The Effective Theory of Inflation and the Dark Energy in the Standard Model of the Universe

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Standard Cosmological Model: ΛCDM

ΛCDM = Cold Dark Matter + Cosmological Constant
Explains the Observations:

- 3 years WMAP data
- Light Elements Abundances
- Large Scale Structures (LSS) Observations
- Supernova Luminosity/Distance Relations (Acceleration of the Universe expansion)
- Gravitational Lensing Observations
- Lyman α Forest Observations
- Hubble Constant (H_0) Measurements
- Properties of Clusters of Galaxies
- ....

Standard Cosmological Model: Concordance Model

$ds^2 = dt^2 - a^2(t)\, d\vec{x}^2$: spatially flat geometry.

The Universe starts by an **INFLATIONARY ERA**.

Inflation = Accelerated Expansion: $\frac{d^2a}{dt^2} > 0$.

During inflation the universe expands by at least sixty efolds: $e^{60} \simeq 10^{26}$. Inflation lasts $\simeq 10^{-34}$sec and ends by $z \sim 10^{28}$ followed by a radiation dominated era.

Energy scale when inflation starts $\sim 10^{16}$ GeV.

This energy scale **coincides** with the GUT scale ($\Longleftarrow$ CMB anisotropies).

Matter can be effectively described during inflation by an Scalar Field $\phi(t, \boldsymbol{x})$: the Inflaton.

Lagrangean: $\mathcal{L} = a^3(t) \left[\frac{\dot{\phi}^2}{2} - \frac{(\nabla\phi)^2}{2\, a^2(t)} - V(\phi)\right]$.

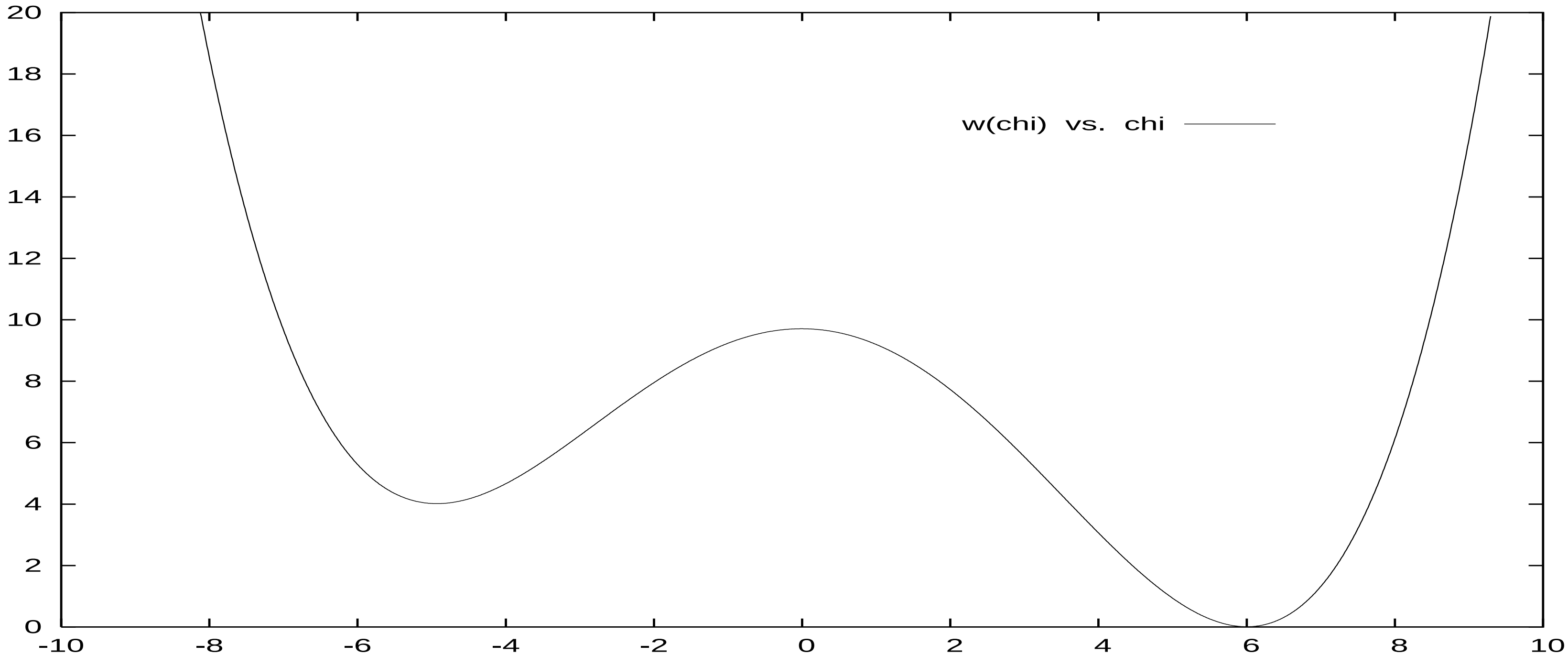
Friedmann eq.: $H^2(t) = \frac{1}{3\, M_{Pl}^2} \left[\frac{\dot{\phi}^2}{2} + V(\phi)\right]$, $H(t) \equiv \dot{a}(t)/a(t)$.

What is the Inflaton?

It is an effective field.
It can describe a fermion-antifermion pair condensate:
$\phi = < \bar\psi \psi >$, ψ = GUT fermion,
Such condensate can dominate the expectation value of the hamiltonian and therefore govern the cosmological expansion. [Recall that $< \psi > = 0$].
Relevant effective theories in physics:

- Ginsburg-Landau theory of superconductivity. It is an effective theory for Cooper pairs in the microscopic BCS theory of superconductivity.
- The $O(4)$ sigma model for pions, the sigma and photons at energies $\lesssim 1$ GeV. The microscopic theory is QCD: quarks and gluons. $\pi \simeq \bar q q \; , \; \sigma \simeq \bar q q$.

Slow Roll Inflaton Models



$V(\text{Min}) = V'(\text{Min}) = 0$: inflation ends after a finite number of efolds. Universal form of the slow-roll inflaton potential:

$$V(\phi) = N\ M^4\ w\left(\frac{\phi}{\sqrt{N}\ M_{Pl}}\right)$$

$N \sim 50$ number of efolds since horizon exit till end of inflation. $M =$ energy scale of inflation.

SLOW and Dimensionless Variables

$$\chi = \frac{\phi}{\sqrt{N}\; M_{Pl}} \quad , \quad \tau = \frac{m\, t}{\sqrt{N}} \quad , \quad \mathcal{H}(\tau) = \frac{H(t)}{m\,\sqrt{N}} \quad , \quad \left(m \equiv \frac{M^2}{M_{Pl}} \right)$$

slow inflaton, slow time, slow Hubble.

χ and $w(\chi)$ are of order one.

Evolution Equations:

$$\mathcal{H}^2(\tau) = \frac{1}{3}\left[\frac{1}{2\,N}\left(\frac{d\chi}{d\tau}\right)^2 + w(\chi)\right] \quad ,$$

$$\frac{1}{N}\,\frac{d^2\chi}{d\tau^2} + 3\,\mathcal{H}\,\frac{d\chi}{d\tau} + w'(\chi) = 0 \quad . \tag{1}$$

$1/N$ terms: corrections to slow-roll

Higher orders in slow-roll are obtained systematically by expanding the solutions in $1/N$.

Primordial Power Spectrum

Adiabatic Scalar Perturbations: $P(k) = |\Delta_{k\ ad}^{(S)}|^2 \; k^{n_s-1}$.
To dominant order in slow-roll:

$$|\Delta_{k\ ad}^{(S)}|^2 = \frac{N^2}{12\,\pi^2} \; \left(\frac{M}{M_{Pl}}\right)^4 \; \frac{w^3(\chi)}{w'^2(\chi)} \; .$$

Hence, for all slow-roll inflation models:

$$|\Delta_{k\ ad}^{(S)}| \sim \frac{N}{2\,\pi\sqrt{3}} \left(\frac{M}{M_{Pl}}\right)^2$$

The WMAP result $|\Delta_{k\ ad}^{(S)}| = (0.467 \pm 0.023) \times 10^{-4}$
determines the scale of inflation M (using $N \sim 50$)

$$\left(\frac{M}{M_{Pl}}\right)^2 = 1.02 \times 10^{-5} \longrightarrow M = 0.77 \times 10^{16} \text{ GeV}$$

The inflation energy scale turns to be the grand unification energy scale !!
This statement is model independent [independent of the shape of $w(\chi)$].

spectral index n_s, its running and the ratio r

$$n_s - 1 = -\frac{3}{N} \left[\frac{w'(\chi)}{w(\chi)}\right]^2 + \frac{2}{N} \frac{w''(\chi)}{w(\chi)} \quad ,$$

$$\frac{d n_s}{d \ln k} = -\frac{2}{N^2} \frac{w'(\chi)\, w'''(\chi)}{w^2(\chi)} - \frac{6}{N^2} \frac{[w'(\chi)]^4}{w^4(\chi)} + \frac{8}{N^2} \frac{[w'(\chi)]^2\, w''(\chi}{w^3(\chi)}$$

$$r = \frac{8}{N} \left[\frac{w'(\chi)}{w(\chi)}\right]^2 \quad .$$

χ is the inflaton field at horizon exit.
$n_s - 1$ and r are always of order $1/N \sim 0.02$ (model indep.)
Running of n_s of order $1/N^2 \sim 0.0004$ (model independent).

D. Boyanovsky, H. J. de Vega, N. G. Sanchez,
Phys. Rev. D 73, 023008 (2006), astro-ph/0507595.

Ginsburg-Landau Approach

We choose a polynomial for $w(\chi)$. A quartic $w(\chi)$ is renormalizable. Higher order polynomials are acceptable since inflation is an effective theory.

$$w(\chi) = w_o \pm \frac{\chi^2}{2} + G_3\, \chi^3 + G_4\, \chi^4 \quad , \quad G_3 = \mathcal{O}(1) = G_4$$

$$V(\phi) = N\; M^4\; w\left(\frac{\phi}{\sqrt{N}\; M_{Pl}}\right) = V_o \pm \frac{m^2}{2}\; \phi^2 + g\; \phi^3 + \lambda\; \phi^4 \; .$$

$$m = \frac{M^2}{M_{Pl}} \quad , \quad g = \frac{m}{\sqrt{N}} \left(\frac{M}{M_{Pl}}\right)^2 \; G_3 \quad , \quad \lambda = \frac{G_4}{N} \; \left(\frac{M}{M_{Pl}}\right)^4$$

Notice that

$$\left(\frac{M}{M_{Pl}}\right)^2 \simeq 10^{-5} \quad , \quad \left(\frac{M}{M_{Pl}}\right)^4 \simeq 10^{-10} \quad , \quad N \simeq 50 \; .$$

- Small couplings arise naturally as ratio of two energy scales: inflation and Planck.
- The inflaton is a light particle:
  $$m = M^2/M_{Pl} \simeq 0.003\; M \quad , \quad m = 2.5 \times 10^{13} \mathrm{GeV}$$

The number of efolds in Slow-roll

The number of e-folds $N[\chi]$ since the field χ exits the horizon till the end of inflation is:

$N[\chi] = N \int_{\chi_{end}}^{\chi} \frac{w(\chi)}{w'(\chi)} \, d\chi$. We choose then $N = N[\chi]$.

The spontaneously broken symmetric potential:

$$w(\chi) = \frac{y}{32} \left(\chi^2 - \frac{8}{y}\right)^2$$

produces inflation with $0 < \sqrt{y} \, \chi_{initial} \ll 1$ and $\chi_{end} = \sqrt{\frac{8}{y}}$.

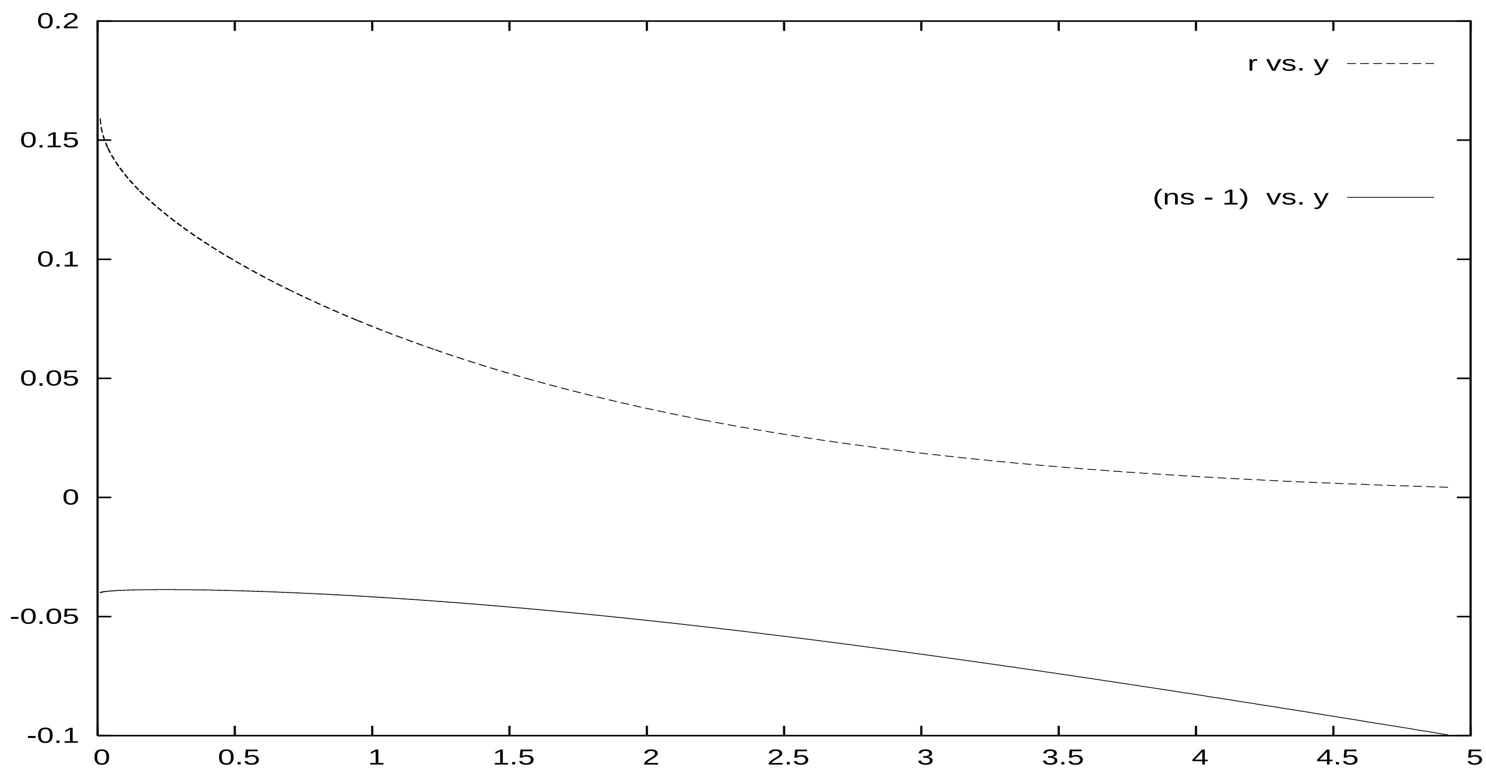
This is small field inflation.

From the above integral: $y = z - 1 - \log z$ where $z \equiv y \, \chi^2/8$
This defines $\chi = \chi(y)$. [$1 > z > 0$ for $0 < y < \infty$].
Spectral index n_s and the ratio r as functions of y:

$$n_s = 1 - \frac{y}{N} \frac{3\,z+1}{(z-1)^2} \quad , \quad r = \frac{16\,y}{N} \frac{z}{(z-1)^2}$$

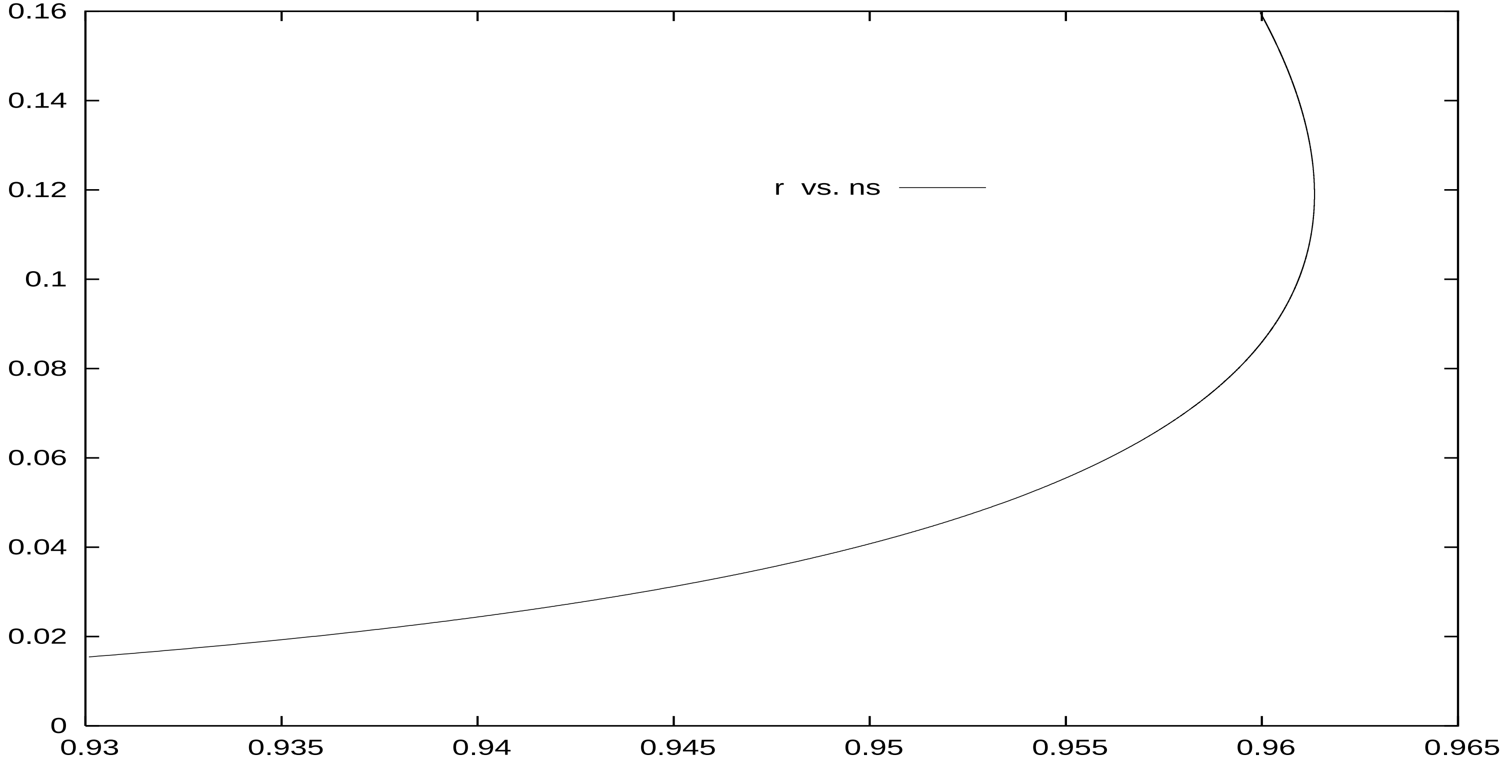
Binomial New Inflation: (y = coupling).

r decreases monotonically with y :
(strong coupling) $0 < r < \frac{8}{N} = 0.16$ (zero coupling).



n_s first grows with y, reaches a maximum value $n_{s,maximum} = 0.96139\ldots$ at $y = 0.2387\ldots$ and then n_s decreases monotonically with y.

Binomial New Inflation



$r = \frac{8}{N} = 0.16$ and $n_s = 1 - \frac{2}{N} = 0.96$ at $y = 0$.

r is a double valued function of n_s.

Trinomial Inflationary Models

- Trinomial Chaotic inflation:
  $$w(\chi) = \frac{1}{2}\,\chi^2 + \frac{h}{3}\,\sqrt{\frac{y}{2}}\,\chi^3 + \frac{y}{32}\,\chi^4 \; .$$
- Trinomial New inflation:
  $$w(\chi) = -\frac{1}{2}\,\chi^2 + \frac{h}{3}\,\sqrt{\frac{y}{2}}\,\chi^3 + \frac{y}{32}\,\chi^4 + \frac{2}{y}\;F(h) \; .$$

h = asymmetry parameter. $w(\min) = w'(\min) = 0$,
$y =$ quartic coupling, $F(h) = \frac{8}{3}\,h^4 + 4\,h^2 + 1 + \frac{8}{3}\,|h|\;(h^2+1)^{\frac{3}{2}}$.

H. J. de Vega, N. G. Sanchez, Single Field Inflation models allowed and ruled out by the three years WMAP data. Phys. Rev. D 74, 063519 (2006), astro-ph/0604136.

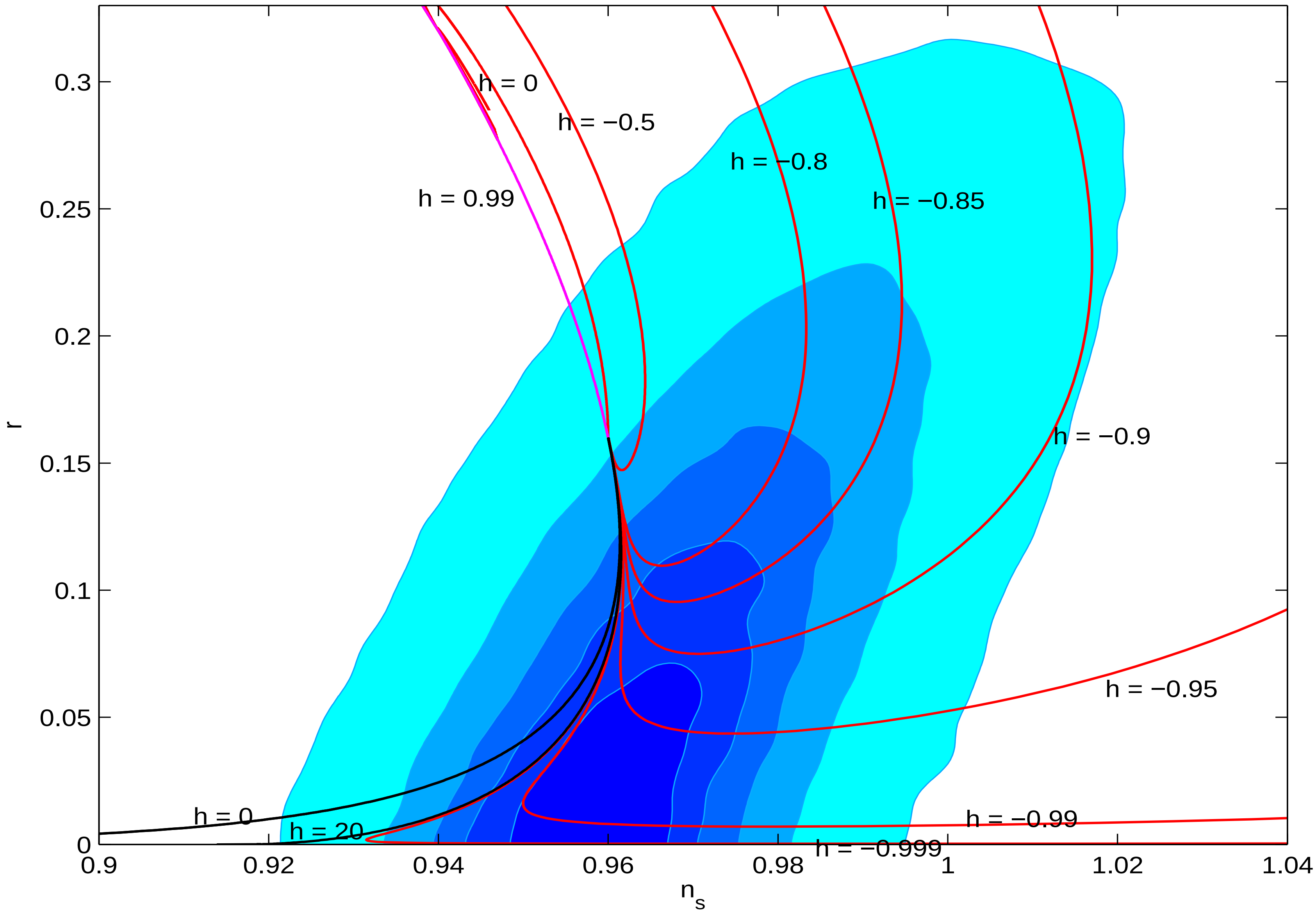
Monte Carlo Markov Chains Analysis of Data: MCMC.

MCMC is an efficient stochastic numerical method to find the probability distribution of the theoretical parameters that describe a set of empirical data.

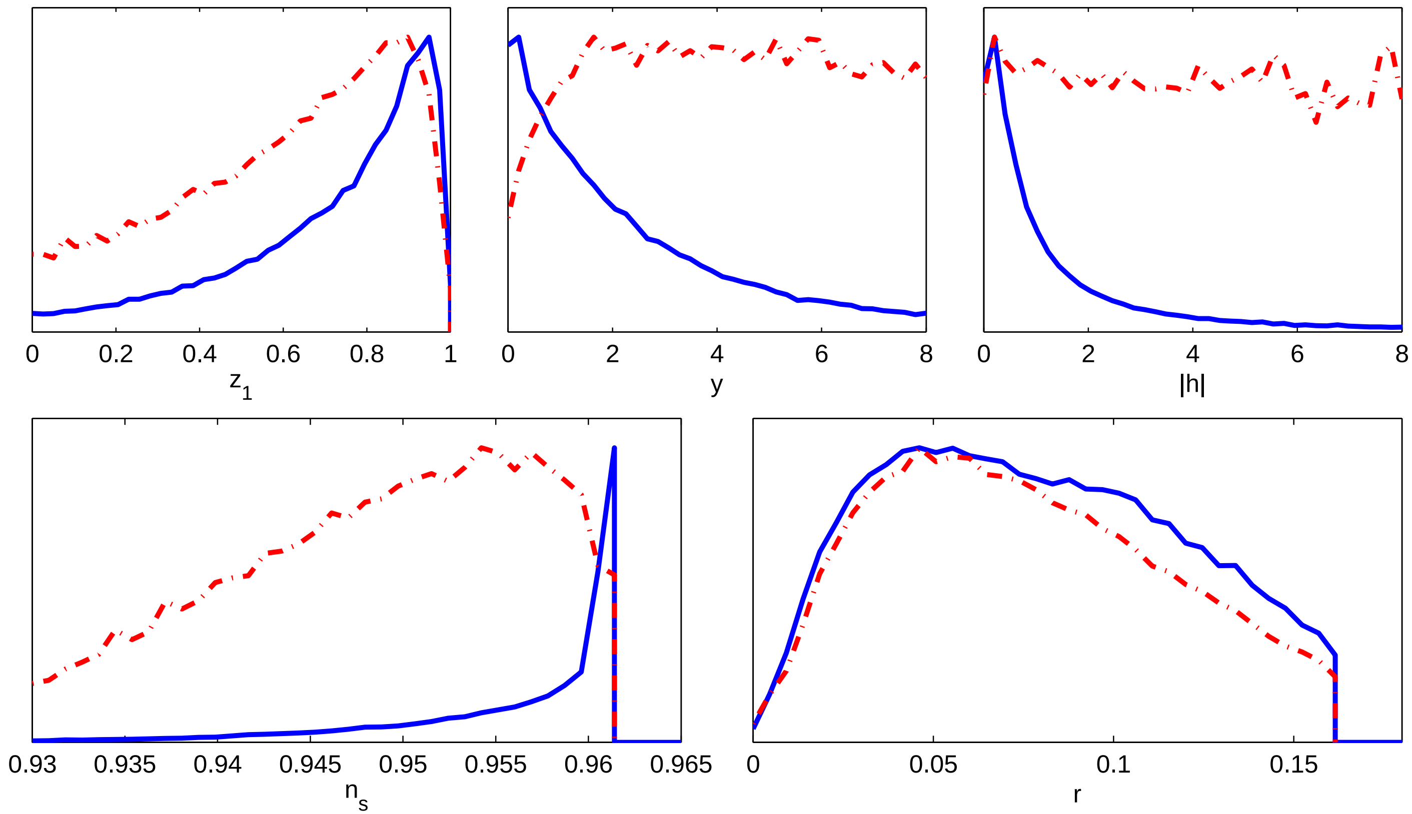
We found n_s and r and the couplings y and h by MCMC.
NEW: We imposed as a hard constraint that r and n_s are given by the trinomial potential.
Our analysis differs in this crucial aspect from previous MCMC studies of the WMAP data.

We ignore running of the spectral index since $dn_s/d\ln k \sim 0.0004$ in slow roll.
Adding the running made insignificant changes on the fit of n_s and r.

MCMC Results for Trinomial New Inflation.



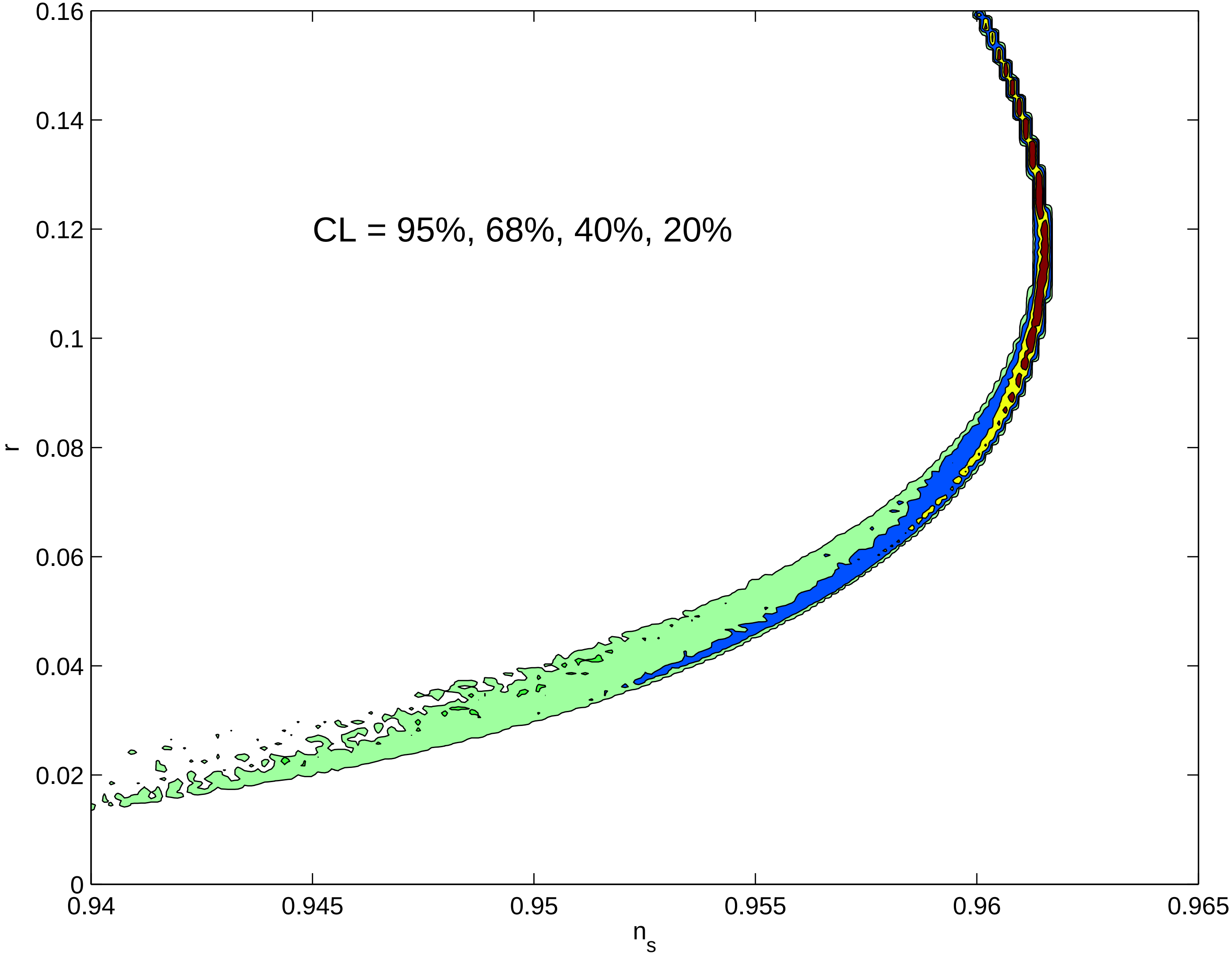
Probability Distributions. Trinomial New Inflation.



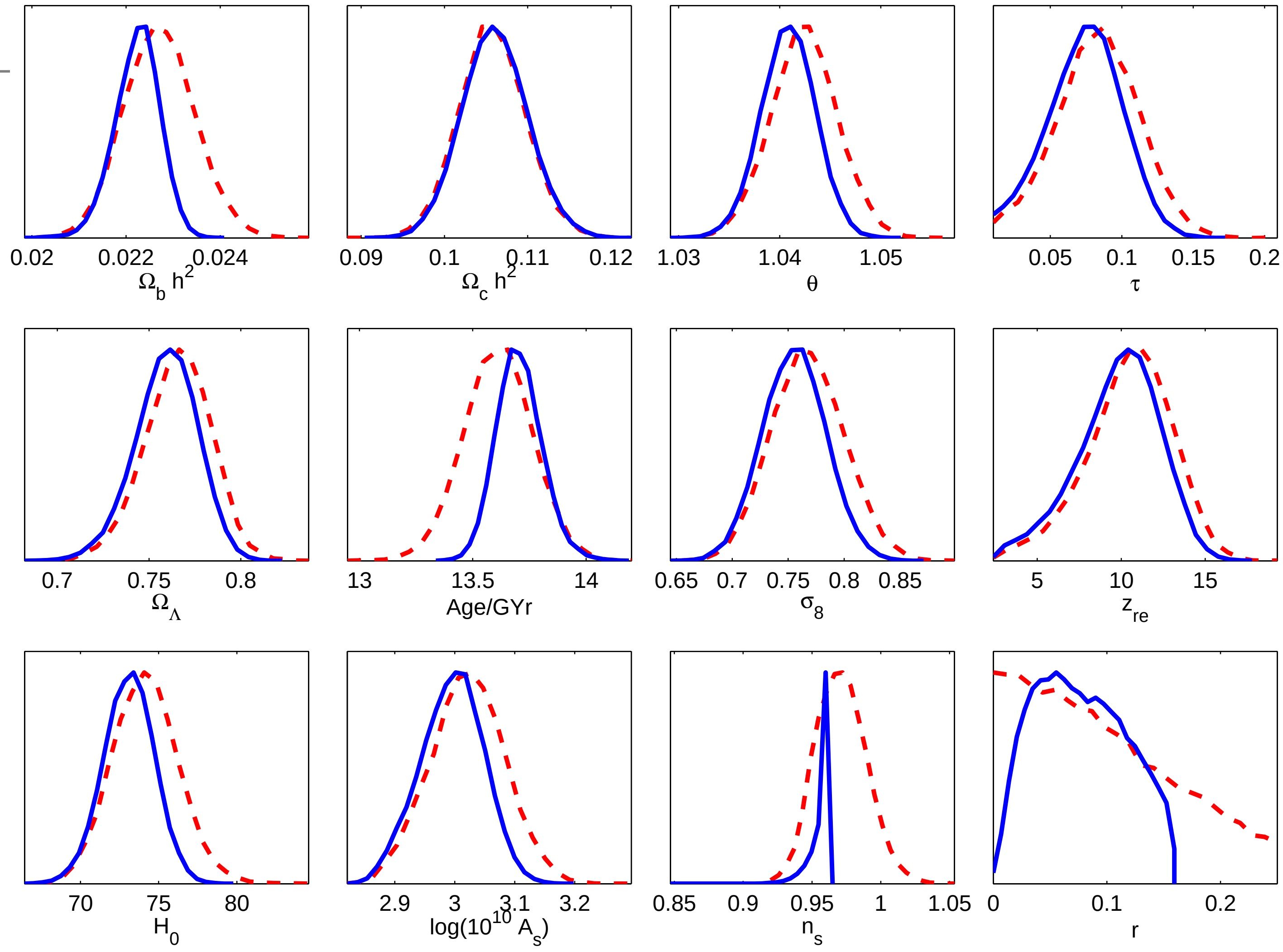
Probability distributions: solid blue curves
Mean likelihoods: dot-dashed red curves.

$$z_1 = 1 - \frac{y}{8\left(|h|+\sqrt{h^2+1}\right)^2}\ \chi^2 \ .$$

r vs. n_s data within the Trinomial New Inflation Region.

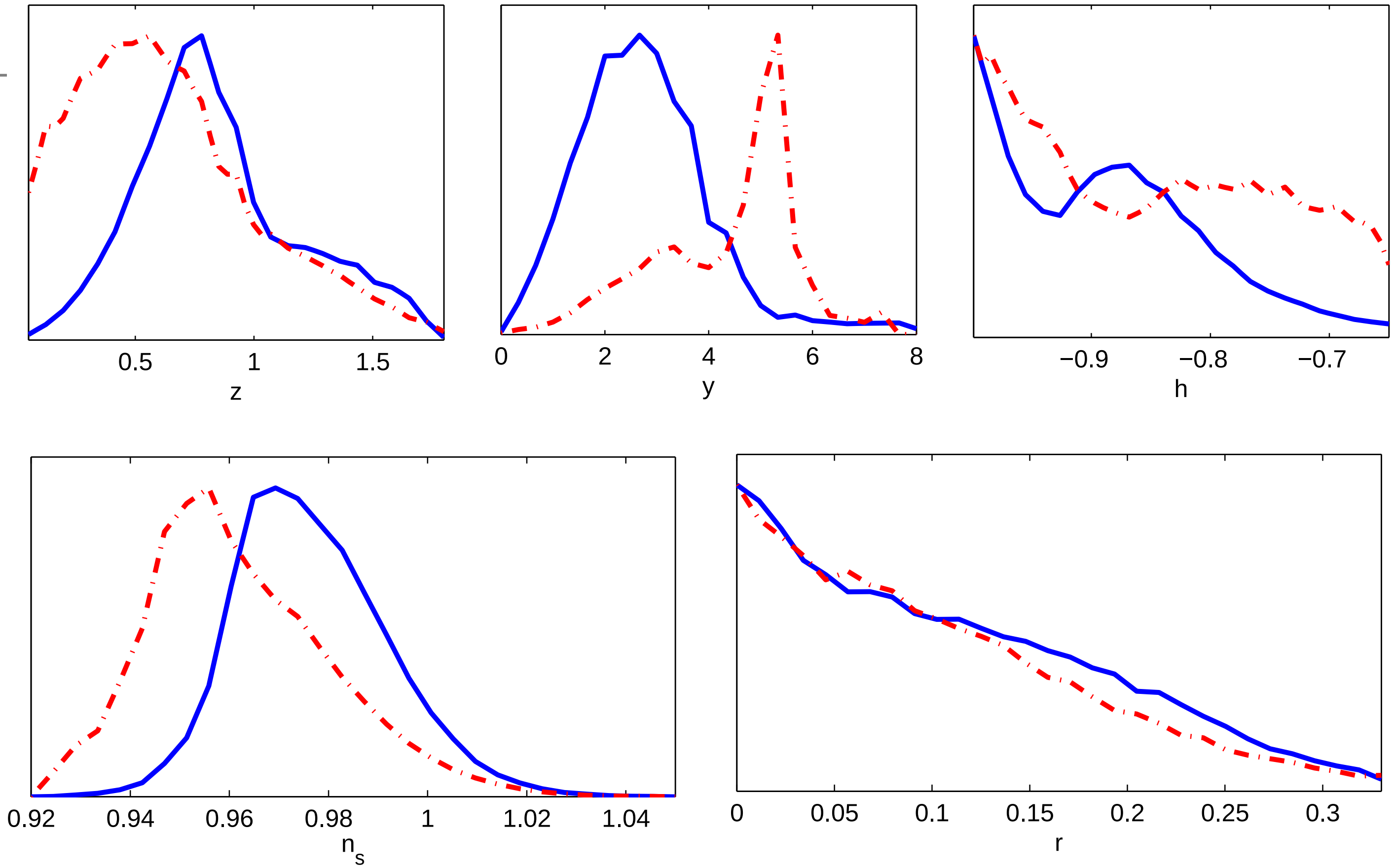


Marginalized probability distributions. New Inflation.



Imposing the trinomial potential (solid blue curves) and just the ΛCDM+r model (dashed red curves).
(curves normalized to have the maxima equal to one).

Probability Distributions. Trinomial Chaotic Inflation.



Probability distributions (solid blue curves) and mean likelihoods (dot-dashed red curves).

The data request a strongly asymmetric potential in chaotic inflation almost having two minima. That is, a strong breakdown of the $\chi \to -\chi$ symmetry.

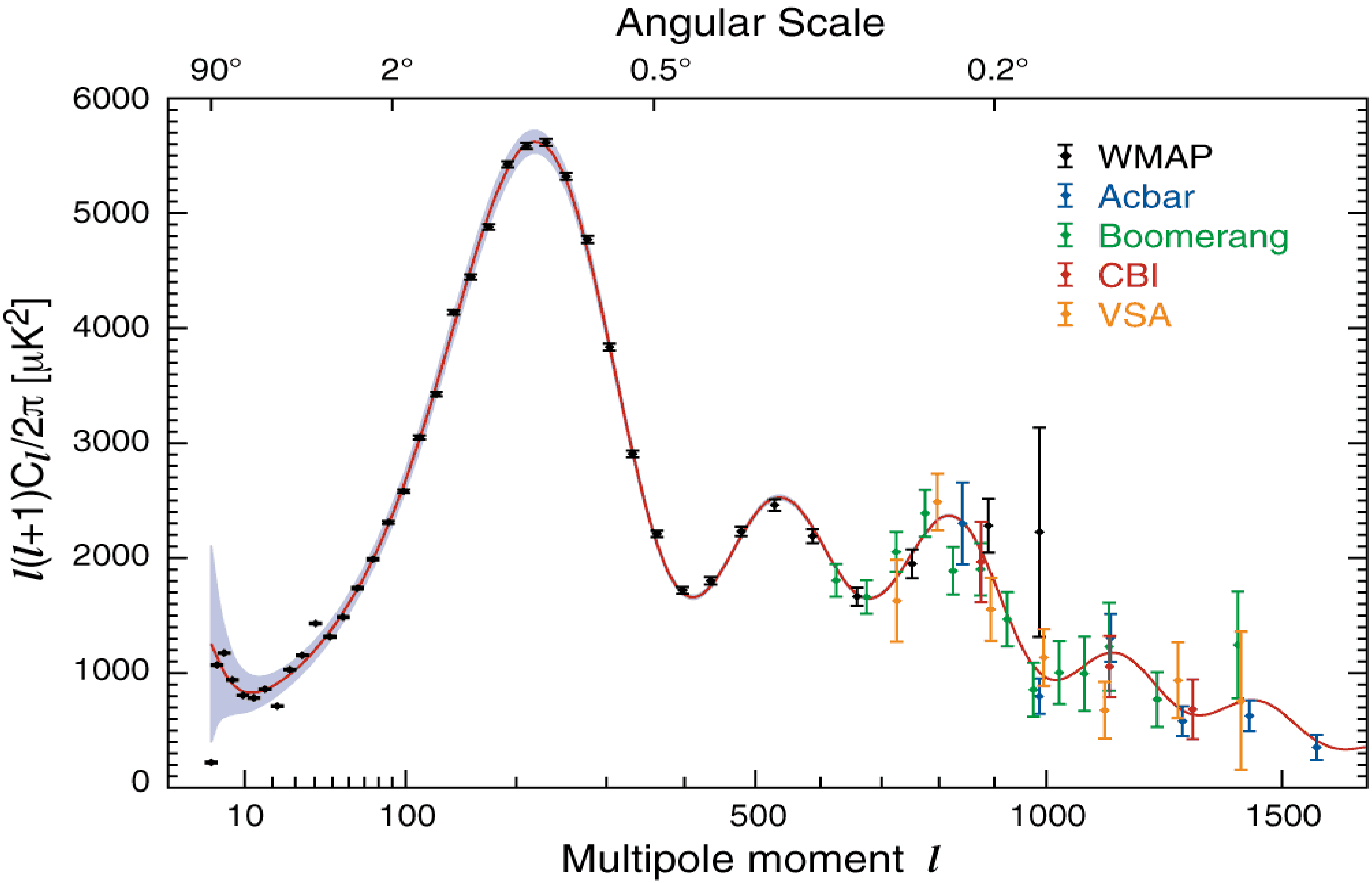
MCMC Results for Trinomial New Inflation.

Bounds: $r > 0.016 \; (95\% \text{ CL}) \quad , \quad r > 0.049 \; (68\% \text{ CL})$
Most probable values: $n_s \simeq 0.956 \quad , \quad r \simeq 0.055$.
The most probable trinomial potential for new inflation is symmetric and has a moderate nonlinearity with the quartic coupling $y \simeq 2.01 \ldots$ and $h \simeq 0.3$.
The $\chi \to -\chi$ symmetry is here spontaneously broken since the absolute minimum of the potential is at $\chi \neq 0$.

$$w(\chi) = \frac{y}{32} \left(\chi^2 - \frac{8}{y}\right)^2$$

C. Destri, H. J. de Vega, N. Sanchez, MCMC analysis of WMAP3 data points to broken symmetry inflaton potentials and provides a lower bound on the tensor to scalar ratio, astro-ph/0703417.

WMAP 3 years data plus others.



Theory and observations nicely agree except for the lowest multipoles: the quadrupole suppression.

Quadrupole Suppression and Fast Roll

Slow-roll inflation is generically preceded by a fast-roll stage where $\dot{\phi}^2 \sim V(\phi)$. Fast-Roll typically lasts 1 efold.

The slow-roll regime is an attractor with a large basin of attraction.

During fast roll curvature and tensor perturbations feel a potential equal to the slow-roll potential plus an extra attractive piece. This new piece suppresses the low multipoles as $1/l^2$.
If the quadrupole modes ($\sim$ Hubble radius today) exited the horizon 1.5 efolds after the beginning of fast roll, then the quadrupole modes get suppresed $\sim 20\%$ in agreement with the observations. $\Longrightarrow N_{total\ efolds} \simeq 60 + 1.5$.

D. Boyanovsky, H. J. de Vega, N. G. Sanchez, Phys. Rev. **D74**, 123006 and123007 (2006).

Loop Quantum Corrections to Slow-Roll Inflation

$$\phi(\vec{x},t) = \Phi_0(t) + \varphi(\vec{x},t), \quad \Phi_0(t) \equiv < \phi(\vec{x},t) >, \quad < \varphi(\vec{x},t) > = 0$$

$$\varphi(\vec{x},t) = \frac{1}{a(\eta)} \int \frac{d^3k}{(2\,\pi)^3} \left[a_{\vec{k}}\, \chi_k(\eta)\; e^{i\vec{k}\cdot\vec{x}} + \mathrm{h.c.} \right],$$

$a^\dagger_{\vec{k}}$, $a_{\vec{k}}$ are creation/annihilation operators,
$\chi_k(\eta)$ are mode functions. $\eta =$ conformal time.
To one loop order the equation of motion for the inflaton is

$$\ddot{\Phi}_0(t) + 3\,H\;\dot{\Phi}_0(t) + V'(\Phi_0) + g(\Phi_0)\;\langle[\varphi(\boldsymbol{x},t)]^2\rangle = 0$$

where $g(\Phi_0) = \frac{1}{2}\,V^{'''}(\Phi_0)$.
The mode functions obey:

$$\chi^{''}_k(\eta) + \left[k^2 + M^2(\Phi_0)\;a^2(\eta) - \frac{a^{''}(\eta)}{a(\eta)} \right] \chi_k(\eta) = 0$$

where $M^2(\Phi_0) = V''(\Phi_0) = 3\;H_0^2\;\eta_V + \mathcal{O}(1/N^2)$

Quantum Corrections to the Friedmann Equation

The mode functions equations for slow-roll become,

$$\chi_k''(\eta) + \left[k^2 - \frac{\nu^2 - \frac{1}{4}}{\eta^2}\right] \chi_k(\eta) = 0 \quad , \quad \nu = \frac{3}{2} + \epsilon_V - \eta_V + \mathcal{O}(1/N^2).$$

The scale factor during slow roll is $a(\eta) = -\frac{1}{H_0 \, \eta \, (1 - \epsilon_V)}$.

Scale invariant case: $\nu = \frac{3}{2}$. $\Delta \equiv \frac{3}{2} - \nu = \eta_V - \epsilon_V$ controls the departure from scale invariance.

Explicit solutions in slow-roll:

$\chi_k(\eta) = \frac{1}{2} \sqrt{-\pi \eta} \; i^{\nu + \frac{1}{2}} \; H^{(1)}_\nu(-k\eta), \quad H^{(1)}_\nu(z) =$ Hankel function

Quantum fluctuations: $\langle [\varphi(\boldsymbol{x}, t)]^2 \rangle = \frac{1}{a^2(\eta)} \int \frac{d^3k}{(2\pi)^3} \, |\chi_k(\eta)|^2$

$$\frac{1}{2} \langle [\varphi(\boldsymbol{x}, t)]^2 \rangle = \left(\frac{H_0}{4\,\pi}\right)^2 \left[{\Lambda_p}^2 + \ln \Lambda_p^2 + \frac{1}{\Delta} + 2\,\gamma - 4 + \mathcal{O}(\Delta)\right]$$

UV cutoff $\Lambda_p = \text{physical cutoff}/H, \quad \frac{1}{\Delta} =$ infrared pole.

$\left\langle \dot{\varphi}^2 \right\rangle , \quad \left\langle (\nabla \varphi)^2 \right\rangle$ are infrared finite. We thus compute $\langle T_{00} \rangle$.

Quantum Corrections to the Inflaton Potential

Upon UV renormalization the Friedmann equation results

$$H^2 = \frac{1}{3\,M_{Pl}^2}\left[\frac{1}{2}\,\dot{\Phi_0}^{\,2} + V_R(\Phi_0) + \left(\frac{H_0}{4\,\pi}\right)^2 \frac{V_R^{''}(\Phi_0)}{\Delta} + \mathcal{O}\left(\frac{1}{N}\right)\right]$$

Quantum corrections are proportional to $\left(\frac{H}{M_{Pl}}\right)^2 \sim 10^{-9}$!!

The Friedmann equation gives for the effective potential:

$$V_{eff}(\Phi_0) = V_R(\Phi_0) + \left(\frac{H_0}{4\,\pi}\right)^2 \frac{V_R^{''}(\Phi_0)}{\Delta}$$

$$V_{eff}(\Phi_0) = V_R(\Phi_0)\left[1 + \left(\frac{H_0}{4\,\pi\,M_{Pl}}\right)^2 \frac{\eta_V}{\eta_V - \epsilon_V}\right]$$

in terms of slow-roll parameters

Very DIFFERENT from the one-loop effective potential in Minkowski space-time:

$$V_{eff}(\Phi_0) = V_R(\Phi_0) + \frac{[V_R^{''}(\Phi_0)]^2}{64\,\pi^2} \ln \frac{V_R^{''}(\Phi_0)}{M^2}$$

Quantum Fluctuations:

Scalar Curvature(C), Tensor (t), Fermion (F), Light Scalar(s)

All these quantum fluctuations contribute to the inflaton potential and to the primordial power spectra.

In de Sitter space-time: $< T_{\mu\nu} >= \frac{1}{4}\, g_{\mu\nu} \;\; < T_\alpha^\alpha >$

Hence, $V_{eff} = V_R + < T_0^0 >= V_R + \frac{1}{4} < T_\alpha^\alpha >$

Sub-horizon (Ultraviolet) contributions appear through the trace anomaly and only depend on the spin of the particle. Superhorizon (Infrared) contributions are of the order N^0 and can be expressed in terms of the slow-roll parameters.

$$V_{eff}(\Phi_0) = V(\Phi_0)\left[1 + \frac{H_0^2}{3\,(4\pi)^2\, M_{Pl}^2}\left(\frac{\eta_v - 4\,\epsilon_v}{\eta_v - 3\,\epsilon_v} + \frac{3\,\eta_s}{\eta_s - \epsilon_v} + \mathcal{T}\right)\right]$$

$\mathcal{T} = \mathcal{T}_C + \mathcal{T}_s + \mathcal{T}_t + \mathcal{T}_F = -\frac{2903}{20}$ is the total trace anomaly.

$\mathcal{T}_C = \mathcal{T}_s = -\frac{29}{30}, \;\mathcal{T}_t = -\frac{717}{5}, \;\mathcal{T}_F = \frac{11}{60}$

$\longrightarrow$ the graviton (t) dominates.

Corrections to the Primordial Scalar and Tensor Power

$$|\Delta^{(S)}_{k,eff}|^2 = |\Delta^{(S)}_{k}|^2 \{1+$$
$$+\frac{1}{3}\left(\frac{H_0}{4\,\pi\,M_{Pl}}\right)^2\left[2+\frac{\frac{3}{4}\,r\,(n_s-1)+4\,\frac{dn_s}{d\ln k}}{(n_s-1)^2}+\frac{2903}{20}\right]\right\}$$
$$|\Delta^{(T)}_{k,eff}|^2 = |\Delta^{(T)}_{k}|^2 \left\{1-\frac{1}{3}\left(\frac{H_0}{4\,\pi\,M_{Pl}}\right)^2\left[-1+\frac{1}{8}\,\frac{r}{n_s-1}+\frac{2903}{20}\right]\right\}.$$

The anomaly contribution $-\frac{2903}{20}=-145.15$ DOMINATES (as long as the number of fermions is less than 783).

The scalar curvature fluctuations $|\Delta^{(S)}_{k}|^2$ are ENHANCED and the tensor fluctuations $|\Delta^{(T)}_{k}|^2$ REDUCED.

However, $\left(\frac{H}{M_{Pl}}\right)^2 \sim 10^{-9}$.

D. Boyanovsky, H. J. de Vega, N. G. Sanchez, Phys. Rev. D 72, 103006 (2005), astro-ph/0507596.

The Energy Scale of Inflation

Grand Unification Idea (GUT)

- Renormalization group running of electromagnetic, weak and strong couplings shows that they all meet at $E_{GUT} \simeq 2 \times 10^{16}$ GeV
- Neutrino masses are explained by the see-saw mechanism: $m_\nu \sim \frac{M^2_{\rm Fermi}}{M_R}$ with $M_R \sim 10^{16}$ GeV.
- Inflation energy scale: $M \simeq 10^{16}$ GeV.

Conclusion: the GUT energy scale appears in at least three independent ways.

Moreover, moduli potentials: $V_{moduli} = M^4_{\rm SUSY} \; v\left(\frac{\phi}{M_{Pl}}\right)$

ressemble inflation potentials provided $M_{\rm SUSY} \sim 10^{16}$GeV.
First observation of SUSY in nature??

De Sitter Geometry and Scale Invariance

The De Sitter metric is scale invariant:

$$ds^2 = \frac{1}{(H\ \eta)^2}\left[(d\eta)^2 - (d\vec{x})^2\right] \ .$$

$\eta =$ conformal time.

But inflation only lasts for N efolds !

Corrections to scale invariance:

$|n_s - 1|$ as well as the ratio r are of order $\sim 1/N$,

$n_s = 1$ and $r = 0$ correspond to a critical point.

It is a gaussian fixed point around which the inflation model hovers in the renormalization group (RG) sense with an almost scale invariant spectrum during the slow roll stage.

The quartic coupling:

$$\lambda = \frac{G_4}{N}\ \left(\frac{M}{M_{Pl}}\right)^4 \quad , \quad N = \log\frac{a(\text{inflation end})}{a(\text{horizon exit})}$$

runs like in four dimensional RG in flat euclidean space.

Dark Energy

$76 \pm 5\%$ of the present energy of the Universe is Dark!
Current observed value:
$\rho_\Lambda = \Omega_\Lambda \, \rho_c = (2.39 \text{ meV})^4 \; , \; 1 \text{ meV} = 10^{-3} \text{ eV}$.
Equation of state $p_\Lambda = -\rho_\Lambda$ within observational errors.
Quantum zero point energy. Renormalized value is finite.
Bosons (fermions) give positive (negative) contributions.
Mass of the lightest fermion ~ 1 meV is in the right scale.
Spontaneous symmetry breaking of continuous symmetries produces massless scalars as Goldstone bosons. A small symmetry breaking provide light scalars: axions, familons, majorons .....
Observational Axion window $10^{-3} \text{ meV} \lesssim M_{\text{axion}} \lesssim 10 \text{ meV}$.
Dark energy can be a cosmological analogue to the Casimir effect in Minkowski with non-trivial boundaries.
We need to learn the physics of light particles (< 1 MeV), also to understand dark matter !!

Summary and Conclusions

- Inflation can be formulated as an effective field theory in the Ginsburg-Landau spirit with energy scale $M \sim M_{GUT} \sim 10^{16}$GeV $\ll M_{Pl}$.
- Effective theory does work because: $H \ll M \ll M_{Pl}$. Inflaton mass small: $m \sim H/\sqrt{N}$. Infrared regime!
- The slow-roll approximation is a $1/N$ expansion, $N \sim 50$
- MCMC analysis of WMAP+LSS data plus the Trinomial Inflation potential indicates a spontaneously symmetry breaking potential (new inflation): $w(\chi) = \frac{y}{32}\left(\chi^2 - \frac{8}{y}\right)^2$.
- Lower Bounds: $r > 0.016\ (95\%\ \text{CL})\ ,\ r > 0.049\ (68\%\ \text{CL})$. The most probable values are $n_s \simeq 0.956\ ,\ r \simeq 0.055$ with a quartic coupling $y \simeq 2$.

Summary and Conclusions 2

- The quadrupole suppression may be explained by the effect of fast roll inflation provided the today's horizon size modes exited 1.5 efolds after the beginning of inflation.
- Quantum (loop) corrections in the effective theory are of the order $(H/M_{Pl})^2 \sim 10^{-9}$.

D. Boyanovsky, H. J. de Vega, N. G. Sanchez,

Quantum corrections to the inflaton potential and the power spectra from superhorizon modes and trace anomalies, Phys. Rev. D 72, 103006 (2005), astro-ph/0507596.

Quantum corrections to slow roll inflation and new scaling of superhorizon fluctuations. Nucl. Phys. B 747, 25 (2006), astro-ph/0503669.

THANK YOU VERY MUCH

FOR YOUR ATTENTION!!

Fast and Slow Roll Inflation

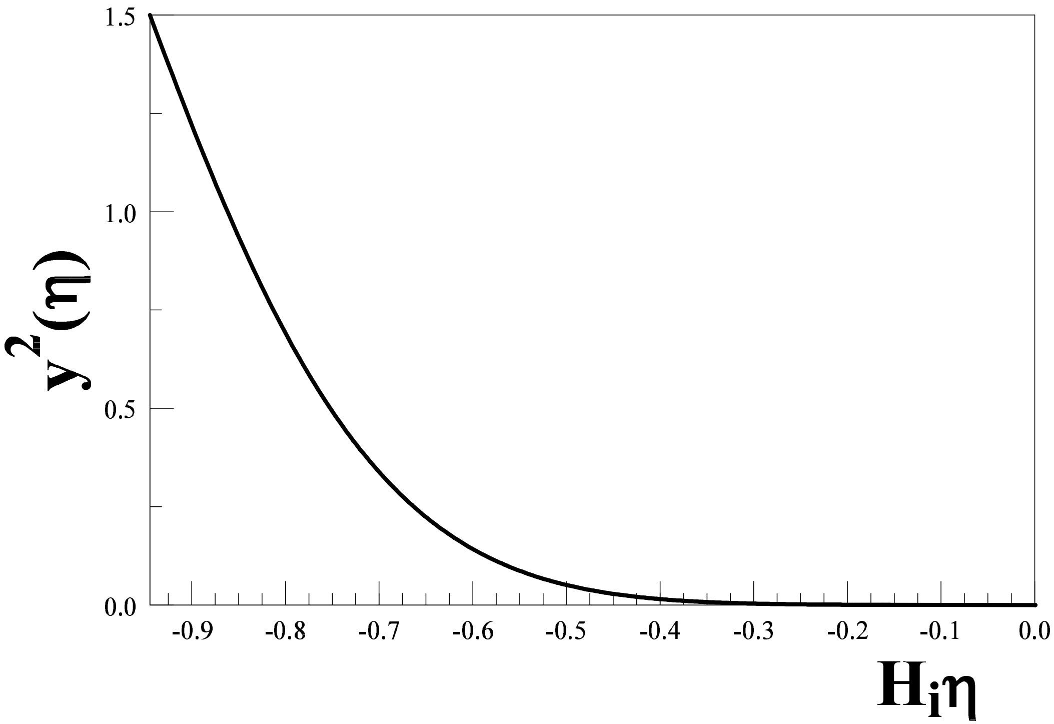
$$H^2 = \frac{1}{3\,M_{PL}^2}\left[\frac{1}{2}\,\dot{\Phi}^2 + V(\Phi)\right] \; ,$$
$$\ddot{\Phi} + 3\,H\,\dot{\Phi} + V'(\Phi) = 0 \; .$$

Slow-roll corresponds to $\dot{\Phi}^2 \ll V(\Phi)$.

Generically, we can have $\dot{\Phi}^2 \sim V(\Phi)$ to start.
That is, FAST ROLL inflation.

However, slow-roll is an attractor with a large basin.

Fast roll for new inflation



$$y^2 = \frac{\dot{\Phi}^2}{2\, M_{Pl}^2\, H^2} = 3\left[1 - \frac{V(\Phi)}{3\, M_{Pl}^2\, H^2}\right]\ ,\ 0 \leq y^2 \leq 3\ , \qquad N \sim 50.$$

$\eta =$ conformal time.

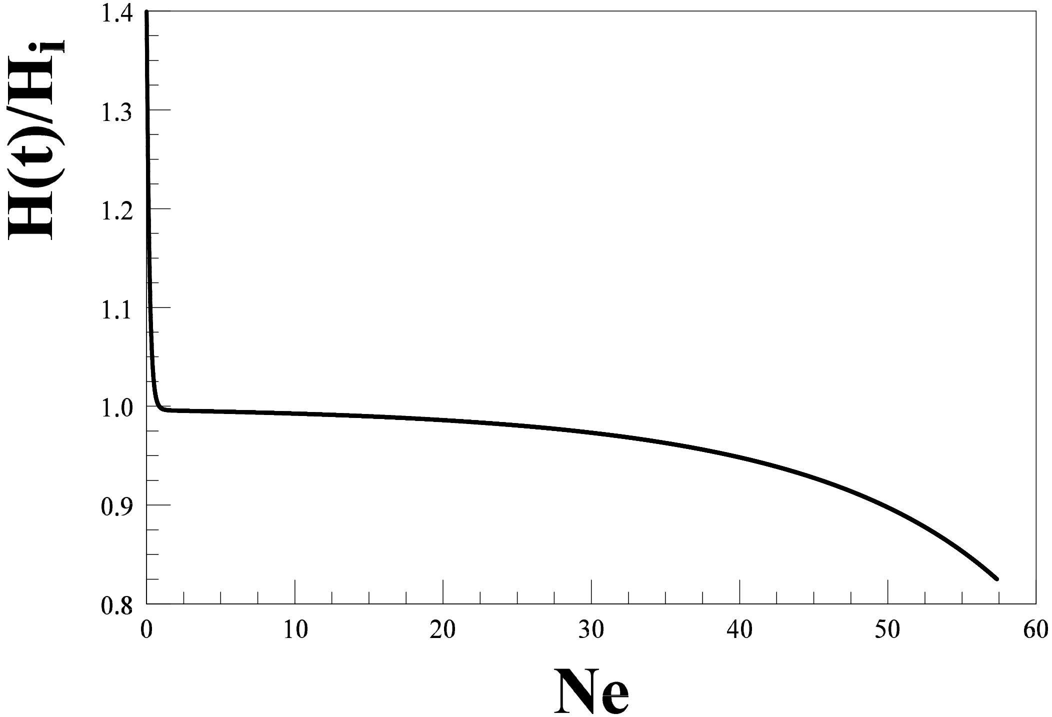
$H_i =$ Hubble at the beginning of slow-roll.

$y^2 \sim 1 =$ Fast-roll for $H_i\, \eta < -0.5$.

$y^2 = \frac{1}{N} \ll 1 =$ slow-roll for $H_i\, \eta > -0.5$.

[$y^2 = \epsilon_V$ during slow-roll.]

Hubble vs. number of efolds



$H_i =$ Hubble at the beginning of slow-roll.
Fast-roll lasts about one-efold.

Extreme fast roll solution ($y^2 = 3$) in cosmic time:

$$H = \frac{1}{3\,t} \quad , \quad a(t) = a_0\, t^{\frac{1}{3}} \quad , \quad \Phi = -M_{Pl}\,\sqrt{\frac{2}{3}}\,\log(m\,t)\ .$$

Gauge Invariant Curvature Perturbations

$\mathcal{R}(\boldsymbol{x},t) = -\psi(\boldsymbol{x},t) - \frac{H(t)}{\dot{\Phi}(t)}\,\phi(\boldsymbol{x},t)$

$\phi(\boldsymbol{x},t) =$ inflaton fluctuations. $\psi(\boldsymbol{x},t) =$ newtonian potential.

These fluctuations around the FRW geometry are responsible of the CMB anisotropies and the LSS formation.

Gauge invariant potential

$u(\boldsymbol{x},t) \equiv -z(t)\,\mathcal{R}(\boldsymbol{x},t)\;,\; z(t) \equiv a(t)\,\frac{\dot{\Phi}(t)}{H(t)}$

In Fourier space: $u(\boldsymbol{k},\eta) = \alpha_{\mathcal{R}}(\boldsymbol{k})\,S_{\mathcal{R}}(k;\eta) + \alpha^{\dagger}_{\mathcal{R}}(\boldsymbol{k})\,S^{*}_{\mathcal{R}}(k;\eta)$

$\alpha^{\dagger}_{\mathcal{R}}(\boldsymbol{k})$ and $\alpha_{\mathcal{R}}(\boldsymbol{k})$ are creation and annihilation operators.

The mode functions obey a Schrödinger-like equation,

$$\left[\frac{d^2}{d\eta^2} + k^2 - W_{\mathcal{R},\mathcal{T}}(\eta)\right] S_{\mathcal{R},\mathcal{T}}(k;\eta) = 0\;.$$

Scalar Curvature and tensor fluctuations

$W_{\mathcal{R}}(\eta) = \frac{1}{z}\frac{d^2 z}{d\eta^2}$ for scalar, $\quad W_{\mathcal{T}}(\eta) = \frac{1}{a}\frac{d^2 a}{d\eta^2}$ for tensor.

$W_{\mathcal{R},\mathcal{T}}(\eta) = \frac{\nu^2_{\mathcal{R},\mathcal{T}} - \frac{1}{4}}{\eta^2} + \mathcal{V}_{\mathcal{R},\mathcal{T}}(\eta)$.

Like a centrifugal barrier plus $\mathcal{V}_{\mathcal{R},\mathcal{T}}(\eta)$.

scalar: $\nu_{\mathcal{R}} = \frac{3}{2} + 3\,\epsilon_V - \eta_V \quad , \quad$ tensor: $\nu_T = \frac{3}{2} + \epsilon_V$

$$\epsilon_V = \frac{1}{2N} \left[\frac{w'(\chi)}{w(\chi)}\right]^2 \quad , \quad \eta_V = \frac{1}{N}\,\frac{w''(\chi)}{w(\chi)} \; .$$

$\mathcal{V}(\eta) = 0$ during slow-roll, $\mathcal{V}(\eta) \neq 0$ during fast-roll.

During slow-roll: $S(k;\eta) = A(k)\,g_\nu(k;\eta) + B(k)\,f_\nu(k;\eta)$

$g_\nu(k;\eta) = \frac{1}{2}\,i^{\nu+\frac{1}{2}}\,\sqrt{-\pi\eta}\,H^{(1)}_\nu(-k\eta)\;,\;\; f_\nu(k;\eta) = [g_\nu(k;\eta)]^*$

$H^{(1)}_\nu(z)$: Hankel function.

Scale invariant limit: $g_{\frac{3}{2}}(k;\eta) = \frac{e^{-ik\eta}}{\sqrt{2k}}\left[1 - \frac{i}{k\eta}\right]$.

The effect of $\mathcal{V}_{\mathcal{R},\mathcal{T}}(\eta)$ during the fast roll

The initial conditions on the modes $S(k;\eta)$ plus $\mathcal{V}_{\mathcal{R},\mathcal{T}}(\eta)$ determine the coefficients $A_{\mathcal{R},\mathcal{T}}(k)$ and $B_{\mathcal{R},\mathcal{T}}(k)$.

We choose Bunch-Davies initial conditions:

$$S_\nu(k;\eta) \overset{\eta\to-\infty}{=} \frac{1}{\sqrt{2\,k}}\, e^{-ik\eta}$$

$$\mathcal{V}_{\mathcal{R},\mathcal{T}}(\eta) = 0 \longrightarrow A(k) = 1,\ B(k) = 0$$

$\mathcal{V}_{\mathcal{R},\mathcal{T}}(\eta) \neq 0$ is analogous to a one dimensional scattering problem in the η-axis.

D. Boyanovsky, H. J. de Vega, N. Sanchez,
CMB quadrupole suppression:
I. Initial conditions of inflationary perturbations,
II. The early fast roll stage,
Phys.Rev. D74 (2006) 123006 and 123007,
astro-ph/0607508 and astro-ph/0607487.

Primordial Power Spectrum

$$P_{\mathcal{R}}(k) \overset{\eta\to 0^-}{=} \frac{k^3}{2\,\pi^2}\left|\frac{S_{\mathcal{R}}(k;\eta)}{z(\eta)}\right|^2 = P_{\mathcal{R}}^{sr}(k)\Big[1+D_{\mathcal{R}}(k)\Big]\;,$$

$$P_{T}(k) \overset{\eta\to 0^-}{=} \frac{k^3}{2\,\pi^2}\left|\frac{S_{T}(k;\eta)}{a(\eta)}\right|^2 = P_{T}^{sr}(k)\Big[1+D_{T}(k)\Big]\;.$$

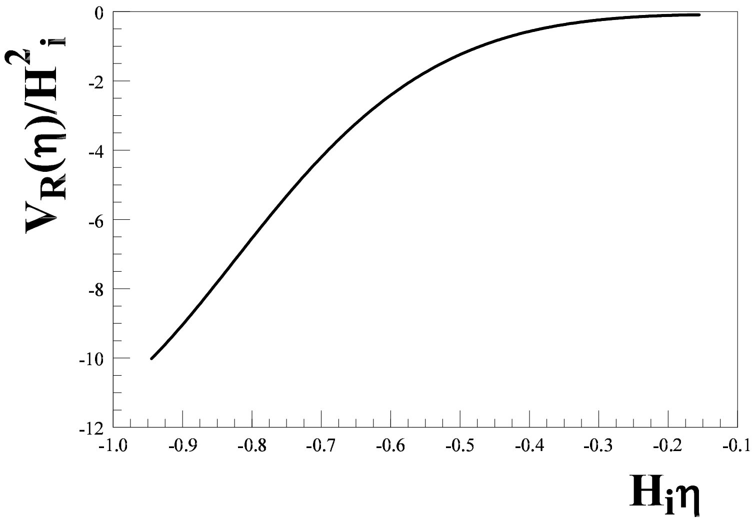
Standard slow roll power spectrum:

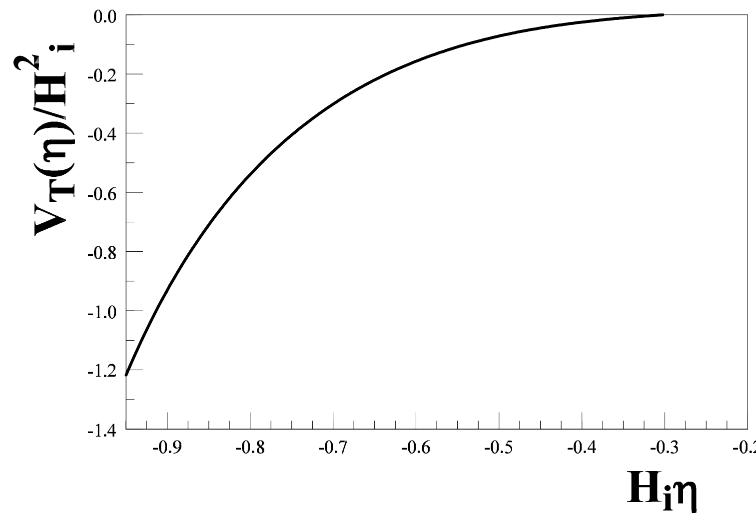
$$P_{\mathcal{R}}^{sr}(k) = \mathcal{A}_{\mathcal{R}}^2\;\left(\frac{k}{k_0}\right)^{n_s-1}\;,\;P_{T}^{sr}(k) = \mathcal{A}_{T}^2\;\left(\frac{k}{k_0}\right)^{n_T}$$

$$D(k) = 2\,|B(k)|^2 - 2\;\mathrm{Re}\left[A(k)\;B^*(k)\,i^{2\nu-3}\right]$$

$D_{\mathcal{R}}(k)$ and $D_T(k)$ are the transfer functions of curvature and tensor perturbations taking into account the effect of the fast-roll stage.

Potential felt by the Scalar and by the Tensor Fluctuations





$H_i =$ Hubble at the beginning of slow-roll.

Both $\mathcal{V}_{\mathcal{R}}(\eta)$ and $\mathcal{V}_{\mathcal{T}}(\eta)$ are ATTRACTIVE potentials.

Potential felt by tensor fluctuations much smaller:

$$\mathcal{V}_{\mathcal{T}}(\eta) \sim \frac{1}{10} \; \mathcal{V}_{\mathcal{R}}(\eta)$$

Change in the C_l due to fast roll

$$C_l \equiv C_l^{sr} + \Delta C_l \quad , \qquad \frac{\Delta C_l}{C_l} = \frac{\int_0^\infty D_{\mathcal{R},\mathcal{T}}(\kappa\, x) \;\; f_l(x)\, dx}{\int_0^\infty f_l(x)\, dx}$$

$$\kappa \equiv a_0 \; H_0/3.3 = a_{sr} \; H_i/3.3 \quad , \qquad f_l(x) \equiv x^{n_s - 2}[j_l(x)]^2 \; .$$

Since $\mathcal{V}_{\mathcal{R},\mathcal{T}}(\eta)$ are quite small we can compute the transfer functions in the Born approximation:

$$D_{\mathcal{R},\mathcal{T}}(k) = \int_{-\infty}^0 d\eta \mathcal{V}_{\mathcal{R},\mathcal{T}}(\eta)[\sin(2k\eta)\left(1 - \frac{1}{k^2\eta^2}\right) + \frac{2}{k\eta} \; \cos(2k\eta)]/k$$

and then, $\quad \frac{\Delta C_2}{C_2} = \frac{1}{\kappa} \int_{-\infty}^0 d\eta \; \mathcal{V}_{\mathcal{R},\mathcal{T}}(\eta) \; \Psi(\kappa \; \eta)$

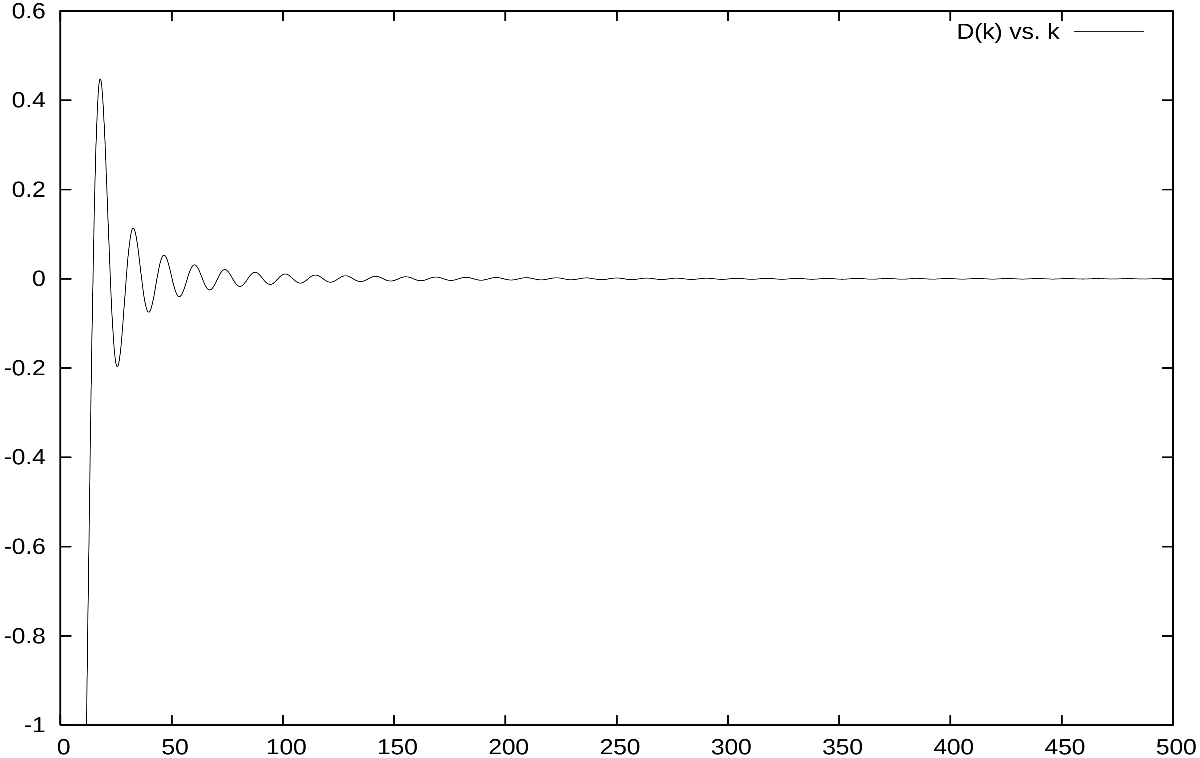
where $\Psi(\kappa \; \eta) > 0$ for $\eta < 0$.

ATTRACTIVE $\mathcal{V}_{\mathcal{R},\mathcal{T}}(\eta) < 0$ implies $\Delta C_2 < 0$.

$\longrightarrow$ QUADRUPOLE SUPPRESSION.

In general, $0 > \frac{\Delta C_l}{C_l} = \mathcal{O}\left(\frac{1}{l^2}\right) \; .$

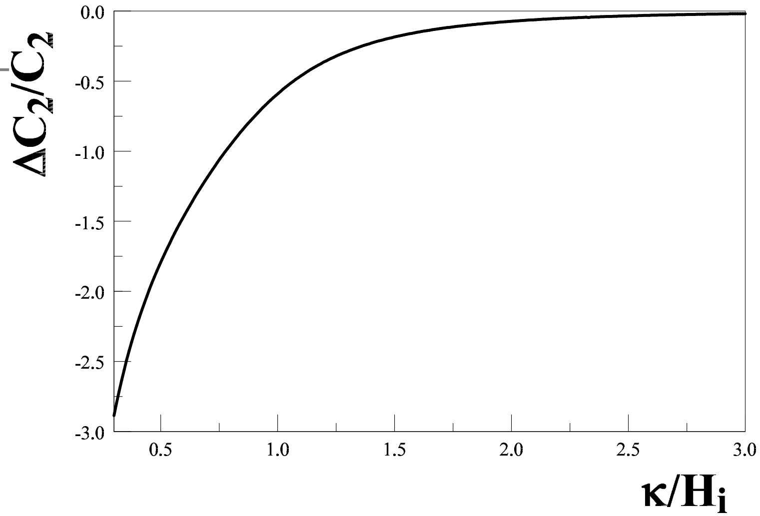
The Transfer Function $D(k)$ for the scalar fluctuations.

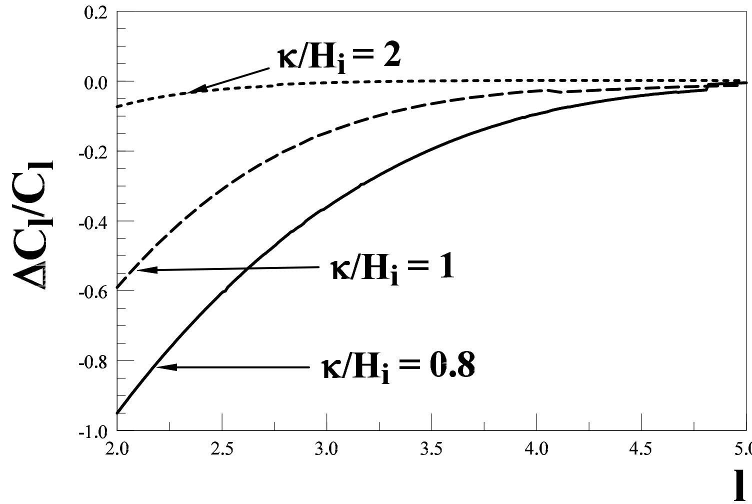


The transfer function $D_{\mathcal{R}}(k)$ computed in the Born approximation for trinomial new inflation $y \simeq 2, \ h = 0$.

$$P_{\mathcal{R}}(k) = P^{sr}_{\mathcal{R}}(k)\left[1 + D_{\mathcal{R}}(k)\right]$$

Quadrupole Suppression vs. Fast Roll

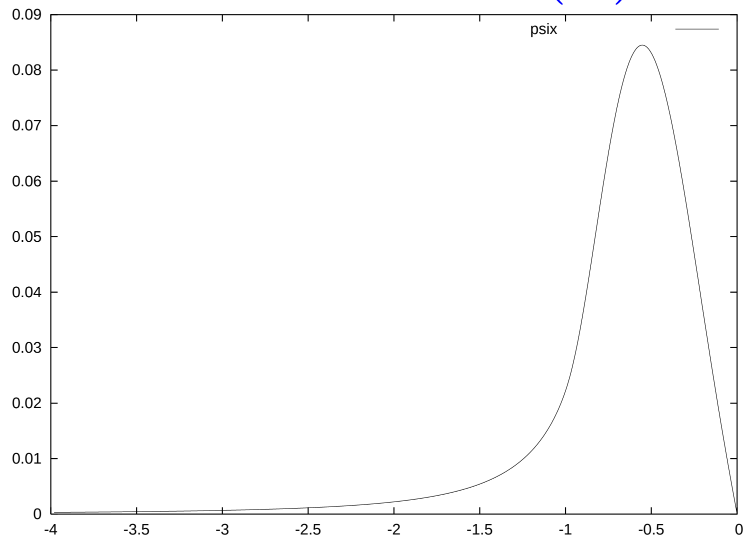




$\frac{\kappa}{H_i} = \frac{a_{sr}}{3.3}$. The Quadrupole is suppressed 20% for $a_{sr} \simeq 4.6 \simeq e^{1.5} \longrightarrow$ the quadrupole modes should exit the horizon $\simeq 1.5$ efolds after fast-roll starts

Quadrupole Suppression Explanation:
Inflation starts with fast roll: 0 efolds.
Fast-roll ends and slow-roll begins: 1 efold.
Today Horizon size modes exit the horizon by 1.5 efolds.
Inflation ends at the minimal number of efolds plus $\simeq 1.5$.
$[N_T \simeq 60 + 1.5]$

$\Psi(x)$ is an odd function.



$$\Psi(x) \equiv 3\int_0^\infty \frac{dy}{y^4}\,[j_2(y)]^2 \left[\left(y^2 - \frac{1}{x^2}\right)\sin(2\,y\,x) + \frac{2\,y}{x}\;\cos(2\,y\,x)\right] =$$
$$= \frac{1}{105\,x^2}\left[p(x)\;(1-x)^3\;\log\left|1-\frac{1}{x}\right| - (x \to -x)\right] + \frac{2}{105\,x} - \frac{13\,x}{126} + \frac{22\,x^3}{105} - \frac{2\,x^5}{21}\;,$$

$p(x)$ is the sixth order polynomial:

$$p(x) \equiv 10\,x^6 + 30\,x^5 + 33\,x^4 + 19\,x^3 + 9\,x^2 + 3\,x + 1\;.$$

$\Psi(x) < 0$ for $x > 0$.

$$\Psi(x) \overset{x\to 0}{=} -\frac{x}{6} + \mathcal{O}(x^3) \quad , \quad \Psi(x) \overset{x\to\infty}{=} -\frac{1}{60\,x^3} + \mathcal{O}\left(\frac{1}{x^5}\right)\;.$$