Gravitational dynamics, galaxy structures and the new SDSS data

Francesco Sylos Labini

Enrico Fermi Center, Rome (Italy)
&
Institute for complex systems, CNR, Rome (Italy)

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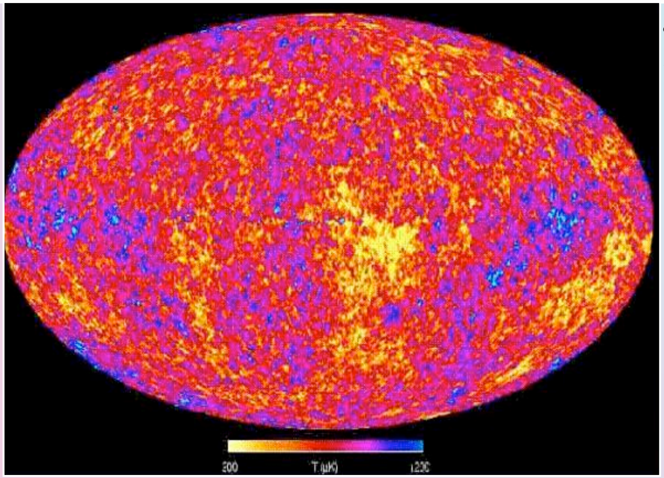
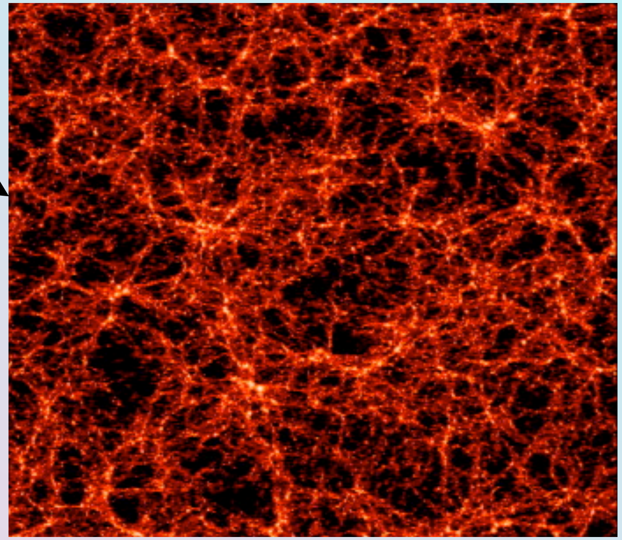
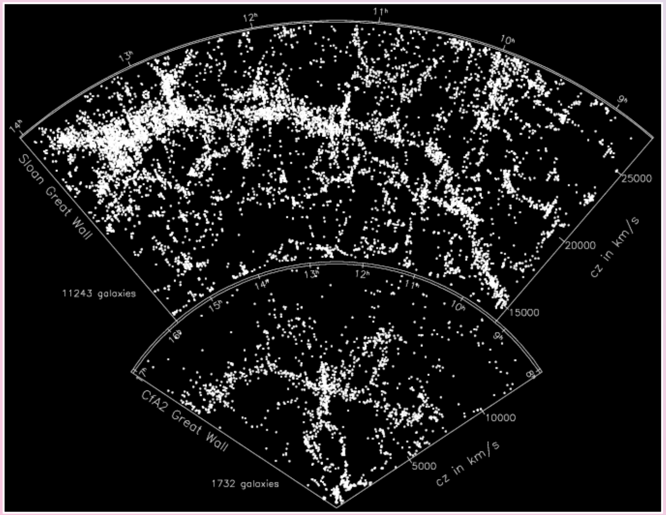
The problem of structure formation



Initial conditions: uniform distribution

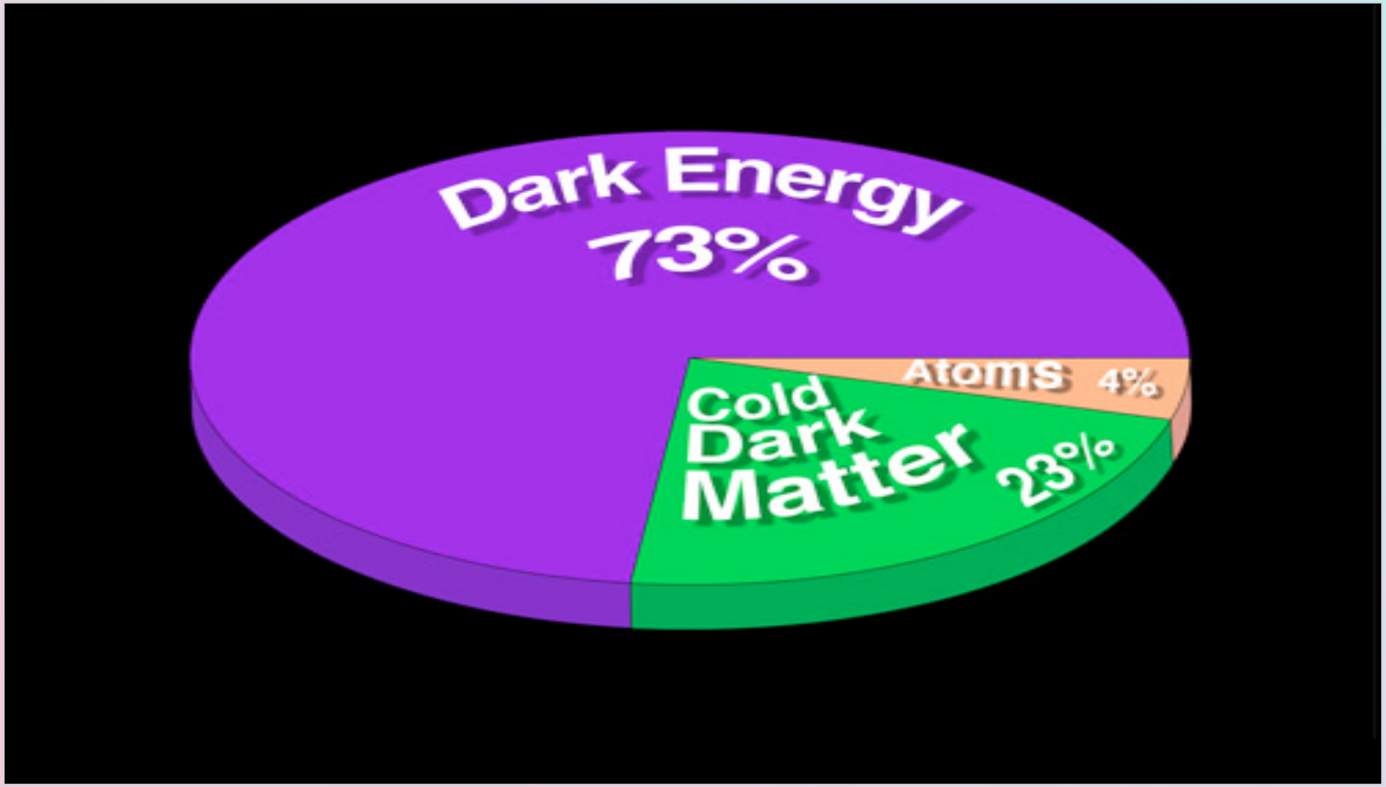
Dynamics: self-gravitating infinite system





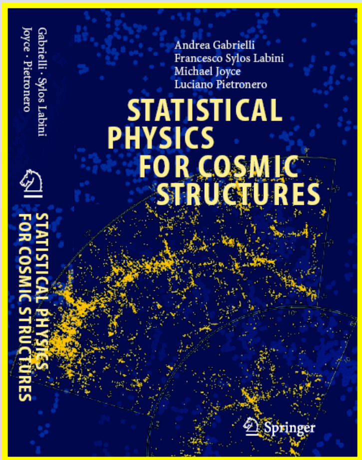
Final conditions: Stronlgy clustered power-law correlations

Dark Matter in cosmology



Dark Matter:
a statistical physicist point of view

- Fundamental properties of (dark matter) density fields in the framework of FRW models
- Study of galaxy correlations and relation to the (dark matter) underlying density field
- Study of the gravitational many-body problem by means of simplified simulations



Fluctuations in FRW models

$$\lim_{R\to\infty}\sigma^2(R)=0\approx\lim_{R\to\infty}\frac{1}{V(R)}\int_{V(R)}\xi(r)d^3r$$

$$\lim_{R\to\infty}\int_{V(R)}\tilde{\xi}(r)d\vec{r}=P(0)=0$$

$$\sigma^2(R=R_H(t))=const.$$ $\rightarrow$ $$\sigma_\phi{}^2(R)\approx\frac{1}{2}P_\phi(k)k^3\Big|_{k=1/R}\approx const.$$

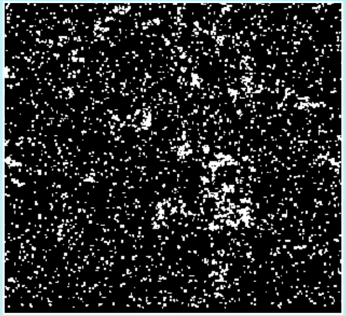
$\rightarrow$ $$\nabla^2\delta\phi(\vec{r})=-4\pi G\delta\rho(\vec{r})$$ $\rightarrow$ $$P_\phi(k)\propto\frac{P(k)}{k^4}\qquad P(k)\propto k$$

Fluctuations in FRW models

Substantially Poisson (finite correlation length)

$$P(0) \approx \int \tilde{\xi}(r) d^3 r = const. > 0$$

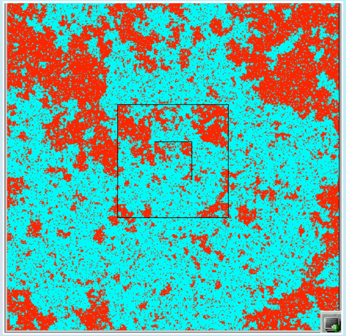
$$< \Delta M(r)^2 > \propto < M(r) >$$



Super-Poisson (infinite correlation length)

$$P(0) \approx \int \tilde{\xi}(r) d^3 r = \infty$$

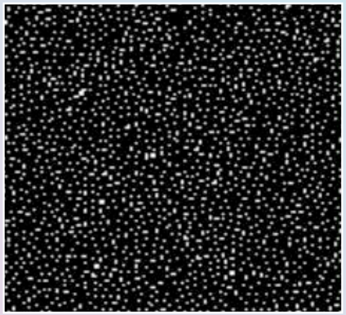
$$< \Delta M(r)^2 > \propto < M(r) >^{\beta} \quad 1 < \beta < 2$$



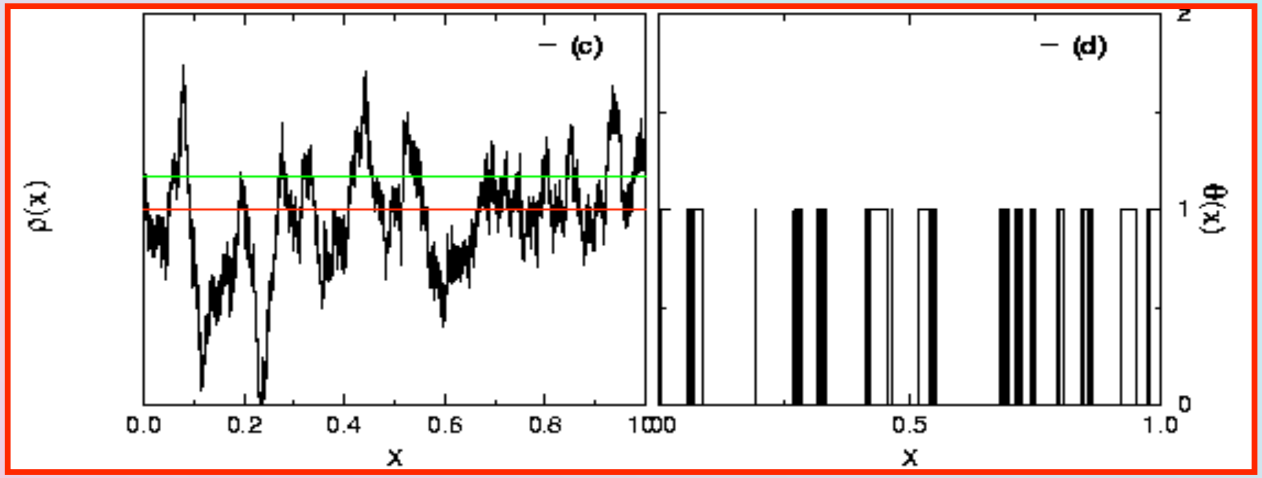
Sub-Poisson (ordered or super-homogeneous)

$$P(0) \approx \int \tilde{\xi}(r) d^3 r = 0$$

$$< \Delta M(r)^2 > \propto < M(r) >^{\beta} \quad 2/3 < \beta < 1$$



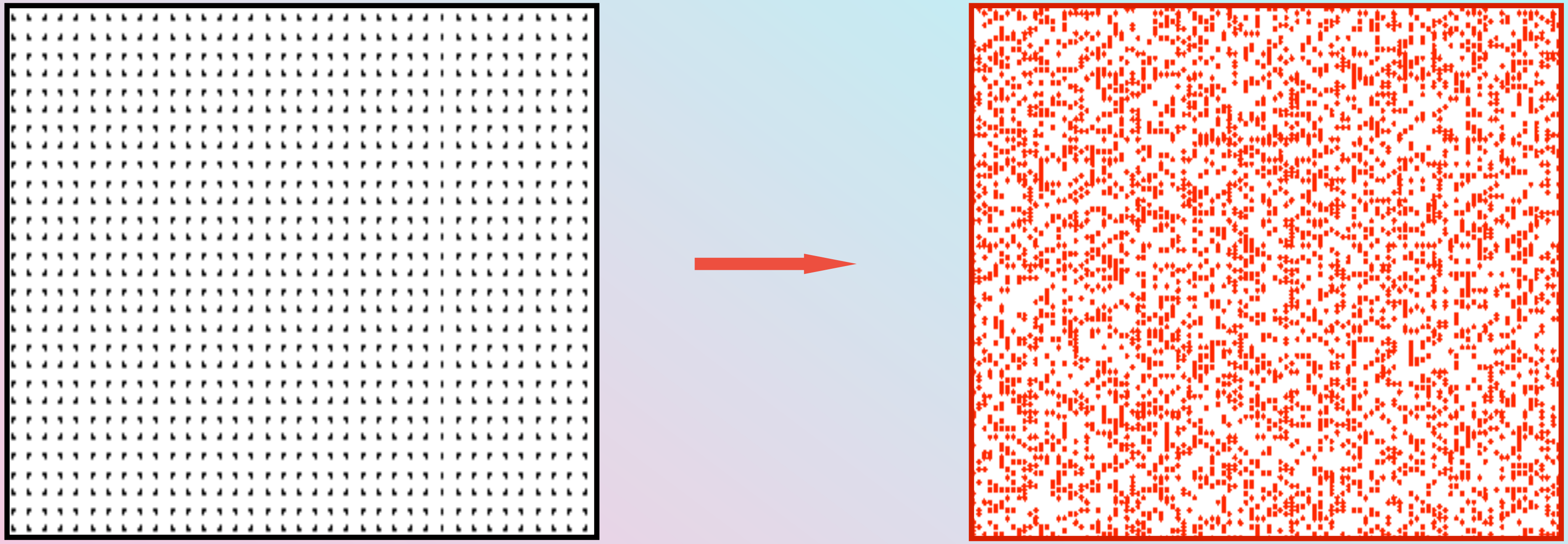
Fluctuations in FRW models



$$\theta_\nu(\vec{x}) \equiv \theta(\delta(\vec{x}) - \nu\sigma) = \begin{cases} 1 \ \text{if} \ \delta(\vec{x}) \geq \nu\sigma \\ 0 \ \text{if} \ \delta(\vec{x}) \leq \nu\sigma \end{cases}$$

$$\xi_\nu(r) = f(\nu, \xi_c(r))$$

Fluctuations in FRW models



$$P(0) = 0$$

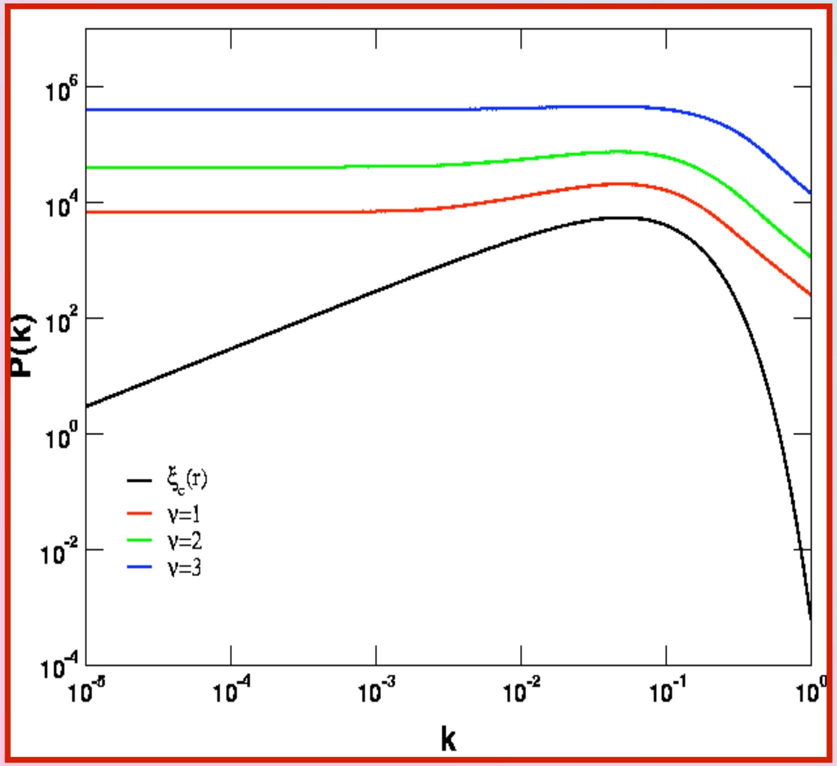
$$\sigma^2(R) \approx R^{-4}$$

$$P(0) = const.$$

$$\sigma^2(R) \approx R^{-3}$$

R. Durrer, A. Gabrielli, M. Joyce and F. Sylos Labini, ApJL ,585, L1 (2003)

Fluctuations in FRW models



$$P_c(0) = const \Rightarrow P_\nu(0) = const$$

$$P_c(0) = \infty \Rightarrow P_\nu(0) = \infty$$

$$P_c(0) = 0 \Rightarrow P_\nu(0) = const$$

$$P_\nu(k) \neq \nu^2 P_c(k) \forall k$$

$$\xi_\nu(r) \cong \nu^2 \xi_c(r)$$

HZ critical feature $\xi(r) \approx -r^{-4}$

Fluctuations in galaxy distributions

$$\xi(r) = \frac{< n(r)n(0) >}{< n >^2} - 1$$

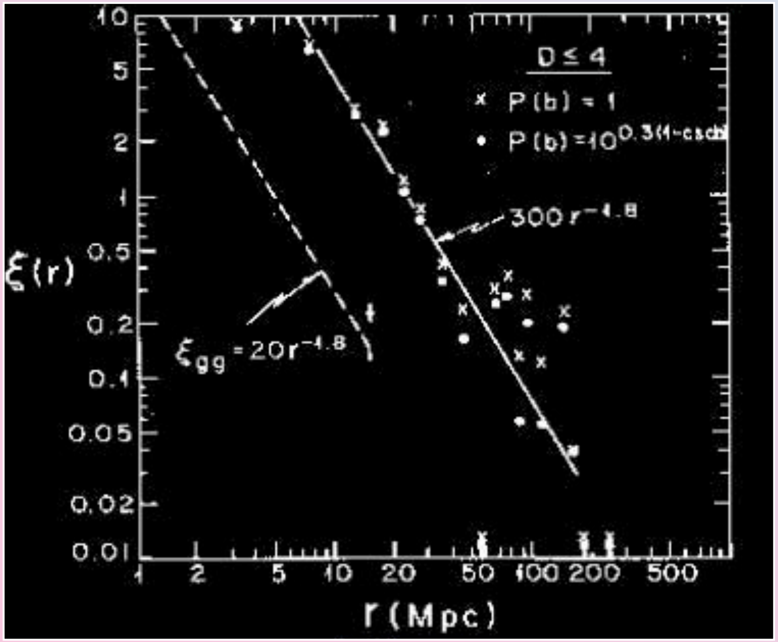
$$\xi(r) = Ar^{-1.7}$$

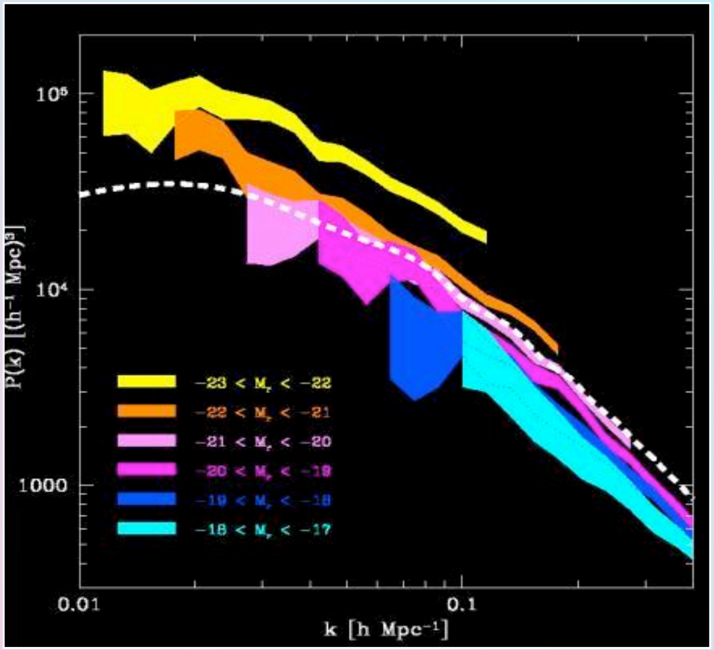
$$\xi_1(r) = const \times \xi_2(r)$$

$$P(k) = FT[\xi(r)]$$

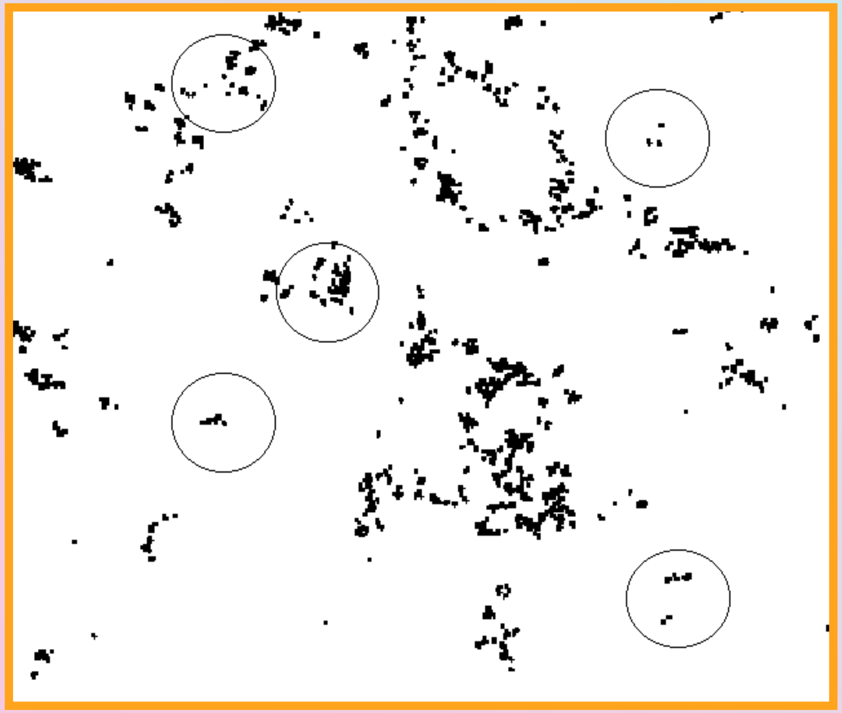
$$P(k) = Bk^{-1.3}$$

$$P_2(k) = const. \times P_1(k)$$





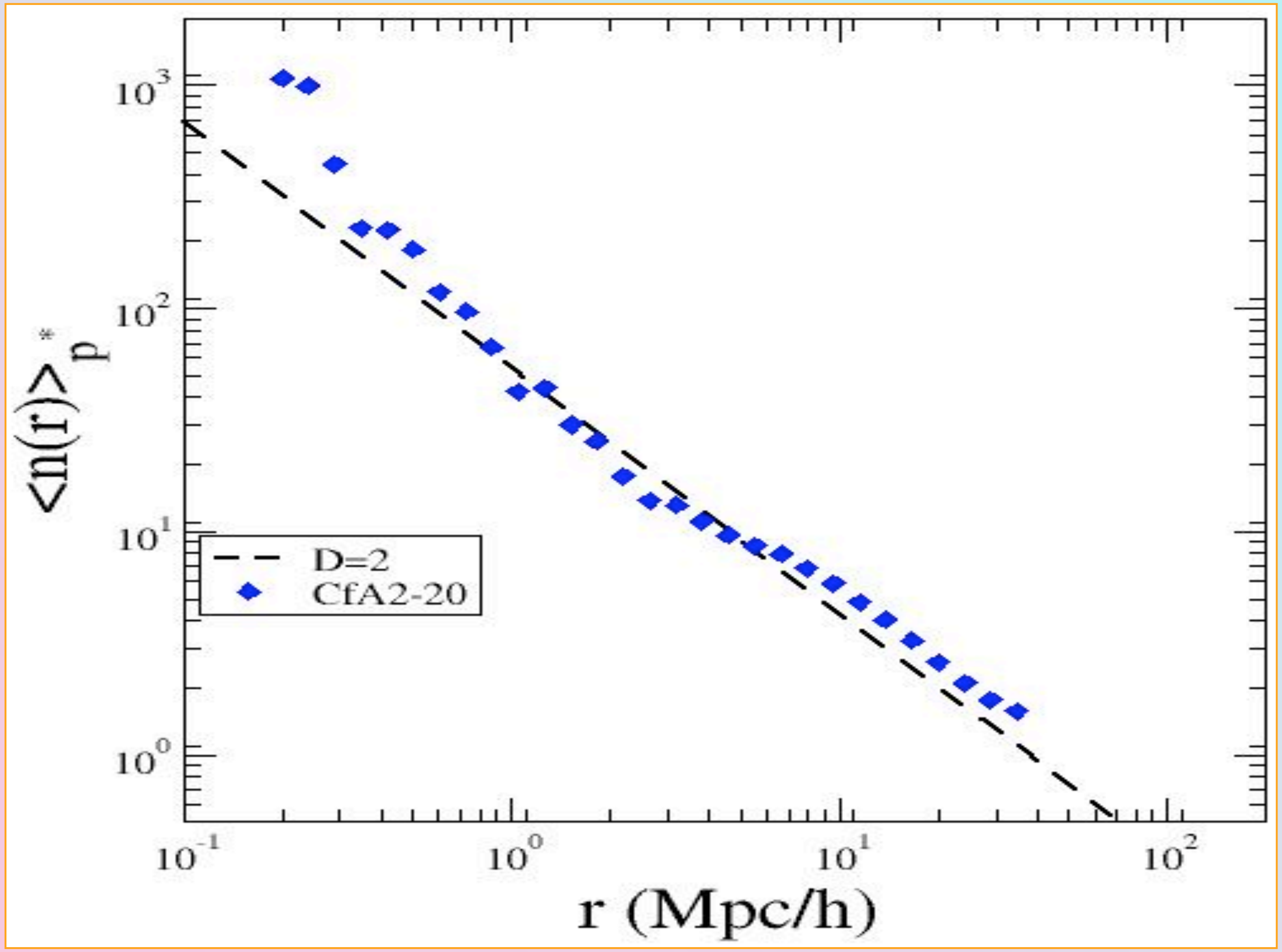
Fluctuations in galaxy distributions



$$\langle N(r)\rangle_P = \sum_{i=1}^{M} N_i(r) = Br^D \quad D \le 3$$

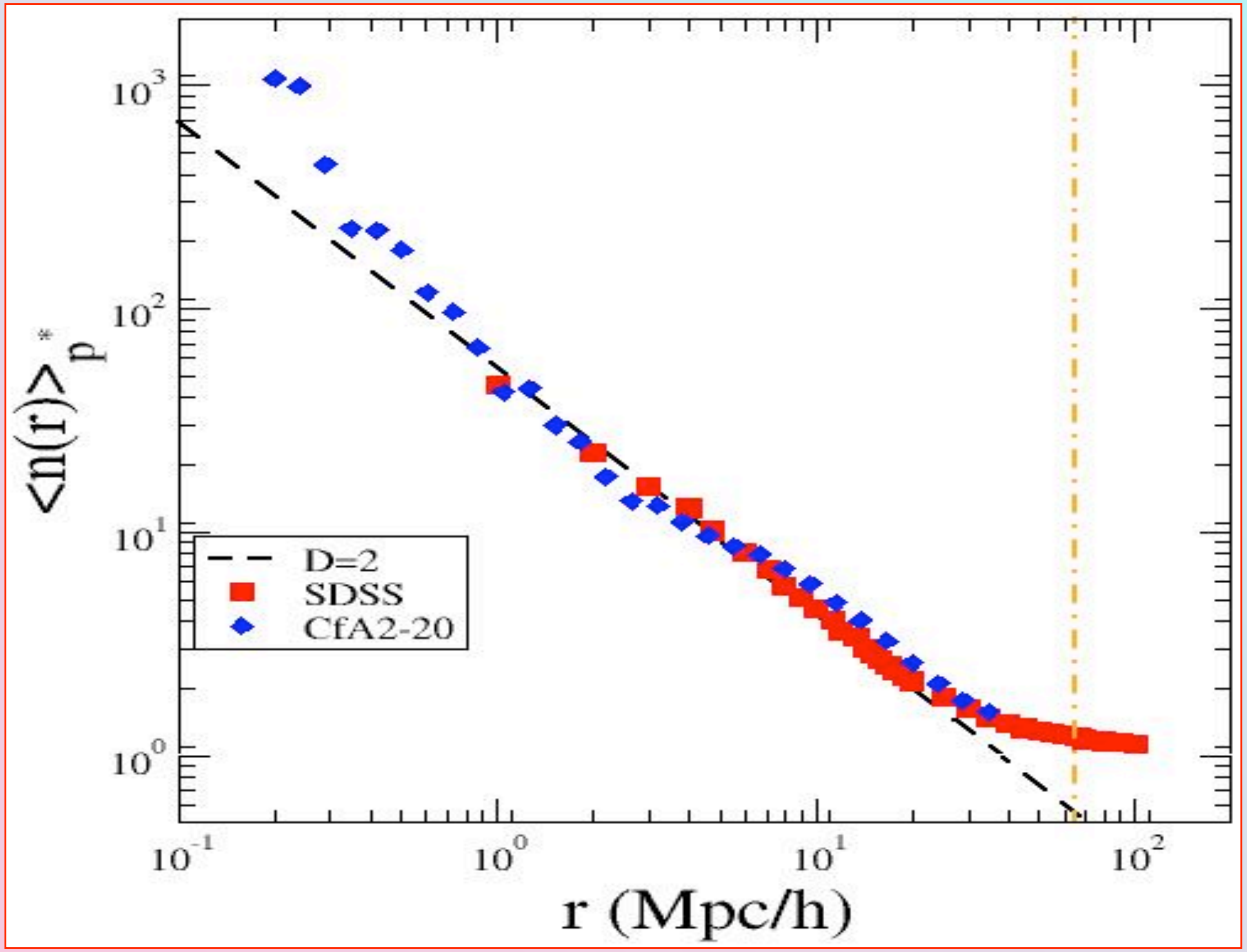
$$\langle n(r)\rangle_P = \frac{\langle N(r)\rangle_p}{V(r)} = \frac{3B}{4\pi} r^{D-3}$$

Fluctuations in galaxy distributions



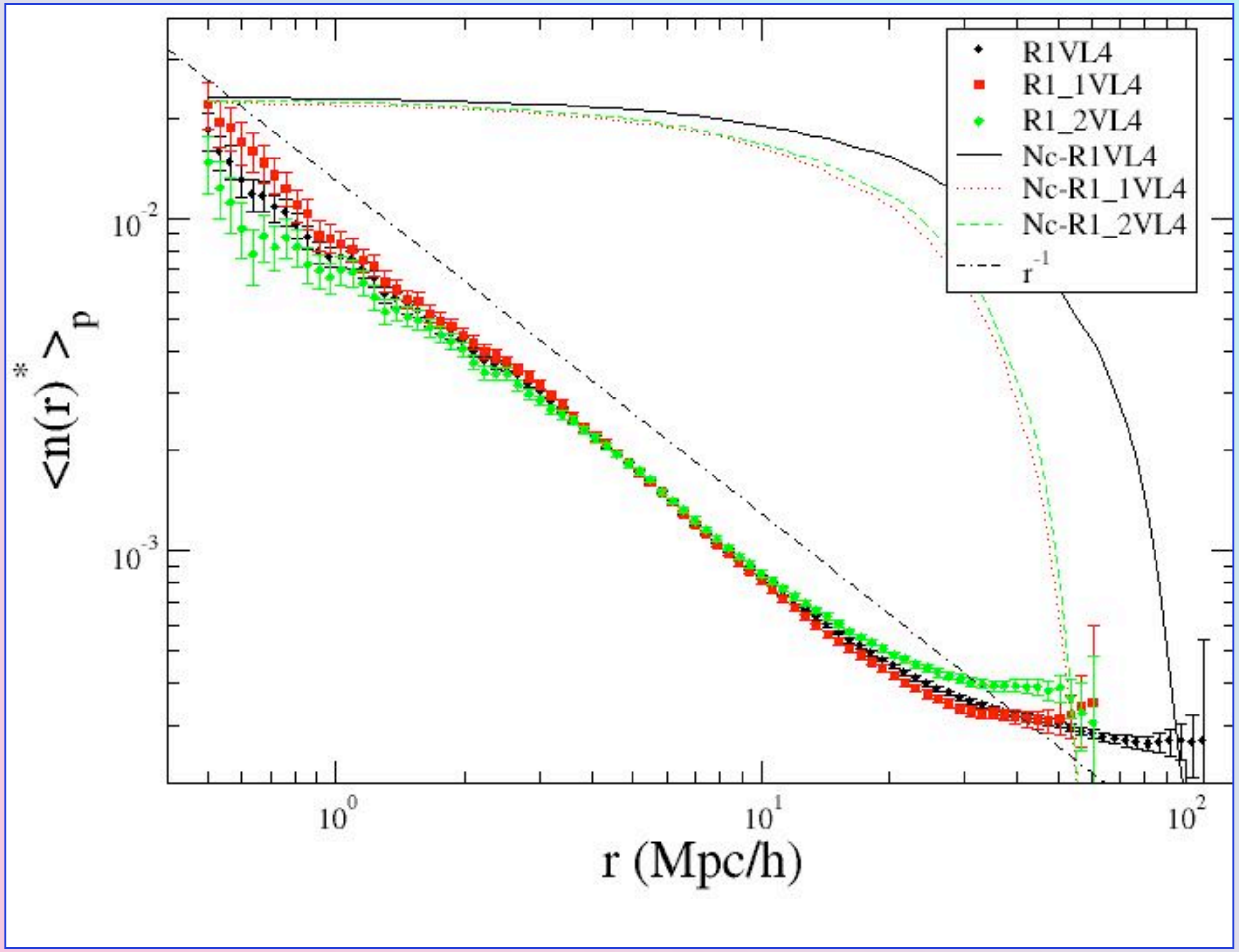
Sylos Labini, F., Montuori M. & Pietronero L. Phys Rep, 293, 66 (1998)

Fluctuations in galaxy distributions



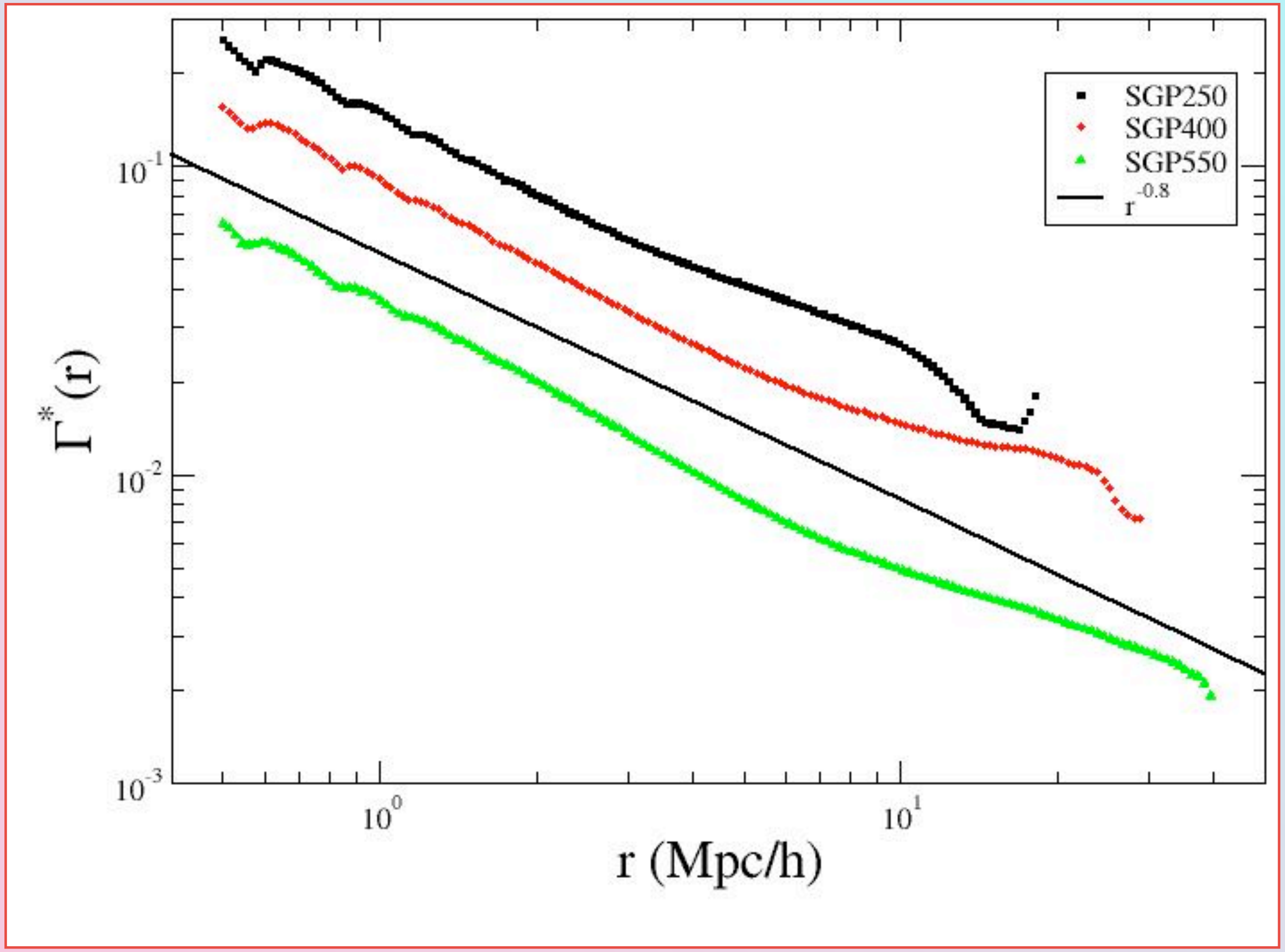
Hogg et al. (SDSS Collaboration). ApJ, 624, 54 (2005)

Fluctuations in galaxy distributions



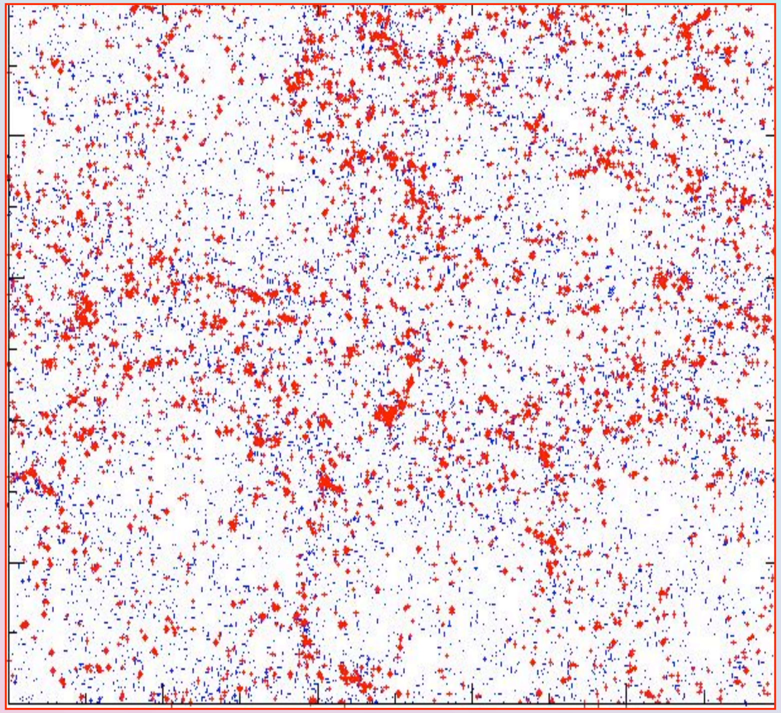
SDSS DR4: Sylos Labini, Vasilyev, Baryshev, A&A 2006

Fluctuations in galaxy distributions



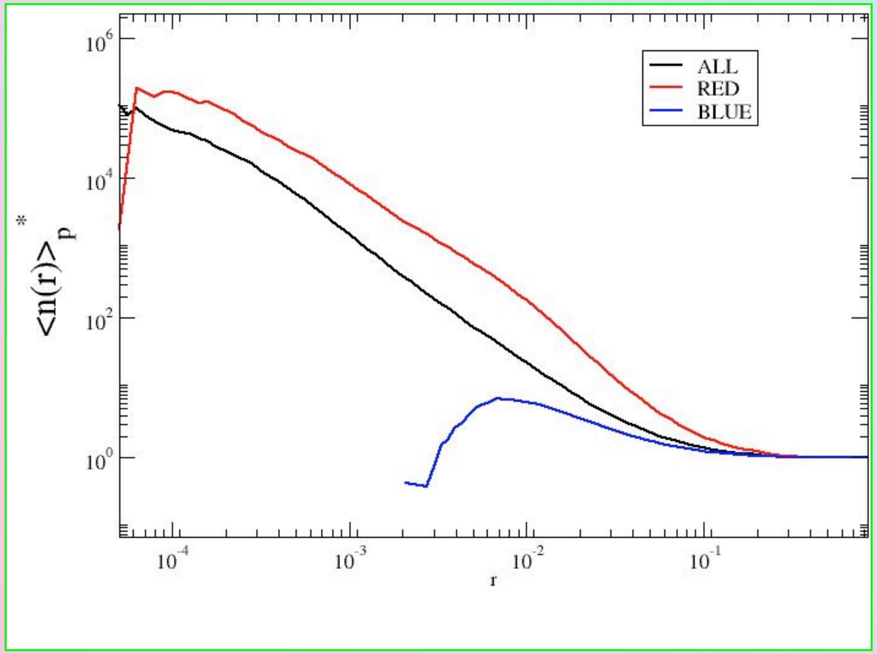
2dF Vasilyev, Baryshev, Sylos Labini A&A 447, 431, 2005

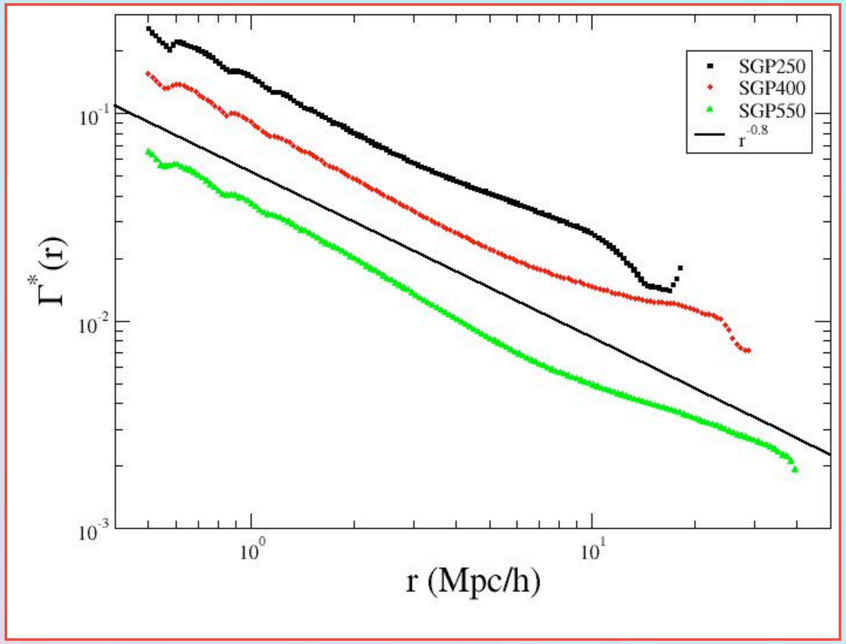
Comparing with mock catalogs



- Selection of objects and the “bias” problem
- Formation of non-linear structures….

Comparing with mock catalogs

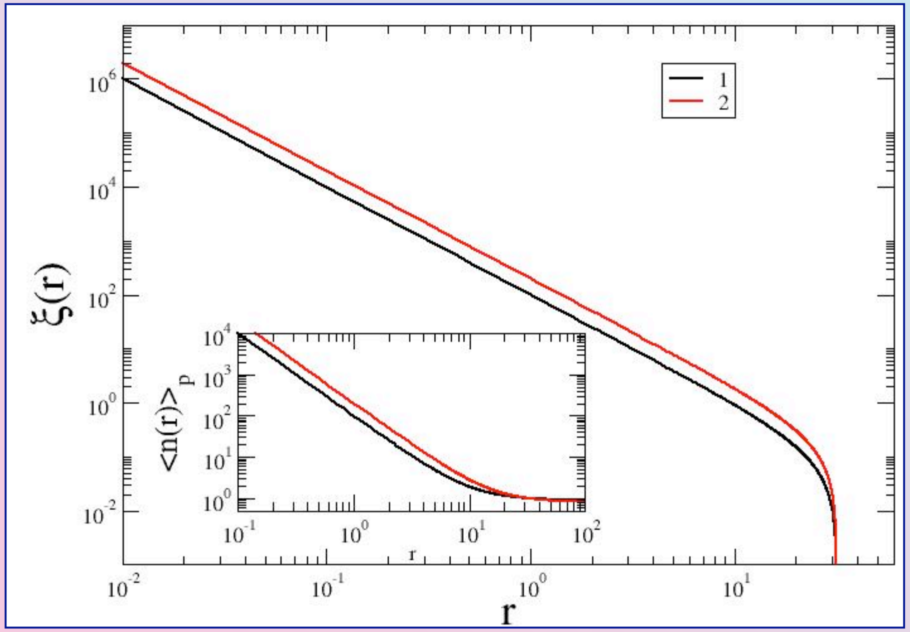


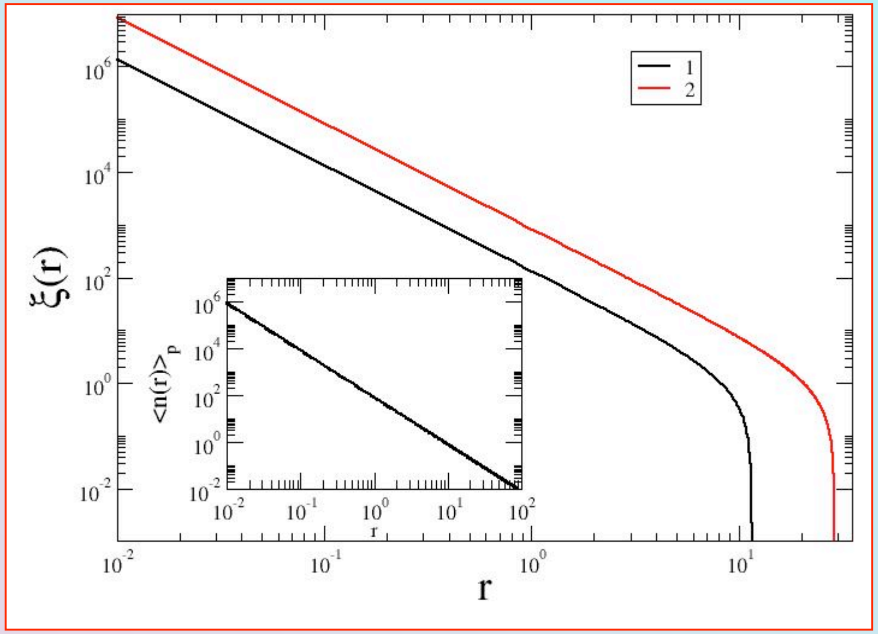


Correlation of “galaxies“ of different “luminosity” is different

Galaxies of different luminosity present the same correlations

Comparing with mock catalogs





?

Comparing with mock catalogs

$$\langle N(R_s)\rangle_P = BR_s^{\,D}$$

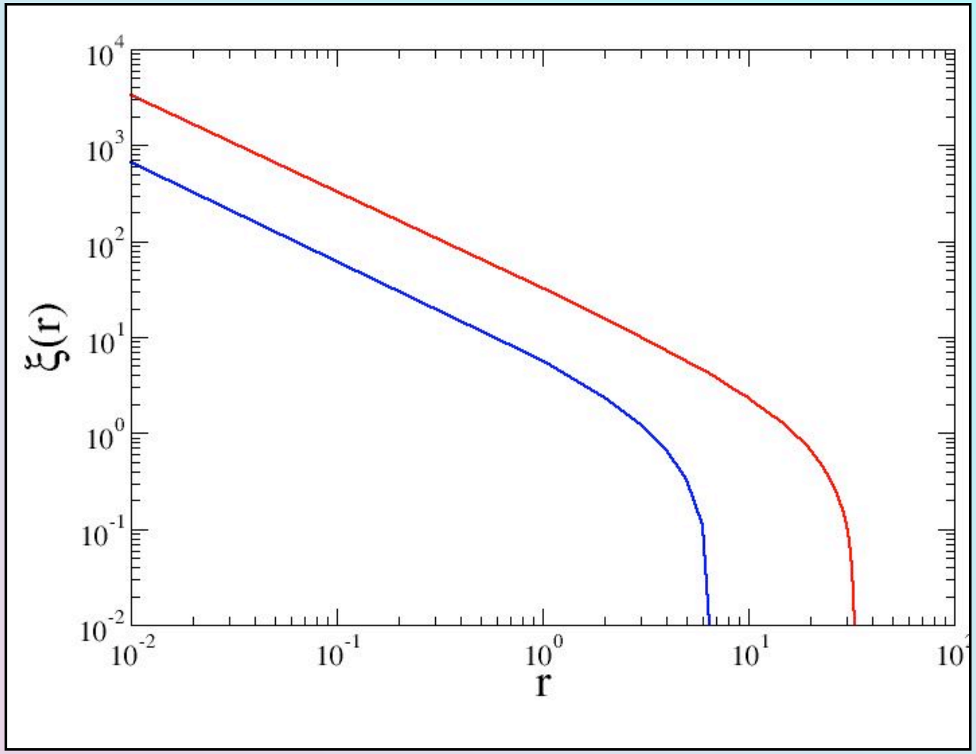
$$V(R_s) = \frac{4\pi}{3}R_s^{\,3}$$

$$\langle n\rangle_E = \frac{\langle N(R_s)\rangle_P}{V(R_s)} = \frac{3B}{4\pi}R_s^{\,D-3}$$

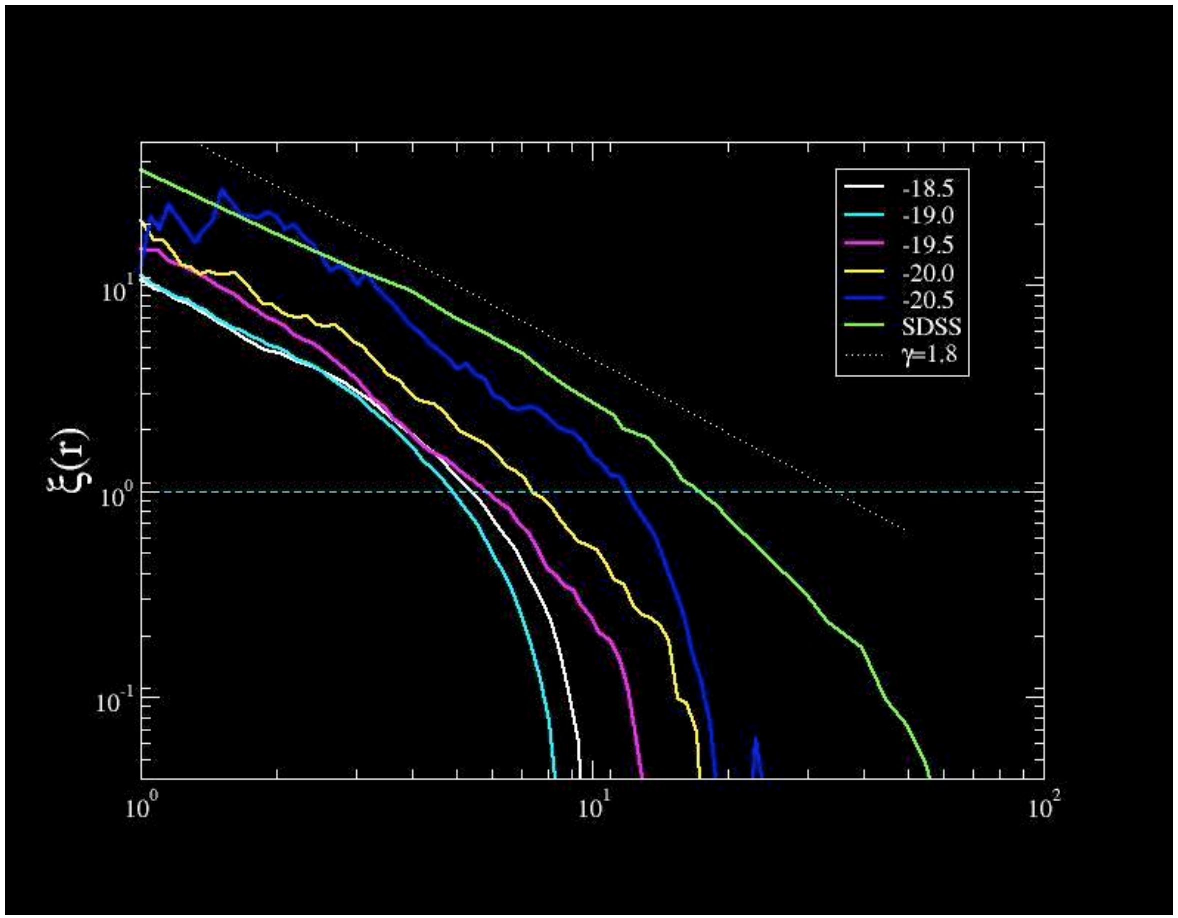
$$\langle n(r)\rangle_P = \frac{\langle N(r)\rangle_p}{V(r)} = \frac{3B}{4\pi}r^{D-3}$$

$$\xi_E(r) = \frac{\langle n(r)\rangle_P}{\langle n\rangle_E} - 1 = \frac{D}{3}\left(\frac{r}{R_s}\right)^{D-3} - 1$$

$$\xi_E(r_0) = 1 \Rightarrow r_0 = \left(\frac{D}{6}\right)^{1/(D-3)} R_s$$



Comparing with mock catalogs



$$\xi(r) = Ar^{-1.7}$$

$$\xi_1(r) = const \times \xi_2(r)$$

$$P(k) = Bk^{-1.3}$$

$$P_2(k) = const. \times P_1(k)$$

$$const = f(R_s)$$

Comparing with mock catalogs

- *Bias: this is not only a parameter but a function b(r).* Is this the same problem of galaxy catalogs ?

- Non-linear regime: no theoretical predictions (crucial for comparison with observations)

- Linear regime: Problem of characterization of real space properties in the regime of small fluctuations (PS vs CF). Problem of genereting IC…

→ How much discreteness is important ?
statistical and dynamical effects

Gravitational clustering: theoretical framework

- Newtonian approximation in an expanding universe
- Collisionless gas (at the scale of interest): Vlasov-Poisson

$$\frac{\partial f}{\partial t} + \frac{\mathbf{v}}{a^2} \cdot \nabla f - \nabla \Phi \frac{\partial f}{\partial \mathbf{v}} = 0$$

$$\nabla^2 \Phi(\mathbf{x}) = \frac{4\pi G}{a^3} \int (f - f_0) \, d\mathbf{v}$$

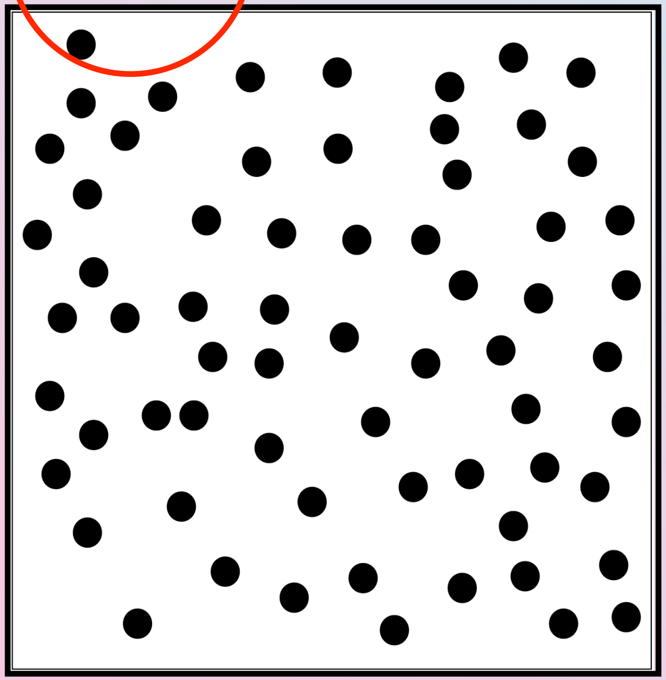
Very often the Vlasov equation is simplified into fluid equations

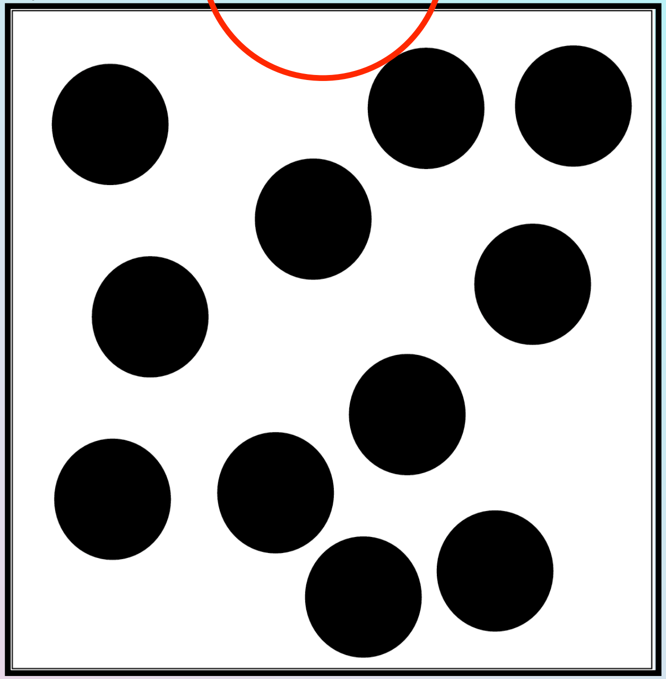
Solution of the equation of motion

- Perturbative solution of fluid equations
  - → Only valid for small density perturbations or large scales
- Numerical simulations
  - → Direct solution Vlasov not feasible
  - → Discretization into "N-bodies"

Discreteness effects

10^{80} particles SuperComp max 10^{10} particles

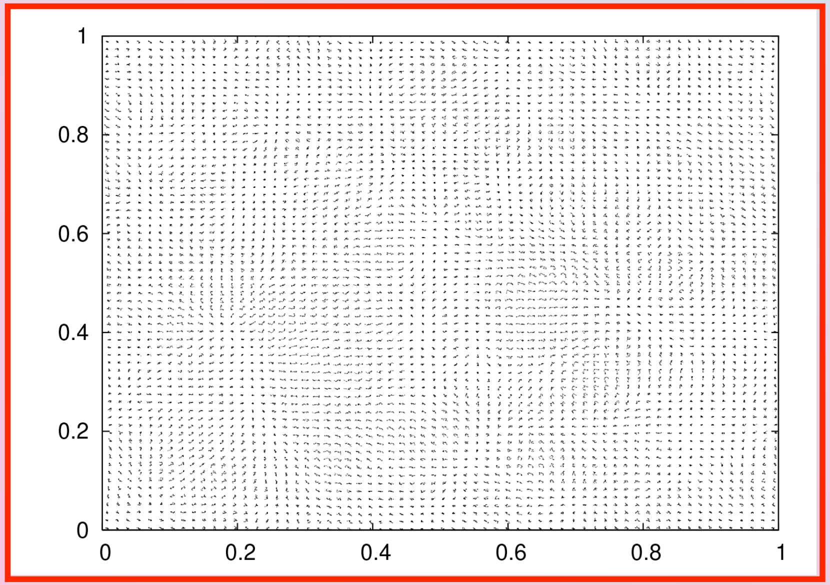




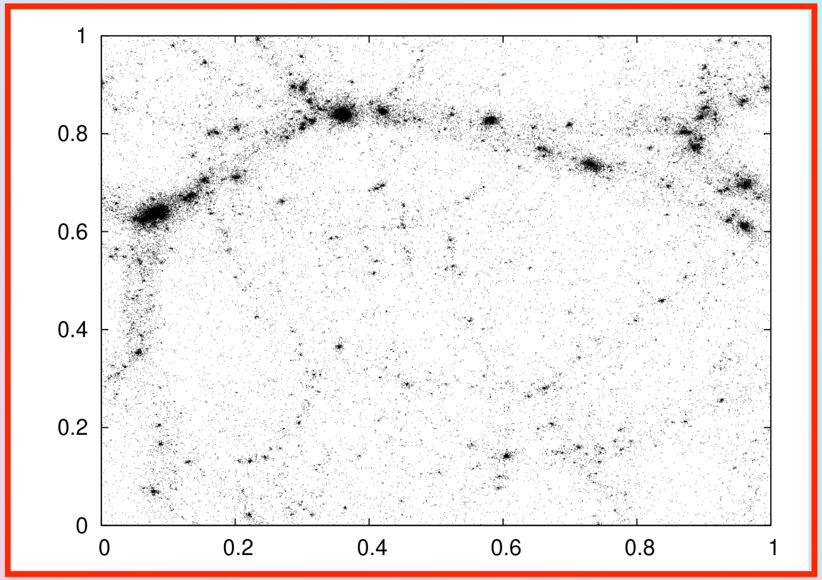
Do these systems have the same evolution ?

Discreteness effects

- There is no rigorous theory of discreteness effects (only convergency "tests")
- The main reason is that there is not a single discretization scale at all times



Relatively easy (we did it!)



Very difficult!

Discreteness effects in the linear regime

- Construct an analogous perturbation theory than the standard fluid one ($\delta\rho/\rho<<1$).
- Full evolution

$$\ddot{\mathbf{x}}_i + 2H(t)\dot{\mathbf{x}}_i = -\frac{1}{a^3}\sum_{i\neq j}\frac{Gm_j(\mathbf{x}_i-\mathbf{x}_j)}{\left|\mathbf{x}_i-\mathbf{x}_j\right|^3}$$

$$a(t)=1;\;\; H(t)=0;\;\; Gm^2=-e^2 \Rightarrow \text{ Wigner Crystal}$$

- Linearization

$$\mathbf{x}_i(t)=\mathbf{R}+\mathbf{u}(\mathbf{R},t) \qquad \mathbf{F}(\mathbf{r})=\sum_{\mathbf{R}'}\mathbf{D}(\mathbf{R}-\mathbf{R}')\mathbf{u}(\mathbf{R}')$$

$$\ddot{\mathbf{u}}(\mathbf{R},t)+2H\dot{\mathbf{u}}(\mathbf{R},t)=-\frac{1}{a^3}\sum_{\mathbf{R}'}\mathbf{D}(\mathbf{R}-\mathbf{R}')\mathbf{u}(\mathbf{R}',t)$$

Discreteness effects in the linear regime

Bloch Theorem: diagonalization by plane waves

$$\mathbf{u}(\mathbf{R},t) = \frac{1}{N}\sum_{\mathbf{k}} \tilde{\mathbf{u}}(\mathbf{k},t)\exp(i\mathbf{k}\mathbf{R}) \qquad \tilde{\mathbf{D}}(\mathbf{k}) = FT[\mathbf{D}(\mathbf{R})]$$

$$\ddot{\tilde{\mathbf{u}}}(\mathbf{k},t) + 2H\dot{\tilde{\mathbf{u}}}(\mathbf{k},t) = -\frac{1}{a^3}\mathbf{D}(\mathbf{k})\tilde{\mathbf{u}}(\mathbf{k},t)$$

System of vectorial 2nd order differential equations

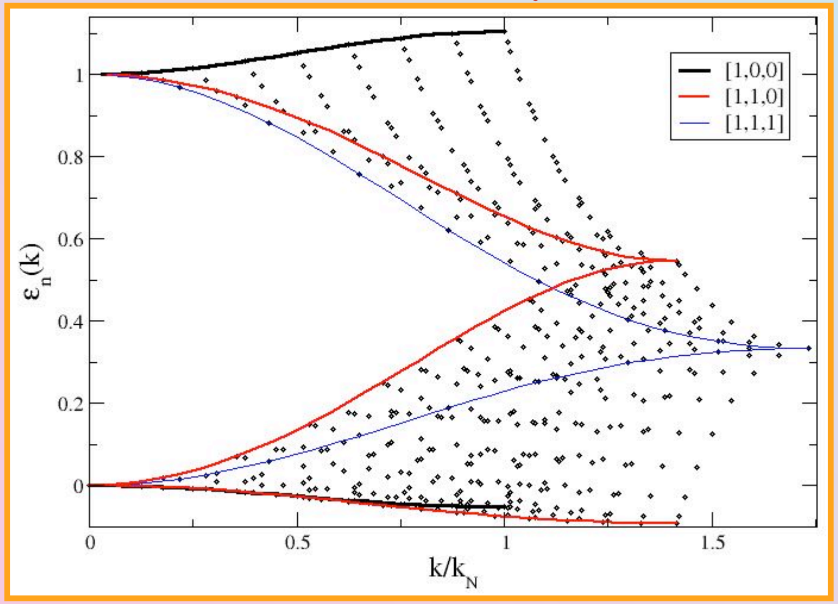
$$\forall \mathbf{k} \text{ Eigenvalue Eq. } \tilde{\mathbf{D}}(\mathbf{k})\mathbf{e}_{\mathbf{n}}(\mathbf{k}) = \omega_n^2(\mathbf{k})\mathbf{e}_{\mathbf{n}}(\mathbf{k}) \qquad n = 1,2,3$$

Kohn sum rule

$$\varepsilon_{\mathbf{n}}(\mathbf{k}) = \frac{\omega_n^2(\mathbf{k})}{4\pi G\rho_0} \Rightarrow \forall \mathbf{k} \sum_{n=1,2,3} \varepsilon_{\mathbf{n}}(\mathbf{k}) = 1$$

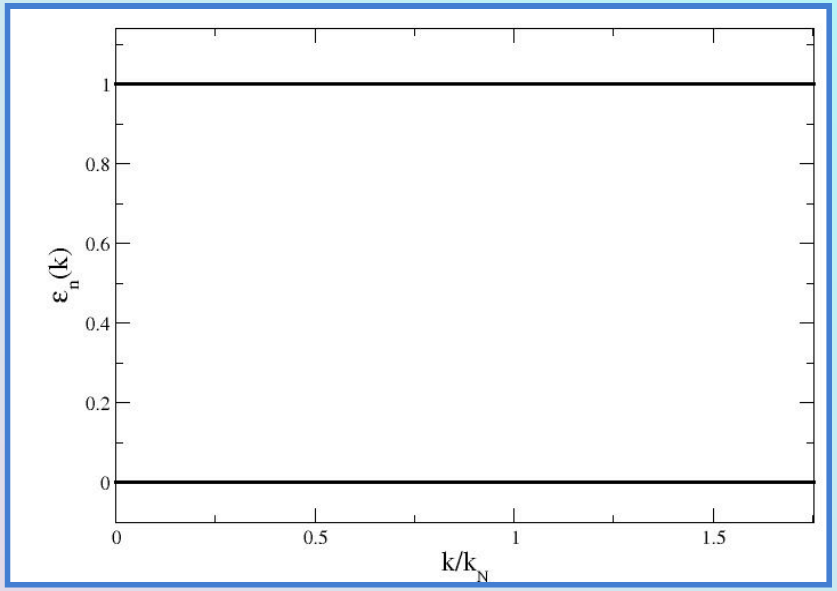
Discreteness effects in the linear regime

Nbody



$$\frac{\delta\rho}{\rho}(k) \sim \exp\left(\sqrt{4\pi G \rho_0 \mathbf{e_n}(\mathbf{k})}\,t\right)$$

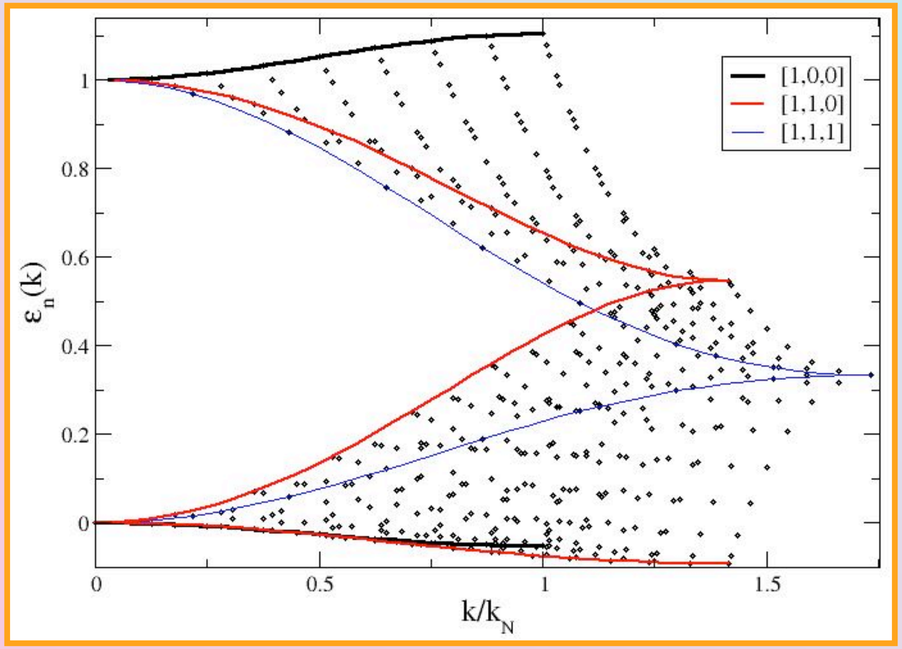
Fluid



$$\frac{\delta\rho}{\rho}(k) \sim \exp\left(\sqrt{4\pi G \rho_0}\,t\right)$$

Coulomb crystal: $t \rightarrow it$ Plasma oscillations $\omega_P^2 = \frac{4\pi e^2 n_0}{m}$

Discreteness effects in the linear regime



- Explicit fluid limit
- Oscillatory modes
- "**Super-fluid**" modes
- Anisotropy
- Discreteness effects depend on **time**
- Explicit differentiation between discreteness and non-linear fluid effects

M. Joyce, B. Marcos, A. Gabrielli, T. Baertschiger, F. Sylos Labini
Phys. Rev. Lett. 95, 011305 (2005)
B. Marcos, T. Baertschiger, M. Joyce, A. Gabrielli, F. Sylos Labini
Phys. Rev. D 73, 103507 (2006)

Conclusions

- **HZ tail in real space:** distinctive feature of FRW-IC in matter distribution is the behavior of the large scales tail of the correlation function. *Note yet observed in galaxy distributions*

- **Homogeneity scale:** *not yet identified with galaxy distribution*

- **Structures in N-Body simulations:** Problem of identification of "galaxies" and problem of size. Underlying theoretical problem: *discreteness and its effects in the non-linear regime*

Selected references

- **A.Gabrielli, F. Sylos Labini, M. Joyce, L. Pietronero Statistical physics for cosmic structures** Springer Verlag 2005

- M. Joyce, B. Marcos, A. Gabrielli, T. Baertschiger, F. Sylos Labini **Gravitational evolution of a perturbed lattice and its fluid limit** Phys.Rev.Lett. 95 011304 2005

- Gabrielli, M. Joyce and F. Sylos Labini **The Glass-like universe: real space statistical properties of standard cosmological models**, Phys.Rev.D, 65, 083523 (2002)

- M. Joyce, F. Sylos Labini, et al. **Basic properties of galaxy clustering in the light of recent results from the Sloan Digital Sky Survey** A&A. 443, 11, (2005)