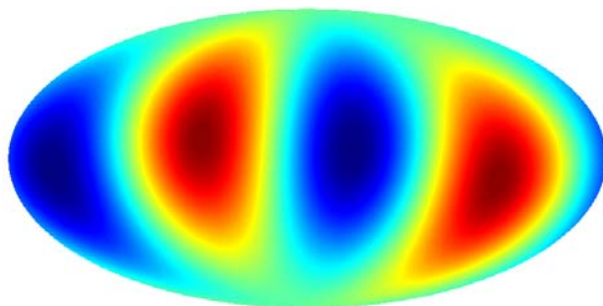


# Bianchi VIIh ?





ILC

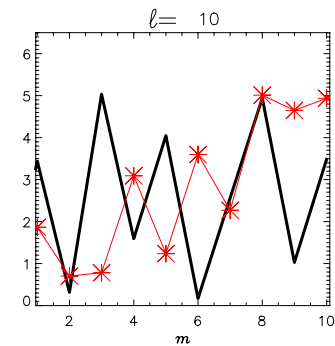
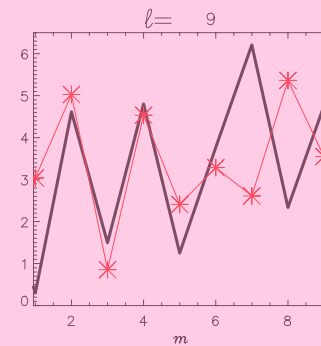
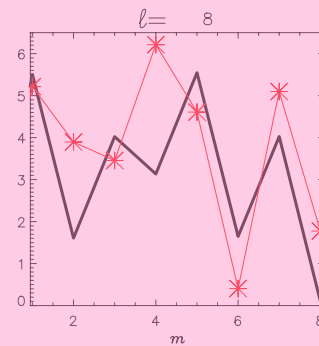
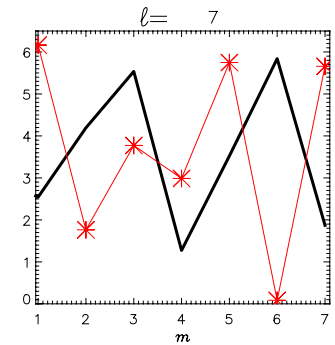
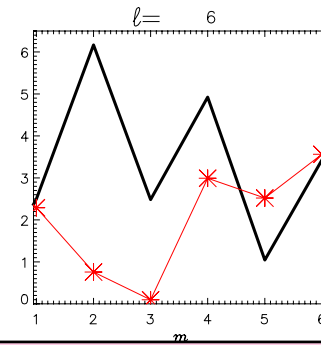
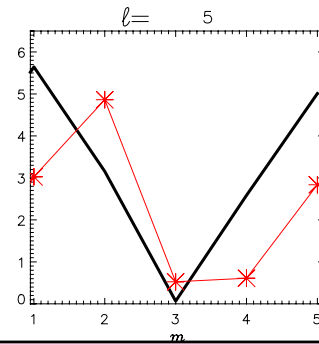
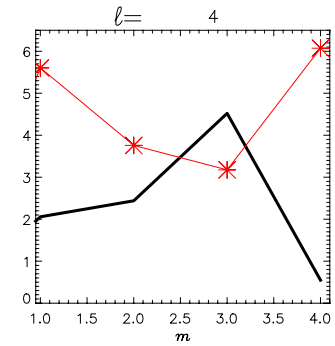
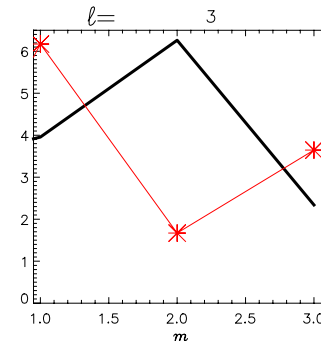
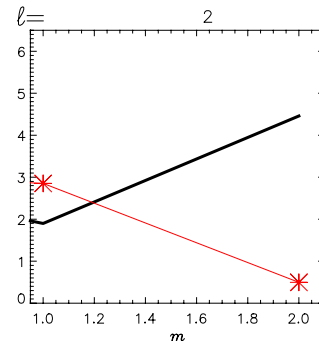




# Why the Bianchi VIIh was useful for correction of the ILC ?

1.

**Phases of the Bianchi  
VIIh-induced CMB  
versus the Wfgd**



## Properties of the Bianchi VIIh phases

$$\Psi_{l,m} = \tan^{-1} \left[ \frac{a-b}{a+b} \tan(\phi_{l,m=1} - \gamma) \right] - \alpha m$$

$$a = d^l_{m,1}(\beta),$$

$$b = (-1)^m d^l_{m,-1}(\beta)$$

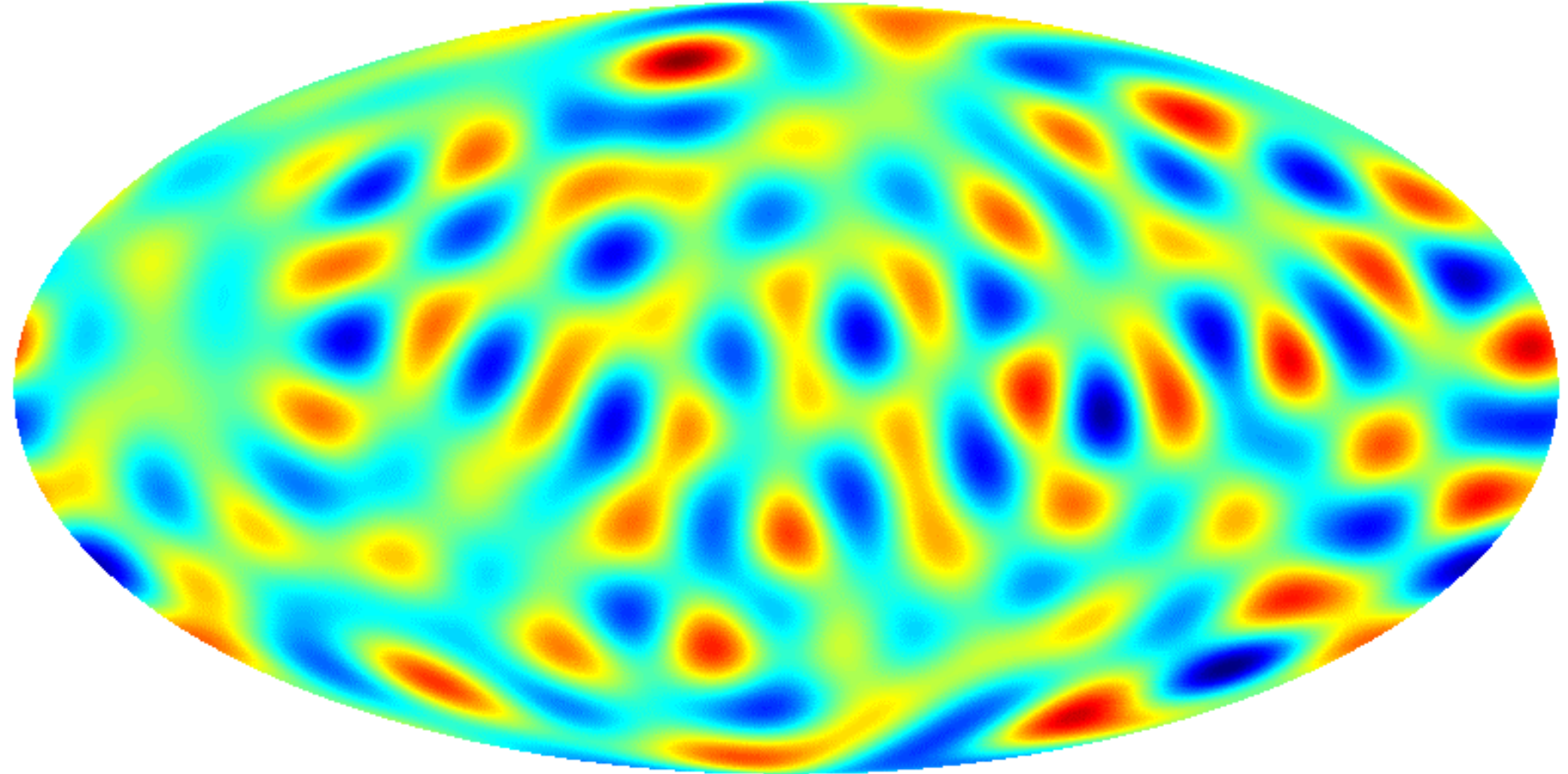
$$D^l_{m,m'}(\alpha, \beta, \gamma) = \exp(-im\alpha - im'\gamma) d^l_{m,m'}(\beta)$$

Linear term

Non-linear modulation

# Non-Bianchi VIIh “toy model”

$$\Psi_{l,m} = \pi / 2 + m\pi,$$



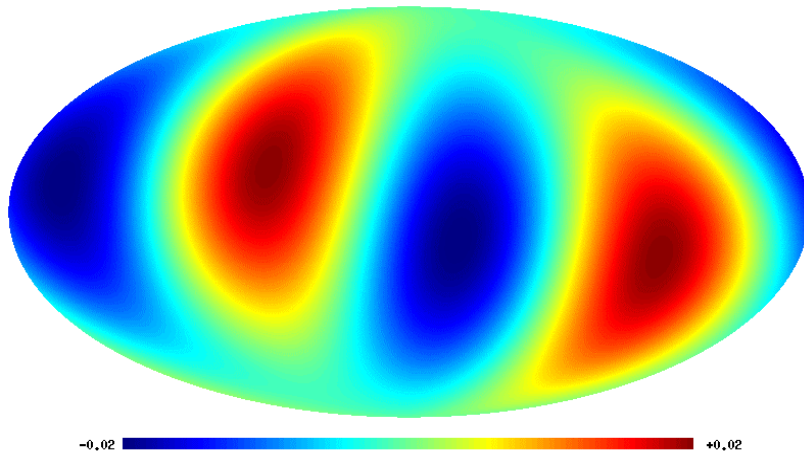
-0.04

+0.04

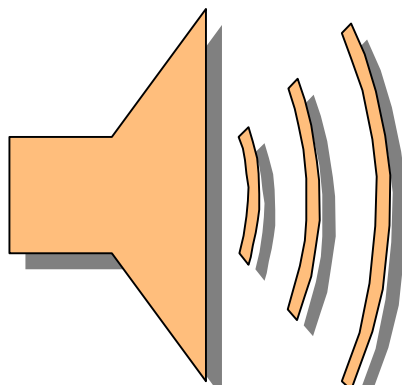
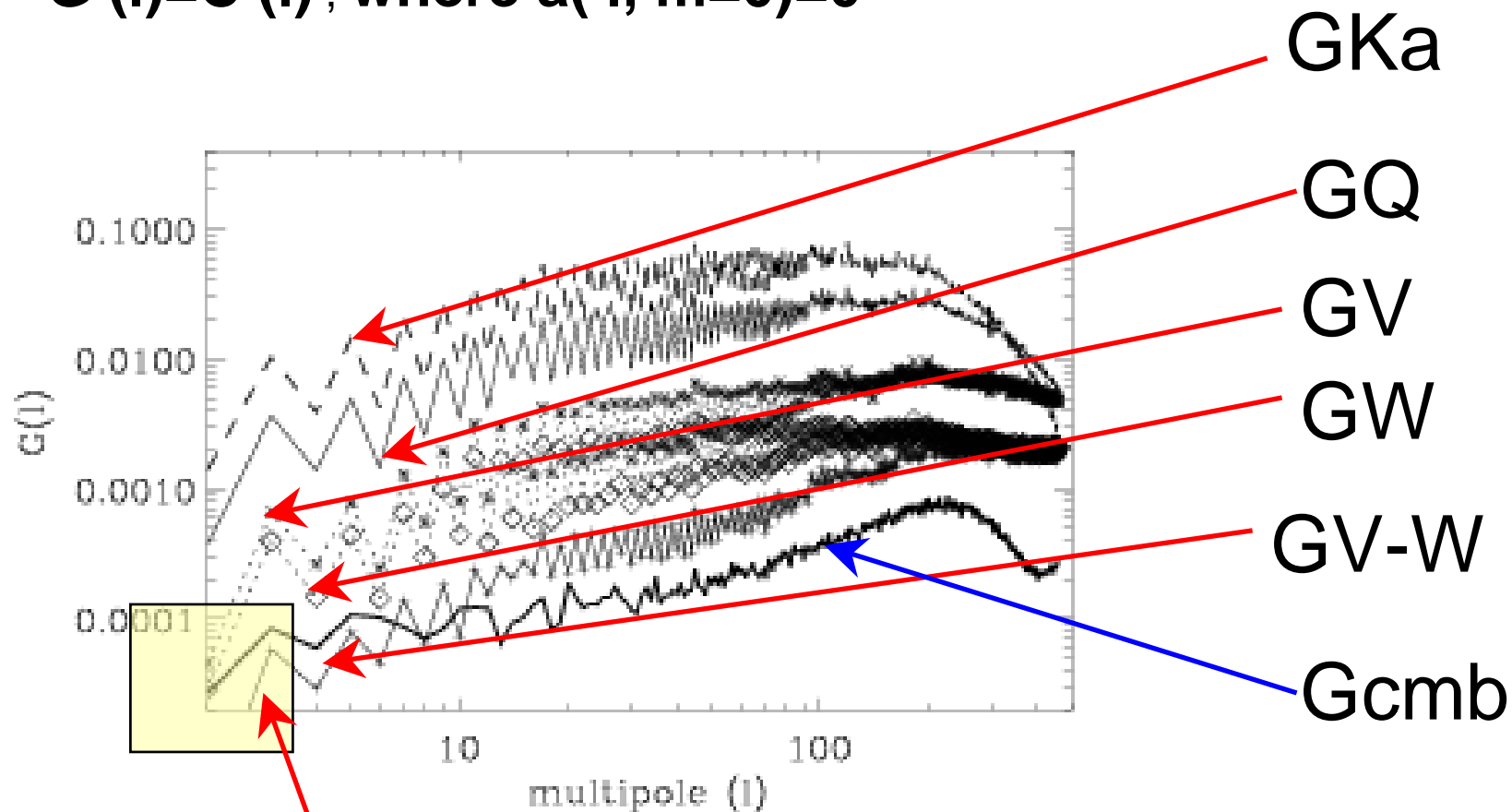
# Conclusion

1. Phases of low multipoles reveal strong correlations with the phases of foregrounds
2. To reject a null hypothesis
3. Effects of systematic ?

*MMM..... Nobel Prize after George?*



$G(l) = C(l)$ , where  $a(l, m=0)=0$



Possible instability for  
Components separation  
(CMB +Foregrounds)

