Effective Field Theory of inflation:
MCMC analysis of current experimental data

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The 12th Paris Cosmology Colloquium 2008: CMB, DM, DE, Dark Ages and LSS in the Standard Model of the Universe

Outline

1. The framework
   - Observational data and models
   - On cosmic variance
   - Monte Carlo Markov Chains
2. The theory
   - EFT of Inflation
   - Trinomial Chaotic and New Inflation
   - Early fast-roll and primordial fluctuations
   - The transfer function $D(k)$
3. MCMC results
   - Trinomial Chaotic and Trinomial New Inflation
   - Binomial New Inflation
   - Sharp-cut vs. fast-roll

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The problem:

Experimental data

CMB, LSS, SN Lensing, Ly$-\alpha$ ...

$\longleftrightarrow$

Cosmological parameters

Ω_b, Ω_c, H_0, τ, A_s, n_s, r, ...

The case of CMB:

CMB central observables

TT, TE, EE, (BB) correlation multipoles C_ℓ

- from raw data to $C_\ell^{(data)}$ with much systematics and complex likelihood code;
- from theory to $C_\ell^{(model)}$ (fully independent and chi-square distributed if primordial fluctuations are gaussian) through CMBFAST, CAMB, ...;
- Markov Chain Monte Carlo analysis to assign likelihood to model parameters with CosmoMC (possibly including LSS, SN, Lensing, ...).

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- Publicly available F90 open source code
- Comes with the experimental data (except WMAP)
- Interfaces directly with the WMAP1-3-5 likelihood software
- Theoretical calculations with CAMB (derived from CMBFAST)
- Quite accurate, easy to use, fairly well documented
- Comes with tool for analyzing results and useful MATLAB scripts
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- poorly modular, too many globals

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Neglecting all uncertainities but cosmic variance:

Let $x = C_\ell^{(data)}/C_\ell^{(model)}$; then

$p_\ell(x|model) \propto \frac{1}{x}\left(xe^{-x}\right)^{\ell+1/2}$ (chi-square distribution) sets the probability density for $C_\ell^{(data)}$ given the model, with

$$\langle x\rangle = 1 \text{ and } (x)_{ML} = \frac{2\ell-1}{2\ell+1}$$

At the same time, if $y = 1/x = C_\ell^{(model)}/C_\ell^{(data)}$, then

$p_\ell(y|data) \propto \left(e^{-1/y}/y\right)^{\ell+1/2}$ sets the probability density for $C_\ell^{(model)}$ given the data (assuming flat priors), with

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An example: lowest 9 TT multipoles

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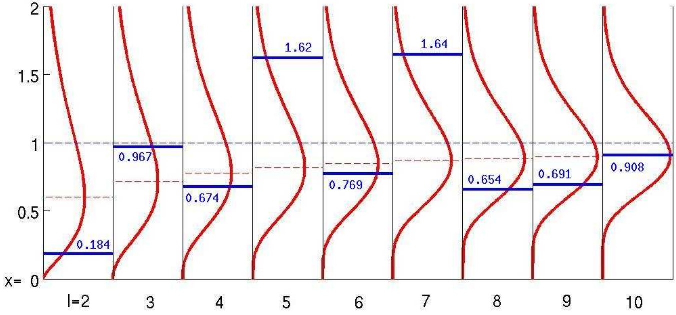
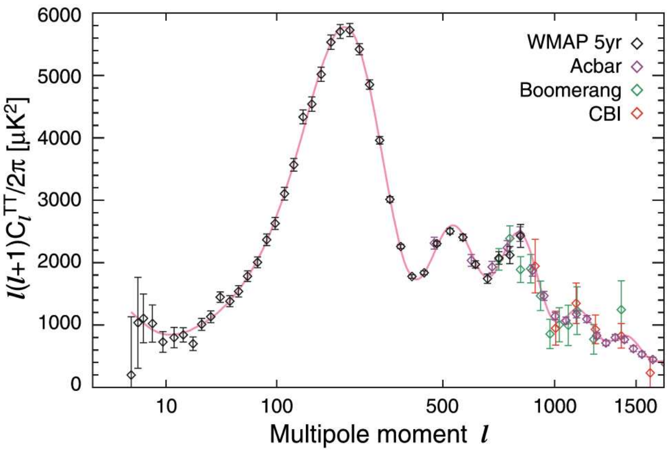
probability curves from best fit ΛCDM
WMAP5 data



$$\text{Prob}[C_2^{(data)} < 0.184\, C_2^{(model)}] = 0.031\ldots$$

1σ uncertainities based on $p_\ell(y|data)$



from "M. R. Nolta et al.", arXiv:0803.0593 [astro-ph] 5 Mar 2008

Unbiased(?) estimation

Probability that at least one multipole, regardless of ℓ, is smaller than $\bar{x}$ times its theoretical expectation value:

$$P(\bar{x}, \ell_{max}) = 1 - \prod_{\ell=2}^{\ell_{max}} \left[1 - \int_0^{\bar{x}} dx\, p(x|model) \right]$$

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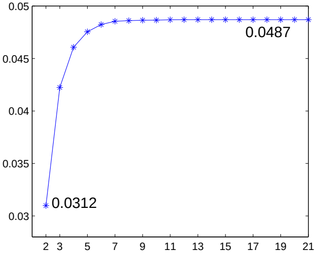

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The setup

- Observational CMB data $\Longrightarrow$ likelihood on $C_\ell^{(model)}$;
- Model with cosmological parameters $\lambda = \{\lambda_1, \lambda_2, \dots \lambda_n\} \Longrightarrow C_\ell^{(model)}(\lambda)$ (CMBFAST, CAMB,...);

$$\Longrightarrow \quad \text{likelihood } L(\lambda) = \exp[-\chi^2(\lambda)/2]$$

The **MCMC** method produces sequences distributed as $L(\lambda)$ ($\times$ the **prior probability**), through an acceptance/rejection one-step algorithm (*e.g.* **Metropolis**)

$$W(\lambda^{(k+1)}, \lambda^{(k)}) = g(\lambda^{(k+1)}, \lambda^{(k)}) \min\left\{1, \frac{L(\lambda^{(k+1)})g(\lambda^{(k+1)}, \lambda^{(k)})}{L(\lambda^{(k)})g(\lambda^{(k)}, \lambda^{(k+1)})}\right\}$$

runs made on a Linux cluster (Turing) with 8 to 16 parallel chains, repeated up to 4 times for each setup, with $R - 1 < 0.03$

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The ΛCDM+r reference model

MCMC parameters:	ω_b, ω_c, θ, τ, (slow), A_s, n_s, r (fast)
Context:	$\Omega_\nu = 0, \ldots$; standard priors, $k_{pivot} = 0.05$, no SZ, lensed CMB, linear mpk, ...
Datasets:	WMAP5, SDSS

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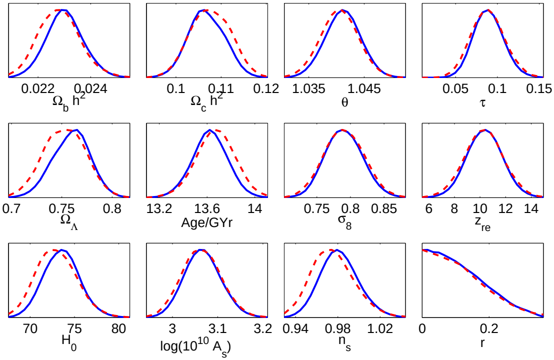
param	best fit
$100\Omega_b h^2$	2.250
$\Omega_c h^2$	0.107
θ	1.038
100τ	8.824
H_0	72.2
σ_8	0.784
$\ln[10^{10} A_s]$	3.06
n_s	0.963
r	0.01
$-\log(L)$	1340.94



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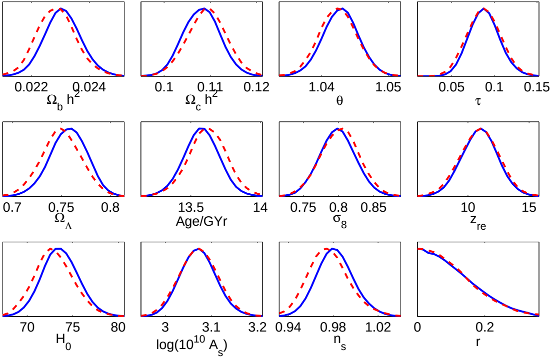
param	best fit
$100\Omega_b h^2$	2.271
$\Omega_c h^2$	0.111
θ	1.042
100τ	8.81
H_0	71.87
σ_8	0.81
$\ln[10^{10} A_s]$	3.08
n_s	0.969
r	0.01
$-\log(L)$	1354.30



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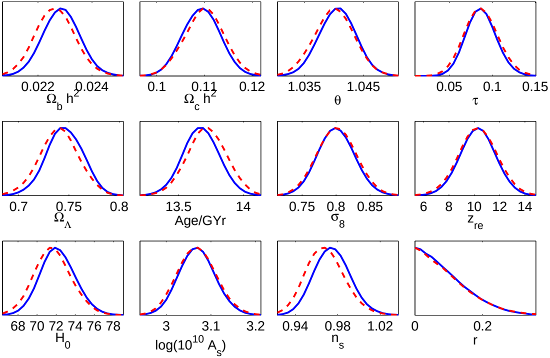
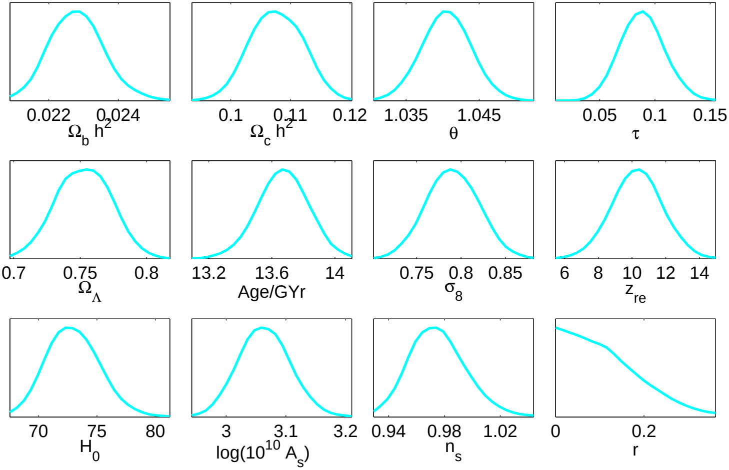
param	best fit
$100\Omega_b h^2$	2.252
$\Omega_c h^2$	0.11
θ	1.04
100τ	7.85
H_0	71.3
σ_8	0.79
$\ln[10^{10} A_s]$	3.048
n_s	0.962
r	0.01
$-\log(L)$	1495.98



ΛCDM$+r$ marginalized mean likelihoods

WMAP5 + SDSS



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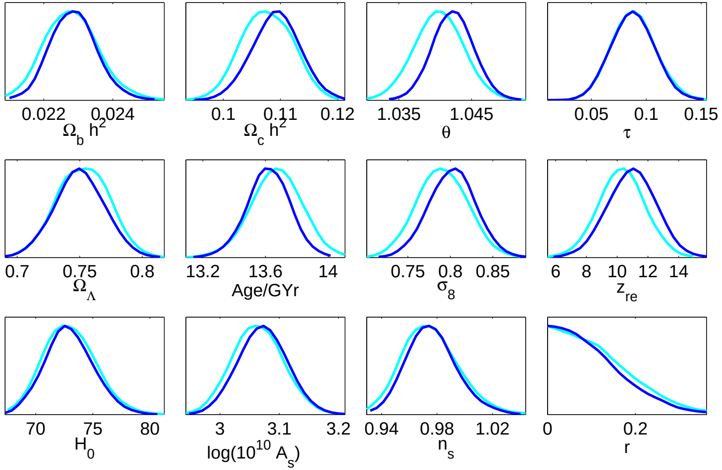

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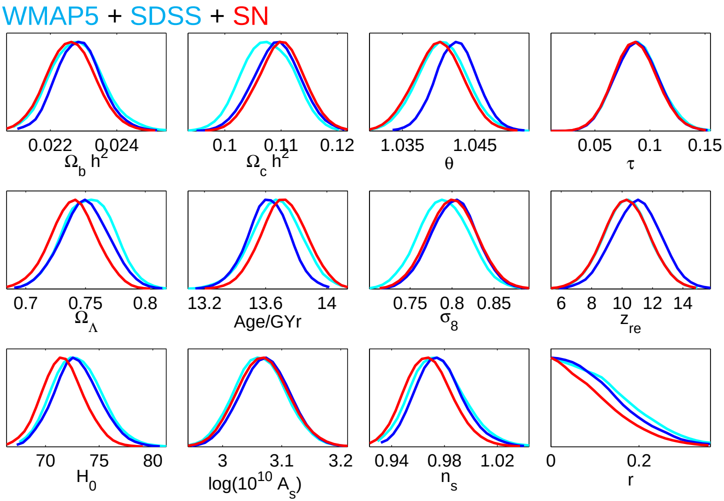
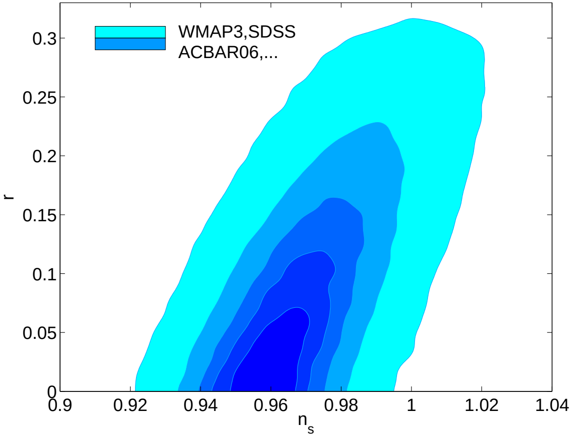


(n_s, r) joint probability distribution



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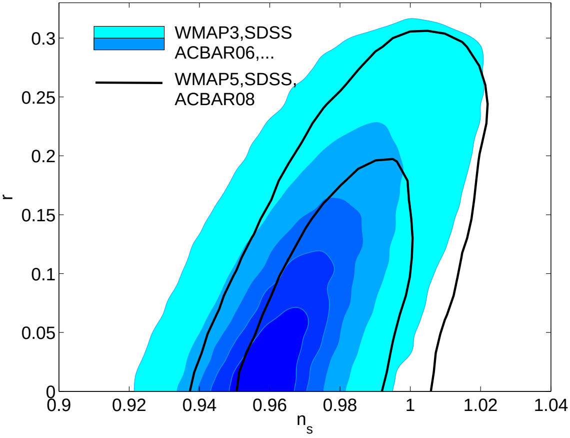

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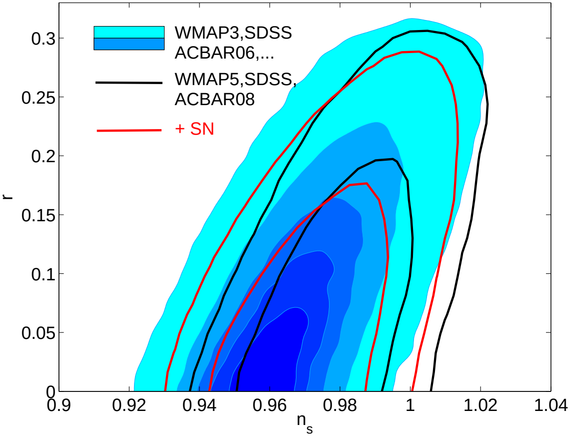

Outline

EFT of (single field) inflation (à la Boyanowski–de Vega–Sanchez)

Inflaton potential ($\hbar = 1$, $c = 1$, $M_{PL} = 2.4 \times 10^{18}$ GeV)

$$V(\phi) = N\, M^4\, w(\chi)\ , \quad \chi = \frac{\phi}{\sqrt{N}\, M_{PL}}\ , \quad w(\chi) \sim \mathscr{O}(1)\ , \quad N \sim 50 - 60$$

Energy scale of inflation and inflaton mass

$$M \simeq 0.57 \times 10^{16}\ \text{GeV} \sim M_{\rm GUT}, \quad m = M^2/M_{PL} \sim 1.3 \times 10^{13}\ \text{GeV}$$

Hubble parameter and quantum corrections

$$H \sim 5\, m \ll M_{PL}\ , \qquad \text{loops} \to (H/M_{PL})^2 \sim 10^{-9}$$

Number of efolds kept fixed in MCMC analysis

$$N = \log \frac{a(t_{end})}{a(t_{exit})}\ , \qquad w(\chi_{end}) = w'(\chi_{end}) = 0$$

t_{exit}: the mode with comoving k_0 becomes superhorizon ($\to N = N(k_0)$)

- self–consistency of inflation $\longrightarrow N$ (coarse–grained)
- Understanding beginning/end of inflation $\longrightarrow N$ (fine–grained)

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$$M \simeq 0.57 \times 10^{16} \text{ GeV} \sim M_{\rm GUT}, \quad m = M^2/M_{PL} \sim 1.3 \times 10^{13} \text{ GeV}$$

Hubble parameter and quantum corrections

$$H \sim 5m \ll M_{PL} \; , \quad \text{loops} \to (H/M_{PL})^2 \sim 10^{-9}$$

Number of efolds kept fixed in MCMC analysis

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t_{exit}: the mode with comoving k_0 becomes superhorizon ($\to N = N(k_0)$)

- self–consistency of inflation $\longrightarrow N$ (coarse–grained)
- Understanding beginning/end of inflation $\longrightarrow N$ (fine–grained)

EFT of (single field) inflation (à la Boyanowski–de Vega–Sanchez)

Inflaton potential ($\hbar = 1$, $c = 1$, $M_{PL} = 2.4 \times 10^{18}$ GeV)

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The number of efolds N (coarse-grained: constant H, sharp ΛD$\rightarrow$ RD)

Friedmann's equation right after inflation

$$\frac{H^2}{H_0^2} = \Omega_\Lambda + \frac{\Omega_m}{a^3} + \frac{\Omega_r}{a^4} \simeq \frac{\Omega_r}{a^4} \,, \qquad \frac{H}{H_0} \simeq 4.2 \times 10^{55} \,, \qquad (H \simeq 2.8 \times 10^{-5} M_{Pl})$$

$$\Longrightarrow \qquad \log \frac{1}{a} \simeq \frac{1}{2} \log \frac{H}{H_0 \sqrt{\Omega_r}}$$

Inflation efolds for the quadrupole scale ($k_q = 0.982 H_0$)

$$N_q \simeq \log \frac{H}{H_0} - \log \frac{1}{a} \simeq \log \frac{H\sqrt{\Omega_r}}{H_0} \simeq 61.6$$

$$N = N_q - \log \frac{k_0}{k_q} \simeq 57 \,, \qquad k_0 = 0.05 \text{Mpc}^{-1} \text{ (CosmoMC)}$$

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Dimensionless setup

Stretched time and dimensionless Hubble parameter

$$\tau = \frac{t\, M^2}{M_{PL}\sqrt{N}} \;, \quad \mathscr{H} = \frac{H\, M_{Pl}}{\sqrt{N}\, M^2} = \mathscr{O}(1)$$

Equations of motion

$$\mathscr{H}^2(\tau) = \frac{1}{3}\left[\frac{1}{2N}\dot{\chi}^2 + w(\chi)\right] \;, \quad \frac{1}{N}\ddot{\chi} + 3\mathscr{H}\dot{\chi} + w'(\chi) = 0$$

Energy density and pressure

$$\rho = N\, M^4\left[\frac{1}{2N}\dot{\chi}^2 + w(\chi)\right] \;, \quad p = N\, M^4\left[\frac{1}{2N}\dot{\chi}^2 - w(\chi)\right]$$

From fast-roll to slow-roll (attractive regime)

$$\frac{1}{2N}\dot{\chi}^2 \sim w(\chi) \;, \quad \frac{1}{2N}\dot{\chi}^2 \ll w(\chi)$$

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Main observables to first order in $1/N$ (slow roll)

Amplitude of adiabatic scalar fluctuations

$$|\Delta_0|^2 = \frac{N^2}{12\pi^2}\left(\frac{M}{M_{\rm Pl}}\right)^4 \frac{w^3(\chi)}{w'^2(\chi)} \quad , \qquad \chi \equiv \chi_{\rm exit}(k_0)\,. \quad k_0 \equiv k_{pivot}$$

Spectral indexes

$$n_s = 1 - \frac{1}{N}\left\{3\left[\frac{w'(\chi)}{w(\chi)}\right]^2 - 2\,\frac{w''(\chi)}{w(\chi)}\right\} , \quad n_t = -r/8$$

Tensor to scalar amplitude ratio and running

$$r = \frac{8}{N}\left[\frac{w'(\chi)}{w(\chi)}\right]^2 \quad , \qquad \frac{d\,n_s}{d\ln k} = \mathcal{O}\left(\frac{1}{N^2}\right)$$

Trinomial realization (minimality + Ginsburg-Landau)

$$V(\phi) = N\,M^4\,w(\chi) = V_0 \pm \frac{1}{2}m^2\phi^2 - \frac{1}{3}mg\phi^3 + \frac{1}{4}\lambda\phi^4$$

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Outline

Trinomial inflation

$$w(\chi) = \pm\frac{1}{2}\chi^2 + \frac{1}{3}h\sqrt{\frac{y}{2}}\chi^3 + \frac{1}{32}y\chi^4 + \text{const}$$

with h and y of order 1

$$g = -h\sqrt{\frac{y}{2N}}\left(\frac{M}{M_{Pl}}\right)^2 \sim 10^{-9}\,, \quad \lambda = \frac{y}{8N}\left(\frac{M}{M_{Pl}}\right)^4 \sim 10^{-12}$$

Trinomial chaotic (= large field) inflation

$$w(\chi) = \frac{1}{2}\chi^2 - \frac{1}{3}h\sqrt{\frac{y}{2}}\chi^3 + \frac{1}{32}y\chi^4 \,, \quad -1 < h < 0 \,, \quad \Delta = \sqrt{1-h^2}$$

Fixing N $\longrightarrow$

$$y = z + \frac{4}{3}h\sqrt{z} + \left(1 - \frac{4}{3}h^2\right)\log\left(1 + 2h\sqrt{z} + z\right)$$
$$- \frac{4h}{3\Delta}\left(\frac{5}{2} - 2h^2\right)\left[\arctan\left(\frac{h+\sqrt{z}}{\Delta}\right) - \arctan\left(\frac{h}{\Delta}\right)\right]$$

Effective coupling

$$z = \frac{y}{8}\chi^2_{exit} \quad ; \qquad 0 \leq z < \infty \,, \quad 0 \leq y < \infty$$

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$$w(\chi) = -\frac{1}{2}\chi^2 - \frac{1}{3}h\sqrt{\frac{y}{2}}\chi^3 + \frac{1}{32}y\chi^4 + \frac{2}{y}F(h)\,, \quad h>0\,, \quad \Delta = \sqrt{1+h^2}$$

with

$$F(h) = \frac{y\,V_0}{2NM^4} = \frac{8}{3}h^4 + 4h^2 + 1 + \frac{8}{3}h\Delta^3 \longrightarrow w(\chi_{end}) = w'(\chi_{end}) = 0$$

$$\chi_{end} = \sqrt{\frac{8}{y}}(\Delta+h) > 0$$

Fixing N $\longrightarrow$

$$y = z - 2h^2 - 1 - 2h\sqrt{h^2+1} + \frac{4}{3}h(h+\Delta-\sqrt{z})$$
$$+\frac{16}{3}h(\Delta+h)\Delta^2\log\left[\frac{1}{2}\left(1+\frac{\sqrt{z}-h}{\Delta}\right)\right] - 2F(h)\log\left[\sqrt{z}(\Delta-h)\right]$$

Effective coupling and normalized *effective coupling*

$$z \equiv \frac{y}{8}\chi_{exit}^2 \quad ; \qquad z_1 = 1 - \frac{z}{(\Delta+h)^2}\,, \quad 0 \le z_1 < 1\,, \quad 0 \le y < \infty$$

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Trinomial chaotic inflation

Scalar spectral index and tensor–scalar ratio

$$n_s = 1 - \frac{y}{2Nz}\left[3\frac{\left(1+2h\sqrt{z}+z\right)^2}{\left(1+\frac{4}{3}h\sqrt{z}+\frac{1}{2}z\right)^2} - \frac{1+4h\sqrt{z}+3z}{1+\frac{4}{3}h\sqrt{z}+\frac{1}{2}\,z}\right]$$

$$r = \frac{4y}{Nz}\,\frac{\left(1+2h\sqrt{z}+z\right)^2}{\left(1+\frac{4}{3}h\sqrt{z}+\frac{1}{2}z\right)^2}$$

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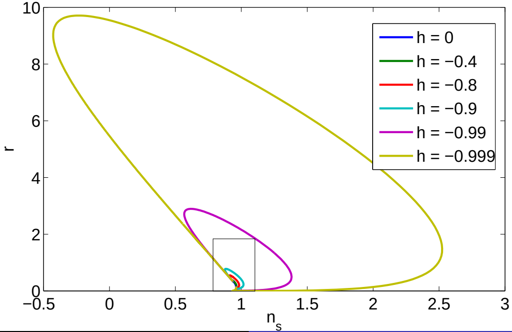

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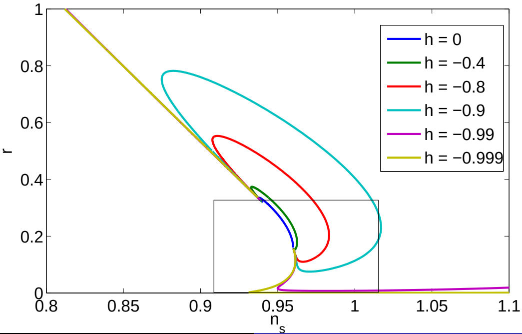

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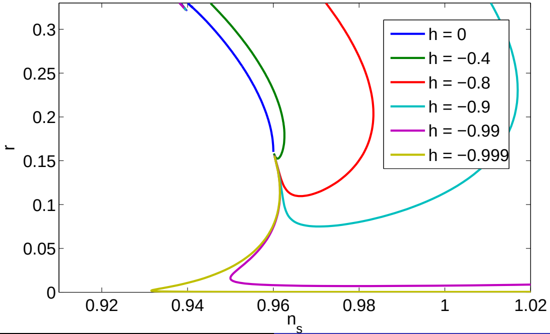

Trinomial new inflation

Scalar spectral index and tensor–scalar ratio

$$n_s = 1 - \frac{6y}{N} \frac{z(z+2h\sqrt{z}-1)^2}{\left[F(h)-2z+\frac{8}{3}hz^{3/2}+z^2\right]^2} + \frac{y}{N} \frac{3z+4h\sqrt{z}-1}{F(h)-2z+\frac{8}{3}hz^{3/2}+z^2}$$

$$r = \frac{16y}{N} \frac{z(z+2h\sqrt{z}-1)^2}{\left[F(h)-2z+\frac{8}{3}hz^{3/2}+z^2\right]^2}$$

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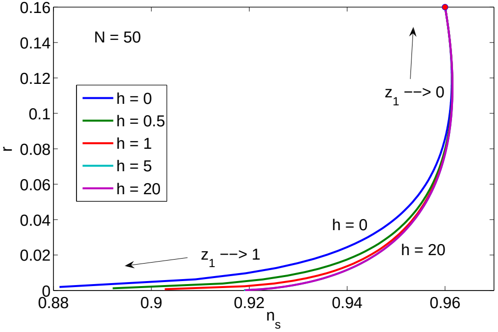

Binomial New Inflation

Setting the asymmentry $h = 0$ in TNI

$$w(\chi) = \frac{y}{32}\left(\chi^2 - \frac{8}{y}\right)^2$$

Scalar spectral index and tensor–scalar ratio

$$n_s = 1 - \frac{y}{N}\,\frac{3z+1}{(1-z)^2}\,, \quad r = \frac{16\,y}{N}\,\frac{z}{(1-z)^2}\,, \quad y = z - 1 - \log z\,, \quad 0 < z < 1$$

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Outline

Scalar fluctuations

Gauge–invariant quantum perturbation field

$$u(x,t) = -\xi(t) R(x,t) = \int \frac{d^3k}{(2\pi)^{3/2}} \left[\alpha_k S_k(\eta) e^{ik\cdot x} + \alpha_k^\dagger S_k^*(\eta) e^{-ik\cdot x}\right]$$

$$\left[\alpha_k, \alpha_{k'}^\dagger\right] = \delta^{(3)}(k-k') , \quad \xi(t) = \frac{a(t)}{H(t)} \dot{\phi}(t) , \quad \eta = \int \frac{dt}{a(t)}$$

Schroedinger–like dynamics

$$\left[-\frac{d^2}{d\eta^2} - k^2 + U(\eta)\right] S_k(\eta) = 0 , \quad U(\eta) = \frac{1}{\xi} \frac{d^2\xi}{d\eta^2}$$

Standard parametrization in dimensionless setup

$$U(\eta) = a^2 \mathscr{H}^2 m^2 N \left[2 - 7\,\varepsilon_v + 2\varepsilon_v^2 - \sqrt{\frac{8\varepsilon_v}{N}} \frac{w'}{\mathscr{H}^2} - \eta_v (3 - \varepsilon_v)\right]$$

$$\varepsilon_v = \frac{1}{2N} \frac{1}{\mathscr{H}^2} \left(\frac{d\chi}{d\tau}\right)^2 , \quad \eta_v = \frac{1}{N} \frac{w''(\chi)}{w(\chi)}$$

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Schroedinger–like dynamics

$$\left[-\frac{d^2}{d\eta^2} - k^2 + U(\eta)\right] S_k(\eta) = 0 , \quad U(\eta) = \frac{1}{\xi} \frac{d^2\xi}{d\eta^2}$$

Standard parametrization in dimensionless setup

$$U(\eta) = a^2 \mathscr{H}^2 m^2 N \left[2 - 7\,\varepsilon_v + 2\varepsilon_v^2 - \sqrt{\frac{8\varepsilon_v}{N}}\,\frac{w'}{\mathscr{H}^2} - \eta_v (3-\varepsilon_v)\right]$$

$$\varepsilon_v = \frac{1}{2N} \frac{1}{\mathscr{H}^2} \left(\frac{d\chi}{d\tau}\right)^2 , \quad \eta_v = \frac{1}{N} \frac{w''(\chi)}{w(\chi)}$$

Scalar fluctuations

Gauge–invariant quantum perturbation field

$$u(x,t) = -\xi(t) R(x,t) = \int \frac{d^3k}{(2\pi)^{3/2}} \left[\alpha_k S_k(\eta) e^{ik\cdot x} + \alpha_k^\dagger S_k^*(\eta) e^{-ik\cdot x}\right]$$

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Slow–roll approximation

$$\epsilon_V = \frac{1}{2N}\left[\frac{w'(\chi_{exit})}{w(\chi_{exit})}\right]^2 + \mathcal{O}\left(\frac{1}{N^2}\right) \; , \quad \eta_V = \frac{1}{N}\,\frac{w''(\chi_{exit})}{w(\chi_{exit})} + \mathcal{O}\left(\frac{1}{N^2}\right)$$

$$U(\eta) = \frac{\nu^2 - \frac{1}{4}}{\eta^2} \; , \quad \nu = \frac{3}{2} + 3\,\epsilon_V - \eta_V$$

Fast–roll correction

$$U(\eta) = \frac{\nu^2 - \frac{1}{4}}{\eta^2} + \mathcal{V}(\eta)$$

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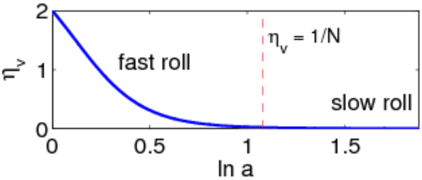

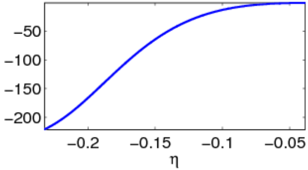

for Binomial New Inflation, $w(\chi) = \frac{y}{32}\left(\chi^2 - \frac{8}{y}\right)^2$ and $y = 1.32$

Fast–roll = special initial conditions for slow–roll

Solution when $\mathscr{V}(\eta) = 0$

$$S_k(\eta) = A(k) g_\nu(k;\eta) + B(k) g^*_\nu(k;\eta) \,,\; g_\nu(k;\eta) = \frac{1}{2} i^{\nu+\frac{1}{2}} \sqrt{-\pi\eta}\, H^{(1)}_\nu(-k\eta)$$

"Scattering" amplitudes (or Bogoliubov coefficients)

$$A(k) = 1 + i \int_{-\infty}^{0} d\eta\, g^*_\nu(k;\eta)\, \mathscr{V}(\eta)\, S_k(\eta)$$
$$B(k) = -i \int_{-\infty}^{0} d\eta\, g_\nu(k;\eta)\, \mathscr{V}(\eta)\, S_k(\eta)$$

Primordial power spectrum

$$P(k) = lim_{\eta\to 0^-} \frac{k^3}{2\pi^2} \left| \frac{S_k(\eta)}{\xi(\eta)} \right|^2 = P_{sr}(k) \Big[1 + D(k) \Big]$$

Fast-roll transfer function

$$D(k) = 2|B(k)|^2 - 2\mathscr{R}e \Big[A(k) B^*(k)\, i^{2\nu-3} \Big]$$

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Outline

1. The framework
   - Observational data and models
   - On cosmic variance
   - Monte Carlo Markov Chains
2. The theory
   - EFT of Inflation
   - Trinomial Chaotic and New Inflation
   - Early fast-roll and primordial fluctuations
   - The transfer function $D(k)$
3. MCMC results
   - Trinomial Chaotic and Trinomial New Inflation
   - Binomial New Inflation
   - Sharp-cut vs. fast-roll

The transfer function $D(k)$ for BNI

Born's approximation

$$D(k) = \frac{1}{k} \int_{-\infty}^{0} d\eta \, \mathscr{V}(\eta) \left[\sin(2k\eta)\left(1 - \frac{1}{k^2\eta^2}\right) + \frac{2}{k\eta} \cos(2k\eta)\right]$$

Scaling properties and the crossover wavenumber k_1

$$\mathscr{V}(\eta) = k_1^2 \, Q(k_1 \eta) \quad , \qquad D(k) = F\left(\frac{k}{k_1}\right)$$

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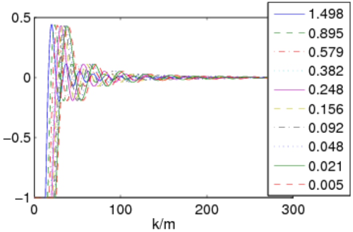
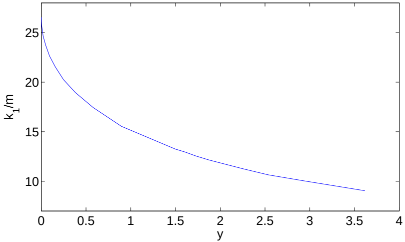
For Binomial New Inflation:





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Outline

Main results

- TCI fits data very well, but only in a small corner of parameter space (Ginsburg-Landau unsafe);
- TNI fits data well throughout parameter space (Ginsburg-Landau safe) and provides a lower bound for r.

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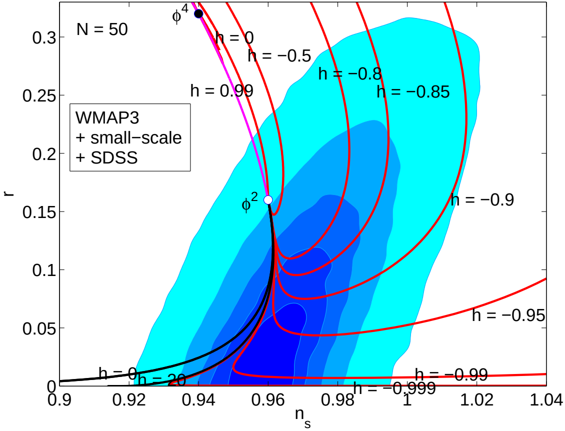
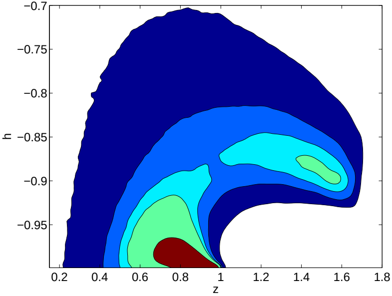
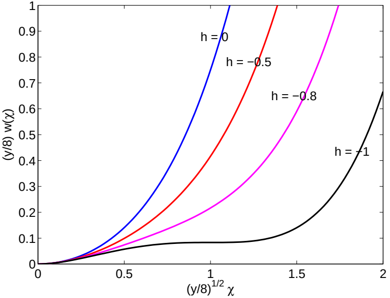


Trinomial Chaotic Inflation

The marginalized $z - h$ probability
CL: 12%, 27%, 45%, 68%, 95%



The inflaton potential



The ΛCDM_TNI(z_1,h) model

MCMC parameters:	ω_b, ω_c, θ, τ, (slow), A_s, z_1, h (fast)
Context:	$N = 60$, $\Omega_\nu = 0$, ... ; standard priors, no SZ, lensed CMB, linear mpk, ...
Datasets:	WMAP5, SDSS, SN

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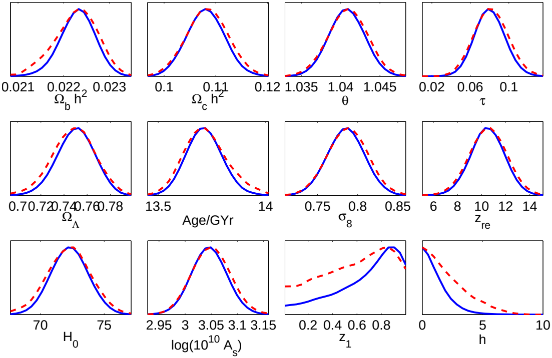
Datasets: WMAP5, SDSS, SN

param	best fit
$100\Omega_b h^2$	2.237
$\Omega_c h^2$	0.108
θ	1.041
100τ	7.885
H_0	72.24
σ_8	0.786
$\log[10^{10} A_s]$	3.045
z_1	0.867
h	0.666
$-\log(L)$	1496.01

flat $0 < z_1 < 1$ prior



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Context: $N = 60$, $\Omega_\nu = 0$, ... ; standard priors,
no SZ, lensed CMB, linear mpk, ...
Datasets: WMAP5, SDSS, SN

flat $0 < r < 2/15$ prior

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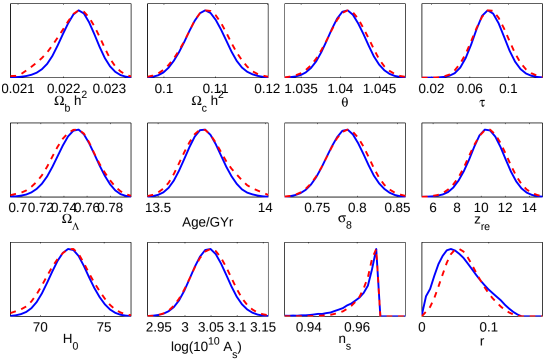
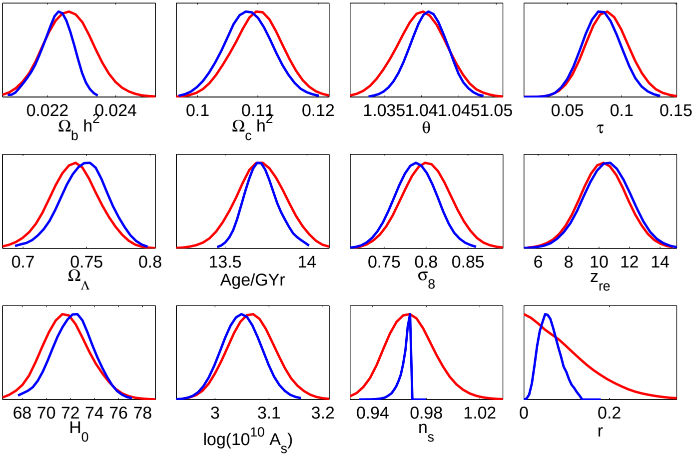
$\Omega_c h^2$	0.108
θ	1.041
100τ	7.885
H_0	72.24
σ_8	0.786
$\log[10^{10} A_s]$	3.045
n_s	0.963
r	0.043
$-\log(L)$	1496.01



Comparing marginalized mean likelihoods

ΛCDM_TNI(z_1,h) vs. ΛCDM$+r$



$\Delta\chi^2 = 0.08$, CL 95%: $r > 0.026$

Outline

BNI on the $n_s - r$ plane

$$n_s = 1 - \frac{y}{N}\,\frac{3z+1}{(1-z)^2}\,, \quad r = \frac{16\,y}{N}\,\frac{z}{(1-z)^2}\,, y = z - 1 - \log z\,, \quad 0 < z < 1$$

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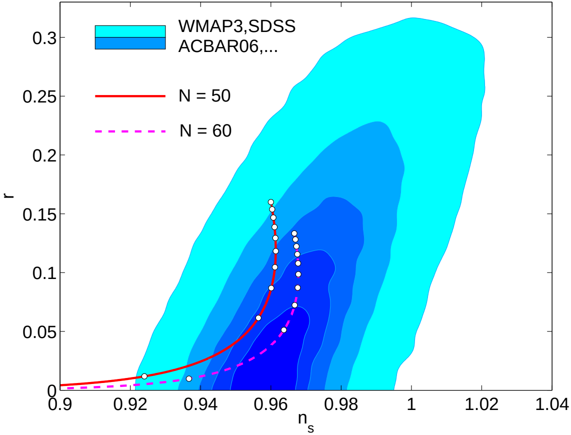

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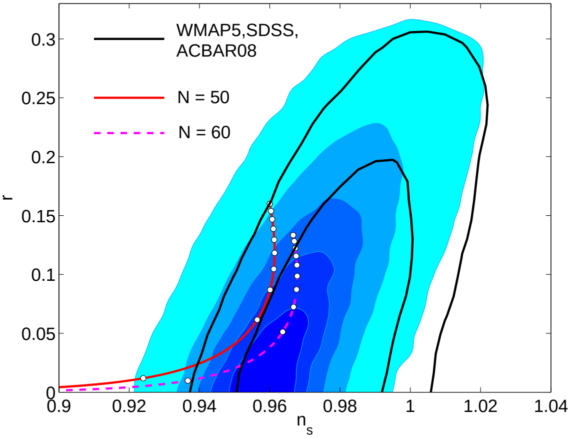

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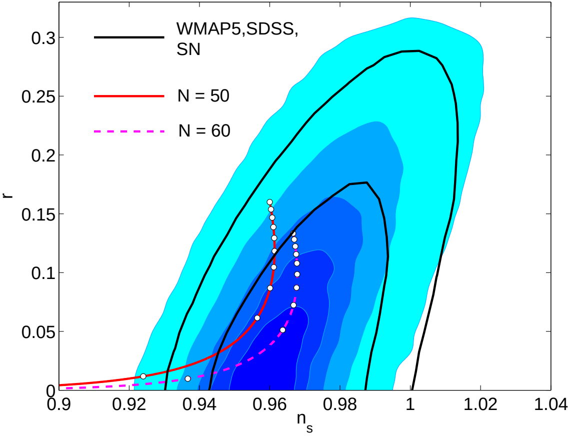

The ΛCDM_BNI(z) model

MCMC parameters:	ω_b, ω_c, θ, τ, (slow), A_s, z (fast)
Context:	$N = 60$, $\Omega_\nu = 0$, ... ; standard priors, no SZ, lensed CMB, linear mpk, ...
Datasets:	WMAP5, SDSS, ACBAR08

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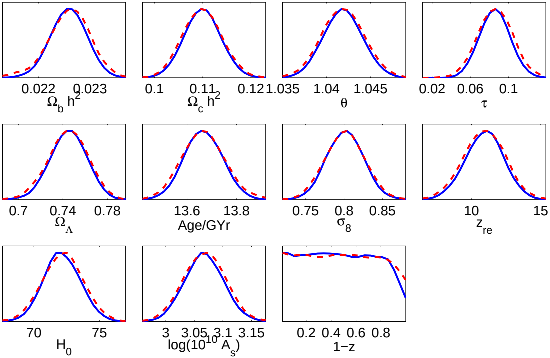
MCMC parameters: ω_b, ω_c, θ, τ, (slow), A_s, z (fast)
Context: $N = 60$, $\Omega_\nu = 0$, … ; standard priors,
no SZ, lensed CMB, linear mpk, …
Datasets: WMAP5, SDSS, ACBAR08

param	best fit
$100\Omega_b h^2$	2.226
$\Omega_c h^2$	0.111
θ	1.042
100τ	8.78
H_0	71.9
σ_8	0.807
$\log[10^{10} A_s]$	0.307
z	0.553
$-\log(L)$	1354.297

flat $0 < z < 1$ prior



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flat $0 < r < 2/15$ prior

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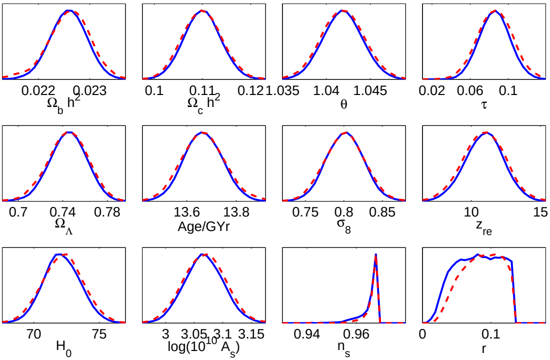
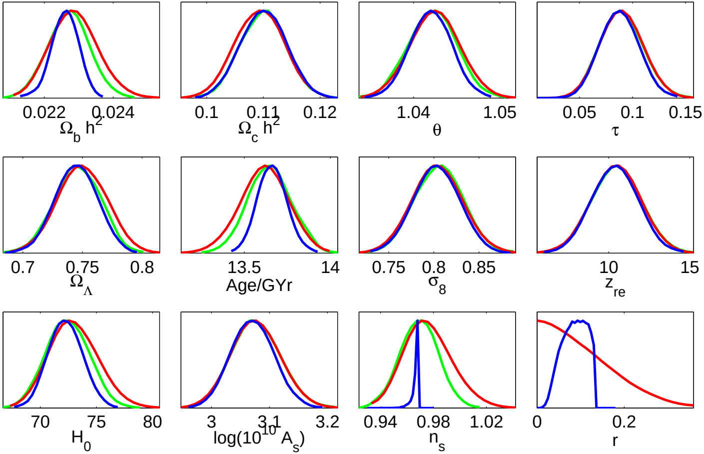
θ	1.042
100τ	8.78
H_0	71.9
σ_8	0.807
$\log[10^{10} A_s]$	0.307
n_s	0.968
r	0.107
$-\log(L)$	1354.297



Comparing marginalized mean likelihoods (WMAP5+SDSS+ACBAR08)

ΛCDM_BNI(z) vs. ΛCDM vs. ΛCDM$+r$



$\Delta\chi^2 = +0.01$, $\Delta\chi^2 = -0.03$, CL 95%: $r > 0.039$

The ΛCDM_BNI(z) model

MCMC parameters:	ω_b, ω_c, θ, τ, (slow), A_s, z (fast)
Context:	$N = 60$, $\Omega_\nu = 0$, ... ; standard priors, no SZ, lensed CMB, linear mpk, ...
Datasets:	WMAP5, SDSS, SN

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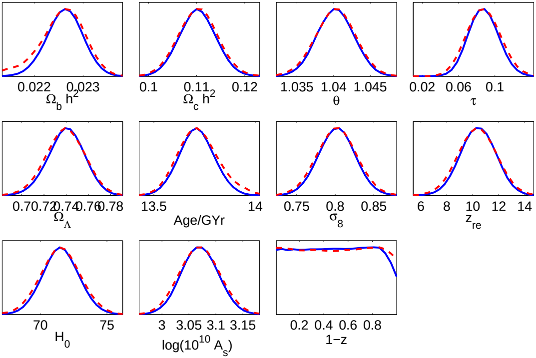
MCMC parameters: ω_b, ω_c, θ, τ, (slow), A_s, z (fast)
Context: $N = 60$, $\Omega_\nu = 0$, ... ; standard priors, no SZ, lensed CMB, linear mpk, ...
Datasets: WMAP5, SDSS, SN

flat $0 < z < 1$ prior

param	best fit
$100\Omega_b h^2$	2.226
$\Omega_c h^2$	0.111
θ	1.040
100τ	8.67
H_0	71.3
σ_8	0.805
$\log[10^{10} A_s]$	0.307
z	0.148
$-\log(L)$	1495.83



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Datasets: WMAP5, SDSS, SN

flat $0 < r < 2/15$ prior

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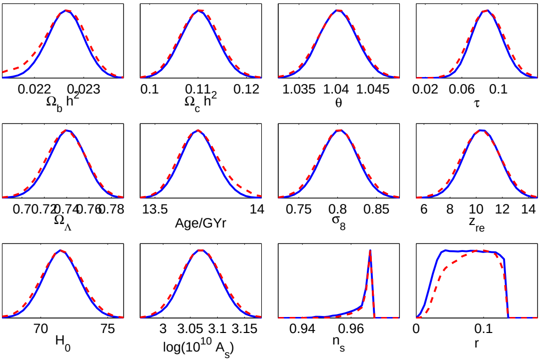
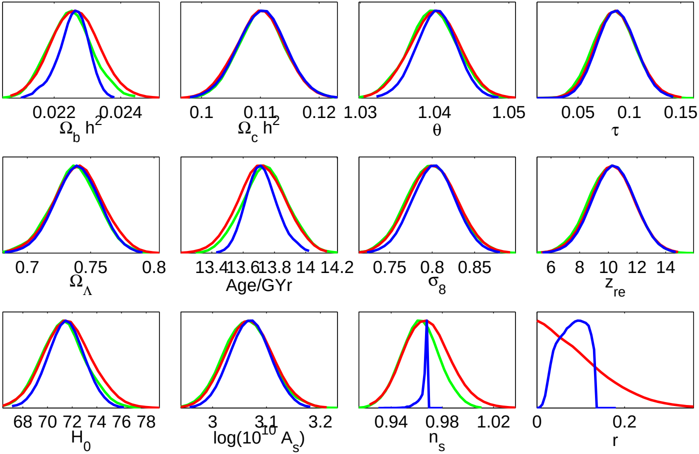
$100\Omega_b h^2$	2.226
$\Omega_c h^2$	0.111
θ	1.040
100τ	8.67
H_0	71.3
σ_8	0.805
$\log[10^{10} A_s]$	0.307
n_s	0.965
r	0.057
$-\log(L)$	1495.83



Comparing marginalized mean likelihoods (WMAP5+SDSS+SN)

ΛCDM_BNI(z) vs. ΛCDM vs. ΛCDM$+r$



$\Delta\chi^2 = -0.07$, $\Delta\chi^2 = -0.15$, CL 95%: $r > 0.032$

Outline

BNI+sharpcut vs. BNI+fastroll

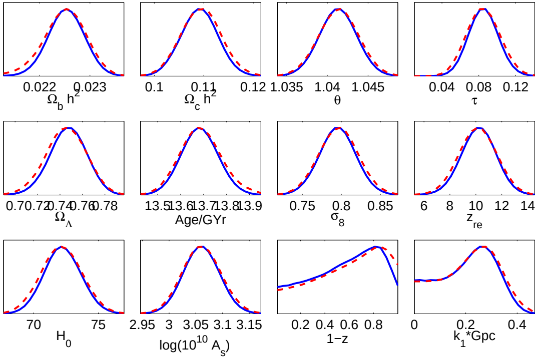
MCMC parameters:	ω_b, ω_c, θ, τ, (slow), A_s, z, k_1 (fast)
Context:	$N = 60$, $\Omega_\nu = 0$, ... ; standard priors, no SZ, lensed CMB, linear mpk, ...
Datasets:	WMAP5, SDSS, ACBAR08

BNI+sharpcut vs. BNI+fastroll

MCMC parameters:	ω_b, ω_c, θ, τ, (slow), A_s, z, k_1 (fast)
Context:	$N = 60$, $\Omega_\nu = 0$, ... ; standard priors, no SZ, lensed CMB, linear mpk, ...
Datasets:	WMAP5, SDSS, ACBAR08

param	best fit
$100\Omega_b h^2$	2.256
$\Omega_c h^2$	0.110
θ	1.041
100τ	8.83
H_0	71.82
σ_8	0.803
$\log[10^{10} A_s]$	0.307
z	0.162
k_1	0.260
$-\log(L)$	1253.96

sharp-cut flat $0 < z < 1$ prior



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$\Omega_c h^2$	0.109
θ	1.041
100τ	8.42
H_0	72.00
σ_8	0.794
$\log[10^{10} A_s]$	0.306
z	0.102
k_1	0.284
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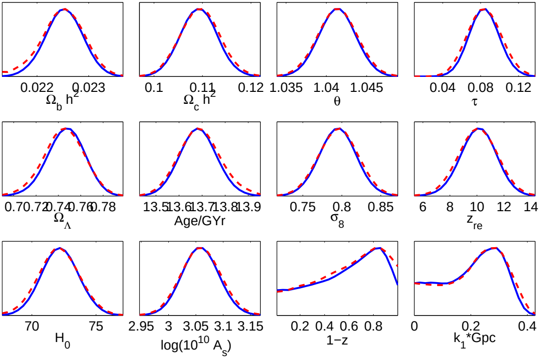

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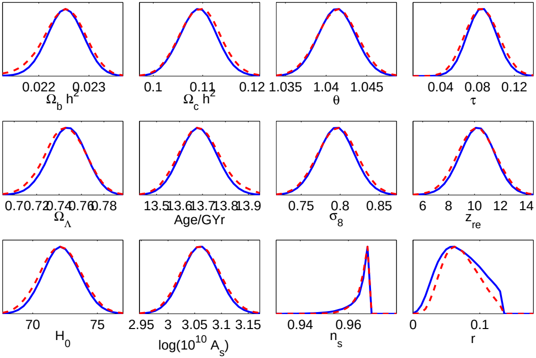

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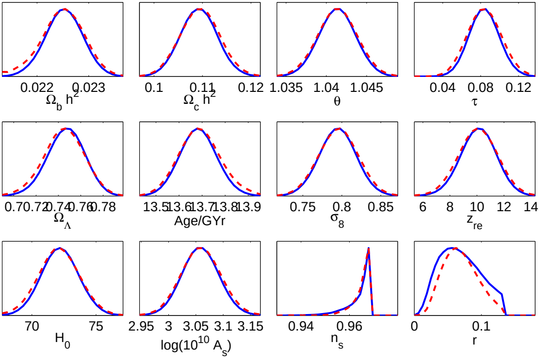


$\Delta\chi^2$ w.r.t. ΛCDM$+r$

	WMAP5	+SDSS+ACBAR08	+SDSS+SN
BNI+sharpcut	-1.07	-0.71	-1.02
BNI+fastroll	-1.15	-0.99	-1.45

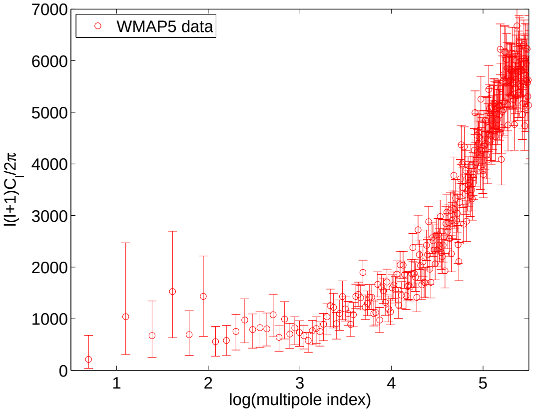
95% lower bound on r

	WMAP5	+SDSS+ACBAR08	+SDSS+SN
BNI+sharpcut	0.028	0.033	0.028
BNI+fastroll	0.028	0.032	0.027

most likely value of k_1 (in Gpc^{-1})

	WMAP5	+SDSS+ACBAR08	+SDSS+SN
BNI+sharpcut	0.258	0.260	0.244
BNI+fastroll	0.298	0.284	0.291

Comparing TT multipoles



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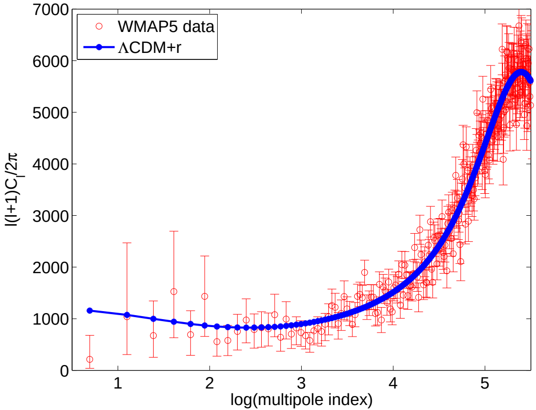

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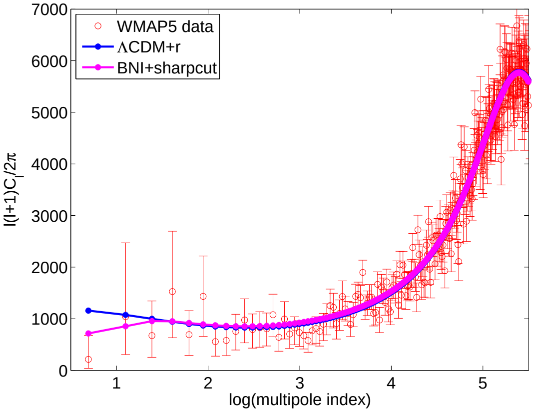

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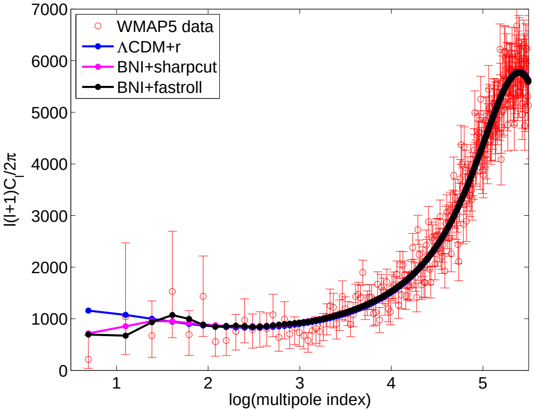

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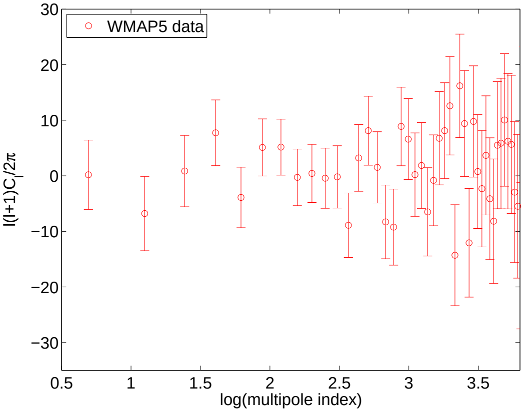

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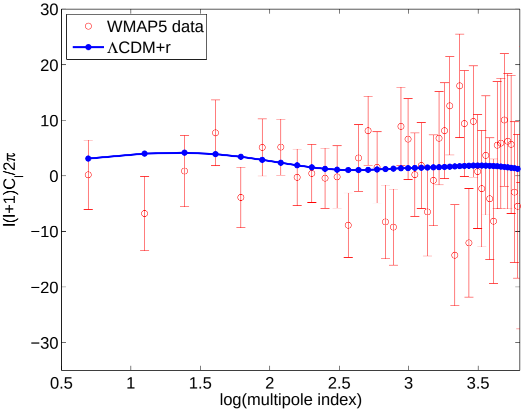

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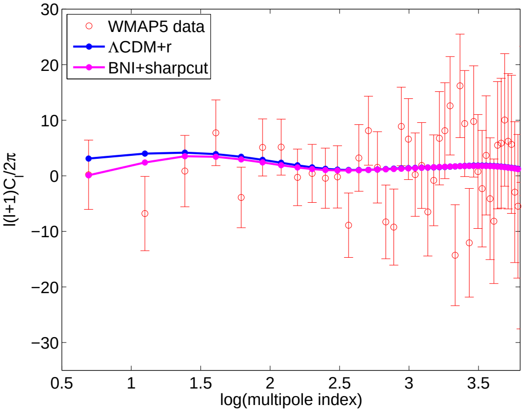

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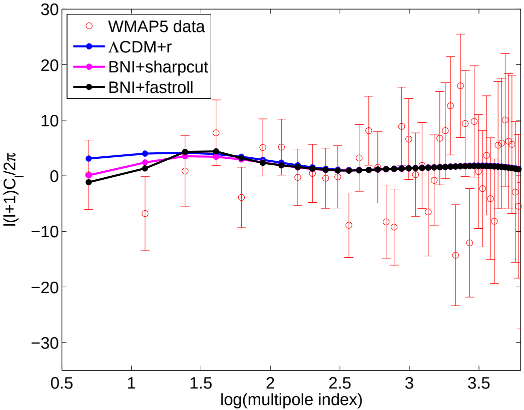
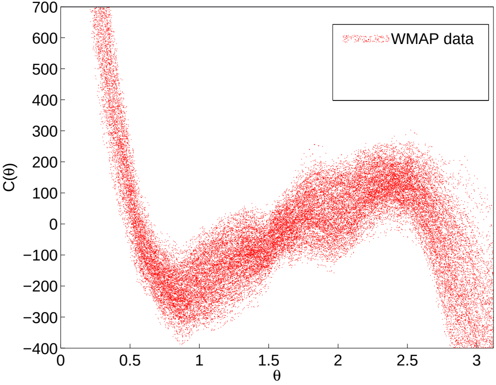


Comparing real-space TT correlations



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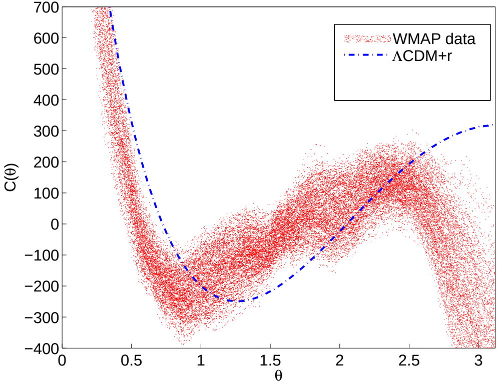

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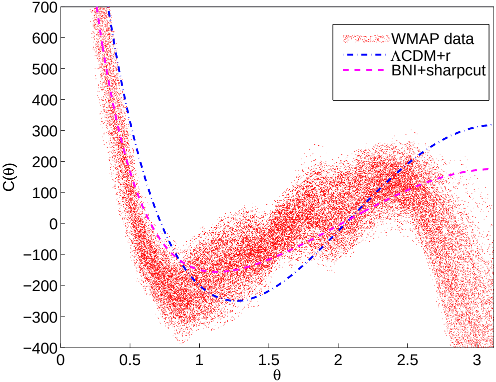

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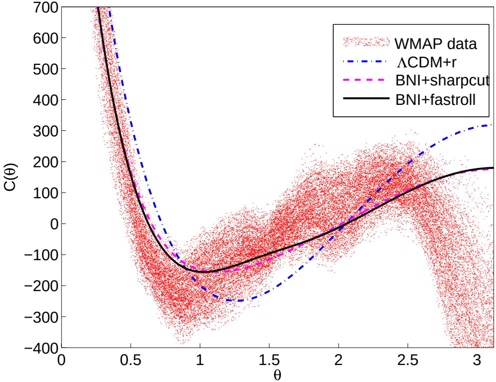


Highlights of BNI+fastroll

k_1 corresponds to the crossover comoving wavenumber

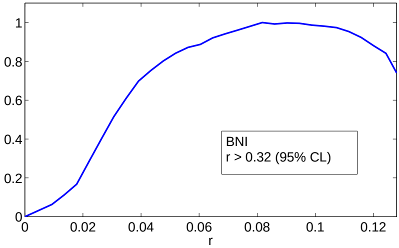
it exits the horizon when fast-roll ends and slow-roll starts

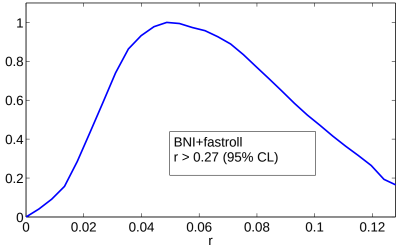
the quadrupole wavenumber

$k_q \simeq 0.83\,k_1$ and exits roughly 1/10 of an efold before k_1

Highlights of BNI+fastroll

The probability distribution for r (WMAP5+SDSS+SN):





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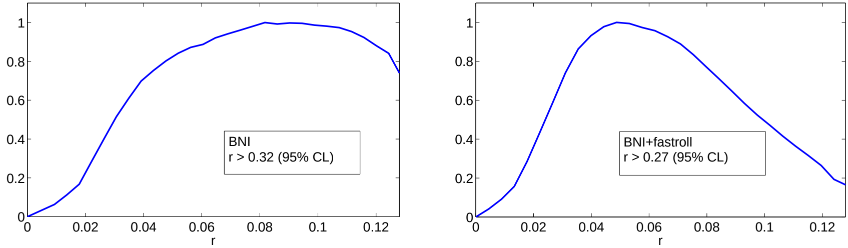

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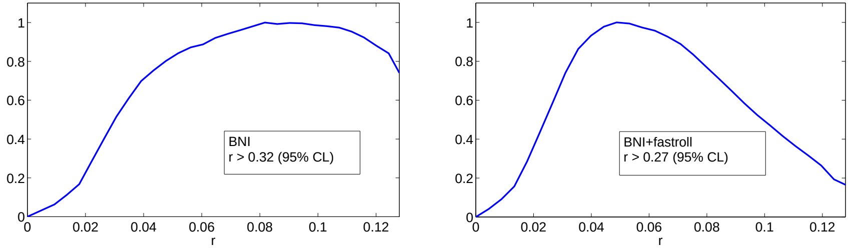

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Checking the total number of inflation efolds
(using best fit $y = 1.26$ with WMAP5 and $N = 60$ (it was $y = 1.32$ with WMAP3 and $N = 50\ldots$))

The quadrupole wavenumber at the beginning of inflation

$$k_q^{(then)} \simeq 0.83\, k_1^{(then)} \simeq 11\, m \simeq 1.4 \times 10^{14}\, GeV$$

The total expansion since k_q exits is OK
NB: H is still rapidly decreasing when fast-roll $\to$ slow-roll

$$k_q^{(then)} / k_q^{(now)} \simeq 0.9 \times 10^{56} \Longrightarrow N \simeq 57$$

compare

Hence, assuming sharp ΛD$\to$ RD

Repeat MCMC with $N = 57$ or use $N = \log \frac{H\sqrt{\Omega_r}}{H_0} - \log \frac{k_0}{k_q}$

or really understand reheating or be satisfied with $57 < N < 76$

Another interesting refinement

Study fast-roll $\to$ slow-roll more accurately and use $k_1 = k_1(y)$ in a 6$-$parameter model with builtin quadrupole depression

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Summary

- ΛCDM_BNI(z) is the true inflationary ΛCDM model.
- It provides lower bounds for r.
- Early fast-roll inflation is generic and provides a quadrupole. depression
- BNI+fastroll significantly improves the fit w.r.t. ΛCDM$+r$.
- BNI+fastroll improves the fits also w.r.t. BNI+sharpcut.
- Fast-roll puts the lower bounds $N > 57$ on the number of efolds.
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- Outlook
  - Improve the theory (expecially concerning reheating)
  - Wait for better data (Plank and new gound/balloon experiments)
  - Refine, refine, refine

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