The Effective Theory of Inflation, Dark Matter and Dark Energy in the Standard Model of the Universe

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Standard Model of the Universe: ΛCDM

ΛCDM = Cold Dark Matter + Cosmological Constant
Explains the Observations:

- 5 years WMAP data and previous CMB data
- Light Elements Abundances
- Large Scale Structures (LSS) Observations
- Supernova Luminosity/Distance Relations (Acceleration of the Universe expansion)
- Gravitational Lensing Observations
- Lyman α Forest Observations
- Hubble Constant (H_0) Measurements
- Properties of Clusters of Galaxies
- ....

Standard Model of the Universe: Concordance Model

$ds^2 = dt^2 - a^2(t)\; d\vec{x}^{\,2}$: spatially flat geometry.

The Universe starts by an **INFLATIONARY ERA**.

Inflation = Accelerated Expansion: $\frac{d^2a}{dt^2} > 0$.

During inflation the universe expands by at least sixty or so efolds: $e^{66} \simeq 10^{29}$. Inflation lasts $\simeq 10^{-34}$ sec and ends by $z \sim 10^{27}$ followed by a radiation dominated era.

Energy scale when inflation starts $\sim 10^{16}$ GeV ($\Longleftarrow$ CMB anisotropies) which **coincides** with the GUT scale.

Matter can be effectively described during inflation by an Scalar Field $\phi(t, \boldsymbol{x})$: the Inflaton.

Lagrangean: $\mathcal{L} = a^3(t) \left[\frac{\dot{\phi}^2}{2} - \frac{(\nabla\phi)^2}{2\;a^2(t)} - V(\phi)\right]$.

Friedmann eq.: $H^2(t) = \frac{1}{3\;M_{Pl}^2}\left[\frac{\dot{\phi}^2}{2} + V(\phi)\right],\; H(t) \equiv \dot{a}(t)/a(t)$.

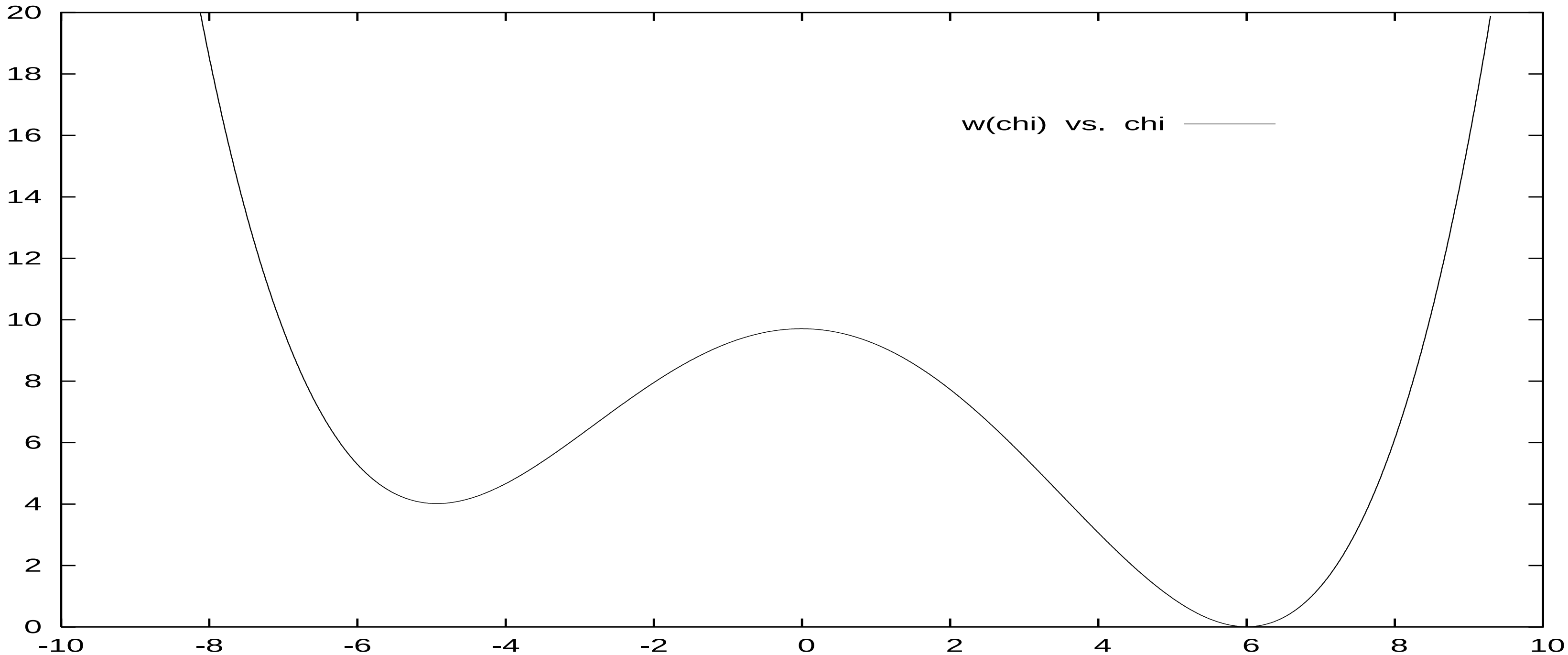
The Theory of Inflation

The inflaton is an effective field in the Ginsburg-Landau sense.

Relevant effective theories in physics:

- Ginsburg-Landau theory of superconductivity. It is an effective theory for Cooper pairs in the microscopic BCS theory of superconductivity.
- The $O(4)$ sigma model for pions, the sigma and photons at energies $\lesssim 1$ GeV. The microscopic theory is QCD: quarks and gluons. $\pi \simeq \bar{q}q \; , \; \sigma \simeq \bar{q}q$.
- The theory of second order phase transitions à la Landau-Kadanoff-Wilson... (ferromagnetic, antiferromagnetic, liquid-gas, Helium 3 and 4, ...)

Slow Roll Inflaton Models



$V(\text{Min}) = V'(\text{Min}) = 0$: inflation ends after a finite number of efolds. Universal form of the slow-roll inflaton potential:

$$V(\phi) = N\ M^4\ w\left(\frac{\phi}{\sqrt{N}\ M_{Pl}}\right)$$

$N \sim 60$: number of efolds since horizon exit till end of inflation. $M =$ energy scale of inflation.

SLOW and Dimensionless Variables

$$\chi = \frac{\phi}{\sqrt{N}\; M_{Pl}} \quad , \quad \tau = \frac{m\,t}{\sqrt{N}} \quad , \quad \mathcal{H}(\tau) = \frac{H(t)}{m\,\sqrt{N}} \quad , \quad \left(m \equiv \frac{M^2}{M_{Pl}}\right)$$

slow inflaton, slow time, slow Hubble.

χ and $w(\chi)$ are of order one.

Evolution Equations:

$$\mathcal{H}^2(\tau) = \frac{1}{3}\left[\frac{1}{2\,N}\left(\frac{d\chi}{d\tau}\right)^2 + w(\chi)\right] \quad ,$$

$$\frac{1}{N}\,\frac{d^2\chi}{d\tau^2} + 3\,\mathcal{H}\,\frac{d\chi}{d\tau} + w'(\chi) = 0 \quad . \qquad (1)$$

$1/N$ terms: corrections to slow-roll

Higher orders in slow-roll are obtained systematically by expanding the solutions in $1/N$.

Primordial Power Spectrum

Adiabatic Scalar Perturbations: $P(k) = |\Delta_{k\ ad}^{(S)}|^2\ \ k^{n_s - 1}$.

To dominant order in slow-roll:

$$|\Delta_{k\ ad}^{(S)}|^2 = \frac{N^2}{12\,\pi^2}\ \left(\frac{M}{M_{Pl}}\right)^4\ \frac{w^3(\chi)}{w'^2(\chi)}\ .$$

Hence, for all slow-roll inflation models:

$$|\Delta_{k\ ad}^{(S)}| \sim \frac{N}{2\,\pi\sqrt{3}}\left(\frac{M}{M_{Pl}}\right)^2$$

The WMAP5 result: $|\Delta_{k\ ad}^{(S)}| = (0.470 \pm 0.09) \times 10^{-4}$

determines the scale of inflation M (using $N \simeq 60$)

$$\left(\frac{M}{M_{Pl}}\right)^2 = 0.85 \times 10^{-5}\ \longrightarrow\ M = 0.70 \times 10^{16}\ \text{GeV}$$

The inflation energy scale turns to be the grand unification energy scale !!

We find the scale of inflation without knowing r !!

The scale M is independent of the shape of $w(\chi)$.

spectral index n_s , its running and the ratio r

$$n_s - 1 = -\frac{3}{N} \left[\frac{w'(\chi)}{w(\chi)}\right]^2 + \frac{2}{N} \frac{w''(\chi)}{w(\chi)} \quad , \quad r = \frac{8}{N} \left[\frac{w'(\chi)}{w(\chi)}\right]^2$$

$$\frac{dn_s}{d\ln k} = -\frac{2}{N^2} \frac{w'(\chi)\, w'''(\chi)}{w^2(\chi)} - \frac{6}{N^2} \frac{[w'(\chi)]^4}{w^4(\chi)} + \frac{8}{N^2} \frac{[w'(\chi)]^2\, w''(\chi)}{w^3(\chi)} \, ,$$

χ is the inflaton field at horizon exit.
$n_s - 1$ and r are always of order $1/N \sim 0.02$ (model indep.)
Running of n_s of order $1/N^2 \sim 0.0003$ (model independent).

D. Boyanovsky, H. J. de Vega, N. G. Sanchez,
Phys. Rev. D 73, 023008 (2006), astro-ph/0507595.

Ginsburg-Landau Approach

We choose a polynomial for $w(\chi)$. A quartic $w(\chi)$ is renormalizable. Higher order polynomials are acceptable since inflation is an effective theory.

$$w(\chi) = w_o \pm \frac{\chi^2}{2} + G_3\,\chi^3 + G_4\,\chi^4 \quad , \quad G_3 = \mathcal{O}(1) = G_4$$

$$V(\phi) = N\;M^4\;w\left(\frac{\phi}{\sqrt{N}\;M_{Pl}}\right) = V_o \pm \frac{m^2}{2}\,\phi^2 + g\,\phi^3 + \lambda\,\phi^4\;.$$

$$m = \frac{M^2}{M_{Pl}} \quad , \quad g = \frac{m}{\sqrt{N}}\left(\frac{M}{M_{Pl}}\right)^2 G_3 \quad , \quad \lambda = \frac{G_4}{N}\;\left(\frac{M}{M_{Pl}}\right)^4$$

Notice that

$$\left(\frac{M}{M_{Pl}}\right)^2 \simeq 10^{-5} \quad , \quad \left(\frac{M}{M_{Pl}}\right)^4 \simeq 10^{-10} \quad , \quad N \simeq 60\;.$$

- Small couplings arise naturally as ratio of two energy scales: inflation and Planck.
- The inflaton is a light particle:
  $m = M^2/M_{Pl} \simeq 0.003\;M \quad , \quad m = 2 \times 10^{13}\;\mathrm{GeV}$

The number of efolds in Slow-roll

The number of e-folds $N[\chi]$ since the field χ exits the horizon till the end of inflation is:

$N[\chi] = N \int_{\chi_{end}}^{\chi} \frac{w(\chi)}{w'(\chi)} \, d\chi$. We choose then $N = N[\chi]$.

The spontaneously broken symmetric potential:

$$w(\chi) = \frac{y}{32} \left(\chi^2 - \frac{8}{y}\right)^2$$

produces inflation with $0 < \sqrt{y} \, \chi_{initial} \ll 1$ and $\chi_{end} = \sqrt{\frac{8}{y}}$.

This is small field inflation.

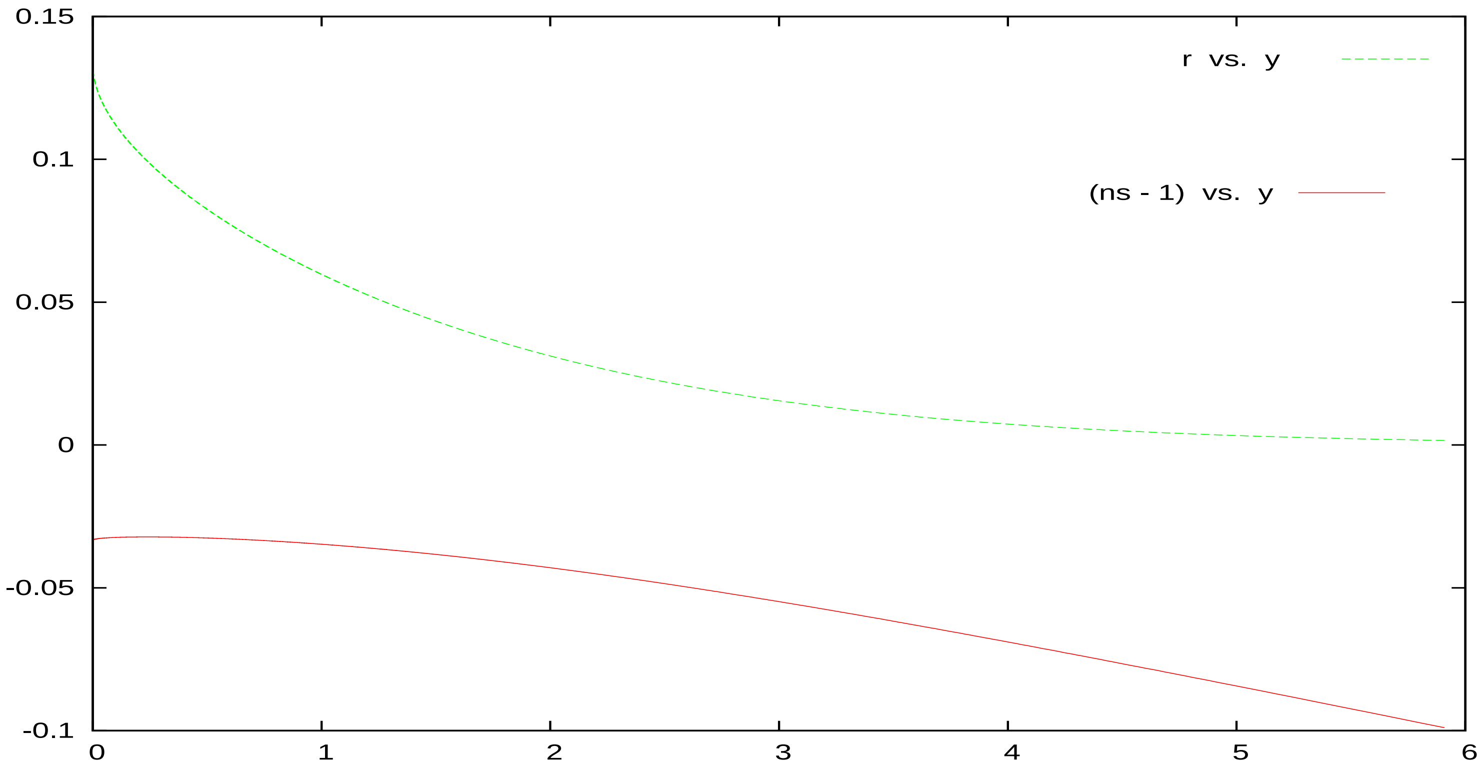
From the above integral: $y = z - 1 - \log z$ where $z \equiv y \, \chi^2/8$
This defines $\chi = \chi(y)$. $[1 > z > 0$ for $0 < y < \infty]$.
Spectral index n_s and the ratio r as functions of y:

$n_s = 1 - \frac{y}{N} \frac{3\,z+1}{(z-1)^2} \quad , \quad r = \frac{16\,y}{N} \frac{z}{(z-1)^2}$

Binomial New Inflation: (y = coupling).

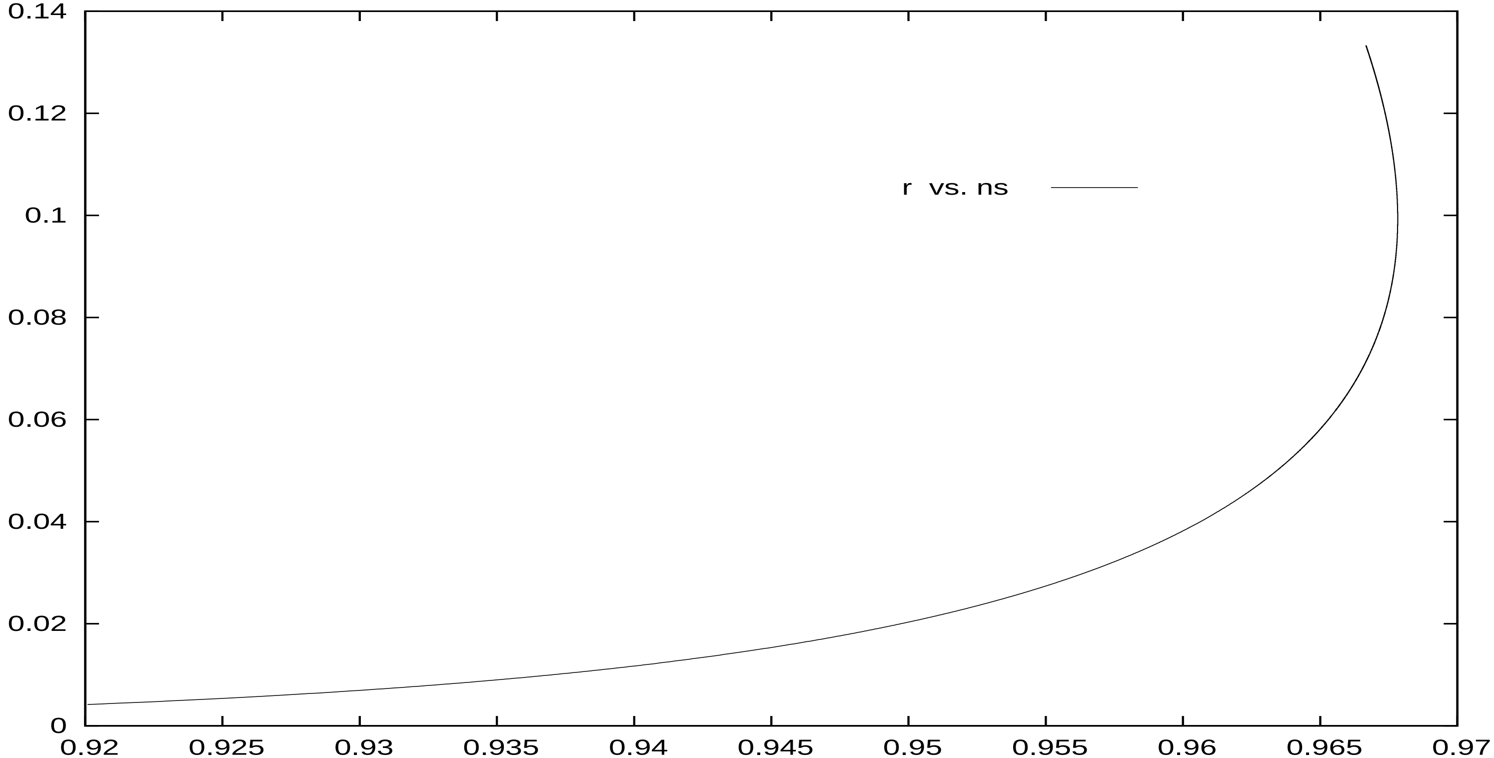
r decreases monotonically with y :

(strong coupling) $0 < r < \frac{8}{N} = 0.133...$ (zero coupling).



n_s first grows with y, reaches a maximum value $n_{s,maximum} = 0.96782\ldots$ at $y = 0.2387\ldots$ and then n_s decreases monotonically with y.

Binomial New Inflation



$r = \frac{8}{N} = 0.133...$ and $n_s = 1 - \frac{2}{N} = 0.967...$ at $y = 0$, $N = 60$.

r is a double valued function of n_s.

Trinomial Inflationary Models

- Trinomial Chaotic inflation:
  $w(\chi) = \frac{1}{2}\,\chi^2 + \frac{h}{3}\,\sqrt{\frac{y}{2}}\,\chi^3 + \frac{y}{32}\,\chi^4 \; .$
- Trinomial New inflation:
  $w(\chi) = -\frac{1}{2}\,\chi^2 + \frac{h}{3}\,\sqrt{\frac{y}{2}}\,\chi^3 + \frac{y}{32}\,\chi^4 + \frac{2}{y}\;F(h) \; .$

h = asymmetry parameter. $w(\min) = w'(\min) = 0$,
$y =$ quartic coupling, $F(h) = \frac{8}{3}\,h^4 + 4\,h^2 + 1 + \frac{8}{3}\,|h|\;(h^2+1)^{\frac{3}{2}} \; .$

H. J. de Vega, N. G. Sanchez, Single Field Inflation models allowed and ruled out by the three years WMAP data. Phys. Rev. D 74, 063519 (2006), astro-ph/0604136.

Monte Carlo Markov Chains Analysis of Data: MCMC.

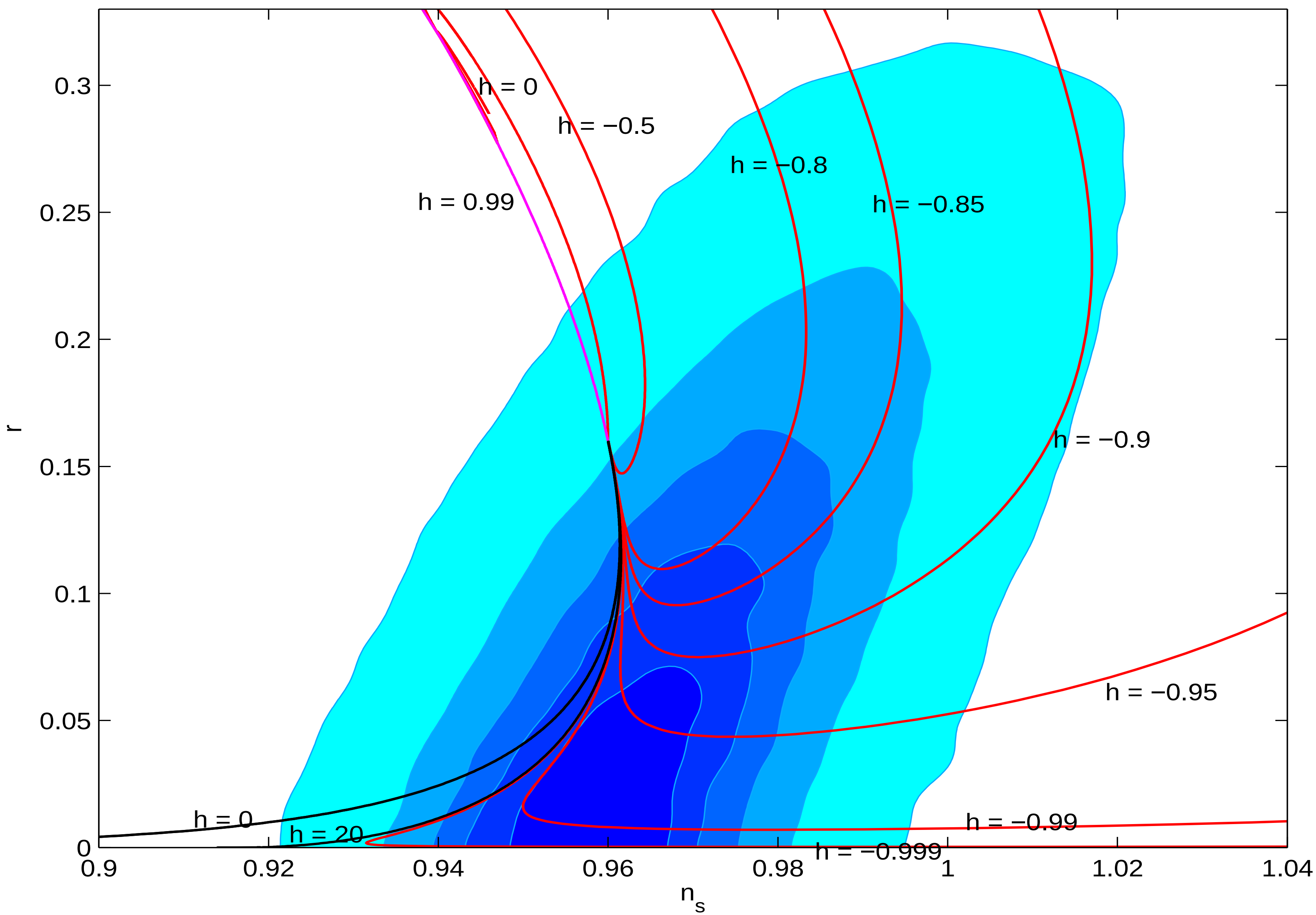
MCMC is an efficient stochastic numerical method to find the probability distribution of the theoretical parameters that describe a set of empirical data.

We found n_s and r and the couplings y and h by MCMC.
NEW: We imposed as a hard constraint that r and n_s are given by the trinomial potential.
Our analysis differs in this crucial aspect from previous MCMC studies of the WMAP data.

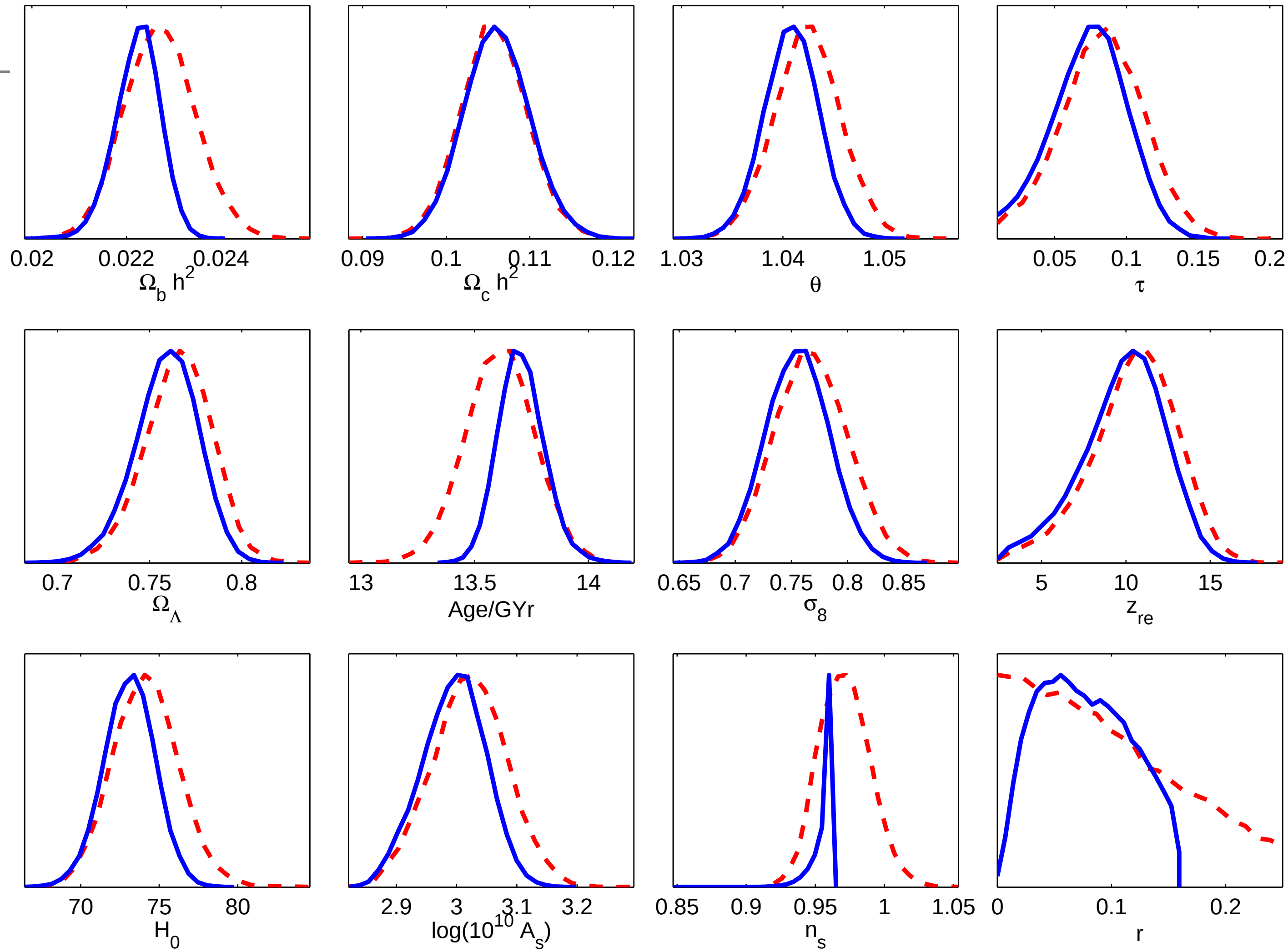
We ignore running of the spectral index since $dn_s/d\ln k \sim 0.0003$ in slow roll.

Adding the running made insignificant changes on the fit of n_s and r.

MCMC Results for Trinomial New Inflation.



Marginalized probability distributions. New Inflation.



Imposing the trinomial potential (solid blue curves) and just the ΛCDM+r model (dashed red curves).
(curves normalized to have the maxima equal to one).

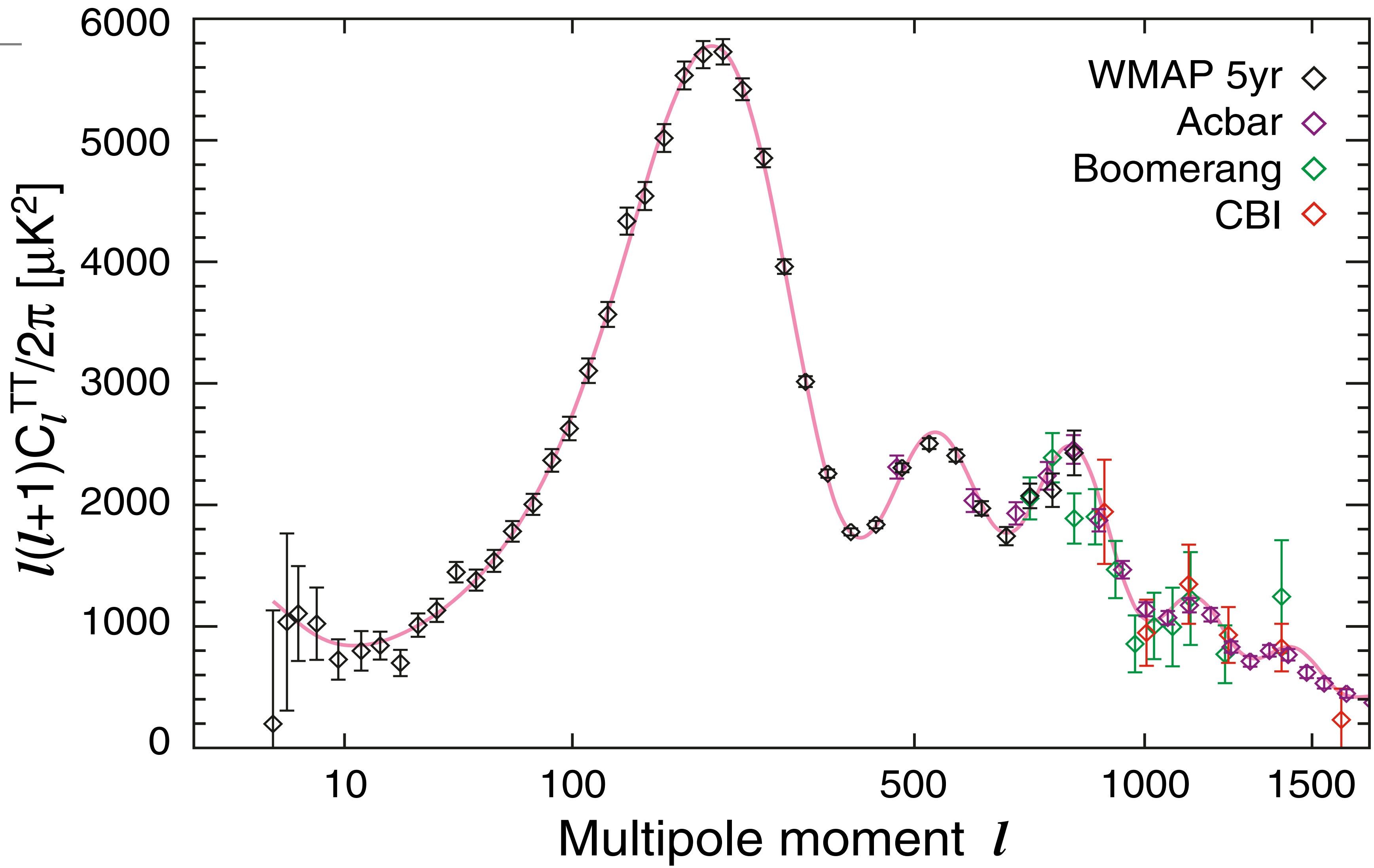
MCMC Results for Trinomial New Inflation.

Bounds: $r > 0.016$ (95% CL) , $r > 0.049$ (68% CL)
Most probable values: $n_s \simeq 0.956, \; r \simeq 0.055 \Leftarrow$measurable!!
The most probable trinomial potential for new inflation has a moderate nonlinearity with the quartic coupling $y \simeq 1.5 \ldots$ and $h < 0.3$.
We can choose $h = 0$ and we then we find $y \simeq 1.322 \ldots$.
The $\chi \to -\chi$ symmetry is here spontaneously broken since the absolute minimum of the potential is at $\chi \neq 0$.

$$w(\chi) = \frac{y}{32} \left(\chi^2 - \frac{8}{y}\right)^2$$

C. Destri, H. J. de Vega, N. Sanchez, MCMC analysis of WMAP3 data points to broken symmetry inflaton potentials and provides a lower bound on the tensor to scalar ratio, Phys. Rev. D77, 043509 (2008), astro-ph/0703417.
Similar results from WMAP5 data. Acbar08 data slightly increases $n_s < 1$ and r.

WMAP 5 years data plus further data



Theory (ΛCDM) and observations nicely agree except for the lowest multipoles: the quadrupole suppression.

Quadrupole Suppression and Fast-Roll

Slow-roll inflation is generically preceded by a fast-roll stage where $\dot{\phi}^2 \sim V(\phi)$. Fast-Roll typically lasts 1 efold.

The slow-roll regime is an attractor with a large basin of attraction.

During fast-roll curvature and tensor perturbations feel a potential equal to the slow-roll potential plus an extra attractive piece. This new piece suppresses the low multipoles as $1/\ell^2$.

If the quadrupole modes ($\sim$ Hubble radius today) exited the horizon about the end of fast-roll, then the quadrupole modes get suppresed $\sim 20\%$ in agreement with the observations. Upper bound on the total number of inflation efolds: $N_{total} < 82$. Favoured value: $N_{total} \simeq 66$.

C. Destri, H. J. de Vega, N. Sanchez, arXiv:0804.2387, Phys.Rev. D78, 023013(2008), D. Boyanovsky, H. J. de Vega, N. Sanchez, Phys.Rev.D74,123006 and 007(2006).

The Energy Scale of Inflation

Grand Unification Idea (GUT)

- Renormalization group running of electromagnetic, weak and strong couplings shows that they all meet at $E_{GUT} \simeq 2 \times 10^{16}$ GeV.
- Neutrino masses are explained by the see-saw mechanism: $m_\nu \sim \frac{M^2_{\rm Fermi}}{M_R}$ with $M_R \sim 10^{16}$ GeV.
- Inflation energy scale: $M \simeq 10^{16}$ GeV.

Conclusion: the GUT energy scale appears in at least three independent ways.

Moreover, moduli potentials: $V_{moduli} = M^4_{\rm SUSY} \; v\left(\frac{\phi}{M_{Pl}}\right)$

ressemble inflation potentials provided $M_{\rm SUSY} \sim 10^{16}$ GeV.
First observation of SUSY in nature ??

De Sitter Geometry and Scale Invariance

The De Sitter metric is scale invariant:

$$ds^2 = \frac{1}{(H\,\eta)^2}\left[(d\eta)^2 - (d\vec{x})^2\right] \ .$$

$\eta =$ conformal time.
But inflation only lasts for N efolds !
Corrections to scale invariance:
$|n_s - 1|$ as well as the ratio r are of order $\sim 1/N$,
$n_s = 1$ and $r = 0$ correspond to a critical point.
It is a gaussian fixed point around which the inflation model hovers in the renormalization group (RG) sense with an almost scale invariant spectrum during the slow roll stage.
The quartic coupling:

$$\lambda = \frac{G_4}{N}\left(\frac{M}{M_{Pl}}\right)^4 \quad , \quad N = \log\frac{a(\text{inflation end})}{a(\text{horizon exit})}$$

runs like in four dimensional RG in flat euclidean space.

Dark Matter

DM must be non-relativistic by structure formation ($z < 30$) in order to reproduce the observed small structure at $\sim 2-3$ kpc. DM particles can decouple being ultrarelativistic (UR) at $T_d \gg m$.

Consider particles that decouple at or out of LTE (LTE = local thermal equilibrium).

Distribution function: $f_d[a(t)\ P_f(t)] = f_d[p_c]$.

$P_f(t) = p_c/a(t) =$ Physical momentum.

$p_c =$ comoving momentum.

DM decoupling at LTE: $f_d(p_c) = 1/[\exp[\sqrt{m^2+p_c^2}/T_d] \pm 1]$

In general (out of equilibrium): $f_d(p_c) = f_d\left(\frac{p_c}{T_d}; \frac{m}{T_d}; \ldots\right)$

Velocity fluctuations: $y = P_f(t)/T_d(t) = p_c/T_d$

$$\langle \vec{V}^2(t) \rangle = \langle \frac{\vec{P}_f^2(t)}{m^2} \rangle = \frac{\int \frac{d^3 P_f}{(2\pi)^3} \frac{\vec{P}_f^2}{m^2}\ f_d[a(t)\ P_f]}{\int \frac{d^3 P_f}{(2\pi)^3}\ f_d[a(t)\ P_f]} = \left[\frac{T_d(t)}{m}\right]^2 \frac{\int_0^\infty y^4 f_d(y) dy}{\int_0^\infty y^2 f_d(y) dy} .$$

Velocity Dispersion of Dark Matter particles

Using entropy conservation: $T_d(t) = \left(\frac{2}{g_d}\right)^{\frac{1}{3}} \; T_\gamma \, [1 + z(t)]$

$g_d =$ effective $\#$ of UR degrees of freedom at decoupling,

$$\sqrt{\langle \vec{V}^2 \rangle}(z) = 0.08875 \; \frac{1+z}{g_d^{\frac{1}{3}}} \; \frac{\text{keV}}{m} \; \left[\frac{\int_0^\infty y^4 \; f_d(y) \; dy}{\int_0^\infty y^2 \; f_d(y) \; dy}\right]^{\frac{1}{2}} \; \frac{\text{km}}{\text{s}}$$

Energy Density: $\rho_{DM}(t) = g \int \frac{d^3 P_f}{(2\pi)^3} \sqrt{m^2 + P_f^2} \; f_d[a(t) \, P_f]$

$\rho_{DM}(t) = m \; g \; T_d^3(t) \; \int_0^\infty y^2 \; f_d(y) \; \frac{dy}{2\pi^2}$ for $m \gg T_d(t)$.

Today $\Omega_{DM} = \rho_{DM}(0)/\rho_c = 0.105/h^2$ and therefore:

$$m = 6.46 \text{ eV} \; g_d/[g \; \int_0^\infty y^2 \; f_d(y) \; dy]$$

For Fermions decoupling at LTE:

$f_d(y) = 1/[e^y + 1]$ and $m = 3.593 \text{ eV} \; g_d/g$.

The formula for m

m increases:

a) if the DM particle decouples earlier because g_d increases.
b) if it decouples out of LTE, $f_d(y)$ can favour small momenta and increase $1/[\int_0^\infty y^2 \, f_d(y) \, dy]$.

Special Cases of the formula for m :

Particles decoupling non-relativistically $\Longrightarrow$ Lee-Weinberg (1977) lower bound.

Particles decoupling ultrarelativistically $\Longrightarrow$ Cowsik-McClelland (1972) upper bound.

Out of equilibrium Decoupling

Thermalization mechanism: k-modes cascade towards the UV till the thermal distribution is attained.
[D. Boyanovsky, C. Destri, H. J. de Vega, PRD69, 045003 (2004). C. Destri, H. J. de Vega, PRD73, 025014 (2006)]

Hence, before LTE is reached: lower momenta are more populated than at LTE.
An approximate description:

$$f_d(y) = f_{equil}(y/\xi)\ \theta(y_0 - y), \quad \xi < 1 \text{ out of equilibrium}$$

Modes with $p_c > y_0\, T_d$ are empty. [$y = p_c/T_d$].

For fermions: $m = 6.46 \text{ eV } (g_d/g)\ F(\infty)/[\xi^3\ F(y_0/\xi)]$

$F(s) \equiv \int_0^s f_{equil}(w)\ w^2\ dw \quad , \quad F(\infty)/[\xi^3\ F(y_0/\xi)] > 1.$

Phase-space density invariant under universe expansion

$$\mathcal{D} \equiv \frac{n(t)}{\langle \vec{P}^2_{phys}(t)\rangle^{\frac{3}{2}}} \stackrel{\text{non-rel}}{=} \frac{\rho_{DM}}{m^4\ \sigma^3_{DM}} \quad , \quad \sigma_{DM} \equiv \sqrt{\langle \vec{V}^{\,2}\rangle} =$$

computed theoretically from equilibrium distributions.

$\rho_{DM} = 1.107 \times \text{keV}/\text{cm}^3 =$ observed value today.

$$\frac{\rho_{DM}}{\sigma^3_{DM}} \sim 10^3\ \frac{\text{keV}/\text{cm}^3}{(\text{km}/\text{s})^3}\ \left(\frac{m}{\text{keV}}\right)^3\ g_d \begin{cases} 0.177 & \text{Fermions} \\ 0.247 & \text{Bosons} \end{cases} .$$

$g_d = \#$ of UR degrees of freedom at decoupling.

Observing dwarf spheroidal satellite galaxies in the Milky Way (dSphs) yields: $\frac{\rho_s}{\sigma_s^3} \sim 5 \times 10^3\ \frac{\text{keV}/\text{cm}^3}{(\text{km}/\text{s})^3}$ Gilmore et al. 07.

Theorem: The phase-space density $\mathcal{D}$ can only decrease under self-gravity interactions (gravitational clustering) [Lynden-Bell, Tremaine, Henon, 1986].

N-body simulations results: $\frac{\rho_s}{\sigma_s^3} \sim 10^{-2}\ \frac{\rho_{DM}}{\sigma^3_{DM}}$.

Mass Estimates of DM particles

Collecting all formulas yields for relics decoupling at LTE:

$$m \sim \frac{2}{g^{\frac14}} \text{ keV} \quad , \qquad g_d \geq 500 \; g^{\frac34} \quad ,$$

Hence, $T_d > 100$ GeV. $[g = 1 - 4]$.

g_d can be smaller for relics decoupling out of LTE

Let us consider now WIMPS (weakly interactive massive particles): $m \sim 100$ GeV, $T_d \sim 10$ MeV. We find:

$$\frac{\rho_{wimp}}{\sigma^3_{wimp}} \sim 10^{21} \; \frac{\mathrm{keV/cm^3}}{(\mathrm{km/s})^3} \; \left(\frac{\sqrt{m \; T_d}}{1 \text{ GeV}}\right)^3 \; g_d \; .$$

Eighteen orders of magnitude larger than the observations in dShps.

D. Boyanovsky, H. J. de Vega, N. Sanchez,
Phys. Rev. D 77, 043518 (2008), arXiv:0710.5180.

Dark Energy

$76 \pm 5\%$ of the present energy of the Universe is Dark!
Current observed value:
$\rho_\Lambda = \Omega_\Lambda \, \rho_c = (2.39 \text{ meV})^4 \quad , \quad 1 \text{ meV} = 10^{-3} \text{ eV}$.
Equation of state $p_\Lambda = -\rho_\Lambda$ within observational errors.
Quantum zero point energy. Renormalized value is finite.
Bosons (fermions) give positive (negative) contributions.
Mass of the lightest particles ~ 1 meV is in the right scale.
Spontaneous symmetry breaking of continuous symmetries produces massless scalars as Goldstone bosons. A small symmetry breaking provide light scalars: axions,majorons...
Observational Axion window $10^{-3} \text{ meV} \lesssim M_{\text{axion}} \lesssim 10 \text{ meV}$.
Dark energy can be a cosmological zero point effect. (As the Casimir effect in Minkowski with non-trivial boundaries).
We need to learn the physics of light particles (< 1 MeV), also to understand dark matter !!

Summary and Conclusions

- Inflation can be formulated as an effective field theory in the Ginsburg-Landau spirit with energy scale $M \sim M_{GUT} \sim 10^{16}$ GeV $\ll M_{Pl}$.
  Inflaton mass small: $m \sim H/\sqrt{N} \sim M^2/M_{Pl} \ll M$.
  Infrared regime !!
- The slow-roll approximation is a $1/N$ expansion, $N \sim 60$.
- MCMC analysis of WMAP+LSS data plus the Trinomial Inflation potential indicates a spontaneously symmetry breaking potential (new inflation): $w(\chi) = \frac{y}{32}\left(\chi^2 - \frac{8}{y}\right)^2$.
- Lower Bounds: $r > 0.016 \; (95\% \text{ CL})$, $r > 0.049 \; (68\% \text{ CL})$.
  The most probable values are
  $r \simeq 0.055$($\Leftarrow$measurable!!) $n_s \simeq 0.956$ with a quartic coupling $y \simeq 1.3$.

Summary and Conclusions 2

- The quadrupole suppression may be explained by the effect of fast-roll inflation provided the today's horizon size modes exited 0.1 efolds before the end of fast-roll inflation.
- Quantum (loop) corrections in the effective theory are of the order $(H/M_{Pl})^2 \sim 10^{-9}$.

D. Boyanovsky, H. J. de Vega, N. G. Sanchez,

Quantum corrections to the inflaton potential and the power spectra from superhorizon modes and trace anomalies, Phys. Rev. D 72, 103006 (2005), astro-ph/0507596.

Quantum corrections to slow roll inflation and new scaling of superhorizon fluctuations. Nucl. Phys. B 747, 25 (2006), astro-ph/0503669.

Future Perspectives

The Golden Age of Cosmology and Astrophysics continues.

A wealth of data from WMAP (6 yr), Planck, Atacama Cosmology Tel and further experiments are coming.

Galaxy formation. Gigantic black-holes ($M \sim 10^9 \; M_\odot$) as galaxy nuclei, early star formation...

The Dark Ages...Reionisation...the 21cm line...

Nature of Dark Energy? 76% of the energy of the universe.

Nature of Dark Matter? 83% of the matter in the universe.

Light DM particles are strongly favoured $m_{DM} \sim 2$ keV.

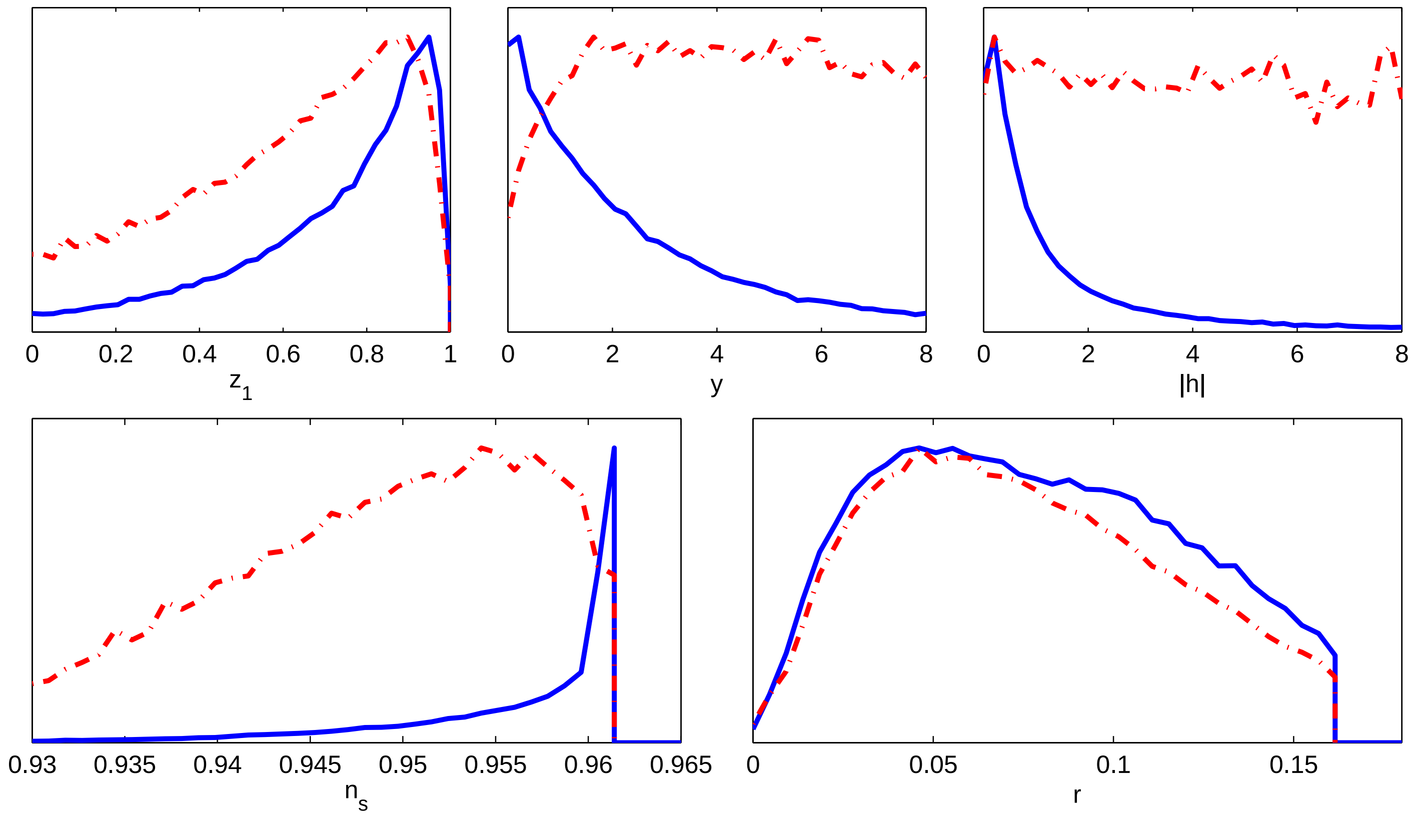
Sterile neutrinos? Some unknown light particle ??

Need to learn about the physics of light particles (< 1 MeV).

THANK YOU VERY MUCH

FOR YOUR ATTENTION !!

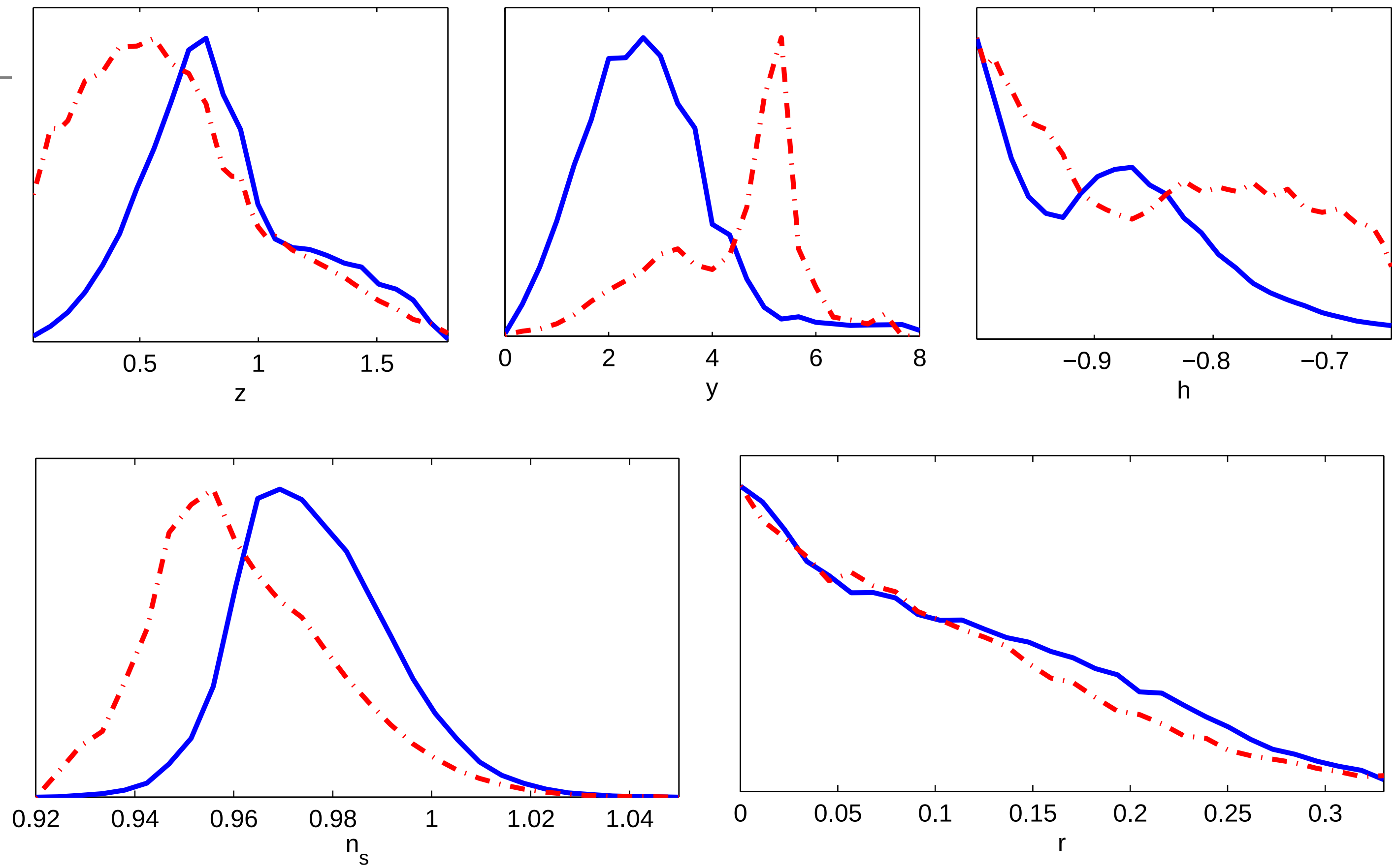
Probability Distributions. Trinomial New Inflation.



Probability distributions: solid blue curves
Mean likelihoods: dot-dashed red curves.

$$z_1 = 1 - \frac{y}{8\left(|h|+\sqrt{h^2+1}\right)^2}\ \chi^2 \ .$$

Probability Distributions. Trinomial Chaotic Inflation.



Probability distributions (solid blue curves) and mean likelihoods (dot-dashed red curves).

The data request a strongly asymmetric potential in chaotic inflation almost having two minima. That is, a strong breakdown of the $\chi \to -\chi$ symmetry.

Gauge Invariant Curvature Perturbations

$\mathcal{R}(\boldsymbol{x},t) = -\psi(\boldsymbol{x},t) - \frac{H(t)}{\dot{\Phi}(t)}\,\phi(\boldsymbol{x},t)$

$\phi(\boldsymbol{x},t) =$ inflaton fluctuations. $\psi(\boldsymbol{x},t) =$ newtonian potential.

These fluctuations around the FRW geometry are responsible of the CMB anisotropies and the LSS formation.

Gauge invariant potential

$u(\boldsymbol{x},t) \equiv -z(t)\,\mathcal{R}(\boldsymbol{x},t)\ ,\ z(t) \equiv a(t)\,\frac{\dot{\Phi}(t)}{H(t)}$

In Fourier space: $u(\boldsymbol{k},\eta) = \alpha_{\mathcal{R}}(\boldsymbol{k})\,S_{\mathcal{R}}(k;\eta) + \alpha^{\dagger}_{\mathcal{R}}(\boldsymbol{k})\,S^{*}_{\mathcal{R}}(k;\eta)$

$\alpha^{\dagger}_{\mathcal{R}}(\boldsymbol{k})$ and $\alpha_{\mathcal{R}}(\boldsymbol{k})$ are creation and annihilation operators.

The mode functions obey a Schrödinger-like equation,

$$\left[\frac{d^2}{d\eta^2} + k^2 - W_{\mathcal{R},\mathcal{T}}(\eta)\right] S_{\mathcal{R},\mathcal{T}}(k;\eta) = 0\ .$$

Scalar Curvature and tensor fluctuations

$W_{\mathcal{R}}(\eta) = \frac{1}{z}\frac{d^2 z}{d\eta^2}$ for scalar, $\quad W_{\mathcal{T}}(\eta) = \frac{1}{a}\frac{d^2 a}{d\eta^2}$ for tensor.

$W_{\mathcal{R},\mathcal{T}}(\eta) = \frac{\nu^2_{\mathcal{R},\mathcal{T}} - \frac{1}{4}}{\eta^2} + \mathcal{V}_{\mathcal{R},\mathcal{T}}(\eta)$.

Like a centrifugal barrier plus $\mathcal{V}_{\mathcal{R},\mathcal{T}}(\eta)$.

scalar: $\nu_{\mathcal{R}} = \frac{3}{2} + 3\,\epsilon_V - \eta_V \quad , \quad$ tensor: $\nu_T = \frac{3}{2} + \epsilon_V$

$$\epsilon_V = \frac{1}{2N} \left[\frac{w'(\chi)}{w(\chi)}\right]^2 \quad , \quad \eta_V = \frac{1}{N} \frac{w''(\chi)}{w(\chi)} \ .$$

$\mathcal{V}(\eta) = 0$ during slow-roll, $\mathcal{V}(\eta) \neq 0$ during fast-roll.

During slow-roll: $S(k;\eta) = A(k)\, g_\nu(k;\eta) + B(k)\, f_\nu(k;\eta)$

$g_\nu(k;\eta) = \frac{1}{2}\, i^{\nu+\frac{1}{2}} \sqrt{-\pi\eta}\, H^{(1)}_\nu(-k\eta)\ ,\ f_\nu(k;\eta) = [g_\nu(k;\eta)]^*$

$H^{(1)}_\nu(z)$: Hankel function.

Scale invariant limit: $g_{\frac{3}{2}}(k;\eta) = \frac{e^{-ik\eta}}{\sqrt{2k}} \left[1 - \frac{i}{k\eta}\right] \ .$

The effect of $\mathcal{V}_{\mathcal{R},\mathcal{T}}(\eta)$ during the fast-roll

The initial conditions on the modes $S(k;\eta)$ plus $\mathcal{V}_{\mathcal{R},\mathcal{T}}(\eta)$ determine the coefficients $A_{\mathcal{R},\mathcal{T}}(k)$ and $B_{\mathcal{R},\mathcal{T}}(k)$.

We choose Bunch-Davies initial conditions:

$$S_\nu(k;\eta) \overset{\eta\to-\infty}{=} \frac{1}{\sqrt{2\,k}}\, e^{-ik\eta}$$

$$\mathcal{V}_{\mathcal{R},\mathcal{T}}(\eta) = 0 \longrightarrow A(k) = 1,\; B(k) = 0$$

$\mathcal{V}_{\mathcal{R},\mathcal{T}}(\eta) \neq 0$ is analogous to a one dimensional scattering problem in the η-axis.

D. Boyanovsky, H. J. de Vega, N. Sanchez,
CMB quadrupole suppression:
I. Initial conditions of inflationary perturbations,
II. The early fast-roll stage,
Phys.Rev. D74 (2006) 123006 and 123007,
astro-ph/0607508 and astro-ph/0607487.

Primordial Power Spectrum

$$P_{\mathcal{R}}(k) \stackrel{\eta\to 0^-}{=} \frac{k^3}{2\,\pi^2} \left|\frac{S_{\mathcal{R}}(k;\eta)}{z(\eta)}\right|^2 = P^{sr}_{\mathcal{R}}(k)\Big[1 + D_{\mathcal{R}}(k)\Big] \; ,$$

$$P_{T}(k) \stackrel{\eta\to 0^-}{=} \frac{k^3}{2\,\pi^2} \left|\frac{S_{T}(k;\eta)}{a(\eta)}\right|^2 = P^{sr}_{T}(k)\Big[1 + D_{T}(k)\Big] \; .$$

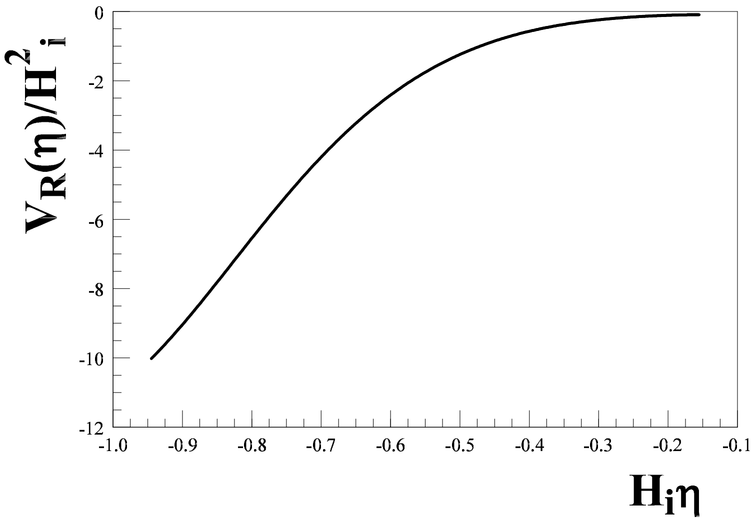
Standard slow roll power spectrum:

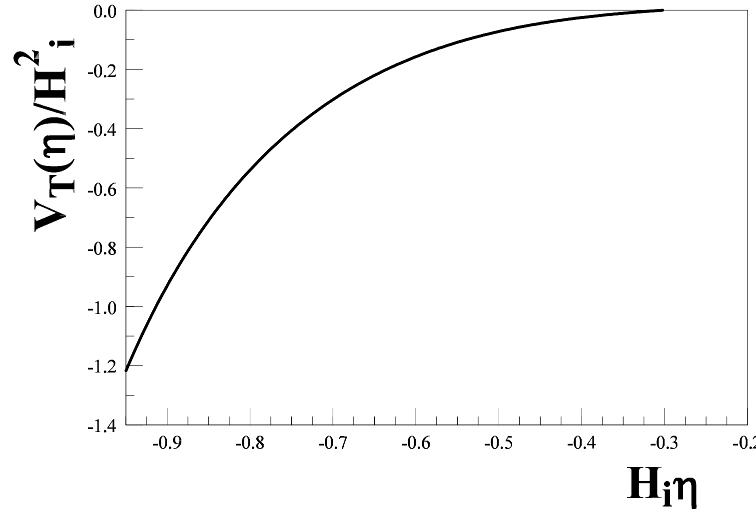
$$P^{sr}_{\mathcal{R}}(k) = \mathcal{A}^2_{\mathcal{R}} \left(\frac{k}{k_0}\right)^{n_s-1} \; , \; P^{sr}_{T}(k) = \mathcal{A}^2_{T} \left(\frac{k}{k_0}\right)^{n_T}$$

$$D(k) = 2\,|B(k)|^2 - 2\,\mathrm{Re}\left[A(k)\;B^*(k)\,i^{2\nu-3}\right]$$

$D_{\mathcal{R}}(k)$ and $D_T(k)$ are the transfer functions of curvature and tensor perturbations taking into account the effect of the fast-roll stage.

Potential felt by the Scalar and by the Tensor Fluctuations





$H_i =$ Hubble at the beginning of slow-roll.

Both $\mathcal{V}_{\mathcal{R}}(\eta)$ and $\mathcal{V}_{\mathcal{T}}(\eta)$ are ATTRACTIVE potentials.

Potential felt by tensor fluctuations much smaller:

$$\mathcal{V}_{\mathcal{T}}(\eta) \sim \frac{1}{10} \ \mathcal{V}_{\mathcal{R}}(\eta)$$

Change in the C_l due to fast-roll

$$C_l \equiv C_l^{sr} + \Delta C_l \quad , \quad \frac{\Delta C_l}{C_l} = \frac{\int_0^\infty D_{\mathcal{R},\mathcal{T}}(\kappa\, x)\;\, f_l(x)\, dx}{\int_0^\infty f_l(x)\, dx}$$

$$\kappa \equiv a_0 \; H_0/3.3 = a_{sr} \; H_i/3.3 \quad , \quad f_l(x) \equiv x^{n_s - 2}[j_l(x)]^2 \; .$$

Since $\mathcal{V}_{\mathcal{R},\mathcal{T}}(\eta)$ are quite small we can compute the transfer functions in the Born approximation:

$$D_{\mathcal{R},\mathcal{T}}(k) = \int_{-\infty}^0 d\eta \mathcal{V}_{\mathcal{R},\mathcal{T}}(\eta)[\sin(2k\eta)\left(1 - \frac{1}{k^2\eta^2}\right) + \frac{2}{k\eta}\; \cos(2k\eta)]/k$$

and then, $\quad \frac{\Delta C_2}{C_2} = \frac{1}{\kappa} \int_{-\infty}^0 d\eta \; \mathcal{V}_{\mathcal{R},\mathcal{T}}(\eta) \; \Psi(\kappa\; \eta)$

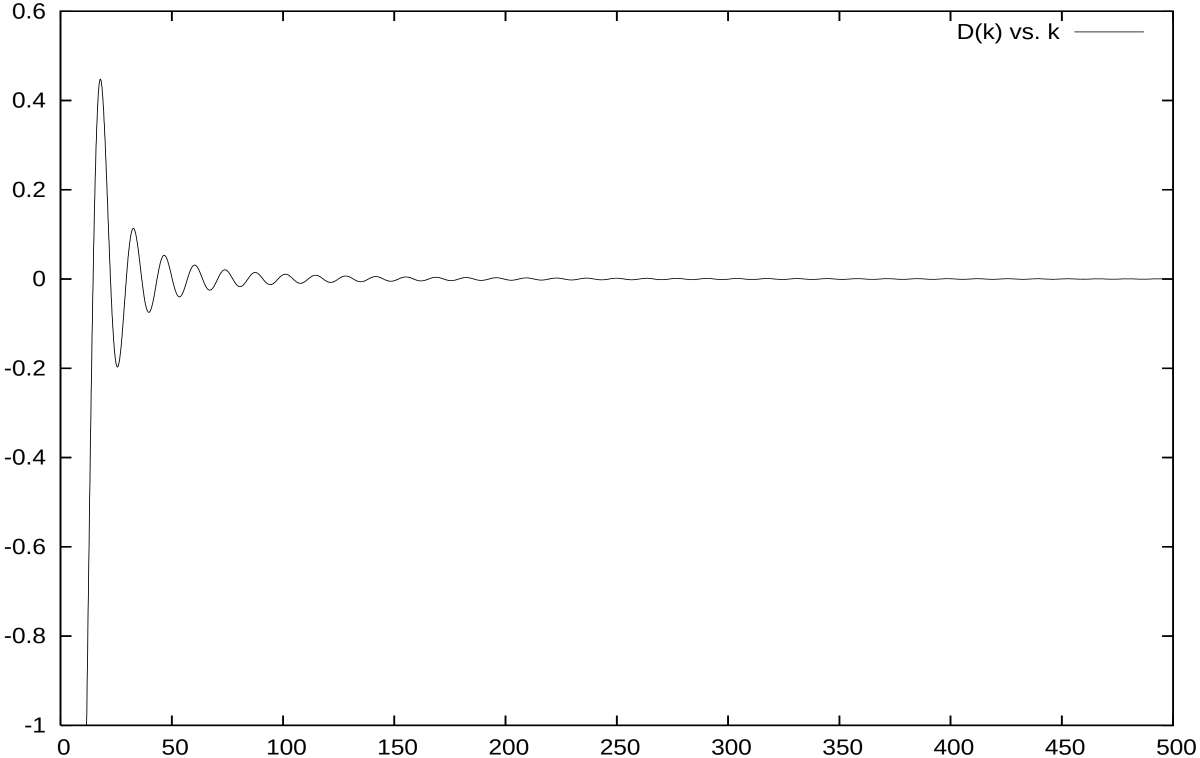
where $\Psi(\kappa\; \eta) > 0$ for $\eta < 0$.

ATTRACTIVE $\mathcal{V}_{\mathcal{R},\mathcal{T}}(\eta) < 0$ implies $\Delta C_2 < 0$.

$\longrightarrow$ QUADRUPOLE SUPPRESSION.

In general, $0 > \frac{\Delta C_l}{C_l} = \mathcal{O}\left(\frac{1}{l^2}\right) \; .$

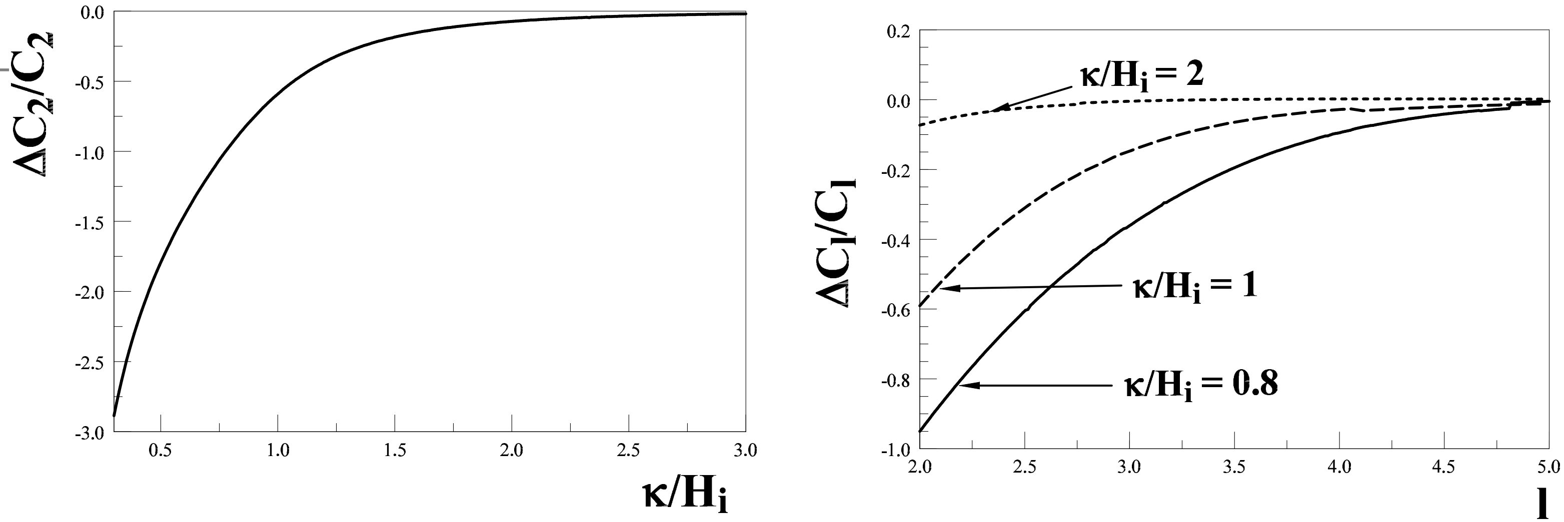
The Transfer Function $D(k)$ for the scalar fluctuations.



The transfer function $D_{\mathcal{R}}(k)$ computed in the Born approximation for trinomial new inflation $y \simeq 2, \; h = 0$.

$$P_{\mathcal{R}}(k) = P^{sr}_{\mathcal{R}}(k)\left[1 + D_{\mathcal{R}}(k)\right]$$

Quadrupole Suppression vs. Fast-Roll



$\frac{\kappa}{H_i} = \frac{a_{sr}}{3.3}$. The Quadrupole is suppressed 20% for

The quadrupole modes should exit the horizon $\simeq 1$ efolds after inflation starts. That is 0.1 efolds before fast-roll ends.

Quadrupole Suppression Explanation:

Inflation starts with fast-roll: 0 efolds.

Today Horizon size modes (quadrupole) exit the horizon by 1 efolds.

Fast-roll ends and slow-roll begins: 1.1 efold.

Inflation ends by a number of efolds $N_T \simeq 66$

Loop Quantum Corrections to Slow-Roll Inflation

$$\phi(\vec{x},t) = \Phi_0(t) + \varphi(\vec{x},t), \quad \Phi_0(t) \equiv < \phi(\vec{x},t) >, \quad < \varphi(\vec{x},t) > = 0$$

$$\varphi(\vec{x},t) = \frac{1}{a(\eta)} \int \frac{d^3k}{(2\,\pi)^3} \left[a_{\vec{k}}\, \chi_k(\eta)\, e^{i\vec{k}\cdot\vec{x}} + \text{h.c.} \right],$$

$a^\dagger_{\vec{k}},\ a_{\vec{k}}$ are creation/annihilation operators,
$\chi_k(\eta)$ are mode functions. $\eta =$ conformal time.
To one loop order the equation of motion for the inflaton is

$$\ddot{\Phi}_0(t) + 3\,H\,\dot{\Phi}_0(t) + V'(\Phi_0) + g(\Phi_0)\,\langle[\varphi(\boldsymbol{x},t)]^2\rangle = 0$$

where $g(\Phi_0) = \frac{1}{2}\,V'''(\Phi_0)$.
The mode functions obey:

$$\chi''_k(\eta) + \left[k^2 + M^2(\Phi_0)\,a^2(\eta) - \frac{a''(\eta)}{a(\eta)} \right] \chi_k(\eta) = 0$$

where $M^2(\Phi_0) = V''(\Phi_0) = 3\,H_0^2\,\eta_V + \mathcal{O}(1/N^2)$

Quantum Corrections to the Friedmann Equation

The mode functions equations for slow-roll become,

$$\chi_k''(\eta) + \left[k^2 - \frac{\nu^2 - \frac{1}{4}}{\eta^2}\right] \chi_k(\eta) = 0 \quad , \quad \nu = \frac{3}{2} + \epsilon_V - \eta_V + \mathcal{O}(1/N^2).$$

The scale factor during slow roll is $a(\eta) = -\frac{1}{H_0 \, \eta \, (1-\epsilon_V)}$.

Scale invariant case: $\nu = \frac{3}{2}$. $\Delta \equiv \frac{3}{2} - \nu = \eta_V - \epsilon_V$ controls the departure from scale invariance.

Explicit solutions in slow-roll:

$$\chi_k(\eta) = \frac{1}{2} \sqrt{-\pi\eta} \, i^{\nu+\frac{1}{2}} \, H_\nu^{(1)}(-k\eta), \quad H_\nu^{(1)}(z) = \text{Hankel function}$$

Quantum fluctuations: $\langle [\varphi(\boldsymbol{x}, t)]^2 \rangle = \frac{1}{a^2(\eta)} \int \frac{d^3k}{(2\pi)^3} \, |\chi_k(\eta)|^2$

$$\frac{1}{2} \langle [\varphi(\boldsymbol{x}, t)]^2 \rangle = \left(\frac{H_0}{4\,\pi}\right)^2 \left[{\Lambda_p}^2 + \ln \Lambda_p^2 + \frac{1}{\Delta} + 2\,\gamma - 4 + \mathcal{O}(\Delta)\right]$$

UV cutoff $\Lambda_p = \text{physical cutoff}/H$, $\quad \frac{1}{\Delta} =$ infrared pole.

$\langle \dot{\varphi}^2 \rangle \quad , \quad \left\langle (\nabla\varphi)^2 \right\rangle$ are infrared finite

Quantum Corrections to the Inflaton Potential

Upon UV renormalization the Friedmann equation results

$$H^2 = \frac{1}{3\,M_{Pl}^2}\left[\frac{1}{2}\,\dot{\Phi_0}^2 + V_R(\Phi_0) + \left(\frac{H_0}{4\,\pi}\right)^2 \frac{V_R^{''}(\Phi_0)}{\Delta} + \mathcal{O}\left(\frac{1}{N}\right)\right]$$

Quantum corrections are proportional to $\left(\frac{H}{M_{Pl}}\right)^2 \sim 10^{-9}$!!

The Friedmann equation gives for the effective potential:

$$V_{eff}(\Phi_0) = V_R(\Phi_0) + \left(\frac{H_0}{4\,\pi}\right)^2 \frac{V_R^{''}(\Phi_0)}{\Delta}$$

$$V_{eff}(\Phi_0) = V_R(\Phi_0)\left[1 + \left(\frac{H_0}{4\,\pi\,M_{Pl}}\right)^2 \frac{\eta_V}{\eta_V - \epsilon_V}\right]$$

in terms of slow-roll parameters

Very DIFFERENT from the one-loop effective potential in Minkowski space-time:

$$V_{eff}(\Phi_0) = V_R(\Phi_0) + \frac{[V_R^{''}(\Phi_0)]^2}{64\,\pi^2}\ln\frac{V_R^{''}(\Phi_0)}{M^2}$$

Quantum Fluctuations:

Scalar Curvature, Tensor, Fermion, Light Scalar.

All these quantum fluctuations contribute to the inflaton potential and to the primordial power spectra.

In de Sitter space-time: $< T_{\mu\nu} >= \frac{1}{4} \, g_{\mu\nu} \; < T_\alpha^\alpha >$

Hence, $V_{eff} = V_R + < T_0^0 >= V_R + \frac{1}{4} < T_\alpha^\alpha >$

Sub-horizon (Ultraviolet) contributions appear through the trace anomaly and only depend on the spin of the particle. Superhorizon (Infrared) contributions are of the order N^0 and can be expressed in terms of the slow-roll parameters.

$$V_{eff}(\Phi_0) = V(\Phi_0)\left[1 + \frac{H_0^2}{3\,(4\pi)^2\,M_{Pl}^2}\left(\frac{\eta_v - 4\,\epsilon_v}{\eta_v - 3\,\epsilon_v} + \frac{3\,\eta_\sigma}{\eta_\sigma - \epsilon_v} + \mathcal{T}\right)\right]$$

$\mathcal{T} = \mathcal{T}_\Phi + \mathcal{T}_s + \mathcal{T}_t + \mathcal{T}_F = -\frac{2903}{20}$ is the total trace anomaly.

$\mathcal{T}_\Phi = \mathcal{T}_s = -\frac{29}{30}, \; \mathcal{T}_t = -\frac{717}{5}, \; \mathcal{T}_F = \frac{11}{60}$

$\longrightarrow$ the graviton (t) dominates.

Corrections to the Primordial Scalar and Tensor Power

$$|\Delta^{(S)}_{k,eff}|^2 = |\Delta^{(S)}_k|^2 \{1+$$
$$+\frac{2}{3}\left(\frac{H_0}{4\,\pi\,M_{Pl}}\right)^2\left[1+\frac{\frac{3}{8}\,r\,(n_s-1)+2\,\frac{dn_s}{d\ln k}}{(n_s-1)^2}+\frac{2903}{40}\right]\Bigg\}$$
$$|\Delta^{(T)}_{k,eff}|^2 = |\Delta^{(T)}_k|^2\left\{1-\frac{1}{3}\left(\frac{H_0}{4\,\pi\,M_{Pl}}\right)^2\left[-1+\frac{1}{8}\,\frac{r}{n_s-1}+\frac{2903}{20}\right]\right\}.$$

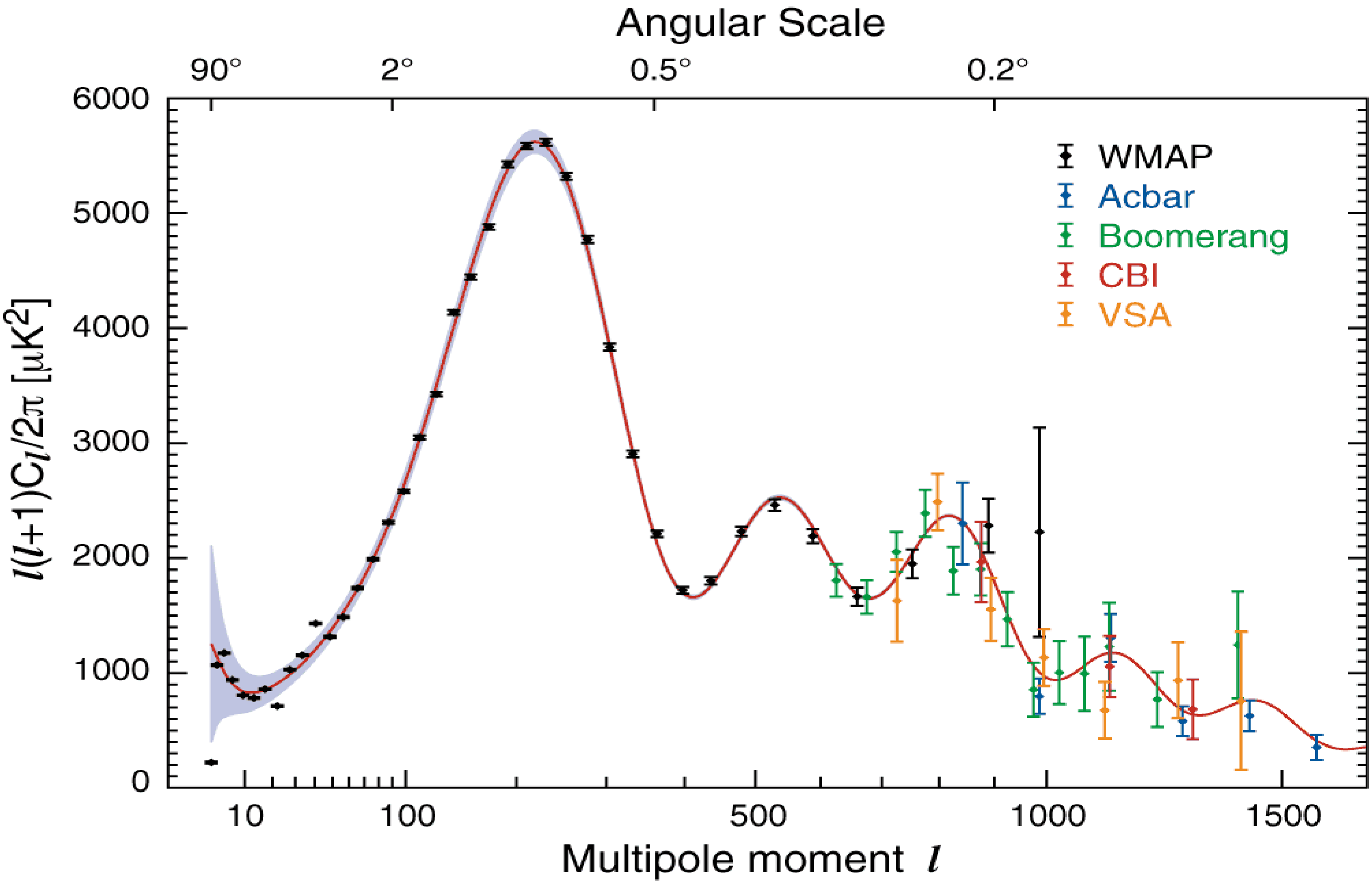
The anomaly contribution $-\frac{2903}{20} = -145.15$ DOMINATES as long as the number of fermions less than 783.

The scalar curvature fluctuations $|\Delta^{(S)}_k|^2$ are ENHANCED and the tensor fluctuations $|\Delta^{(T)}_k|^2$ REDUCED.

However, $\left(\frac{H}{M_{Pl}}\right)^2 \sim 10^{-9}$.

D. Boyanovsky, H. J. de Vega, N. G. Sanchez, Phys. Rev. D 72, 103006 (2005), astro-ph/0507596.

WMAP 3 years data plus others.



Theory and observations nicely agree except for the lowest multipoles: the quadrupole suppression.