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"EARLY FAST ROLL INFLATION: THE CMB QUADRUPOLE SUPPRESSION"

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Contents

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I. INTRODUCTION

Scalar curvature and tensor (gravitational wave) quantum fluctuations generated during the inflationary stage determine the power spectrum of the anisotropies in the cosmic microwave background (CMB) providing the seeds for large scale structure (LSS) formation. Curvature and tensor fluctuations obey a wave equation, and the choice of a particular solution entails a choice of initial conditions[1]-[9]. The power spectra of these fluctuations depend in general on the initial conditions that define the particular solutions. These are usually chosen as Bunch-Davies[10] initial conditions, which select positive frequency modes asymptotically with respect to conformal time. The quantum states in the Fock representation associated with these initial conditions are known as Bunch-Davies states, the vacuum state being invariant under the maximal symmetry group $O(4,1)$ of de Sitter space-time. In earlier studies alternative initial conditions were also considered[11]. The requirement that the energy momentum tensor be renormalizable constrains the UV asymptotic behaviour of the Bogoliubov coefficients that encode different initial conditions[12]. The availability of high precision cosmological data motivated a substantial effort to study the effect of different initial conditions upon the angular power spectrum of CMB anisotropies, focusing primarily in the high-l region near the acoustic peaks[13]. However, the exhaustive analysis of the three year WMAP data[14]-[16] render much less statistical significance to possible effects on small angular scales from alternative initial conditions.

Although there are no statistically significant departures from the slow roll inflationary scenario at small angular scales ($l \gtrsim 100$), the third year WMAP data again confirms the surprinsingly low quadrupole C_2 [14]-[16] and suggests that it cannot be completely explained by galactic foreground contamination. The low value of the quadrupole has been an intriguing feature on large angular scales since first observed by COBE/DMR [17], and confirmed by the WMAP data [14]-[22].

In a companion article [23], we reported on our study of the effect of general initial conditions on the power spectra of curvature and gravitational wave perturbations. General initial conditions are related to the Bunch-Davies initial conditions by a Bogoliubov transformation and their effect on the power spectra is encoded in a *transfer function* $D(k)$ whose large wavevector behavior is constrained by renormalizability and small backreaction[23]. The rapid fall off $D(k) \leq \mathcal{O}(1/k^2)$ for large k entail that observable effects from initial conditions are more pronounced for *low multipoles*, namely in the region of the angular power spectra corresponding to the Sachs-Wolfe plateau.

In ref. [23] we formulate the problem of initial conditions established at the beginning of slow roll, in terms of a scattering by a potential in the wave equations for the mode functions of curvature and tensor perturbations. Such potential is localized in conformal time prior to slow roll and determines the initial conditions for the mode functions. Implementing methods from potential scattering allowed us to establish that such potential yields a transfer function $D(k)$ that automatically satisfies the stringent constraints from renormalizability and backreaction. The results of this previous study reveal that an attractive potential localized just prior to the onset of slow roll and with a scale determined by the energy scale during slow roll inflation yield a *suppression* of the quadrupole for curvature perturbations consistent with the data $\sim 10 - 20\%$ and predicts a small quadrupole suppression for tensor perturbations.

In this article we discuss the *origin of this attractive potential within the effective field theory of inflation.* We argue that such potential is a *generic* feature of a brief *fast roll* stage that merges smoothly with slow roll inflation. This stage is a consequence of an initial condition *for the classical inflaton dynamics* in which the kinetic and potential energy of the inflaton are of the same order, namely, the energy scale of slow roll inflation. During the early fast roll stage the inflaton evolves rapidly during a brief period, but slows down by the cosmological expansion settling in the slow roll stage in which the kinetic energy of the inflaton is much smaller than its potential energy. The scale of the attractive potential is determined by the energy scale during the slow roll stage, which in the effective field theory description[24, 25] is of the order of the grand unification scale, $M \sim 10^{16}$GeV, well below the Planck scale $M_{Pl} \sim 10^{19}$GeV, and no other energy scales are involved. Hence, we emphasize that there is **no need** to advocate transplanckian physics in this context.

Brief summary of results :

In this article we combine the **dynamical** origin of the potential within the effective field theory of inflation, with the results obtained in ref.[23] and show that the early fast roll stage leads to a suppression of the CMB quadrupole . Our main results are the following:

(1)
- Within the effective field theory of inflation with the same inflaton potentials that fulfill the slow roll conditions, we find that an initial state of the inflaton with almost *equipartition* between kinetic and potential inflaton energies yields an *attractive* potential for the mode functions of the fluctuations. This potential emerges from a brief stage in which the inflaton rolls fast, hence we call this the fast roll stage. This early stage only lasts approximately one e-fold and merges smoothly with the slow roll stage. This fast roll stage prior to slow roll is a *generic* feature of an initial condition for cosmological dynamics in which there is an approximate equipartition

(2

between the kinetic and potential energy of the inflaton. The initial conditions for the fluctuations *prior* to the fast roll stage are chosen to be the usual Bunch-Davies conditions. However, the potential that results from the fast roll dynamics of the inflaton lead to non-Bunch Davis conditions for the curvature and tensor perturbations at the beginning of the slow roll stage. The Bogoliubov coefficients and transfer function $D(k)$ automatically satisfy the constraints from renormalizability and small backreaction.

(2)
- We have investigated a large variety of inflationary models with initial inflaton dynamics featuring an approximate equipartition between inflaton kinetic and potential energies. This study leads us to conclude quite generally that the scale of the potential during fast roll is completely determined by the Hubble scale during the subsequent slow roll stage. The effect of this potential during the fast roll evolution of the scale factor leads to modifications of the primordial power spectrum. This potential is *attractive* both for curvature and tensor fluctuations, and leads to a **suppression** of their primordial power spectra on large scales.

(3)
- From a comprehensive numerical study of different inflationary scenarios within the effective field theory approach, we find a 10 − 20% *suppression* of the CMB quadrupole and about a 2 − 4% suppression of the B-mode quadrupole (tensor fluctuations). This CMB quadrupole corresponds to the wavevector k_Q whose physical wavelength is of the order of the Hubble radius *today* and exits the horizon during slow roll inflation just 1 − 2 e-folds after the brief fast roll stage. The suppression on higher l-multipoles reduce considerably following a $1/l^2$ law.

(4)
- The attractive potential resulting from the fast roll stage accounts for the observed suppression of the CMB quadrupole *if* the wavevector k_Q whose wavelength corresponds to the Hubble radius *today* exits 2-3 e-folds after the end of the fast roll stage, which lasts ≈ 1 e-fold. The quadrupole corresponds to the wavevector k_Q that exits the horizon $N_Q = 55$ efolds before the end of inflation, hence our results successfully explain the CMB quadrupole suppression within the effective field theory if inflation lasts at most $N_{tot} \lesssim N_Q + 4 = 59$ efolds. This result establishes an **upper bound** to the number of efolds during inflation.

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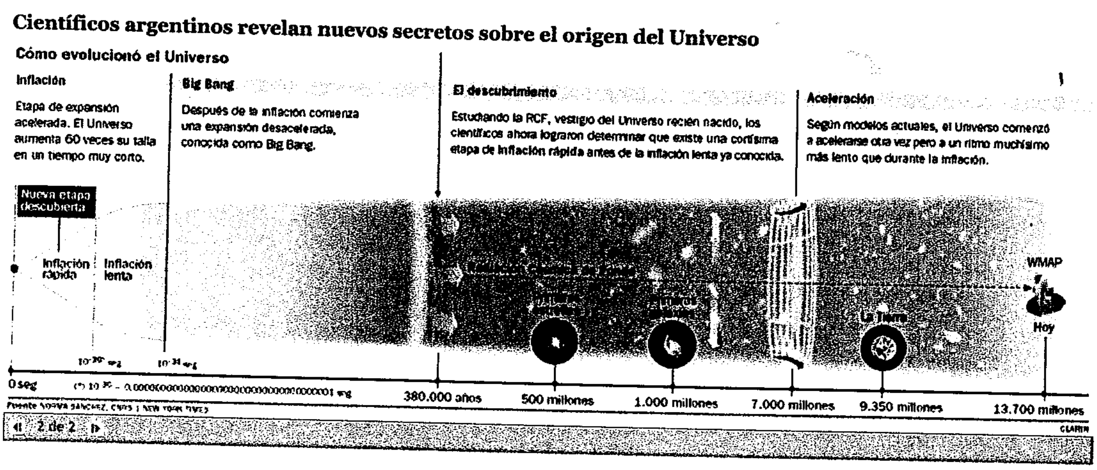

initial conditions

FAST ROLL

SLOW ROLL

INFLATION

QUADRUPOLE CMB SOPPRESSION

(STANDARD) CONCORDANCE MODEL

(Clarin, Buenos Aires, 23/10/2006)

II. INITIAL CONDITIONS OF INFLATIONARY FLUCTUATIONS FROM THE SCATTERING BY A POTENTIAL

In the companion article [23] we have systematically analyzed the consequences of generic initial conditions different from Bunch-Davies, under the conditions that these are UV allowed and yield small backreaction effects. Here we address the *origin* of these initial conditions, beginning by gathering relevant ingredients from[23].

As shown in[23] in a cosmological space-time geometry

COSMIC TIME

$$ds^2 = dt^2 - a^2(t)(d\vec{x})^2 = C^2(\eta)[d\eta^2 - (d\vec{x})^2] \; ,$$

CONFORMAL TIME

where t and η stand for cosmic and conformal time respectively, the wave equations for the mode functions of gaussian curvature and tensor perturbations are of the form of the Schrödinger equation in one dimension

CURVATURE AND TENSOR FLUCTUATIONS

$$\left[\frac{d^2}{d\eta^2} + k^2 - W(\eta)\right] S(k;\eta) = 0 \tag{2.1}$$

WAVE EQUATION (in η)

with η the coordinate, k^2 the energy and $W(\eta)$ a potential that depends on the coordinate η. In the cases under consideration

POTENTIAL $W(\eta)$

$$W(\eta) = \begin{cases} W_{\mathcal{R}}(\eta) = z''/z & \text{for curvature perturbations ,} \\ W_T(\eta) = C''/C & \text{for tensor perturbations .} \end{cases} \tag{2.2}$$

where prime stands for derivative with respect to the conformal time and

$$z = a(t)\, \frac{\dot{\Phi}}{H} \; , \tag{2.3}$$

$\dot{\Phi}$ stands for the derivative of the inflaton field Φ with respect to the cosmic time t.

It is convenient to explicitly separate the behavior of $W(\eta)$ during the slow roll stage by writing

$\mathcal{V}(\eta)$

$$W(\eta) = \mathcal{V}(\eta) + \frac{\nu^2 - \frac{1}{4}}{\eta^2} \tag{2.4}$$

SLOW ROLL PART (REPULSIVE BARRIER)

where

IN TERMS OF SLOW ROLL PARAMETERS

$$\nu = \begin{cases} \nu_{\mathcal{R}} = \frac{3}{2} + 3\,\epsilon_v - \eta_v + \mathcal{O}\left(\frac{1}{N^2}\right) & \text{for curvature perturbations} \\ \nu_T = \frac{3}{2} + \epsilon_v + \mathcal{O}\left(\frac{1}{N^2}\right) & \text{for tensor perturbations .} \end{cases}$$

Here ϵ_v and η_v stand for the slow roll parameters

$$\epsilon_v = \frac{\dot{\Phi}^2}{2\, M_{Pl}^2\, H^2} = \frac{M_{Pl}^2}{2}\left[\frac{V'(\Phi)}{V(\Phi)}\right]^2 + \mathcal{O}\left(\frac{1}{N^2}\right) = \mathcal{O}\left(\frac{1}{N}\right) \quad , \quad \eta_v = M_{Pl}^2\, \frac{V''(\Phi)}{V(\Phi)} = \mathcal{O}\left(\frac{1}{N}\right) \; ,$$

SLOW ROLL: CENTRIFUGAL BARRIER

and $N \sim 55$ stands for the number of efolds from horizon exit until the end of inflation [24].

The slow roll dynamics acts through the term $[(\nu^2 - 1/4)/(\eta^2)]$ which is a *repulsive* centrifugal barrier.

We anticipate that the potential $\mathcal{V}(\eta)$ is localized in the fast roll stage *prior* to slow roll (during which cosmolo relevant modes cross out of the Hubble radius) where $\mathcal{V}(\eta)$ vanishes. Including the potential $\mathcal{V}(\eta)$ the equatic the quantum fluctuations are

FLUCTUATIONS EQUATION

$$\left[\frac{d^2}{d\eta^2} + k^2 - \frac{\nu^2 - \frac{1}{4}}{\eta^2} - \mathcal{V}(\eta)\right] S(k;\eta) = 0 \; .$$

FAST ROLL

During the slow roll stage $\mathcal{V}(\eta) = 0$ and the mode equations simplify to

$\mathcal{V}(\eta) = 0$

$$\left[\frac{d^2}{d\eta^2} + k^2 - \frac{\nu^2 - \frac{1}{4}}{\eta^2}\right] S(k,\eta) = 0 \; .$$

SLOW ROLL

To leading order in slow roll, ν is constant and for general initial conditions the solution is,

$$S(k;\eta) = A(k)\, g_\nu(k;\eta) + B(k)\, f_\nu(k;\eta) \; ,$$

where two linearly independent solutions of eq.(2.8) are,

$$g_\nu(k;\eta) \;=\; \frac{1}{2}\, i^{\nu+\frac{1}{2}}\, \sqrt{-\pi\eta}\, H^{(1)}_\nu(-k\eta) \; ,$$

where two linearly independent solutions of eq.(2.8) are,

$$\begin{cases} g_\nu(k;\eta) = \frac{1}{2}\, i^{\nu+\frac{1}{2}} \sqrt{-\pi\eta}\, H^{(1)}_\nu(-k\eta) \,, & (2.10) \\ f_\nu(k;\eta) = [g_\nu(k;\eta)]^* \,, & (2.11) \end{cases}$$

$H^{(1)}_\nu(z)$ are Hankel functions. These solutions are normalized so that their Wronskian is given by

$$\boxed{W[g_\nu(k;\eta), f_\nu(k;\eta)] = g'_\nu(k;\eta)\, f_\nu(k;\eta) - g_\nu(k;\eta)\, f'_\nu(k;\eta) = -i \,.} \qquad (2.12)$$

The mode functions and coefficients $A(k)$, $B(k)$ will feature a subscript index $\mathcal{R}$, T, for curvature or tensor perturbations, respectively.

For wavevectors deep inside the Hubble radius $|k\,\eta| \gg 1$, the mode functions have the Bunch-Davies asymptotic behavior

BUNCH DAVIES

$$\boxed{g_\nu(k;\eta) \overset{\eta\to-\infty}{=} \frac{1}{\sqrt{2\,k}}\, e^{-ik\eta} \quad , \quad f_\nu(k;\eta) \overset{\eta\to-\infty}{=} \frac{1}{\sqrt{2k}}\, e^{ik\eta} \,,} \qquad (2.13)$$

CONDITIONS

and for $\eta \to 0^-$, the mode functions behave as:

$$g_\nu(k;\eta) \overset{\eta\to0^-}{=} \frac{\Gamma(\nu)}{\sqrt{2\,\pi\,k}} \left(\frac{2}{i\,k\,\eta}\right)^{\nu-\frac{1}{2}} . \qquad (2.14)$$

The complex conjugate formula holds for $f_\nu(k;\eta)$.

In particular, in the scale invariant case $\nu = \frac{3}{2}$ which is the leading order in the slow roll expansion, the mode functions eqs.(2.10) simplify to

SCALE INVARIANT ($\nu = \frac{3}{2}$ CASE) $\rightarrow$

$$g_{\frac{3}{2}}(k;\eta) = \frac{e^{-ik\eta}}{\sqrt{2k}} \left[1 - \frac{i}{k\eta}\right] . \qquad (2.15)$$

$\leftarrow$ IN PARTICULAR

$\mathcal{V}(\eta)$ is localized in the fastroll regime

The mode equation (2.7) can be written as an integral equation,

general solution $\rightarrow$

$$S(k;\eta) = g_\nu(k;\eta) + i\, g_\nu(k;\eta) \int_{-\infty}^{\eta} g^*_\nu(k;\eta')\, \mathcal{V}(\eta')\, S(k;\eta')\, d\eta' - i\, g^*_\nu(k;\eta) \int_{-\infty}^{\eta} g_\nu(k;\eta')\, \mathcal{V}(\eta')\, S(k;\eta')\, d\eta' \quad . \quad (2$$

This solution has the Bunch-Davies asymptotic condition

$$S(k;\eta \to -\infty) = \frac{e^{-ik\eta}}{\sqrt{2k}} \,. \qquad (2$$

We formally consider here the conformal time starting at $\eta = -\infty$. However, it is natural to consider that inflationary evolution of the universe starts at some negative value $\eta_i < \bar\eta$, where $\bar\eta$ is the conformal time when roll ends and slow roll begins.

$\mathcal{V}(\eta) = 0$ for $\eta > \bar\eta$

Since $\mathcal{V}(\eta)$ vanishes for $\eta > \bar\eta$, the mode functions $S(k;\eta)$ can be written for $\eta > \bar\eta$ as linear combinations of mode functions $g_\nu(k;\eta)$ and $g^*_\nu(k;\eta)$,

$$\boxed{S(k;\eta) = A(k)\, g_\nu(k;\eta) + B(k)\, g^*_\nu(k;\eta) \quad , \quad \eta > \bar\eta \quad ,} \qquad (2$$

where the coefficients $A(k)$ and $B(k)$ can be read from eq.(2.16),

BOGOLIUBOV COEFFICIENTS $\rightarrow$

$$\boxed{\begin{aligned} A(k) &= 1 + i \int_{-\infty}^{0} g^*_\nu(k;\eta)\, \mathcal{V}(\eta)\, S(k;\eta)\, d\eta \\ B(k) &= -i \int_{-\infty}^{0} g_\nu(k;\eta)\, \mathcal{V}(\eta)\, S(k;\eta)\, d\eta \,. \end{aligned}} \qquad (2$$

COMPUTED FROM THE DYNAMICS

The coefficients $A(k)$ and $B(k)$ are therefore calculated from the dynamics before slow roll [recall that $\mathcal{V}(\eta)$ for $\eta > \bar\eta$ during slow roll.]

The constancy of the Wronskian $W[g_\nu(\eta), g^*_\nu(\eta)] = -i$ and eq.(2.18) imply the constraint,

$$|A(k)|^2 - |B(k)|^2 = 1 \quad .$$

This relation permits to represent the coefficients $A(k)$; $B(k)$ as [23]

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This relation permits to represent the coefficients $A(k)$; $B(k)$ as [23]

$$A(k) = \sqrt{1+N(k)}\, e^{i\theta_A(k)} \quad ; \quad B(k) = \sqrt{N(k)}\, e^{i\theta_B(k)} \,, \tag{2.20}$$

where $N(k)$, $\theta_{A,B}(k)$ are real.

Starting with Bunch-Davies initial conditions for $\eta \to -\infty$, the action of the potential generates a mixture of the two linearly independent mode functions that result in the mode functions eq.(2.18) for $\eta > \bar{\eta}$ when the potential vanishes. This is clearly equivalent to starting the evolution of the fluctuations at the *beginning* of slow roll $\eta = \bar{\eta}$ with initial conditions defined by the Bogoliubov coefficients $A(k)$ and $B(k)$ given by eq.(2.19).

As shown in ref.[23] the power spectrum of curvature and tensor perturbations for the general fluctuations eq.(2.18) takes the form,

POWER SPECTRUM

FOR GENERAL INITIAL CONDITIONS

$$P_{\mathcal{R}}(k) \overset{\eta\to 0^-}{=} \frac{k^3}{2\,\pi^2}\, {}_S\langle 0|\left|\frac{S_{\mathcal{R}}(k;\eta)}{z}\right|^2|0\rangle_S = P^{sr}_{\mathcal{R}}(k)\Big[1+D_{\mathcal{R}}(k)\Big] \,,$$
$$P_T(k) \overset{\eta\to 0^-}{=} \frac{k^3}{2\,\pi^2}\, {}_S\langle 0|\left|\frac{S_T(k;\eta)}{C(\eta)}\right|^2|0\rangle_S = P^{sr}_T(k)\Big[1+D_T(k)\Big] \,. \tag{2.21}$$

where $D_{\mathcal{R}}(k)$ and $D_T(k)$ are the transfer functions for the initial conditions of curvature and tensor perturbations introduced in ref.[23]:

$$D_{\mathcal{R}}(k) = 2\,|B_{\mathcal{R}}(k)|^2 - 2\,\mathrm{Re}\left[A_{\mathcal{R}}(k)\,B^*_{\mathcal{R}}(k)\,i^{2\nu_R-3}\right] = 2\,N_{\mathcal{R}}(k) - 2\,\sqrt{N_{\mathcal{R}}(k)[1+N_{\mathcal{R}}(k)]}\,\cos\left[\theta^{\mathcal{R}}_k - \pi\left(\nu_R - \frac{3}{2}\right)\right]$$
$$D_T(k) = 2\,|B_T(k)|^2 - 2\,\mathrm{Re}\left[A_T(k)B^*_T(k)\,i^{2\nu_h-3}\right] = 2\,N_T(k) - 2\,\sqrt{N_T(k)[1+N_T(k)]}\,\cos\left[\theta^T_k - \pi\left(\nu_T - \frac{3}{2}\right)\right] \tag{2.22}$$

here $\theta_k \equiv \theta_B(k) - \theta_A(k)$. The standard slow roll power spectrum is given by [3, 7]:

$$P^{sr}_{\mathcal{R}}(k) = \left(\frac{k}{2\,k_0}\right)^{n_s-1} \frac{\Gamma^2(\nu)}{\pi^3}\,\frac{H^2}{2\,\epsilon_v\,M^2_{Pl}} \equiv \mathcal{A}^2_{\mathcal{R}}\,\left(\frac{k}{k_0}\right)^{n_s-1} \,,$$

TRANSFERT FUNCTION OF INITIAL CONDITIONS

$D_R(k)$, $D_T(k)$

SLOW ROLL

ratio

TENSOR

$$P^{sr}_T(k) = \mathcal{A}^2_T\,\left(\frac{k}{k_0}\right)^{n_T} \quad , \quad n_T = -2\,\epsilon_v \quad , \quad \frac{\mathcal{A}^2_T}{\mathcal{A}^2_{\mathcal{R}}} = r = 16\,\epsilon_v \,. \tag{2.23}$$

As shown in ref. [23], the relative change in the $C_l's$ for the general fluctuations eq.(2.18) with respect to the standard slow roll result is given by

CHANGE IN THE C_ℓ' :

$$C_l \equiv C^{sr}_l + \Delta C_l \quad , \quad \frac{\Delta C_l}{C_l} = \frac{\int_0^\infty D(\kappa\,x)\;f_l(x)\,dx}{\int_0^\infty f_l(x)\,dx} \,, \tag{2.24}$$

where $x = k/\kappa$ and

$$\kappa \equiv a_0\,H_0/3.3 \,. \tag{2.25}$$

$D(\kappa\,x)$ is the transfer function of initial conditions for the corresponding perturbation,

$$f_l(x) = x^{n_s-2}[j_l(x)]^2 \,. \tag{2.26}$$

and the $j_l(x)$ are spherical Bessel functions [26]. We derived in ref.[23] an estimate of the corrections, for the maximal asymptotic decay of the occupation numbers

$$N_k = N_\mu\,\left(\frac{\mu}{k}\right)^{4+\delta} \quad ; \quad 0<\delta\ll 1 \tag{2.27}$$

RELATIVE CHANGE

with the result,

$$\frac{\Delta C_l}{C_l} \approx -\frac{4}{3}\,\sqrt{N_\mu}\,\left(\frac{3.3\,\mu}{a_0\,H_0}\right)^2\,\frac{\overline{\cos\theta}}{(l-1)(l+2)} \,. \tag{2.28}$$

where we have taken $\nu = 3/2$ and $\cos\theta_k \approx \overline{\cos\theta}$ (see ref.[23] for details). The $\sim 1/l^2$ behavior is a result of the $1/k^2$ fall off of $D(k)$, a consequence of the renormalizability condition on the occupation number. For the quadrupole, the

$$\frac{\Delta C_l}{C_l} \approx -\frac{4}{3}\sqrt{N_\mu}\left(\frac{3.3\,\mu}{a_0\,H_0}\right)\frac{\cos\theta}{(l-1)(l+2)}\,. \tag{2.28}$$

where we have taken $\nu = 3/2$ and $\cos\theta_k \approx \overline{\cos\theta}$ (see ref.[23] for details). The $\sim 1/l^2$ behavior is a result of the $1/k^2$ fall off of $D(k)$, a consequence of the renormalizability condition on the occupation number. For the quadrupole, the relevant wave-vectors correspond to $x \sim 2$, namely $k_Q \sim a_0\,H_0$. It is convenient to write

$$k_Q = a_{sr}\,H_i = a_0\,H_0\,, \tag{2.29}$$

where a_{sr} and H_i are the scale factor and the Hubble parameter during the slow roll stage of inflation when the wavelength corresponding to *today*'s Hubble radius exits the horizon.

III. THE ORIGIN OF THE POTENTIAL $\mathcal{V}(\eta)$: A *FAST ROLL* STAGE BEFORE SLOW ROLL INFLATION.

The mode functions of perturbations obey the general evolution equation (2.1) where $W(\eta)$ is given by eq.(2.2) and the slow roll part is explicitly separated in eq.(2.4). A full expression for $W(\eta)$ and therefore for the potential $\mathcal{V}(\eta)$ is obtained from the Friedmann equation and the evolution equation of the inflaton

INFLATION EQS OF MOTION

$$H^2 = \frac{1}{3\,M_{PL}^2}\left[\frac{1}{2}\,\dot\Phi^2 + V(\Phi)\right]\,, \tag{3.1}$$

FRIEDMAN EQ.

$$\ddot\Phi + 3\,H\,\dot\Phi + V'(\Phi) = 0\,, \tag{3.2}$$

INFLATON EQ.

The exact potential is obtained by using the equations (3.1)-(3.2). For this purpose it is convenient to introduce a dimensionless variable y^2 as

y^2 VARIABLE

$$\boxed{y^2 \equiv \frac{\dot\Phi^2}{2\,M_{Pl}^2\,H^2} = 3\left[1 - \frac{V(\Phi)}{3\,M_{Pl}^2\,H^2}\right]\quad,\quad 0 \leq y^2 \leq 3\,,} \tag{3.3}$$

in terms of which the equations of motion (3.1) and (3.2) are written in the simple form,

SIMPLER FORM

$$\boxed{\dot\Phi = \text{sign}(\dot\Phi)\,M_{Pl}\,H\,\sqrt{2}\,|y|\quad,\quad \frac{\dot H}{H^2} = -y^2\,.} \tag{3.4}$$

EQUIVALENT

$y^2 = \epsilon_v$ during slow roll , ($y^2 \ll 1$ in slow roll)

In particular, during the slow roll stage: $y^2 = \epsilon_v$ [see eq.(2.6)], but in general, in a stage in which the slow roll approximation is not valid, the kinetic term of the inflaton is not small. The slow roll parameters eqs.(2.6) are $\epsilon_v \ll 1$, $\eta_v \ll 1$ to correctly describe the slow roll stage. But, besides the slow roll stage, in which $y^2 \ll 1$, there is a prior stage in which y^2 is *not small* but $y^2 \sim \mathcal{O}(1)$: in this case the kinetic term of the inflaton is of the same order as the potential $V(\Phi)$. That is, the initial energy of the inflaton is distributed between kinetic and potential energy with approximate *equipartition*.

BUT $y^2 \sim O(1)$ IN FAST ROLL

Thus, there are two distinct regimes determined by the dimensionless variable y^2: (i) $y^2 = \mathcal{O}\left(\frac{1}{N}\right) \ll 1$ corresponds to the usual slow roll regime $\dot\Phi^2 \ll V(\Phi)$; (ii) in contrast, $y^2 \gtrsim 1$ in which $\dot\Phi^2 \sim V(\Phi)$ describes a fast roll regime. Inflation requires:

(1) $y^2 = O(\frac{1}{N}) \ll 1$, $\dot\phi^2 \ll V(\phi)$ } SLOW ROLL

$$\boxed{\frac{\ddot a}{a} = H^2\,(1 - y^2) > 0\quad,} \tag{3.5}$$

← for INFLATION

thus, the range of the variable y^2 for inflationary evolution is $0 < y^2 < 1$.

(2) $y^2 \gtrsim 1$, $\dot\phi^2 \sim V(\phi)$ } FAST ROLL

A. Fast Roll Dynamics

$0 < y^2 < 1$ INFLATION

Notice that the same description of inflation (the same inflaton potential) gives rises to the two diferent regimes: *fast* roll and slow roll regimes. The dynamics in the effective field theory of inflation giving rise to a fast roll stage followed by the slow roll stage is simple: consider an initial condition on the inflaton field and its first derivative that corresponds to an initial value of $y^2 \sim 1$. The potential and kinetic energy of the inflaton in this state are of the same order, this is the beginning of the *fast roll* stage. The strong friction term in the equation of motion for the inflaton eq.(3.1) results in that if initially $\dot\Phi \neq 0$ and large, the kinetic energy of the inflaton dissipates away and $\dot\Phi$ diminishes. This means that when y^2 begins with a large value $y^2 \sim 1$ the dynamics drives it towards smaller values.

Even if initially $y^2 > 1$ produces a non-inflationary stage [see eq.(3.5)], this only occurs for a short period of time until $y^2 < 1$ where the evolution becomes inflationary. The inflaton friction term continues to dissipate away the kinetic energy and when $y^2 = \mathcal{O}(1/N) \ll 1$ the dynamics enters the slow roll inflationary regime in earnest.

We have restricted the above discussion to the case of homogeneous inflaton fields, where the energy is carried by the zero mode of the inflaton up to small quantum fluctuations. However, a fast roll stage prior to slow roll has

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A. Fast Roll Dynamics

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We have restricted the above discussion to the case of homogeneous inflaton fields, where the energy is carried by the zero mode of the inflaton up to small quantum fluctuations. However, a fast roll stage prior to slow roll has also been studied in ref.[27], where a large amplitude inhomogeneous condensate (tsunami inflation) was considered. In that case modes with wavevectors of the order of the inflaton mass were initially excited with large amplitude, the resulting non-perturbative evolution of this initial state also leads to a fast roll stage which smoothly merges with the standard de Sitter regime[27]. The rapid redshift of non-homogeneous modes leads to the formation of an effective homogeneous condensate after a few e-folds. Therefore, a fast roll regime prior to the standard slow roll regime is a rather generic feature, either a result of an almost equipartition between kinetic and potential energies for a *homogeneous* inflaton condensate, or from an inhomogeneous non-perturbative condensate.

B. Curvature perturbations during the fast roll stage

For curvature perturbations, from eq.(2.1)

FAST ROLL PART

SLOW ROLL PART (REPULSIVE BARRIER)

TOTAL POTENTIAL

$$W_{\mathcal{R}}(\eta) \equiv \frac{1}{z}\frac{d^2 z}{d\eta^2} = \mathcal{V}_{\mathcal{R}}(\eta) + \frac{\nu_{\mathcal{R}}^2 - \frac{1}{4}}{\eta^2} \tag{3.6}$$

where $\nu_{\mathcal{R}} = \frac{3}{2} + 3\,\epsilon_v - \eta_v$ [see eq.(2.5)] and z is defined by eq.(2.3).

In order to compute $W_{\mathcal{R}}(\eta)$, it is more convenient to pass to cosmic time, in terms of which,

FELT BY THE FLUCTUATIONS W(η)

With the notation defined by eqs.(2.6) and (3.3) we find,

EXACT W(η)

$$W_{\mathcal{R}}(\eta) = C^2(\eta)\, H^2 \left[2 - 7\,y^2 + 2\,y^4 - (3 - y^2)(4\,\sqrt{\epsilon_v}\;|y|\;\text{sign}(\dot{\Phi}) + \eta_v)\right] . \tag{3.9}$$

In order to clearly exhibit the natural scale of the potential $W_{\mathcal{R}}(\eta)$ it is convenient to use the variables [24]

$$V = N\, m^2\, M_{Pl}^2\; w(\chi) \quad , \quad \Phi = \sqrt{N}\, M_{Pl}\, \chi \quad , \quad H = m\, h\, \sqrt{N} \quad , \quad t = \frac{\sqrt{N}}{m}\, \tau\,, \tag{3.10}$$

where $N \sim 55$ is the number of efolds during slow roll and m (the inflaton mass) defines the scale of the Hubble parameter during the stage of slow roll inflation.

This rescaling builds in the natural scales and results in that $w(\chi) \sim 1$, $h \sim 1$ during the slow roll stage of inflation. Furthermore, as shown in ref.[24], the hierarchy of slow roll parameters is actually a hierarchy in powers of $1/N$, for example

HIERARCHY IN POWERS OF 1/N

$$\epsilon_v = \frac{1}{2N}\left(\frac{w'}{w}\right)^2 \quad , \quad \eta_v = \frac{1}{N}\,\frac{w''}{w}\,. \tag{3.11}$$

($\frac{1}{N}$ EXPANSION)

In terms of these variables we obtain for the exact potential,

EXACT W(η)

$$W_{\mathcal{R}}(\eta) \;=\; C^2(\eta)\; h^2\; m^2\; N\Big[2 - 7\,y^2 + 2\,y^4 - 2\,\sqrt{\frac{2}{N}}\,\frac{w'}{h^2}\;|y|\;\text{sign}(\dot{\Phi}) - \frac{w''}{h^2\,N}\Big] \quad ,$$

$$y^2 \;=\; 3\left(1 - \frac{w}{3\,h^2}\right) = \frac{\dot{\chi}^2}{2\,h^2} > 0\,. \tag{3.12}$$

$O(\frac{1}{N})$ $O(\frac{1}{N})$

displaying that for $y \sim \mathcal{O}(1)$ the last two terms in $W_{\mathcal{R}}(\eta)$ eq.(3.12) are of order $\mathcal{O}(1/N) \ll 1$ and can be neglected.

The above expressions in terms of the variable y are exact and allow to analyze, besides slow roll inflation, other regimes for inflation different from slow roll. Recall the expression for $W(\eta)$ in terms of the slow roll parameters as given by :

$$W_{\mathcal{R}}(\eta) = a^2\; h^2 m^2 N^2\; \left[2 + 2\,\epsilon_v - 3\,\eta_H + 2\,\epsilon_v^2 - 4\,\epsilon_v\,\eta_H + \eta_H^2 + \psi_H^2\right] \,, \tag{3.13}$$

... $\frac{1}{\ }\; w'w'''$ This expression is exact and appropriate in the slow

$$W_{\mathcal{R}}(\eta) = a^2\, h^2 m^2 N^2\, \left[2 + 2\,\epsilon_v - 3\,\eta_H + 2\,\epsilon_v^2 - 4\,\epsilon_v\,\eta_H + \eta_H^2 + \psi_H^2\right] , \tag{3.13}$$

where, $\eta_H = \eta_v - \epsilon_v$, $\psi_H = \psi_v - 3\,\epsilon_v\,\eta_v + 3\,\epsilon_v^2$, $\psi_v = \frac{1}{N^2}\,\frac{w' w'''}{w^2}$. This expression is exact and appropriate in the slow roll approximation, but it is not convenient in regimes different from slow roll.

In the slow roll approximation,

$$y^2 = \epsilon_v = \mathcal{O}(1/N) \ll 1 \;\; ; \;\; C(\eta) = -\frac{1}{\eta\, H\,(1-\epsilon_v)} , \tag{3.14}$$

and we recover

IN SLOW ROLL WE RECOVER

$$W_{\mathcal{R}}^{sr}(\eta) = \frac{2}{\eta^2}\left[1 + \frac{3}{2}\,(3\,\epsilon_v - \eta_v)\right] \Rightarrow \mathcal{V}_{\mathcal{R}}^{sr}(\eta) = 0\,. \tag{3.15}$$

$\mathcal{V}_{\mathcal{R}}^{sr} = 0$

As shown in eq.(3.5) the range of the variable y^2 for inflationary evolution is $0 < y^2 < 1$, which in turn implies:

RANGE FOR INFLATIONARY EVOLUTION

$$3 > \frac{w}{h^2} > 2 \quad \text{or} \quad \sqrt{\frac{w}{3}} < h < \sqrt{\frac{w}{2}}\,. \tag{3.16}$$

The expression of the potential eq.(3.12) in terms of the variable y^2 is very instructive. General properties of $W(\eta)$, such as the sign of the potential, can be analyzed from this expression revealing different regimes. In the fast roll stage (y^2, w'/h^2, $w''/h^2) = \mathcal{O}(1)$ and the dominant part of $W(\eta)$ is given by the polynomial in y, the terms in the derivatives w' and w'' are of order $O(1/\sqrt{N})$ and $O(1/N)$ respectively, namely:

FAST ROLL

$$W_{\mathcal{R}}(\eta) = C^2 H^2 \left[2 - 7\,y^2 + 2\,y^4 + \mathcal{O}\left(\frac{1}{\sqrt{N}}\right)\right] \tag{3.17}$$

The roots of $W_{\mathcal{R}}(\eta)$ are up to corrections $\mathcal{O}\left(\frac{1}{\sqrt{N}}\right)$

ROOTS OF $W_{\mathcal{R}}(\eta)$ y_+, y_-

$$y_-^2 = \frac{7-\sqrt{33}}{4} = 0.31386\ldots \quad , \quad y_+^2 = \frac{7+\sqrt{33}}{4} = 3.13859\ldots .$$

The potential $\mathcal{V}_{\mathcal{R}}(\eta)$ is obtained by subtracting the slow roll contribution from $W(\eta)$, namely

$\mathcal{V}_{\mathcal{R}}(\eta)$

$$\mathcal{V}_{\mathcal{R}}(\eta) = W_{\mathcal{R}}(\eta) - \frac{2 + 9\,\epsilon_v - 3\,\eta_v + \mathcal{O}\left(\frac{1}{N^2}\right)}{\eta^2} , \tag{3.18}$$

in the fast roll stage

FAST ROLL

$$\mathcal{V}_{\mathcal{R}}(\eta) = C^2 H^2 \left[2 - 7\,y^2 + 2\,y^4 + \mathcal{O}\left(\frac{1}{\sqrt{N}}\right)\right] - \frac{2 + 9\,\epsilon_v - 3\,\eta_v + \mathcal{O}\left(\frac{1}{N^2}\right)}{\eta^2}\,. \tag{3.19}$$

Thus, the full range $0 \leq y^2 \leq 3$, the range $y^2 < 1$ for which inflation occurs and the roots of $W(\eta)$ allow to identify three different regimes:

(1)
- $0 < y^2 < y_-^2$, in this region the potential $\mathcal{V}_{\mathcal{R}}(\eta)$ is repulsive and small. This regime includes slow roll inflation for $y^2 = \mathcal{O}\left(\frac{1}{N}\right) \ll 1$.

(2)
- $y_-^2 < y^2 < 1$, corresponds to a *fast roll* inflationary regime in which $W_{\mathcal{R}}(\eta)$ is attractive and consequently $\mathcal{V}_{\mathcal{R}}(\eta)$ is attractive.

(3)
- $1 < y^2 \leq 3 < y_+^2$ describes a fast roll but non-inflationary regime in which the potentials $W_{\mathcal{R}}(\eta)$ and $\mathcal{V}_{\mathcal{R}}(\eta)$ are attractive.

In summary, when the initial value of y^2 is $\gtrsim 1$ the dynamics drives it monotonically towards smaller values. The inflaton friction term continues to dissipate away the kinetic energy and when $y^2 < y_-^2$, the potential $\mathcal{V}_{\mathcal{R}}(\eta)$ becomes repulsive but small and finally when $y^2 \ll y_-^2$ the dynamics enters the slow roll inflationary regime in earnest. Unless the initial conditions on the inflaton determine that $y^2 < y_-^2$, there is *always* a period of *fast roll* inflation during which the potential $\mathcal{V}(\eta)$ for both curvature and tensor perturbations is **attractive**. As we will see below, this attractive fast roll potential $\mathcal{V}(\eta)$ produces **a suppression** of the quadrupole contributions to the angular power spectrum.

FAST ROLL STAGE

C. Tensor perturbations during the fast roll stage

TENSOR PERTURBATION (GRAVITONS)

The mode functions for tensor perturbations (gravitons) obey eq.(2.1) with

in conformal time η
$$W_T(\eta) \equiv C''(\eta)/C(\eta)\,.$$

Again, it is convenient to pass to cosmic time in terms of which,

in cosmic time t
$$W_T(\eta) = a^2(t)\,H^2(t)\left[2+\frac{\dot H}{H^2}\right] = C^2(\eta)\,H^2\,(2-y^2)\,, \tag{3.20}$$

$W(\eta)$ POTENTIAL FELT BY THE FLUCTUATION $W_T(\eta)$

where we used the equation of motion (3.4).

In the slow roll limit $y = \epsilon_v = \mathcal{O}\left(\frac{1}{N}\right) \ll 1$, $\mathcal{V}_T(\eta) = 0$ and eq.(2.7) becomes a Bessel equation,

in the SLOW ROLL limit:
$$\left[\frac{d^2}{d\eta^2} + k^2 - \frac{\nu_T^2 - \frac{1}{4}}{\eta^2}\right] S(k;\eta) = 0\,,$$

where

slow roll:
$$\nu_T = \frac{3}{2} + \epsilon_v + \mathcal{O}\left(\frac{1}{N^2}\right)\quad,\quad W_T^{sr}(\eta) \simeq \frac{2+3\,\epsilon_v}{\eta^2}\quad \text{and}\quad \mathcal{V}_T^{sr}(\eta) = 0\,.$$

$\mathcal{V}_T(\eta)$

- Notice that ν_T differs from the index $\nu_{\mathcal{R}}$ of the scalar fluctuations at order $\mathcal{O}\left(\frac{1}{N}\right)$ [see eq.(2.5)].

During the fast roll stage previous to the slow roll regime, $y > 0$ is not small and introduces an **attractive potential** $\mathcal{V}_T(\eta)$,

FAST ROLL
$$\mathcal{V}_T(\eta) = W_T(\eta) - \frac{2+3\,\epsilon_v}{\eta^2} < 0\,.$$

$\mathcal{V}_T(\eta) \neq 0$

$\mathcal{V}_T(\eta) < 0$

SLOW ROLL : $\mathcal{V}_T(\eta) = 0$

FAST ROLL INFLATION

10

FAST ROLL DYNAMICS :

D. Fast roll in new and chaotic inflation

We consider models both of new inflation (small inflaton field) and chaotic inflation (large inflaton field) to investigate the fast roll dynamics prior to slow roll and its imprint on the quadrupole mode as well as in the higher l-modes. Let us focus first on new inflation with the inflaton potential

NEW INFLATION
$$V(\Phi) = V(0)\left[1 - \lambda\,\frac{M_{Pl}^2}{m^2}\,\frac{\Phi^2}{M_{Pl}^2}\right]^2 \quad ; \quad V(0) \equiv 3\,H_i^2\,M_{Pl}^2\,, \tag{3.21}$$

where H_i is the Hubble parameter during slow roll inflation. We note that during slow roll $\lambda\,\frac{M_{Pl}^2}{m^2} = -\eta_v/4$ and take $\lambda\,\frac{M_{Pl}^2}{m^2} = 0.008$ as an example for numerical study. We solve the equations (3.1) with the initial conditions $\Phi(0)/M_{Pl} = 0$; $\dot\Phi^2(0)/[2\,V_0] = 1$; $a(0) = 1$. These initial conditions entail *an equipartition* between the kinetic and potential energy of the inflaton field at the initial time. Fig. 1 displays $y^2(\eta)$ (left panel) and $y^2(N_e)$ (right panel) with N_e the number of e-folds from the beginning of the evolution at $t = 0$.

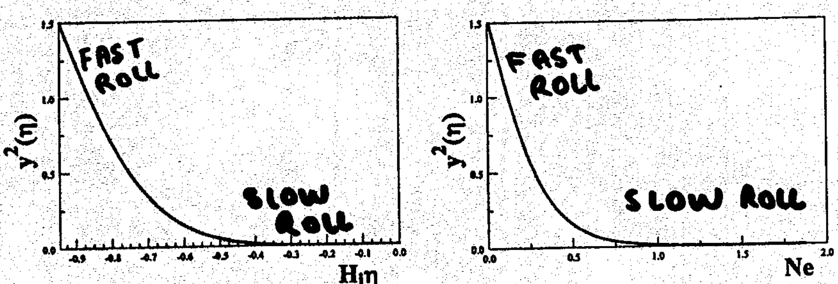


FIG. 1: $y^2(\eta)$ vs. η (left) and $y^2(N_e)$ vs. N_e (right) for initial conditions with kinetic and potential inflaton energy of the same order.

(11

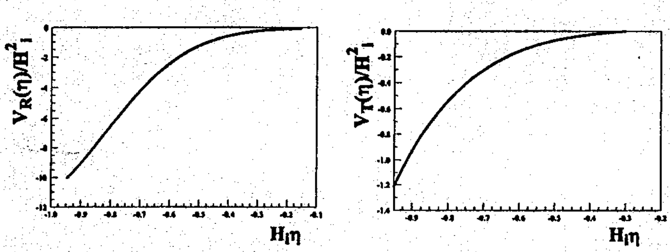
same order.

These conditions initially yield $y^2 > 1$ which produces non-inflationary dynamics, but after a very short time (about one e-fold) y^2 drops below one and so inflationary dynamics begins in the *fast roll* regime $y = \mathcal{O}(1)$, and after about one half e-fold when $y^2 \sim 0.02$ slow roll inflation begins in earnest.

The potentials $\mathcal{V}_{\mathcal{R}}(\eta)$ (left panel) and $\mathcal{V}_T(\eta)$ (right panel) are shown in fig. 2, and the evolution of the Hubble parameter is displayed in fig. 3. Figures (1) and (2) show two distinct time scales: $\eta_0 \approx -1/H_i$ at which the potential is localized and features its minimum, this is the beginning of the fast roll stage, and $\overline{\eta} \sim -0.3/H_i$ at which the potential vanishes, $y^2 \approx \epsilon_v$ and slow roll begins. The brief *fast roll* stage is clearly seen from these figures to correspond to the first e-fold of evolution. Fig. (3) confirms that the fast roll stage lasts approximately one e-fold and that $\overline{\eta}$ corresponds to about 56-57 e-folds before the end of inflation, namely $1-2$ e-folds before the modes corresponding to today's Hubble radius exit the horizon during inflation.

For these parameters, the height of the potentials are approximately $|\mathcal{V}_{\mathcal{R}}| \sim 10\, H_i^2$; $|\mathcal{V}_T| \sim 1.2\, H_i^2$. The widths of the potentials are approximately the same in both cases $|\Delta/\eta_0| \sim \Delta\, H_i \sim 0.2$, see fig. 2.

$\mathcal{V}_R(\eta)$ POTENTIAL FELT BY CURVATURE PERTURBATIONS



$\mathcal{V}_T(\eta)$ POTENTIAL FELT BY TENSOR PERTURBATIO

FIG. 2: The potentials $\mathcal{V}_{\mathcal{R}}(\eta)/H_i^2$ (left panel) and $\mathcal{V}_T(\eta)/H_i^2$ (right panel) felt by curvature and tensor perturbations respectively vs $H_i\,\eta$, H_i being the Hubble parameter during the slow roll stage (see fig.3).

We have carried out analogous numerical studies in scenarios of chaotic inflation with similar results: if the initial kinetic energy of the inflaton is of the same order as the potential energy, *a fast roll stage is always present*. The evolution of y^2 and the potentials for curvature and tensor perturbations, $\mathcal{V}_{\mathcal{R}}(\eta)$ and $\mathcal{V}_T(\eta)$ are again similar to those for new inflation and they are always *attractive* during the fast roll stage.

An initial state for the *inflaton* (inflaton classical dynamics) with approximate *equipartition* between kinetic and potential energies is a more *general* initialization of cosmological dynamics in the effective field theory than slow roll which requires that the inflaton kinetic energy is much more smaller than its potential energy. Therefore, we conclude that the most *generic* initialization of the inflaton dynamics in the effective field theory leads to a *fast roll* stage followed by slow roll inflation.

QUADRUPOLE

IV. QUADRUPOLE SUPPRESSION

SUPPRESSION

In the Born approximation, the Bogoliubov coefficients eqs.(2.19) are given by [23],

(1)

$$A(k) = 1 + i \int_{-\infty}^{0} \mathcal{V}(\eta)\, |g_\nu(k;\eta)|^2\, d\eta \quad , \quad B(k) = -i \int_{-\infty}^{0} \mathcal{V}(\eta)\, g_\nu^2(k;\eta)\, d\eta\,. \tag{4.1}$$

The transfer function of initial conditions given by eq.(2.22) can be computed in the Born approximation, which is appropriate in this situation, using eqs.(4.1) for the Bogoliubov coefficients $A(k)$ and $B(k)$,

TRANSFERT FUNCTION

$$D(k) = \frac{1}{k} \int_{-\infty}^{0} d\eta\, \mathcal{V}(\eta) \left[\sin(2\,k\,\eta) \left(1 - \frac{1}{k^2\,\eta^2}\right) + \frac{2}{k\,\eta} \cos(2\,k\,\eta) \right]. \tag{4.2}$$

OF INITIAL CONDITIONS

(2) The fractional change in the C_l's is obtained by inserting this transfer function in the expression (2.24). We take the lower limit in the integral in eq.(4.2) to be $\eta_0 \sim -1/H_i$ at which the fast roll stage begins. The results of the

(3) numerical integrations for the quadrupole $l = 2$ and the higher multipoles are shown in fig.4.

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The results displayed in this figure are strikingly similar to those found in the examples studied in sections V.B and V.C of ref.[23] lending support to the conclusion that the quadrupole suppression as a consequence of the attractive fast roll potential $\mathcal{V}(\eta)$ is robust.

From eq.(2.29), the relevant dimensionless ratio $\frac{\kappa}{H_i}$ that governs the multipole suppression $\Delta C_l/C_l$, is

$$\frac{\kappa}{H_i} = \frac{a_{sr}}{3.3} , \tag{4.3}$$

where a_{sr} is the scale factor when the mode corresponding to the quadrupole wave vector k_Q exits the Hubble radius during inflation.

We have fixed the initial value for the evolution to be at $\eta = \eta_0$ with $C(\eta_0) \equiv 1$, thus $a_{sr} > 1$ is the logarithm of the number of e-folds between the initial time of the evolution and horizon crossing of k_Q. The left panel of fig. 4 clearly shows that the largest suppression for the quadrupole corresponds to smallest values of a_{sr}, with a $10 - 20\%$ suppression for $2 \leq \kappa/H_i < 3$. This precisely corresponds to $2-3$ e-folds between the onset of the fast roll stage at η_0 and horizon crossing of the mode corresponding to today's Hubble radius. The fast roll stage itself only lasts about one e-fold and is followed by slow roll.

Thus, we conclude that there is a substantial suppression of the quadrupole $\sim 10 - 20\%$ consistent with the observations, if k_Q exits the horizon within a couple of e-folds after the beginning of the slow roll stage, preceded by a short fast roll stage. Therefore, the observed quadrupole suppression is successfully explained by the inflationary dynamics –fast roll followed by slow roll – if inflation lasts not much more than approximately $N_{tot} \sim 59$ e-folds.

The similar form of the tensor potential $\mathcal{V}_T$ leads to a similar behavior in the change of the $C_l's$ for the B-modes, and the fractional change for the quadrupole of tensor modes is smaller by almost an order of magnitude as gleaned from the potentials displayed in fig. 2. This is a *general* prediction, again a consequence of a fast roll stage prior to slow roll.

A numerical analysis reveals that $\Delta C_l/C_l \sim 1/l^2$ in agreement with the result of eq.(2.28), therefore the suppression in the higher multipoles falls below the band of irreducible cosmic variance and it is too small to be observable within the present data.

A numerical fit to the curvature potential $\mathcal{V}_\mathcal{R}(\eta)$ yields

EXPLICIT

$$\mathcal{V}_\mathcal{R}(\eta) \simeq \mathcal{V}_\mathcal{R}(\eta_0)\, e^{-(\eta-\eta_0)/\Delta} , \tag{4.4}$$

$\mathcal{V}_\mathcal{R}(\eta)$

ANALYTIC EXPRESSIONS

$$\mathcal{V}_\mathcal{R}(\eta) \simeq \mathcal{V}_\mathcal{R}(\eta_0)\, e^{-(\eta-\eta_0)/\Delta} , \tag{4.4}$$

$\mathcal{V}_\mathcal{R}(\eta)$

with $\eta_0 \sim -1/H_i$ and $|\Delta/\eta_0| \sim 0.2$. With this analytic expression which provides an excellent fit, we obtain the following asymptotic behavior of the transfer function $D_\mathcal{R}(k)$ and distribution function $N_\mathcal{R}(k)$ for large momenta:

TRANSFERT FUNCTIONS $D_\mathcal{R}(k)$

$$D_\mathcal{R}(k) \overset{k\to\infty}{=} \frac{\mathcal{V}_\mathcal{R}(\eta_0)}{4\,k^2} \quad , \quad N_\mathcal{R}(k) \overset{k\to\infty}{=} \frac{\mathcal{V}^2_\mathcal{R}(\eta_0)}{16\,k^4} , \tag{4.5}$$

NUMBER MODE FUNCTION $N_\mathcal{R}(k)$

clearly indicating that these initial conditions are indeed ultraviolet allowed and consistent with the form eq.(2.27). $\mathcal{V}_\mathcal{R}(\eta)$ to We notice from figs. 1 and 2 that indeed $\mathcal{V}_\mathcal{R}(\eta)$ vanishes when the slow roll regime $y^2 \ll 1$ is reached.

From eq.(3.18) with $y^2(\eta_0) \sim 1$, $C(\eta_0) = 1$ and taking the initial conditions on the inflaton with approximate equipartition between potential and kinetic energies, implies that $H^2(\eta_0) \sim 2\, H_i^2$ yielding

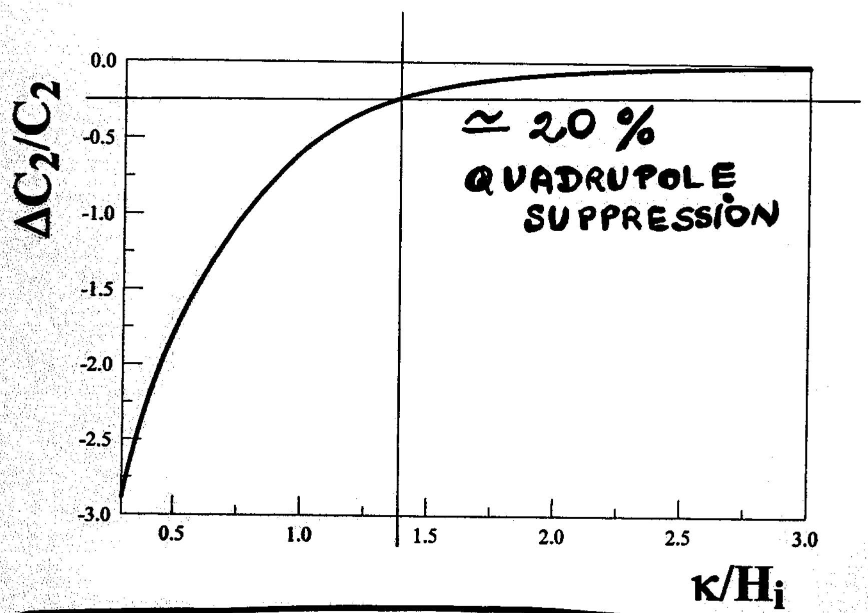
$$\mathcal{V}_\mathcal{R}(\eta_0) \sim -10\, H_i^2 \quad , \quad \mathcal{V}_T(\eta_0) \sim -2\, H_i^2 , \tag{4.6}$$

which is consistent with fig. 4. Comparing with the form eq.(2.27), and taking as an example $N_\mu \sim 0.01$, indicates that the characteristic asymptotic k-scale μ at which the asymptotic form eq.(2.27) sets is $\mu \approx 10\, H_i$, namely a few times the Hubble scale during slow roll inflation. This shows that the energy scales involved in the quadrupole suppression are of the *same* order as the scale of inflation.

Therefore, the condition for observable suppression of the quadrupole is that the modes with physical wavelengths of the order of the Hubble radius today must cross the horizon during inflation just $1-2$ e-folds after the beginning of the slow roll stage. This condition is easily understood from the approximate form eq.(4.5) of the transfer function $D(k)$. Since $D(k)$ is strongly suppressed for $k^2 \gg |\mathcal{V}|$, the potential $\mathcal{V}(\eta)$ will substantially influence the modes with wavevector k if $k^2 \lesssim |\mathcal{V}(\eta_0)| \sim 10\, H_i^2$. Since $k = a_{sr}\, H_i$, then clearly only $1-2$ e-folds of evolution between the end of fast roll and the horizon crossing lead to substantial effects on the mode functions from the $\mathcal{V}(\eta)$ potential.

We have also studied chaotic inflationary scenarios with initial conditions on the inflaton characterized by $y^2 \sim 1$ namely with inflaton kinetic energy of the same order as the inflaton potential energy. We find similar results on the fractional variation of low multipoles, the duration of the fast roll stage and the scale of the fast roll potentials $\mathcal{V}_\mathcal{R}(\eta)$, $\mathcal{V}_T(\eta)$ as for new inflation.

QUADRUPOLE SUPPRESSION



$\mathcal{H} = \frac{a_0 H_0}{3.3}$ H_i = HUBBLE during slow roll

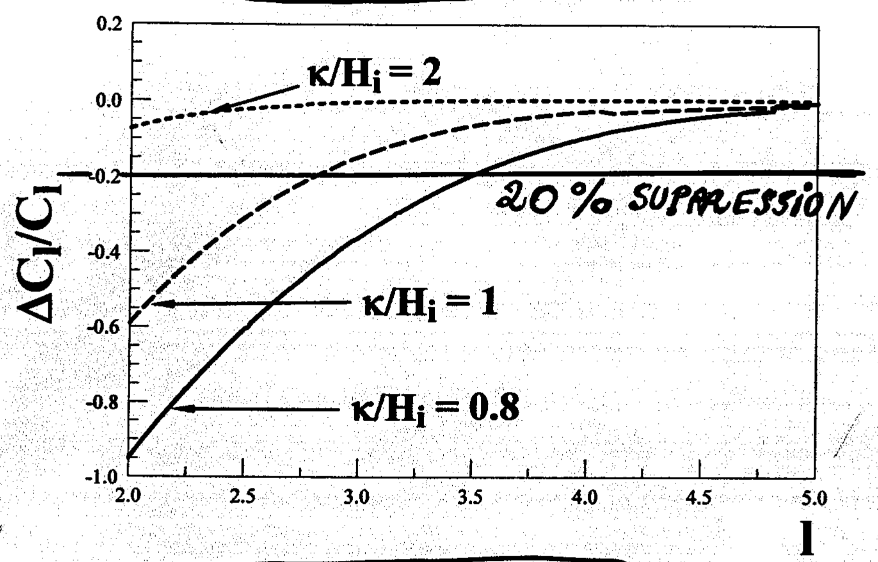
20% QUADRUPOLE SUPPRESSION for $\frac{\mathcal{H}}{H_i} \simeq 1.4$

$k_Q \simeq a_0 H_0 = a_{sr} H_i \Rightarrow \frac{\mathcal{H}}{H_i} = \frac{a_{sr}}{3.3} = 1.4$

$\rightarrow a_{sr} \simeq 4.6 \simeq e^{1.5}$

The Q-Modes should EXIT the Horizon 1.5 e-folds AFTER FAST ROLL STARTS (and 0.5 efold AFTER SLOW ROLL BEGINS)

$\frac{\Delta C_\ell}{C_\ell}$ vs ℓ



$\rightarrow$ $\boxed{\frac{\Delta C_\ell}{C_\ell} \sim \frac{1}{\ell^2}}$ $\leftarrow$

FOR $\frac{\Delta C_2}{C_2} \sim -0.2$,

$\rightarrow \frac{\Delta C_\ell}{C_\ell}$ FOR $\ell > 2$ } CANNOT BE SEEN WITH THE PRESENT DATA

(13

Therefore, we conclude that the phenomena associated with the fast roll stage as a precursor to slow roll are robust, they *do not* depend on the inflationary model, but solely on the scale of inflation and on approximate equipartition between the kinetic and potential energies in the initial condition for the classical dynamics of the inflaton. This initialization of the inflaton dynamics and inflationary potentials that fulfill the slow roll conditions generally guarantee that the dynamical evolution of the inflaton features an initial fast roll stage that merges with the usual slow roll inflationary stage. In turn, the fast roll stage results in an attractive potential in the wave equations for the mode functions of curvature an tensor perturbations, and a consequent suppression of the quadrupole moment in their power spectra.

V. THE EVOLUTION OF PERTURBATIONS AS A SCATTERING PROBLEM.

The equivalence between the equations for the mode functions and the Schrödinger equation with a potential allows us to bring to bear the powerful results of potential scattering theory to provide general statements on the properties of the solutions.

Eq.(2.1) has the form of the radial Schrödinger equation in the radial variable $r \equiv -\eta$, $0 \leq r < \infty$ for the L-wave, being L a real number, $L \equiv \nu - \frac{1}{2}$. We recognize in eq.(2.7) the centrifugal barrier

CENTRIFUGAL BARRIER

$$\boxed{\frac{\nu^2 - \frac{1}{4}}{\eta^2} = \frac{L(L+1)}{\eta^2} \quad , \quad L \equiv \nu - \frac{1}{2}, \quad r \equiv -\eta.}$$

Thus, in the slow roll regime:

$$L \equiv \nu - \frac{1}{2} = 1 + \mathcal{O}\left(\frac{1}{N}\right) .$$

ENERGY

POTENTIAL

Eq.(2.7) takes then the form

SCHRÖDINGER EQUATION

$$\boxed{\left[\frac{d^2}{dr^2} + k^2 - \frac{L(L+1)}{r^2} - \mathcal{V}(r)\right] f_\nu(k,r) = 0 .} \tag{5.1}$$

The scattering solution of eq.(5.1) with unit outgoing amplitude is defined by

$$r \to +\infty \quad ikr \tag{5.2}$$

$$\left[dr^2 \quad r^2 \quad \right]$$

The scattering solution of eq.(5.1) with unit outgoing amplitude is defined by

JOST SOLUTION

$$f_\nu(k,r) \overset{r \to +\infty}{=} e^{ikr} \tag{5.2}$$

This solution $f_\nu(k,r)$ is called the Jost solution in scattering theory [28], it is identical to the Bunch-Davies initial conditions eq.(2.13) up to a normalization factor $\sqrt{2\,k}$.

When $\mathcal{V}(r) = 0$ the Jost solution is given by

$$f_\nu(k,r)_{\mathcal{V}=0} = i^{\nu+\frac{1}{2}} \sqrt{\pi\, k\, r}\; H^{(1)}_\nu(k\, r) .$$

This function coincides with eq.(2.10) up to a normalization factor $\sqrt{2\,k}$. In particular,

$$f_\nu(k,r)_{\mathcal{V}=0} \overset{r \to 0}{=} \frac{\Gamma(\nu)}{\sqrt{\pi}} \left(\frac{k\,r}{2\,i}\right)^{\frac{1}{2}-\nu} . \tag{5.3}$$

For $r \to 0$, eq.(2.7) has two linearly independent solutions of the form: $r^{\frac{1}{2}-\nu}$ and $r^{\frac{1}{2}+\nu}$; since $\nu > 0$ the first solution dominates the behaviour of $f_\nu(k,r)$ for $r \to 0$.

The Jost function of scattering theory is defined as the ratio

JOST FUNCTION

$$F_\nu(k) \equiv \lim_{r \to 0} \frac{f_\nu(k,r)}{f_\nu(k,r)_{\mathcal{V}=0}} = \frac{\sqrt{\pi}}{\Gamma(\nu)} \lim_{r \to 0} \left(\frac{k\,r}{2\,i}\right)^{\nu-\frac{1}{2}} f_\nu(k,r) . \tag{5.4}$$

A. Scattering solutions and the primordial power

By construction, the solution $S(k;\eta)$ fulfils the Bunch-Davies asymptotic condition

$$S(k;\eta) \overset{\eta \to -\infty}{=} \frac{e^{-ik\eta}}{\sqrt{2k}} \tag{5.5}$$

This solution $S(k;\eta)$ is proportional to the scattering Jost solution as

$$S(k;\eta) = \frac{1}{\sqrt{2k}}\, f_\nu(k,r) \quad \text{with} \quad r = -\eta > 0 \quad . \tag{5.6}$$

It can be shown on general grounds that $f_\nu(k,r)$ is an analytic function of k for $\mathrm{Im}k > 0$ and $k \neq 0$ [28]. Moreover, $k^{\nu-\frac{1}{2}}\, f_\nu(k,r)$ as well as $k^\nu\, S(k;\eta)$ are analytic in a neighbourhood of and including $k = 0$.

For $\eta \to 0^-$, eq.(2.7) admits two independent solutions: $(-\eta)^{\frac{1}{2}-\nu}$ and $(-\eta)^{\frac{1}{2}+\nu}$. Since $\nu > 0$, the first solution is the irregular one for $\eta \to 0^-$ and it dominates over the regular solution $(-\eta)^{\frac{1}{2}+\nu}$.
The $\eta \to 0^-$ behaviour of the modes in the $\mathcal{V}(\eta) \equiv 0$ case is given by eq.(2.14), while in the general case $\mathcal{V}(\eta) \neq 0$ we have

$$S(k;\eta) \overset{\eta\to 0^-}{=} \frac{\Gamma(\nu)}{\sqrt{2\,\pi\,k}} \left(\frac{2}{i\,k\,\eta}\right)^{\nu-\frac{1}{2}} F_\nu(k) \,, \tag{5.7}$$

where $F_\nu(k)$ stands for the Jost function. It follows that $F_\nu(k)$ is analytic for $\mathrm{Im}k > 0$ and [28]

$$\lim_{k\to\infty} F_\nu(k) = 1 \,. \tag{5.8}$$

The primordial power spectra are given by eqs.(2.21). Eq.(2.23) for Bunch-Davies (BD) initial conditions is valid when $\mathcal{V}(\eta) = 0$ and the mode functions behave as in eq.(2.14) for $\eta \to 0^-$. From eqs.(2.14) and (5.7) we find for $\mathcal{V} \neq 0$,

PRIMORDIAL POWER SPECTRUM →

$$\boxed{\frac{|S(k;\eta)|^2}{|g_\nu(k;\eta)|^2} \overset{\eta\to 0^-}{=} |F_\nu(k)|^2 \quad .} \tag{5.9}$$

Therefore, we find the equivalence,

$$\boxed{\frac{P(k)_\mathcal{V}}{P^{sr}(k)} = |F_\nu(k)|^2 = 1 + D(k) \quad .} \tag{5.10}$$

Namely, $|F_\nu(k)|^2$ yields the change in the primordial power spectrum due to the potential $\mathcal{V}(\eta)$. This is an important result, which allows to obtain general information on the transfer function of initial conditions $D(k)$ from established results of potential scattering.

We obtain the Jost function $F_\nu(k)$ from the $\eta \to 0^-$ behavior of eq.(2.16) with the result

$$F_\nu(k) = 1 + i^{\frac{1}{2}-\nu}\,\sqrt{\pi}\int_{-\infty}^0 \sqrt{-\eta}\, d\eta\; J_\nu(-k\,\eta)\; \mathcal{V}(\eta)\; S(k;\eta) \quad . \tag{5.11}$$

where $J_\nu(z)$ is Bessel's function.

In the scale invariant case $\nu = \frac{3}{2}$ the Jost function takes the simpler form

IN PARTICULAR, SCALE INVARIANT CASE :

$$F_{\frac{3}{2}}(k) = 1 - i\,\sqrt{\frac{2}{k}}\int_{-\infty}^0 d\eta \left[\frac{\sin(k\,\eta)}{k\,\eta} - \cos(k\,\eta)\right] \mathcal{V}(\eta)\; S(k;\eta) \quad . \tag{5.12}$$

The large k behavior of the Jost solutions and Jost functions follows by solving eqs.(2.16) and (5.11) by iteration. To dominant order we find that the Jost solution is given by the $\mathcal{V}(\eta) = 0$ solution eq.(2.10) while the Jost function equals unity [see eq.(5.8)].

The logarithm of the Jost function has the following asymptotic expansion around $k = \infty$ [29],

ASYMPTOTIC BEHAVIOUR →

$$\boxed{\log F_\nu(k) = -\sum_{n=1}^{\infty} \frac{c_n}{(2\,i\,k)^n} \,,}$$

where the c_n are real coefficients functionals of the potential $\mathcal{V}(\eta)$. The first coefficients take the form,

$$c_1 = \int_{-\infty}^0 d\eta\; \mathcal{V}(\eta) \quad , \quad c_2 = \mathcal{V}(\bar{\eta}) \quad .$$

(15

$|F_\nu(k)|^2 < 1$

$\mathcal{V}(\eta) < 0$

Therefore,

$$\boxed{\log|F_\nu(k)|^2 = \frac{\mathcal{V}(\bar\eta)}{2\,k^2} + \mathcal{O}\left(\frac{1}{k^4}\right) \quad .} \tag{5.13}$$

We see that asymptotically $|F_\nu(k)|^2 < 1$ for a potential which is attractive at the end of fast roll [$\mathcal{V}(\bar\eta) < 0$]. Combined with eq.(5.10) this result shows in general that an attractive potential $\mathcal{V}(\eta)$ suppresses the primordial power.

Computing the $\eta \to 0^-$ behavior of $S(k;\eta)$ from eq.(2.18) permits to relate the Bogoliubov coefficients $A(k)$ and $B(k)$ with the Jost function as

$$\boxed{A(k) + i^{1-2\nu}\,B(k) = F_\nu(k) \quad .} \tag{5.14}$$

where we used eqs.(2.10) and (5.7).

Therefore,

$$\boxed{|F_\nu(k)|^2 - 1 = 2\,|B(k)|^2 - 2\,\text{Re}\,[A(k)\,B^*(k)\,i^{2\nu-3}] = D(k) \; .} \tag{5.15}$$

and we recover the transfer function for the initial conditions $D(k)$ introduced in ref.[23]. Using eq.(2.20), eq.(5.15) reduces *exactly* to eqs.(2.22).

For large k, the mode functions $S(k;\eta)$ as well as the $g_\nu(k;\eta)$ tend to their plane wave asymptotic behaviour

$$S(k;\eta) \overset{k\to\infty}{=} g_\nu(k;\eta) \overset{k\to\infty}{=} \frac{e^{-ik\eta}}{\sqrt{2k}} \; .$$

A look at eq.(2.18) shows that this implies $B(\infty) = 0$, $A(\infty) = 1$. More precisely, we find from eq.(2.19),

$$\boxed{A(k) \overset{k\to\infty}{=} 1 + \frac{i}{2\,k}\int_{-\infty}^{0} \mathcal{V}(\eta)\,d\eta \quad , \quad B(k) \overset{k\to\infty}{=} -\frac{i}{2\,k}\int_{-\infty}^{0} e^{-2\,i\,k\,\eta}\,\mathcal{V}(\eta)\,d\eta} \tag{5.16}$$

According to the Riemann-Lebesgue lemma, $B(k)$ vanishes for $k \to \infty$ faster than any negative power of k. Hence, the convergence at large k in the integrals for the energy momentum tensor is guaranteed

According to the Riemann-Lebesgue lemma, $B(k)$ vanishes for $k \to \infty$ faster than any negative power of k. Hence, the convergence at large k in the integrals for the energy momentum tensor is guaranteed.

The Bogoliubov coefficients $A(k)$ and $B(k)$ are related to the usual transmission (T) and reflection (R) coefficients of scattering theory by the relation,

$$\boxed{T(k) = \frac{1}{A(-k)} \quad ; \quad R(k) = \frac{B(-k)}{A(-k)} \quad , \quad |R(k)|^2 + |T(k)|^2 = 1 \, .} \tag{5.17}$$

We provide with Table I a dictionary to translate from the fluctuations language to the scattering framework.

Fluctuations	Scattering Problem
$-\infty < \eta < 0$	$0 < r < \infty$
Bunch-Davies initial conditions: $S(k;\eta) = \frac{e^{-ik\eta}}{\sqrt{2k}}$ for $\eta \to -\infty$	Jost solutions: $f_\nu(k,r) = e^{ik\,r}$ for $r \to \infty$
Superhorizon modes: $S(k;\eta) \overset{\eta\to 0^-}{\sim} (-\eta)^{\frac12-\nu}$	Jost Function: $F_\nu(k) \equiv \frac{\sqrt{\pi}}{\Gamma(\nu)}\, \lim_{r\to 0}\left(\frac{k\,r}{2\,i}\right)^{\nu-\frac12} f_\nu(k,r)$
Power spectra $\frac{P_\nu(k)}{P^{sr}(k)}$	Modulus Square of the Jost Function = $= |F_\nu(k)|^2$

DICTIONARY

COSMOLOGICAL FLUCTUATIONS, INITIAL CONDITIONS ⟷ SCATTERING FRAMEWORK

TABLE I. Correspondence between the scalar fluctuations as functions of the conformal time $\eta < 0$ and the radial wave functions, of $r > 0$ and angular momentum $L \equiv \nu - \frac{1}{2}$.

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GENERAL RESULTS

ON QUADRUPOLE B. The quadrupole suppression: General results SUPPRESSION

We now implement the exact relations between the scattering problem and the power spectra of perturbations derived in the previous subsection to obtain general results for the quadrupole produced by the potential $\mathcal{V}(\eta)$. From eq.(2.24) for $l = 2$ and to zeroth order in slow roll, the fractional change in the quadrupole is given by,

ANALYTIC

$$\boxed{\frac{\Delta C_2}{C_2} = \frac{\int_0^\infty D(\kappa\, x)\; f_2(x)\, dx}{\int_0^\infty f_2(x)\, dx} = 3 \int_0^\infty \frac{dx}{x}\; D(\kappa\, x)\; [j_2(x)]^2 \;,} \tag{5.18}$$

where $j_2(x)$ is the spherical Bessel function of order two [26]. We compute the transfer function $D(k)$ from the Jost function using eqs.(5.12) and (5.15)in the Born approximation, which turns be an excellent one for this purpose; since in fact the potential $\mathcal{V}(\eta)$ is small. The Jost function in the Born approximation to zeroth order in slow roll is given by

$$F_{\frac32}(k) = 1 + \frac{i}{2\,k} \int_{-\infty}^0 d\eta\, \mathcal{V}(\eta) \left(1 - \frac{i}{k\,\eta}\right) \left[1 + e^{-2\,i\,k\,\eta} - \frac{1 - e^{-2\,i\,k\,\eta}}{i\,k\,\eta}\right] .$$

Therefore up to first order in $\mathcal{V}(\eta)$ (Born approximation) we find

$$\boxed{D(k) = |F_{\frac32}(k)|^2 - 1 = \frac{1}{k} \int_{-\infty}^0 d\eta\; \mathcal{V}(\eta) \left[\sin(2\,k\,\eta) \left(1 - \frac{1}{k^2\,\eta^2}\right) + \frac{2}{k\,\eta} \cos(2\,k\,\eta)\right] .}$$

Inserting this expression for $D(k)$ into eq.(5.18) yields

GENERAL ANALYTIC RESULT

$$\boxed{\frac{\Delta C_2}{C_2} = \frac{1}{\kappa} \int_{-\infty}^0 d\eta\; \mathcal{V}(\eta)\; \Psi(\kappa\,\eta)} \tag{5.19}$$

SUPPRESSION FOR $\mathcal{V}(\eta) < 0$

where

$$\boxed{\Psi(x) \equiv 3 \int_0^\infty \frac{dy}{y^4}\, [j_2(y)]^2 \left[(y^2 - \frac{1}{x^2}) \sin(2\,y\,x) + \frac{2\,y}{x}\, \cos(2\,y\,x)\right]} \tag{5.20}$$

$$\boxed{\Psi(x) \equiv 3 \int_0^\infty \frac{dy}{y^4}\, [j_2(y)]^2 \left[(y^2 - \frac{1}{x^2}) \sin(2\,y\,x) + \frac{2\,y}{x}\, \cos(2\,y\,x)\right]} \tag{5.20}$$

$\Psi(x)$ is an odd function of x. The integral in eq.(5.20) can be computed in terms of elementary functions by using the power series expansion [30]

$$[j_2(x)]^2 = \frac{\sqrt{\pi}}{2} \sum_{k=0}^\infty \frac{(-1)^k\, x^{2k+4}}{k!\,\Gamma(k+\frac72)\,(k+3)(k+4)(k+5)} \,,$$

with the result

$$\Psi(x) = -\frac{3}{4\,x^3} \sum_{k=0}^\infty \frac{1}{x^{2k}} \frac{1}{(k+\frac32)(k+\frac52)(k+4)(k+5)} \left[1 + \frac{1}{(k+\frac12)(k+3)}\right] =$$
$$= -\frac{1}{x^3} \sum_{k=0}^\infty \frac{1}{x^{2k}} \left[\frac{1}{105}\frac{1}{k+\frac12} + \frac{1}{35}\frac{1}{k+\frac32} - \frac{1}{5}\frac{1}{k+3} + \frac{9}{35}\frac{1}{k+4} - \frac{2}{21}\frac{1}{k+5}\right] . \tag{5.21}$$

These series can be summed up explicitly with the result

$$\Psi(x) = \frac{1}{105\,x^2} \left[p(x)\,(1-x)^3 \log\left|1 - \frac{1}{x}\right| - p(-x)\,(1+x)^3 \log\left|1 + \frac{1}{x}\right|\right] + \frac{2}{105\,x} - \frac{13\,x}{126} + \frac{22\,x^3}{105} - \frac{2\,x^5}{21} \tag{5.22}$$

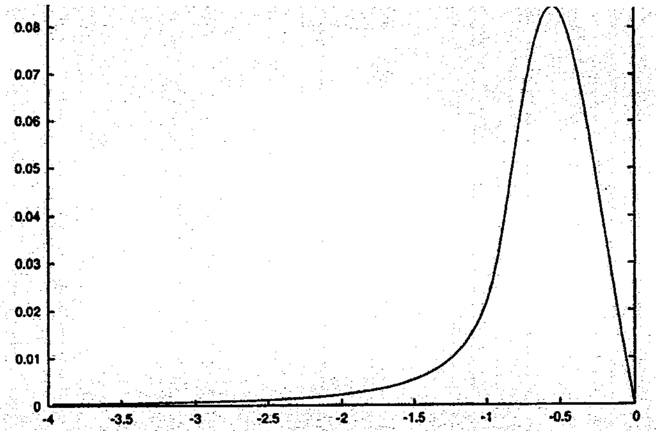
where $p(x)$ is the sixth order polynomial

$$p(x) \equiv 10\,x^6 + 30\,x^5 + 33\,x^4 + 19\,x^3 + 9\,x^2 + 3\,x + 1 \,.$$

The function $\Psi(x)$ is negative for $x > 0$ and positive for $x < 0$. It vanishes for $x \to 0$ and for $x \to \infty$ as,

$$\Psi(x) \overset{x \to 0}{=} -\frac{x}{6} + \mathcal{O}(x^3) \,.$$

(17



$\Psi(-x) = -\Psi(x)$

FIG. 5: The odd function $\Psi(x)$ vs. x for negative x [see eq.(5.20)]. This function convoluted with the potential $\mathcal{V}(\eta)$ yields the change on the quadrupole $\frac{\Delta C_2}{C_2}$ [see eq.(5.19)].

and

$$\Psi(x) \overset{x\to\infty}{=} -\frac{1}{60\,x^3} + \mathcal{O}\left(\frac{1}{x^5}\right) .$$

Fig. 5 displays $\Psi(x)$ as a function of x for negative x . $\Psi(x)$ features a maximum at $x = x_M = -0.555\ldots$ with $\Psi(x_M) = 0.08453\ldots$.

Eq.(5.19) highlights the **general** result that an **attractive** potential $\mathcal{V}(\eta) < 0$ yields a **suppression** of the quadrupole since $\Psi(x) > 0$ for negative values of its argument x.

These results establish unequivocally that the attractive potentials $\mathcal{V}_{\mathcal{R}}(\eta)$; $\mathcal{V}_T(\eta)$, which are a consequence of the fast roll stage, lead to a suppression of the quadrupole for curvature and tensor perturbations.

$\Psi(x)$ is an ODD FUNCTION: $\Psi(-x) = -\Psi(x)$

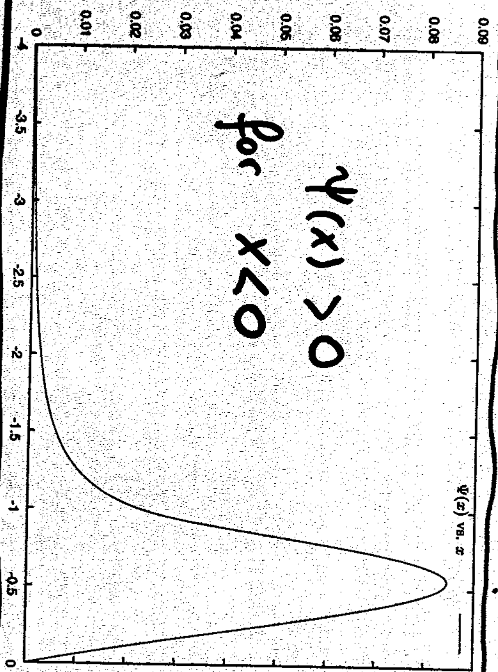


FIG. 1: The odd function $\Psi(x)$ vs. x for negative x [see eq.(5.20)]. This function convoluted with the potential $\mathcal{V}(\eta)$ yields the change on the quadrupole $\frac{\Delta C_2}{C_2}$ [see eq.(5.19)].

$\Psi(x)$ CONVOLUTED with the fluctuations POTENTIAL $\mathcal{V}(\eta)$

→ CHANGE ON THE QUADRUPOLE $\frac{\Delta C_2}{C_2}$

$$\frac{\Delta C_2}{C_2} = \frac{1}{\varkappa}\int_{-\eta_0}^{0} d\eta \, \mathcal{V}(\eta)\, \Psi(\varkappa x) < 0$$

↑ ATTRACTIVE

$\Delta C_\ell < 0 \sim 1/\ell^2$

fast roll stage, lead to a suppression of the quadrupole for

INVERSE PROBLEM

C. The Inverse Problem. Reconstructing the fast roll potential $\mathcal{V}(\eta)$ from the primordial power

In scattering theory, the potential can be obtained from the scattering data, through the Gelfand-Levitan equation. This is a linear integral equation which determines the potential $\mathcal{V}(r)$ from the knowledge of the modulus of the Jost function and the bound states[29].

The Gelfand-Levitan equation can be written as

determines kernel

$$K_\nu(r,r') + G_\nu(r,r') + \int_0^r dr'' \, K_\nu(r,r'') \, G_\nu(r'',r') = 0 \, . \tag{5.23}$$

where $G_\nu(r,r')$ is a known function that can be expressed in terms of the Jost function as follows

$$G_\nu(r,r') = \sqrt{r\,r'} \int_0^\infty k \, dk \, J_\nu(k\,r) \, J_\nu(k\,r') \left[\frac{1}{|F_\nu(k)|^2} - 1\right] \tag{5.24}$$

DEVIATION FROM S-R BD POWER

where the $J_\nu(z)$ are Bessel functions, and the kernel $K_\nu(r,r')$ is obtained by solving eq.(5.23). Once $K_\nu(r,r')$ is computed, the potential follows as

kernel determines $\mathcal{V}(r)$

$$\boxed{\mathcal{V}(r) = 2\,\frac{d}{dr} K_\nu(r,r)} \; . \tag{5.25}$$

the potential $\mathcal{V}(\eta)$

Eq.(5.23) is the Gelfand-Levitan equation in absence of bound states. By bound states we mean solutions of eq.(5.1) which are regular at $r = 0$ and decay exponentially for $r \to \infty$. We will not consider their presence since the analysis in secs. II and III of ref.[23] indicates that bound states are absent in the present case.

$\mathcal{V}_R(\eta) \to W_R(\eta)$ expl. determined from the primordial power data

We have seen in eq.(5.10) that the deviation of the primordial power from slow roll is given by the square modulus of the Jost function. Eqs.(5.23)-(5.25) show that this deviation from the BD-slow roll primordial power explicitly determines the potential $\mathcal{V}(\eta)$. The present quantitative information about the deviation of the primordial power from slow roll is too scarce to feed back into the Gelfand-Levitan equation, but it is important to see that the fast roll potential $\mathcal{V}_{\mathcal{R}}(\eta)$ felt by the fluctuations and hence $W_{\mathcal{R}}(\eta)$ can be explicitly determined from the primordial power data.

VI. CONCLUSIONS

Although the latest analysis of the WMAP data confirms the basic paradigm of slow roll inflation and renders much less statistical significance to potential departures from its basic predictions, the anomalously low quadrupole in the CMB remains a long-standing challenge.

(0) In this article we proposed a mechanism that yields a suppression of the low multipoles both for curvature and tensor perturbations, *within the effective field theory of inflation.* The main premise of our observation is that a more general initialization of the classical dynamics of the inflaton, allowing for approximate equipartition between initial kinetic and potential energy of the inflaton leads to a brief period of ***fast roll dynamics*** that is the precursor to the usual slow roll stage. This early fast roll stage results in an ***attractive potential*** in the wave equation for the mode functions of curvature and tensor perturbations. Implementing the methods and borrowing the results from our companion article[23], we show that this attractive potential yields a transfer function for initial conditions $D(k)$ which fulfills the stringent criteria of renormalizability and small back reaction and yields a $10-20\%$ suppression of the CMB quadrupole consistent with the observational data. We also predict a small $\sim 2-4\%$ quadrupole suppression for B-modes.

Our main results are summarized as follows:

(1)
- Within the framework of the effective field theory of inflation at the GUT scale we show that allowing for an initial state of the inflaton for which its kinetic energy is of the same order as the potential energy, there emerges a brief stage prior to slow roll in which the inflaton rolls *fast.* We call this brief, but consequential stage, the ***fast roll regime***. The inflaton potential fulfills the slow roll conditions and is the same both in the slow roll and in the fast roll regime. We prove that this brief fast roll stage generates an *attractive* localized potential for the mode functions of metric and tensor perturbations. Such potential leads to initial conditions for the fluctuations *during the slow roll stage* which are different from Bunch-Davies and are consistent with renormalization and

in the fast roll regime. We prove that this brief fast roll stage generates an *attractive* localized potential for the
mode functions of metric and tensor perturbations. Such potential leads to initial conditions for the fluctuations
during the slow roll stage which are different from Bunch-Davies and are consistent with renormalization and
negligible backreaction.

(2)

- We provide an exhaustive numerical analysis for several inflationary models with the result that for generic
inflaton initial conditions with *equipartition* between kinetic and potential inflaton energy there is a brief period
of **fast roll** that lasts approximately ~ 1 e-fold. This brief stage translates in a potential $\mathcal{V}(\eta)$ in the wave
equation for the mode functions of curvature and tensor perturbations. The typical scales of these potentials are
$\mathcal{V}_{\mathcal{R}} \sim -10\ H_i^2$ for curvature perturbations and $\mathcal{V}_T \sim -2\ H_i^2$ for tensor perturbations, where H_i is the Hubble
parameter *during slow roll inflation*. A suppression of the CMB quadrupole of about $10-20\%$, consistent with
observation is obtained if the mode corresponding to the quadrupole, whose physical wavelength is of the order
of the Hubble radius today, crossed the horizon within $1-2$ efolds after the beginning of slow roll stage.

(3)

- Our study also predicts a suppression of the quadrupole for the B-modes, with a fractional change of at least
an order of magnitude smaller than that for temperature fluctuations.

(4)

- The evolution of the inflationary perturbations has been shown to be equivalent to the scattering by a potential
and useful expressions between the two sets of solutions and observables have been derived. By implementing
the methods of scattering theory we prove in general that the CMB quadrupole is suppressed by the attractive
potential $\mathcal{V}(\eta)$ which is a consequence of the fast roll stage.

(5)

- Thus, we conclude that generic ultraviolet-finite initial conditions imprinted upon gaussian curvature pertur-
bations from a fast roll stage just prior to slow roll inflation successfully explain the low quadrupole. Such
suppression happens provided the inflationary stage does not last more than $\sim 57-58$ e-folds. Therefore
this suppression mechanism successfully explains the low CMB quadrupole provided there is the *upper bound*
$N_{tot} \sim N_Q + 4 = 59$ on the total number of efolds during inflation. This upper bound results from the following
accounting: the modes corresponding to the quadrupole crossed out of the Hubble radius during the slow roll
stage approximately $N_Q = 55$ e-folds before the end of inflation. However for the potential $\mathcal{V}_{\mathcal{R}}(\eta)$ to influence
these modes, the exit time cannot be more than approximately $2-3$ e-folds after the end of the fast roll stage,
which itself lasts approximately 1 e-fold, yielding a total of about $N_{tot} = N_Q + 4 = 59$ e-folds.

String Dynamics in Curved S-times

String Eqs. of Motion
+ Constraints
+ Quantization

Non Linear σ- models

$\Downarrow$

MASS SPECTRUM

$\Rightarrow$ $m^2(n)$, $m^2 = m_s^2 f(n)$

$\Downarrow$

DENSITY OF LEVELS

$n \Rightarrow$ $d_n(n) = n^{-a} e^{b\sqrt{n}}$

$$\rho_s(m)\, dm = d_n(n)\, dn$$

$\Downarrow$

MICROSCOPIC DENSITY OF STATES OF MASS

$m \Rightarrow$ $\rho_s(m) = \dfrac{m}{m_s}\left[\dfrac{d_n(n)}{f'(n)}\right]_{n=n(m)}$

$\boxed{\rho_s(m)}$ $\left\{\begin{array}{l}\text{in de Sitter (ds)}\\ \text{in Anti de Sitter (AdS)}\\ \text{in Black holes (Bh)}\end{array}\right.$

— explicit expressions —

$\boxed{\Lambda = 0}$ $m^2 = \dfrac{1}{\alpha'} n$, $n = 0, 1, 2, \ldots$

$$\rho(m) = e^{m/T_s}$$

$$T_s = \frac{1}{b}\sqrt{\frac{\hbar}{\alpha' c}}$$

$\boxed{\begin{array}{c}\Lambda > 0\\ \text{D-S}\end{array}}$ $m^2 = \dfrac{1}{\alpha'} n \left(1 - \dfrac{\hbar}{c^3} H^2 \alpha' n\right)$

$$\rho(m) = e^{\frac{m\sqrt{\alpha'}}{\alpha' H^2}\left[1 - \sqrt{1 - m^2 \alpha' H^2}\right]^{1/2}}$$

$$T_s = \frac{c^3}{2\pi K_B} \frac{1}{\alpha' H}$$

$\boxed{\begin{array}{c}\Lambda < 0\\ \text{A-DS}\end{array}}$ $m^2 = H^2 n^2 = \Lambda n^2$

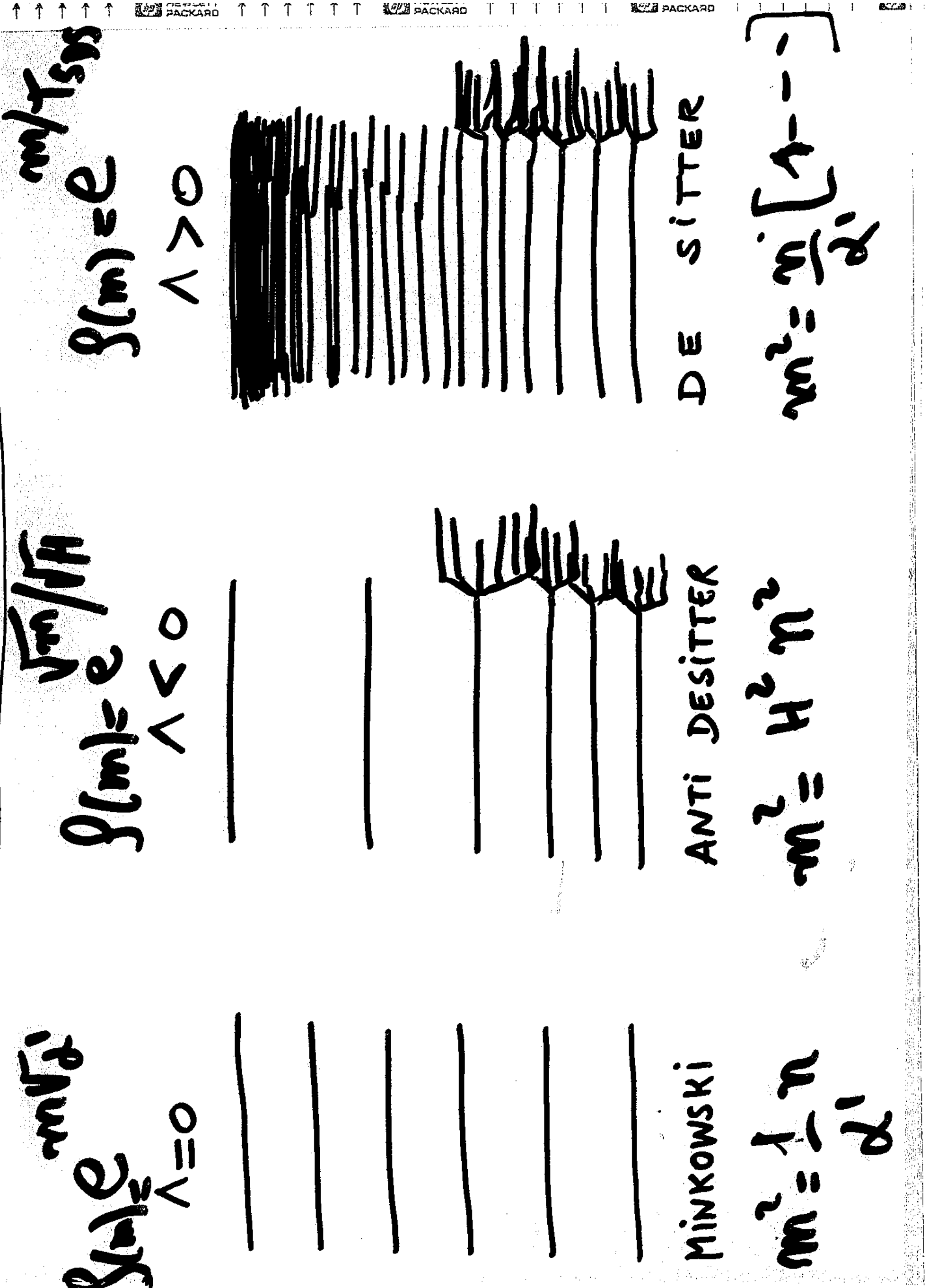
$$J^2 = m^2 / H^2$$

$$\rho(m) = e^{\sqrt{m}/\sqrt{H}} = e^{\sqrt{m/\sqrt{\Lambda}}}$$

$$T_s = \infty$$

STRINGS IN THE PRESENCE OF a Cosmological Constant Λ

STRINGS WITH A COSMOLOGICAL CONSTANT Λ



DISCRETE SPECTRUM FOR H

$$M_n = m_{pl} \sqrt{n}$$

$$n = 0, 1, 2, \dots$$

$$R_n = l_{pl} \sqrt{n}$$

$$\boxed{H_n = \frac{c}{l_{pl}} \frac{1}{\sqrt{n}} \qquad n = 1, 2, \dots}$$

(as for the mass of the black hole (quantization)

Concluding Remarks (1)

- The Hawking temperature, elementary particle and string temperatures are shown to be the same concept in different energy regimes and turn out to be the precise classical-quantum duals of each other.

- This result holds for the black hole decay rate, heavy particle and string decay rates; black hole evaporation ends as quantum string decay into pure (non mixed) non thermal radiation.

- Microscopic density of states and entropies in the two (semi-classical and quantum) gravity regimes are related, an unifying formula for black holes, de Sitter and anti-de Sitter states is provided in the two regimes.

Concluding Remarks (2)

- A phase transition towards the de Sitter string temperature (which is shown to be the precise quantum dual of the semi-classical (Hawking-Gibbons) de Sitter temperature) is found.

- Cosmological evolution goes from a quantum string phase to a semi-classical phase (inflation) and then to the classical (standard Friedman-Robertson-Walker) phase.

- The wave-particle-string duality precisely manifests in this evolution, and can be viewed as a mapping between asymptotic states and so as a scattering -matrix description.