Renormalization Group approach to Density Fluctuations

in collaboration with:

Massimo Pietroni, INFN, Padova

Sabino Matarrese

Dipartimento di Fisica "Galileo Galilei"

Università degli Studi di Padova

email: sabino.matarrese@pd.infn.it

based on:

- S. Matarrese & M. Pietroni 2007, "*Resumming Cosmic Perturbations*", JCAP 06 (2007) 026 (arXiv:astro-ph/0703563)

- S. Matarrese & M. Pietroni 2007, "*Baryonic Acoustic Oscillations via the Renormalization Group*", arXiv:astro-ph/0702653

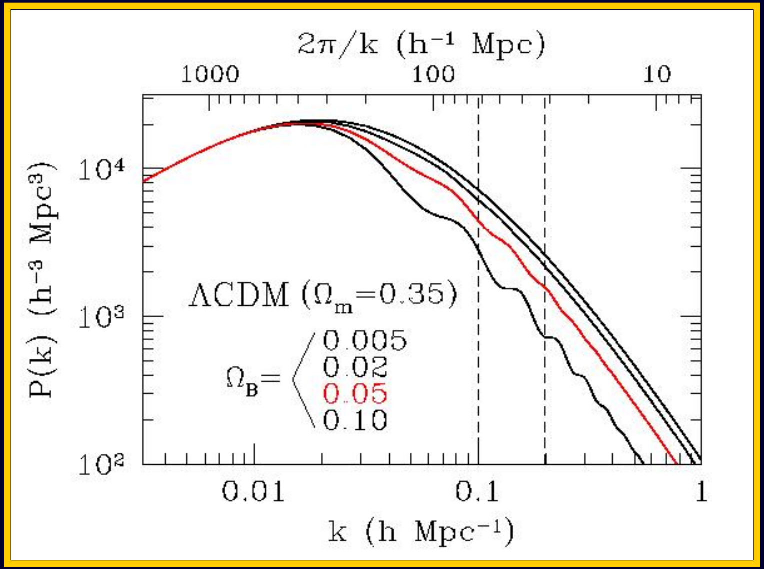
Contents

- Motivations: precision cosmology, acoustic oscillations, and all that
- Eulerian Perturbation Theory: traditional and compact forms
- RG approach: formulation and first results. The emergence of an intrinsic UV cutoff. Accurate representation of the BAO’s region in the non-linear power-spectrum
- Future prospects

Need to go beyond standard perturbation theory

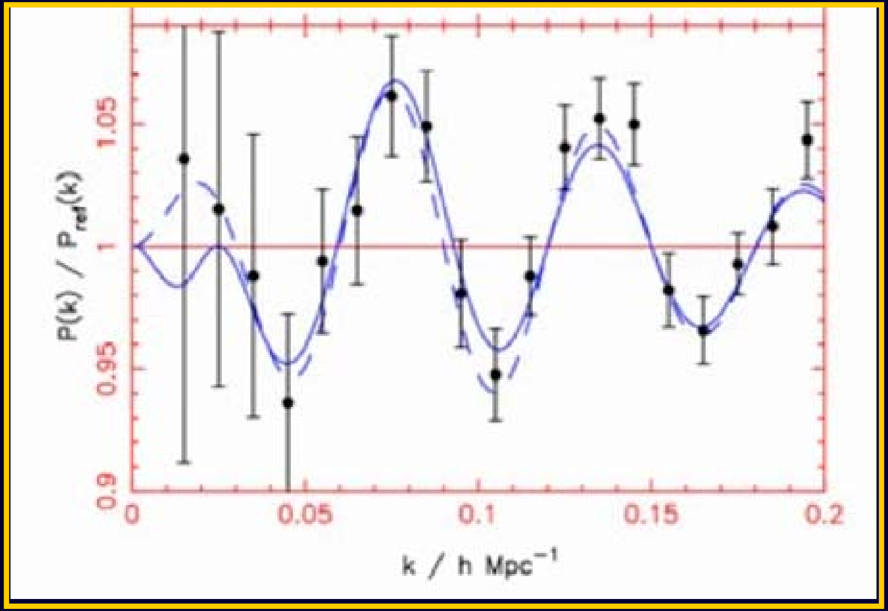
- (Linear) perturbation theory proved extremely successful in dealing with CMB data
- The study of the LSS requires better schemes, owing to the crucial role played by the gravitational instability, which makes the underlying dark matter density field unavoidably non-linear, hence non-Gaussian, on a relevant range of scales.
- Renormalized Perturbation Theory (Crocce & Scoccimarro 2005, 2006)
- Renormalization Group (McDonald 2006; Matarrese & Pietroni 2007)

Future surveys vs. BAO



Acoustic peaks are a small effect: require large surveys to be detected.

$P(k)/P_{ref}(k)$

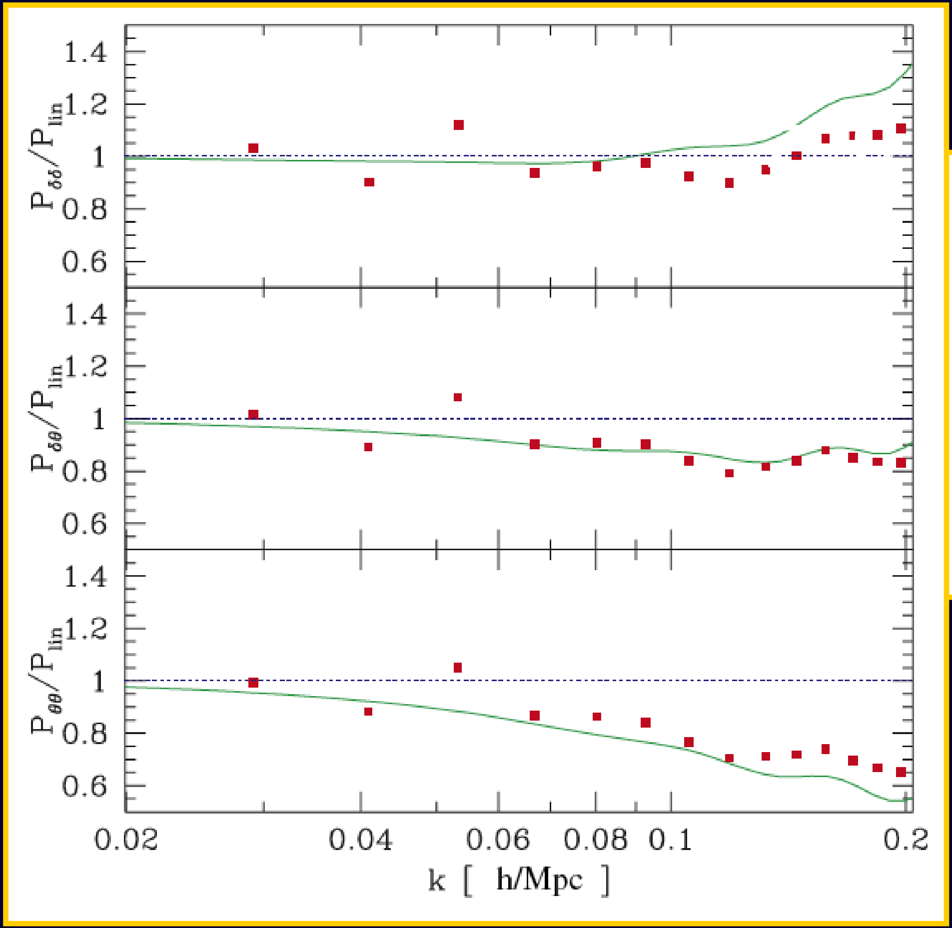


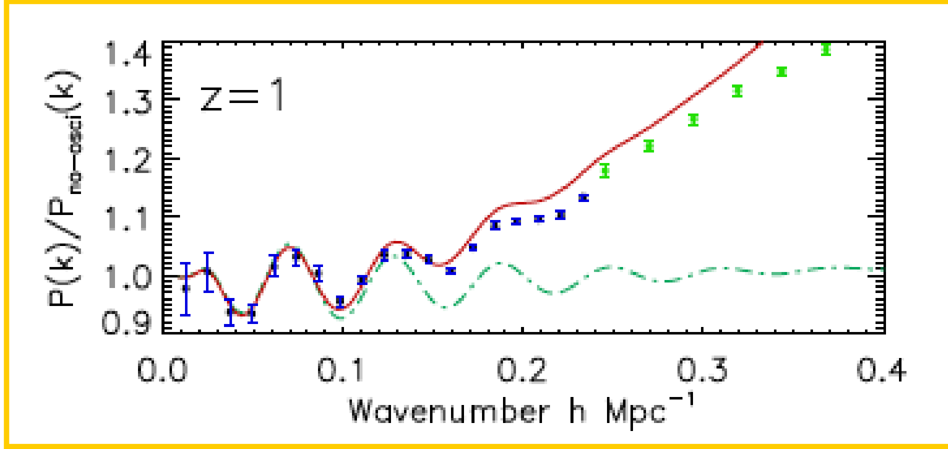
Goal: predict the LSS power spectrum to % accuracy

Standard approach: Perturbation Theory

1-loop PT

Jeong & Komatsu 2006





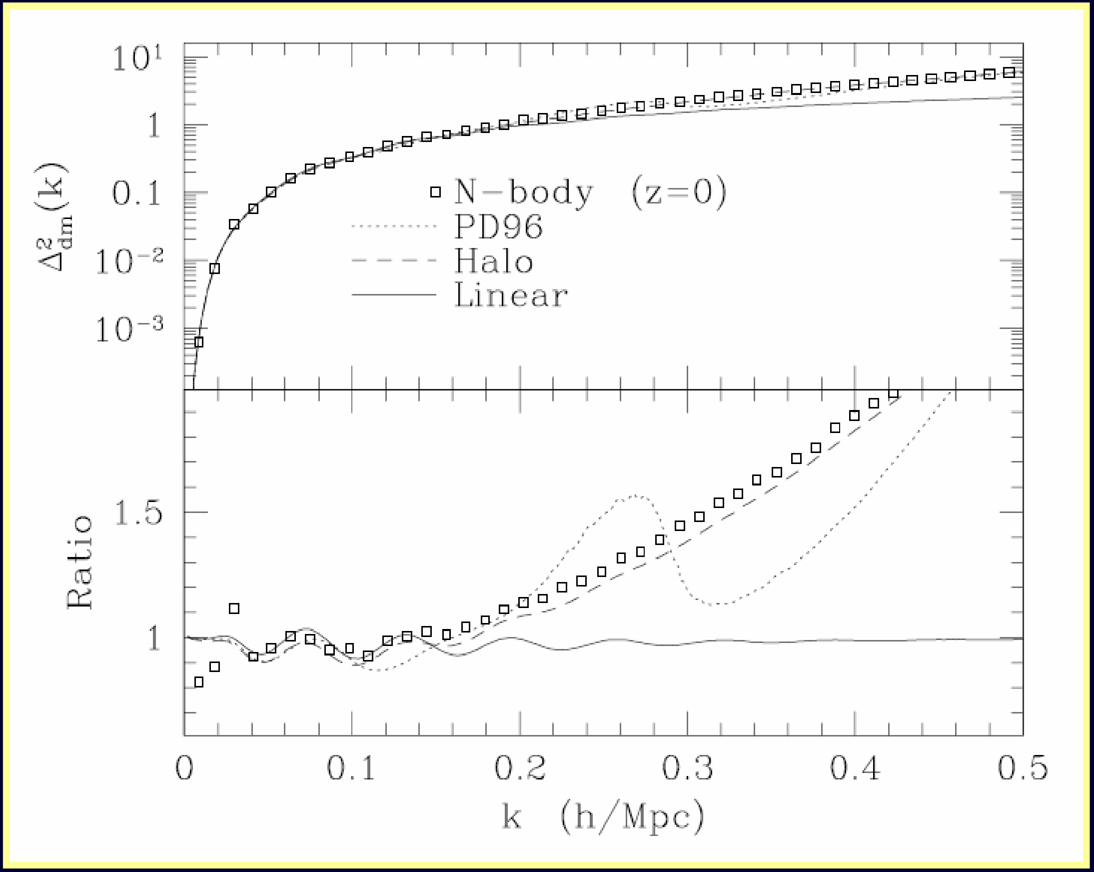
Scoccimarro 2004

- Non-linearities become more and more relevant in the redshift range 0<z<1, which is crucial for Dark Energy studies.

Present status: N-body simulations vs. fitting functions

Huff et al. 2006

Fitting schemes (e.g. Peacock & Dodds '96) evolve the power spectrum by a non-linear and non-local mapping of the linear one, by a smooth interpolation (based on outcomes of N-body simulations) between large scales, where linear theory applies, and very small scales, where stable-clustering is expected to hold. The halo-model assumes that all the matter self-organizes in clumps ("halos") by gravitational instability (→ Press-Schechter theory).



- ✓ ~10% discrepancies between fitting functions and N-body simulations

RG and LSS

- Apply standard Renormalization Group techniques to the study of the dynamics of (cold) dark matter by self-gravity, to accurately follow the (mildly) non-linear regime ($\delta \sim 1$).
- The general idea is that of describing how, starting from large scales (or early times), where linear theory holds true, statistical quantities change when shorter and shorter scales are gradually included.
- Fully general method: it can be applied in principle also to DM + baryons and/or away from the fluid (single-stream) regime, i.e. it can be extended to highly non-linear scales.

Dark Matter hydrodynamics

- DM phase-space distribution function obeys the collisionless Boltzmann (Vlasov) equation (using conformal time)

$$\boxed{\frac{\partial f}{\partial \tau} + \frac{\mathbf{p}}{am} \cdot \nabla f - am \nabla \phi \cdot \nabla_{\mathbf{p}} f = 0}$$

where $\mathbf{p}=am\, d\mathbf{x}/d\tau$ and ϕ is the peculiar gravitational potential, which obeys the cosmological Poisson equation

$$\boxed{\nabla^2 \phi = \frac{3}{2} \Omega_M \mathcal{H}^2 \delta}$$

- Taking moments and neglecting the velocity dispersion tensor (single-stream approximation) yields a <u>pressureless</u> (dust) fluid picture.
- Invalid after shell-crossing, i.e. beyond the mildly non-linear regime.

Cosmological Euler-Poisson system

$$\frac{\partial\,\delta}{\partial\,\tau} + \nabla\cdot[(1+\delta)\mathbf{v}] = 0\,, \qquad \frac{\partial\,\mathbf{v}}{\partial\,\tau} + \mathcal{H}\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla\phi$$

$$\nabla^2\phi = \frac{3}{2}\,\Omega_M\,\mathcal{H}^2\,\delta$$

$$\int d^3\mathbf{p}\, f(\mathbf{x},\mathbf{p},\tau) \equiv \rho(\mathbf{x},\tau) \equiv \overline{\rho}(\tau)[1+\delta(\mathbf{x},\tau)]$$

mass-density

$$\int d^3\mathbf{p}\, \frac{p_i}{am} f(\mathbf{x},\mathbf{p},\tau) \equiv \rho(\mathbf{x},\tau) v_i(\mathbf{x},\tau)$$

streaming velocity

$$\int d^3\mathbf{p}\, \frac{p_i\,p_j}{a^2m^2} f(\mathbf{x},\mathbf{p},\tau) \equiv \rho(\mathbf{x},\tau) v_i(\mathbf{x},\tau) v_j(\mathbf{x},\tau) + \sigma_{ij}(\mathbf{x},\tau)$$
$$\ldots$$

velocity dispersion tensor

Go to Fourier-space

- Defining the velocity divergence (remind: cosmic velocity fields are always irrotational) $\theta(\mathbf{x},\tau) \equiv \nabla \cdot \mathbf{v}(\mathbf{x},\tau)$ one gets, in Fourier-space

$$\frac{\partial\,\delta(\mathbf{k},\tau)}{\partial\,\tau} + \theta(\mathbf{k},\tau) + \int d^3\mathbf{k_1} d^3\mathbf{k_2}\,\delta_D(\mathbf{k}-\mathbf{k_1}-\mathbf{k_2})\alpha(\mathbf{k_1},\mathbf{k_2})\theta(\mathbf{k_1},\tau)\delta(\mathbf{k_2},\tau) = 0$$

$$\frac{\partial\,\theta(\mathbf{k},\tau)}{\partial\,\tau} + \mathcal{H}\,\theta(\mathbf{k},\tau) + \frac{3}{2}\Omega_M\mathcal{H}^2\delta(\mathbf{k},\tau) + \int d^3\mathbf{k_1} d^3\mathbf{k_2}\,\delta_D(\mathbf{k}-\mathbf{k_1}-\mathbf{k_2})\beta(\mathbf{k_1},\mathbf{k_2})\theta(\mathbf{k_1},\tau)\theta(\mathbf{k_2},\tau) = 0$$

- mode-mode coupling is controlled by two functions

$$\alpha(\mathbf{k_1},\mathbf{k_2}) \equiv \frac{(\mathbf{k_1}+\mathbf{k_2})\cdot\mathbf{k_1}}{k_1^2}$$

$$\beta(\mathbf{k_1},\mathbf{k_2}) \equiv \frac{|\mathbf{k_1}+\mathbf{k_2}|^2(\mathbf{k_1}\cdot\mathbf{k_2})}{2k_1^2k_2^2}$$

Linear Perturbation Theory

- Expected to hold at early times and/or on large scales.
- Consists in dropping mode-mode coupling in fluid equations: $\alpha = \beta = 0$

$$\frac{\partial\,\delta(\mathbf{k},\tau)}{\partial\,\tau} + \theta(\mathbf{k},\tau) = 0 \qquad \Omega_M = 1 \rightarrow \mathcal{H} \sim a^{-1/2}$$

$$\frac{\partial\,\theta(\mathbf{k},\tau)}{\partial\,\tau} + \mathcal{H}\,\theta(\mathbf{k},\tau) + \frac{3}{2}\Omega_M \mathcal{H}^2 \delta(\mathbf{k},\tau) = 0$$

$$\begin{aligned} \delta(\mathbf{k},\,\tau) &= \delta(\mathbf{k},\,\tau_i)\left(\frac{a(\tau)}{a(\tau_i)}\right)^m \\ -\frac{\theta(\mathbf{k},\,\tau)}{\mathcal{H}} &= m\,\delta(\mathbf{k},\,\tau) \end{aligned}$$

$$m = \begin{cases} 1 & \text{growing mode} \\ -\frac{3}{2} & \text{decaying mode} \end{cases}$$

Traditional Perturbation Theory

Assume EdS, $\Omega_M = 1$, then solutions have the form

fastest growing mode only

$$\delta(\mathbf{k},\tau) = \sum_{n=1}^{\infty} a^n(\tau)\delta_n(\mathbf{k})$$

$$\theta(\mathbf{k},\tau) = -\mathcal{H}(\tau)\sum_{n=1}^{\infty} a^n(\tau)\theta_n(\mathbf{k})$$

fastest growing mode only

with

$$\delta_n(\mathbf{k}) = \int d^3\mathbf{q_1}\dots d^3\mathbf{q_n}\,\delta_D(\mathbf{k}-\mathbf{q_{1\dots n}})F_n(\mathbf{q_1},\dots,\mathbf{q_n})\delta_0(\mathbf{q_1})\dots\delta_0(\mathbf{q_n})$$

$$\theta_n(\mathbf{k}) = \int d^3\mathbf{q_1}\dots d^3\mathbf{q_n}\,\delta_D(\mathbf{k}-\mathbf{q_{1\dots n}})G_n(\mathbf{q_1},\dots,\mathbf{q_n})\delta_0(\mathbf{q_1})\dots\delta_0(\mathbf{q_n})$$

The Kernels F_n and G_n satisfy recursion relations, with $F_1 = G_1 = 1$, and $\delta_1 = \theta_1 = \delta_0$:

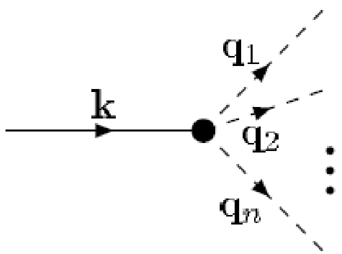
$$F_n(\mathbf{q_1},\dots,\mathbf{q_n}) = \sum_{m=1}^{n-1}\frac{G_m(\mathbf{q_1},\dots,\mathbf{q_m})}{(2n+3)(n-1)} \times\left[(2n+1)\alpha(\mathbf{k_1},\mathbf{k_2})F_{n-m}(\mathbf{q_{m+1}},\dots,\mathbf{q_n})+2\beta(\mathbf{k_1},\mathbf{k_2})G_{n-m}(\mathbf{q_{m+1}},\dots,\mathbf{q_n})\right]$$

$$G_n(\mathbf{q_1},\dots,\mathbf{q_n}) = \cdots$$

where $\mathbf{k_1} = \mathbf{q_1}+\dots+\mathbf{q_m}$, $\quad \mathbf{k_2} = \mathbf{q_{m+1}}+\dots+\mathbf{q_n}$

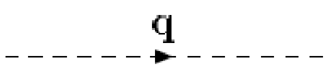
Traditional Diagrammar

Fry, '84
Goroff et al, '86
Wise, '88
Scoccimarro, Frieman, '96
Heavens, Matarrese & Verde, 1998



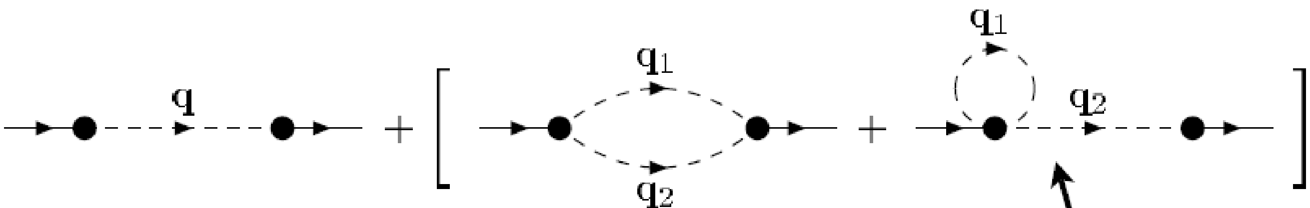
$$\equiv \quad \delta_D(\mathbf{k} - \mathbf{q}_1 - \ldots - \mathbf{q}_n) F_n^{(s)}(\mathbf{q}_1, \ldots, \mathbf{q}_n)$$

An infinite number of basic vertices!
very redundant!!



$$\equiv \quad P_1(q, \tau)$$

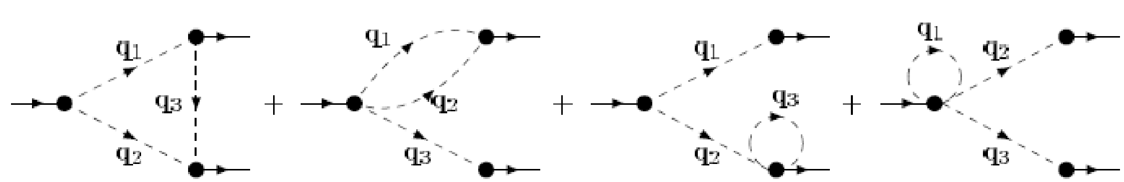
Example: 1-loop correction to the density power spectrum:



a.k.a. "P22"

a.k.a. "P13"

bispectrum:



Compact Perturbation Theory

Crocce, Scoccimarro '05

The hydrodynamical equations for density and velocity perturbations,

$$\frac{\partial\,\delta}{\partial\,\tau} + \nabla\cdot[(1+\delta)\mathbf{v}] = 0\,, \qquad \frac{\partial\,\mathbf{v}}{\partial\,\tau} + \mathcal{H}\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla\phi\,,$$

can be written in a compact form (we assume an EdS model):

$$(\delta_{ab}\partial_\eta + \Omega_{ab})\,\varphi_b(\eta,\mathbf{k}) = e^\eta\gamma_{abc}(\mathbf{k},\,-\mathbf{k_1},\,-\mathbf{k_2})\,\varphi_b(\eta,\mathbf{k_1})\,\varphi_c(\eta,\mathbf{k_2}) \qquad (1)$$

where $\begin{pmatrix} \varphi_1(\eta,\mathbf{k}) \\ \varphi_2(\eta,\mathbf{k}) \end{pmatrix} \equiv e^{-\eta}\begin{pmatrix} \delta(\eta\,,\mathbf{k}) \\ -\theta(\eta\,,\mathbf{k})/\mathcal{H} \end{pmatrix} \qquad \eta = \log\frac{a}{a_{in}} \qquad \mathbf{\Omega} = \begin{pmatrix} 1 & -1 \\ -3/2 & 3/2 \end{pmatrix}$

and the only non-zero components of the vertex are

$$\gamma_{121}(\mathbf{k_1},\,\mathbf{k_2},\,\mathbf{k_3}) = \gamma_{112}(\mathbf{k_1},\,\mathbf{k_3},\,\mathbf{k_2}) = \delta_D(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})\,\frac{(\mathbf{k_2}+\mathbf{k_3})\cdot\mathbf{k_2}}{2k_2^2}$$

$$\gamma_{222}(\mathbf{k_1},\,\mathbf{k_2},\,\mathbf{k_3}) = \delta_D(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})\,\frac{|\mathbf{k_2}+\mathbf{k_3}|^2\,\mathbf{k_2}\cdot\mathbf{k_3}}{2\,k_2^2\,k_3^2}$$

Extend to non-EdS cosmologies

$$\eta = \log \frac{a}{a_{in}} \longrightarrow \eta = \log \frac{D^+}{D^+_{in}}$$

Bouchet et al. '92
Bernardeau '94
Catelan et al. '94
Nusser & Colberg '98

- Where D_+ is the linear growth factor of density fluctuations in the given cosmology
- In doing this we are neglecting the effect of decaying modes in the non-linear regime (Crocce & Scoccimarro '05)
- Initial time $\eta=0$ corresponds to $z=z_{in}$ and is chosen so that perturbations are well inside the linear regime.
- We take $z_{in}=80$

An action principle

Matarrese & Pietroni '07

Eq. (1) can be derived by varying the action S w.r.t. the field χ

$$S = \int d\eta_1 d\eta_2\, \chi_a\, g^{-1}_{ab}\, \varphi_b - \int d\eta\, e^{\eta}\, \gamma_{abc}\, \chi_a\, \varphi_b\, \varphi_c$$

where the <u>auxiliary field</u> $\chi_a(\eta, \mathbf{k})$ has been introduced and $g_{ab}(\eta_1, \eta_2)$ is the <u>retarded propagator</u>:

$$(\delta_{ab}\partial_\eta + \Omega_{ab})\, g_{bc}(\eta, \eta') = \delta_{ac}\, \delta_D(\eta - \eta')$$

so that $\varphi^0_a(\eta, \mathbf{k}) = g_{ab}(\eta, \eta')\varphi^0_b(\eta', \mathbf{k})$ is the solution of the ***linear*** equation

Explicitly, one finds: $\mathbf{g}(\eta_1, \eta_2) = \begin{cases} \mathbf{B} + \mathbf{A}\, e^{-5/2(\eta_1 - \eta_2)} & \eta_1 > \eta_2 \\ 0 & \eta_1 < \eta_2 \end{cases}$

$$\mathbf{B} = \frac{1}{5}\begin{pmatrix} 3 & 2 \\ 3 & 2 \end{pmatrix}$$

$$\mathbf{A} = \frac{1}{5}\begin{pmatrix} 2 & -2 \\ -3 & 3 \end{pmatrix}$$

growing mode

decaying mode

Initial conditions: $\varphi^0_b(\eta', \mathbf{k}) \propto u_b = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \qquad \begin{pmatrix} 1 \\ -3/2 \end{pmatrix}$

A generating functional

The probability of the configuration $\varphi_a(\eta_f)$, given the initial condition $\varphi_a(\eta_i)$, is

$$P[\varphi_a(\eta_f);\varphi_a(\eta_i)] = \delta\left[\varphi_a(\eta_f) - \overline{\varphi}_a[\eta_f;\varphi_a(\eta_i)]\right]$$

solution of the e.o.m.

fixed extrema

$$\sim \int \mathcal{D}''\varphi_a \mathcal{D}\chi_b \exp\left\{ i\int_{\eta_i}^{\eta_f} d\eta\, \chi_a \left[(\delta_{ab}\partial_\eta + \Omega_{ab})\varphi_b - e^\eta \gamma_{abc}\varphi_b\varphi_c\right]\right\}$$

only tree-level (saddle point)

The generating functional ***at fixed initial conditions*** is

$$Z[J_a,\Lambda_b;\varphi_c(\eta_i)] = \int \mathcal{D}\varphi_a(\eta_f) \exp\left\{ i\int_{\eta_i}^{\eta_f} d\eta (J_a\varphi_a + \Lambda_b\chi_b)\right\} P[\varphi_a(\eta_f);\varphi_a(\eta_i)]$$

We are interested in ***statistical*** correlations, ***not in single solutions:***

$$Z[J_a,\Lambda_b;K's]=\int\mathcal{D}\varphi_c(\eta_i)W[\varphi_c(\eta_i);K's]Z[J_a,\Lambda_b;\varphi_c(\eta_i)]$$

where all the initial correlations are contained in

$$W[\varphi_c(\eta_i);K's]=\exp\left\{-\varphi_a(\eta_i;\mathbf{k})K_a(\mathbf{k})-\frac{1}{2}\varphi_a(\eta_i;\mathbf{k_a})K_{ab}(\mathbf{k_a},\mathbf{k_b})\varphi_b(\eta_i;\mathbf{k_b})+\cdots\right\}$$

In the case of Gaussian initial conditions: $(K(\mathbf{k}))^{-1}_{\mathbf{ab}}=\mathbf{P^0_{ab}}(\mathbf{k})\equiv\mathbf{u_a u_b P^0}(\mathbf{k})$

Putting all together...

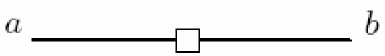
$$\boxed{Z[\mathbf{J},\,\mathbf{\Lambda}]=\int\mathcal{D}\varphi\,\mathcal{D}\chi\,\exp\left\{\int d\eta_1 d\eta_2\left[-\frac{1}{2}\,\chi\,\mathbf{g}^{-1}\mathbf{P^L}\mathbf{g^T}^{-1}\chi+i\,\chi\,\mathbf{g}^{-1}\,\varphi\right]-i\int d\eta\left[\mathbf{e}^{\eta}\gamma\,\chi\varphi\varphi-\mathbf{J}\varphi-\mathbf{\Lambda}\chi\right]\right\}}$$

where the initial conditions are encoded in the linear power spectrum: $P^L_{ab}(\eta,\,\eta';\,\mathbf{k})\equiv\left(\mathbf{g}(\eta)\mathbf{P^0}(\mathbf{k})\mathbf{g}^T(\eta')\right)_{ab}$

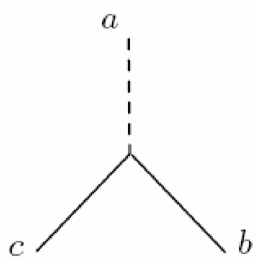
Derivatives of Z w.r.t. the sources J and Λ give all the N-point correlation functions (power spectrum, bispectrum, ...) and the full propagator (k-dependent growth factor)

Compact Diagrammar

propagator (linear growth factor): $-i\, g_{ab}(\eta_a,\, \eta_b)$

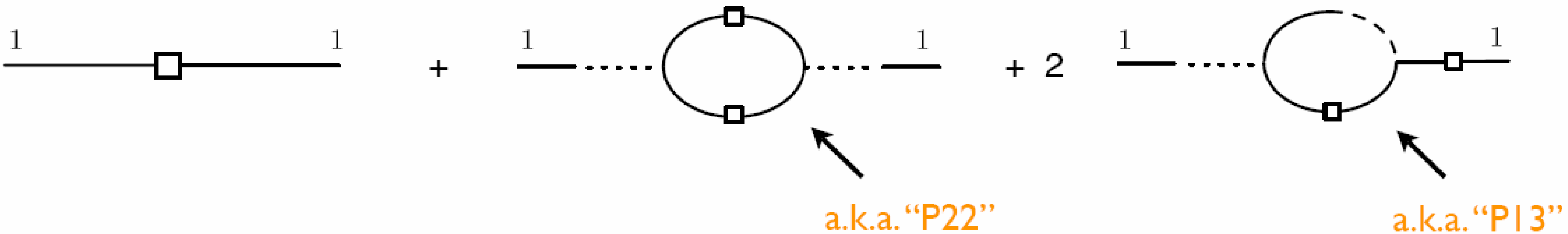


power spectrum: $P^L_{ab}(\eta_a,\, \eta_b;\, \mathbf{k})$



interaction vertex: $-i\, e^{\eta}\, \gamma_{abc}(\mathbf{k_a},\, \mathbf{k_b},\, \mathbf{k_c})$

Example: 1-loop correction to the density power spectrum:



All known results in cosmological perturbation theory are expressible in terms of diagrams in which <u>only a trilinear fundamental interaction</u> appears

Beyond perturbation theory: the renormalization group

Inspired by applications of Wilsonian RG to field theory : the RG parameter is momentum

Modify the primordial power spectrum as: $P^0_\lambda(k) = P^0(k)\,\Theta(\lambda - k)$ (step function)

then, plug it into the generating functional: $Z[\mathbf{J},\,\mathbf{\Lambda}] \longrightarrow Z_\lambda[\mathbf{J},\,\mathbf{\Lambda}]$

$$Z_\lambda[\mathbf{J},\,\mathbf{\Lambda}] = \int \mathcal{D}\varphi\,\mathcal{D}\chi\,\exp\left\{\int d\eta_1 d\eta_2 \left[-\frac{1}{2}\chi\,\mathbf{g}^{-1}\mathbf{P}^{\mathbf{L}}_\lambda \mathbf{g}^{\mathbf{T}^{-1}}\chi + i\,\chi\,\mathbf{g}^{-1}\,\varphi\right] - i\int d\eta\,[\mathbf{e}^{\eta}\gamma\,\chi\varphi\varphi - \mathbf{J}\varphi - \mathbf{\Lambda}\chi]\right\}$$

this object generates all the N-point functions for the Universe in which
there were no primordial perturbations with $k > \lambda$

The evolution from $\lambda = 0$ to $\lambda = \infty$ can be described non-perturbatively by RG equations:

$$\boxed{\frac{\partial}{\partial\lambda} Z_\lambda = \int d\eta\, d\eta' \left[\frac{1}{2}\frac{\partial}{\partial\lambda}\left(g^{-1} P^L_\lambda g^{-1^T}\right)_{ab} \frac{\delta^2 Z_\lambda}{\delta\Lambda_b \delta\Lambda_a}\right]}$$

Generating functionals

Next, define the generator of connected n-point functions:

$$W_\lambda[J,\,\Lambda] = -i\log Z_\lambda[J,\,\Lambda]$$

And, through functional Legendre transform, the effective action (or generator of 1PI diagrams):

λ λ

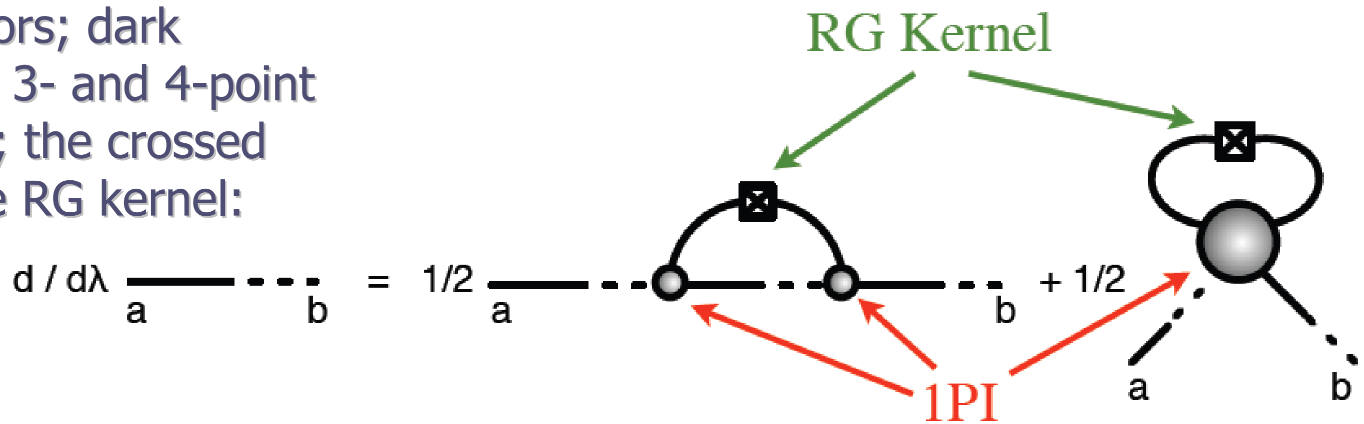
$$\Gamma[\varphi_a,\,\chi_b] = W\,[J_a\ \Lambda_b\,] - \int d\eta\, d^3\mathbf{k}\,(J_a\varphi_a + \Lambda_b\chi_b)$$

The propagator

$$\delta^{(3)}(\mathbf{k}+\mathbf{k}')\,G_{\lambda,ab}(k;\,\eta_a,\,\eta_b) = -\frac{\delta^2 W_\lambda[J,\,\Lambda]}{\delta J_a(\mathbf{k},\,\eta_a)\delta\Lambda_b(\mathbf{k}',\,\eta_b)} \qquad W_\lambda[J,\,\Lambda] = -i\log Z_\lambda[J,\,\Lambda]$$

$$\boxed{\frac{\partial}{\partial\lambda}\,\frac{\delta^2 W_\lambda}{\delta J_a\,\delta\Lambda_b} = \frac{1}{2}\int d\eta_c d\eta_d\, d^3\mathbf{q}\;\delta(\lambda-q)\,\left(g^{-1}P^L g^{-1\,T}\right)_{cd}\;\frac{\delta^4 W_\lambda}{\delta J_a\,\delta\Lambda_b\,\delta\Lambda_c\,\delta\Lambda_d}}$$

Thick lines indicate full propagators; dark blobs 1PI 3- and 4-point functions; the crossed box is the RG kernel:



infinite tower of RGE's

Structure of the RG equations (1)

- The RHS of the RG equation is remarkably simple. The two contributions have the same structure of one-loop diagrams, where tree-level vertices and propagators have been replaced by full, λ-dependent ones. The same holds true for any other quantity of interest.

Structure of the RG equations (2)

- A recipe can then be given to obtain the RG equation for any given quantity.

  - ✓ Write down the 1-loop expression for the quantity of interest, obtained using any needed vertex (for instance, in the case of G_l we have not only the vertex $\chi\varphi\varphi$, but also $\chi\varphi\varphi\varphi$ although it vanishes at tree-level).

  - ✓ Promote the linear propagator, the power-spectrum and the vertices appearing in that expression to full, λ-dependent ones.

  - ✓ Take the λ-derivative of the full expression, by considering only the explicit λ-dependence of the step-function contained in the initial power-spectrum.

Structure of the RG equations (3)

→ The RG equations obtained following these rules are exact, in the sense that they encode all the dynamical and statistical content of the path-integral or, equivalently, of the Euler-Poisson system supplemented by the initial power-spectrum.

Approximation ansatz

- Full λ-dependent propagators G
- Tree-level vertices

$$\frac{\partial}{\partial\lambda} G_{\lambda,\,ab}(k;\eta_a,\,\eta_b) = 4\int d\eta_c d\eta_d\, d^3\mathbf{q}\,\delta(\lambda-q)\,\mathcal{K}_{cd}(q;\,\eta_c,\,\eta_d)\,\gamma_{ecg}(\eta_c;\,-\mathbf{k},\,\mathbf{q},\,\mathbf{k}-\mathbf{q})\,\gamma_{idl}(\eta_d;\,-\mathbf{k}+\mathbf{q},\,-\mathbf{q},\,\mathbf{k})\,G_{\lambda,\,ae}(k;\eta_a,\,\eta_c)G_{\lambda,\,gi}(|\mathbf{k}-\mathbf{q}|;\eta_c,\,\eta_d)G_{\lambda,\,lb}(k;\eta_d,\,\eta_b)$$

- RG Kernel: $\mathcal{K}_{\lambda,\,cd}(q;\,\eta_c,\,\eta_d) = G_{\lambda,\,cm}(q;\,\eta_c,\,0)u_m P^0(q)u_n G^T_{\lambda,\,nd}(q;\,0,\,\eta_d)$

Lowest order approximation: k»λ

- The RG equation for the propagator becomes:

$$\frac{\partial}{\partial \lambda} G_{\lambda,\,ab}(k;\eta_a,\,\eta_b) = -\frac{1}{2}\left(e^{\eta_a} - e^{\eta_b}\right)^2 \frac{k^2}{3}\int d^3\mathbf{q}\;\delta(\lambda - q)\;\frac{P^0(q)}{q^2}\,G_{\lambda,\,ab}(k;\eta_a,\,\eta_b)$$

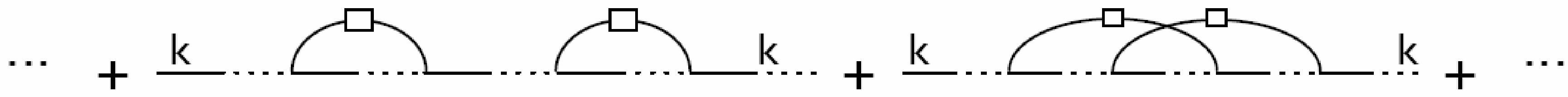
$$G_{\lambda=0,\,ab}(k;\eta_a,\,\eta_b) = g_{ab}(\eta_a,\,\eta_b)$$ boundary condition

- It can be integrated analytically, to yield:

$$G_{\lambda,\,ab}(k;\eta_a,\,\eta_b) = g_{ab}(\eta_a,\,\eta_b)\exp\left[-\frac{k^2\sigma_\lambda^2}{2}\left(e^{\eta_a} - e^{\eta_b}\right)^2\right]$$

- where: $$\sigma_\lambda^2 = \frac{1}{3}\int d^3\mathbf{q}\,\frac{P^0(q)}{q^2}\,\theta(\lambda - q)$$

in perturbation theory, it can be obtained by summing the infinite series of chain diagrams (Crocce Scoccimarro, '06)



physically, it represents the effect of multiple interactions of the k-mode with the surrounding modes

$$G \sim e^{-\frac{k^2\sigma^2}{2} e^{2\eta}}$$

We can remove the k » λ limit and integrate numerically the RG equation. We still get the same UV cutoff

`coherence momentum' $k_{ch} = (\sigma\, e^{\eta})^{-1} \simeq 0.15\, \mathrm{h\, Mpc}^{-1}$

in the BAO range!

A self-generated UV cutoff

Inserting this result in the expression for the RG kernel, we get:

$$\boxed{\mathcal{K}_{\lambda,\,cd}(q;\,\eta_c,\,\eta_d) = u_c u_d P^0(q)\exp\left[-\frac{q^2\sigma_\lambda^2}{2}\left((e^{\eta_c}-1)^2+(e^{\eta_d}-1)^2\right)\right]}$$

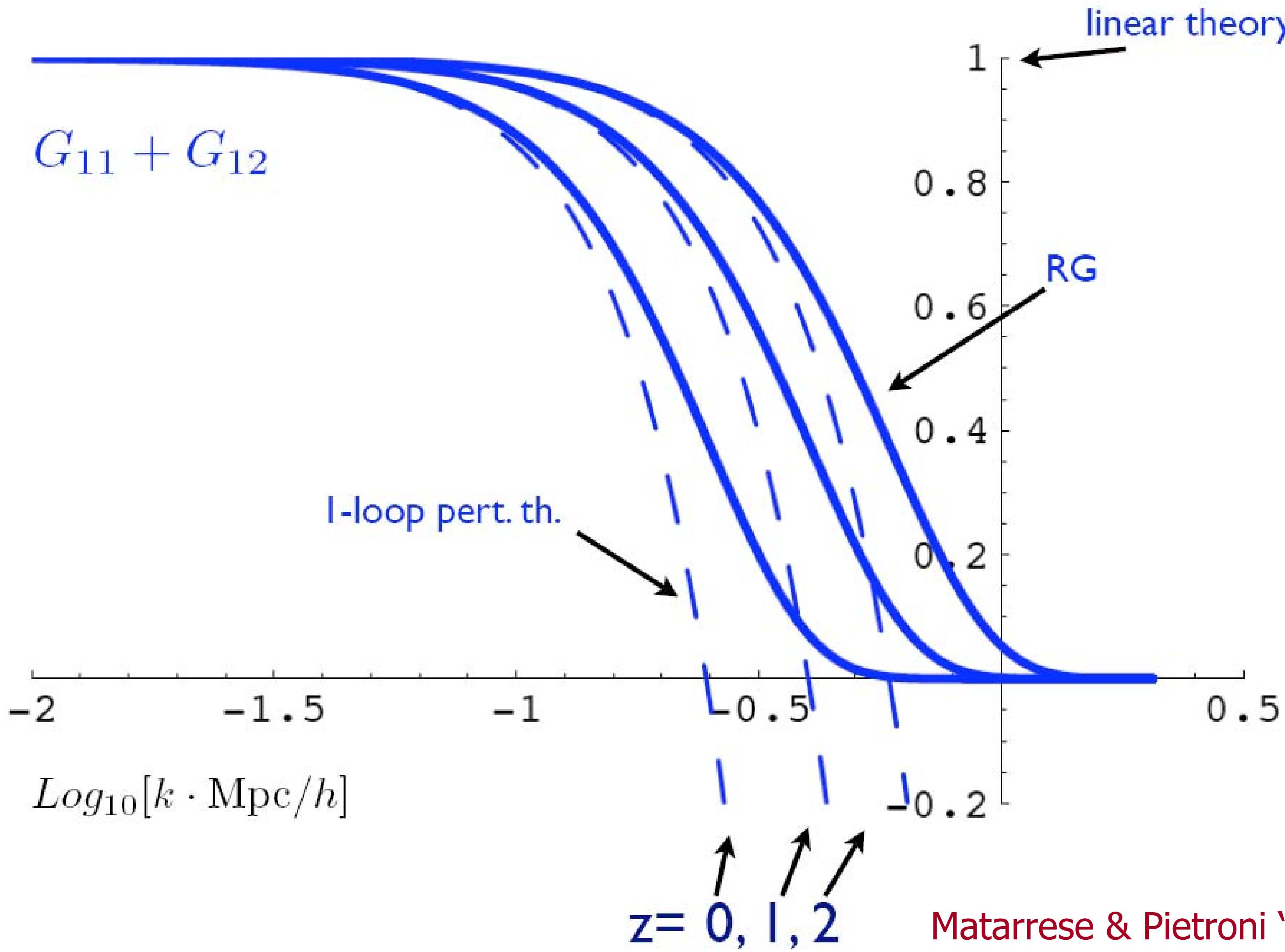
The effect of modes with momenta larger than $\boxed{\sigma_\lambda^{-1}(e^{\eta}-1)^{-1}}$ is exponentially screened.

We can remove the k >> λ approximation. Integrating numerically The RG equation we still get the same UV cutoff.

The UV is much better behaved than one would guess from "usual" perturbation theory!!

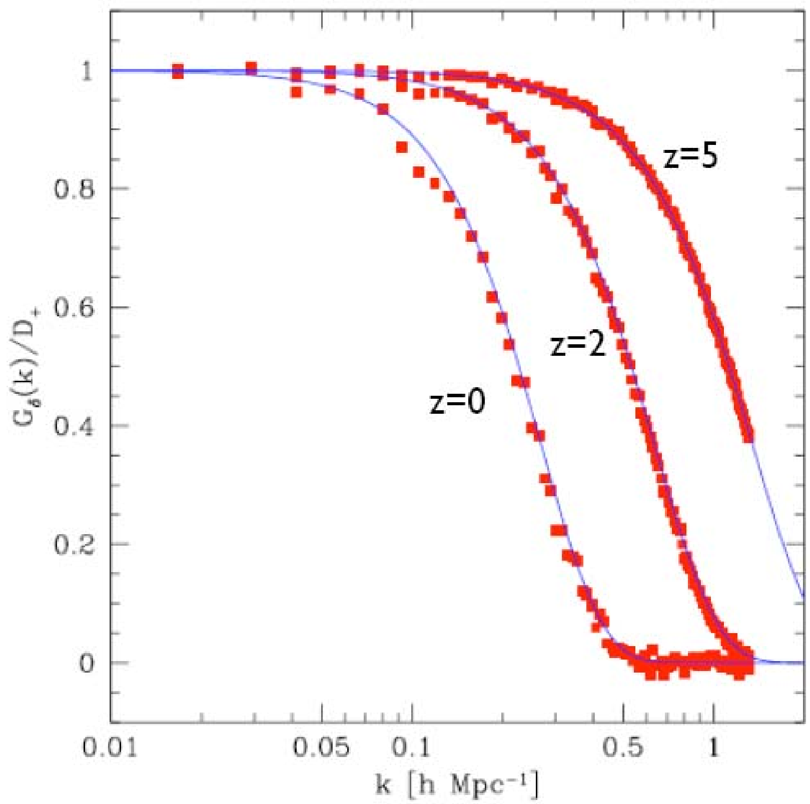
$$\boxed{\left(\mathcal{K}_{\lambda,\,cd}(q;\,\eta_c,\,\eta_d)\to u_c u_d P^0(q)\right)}$$

the effect persists even relaxing the k>>λ condition:



Matarrese & Pietroni '07

Confirmed by N-body simulations



smooth interpolation between the small k (perturbative) and the large k (resummed) results
(from Crocce and Scoccimarro, '06)

A self-generated UV cutoff

Inserting this result in the expression for the RG kernel, we get:

$$\mathcal{K}_{\lambda,\,cd}(q;\,\eta_c,\,\eta_d) = \delta(\lambda - q)\,P^0_{cd}(q)\,\exp\left[-\frac{q^2\sigma_\lambda^2}{2}\left((e^{\eta_c}-1)^2 + (e^{\eta_d}-1)^2\right)\right]$$

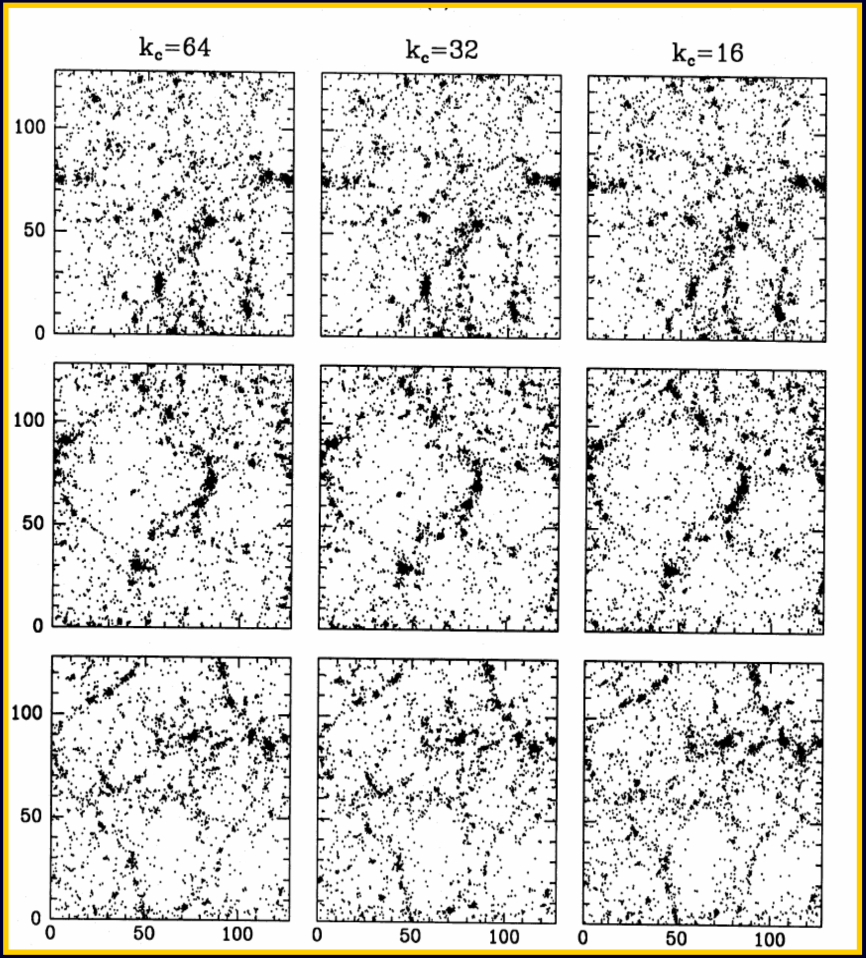
The effect of modes with momenta larger than $\sigma_\lambda^{-1}(e^{\eta}-1)^{-1}$ is exponentially screened.

The UV is much better behaved than one would guess from `usual' perturbation theory ($\mathcal{K}_{\lambda,\,cd}(q;\,\eta_c,\,\eta_d) \to \delta(\lambda - q)P^0_{cd}(q)$) !!

Self-generated UV cutoff

- Same result obtained by Crocce & Scoccimarro 2005 and already noticed in N-body simulations

- The high-frequency modes of the initial (linear) power-spectrum are truncated down to the present-day non-linearity scale. The final (z=0) outputs are left almost unchanged.



Little, Weinberg & Park 1991

The power spectrum

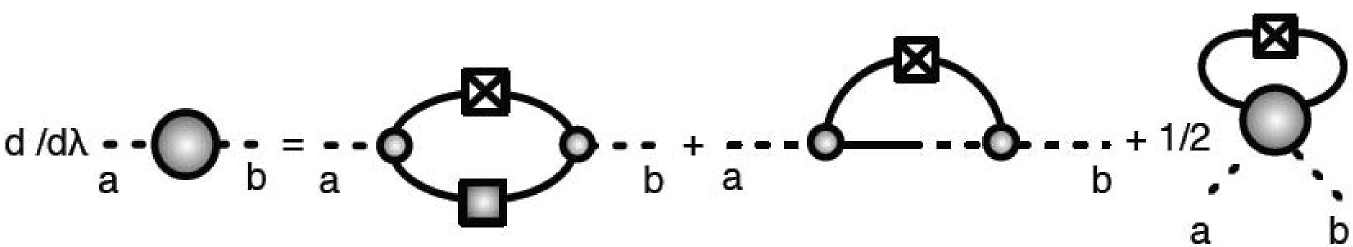
The full PS has the structure: $P_{ab} = P^{I}_{ab} + P^{II}_{ab}$

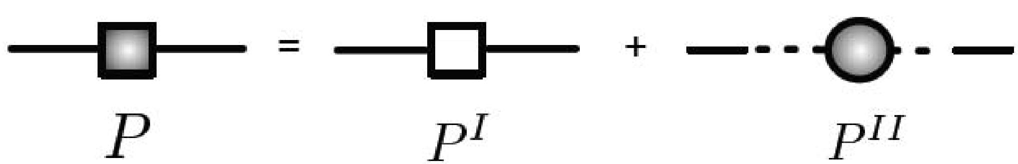
similar to 2-halo term in the halo model

with $$P^{I}_{ab}(k;\eta_a,\eta_b) = G_{ac}(k;\eta_a,0)G_{bd}(k;\eta_b,0)P^{0}_{cd}(k)$$

similar to 1-halo term in the halo model

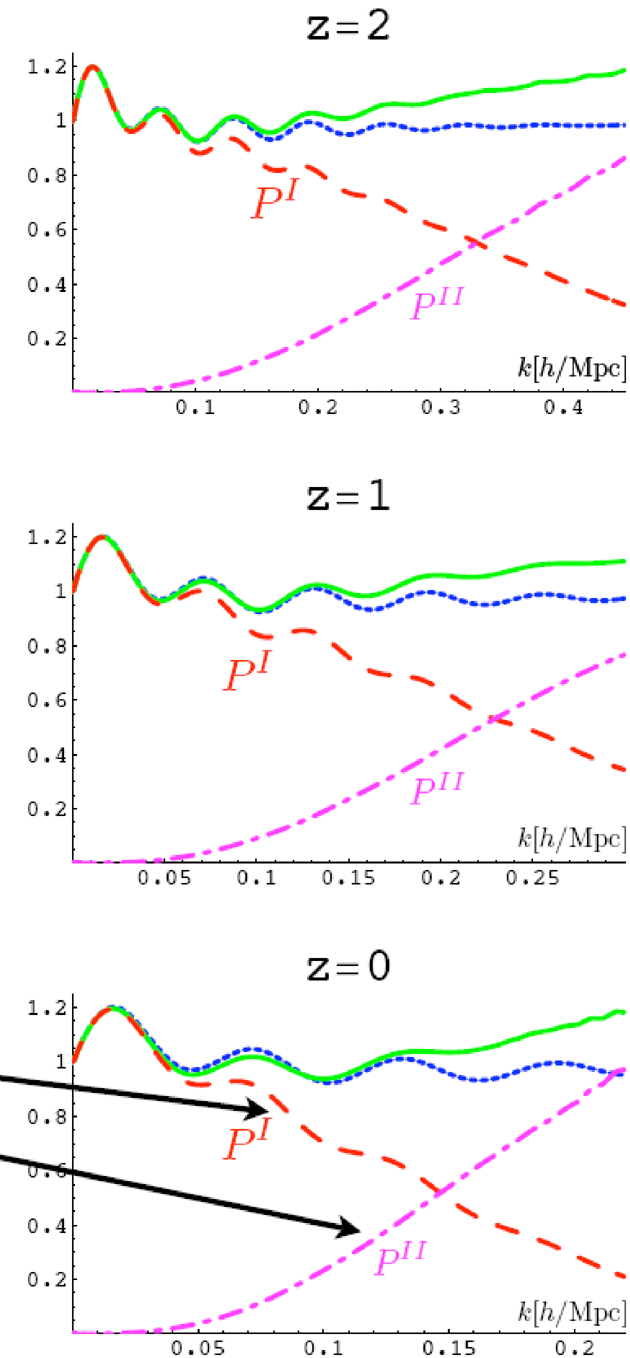
$$P^{II}_{ab}(k;\eta_a,\eta_b) = \int_0^{\eta_a} ds_1 \int_0^{\eta_b} ds_2\, G_{ac}(k;\eta_a,s_1)G_{bd}(k;\eta_b,s_2)\Phi_{cd}(k;s_1,s_2)$$





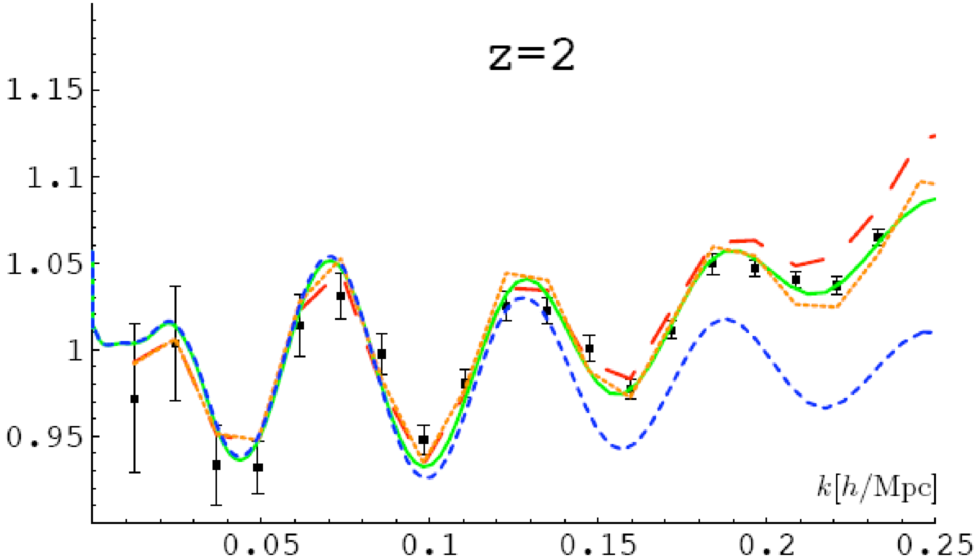
again, tree-level vertices...

$$\partial_\lambda \Phi_{ab,\,\lambda}(k;\, s_1,\, s_2) = 4\, e^{s_1+s_2} \int d^3\mathbf{q}\, \delta(\lambda - q) P^I_{dc,\lambda}(q;\, s_1,\, s_2) \times$$
$$P_{fe,\lambda}(|\mathbf{q}-\mathbf{k}|;\, s_1,\, s_2)\, \gamma_{adf}(\mathbf{k}, -\mathbf{q}, -\mathbf{k}+\mathbf{q}) \gamma_{bce}(-\mathbf{k}, \mathbf{q}, \mathbf{k}-\mathbf{q})$$



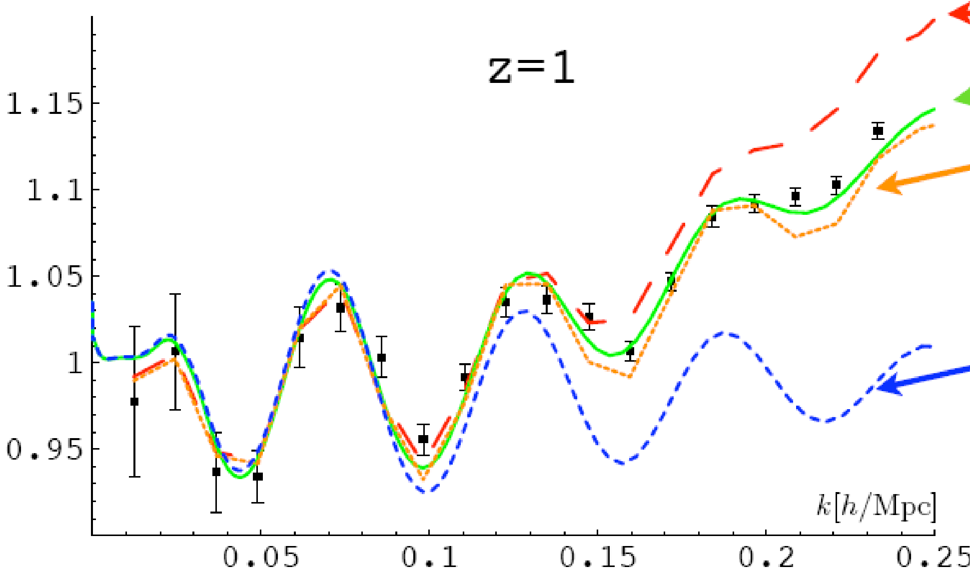
accuracy of linear theory up to k~0.12 h/Mpc is fortuitous: cancellation between two large non-linear effects

comparison with other approaches



ΛCDM model

$\Omega_\Lambda=0.73$, $\Omega_b=0.043$ h=0.7



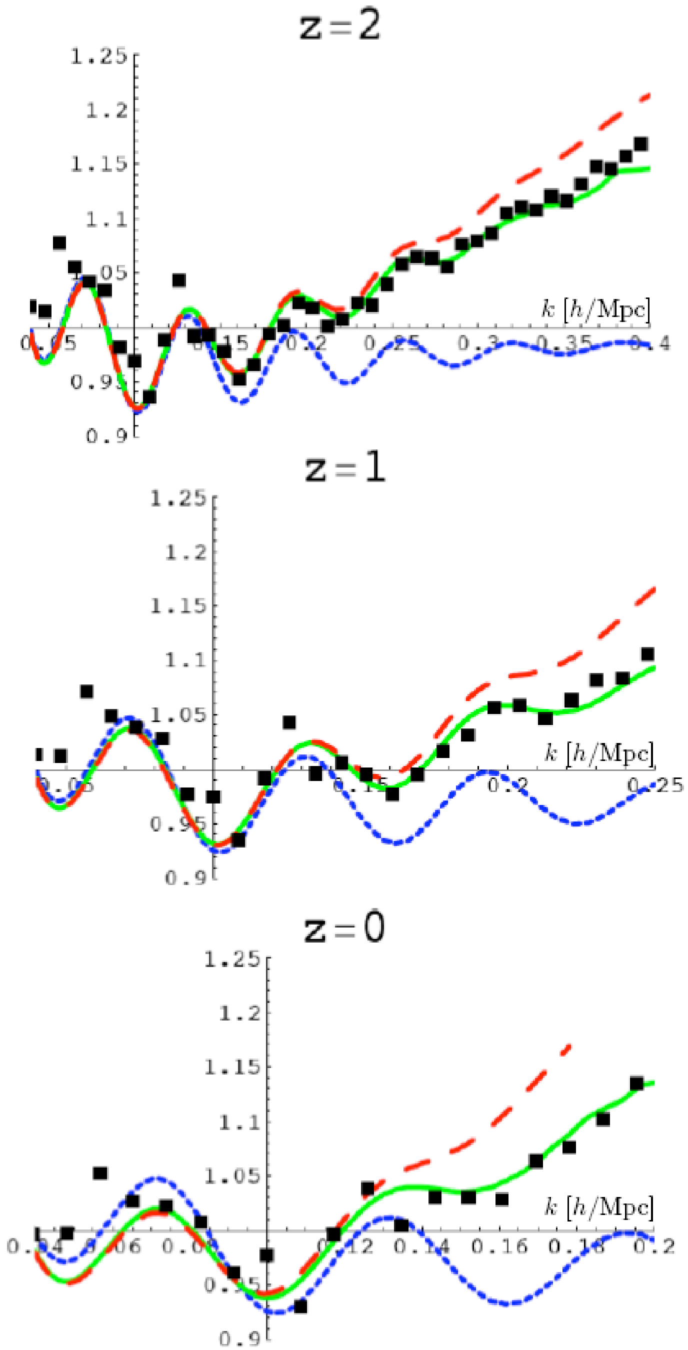
1-loop PT

RG (Matarrese, M.P.)

Halo model

Linear theory

data from N-body simulations of
Jeong and Komatsu, astro-ph/0604075



ΛCDM model

Ω_Λ=0.7, Ω_b=0.046 h=0.72

data from N-body simulations of

Huff et al, Astrop. Phys. 26, 351 (2007)

Short/Mid/Long-Term Goals

- Improve solution by including "running" of the trilinear vertex and write an approximate RG expression for the <u>effective action</u> (S.M. & Pietroni, in progress)
- Make the code publicly available (as soon as the above step is accomplished and numerical integration is refined)
- RG calculation of the bispectrum, accounting for primordial NG (via a quadratic f_{NL} term in the gravitational potential) (S.M., M. Pietroni & A. Riotto, in progress)
- Account for non-linear and non-local halo/galaxy biasing; go to redshift space
- Non-linear mapping for density & velocity fields (?)

Conclusions (1)

- Very important quantifying departures from linear theory to compare cosmological models with future galaxy surveys. The $0<z<1$ range crucial for DE studies.
- The compact perturbation theory formulated by Crocce and Scoccimarro is a very convenient starting point for applying RG techniques to cosmology.
- Exact RG equations can be derived for any kind of correlation function and for the scale-dependent growth factor.
- Systematic approximation schemes, based on truncations of the full hierarchy of equations, can be applied, borrowing the experience from QFT.

Conclusions (2)

- A simple approximation scheme already shows the emergence of an intrinsic UV cutoff in the RG running.
- Excellent agreement of RG power-spectrum predictions down to z=0 with results of N-body simulations in the range of Baryonic Acoustic Oscillations.
- Immediate lines of development include: computation of the bispectrum (including non-Gaussian initial conditions), improved approximations for the propagator, bias and redshift-space distorsions.