M. Giovannini, PRD 73, 101302 (2006);
PRD 74, 063002 (2006);
arXiv:0706.4428[astro-ph];
arXiv:0707.0857[astro-ph];

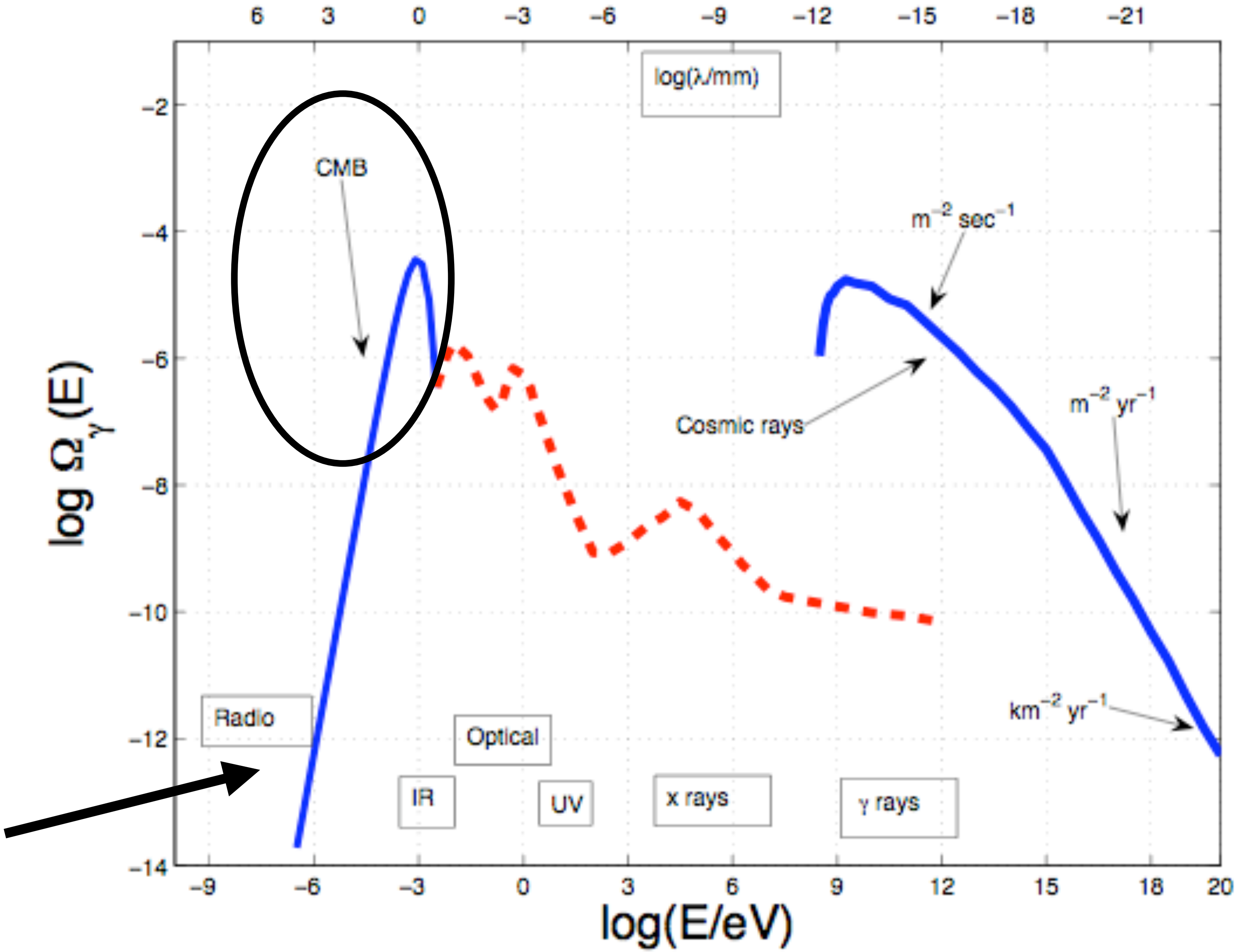
Magnetized CMB anisotropies

Massimo Giovannini (CERN-PH-TH)

Paris, Chalonge School, August 2007

Grand Spectrum

$$\Omega_\gamma(E) = \frac{1}{\rho_{crit}} \frac{d\rho_\gamma}{d\ln E}$$



A Magnetized Universe?

- Large-scale magnetic fields (typical length-scales > 1 A.U.) $1 A.U. = 1.49\ 10^{13}\ cm$

- First speculations: early forties (Alfven) late forties (Fermi, Fermi & Chandrasekar) on cosmic ray physics $1\,\mu G = 0.1\ nT = 10^{-26}\ GeV^2$

-Today: magnetic fields measured with various techniques

Zeeman splitting of radio transitions

$$\Delta\nu_Z = \frac{e\overline{B}_{\parallel}}{2\pi m_e}$$

$$\Delta\nu_{Doppler} \simeq \left(\frac{v_{th}}{c}\right)\nu \gg \Delta\nu_{Zeeman} \simeq \frac{e\overline{B}_{\parallel}}{2\pi m_e}$$

Synchrotron emission

$$\varepsilon(\nu) = 10^{-23}\, n_{er0}\, L\, \xi(\gamma)\, (6.3\times 10^{18})^{(\gamma-1)/2} (B_{\perp})^{(\gamma+1)/2}\, \nu^{(1-\gamma)/2}\ \ erg\ sec^{-1}\ cm^{-2}\ Hz^{-1}$$

Faraday rotation

$$\Delta\phi = \frac{f_e}{2}\left(\frac{\omega_p}{\omega}\right)^2 \omega_B\, \Delta z$$

$$\omega_p = \left(\frac{4\pi n_e e^2}{m_e}\right)^{1/2} \qquad \omega_B = \frac{eB}{mc}$$

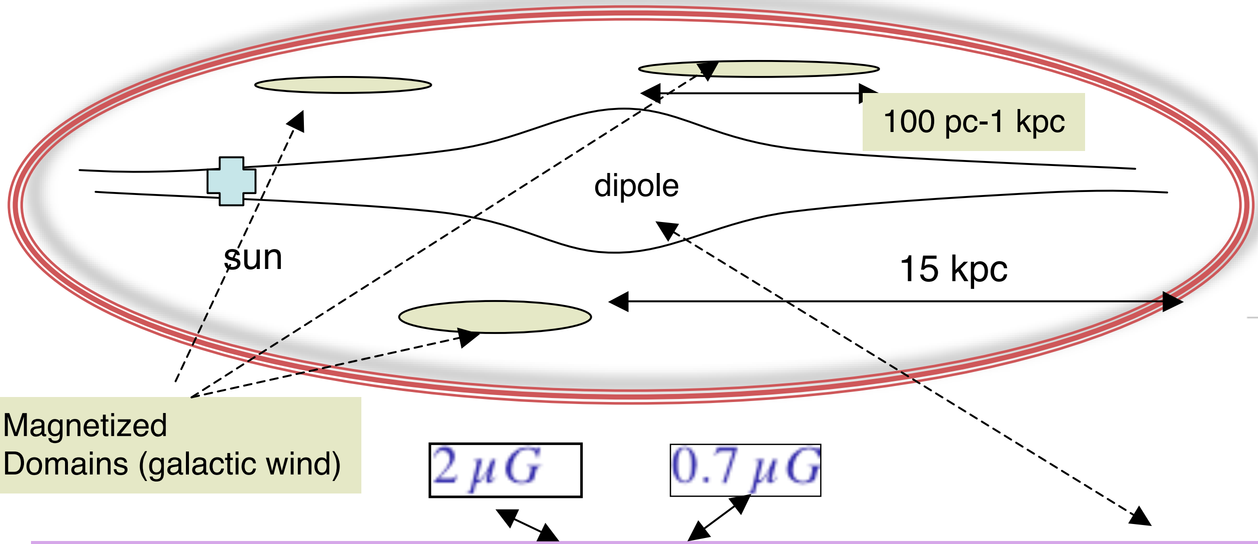
$$\phi = RM\,\lambda^2 + \phi_0$$

$$RM = \frac{\Delta\phi}{\Delta\lambda^2} = 811.9 \int \left(\frac{n_e}{cm^{-3}}\right)\left(\frac{B_{\parallel}}{\mu G}\right) d\left(\frac{\ell}{kpc}\right)\ \frac{rad}{m^2}$$

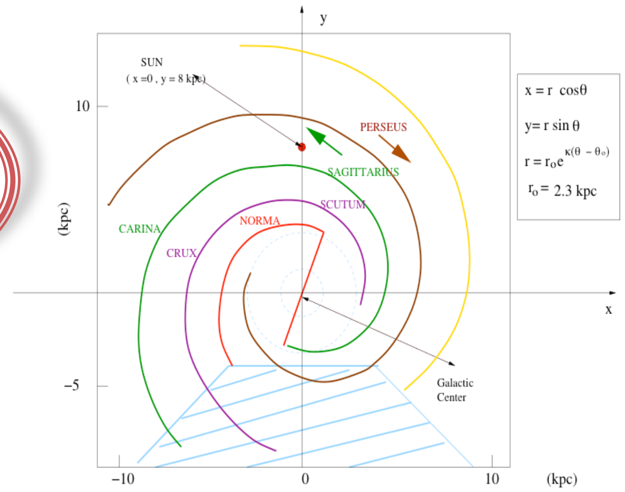
$$\langle B_{\parallel}\rangle = \frac{RM}{DM}$$

$$DM \propto \int n_e\, d\ell$$

Magnetized galaxies, clusters, and superclusters



SGP survey
Parkes



Milky way: toroidal field south/north hemisphere; dipoles (center) $10\ rad/m^2$

Local Group: Andromeda, Magellanic Clouds,… $2 - 7\,\mu G$ (elliptical galaxies: shorter scale)

Abell Clusters (like COMA): magnetic fields inside cluster (VLA+ROSAT) [Faraday RM]
Typical RM: $100\ rad/m^2$ $B \sim 0.5\,\mu G = 500\,nG$ $L \sim 50 - 100\ kpc$

Superclusters: Local Supercluster (Local Group + Virgo Cluster $1.5\,\mu G$
Coma Supercluster (COMA+ Abell 1367) $0.5\,\mu G$?

If true: important for UHECR…

Hercules / Perseus-Pisces $B_L \simeq 0.5\mu G$ $L \simeq 500\ kpc$
$n_e \simeq 10^{-6}\ cm^{-3}$ GRG

Kronberg (2006)

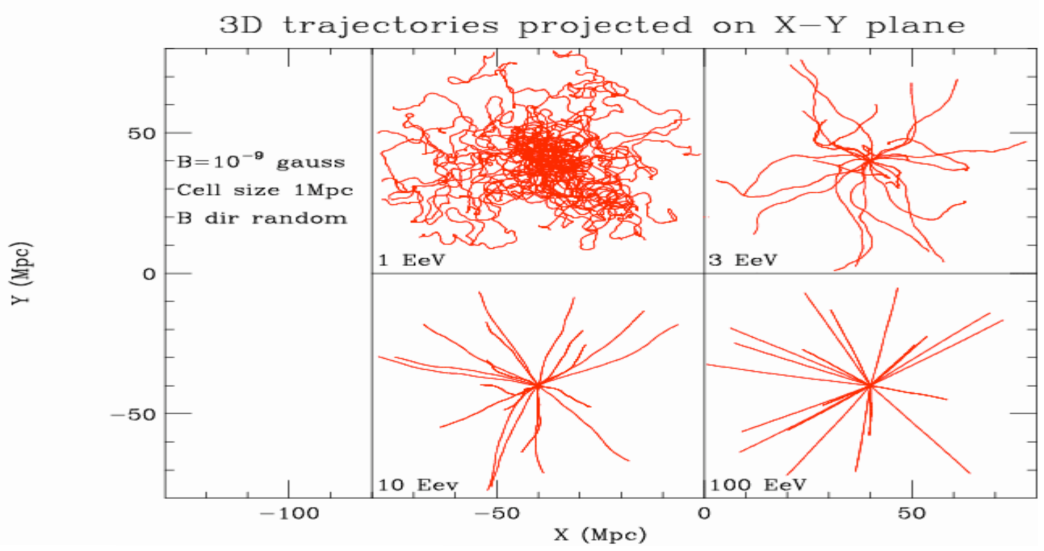
Cosmic Rays: Auger

Recent Auger results:

They reject (to 6 sigma confidence) the hypothesis that the CR spectrum continues in the form of a power law for energies larger than $10^{19.6}$eV

They DO NOT find anisotropies near the direction of the Galactic center for energies between 10^{18}eV and 10^{19}eV

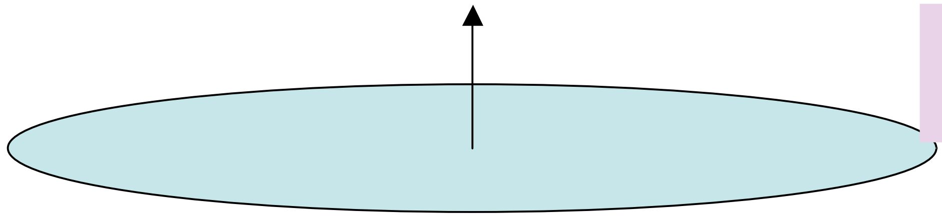
Projected view of 20 trajectories of proton primaries emanating from a point source for several energies.



Dynamo and compressional amplification

Galaxy:

$$\lambda_D \simeq \sqrt{\frac{T}{8\pi n_e e^2}}$$



Charged fluid (globally neutral)

Typical rotation period: $P \sim 3 \times 10^8\ yrs$ age $T \sim 10^{10}\ yrs$

Dynamo instability:

$$\alpha = -\frac{\tau_0}{3}\langle \vec{v}\cdot\vec{\nabla}\times\vec{v}\rangle \quad \sim 9.1 \times 10^6\ \frac{cm}{sec}$$

$$\frac{1}{4\pi\sigma} = 10^{25}\frac{cm^2}{sec}$$

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla}\times(\vec{v}\times\vec{B}) + \frac{1}{4\pi\sigma}\nabla^2\vec{B}$$

Dynamo term

Diffusivity term

$$\frac{\partial <\vec{B}>}{\partial t} = \alpha\vec{\nabla}\times <\vec{B}> + \frac{1}{\sigma}\nabla^2 <\vec{B}>$$

$$1/k \sim L > kpc$$

Maximal and optimistic amplification: $e^{\Gamma t} \sim e^{T/P} \sim e^{N} \sim 10^{13}$

$B_i \sim 10^{-19}\ G$ Over L = 30 kpc

Clash: dynamo versus helicity conservation.
Brandenburg & Subramanian

$$\frac{d}{dt}\int_V d^3x\vec{A}\cdot\vec{B} = -\frac{1}{4\pi\sigma}\int_V d^3x\vec{B}\cdot\vec{\nabla}\times\vec{B} + O\left(\frac{1}{\sigma^2}\right)$$

Compressional amplification:

$$B_b = \left(\frac{\rho_b}{\rho_a}\right)^{2/3} B_a$$

Mpc

30 kpc

$$B_i \geq 10^{-23}\ G \quad over \quad L \sim Mpc$$

Primordial magnetogenesis

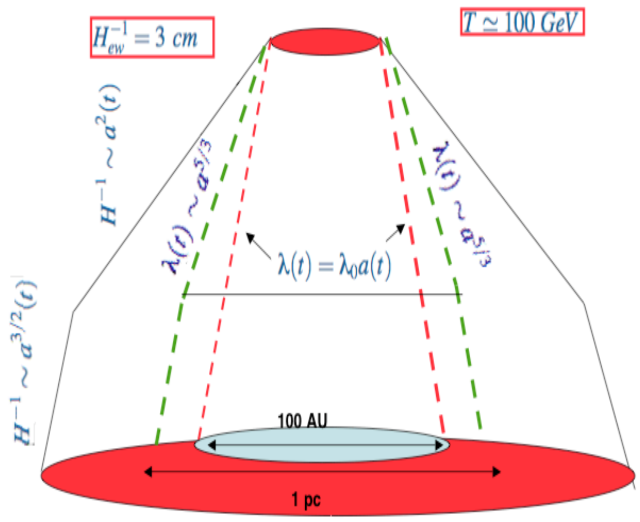
$B_{\rm seed} > 10^{-23}{\rm G}$ → Too optimistic

$B_{\rm seed} \simeq 10^{-11}\,{\rm G} = O(0.01\,{\rm nG})$

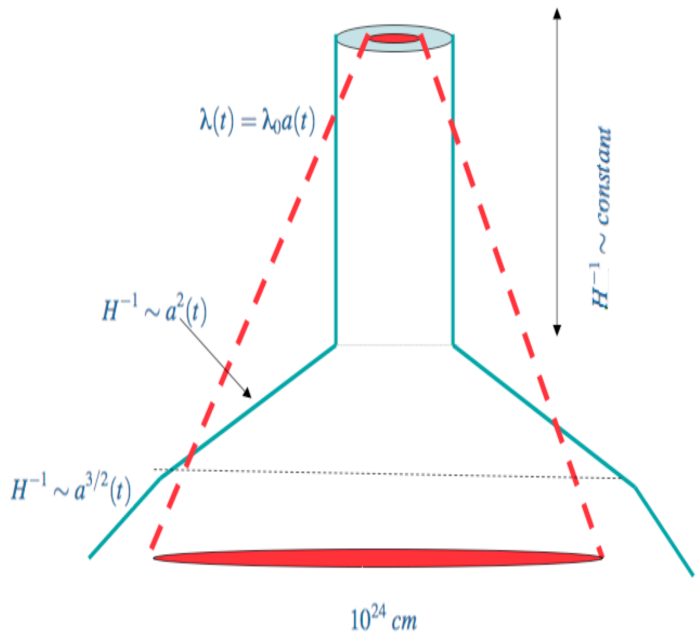
$B_{\rm seed} > 10^{-18}{\rm G}$ → More realistic [flux not exactly conserved, small-scale fields can grow large and swamp dynamo action]

effective e-folds 30->25

CAUSAL mechanisms



``Inflationary'' mechanisms



LET US SUPPOSE....

Uniform magnetic field approximation
[magnetic field along a specific axis].
Simplified estimates
[not so realistic in diverse cases]

FOREGROUNDS & B FIELDS

-- distorsion of the Planckian spectrum
-- shift of the polarization plane of CMB (Faraday rotation)
-- effects on primary anisotropies

Intermediate situation:
uniform magnetic field with
inhomogeneous fluctuations

Fully inhomogeneous
magnetic fields : more
realistic [mathematically
less tractable]

Zeldovich approximation (1965)

Zeldovich ``approximation" : homogeneous field with (weak) breaking of spatial isotropy

Y. Zeldovich
Sov. Phys. JETP 21
656 (1965)

Magnetic fields weakly breaks spatial isotropy: Bianchi-type I paradigm

(generalizations MG PRD 2000)

$$ds^2 = dt^2 - a^2(t)dx^2 - b^2(t)[dy^2 + dz^2]$$

Electromagnetic radiation propagating along x and y will have a different temperature

$$T_x(t) = T_1 \frac{a_1}{a} = T_1 e^{-\int H(t)dt},$$
$$T_y(t) = T_1 \frac{b_1}{b} = T_1 e^{-\int F(t)dt},$$

$$\rightarrow$$

$$\boxed{\frac{\Delta T}{T} \sim \int [H(t) - F(t)]\, dt = \frac{1}{2} \int r(t)\, d\log t}$$

$$r(t) = \frac{3[H(t) - F(t)]}{[H(t) + 2F(t)]}$$

Radiation-dominated case

Shear parameter is conserved and proportional to the magnetic energy density

From "Zeldovich" approximation"

$$\frac{B_0^2}{\rho_\gamma} \leq 10^{-6} \quad \rightarrow \quad B_0 \leq 2.23 \times 10^{-9} \ Gauss$$

More accurate estimates based on modified angular power spectrum lead to quantitatively similar estimates.

G. Chen, et al APJ (2004)

Faraday rotation by a UNIFORM magnetic field

Kosowsky& Loeb ApJ (97)
MG PRD (97), MG(PRD,2005)

From two-fluid description:

$$\Delta\phi = f_e \frac{e}{2m_e}\left(\frac{\omega_p}{\omega}\right)^2 (\vec{B}\cdot\hat{z})\delta z$$

$$\langle(\Delta\phi)^2\rangle^{1/2} \simeq 1.6^0 \left(\frac{B}{B_c}\right)\left(\frac{\omega_M}{\omega}\right)^2$$

$$B_c \sim 10^{-3}\, G$$

$$\omega_F = \frac{d\phi}{d\eta} = \frac{e^3\, n_e\, x_e \vec{B}\cdot\vec{q}}{8\pi^2\, m_e^2\, \nu^2}\frac{a}{a_0}$$

Axial symmetry around k, e.g. B II k (!)

$$\Delta_Q' + (ik\mu + \tau')\Delta_Q - 2\omega_F\Delta_U = \frac{\tau'}{2}[1 - P_2(\mu)]S_Q$$
$$\Delta_U' + (ik\mu + \tau')\Delta_U + 2\omega_F\Delta_Q = 0$$

$$\tau' = x_e\, n_e\, \sigma_T \frac{a}{a_0}$$

$$S_Q = \Delta_{I,2} + \Delta_{Q,0} + \Delta_{Q,2}$$

$$\mathrm{TB} = \omega_F\, TE$$

Visibility function

$$B_0 < 10^{-8}\ Gauss,\ \ @\ \ 30\ GHz$$

$$(\Delta_Q \pm i\Delta_U) = \frac{3}{4}(1-\mu^2)\int_0^{\eta_0} d\eta e^{-ik\mu\Delta\eta} K(\eta)\, S_Q(\eta)\, e^{\mp 2i\omega_F\Delta\eta}$$

From WMAP TE correlations

E-modes are ROTATED into B-modes !

$$(\Delta_Q \pm i\Delta_U)(\hat{n}) = \sum_{\ell m} a_{\pm 2,\ell m}\ {}_{\pm 2}Y_{\ell m}(\hat{n})$$

$$a_{E,\ell m} = -\frac{1}{2}(a_{2,\ell m} + a_{-2,\ell m})$$
$$a_{B,\ell m} = \frac{i}{2}(a_{2,\ell m} - a_{-2,\ell m}).$$

$$E(\hat{n}) = \sum_{lm} a_{E,\ell m} Y_{lm}(\hat{n}), \qquad B(\hat{n}) = \sum_{lm} a_{B,\ell m} Y_{lm}(\hat{n}).$$

Fully inhomogeneous magnetic fields

$$\langle B_i(\vec{k},\tau)B^j(\vec{p},\tau)\rangle = \frac{2\pi^2}{k^3}P_i^j(k)\,\delta^{(3)}(\vec{k}+\vec{p})$$

$$B_i(\vec{k},\tau) = a^2(\tau)\mathcal{B}_i(\vec{k},\tau)$$

Spectral index

$$P_i^j(k) = P_B(k)\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) + \cancel{iQ_B(k)\epsilon_{ij\ell}\frac{k^\ell}{k}}, \qquad P_B(k) = A_B\left(\frac{k}{k_p}\right)^{\varepsilon}, \qquad \cancel{Q_B(k) = \tilde{A}_B k^{\tilde{\varepsilon}}}$$

MHD approach:

$$\vec{J} = \frac{1}{4\pi}\vec{\nabla}\times\vec{B}, \qquad \vec{E} = \frac{\vec{\nabla}\times\vec{B}}{4\pi\sigma} - \vec{v}\times\vec{B}$$

$$\frac{1}{a^4}\vec{\nabla}\cdot[\vec{J}\times\vec{B}] = \frac{1}{4\pi a^4}\vec{\nabla}\cdot[(\vec{\nabla}\times\vec{B})\times\vec{B}]$$

Divergence of "Lorentz Force"

$$\delta_s\mathcal{T}_0^0 = \delta_s\rho_B, \qquad \delta_s\mathcal{T}_i^j = -\delta_s p_B\delta_i^j + \tilde{\Pi}_i^j$$

Not all independent !

$$\delta_s\rho_B = \frac{|B^2(\vec{x},\tau)|^2}{8\pi\,a^4(\tau)}, \qquad \delta_s p_B = \frac{\delta\rho_B}{3}$$

$$\tilde{\Pi}_i^j = \frac{1}{4\pi\,a^4(\tau)}\left[B_i B^j - \frac{B^2}{3}\delta_i^j\right]$$

Magnetic energy density and pressure

Anisotropic stress

Inhomogeneities in FRW models

$$\delta \mathcal{T}_{\mu\nu} = \delta \mathcal{T}_{\mu\nu}^{(S)} + \delta \mathcal{T}_{\mu\nu}^{(V)} + \delta \mathcal{T}_{\mu\nu}^{(T)}$$

$$g_{\mu\nu} \to g_{\mu\nu}(\tau) + \delta g_{\mu\nu}(\vec{x},\tau) \Rightarrow \delta g_{\mu\nu}(\vec{x},\tau) = \delta_s g_{\mu\nu}(\vec{x},\tau) + \delta_v g_{\mu\nu}(\vec{x},\tau) + \delta_t g_{\mu\nu}(\vec{x},\tau)$$

10 degrees of freedom

$$ds^2 = a^2(\tau)[d\tau^2 - d\vec{x}^2]$$

$$\delta_s g_{\mu\nu}(\vec{x},\tau) = a^2(\tau)\begin{pmatrix} 2\phi & -\partial_i B \\ -\partial_i B & 2(\psi\delta_{ij} - \partial_i\partial_j E) \end{pmatrix} \Rightarrow$$

4 d. f.

$$\delta_v g_{\mu\nu}(\vec{x},\tau) = a^2(\tau)\begin{pmatrix} 0 & -Q_i \\ -Q_i & \partial_i W_j + \partial_j W_i \end{pmatrix} \Rightarrow$$

4 d. f.

$$\boxed{\partial_i Q^i = 0, \qquad \partial_i W^i = 0}$$

$$\delta_t g_{\mu\nu}(\vec{x},\tau) = a^2(\tau)\begin{pmatrix} 0 & 0 \\ 0 & -h_{ij} \end{pmatrix} \Rightarrow$$

2 d. f.

$$\boxed{\partial_i h^i_j = 0, \qquad h^i_i = 0}$$

1) Vector modes (easier)

2) Tensor modes (gauge-invariant)

3) Scalar modes (most complicated) $\Rightarrow$

4) CMB anisotropies induced by scalar modes

5) CMB polarization

Magnetized curvature perturbations

Choose a gauge (for instance conformally Newtonian) $\mathcal{H} = \frac{a'}{a}$

$$\zeta = -\psi - \frac{\delta\rho_t + \delta\rho_B}{\rho_t'}\mathcal{H}$$

Hamiltonian constraint

$$\zeta = \mathcal{R} + \frac{\nabla^2\psi}{12\pi G a^2 (p_t + \rho_t)}$$

Density contrast on uniform curvature hypersurfaces

$$\mathcal{R} = -\psi - \frac{\mathcal{H}(\mathcal{H}\phi + \psi')}{\mathcal{H}^2 - \mathcal{H}'}$$

$$\zeta(k,\tau) \simeq \mathcal{R}(k,\tau) + O(|k\tau|^2)$$

Curvature fluctuations on comoving orthogonal hypersurfaces

$\rho_t = \rho_\gamma + \rho_\nu + \rho_b + \rho_c$ $+\Lambda$ (does not fluctuate)

Photons

neutrinos

Baryons

CDM

$$\delta\rho_t = \delta\rho_\gamma + \delta\rho_\nu + \delta\rho_b + \delta\rho_c$$

Evolution equations

Photons and baryons : tightly coupled at early times $\theta_\gamma \simeq \theta_b = \theta_{\gamma b}$

$$\theta'_{\gamma b} + \frac{\mathcal{H} R_b}{(1+R_b)}\theta_{\gamma b} + \frac{\nabla^2 \delta_\gamma}{4(1+R_b)} + \nabla^2\phi = \frac{3}{4}\frac{\vec{\nabla}\cdot[\vec{J}\times\vec{B}]}{a^4\rho_\gamma(1+R_b)}$$

$$\delta_\gamma = \frac{\delta\rho_\gamma}{\rho_\gamma} \qquad \theta_{\gamma b} = \partial_i v^i_{\gamma b}$$

$$R_b = \left(\frac{698}{z+1}\right)\left(\frac{\omega_b}{0.023}\right)\left(\frac{\omega_\gamma}{2.47\times 10^{-5}}\right)^{-1}$$

Neutrinos : collisionless below 1 MeV

$$\theta'_\nu + \frac{1}{4}\nabla^2\delta_\nu + \nabla^2\phi = \nabla^2\sigma_\nu, \qquad \delta'_\nu = 4\psi' - \frac{4}{3}\theta_\nu, \qquad \sigma'_\nu = \frac{4}{15}$$

$$\delta_\nu = \frac{\delta\rho_\nu}{\rho_\nu} \qquad \theta_\nu = \partial_i v^i_\nu$$

CDM : only coupled through metric fluctuations

$$\theta'_c + \mathcal{H}\theta_c + \nabla^2\phi = 0, \qquad \delta'_c = 3\psi' - \theta_c, \qquad \delta_c = \frac{\delta\rho_c}{\rho_c}$$

$$\theta_c = \partial_i v^i_c$$

+ PERTURBED EINSTEIN EQUATIONS COUPLING PLASMA & MAGNETIC FIELDS

Anisotropic stress: important aspect (neutrinos + magnetic fields)

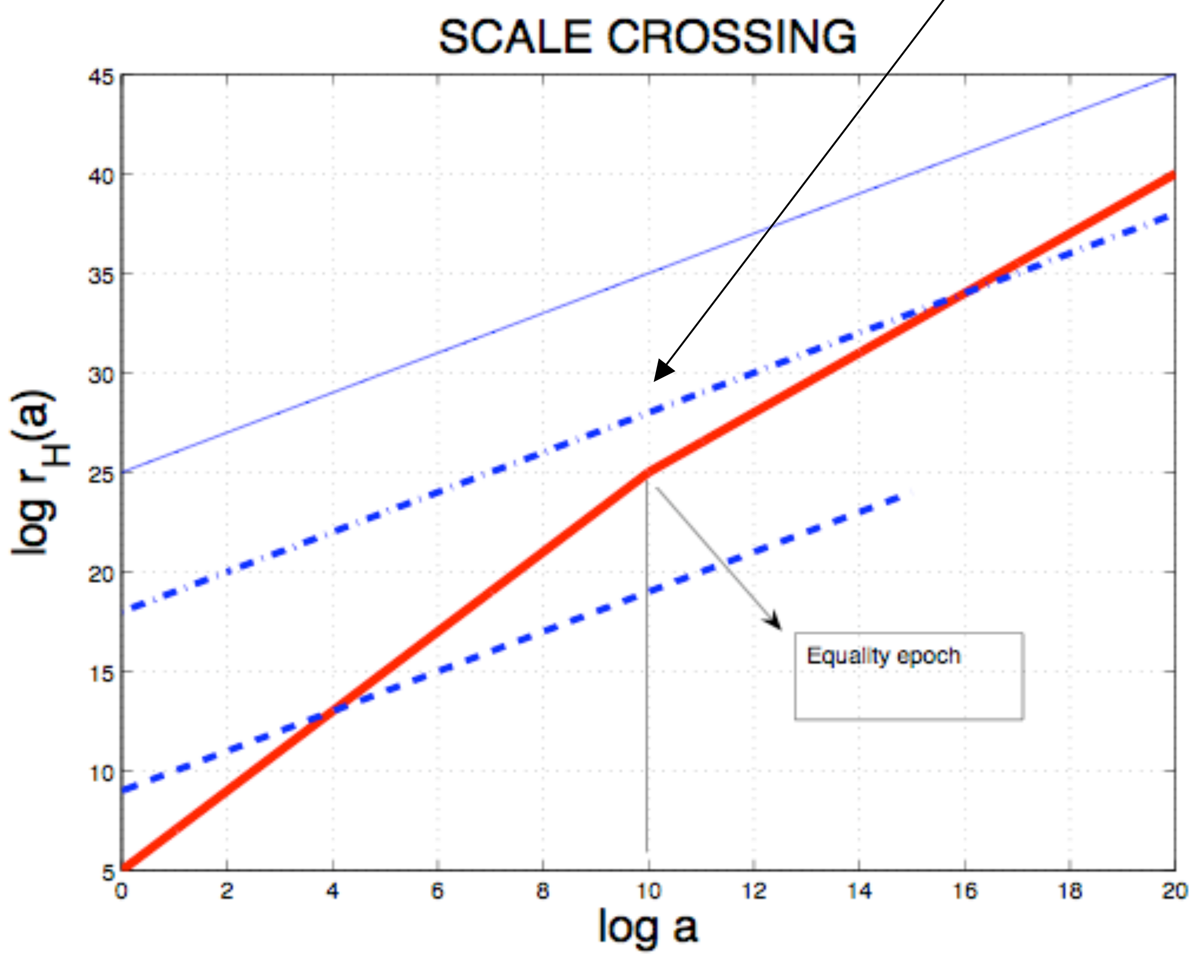
$$\nabla^4(\phi-\psi) = 12\pi G a^2[(p_\nu+\rho_\nu)\nabla^2\sigma_\nu + (p_\gamma+\rho_\gamma)\nabla^2\sigma_B]$$

$$\nabla^2\sigma_B = \frac{3}{16\pi a^4\rho_\gamma}\vec{\nabla}\cdot[(\vec{\nabla}\times\vec{B})\times\vec{B}] + \frac{\nabla^2\Omega_B}{4}$$

$$\Omega_B(\vec{x}) = \frac{\delta\rho_B(\tau,\vec{x})}{\rho_\gamma(\tau)}$$

Crossing of different scales

$$\zeta' = -\frac{\mathcal{H}}{\rho_t(1+w_t)}\delta p_{\mathrm{nad}} + \frac{\mathcal{H}(3c_{\mathrm{st}}^2-1)}{3\rho_t(1+w_t)}\delta\rho_B - \cancel{\frac{\theta_t}{3}}.$$



Magnetized adiabatic mode

$|k\tau| \ll 1$

$$\delta_\gamma = \delta_\nu = -2\phi_i - R_\gamma \Omega_B, \qquad \delta_b = \delta_c = -\frac{3}{2}\phi_i - \frac{3}{4}R_\gamma \Omega_B$$

Density contrasts

$$\theta_{\gamma b} = \frac{k^2\tau}{4}[2\phi_i + R_\nu \Omega_B - 4\sigma_B], \qquad \theta_c = \frac{k^2\tau}{2}\phi_i, \qquad \theta_\nu = \frac{k^2\tau}{2}\left[\phi_i - \frac{R_\gamma \Omega_B}{2}\right] + k^2\tau \frac{R_\gamma}{R_\nu}\sigma_B$$

Peculiar velocities

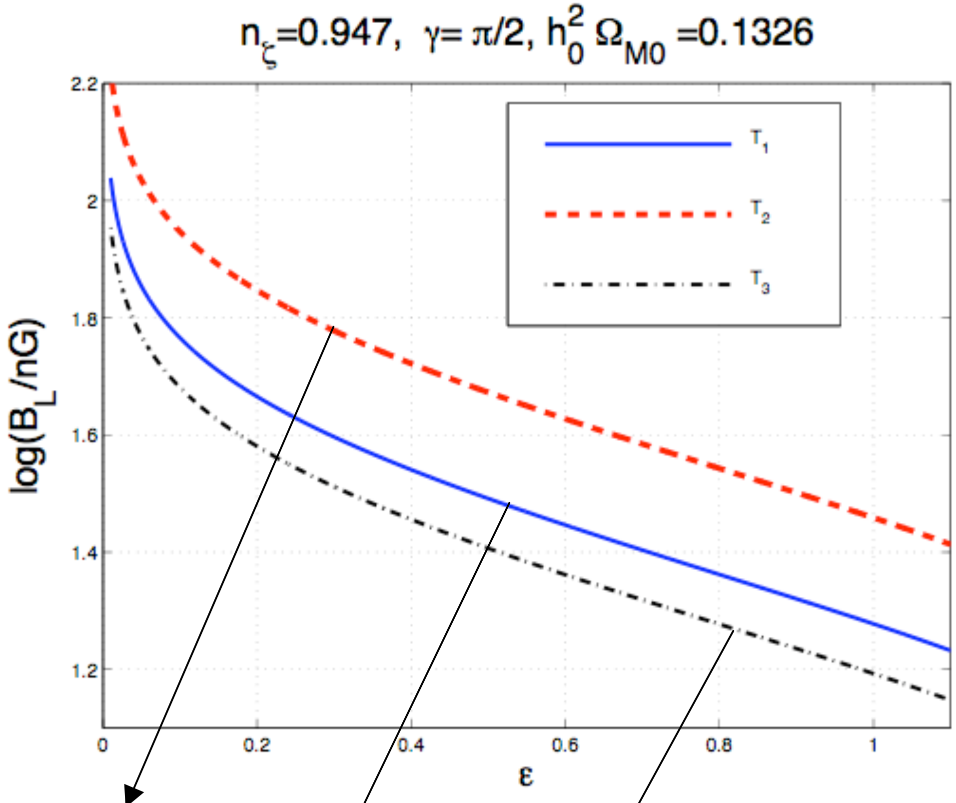
$$\psi_i = \phi_i\left(1 + \frac{2}{5}R_\nu\right) + \frac{R_\gamma}{5}(4\sigma_B - R_\nu \Omega_B), \qquad \sigma_\nu = -\frac{R_\gamma}{R_\nu}\sigma_B + \frac{k^2\tau^2}{6R_\nu}(\psi_i - \phi_i).$$

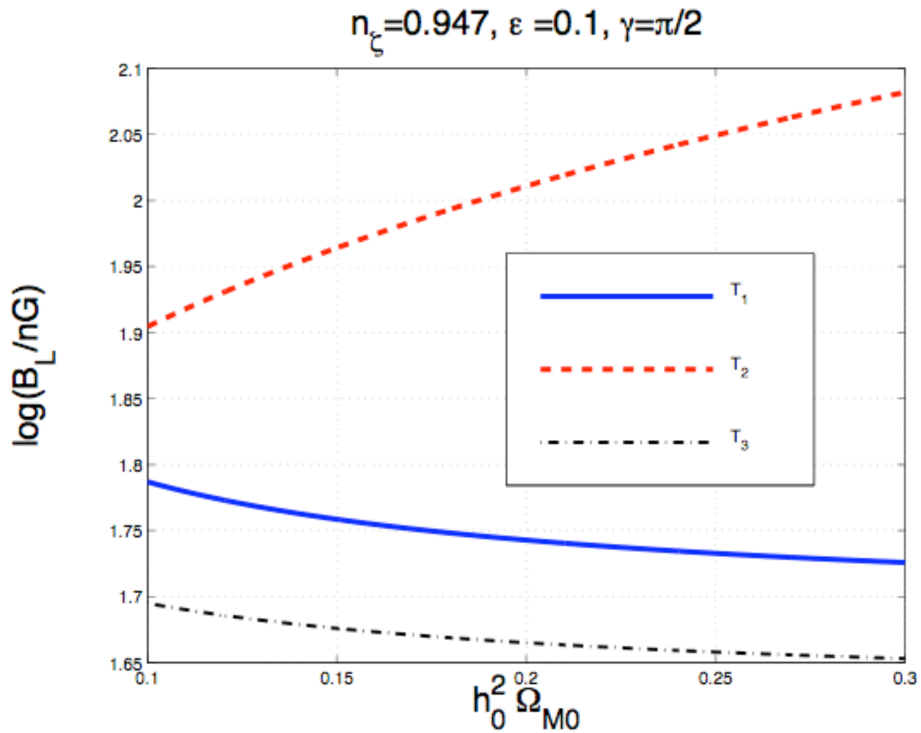
Metric variables & quadrupole moment of neutrino phase space distribution

Notation

$$R_\gamma = 1 - R_\nu, \qquad R_\nu = \frac{r}{1+r}, \qquad r = \frac{7}{8}N_\nu\left(\frac{4}{11}\right)^{4/3} \equiv 0.681\left(\frac{N_\nu}{3}\right)$$

Different thermal histories



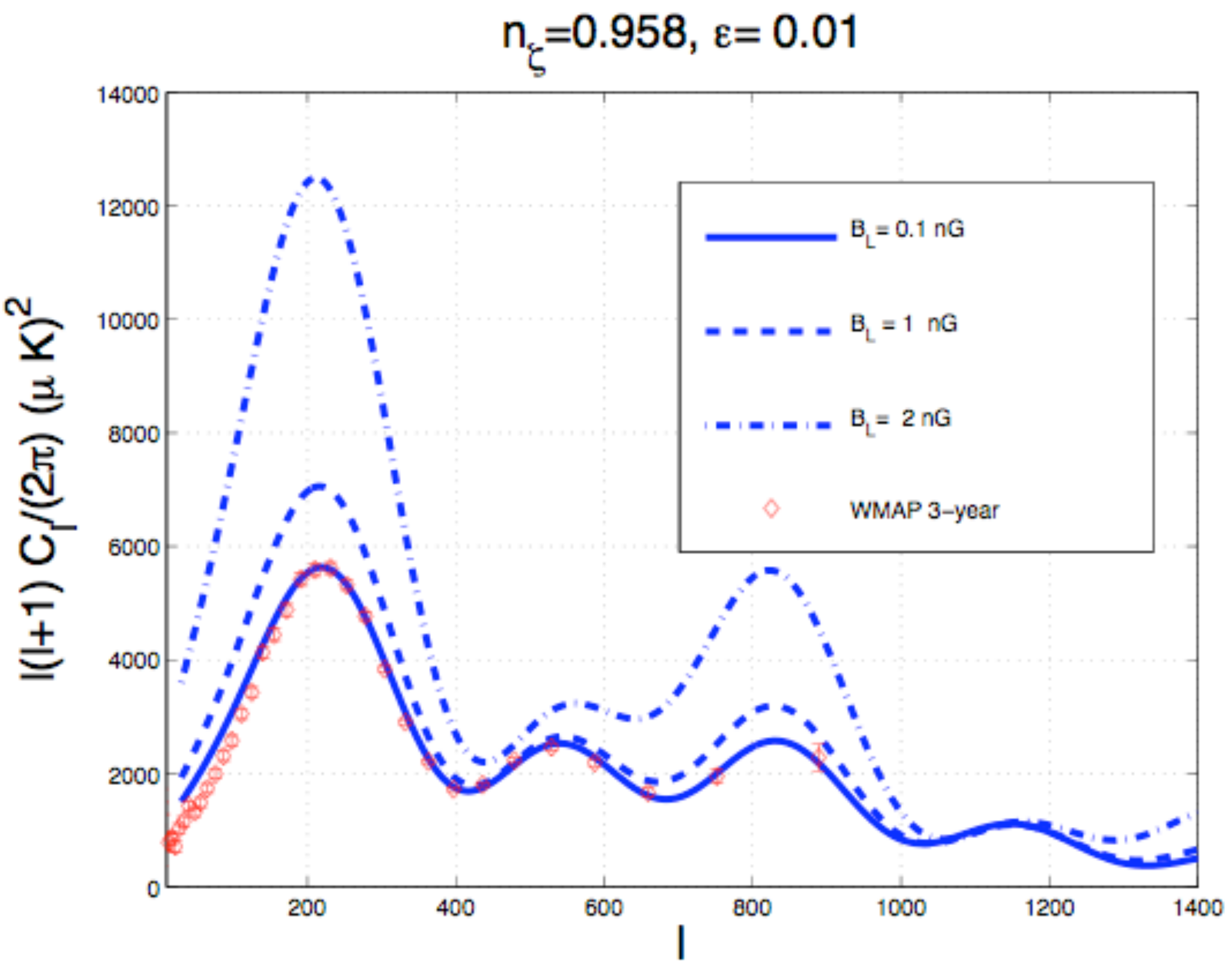


Inflation -> Stiff-> Radiation-> Matter->DE

Inflation ->Radiation -> Matter ->DE

Inflation->prolonged reheating->Radiation ->Matter->DE

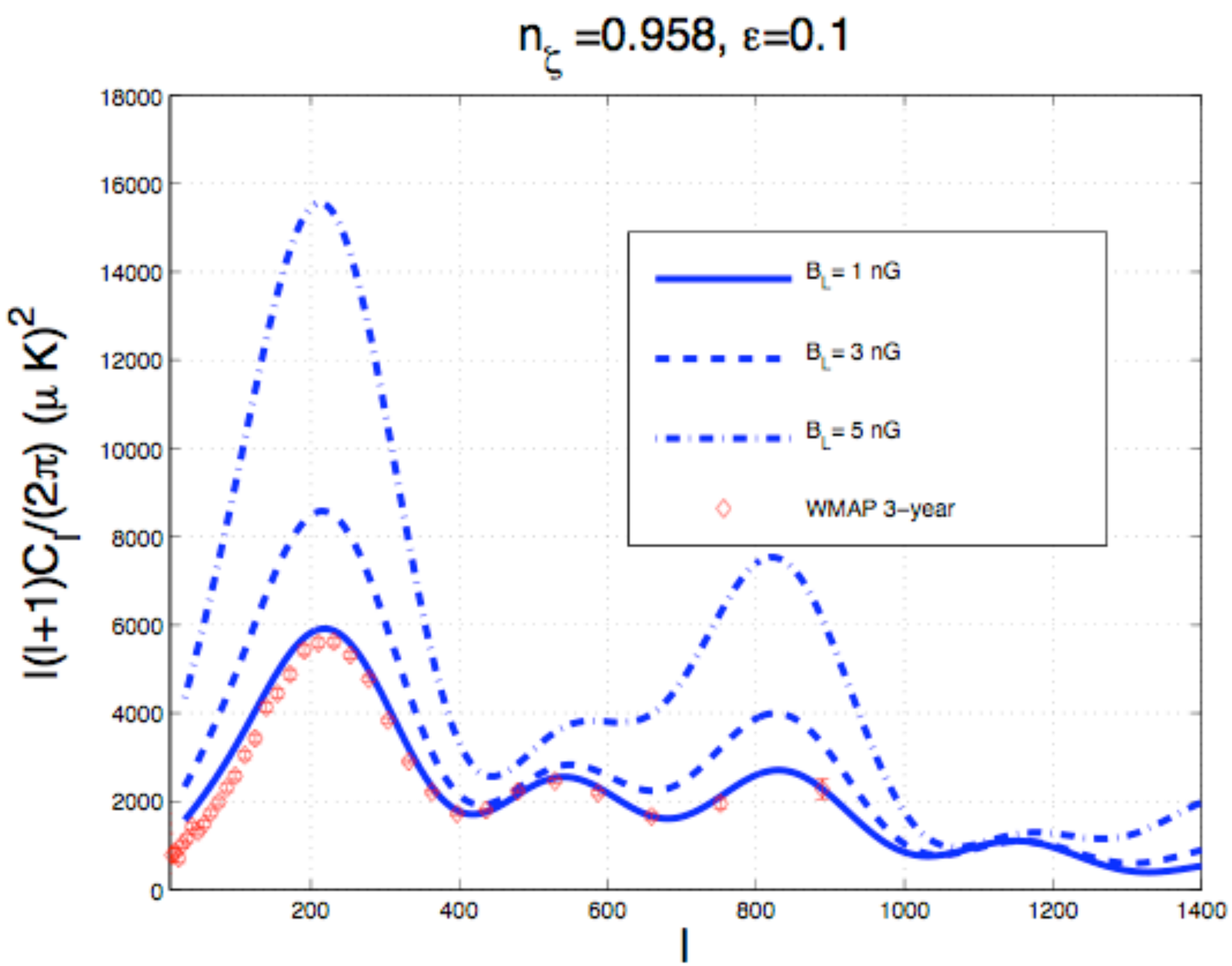
Temperature autocorrelations/1



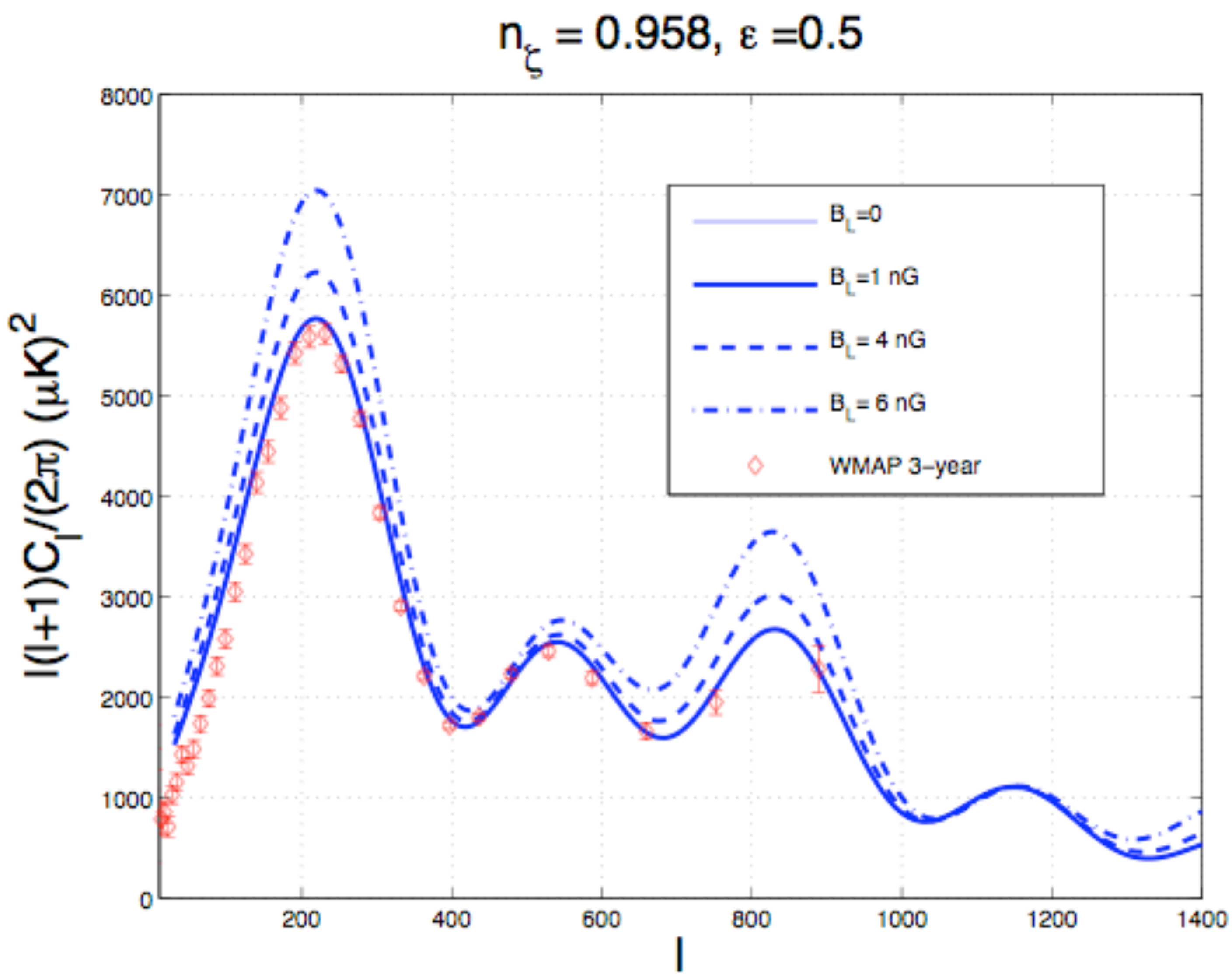
$h_0 = 0.73$ $h_0^2\Omega_{b0} = 0.02229$ $h_0^2\Omega_{c0} = 0.1504$ $h_0^2\Omega_{M0} = 0.1277$

(WMAP + ACBAR +CBI + VSA+ HSTKP+ SDS+ SNLS+SNGS)

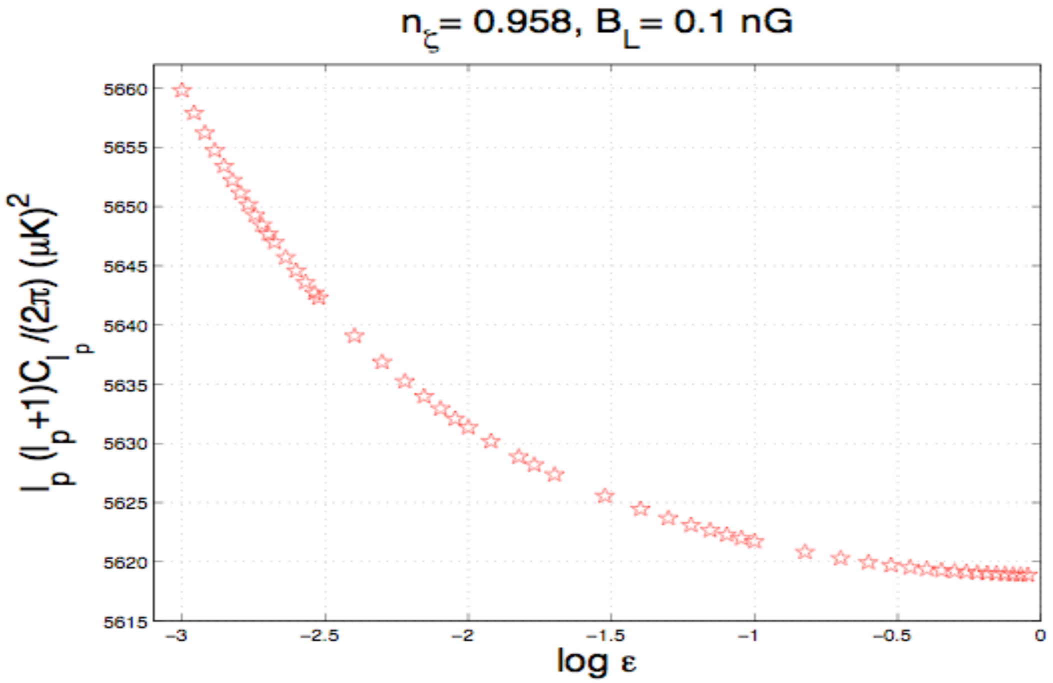
Temperature autocorrelations/2



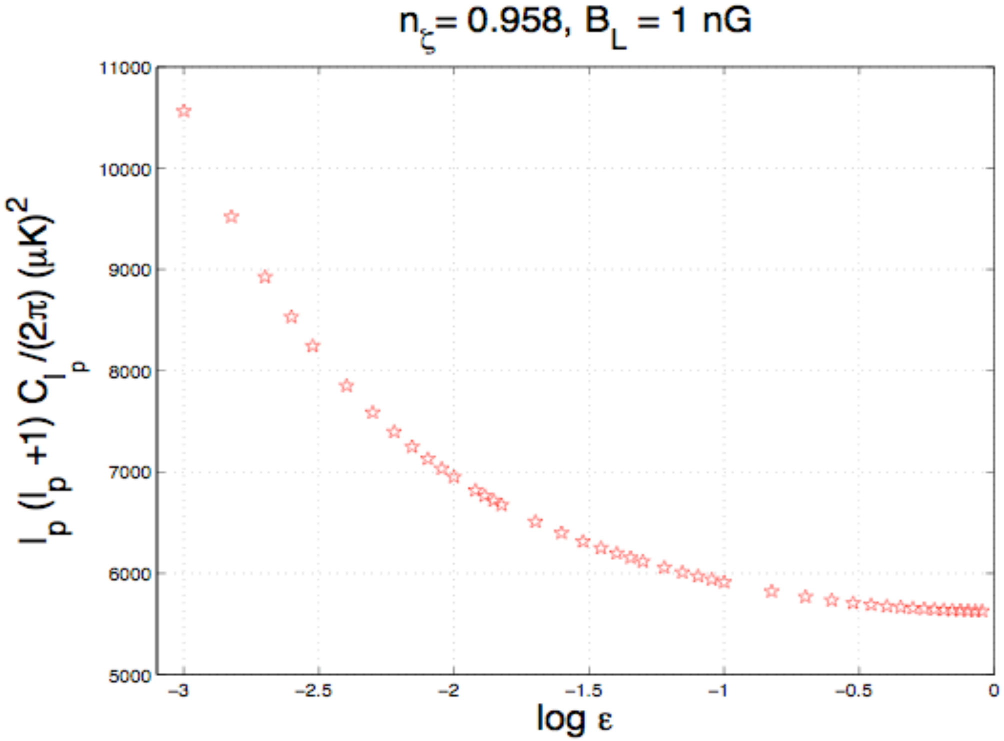
Temperature autocorrelations/3



Bounds/1



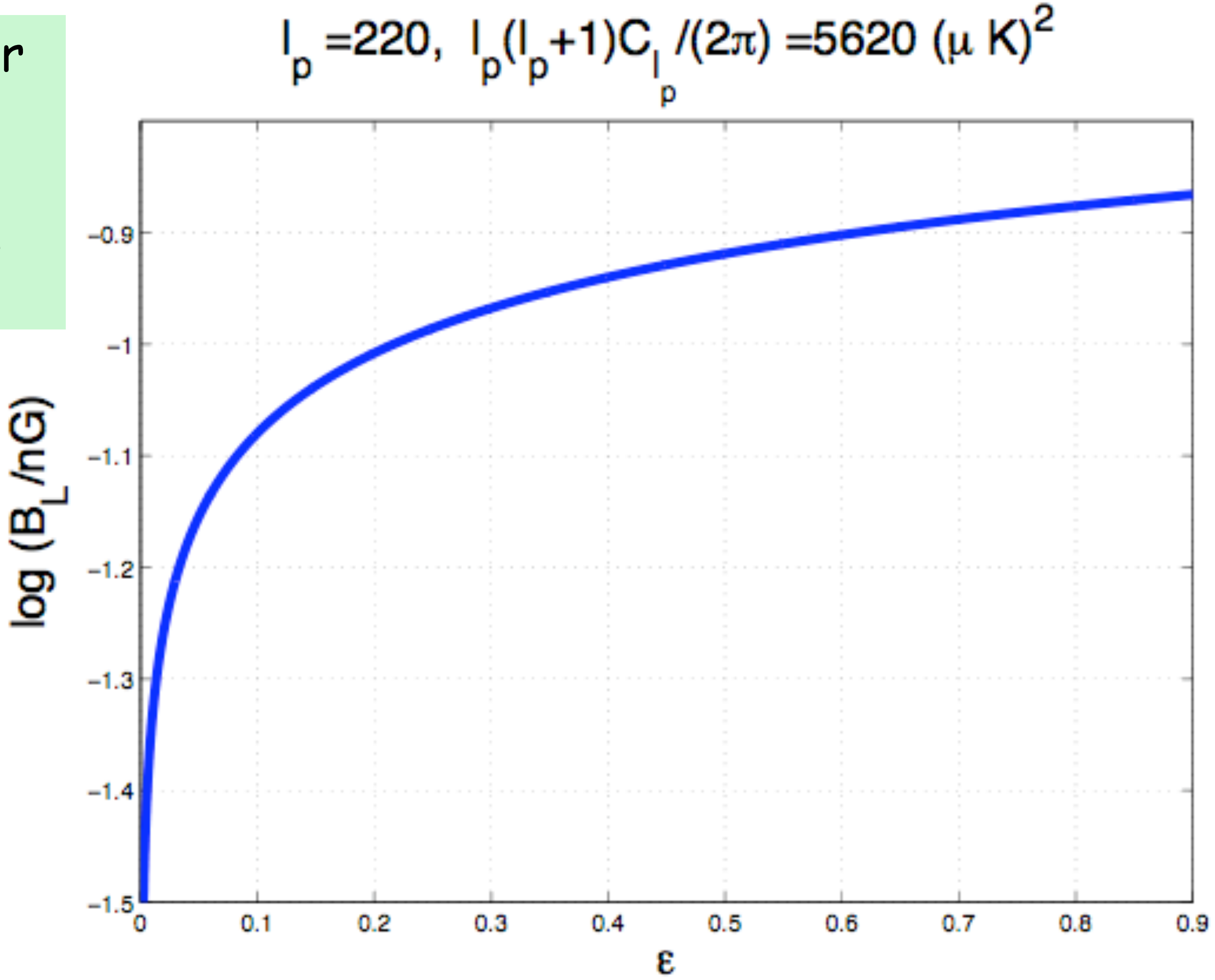
n_ζ= 0.958, B_L= 0.1 nG
$l_p(l_p+1)C_{l_p}/(2\pi)$ $(\mu K)^2$
log ε



n_ζ= 0.958, B_L = 1 nG
$l_p(l_p+1)$ $C_{l_p}/(2\pi)$ $(\mu K)^2$
log ε

Bounds/2

Stronger than Faraday rotation limits!



Concluding remarks

First quantitative estimates of scalar magnetized modes

Sizable magnetic fields @ nG level excluded

Magnetic fields germane to several aspects of CMB physics

Eagerly waiting for PLANCK and (much later ?) SKA ...

develop tools for the analysis
and
reconsider the theoretical ideas.