Dark Matter: from microphysics to Galaxies

❖ Cold (CDM): small velocity dispersion: small structure forms first, ***bottom-up*** hierarchical merger

❖Hot (HDM) : large velocity dispersion: big structure forms first, ***top-down***, fragmentation

❖ Warm (WDM): ``in between''

ΛCDM Concordance Model:

CMB+ LSS + N-body:

DM is COLD and COLLISIONLESS

N-body:

➢``clumpy halo'', large number of satellite galaxies

➢$\rho(r) \sim 1/r$ (NFW)

Observational Evidence:

❑Too few ``satellite'' galaxies
❑dSphS: cores instead of cusps ~ 1 kpc

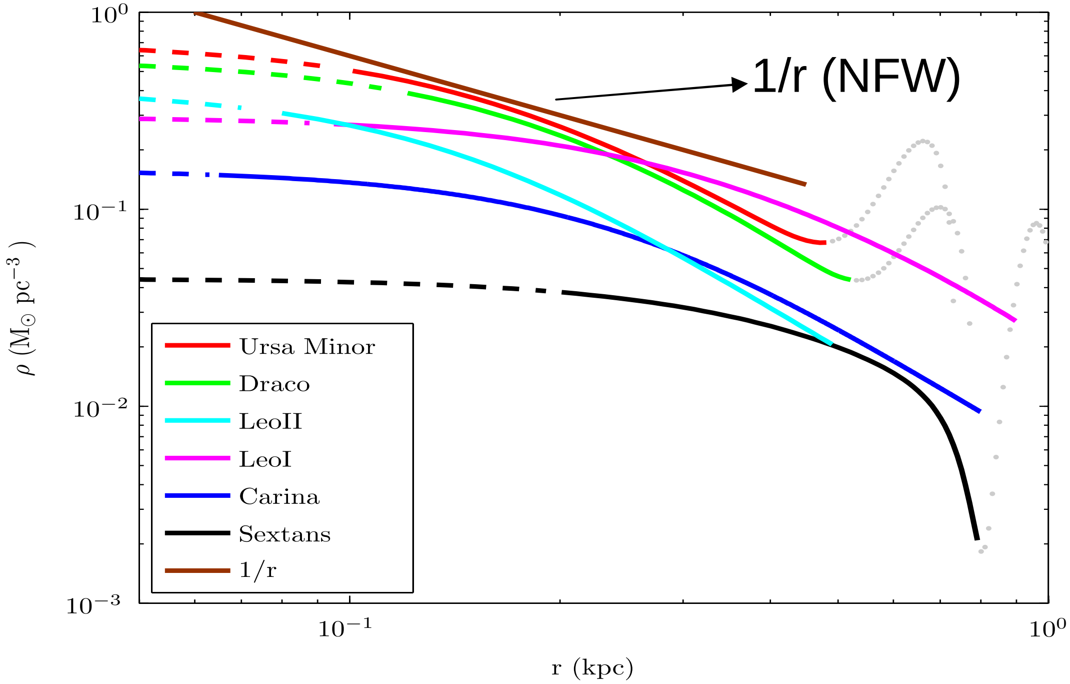


Fig. 4.— Derived inner mass distributions from isotropic Jeans' equation analyses for six dSph galaxies. The modelling is reliable in each case out to radii of log (r)kpc$\sim$ 0.5. The unphysical behaviour at larger radii is explained in the text. The general similarity of the inner mass profiles is striking, as is their shallow profile, and their similar central mass densities. Also shown is an r^{-1} density profile, predicted by many CDM numerical simulations (eg Navarro, Frenk & White 1997). The individual dynamical analyses are described in full as follows: Ursa Minor (Wilkinson et al. (2004)); Draco (Wilkinson et al. (2004)); LeoII (Koch et al. (2007)); LeoI (Koch et al. (2006)); Carina (Wilkinson et al. (2006), and Wilkinson et al in preparation); Sextans (Kleyna et al. (2004)).

Collisionless DM= decoupled particles

$\Gamma > H$ or $\lambda < d_H$ $\longrightarrow$ LTE: Eq. distribution, FD, BE,MB

$\Gamma < H$ or $\lambda > d_H$ $\longrightarrow$ Out of LTE: frozen distribution

Frozen distribution obeys collisionless BE:

$$\frac{d}{dt} f(P_f, t) = 0 \rightarrow \left[\frac{\partial}{\partial t} - H P_f \frac{\partial}{\partial P_f}\right] f(P_f, t) = 0$$

$f(P_f,t) = f(a(t)P_f) = f(p_c)$ $\longrightarrow$ $f(y, x_1, x_2, \cdots)$ Distribution function Of decoupled particle

$y = \frac{p_c}{T_d}$

*$x_1, x_2, \cdots$: **Dimensionless const.(microphysics)***

Non-relativistic particles: m > T(t) $\Longrightarrow$

$$\rho(t) = mn(t) = mg\int \frac{d^3P_f}{(2\pi)^3} f(P_f, T) = mg\,\frac{T_d^3}{2\pi^2\, a^3(t)} \int_0^\infty y^2 f(y)dy$$

$$\langle \vec{V}^{\,2} \rangle = \left\langle \frac{\vec{P}_f^{\,2}}{m^2} \right\rangle = \frac{\int \frac{d^3P_f}{(2\pi)^3} \frac{\vec{P}_f^{\,2}}{m^2} f\,[a(t)P_f\,]}{\int \frac{d^3P_f}{(2\pi)^3} f\,[a(t)P_f\,]} = \left(\frac{T_d}{ma(t)}\right)^2 \frac{\int_0^\infty y^4 f\,(y)dy}{\int_0^\infty y^2 f\,(y)dy}$$

Constraints:

1) ABUNDANCE: Upper bound

$$\Omega_{DM} h^2 = 0.105$$

# of Relativistic d.o.f at decoupling

$$m_a \leq 2.695 \frac{2 g_d\, \xi(3)}{g \int_0^\infty y^2 f(y)dy} \text{ (eV)}$$

2) Phase space density: LOWER BOUND

For decoupled non-relativistic particles the phase space density is ***CONSERVED***

$$\mathcal{D} \equiv \frac{n(t)}{\left\langle \vec{P}_f^2 \right\rangle^{\frac{3}{2}}} = \frac{g}{2\pi^2} \frac{\left[\int_0^\infty y^2 f(y) dy\right]^{\frac{5}{2}}}{\left[\int_0^\infty y^4 f(y) dy\right]^{\frac{3}{2}}}$$

$$\frac{\rho_{DM}}{\sigma_{DM}^3} \equiv 6.611 \times 10^8 \, \mathcal{D} \left[\frac{m}{\text{keV}}\right]^4 \frac{M_\odot/\text{kpc}^3}{\left(\text{km/s}\right)^3}$$

one dimensional velocity dispersion

NR-DM phase space density

OBSERVATIONS: dSphs:

$$0.9 \leq \left[10^{-4} \frac{\rho}{\sigma^3} \frac{(\text{km/s})^3}{M_\odot/(\text{kpc})^3}\right] \leq 20$$

Thm: phase space density diminishes in ``violent'' relaxation (mergers)

$$\underbrace{\frac{[62.36\,\mathrm{eV}]}{\mathcal{D}^{\frac{1}{4}}}\left[10^{-4}\frac{\rho}{\sigma^3}\frac{(\mathrm{km/s})^3}{M_\odot/(\mathrm{kpc})^3}\right]^{\frac{1}{4}}}_{\text{Lower bound from phase space density of dSphs}} < m \le \underbrace{2.695\frac{2g_d\,\xi(3)}{g\int_0^\infty y^2 f(y)dy}\,(\mathrm{eV})}_{\text{Upper bound from abundance}}$$

BOUNDS FOR THERMAL RELICS

1) Relativistic at decoupling: Fermions or Bosons

m ~ 1 keV consistent with CORED profiles for $T_d \le$ 100-300 GeV

for cusps $g_d \ge$ 2000 (beyond SM!!)

2) Non-relativistic at decoupling : Wimps

M ~ 100 GeV, T_d ~ 10 MeV

$$\left.\frac{\rho}{\sigma^3}\right)_{Wimp} = \begin{cases} \left.\frac{\rho}{\sigma^3}\right)_{cusp}\times 10^{15} \\ \left.\frac{\rho}{\sigma^3}\right)_{cored}\times 10^{18} \end{cases}$$

CAN THIS RELAXATION BE POSSIBLE??

CONSTRAINTS: SUMMARY

- ARBITRARY DECOUPLED DISTRIBUTION FUNCTION
- ABUNDANCE $\Rightarrow$ UPPER BOUND
- dSphs (DM dominated) PHASE SPACE $\Rightarrow$ LOWER BOUND
- m ~ keV THERMAL RELICS decoupled when relativistic $\lesssim$ 100-300 GeV consistent with CORES
- Wimps with m ~ 100 GeV, T_d ~ 10 MeV PSD ~ 10^{18}-10^{15} x (dSphs)!!

Transfer function and power spectrum:

- ❑ NR Boltzmann-Vlasov eqn for (DM) density + gravitational perturbation
- ❑ Valid for particles that are NR and modes inside Hubble radius
- ❑ Matter domination $z \leq z_{eq} \sim 3050$
- ❑ All scales relevant for structure formation

What's out?

❖ Photons + Baryons modify T(k) ~ few %

❖ BAO on scales ~ 150 Mpc (acoustic horizon) (interested in MUCH smaller scales!!)

Why?

✓ Study arbitrary distribution functions, couplings, masses

✓ Analytical understanding of small scale properties

✓ No tinkering with codes

$$f(\vec{p};\vec{x};t) = f_0(p) + F_1(\vec{p};\vec{x};t) \qquad \varphi(\vec{x},t) = \varphi_0(\vec{x},t) + \varphi_1(\vec{x},t)$$

Unperturbed decoupled distribution

(DM) perturbation

Unperturbed grav. Potential (FRW)

Grav. perturbation

Linearized B-V Eqn:

$$\frac{1}{a}\frac{\partial F_1}{\partial \tau} + \frac{\vec{p}}{ma^2}\cdot\vec{\nabla}_{\vec{x}} F_1 - m\vec{\nabla}_{\vec{x}}\varphi_1\cdot\vec{\nabla}_{\vec{p}} f_0 = 0$$

Poisson Eqn:

$$\varphi_1(\vec{k};s) = -\frac{4\pi G}{k^2 a(s)}\Delta(\vec{k};s) \quad ; \quad \Delta(\vec{k},s) = m\int \frac{d^3p}{(2\pi)^3} F_1(\vec{k},\vec{p};s)$$

``New'' variable s = $\frac{2u}{H_0\sqrt{\Omega_{DM}}a_{eq}}$; $u = 1 - \left(\frac{a_{eq}}{a}\right)^{\frac{1}{2}} \longrightarrow \frac{ds}{d\tau} = \frac{1}{a}$

Follow the steps…

- Integrate B-V equation (in s)
- Use Poisson's eqn. $\longrightarrow$ Integral eqn: Gilbert's

- Normalize at initial time (t_{eq}): $\Phi(\vec{k},u)=\dfrac{\varphi_1(\vec{k},u)}{\varphi_1(\vec{k},0)}$; $\delta(k,u)=\dfrac{\Delta(k;u)}{\Delta(k;0)}$

$$P_f(k)=T^2(k)P_i(k)$$

$$T(k)=\frac{5}{3}\Phi(k;1)$$

- Normalize the decoupled
- distribution function:

$$\tilde{f}_0(y)=\frac{f_0(y)}{\int_0^\infty y^2 f_0(y)dy} \quad ; \quad y=\frac{p}{T_{0,d}}$$

$p \longrightarrow$ comoving momentum

$T_{0,d} \longrightarrow$ decoupling temp.

- Take 2 derivatives w.r.t. u: $\longrightarrow$

$$\underbrace{\ddot{\delta}(k,u)-\frac{6\,\delta(k,u)}{(1-u)^2}+3\,\gamma^2\,\delta(k,u)}_{\text{Jeans' Fluid equation: replace } C^2_s \text{ by } \langle V^2\rangle}-\underbrace{\int_0^u du'\,K(u-u')\,\frac{\delta(k,u')}{(1-u')^2}}_{\text{Correction to fluid description: memory of gravitational clustering}}=\underbrace{S_0(k;u)}_{\text{Free streaming solution in absence of gravity: INITIAL CONDITIONS}}$$

$$\gamma^2=\frac{2\,k^2}{k^2_{fs}(t_{eq})}\quad;\quad \underbrace{k_{fs}(t_{eq})=\frac{0.0102}{\sqrt{\overline{y^2}}}\left[\frac{g_d}{2}\right]^{\frac{1}{3}}\frac{m}{\mathrm{keV}}[\mathrm{kpc}]^{-1}}_{\text{Free streaming wave vector at matter-radiation equality}}\quad;\quad \overline{y^2}=\int_0^\infty dy\;y^4\;\tilde{f}_0(y)$$

$$k_{fs}(t_{eq})=\begin{cases}\dfrac{5.88}{\mathrm{pc}}\left(\dfrac{g_d}{2}\right)^{\frac{1}{3}}\left(\dfrac{m}{100\,\mathrm{GeV}}\right)^{\frac{1}{2}}\left(\dfrac{T_d}{10\,\mathrm{MeV}}\right)^{\frac{1}{2}} & \text{WIMPs}\\[2ex] 0.00284\left(\dfrac{g_d}{2}\right)^{\frac{1}{3}}\dfrac{m}{\mathrm{keV}}[\mathrm{kpc}]^{-1} & \text{FD thermal relics}\\[2ex] 0.00317\left(\dfrac{g_d}{2}\right)^{\frac{1}{3}}\dfrac{m}{\mathrm{keV}}[\mathrm{kpc}]^{-1} & \text{BE thermal relics}\end{cases}$$

$$K(u-u') = 6\alpha \int_0^{\infty} y\,(\overline{y^2} - y^2)\,\tilde{f}_0(y)\,\sin[\alpha\, y\,(u-u')]\,dy \quad ; \quad \alpha = \sqrt{\frac{3}{\overline{\overline{y^2}}}}\,\gamma$$

DECOUPLED DISTRIBUTION FUNCTION: **STATISTICS**

Properties of K(u-u'):

- ❖ Correction to fluid description
- ❖ Memory of gravitational clustering
- ❖ $f_0(y)$ with larger support for small y $\longrightarrow$ **longer range of memory**
- ❖ Longer range of memory $\longrightarrow$ **larger T(k)**
- ❖ Negligible at **large** scales $k \ll k_{fs}(t_{eq})$
- ❖ Important at **small** scales $k \geq k_{fs}(t_{eq})$

Exact T(k)

$$T(k)=\frac{10}{\sqrt{3}\,\gamma^3}\int_0^1 h_2(u)\Big[\frac{I[\alpha u]}{(1-u)^2}+\frac{1}{6}S_{NB}[\delta;u]\Big]du$$

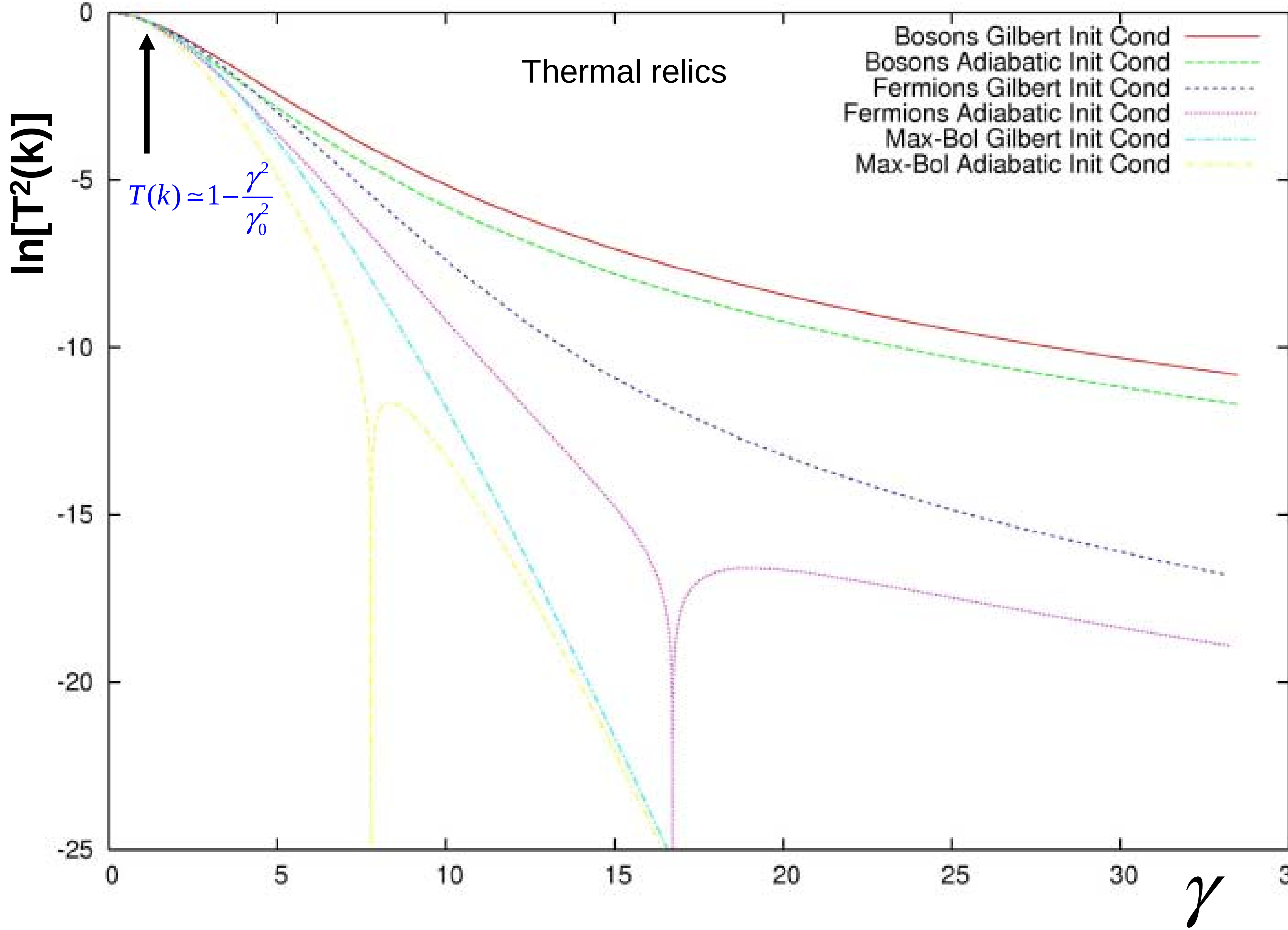
Regular solution of Jeans' **Fluid** eqn.

Free streaming solution
In absence of gravity:
INITIAL CONDITIONS

Memory of gravitational clustering: **K(u-u')**

Features:

- ✓ Systematic Fredholm expansion
- ✓First TWO terms simple and remarkably accurate
- ✓Include memory of gravitational clustering
- ✓***Arbitrary distribution function(statistics+non LTE)***
- ✓***Arbitrary initial conditions***



Thermal relics
Bosons Gilbert Init Cond
Bosons Adiabatic Init Cond
Fermions Gilbert Init Cond
Fermions Adiabatic Init Cond
Max-Bol Gilbert Init Cond
Max-Bol Adiabatic Init Cond
$T(k) \simeq 1 - \frac{\gamma^2}{\gamma_0^2}$
ln[T²(k)]
γ
0
-5
-10
-15
-20
-25
0
5
10
15
20
25
30
35

Sterile neutrinos: a DM candidate

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2}\partial_\mu \chi \partial^\mu \chi - \frac{M^2}{2}\chi^2 + i\bar{\nu}\partial\nu - \frac{Y}{2}\chi\bar{\nu}^c\nu - \frac{m}{2}\bar{\nu}^c\nu - y_\alpha H^\dagger \bar{L}_\alpha \nu - V(H^\dagger H;\chi) + \text{h.c}$$

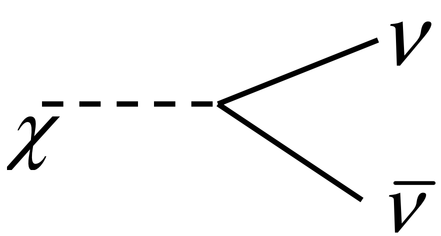
χ = Scalar, gauge singlet

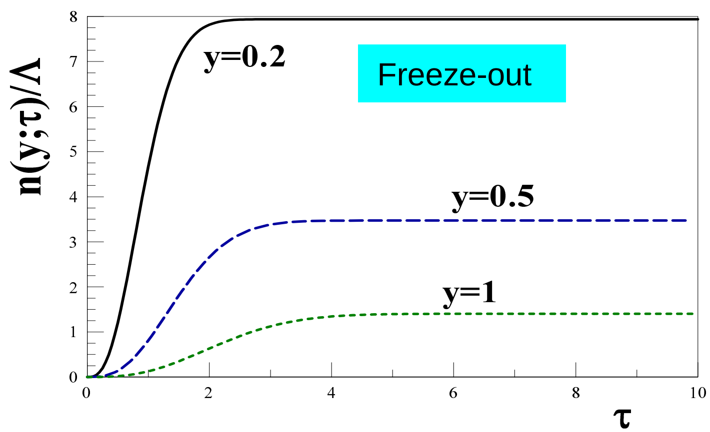
$$M \sim \langle\chi\rangle \sim 100\,\text{GeV} \qquad Y \sim 10^{-8}$$

ν = ``sterile neutrino'' SU(2) singlet

$$m = Y\langle\chi\rangle \sim \text{keV}$$

Production: Scalar decay





$$\Lambda \sim 10^{-2}$$

$$\tau = \frac{M}{T(t)}$$

Frozen distribution

$$f_0(y) = 2\Lambda\sqrt{\pi}\frac{g_{\frac{5}{2}}(y)}{y^{\frac{1}{2}}}$$

$y = p/T_d$

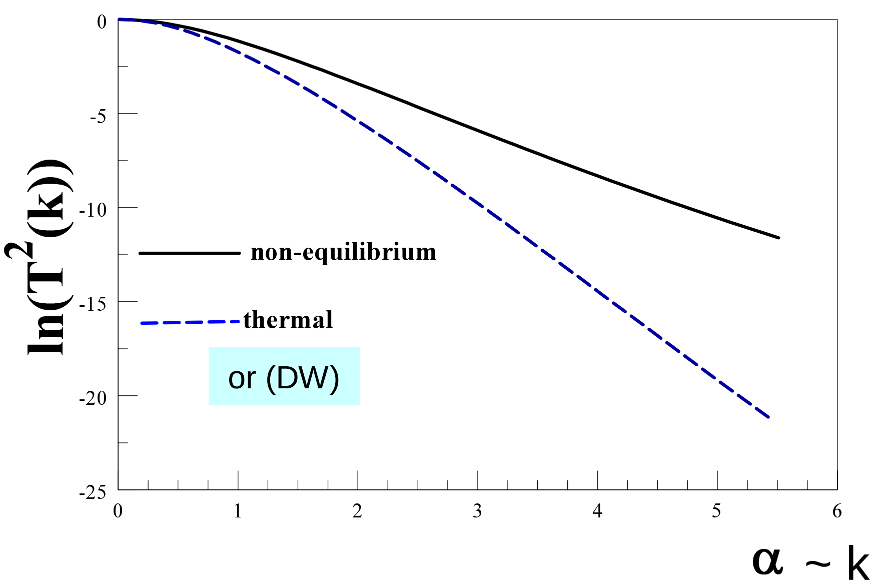
Decoupling at ~ 100 GeV

Strong enhancement of small momentum

Abundance + phase space density constraints:

$$560\,\mathrm{eV} \lesssim m \lesssim 1330\,\mathrm{eV}$$

Consistent with model ``beyond SM''



Enhancement of T(K) at SMALL scales, from long-range memory

$$k_{fs} \sim 490\,\mathrm{kpc}$$

Summary: Roadmap

1) ***Microphysics:*** Particle physics model, kinetics of production, decoupling

$\longrightarrow$ $f(y)$ = decoupled distribution function, $y=p/T_{0,d}$

2) ***Constrain*** mass, couplings, $T_{0,d}$ from abundance + phase space density

$$\underbrace{\frac{100\,\mathrm{eV}}{\mathcal{D}^{\frac{1}{4}}}}_{\text{Lower bound from phase Space density of dSphs}} \leq m \leq 6.5\,\mathrm{eV}\,\underbrace{\frac{g_d}{g\int_0^\infty y^2\, f(y)\,dy}}_{\text{Upper bound from abundance}} \quad ; \quad \mathcal{D} = \frac{g}{2\pi^2}\,\frac{\left[\int_0^\infty y^2 f(y)dy\right]^{\frac{5}{2}}}{\left[\int_0^\infty y^4 f(y)dy\right]^{\frac{3}{2}}}$$

Lower bound from phase Space density of dSphs

Upper bound from abundance

Thermal relics that decouple relativistically: $\mathcal{D} \sim 2\times 10^{-3}$ $\longrightarrow$ m ~ keV

3) ***DM T(k):* exact** $\longrightarrow$ ***simple + accurate approx:***

- ***arbitrary f(y)+ini. conds.***
- ***corrections to fluid+ memory of grav. clustering.***
- ***large f(y) at small y=long memory=large T(k) at small scales.***