Inflation, Dark Matter, Dark Energy in the Standard Model of the Universe: New understanging after WMAP

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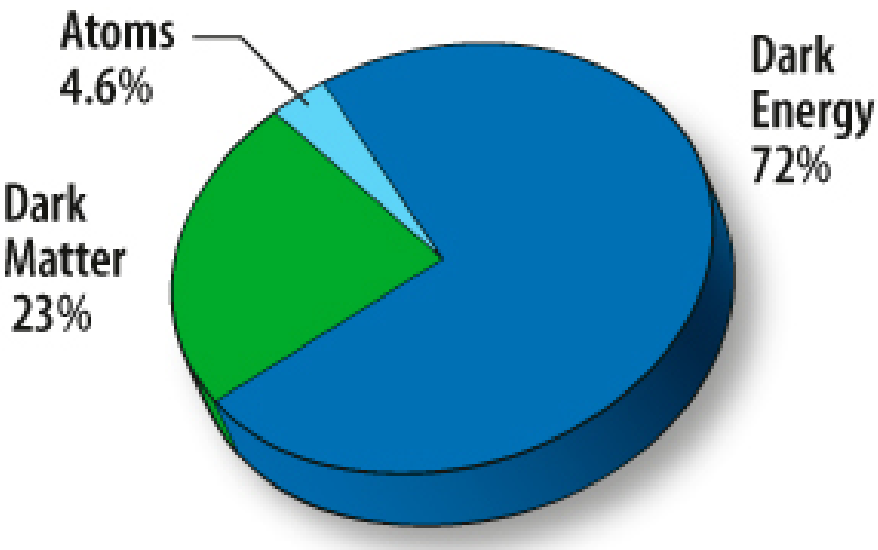
CONTENT OF THE UNIVERSE

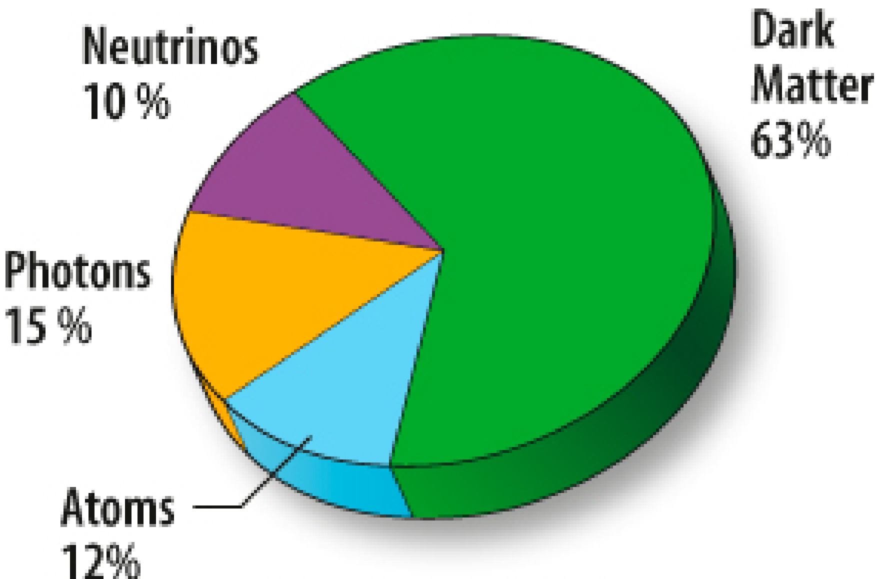
WMAP data reveals that its contents include 4.6% atoms, the building blocks of stars and planets.

Dark matter comprises 23% of the universe.
This matter, different from atoms, does not emit or absorb light.
It has only been detected indirectly by its gravity.

72% of the Universe, is composed of "dark energy", that acts as a sort of an anti-gravity.
This energy, distinct from dark matter, is responsible for the present-day acceleration of the universal expansion.

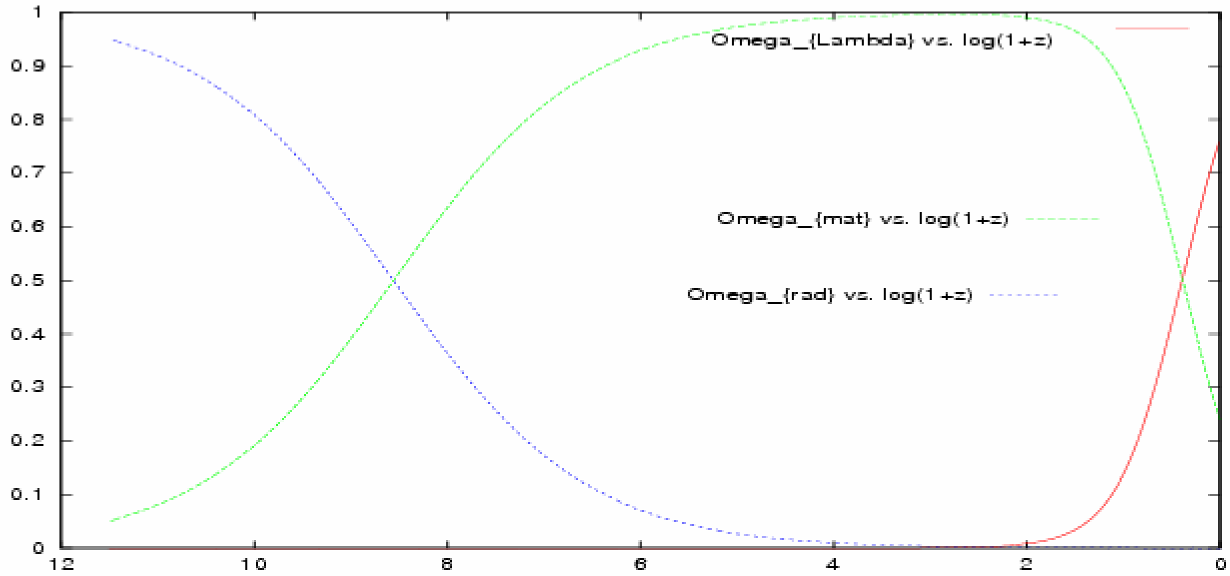


TODAY



13.7 BILLION YEARS AGO
(Universe 380,000 years old)

The Universe is made of radiation, matter and dark energy



Electro-Weak phase transition: $z \sim 10^{15}$, $T_{\rm EW} \sim 100$ GeV.
QCD phase transition (conf.): $z \sim 10^{12}$, $T_{\rm QCD} \sim 170$ MeV.
BBN: $z \sim 10^9$, $\ln(1+z) \sim 21$, $T \simeq 0.1$ MeV.
Rad-Mat equality: $z \simeq 3050$, $\ln(1+z) \simeq 8$, $T \simeq 0.7$ eV.
CMB last scattering: $z \simeq 1100$, $\ln(1+z) \simeq 7$, $T \simeq 0.25$ eV.
Mat-DE equality: $z \simeq 0.47$, $\ln(1+z) \simeq 0.38$, $T \simeq 0.345$ meV.
Today: $z = 0$, $\ln(1+z) = 0$, $T = 2.725$K $= 0.2348$ meV.

Standard Model of the Universe: ΛCDM

ΛCDM = Cold Dark Matter + Cosmological Constant
Explains the Observations:

- 5 years WMAP data and previous CMB data
- Light Elements Abundances
- Large Scale Structures (LSS) Observations
- Supernova Luminosity/Distance Relations (Acceleration of the Universe expansion)
- Gravitational Lensing Observations
- Lyman α Forest Observations
- Hubble Constant (H_0) Measurements
- Properties of Clusters of Galaxies
- ....

Standard Cosmological Model: Concordance Model

$ds^2 = dt^2 - a^2(t)\ d\vec{x}^{\,2}$: spatially flat geometry.

The Universe starts by an **INFLATIONARY ERA**.

Inflation = Accelerated Expansion: $\frac{d^2a}{dt^2} > 0$.
During inflation the universe expands by at least sixty efolds: $e^{60} \simeq 10^{26}$. Inflation lasts $\simeq 10^{-34}$ sec and ends by $z \sim 10^{28}$ followed by a radiation dominated era.
Energy scale when inflation starts $\sim 10^{16}$ GeV.
This energy scale **coincides** with the GUT scale ($\Longleftarrow$ CMB anisotropies).

Matter can be effectively described during inflation by an Scalar Field $\phi(t, x)$: the Inflaton.

Lagrangean: $\mathcal{L} = a^3(t) \left[\frac{\dot{\phi}^2}{2} - \frac{(\nabla\phi)^2}{2\ a^2(t)} - V(\phi) \right]$.

Friedmann eq.: $H^2(t) = \frac{1}{3\ M_{Pl}^2} \left[\frac{\dot{\phi}^2}{2} + V(\phi) \right]$, $H(t) \equiv \dot{a}(t)/a(t)$

The Effective Field Theory (EFT) Approach relies on the separation between the energy scale of Inflation** ***and the higher energy scale of the earlier stage (cutoff scale) which here is the Planck scale.

***Scale of Inflation: Hubble parameter during the relevant stage of inflation (wavelengths of cosmological relevance cross the horizon).**

EFT expansion: defined by the dimensionless ratio $H(\Phi°)/M_{Pl}$. Reliability of the expansion:** ***improves upon dynamical evolution **since the scale of inflation diminishes with time.**

EFT expansion is an excellent one** ***since the amplitudes of scalar and tensor perturbations are given by

$$2\,(2)^{1/2}\ \varepsilon_V\, \Delta_R = H / (\pi\, M_{Pl})\ ,\ \ \Delta_T = (2)^{1/2}\, H / (\pi\, M_{Pl})$$

$\varepsilon_V \ll 1$. WMAP data $\Delta_R = 0.47 \times 10^{-4}$ provide strong observational support to the validity of an effective field theory of inflation well below the Planck scale and to the H / M_{Pl} expansion.

Expected CMB constraints on Δ_T *should still improve* this support.

THE SCALE OF SEMICLASSICAL GRAVITY

Δ_T and Δ_R expressed in terms of the semiclassical and quantum Gravity Temperature scales

$$T_{sem} = h\,H/(2\pi\, k_B) \quad , \quad T_{Pl} = M_{Pl}\, c^2/(2\pi\, k_B)$$

T_{sem} is the semiclassical or Hawking-Gibbons temperature of the initial state (or Bunch-Davies vacuum) of inflation. T_{Pl} is the Planck temparature $10^{32°}$ K.

$$T_{sem} / T_{Pl} = 2\pi\, (2\, \varepsilon_V)^{1/2}\, \Delta_R, \quad T_{sem} / T_{Pl} = \pi\, (2)^{-1/2}\, \Delta_T$$

Therefore, WMAP data yield for the Hawking-Gibbons Temperature of Inflation:

$$\rightarrow \rightarrow \rightarrow \quad T_{sem} \sim (\varepsilon_V)^{1/2}\, 10^{28\,°}\ \text{K.}$$

What is the Inflaton?

It is an effective field.
It can describe a fermion-antifermion pair condensate:
$\phi =< \bar\psi \psi >$, ψ = GUT fermion. ϕ neutral and singlet field.
Such condensate can dominate the expectation value of the hamiltonian and therefore govern the cosmological expansion. [Recall that $< \psi >= 0$].
Relevant effective theories in physics:

- Ginsburg-Landau theory of superconductivity. It is an effective theory for Cooper pairs in the microscopic BCS theory of superconductivity.
- The $O(4)$ sigma model for pions, the sigma and photons at energies $\lesssim 1$ GeV. The microscopic theory is QCD: quarks and gluons. $\pi \simeq \bar q q \; , \; \sigma \simeq \bar q q$.

The Theory of Inflation

It is an effective field in the Ginsburg-Landau sense.
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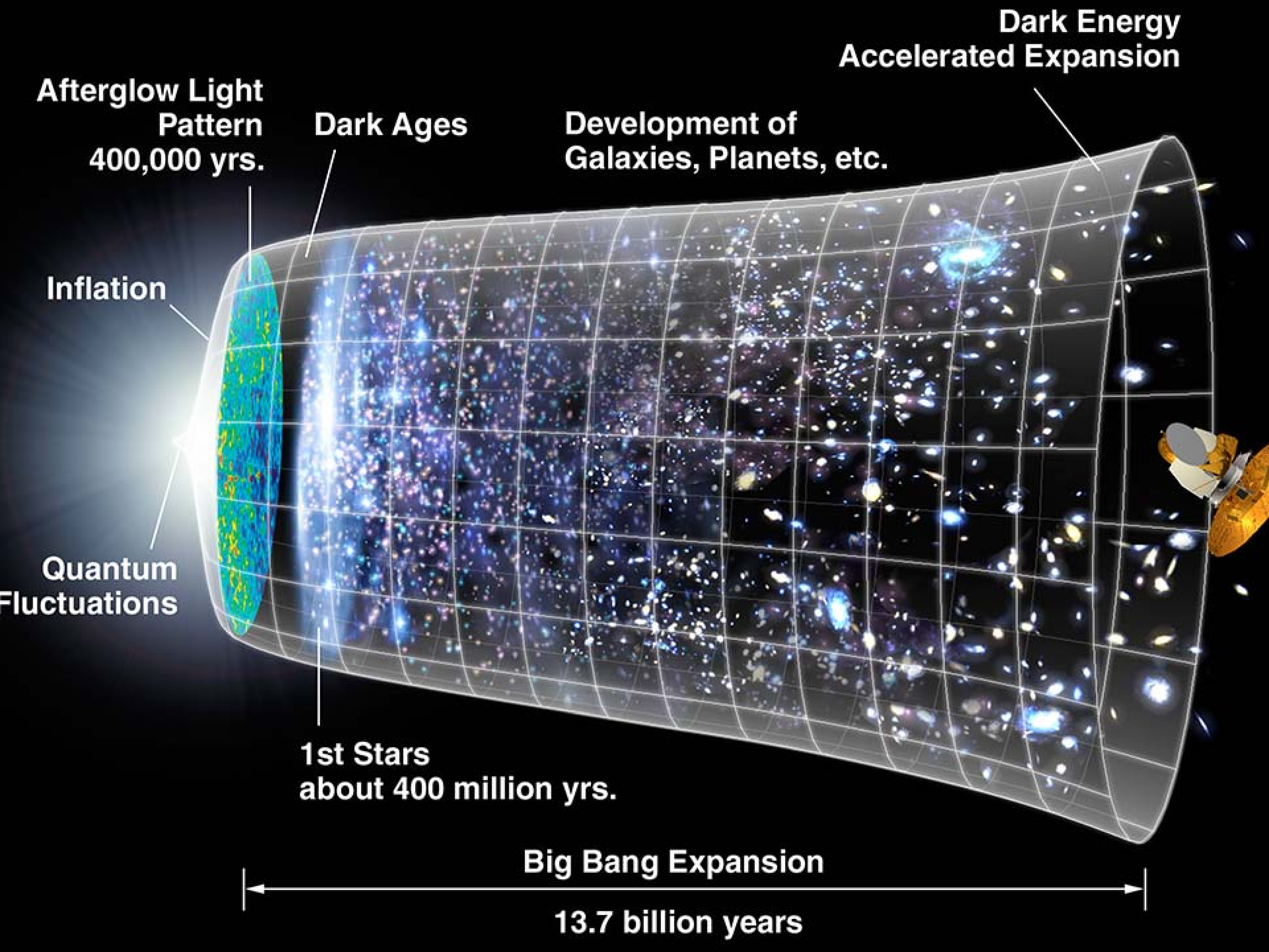
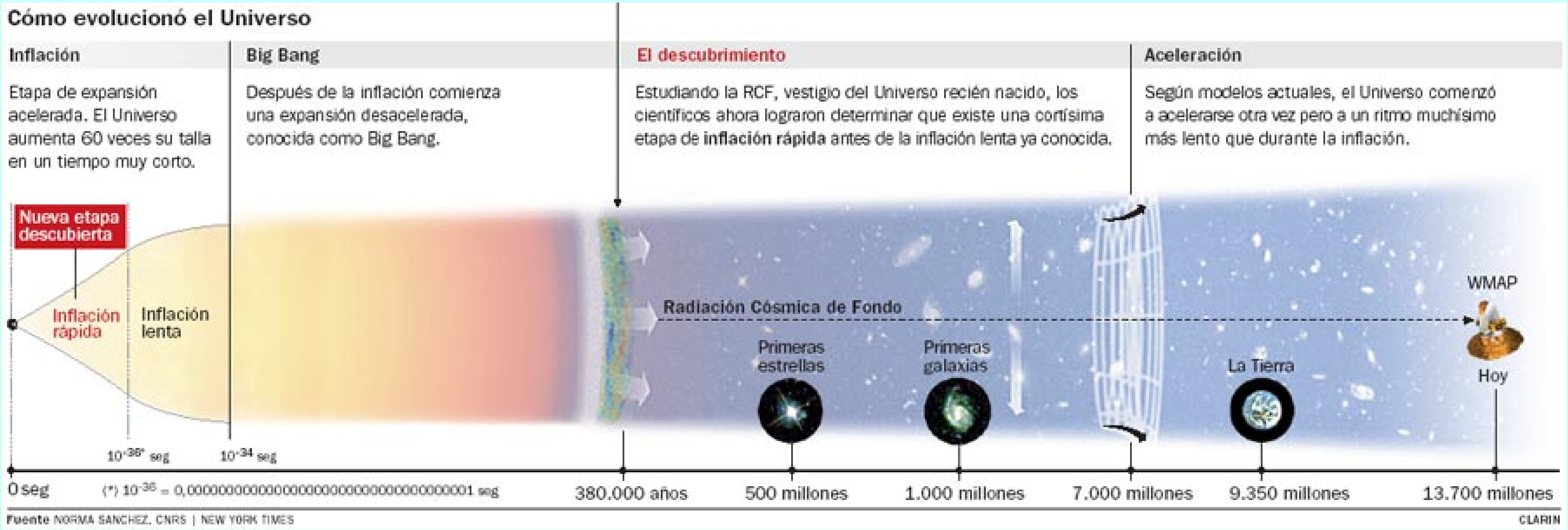
- The theory of second order phase transitions à la Landau-Kadanoff-Wilson... (ferromagnetic, antiferromagnetic, liquid-gas, Helium 3 and 4, ...)


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
400,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years

Fast roll Inflation produces the Observed Quadrupole CMB Suppression

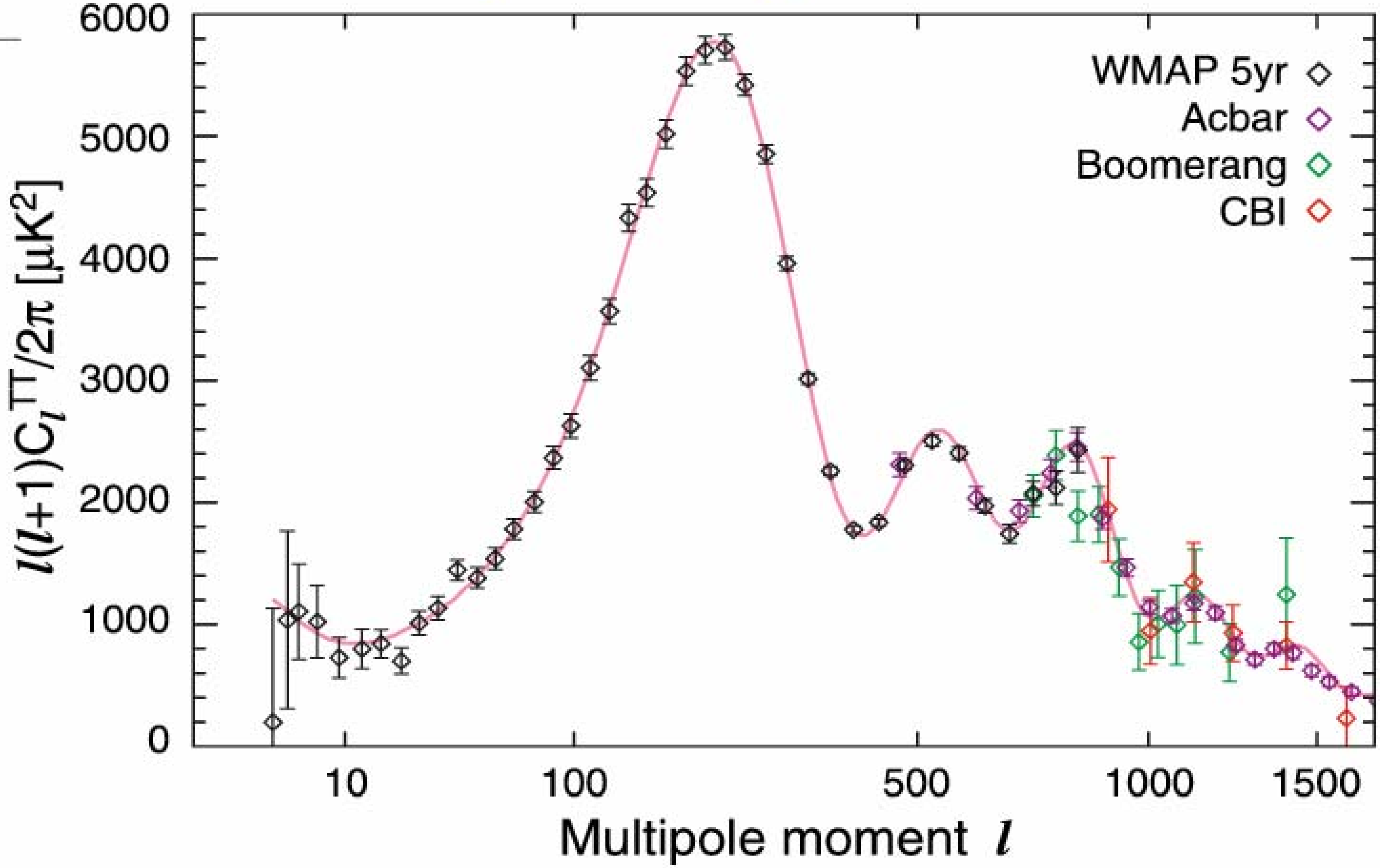


D. Boyanovsky, H. J de Vega and N. G. Sanchez,

” CMB quadrupole suppression II : The early fast roll stage ”

Phys. Rev. D74 , 123006 (2006)

WMAP 5 years data plus further data



Theory (ΛCDM) and observations nicely agree except for the lowest multipoles: the quadrupole suppression.

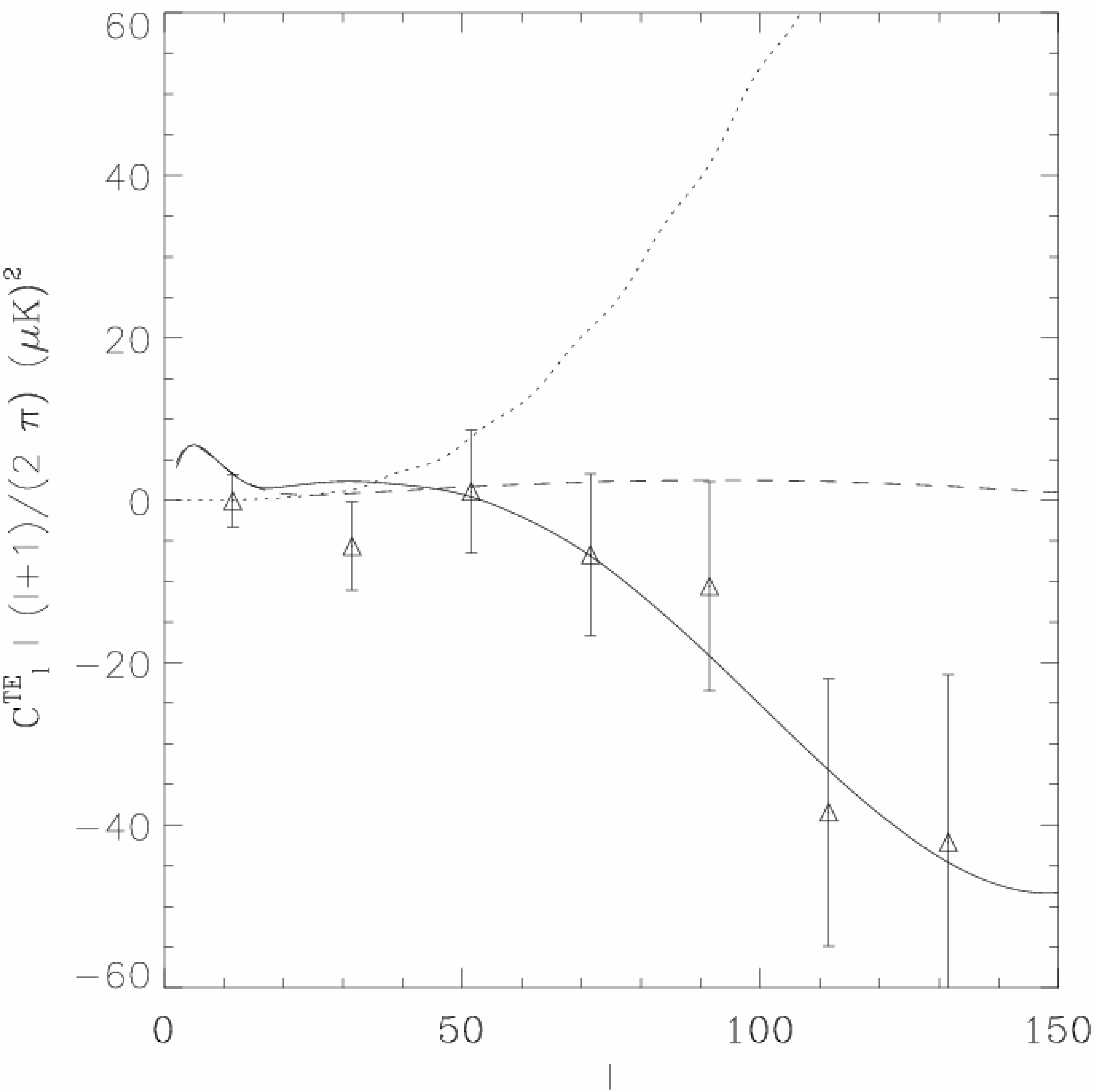
Les résultats de WMAP, des données des structures à grande échelle (LSS) et d'energie noire placent l'Inflation de l'univers primordial dans le cadre du modele standard de l'univers. Théorie effective de l'Inflation de l'univers primordial avec les echelles d'énergie physiques relevantes du problème: l'echelle d'energie de l'inflation, l'echelle de Planck et l'echelle de Hubble de l'inflation.

(i)le cadre naturel pour d'écrire les résultats du CMB+ LSS+ energie noire est une théorie effective de l'inflation avec un potentiel d'une forme simple et universelle bien déterminée sans aucun «fine-tuning». (potentiel quartique générique). Cette clarification essentielle permet de simplifier, comprendre et placer l'inflation dans le cadre de la grande unification et en analogie avec des autres theories effectives, comme par example la supraconductivité.

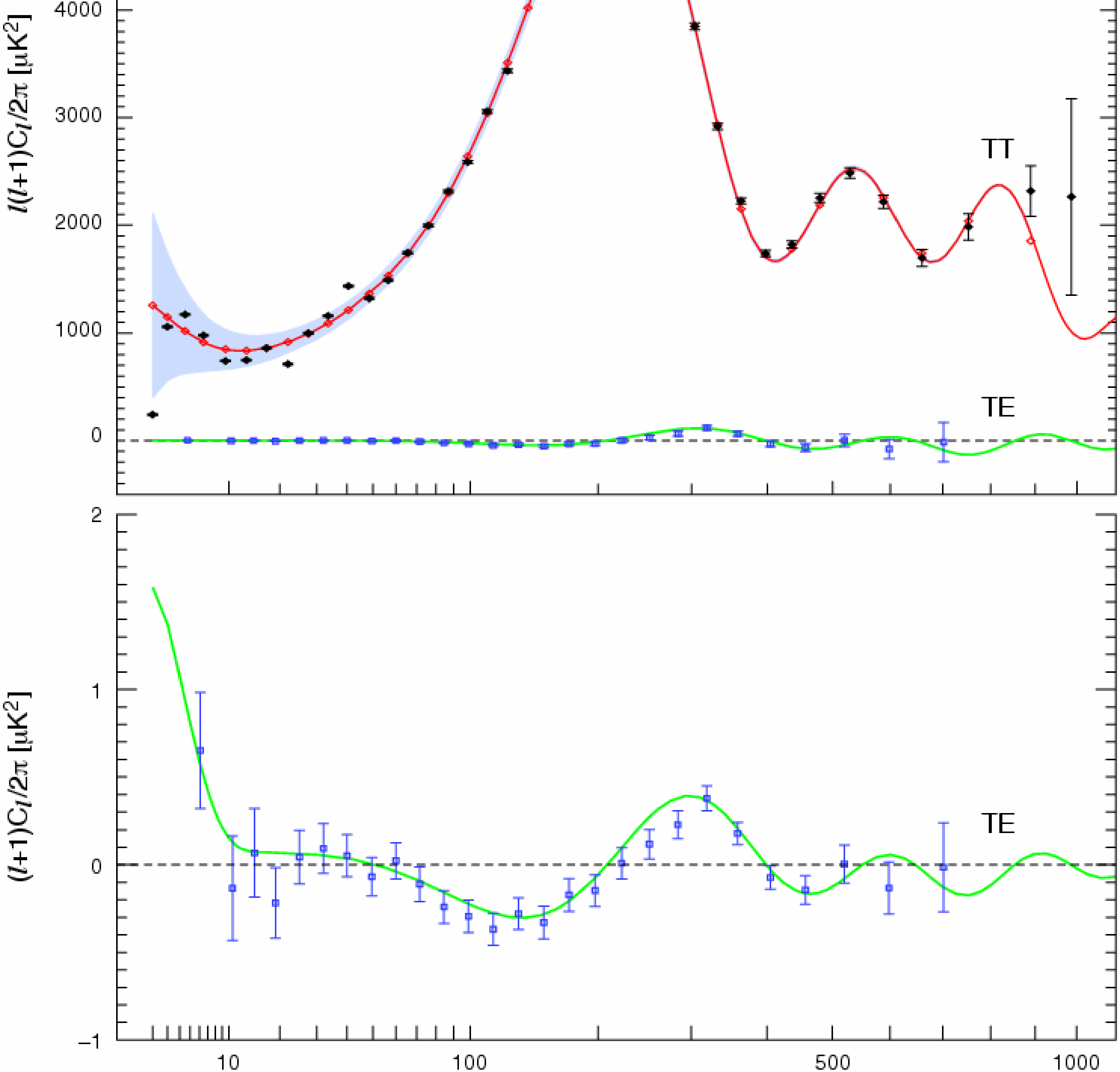
(ii) des prédictions pour les observables CMB : le rapport r = tenseur/scala la variation de l'index spectrale ("running index"), et l'index spectral scalaire ns des fluctuations primordiales.

(iii) une nouvelle analyse des données WMAP avec Monte Carlo Markov Chains et l'input théorique conceptuel de la théorie effective de l'inflation, ainsi une borne inferieure pour le rapport r = tenseur/scalaire est trouvée.

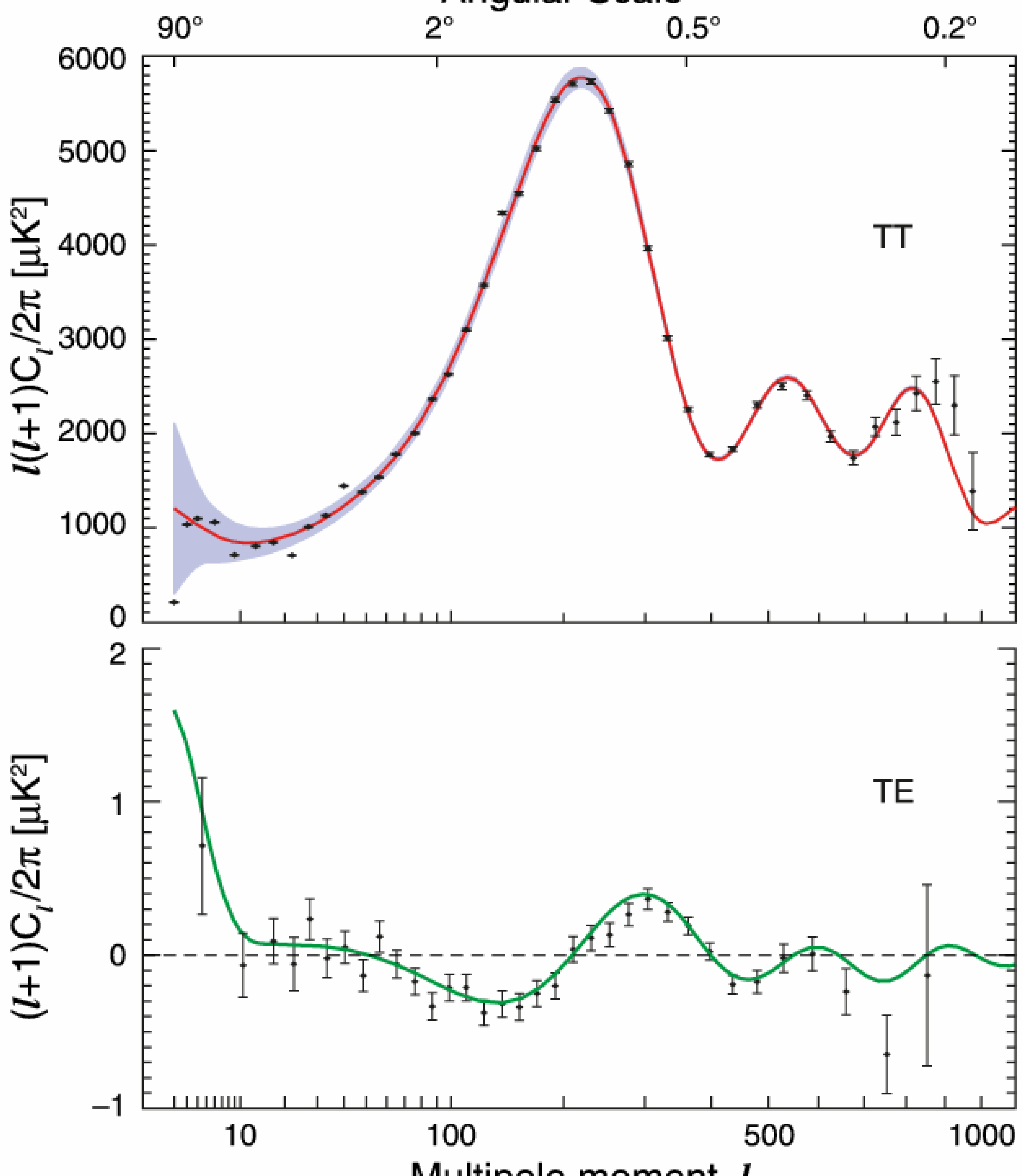
(iv) un éclairage nouveau sur l'inflation et ses prédictions: conditions initiales de l'inflation et suppression du quadrupole du CMB, découverte d'une étape d'inflation rapide (de "fast roll") précédent l'inflation lente (de "slow roll").



60
40
20
0
−20
−40
−60
0
50
100
150
l
C^TE_l l (l+1)/(2 π) (μK)^2



4000
3000
2000
1000
0
$l(l+1)C_l/2\pi\ [\mu K^2]$
TT
TE
2
1
0
−1
$(l+1)C_l/2\pi\ [\mu K^2]$
TE
10
100
500
1000



Angular Scale
90°
2°
0.5°
0.2°
TT
TE
l(l+1)C_l/2π [μK²]
(l+1)C_l/2π [μK²]
Multipole moment l

Effective field theory basics

A single scalar field

$$\mathcal{L} = \frac{1}{2}\partial_\mu \Phi \partial^\mu \Phi + V(\Phi)$$

$$\Phi(\vec{x}, t) = \phi + \delta\Phi(\vec{x}, t)$$

condensate   fluctuation

$$\delta\Psi \sim \frac{\delta T}{T}$$

$$g_{\mu\nu}(\vec{x}, t) = FRW + \delta g_{\mu\nu}(\vec{x}, t)$$

Tensor (grav.waves)   Vectors   Scalar

$$\epsilon_v = \frac{M_P^2}{2}\left[\frac{V^{'}(\phi)}{V(\phi)}\right]^2, \eta_v = M_P^2\,\frac{V^{''}(\phi)}{V(\phi)}, \cdots$$

Slow roll parameters << 1

Curvature perturbations (Gaussian):

Boundary conditions and Quadrupole suppression

COBE, WMAPI, WMAPIII → ***low quadrupole***

$v_k = \delta\Psi_k a(t)\dot{\phi}/H$ **obeys a wave eqn. in conformal time** η

during slow roll

$$\left[\frac{d^2}{d\eta^2} + k^2 - \frac{\nu^2 - \frac{1}{4}}{\eta^2}\right] v_k = 0\ ;\ \nu = \frac{3}{2} + 3\epsilon_v - \eta_v$$

Solutions with Bunch-Davies b.c.

$$g_\nu(k;\eta) = \frac{1}{2}\, i^{\nu+\frac{1}{2}}\, \sqrt{-\pi\eta}\, H_\nu^{(1)}(-k\eta)\ ;\ \ g_\nu(k;\eta) \stackrel{\eta\to-\infty}{=} \frac{1}{\sqrt{2\,k}}\, e^{-ik\eta}$$

Slow Roll expansion: a hierarchy of dimensionless parameters:

$$\epsilon_v = \frac{M_P^2}{2} \left[\frac{V'(\phi)}{V(\phi)} \right]^2 \quad , \quad \eta_v = M_P^2 \, \frac{V''(\phi)}{V(\phi)} \cdots$$

as a 1/Ne expansion:

$$N[\phi(t)] = -\frac{1}{M_P^2} \int_{\phi(t)}^{\phi_{end}} V(\phi) \, \frac{d\phi}{dV} \, d\phi$$

$$\phi = \sqrt{N_e} M_P \, \chi \longleftarrow$$ **Rescale field**

$$V(\phi) = N_e \, M^4 \, w(\chi)$$

$w(\chi) \rightarrow \sim \mathcal{O}(1)$

$M^4 \rightarrow$ **energy scale of inflation**

$$1 = - \int_{\chi_c}^{\chi_{end}} \frac{w(\chi)}{w'(\chi)} \, d\chi$$

value of χ at the END of inflation

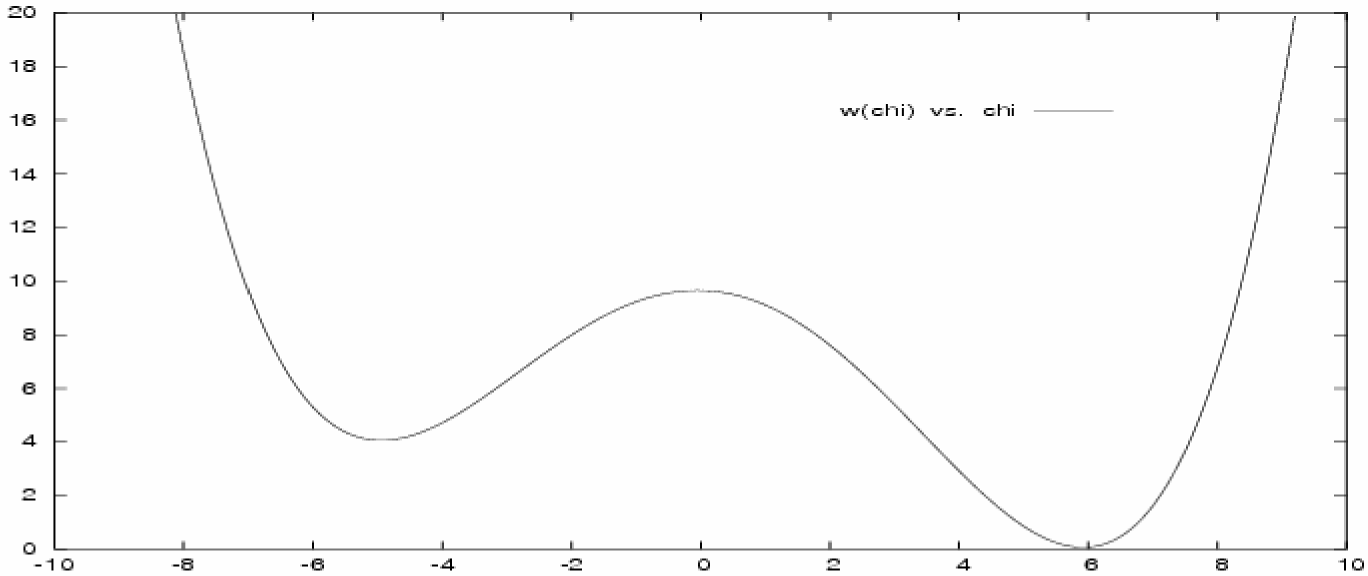
value of χ Ne e-folds before END of inflation

Independent of Ne: an implicit relation between χ and couplings

$$\epsilon_v = \frac{1}{2\,N_e} \left[\frac{w'(\chi_c)}{w(\chi_c)} \right]^2 \quad , \quad \eta_v = \frac{1}{N_e} \, \frac{w''(\chi_c)}{w(\chi_c)}$$

Explicit dependence on Ne: SIMPLE RESCALING

Slow Roll Inflaton Models



$V(\mathrm{Min}) = V'(\mathrm{Min}) = 0$: inflation ends after a finite number of efolds. Universal form of the slow-roll inflaton potential:

$$V(\phi) = N\ M^4\ w\left(\frac{\phi}{\sqrt{N}\ M_{Pl}}\right)$$

$N \sim 50$ number of efolds since horizon exit till end of inflation. $M =$ energy scale of inflation.

Slow-roll is needed to produce enough efolds of inflation.

SLOW and Dimensionless Variables

$$\chi = \frac{\phi}{\sqrt{N}\, M_{Pl}} \quad , \quad \tau = \frac{m\, t}{\sqrt{N}} \quad , \quad \mathcal{H}(\tau) = \frac{H(t)}{m\, \sqrt{N}} \quad , \quad \left(m \equiv \frac{M^2}{M_{Pl}} \right)$$

slow inflaton, slow time, slow Hubble.

χ and $w(\chi)$ are of order one.

Evolution Equations:

$$\mathcal{H}^2(\tau) = \frac{1}{3} \left[\frac{1}{2\, N} \left(\frac{d\chi}{d\tau} \right)^2 + w(\chi) \right] \quad ,$$

$$\frac{1}{N}\, \frac{d^2\chi}{d\tau^2} + 3\, \mathcal{H}\, \frac{d\chi}{d\tau} + w'(\chi) = 0 \quad . \qquad (1)$$

$1/N$ terms: corrections to slow-roll

Higher orders in slow-roll are obtained systematically by expanding the solutions in $1/N$.

Primordial Power Spectrum

Adiabatic Scalar Perturbations: $P(k) = |\Delta^{(S)}_{k\;ad}|^2 \; k^{n_s - 1}$.

To dominant order in slow-roll:

$$|\Delta^{(S)}_{k\;ad}|^2 = \frac{N^2}{12\,\pi^2} \; \left(\frac{M}{M_{Pl}}\right)^4 \; \frac{w^3(\chi)}{w'^2(\chi)} \; .$$

Hence, for all slow-roll inflation models:

$$|\Delta^{(S)}_{k\;ad}| \sim \frac{N}{2\,\pi\sqrt{3}} \left(\frac{M}{M_{Pl}}\right)^2$$

The WMAP result $\quad |\Delta^{(S)}_{k\;ad}| = (0.467 \pm 0.023) \times 10^{-4}$

determines the scale of inflation M

$$\left(\frac{M}{M_{Pl}}\right)^2 = 1.02 \times 10^{-5} \Longrightarrow M = 0.77 \times 10^{16} \text{ GeV}$$

The inflation energy scale turns to be the grand unification energy scale !!

This statement is model independent [independent of the shape of $w(\chi)$].

spectral index n_s and the ratio r

$r \equiv$ ratio of tensor to scalar fluctuations.
tensor fluctuations = primordial gravitons.

$$n_s - 1 = -\frac{3}{N} \left[\frac{w'(\chi)}{w(\chi)}\right]^2 + \frac{2}{N} \frac{w''(\chi)}{w(\chi)} \quad ,$$

$$r = \frac{8}{N} \left[\frac{w'(\chi)}{w(\chi)}\right]^2 \quad . \qquad (2)$$

χ is the inflaton field at horizon exit.

$n_s - 1$ and r are always of order $1/N \sim 0.02$.

D. Boyanovsky, H. J. de Vega, N. G. Sanchez, Phys. Rev. D 73, 023008 (2006), astro-ph/0507595.

New vs. chaotic inflation and reconstruction program: confronting *WMAP 3*

Implement eff. field theory + slow roll as 1/Ne expansion to systematically explore a large ``family'' of inflaton potentials.

WMAP 3 + LSS:

$$n_s = 0.958 \pm 0.016 \quad \text{(assuming } r = 0 \text{ with no running)}$$

$$r < 0.28 \ (95\% \, CL) \quad \text{no running}$$

$$r < 0.67 \ (95\% \, CL) \quad \text{with running}$$

Ginsburg-Landau Approach

We choose a polynomial for $w(\chi)$. A quartic $w(\chi)$ is renormalizable. Higher order polynomials are acceptable since inflation it is an effective theory.

$$w(\chi) = w_o \pm \frac{\chi^2}{2} + G_3\,\chi^3 + G_4\,\chi^4 \quad , \quad G_3 = \mathcal{O}(1) = G_4$$

$$V(\phi) = N\;M^4\;w\left(\frac{\phi}{\sqrt{N}\,M_{Pl}}\right) = V_o \pm \frac{m^2}{2}\,\phi^2 + g\,\phi^3 + \lambda\,\phi^4\,.$$

$$m = \frac{M^2}{M_{Pl}} \quad , \quad g = \frac{m}{\sqrt{N}}\left(\frac{M}{M_{Pl}}\right)^2 G_3 \quad , \quad \lambda = \frac{G_4}{N}\left(\frac{M}{M_{Pl}}\right)^4$$

Notice that

$$\left(\frac{M}{M_{Pl}}\right)^2 \simeq 10^{-5} \quad , \quad \left(\frac{M}{M_{Pl}}\right)^4 \simeq 10^{-10} \quad , \quad N \simeq 50\,.$$

- Small couplings arise naturally as ratio of two energy scales: inflation and Planck.
- The inflaton is a light particle:

$$m = \frac{M^2}{M_{Pl}} \simeq 0.003\,M \quad , \quad m = 2.5\times 10^{13}\mathrm{GeV}$$

The number of efolds in Slow-roll

The number of e-folds $N[\chi]$ since the field χ exits the horizon till the end of inflation is:

$N[\chi] = N \int_{\chi_{end}}^{\chi} \frac{w(\chi)}{w'(\chi)} \, d\chi$. We choose then $N = N[\chi]$.

The spontaneously broken symmetric potential:

$$w(\chi) = \frac{y}{32} \left(\chi^2 - \frac{8}{y}\right)^2$$

produces inflation with $0 < \sqrt{y} \, \chi_{initial} \ll 1$ and $\chi_{end} = \sqrt{\frac{8}{y}}$.

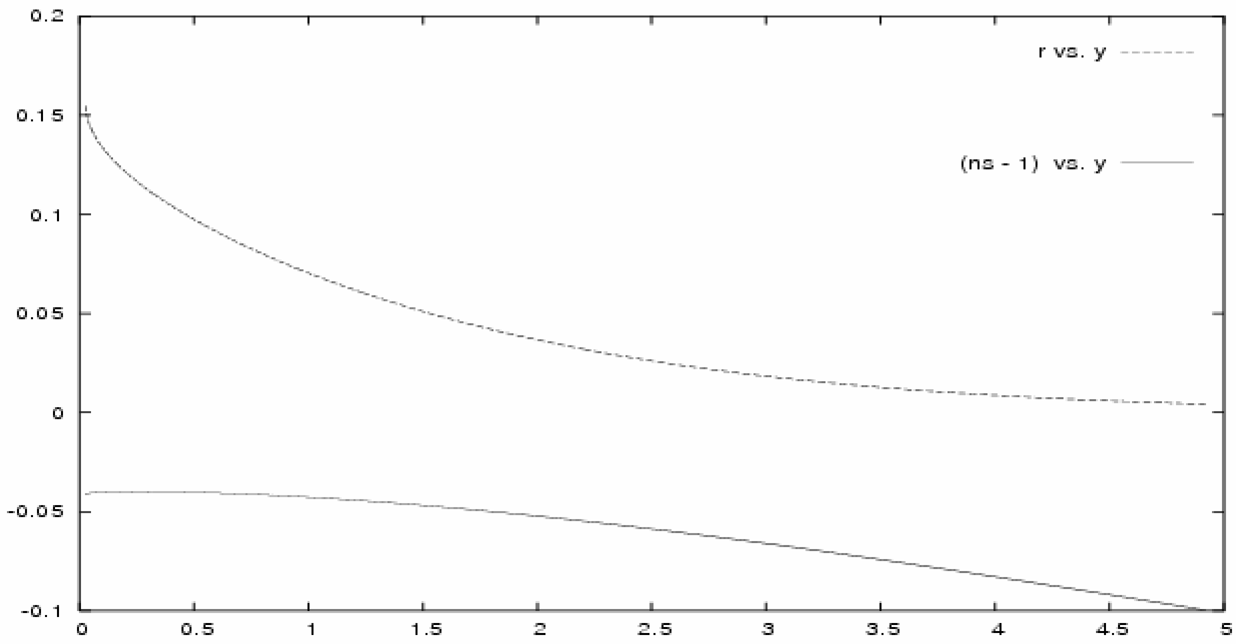
This is small field inflation.

From the above integral: $\quad y = z - 1 - \log z$
where $z \equiv y \, \chi^2/8$ and we have $0 < y < \infty$ for $1 > z > 0$.
Spectral index n_s and the ratio r as functions of y:

$n_s = 1 - \frac{y}{N} \frac{3\,z+1}{(z-1)^2} \quad , \quad r = \frac{16\,y}{N} \frac{z}{(z-1)^2}$

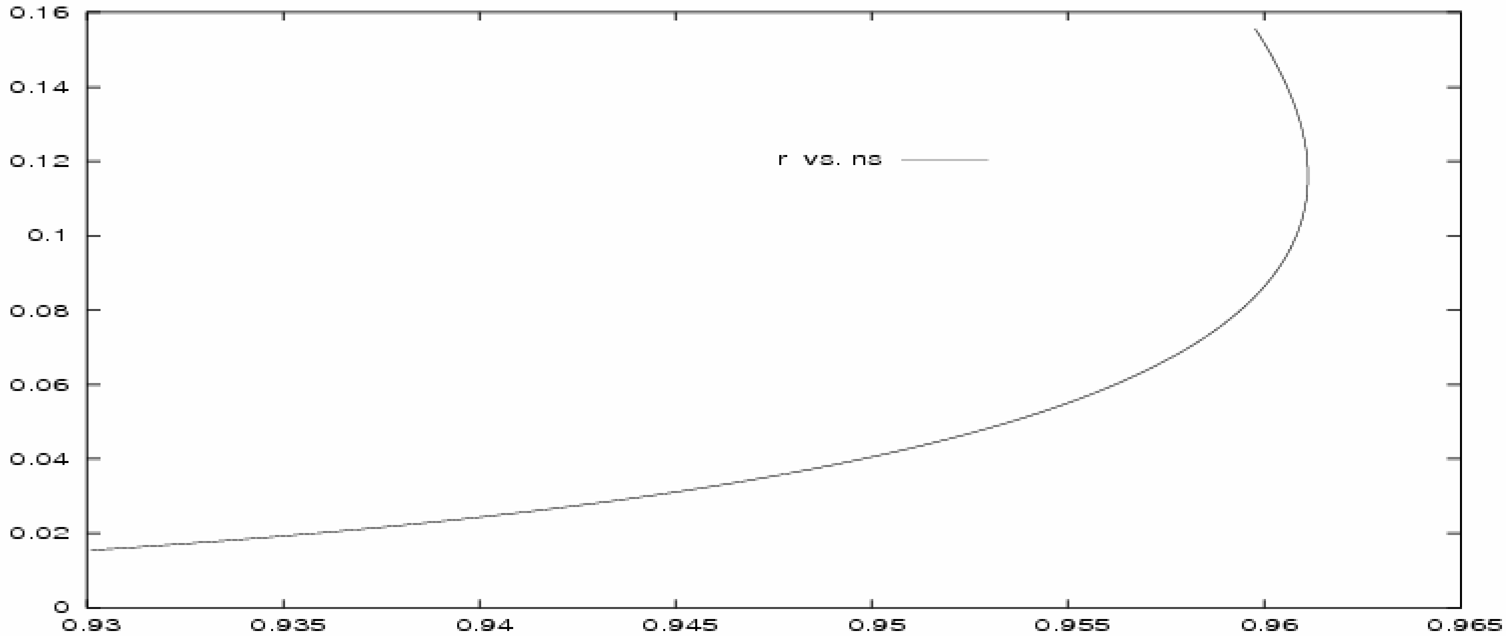
Binomial New Inflation: (y = coupling).

r decreases monotonically with y :
(strong coupling) $0 < r < \frac{8}{N} = 0.16$ (zero coupling).



n_s first grows with y, reaches a maximum value $n_{s,maximum} = 0.96139\ldots$ at $y = 0.2387\ldots$ and then n_s decreases monotonically with y.

Binomial New Inflation



$r = \frac{8}{N} = 0.16$ and $n_s = 1 - \frac{2}{N} = 0.96$ at $y = 0$.

r is a double valued function of n_s.

Trinomial Inflationary Models

- Trinomial Chaotic inflation:
  $$w(\chi) = \frac{1}{2}\,\chi^2 + \frac{h}{3}\,\sqrt{\frac{y}{2}}\,\chi^3 + \frac{y}{32}\,\chi^4 \,.$$
- Trinomial New inflation:
  $$w(\chi) = -\frac{1}{2}\,\chi^2 + \frac{h}{3}\,\sqrt{\frac{y}{2}}\,\chi^3 + \frac{y}{32}\,\chi^4 + \frac{2}{y}\,F(h) \,.$$

h = asymmetry parameter. $w(\mathrm{min}) = w'(\mathrm{min}) = 0$,
$y =$ quartic coupling, $F(h) = \frac{8}{3}\,h^4 + 4\,h^2 + 1 + \frac{8}{3}\,|h|\,(h^2+1)^{\frac{3}{2}}$.

H. J. de Vega, N. G. Sanchez, Single Field Inflation models allowed and ruled out by the three years WMAP data. Phys. Rev. D 74, 063519 (2006), astro-ph/0604136.

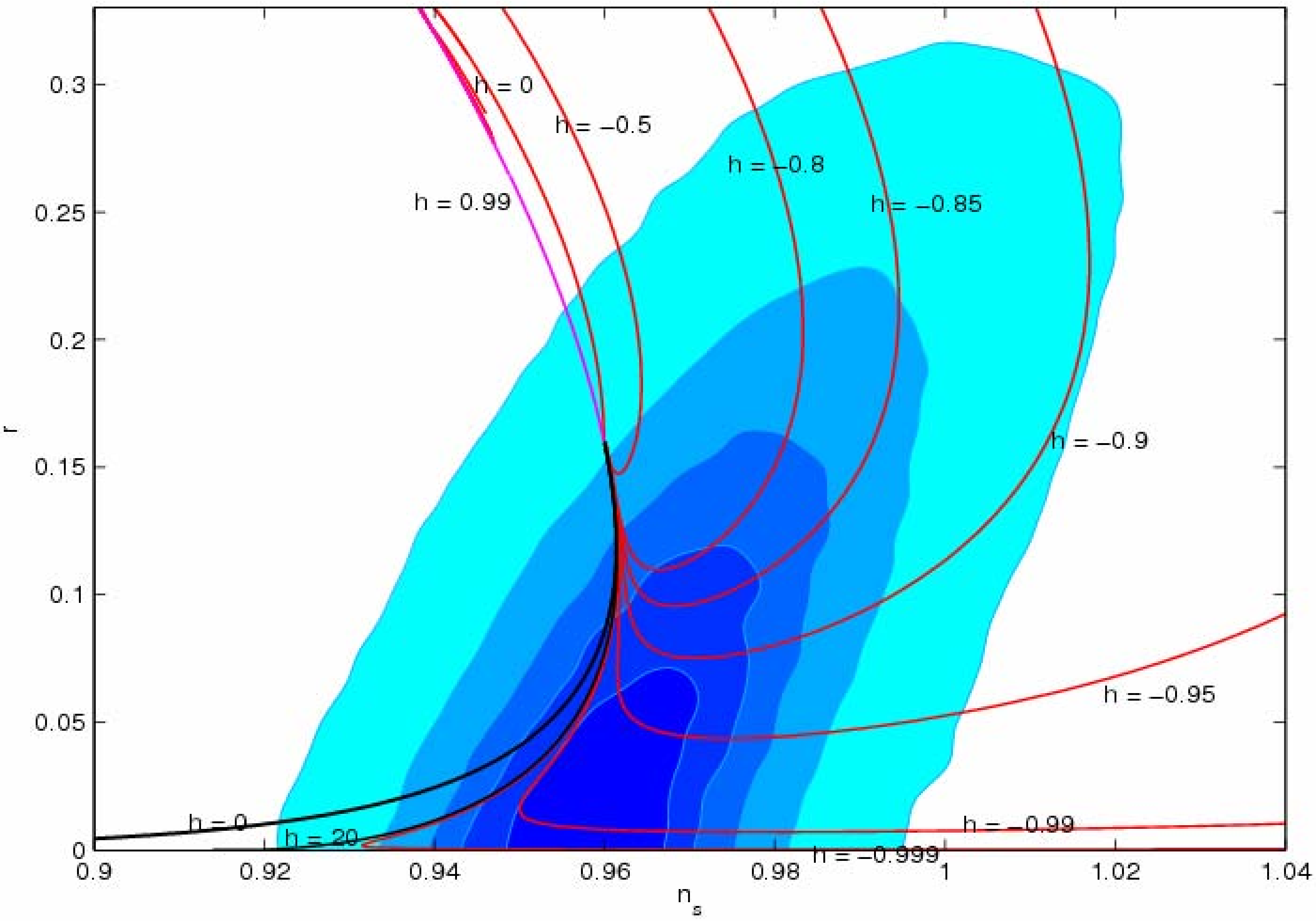
Monte Carlo Markov Chains Analysis of Data: MCMC.

MCMC is an efficient stochastic numerical method to find the probability distribution of the theoretical parameters that describe a set of empirical data.

We found n_s and r and the couplings y and h by MCMC.
NEW: We imposed as a hard constraint that r and n_s are given by the trinomial potential.
Our analysis differs in this crucial aspect from previous MCMC studies of the WMAP data.

The color–filled areas correspond to 12%, 27%, 45%, 68% and 95% confidence levels according to the WMAP3 and Sloan data.
The color of the areas goes from the darker to the lighter for increasing CL.

MCMC Results for Trinomial New Inflation.



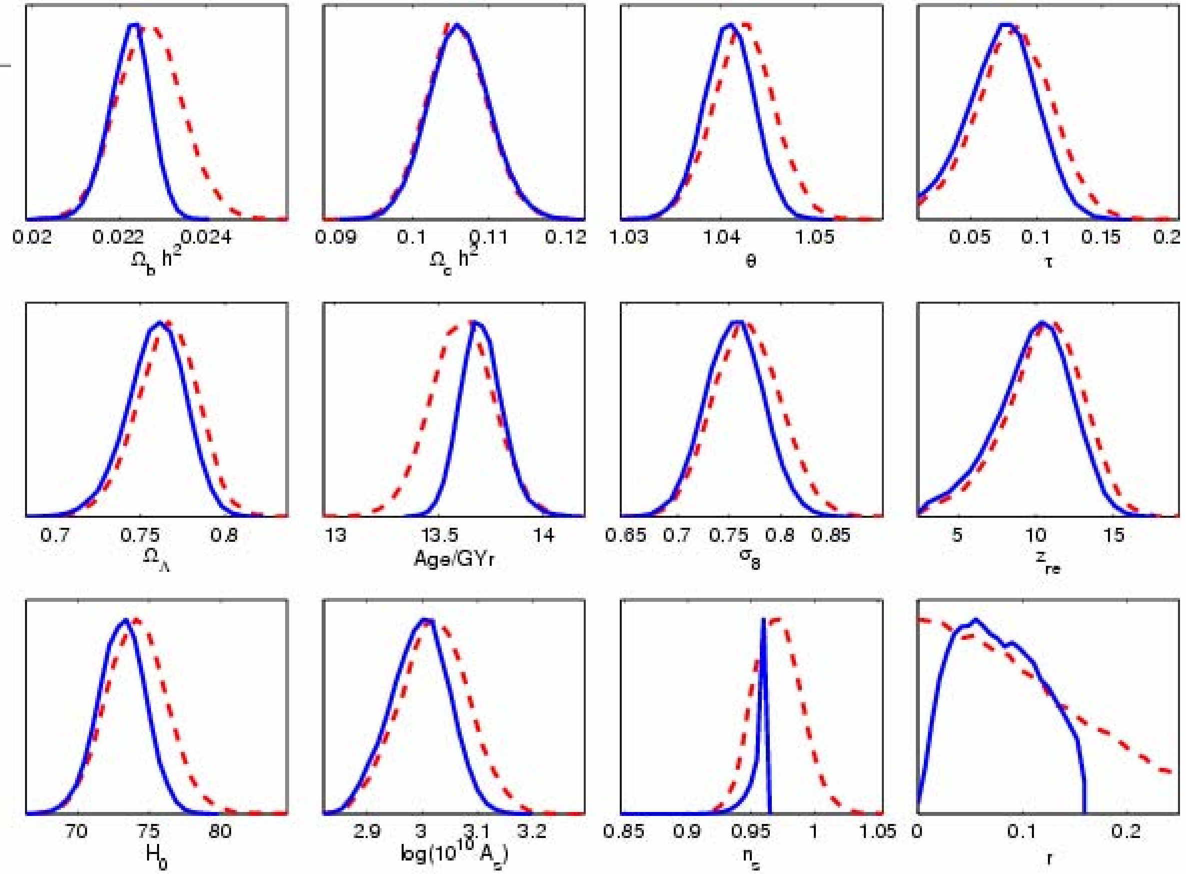
MCMC Results for Trinomial New Inflation.

Bounds: $r > 0.016$ (95% CL) , $r > 0.049$ (68% CL)
Most probable values: $n_s \simeq 0.956$, $r \simeq 0.055$.
The most probable trinomial potential for new inflation is symmetric and has a moderate nonlinearity with the quartic coupling $y \simeq 1.5\ldots$ and $h < 0.3$.
We can choose $h = 0$ and we then find $y \simeq 1.322\ldots$.
The $\chi \to -\chi$ symmetry is here spontaneously broken since the absolute minimum of the potential is at $\chi \neq 0$.

$$w(\chi) = \frac{y}{32}\left(\chi^2 - \frac{8}{y}\right)^2$$

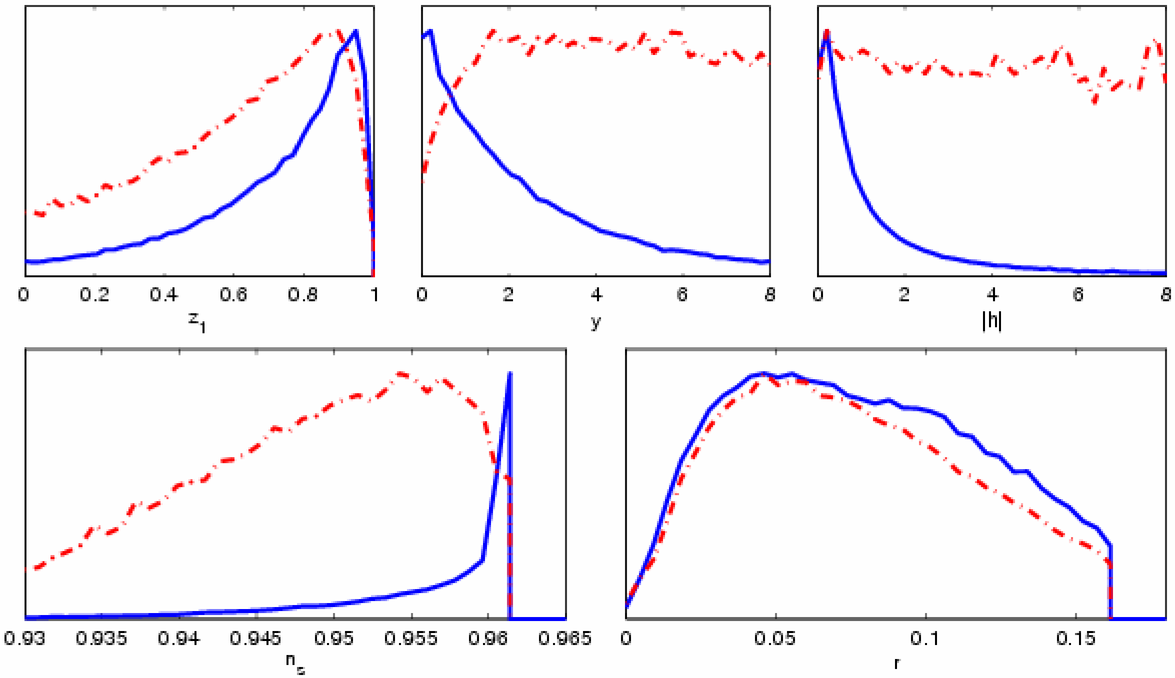
C. Destri, H. J. de Vega, N. Sanchez, MCMC analysis of WMAP3 data points to broken symmetry inflaton potentials and provides a lower bound on the tensor to scalar ratio, Phys. Rev. D77, 043509 (2008), astro-ph/0703417.
Same results from WMAP5 data. Acbar08 data slightly increases $n_s < 1$.

Marginalized probability distributions. New Inflation.



Imposing the trinomial potential (solid blue curves) and just the ΛCDM+r model (dashed red curves).
(curves normalized to have the maxima equal to one).

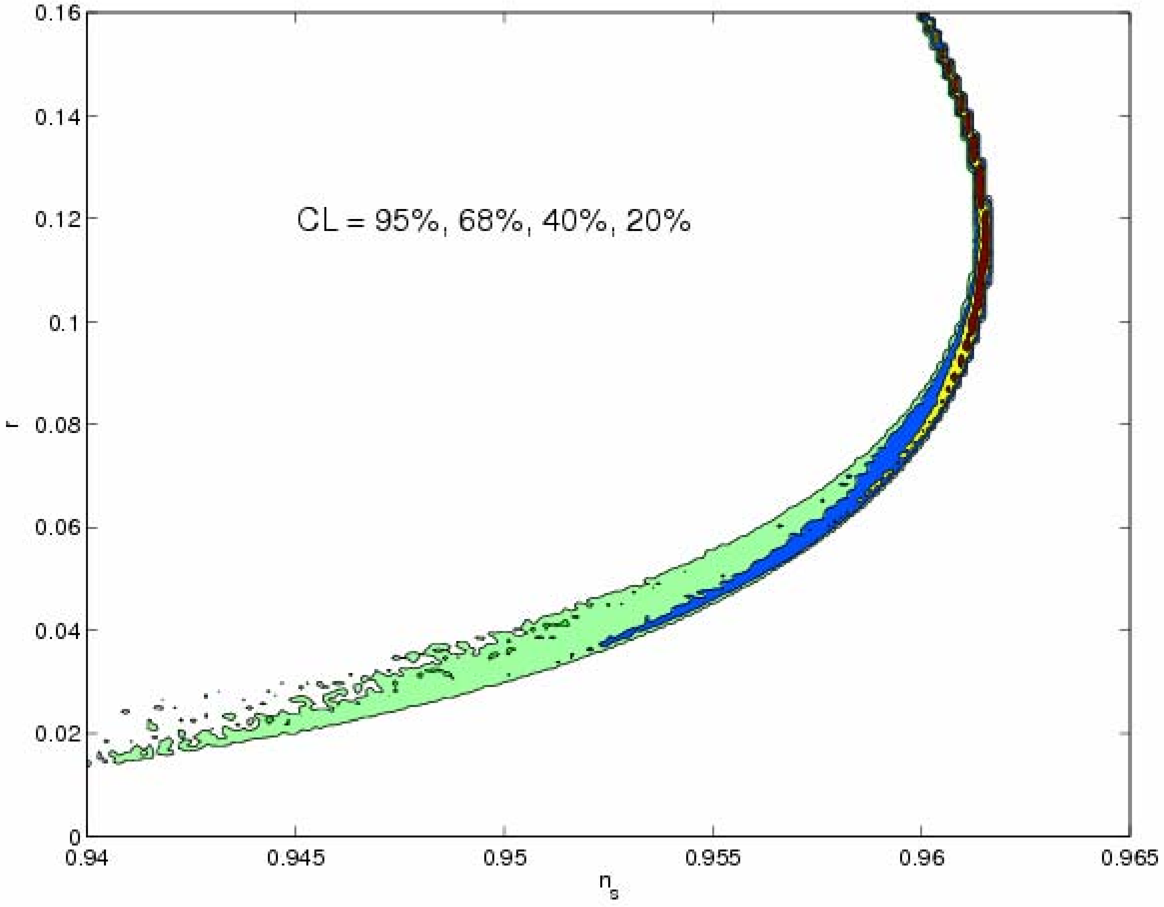
Probability Distributions. Trinomial New Inflation.



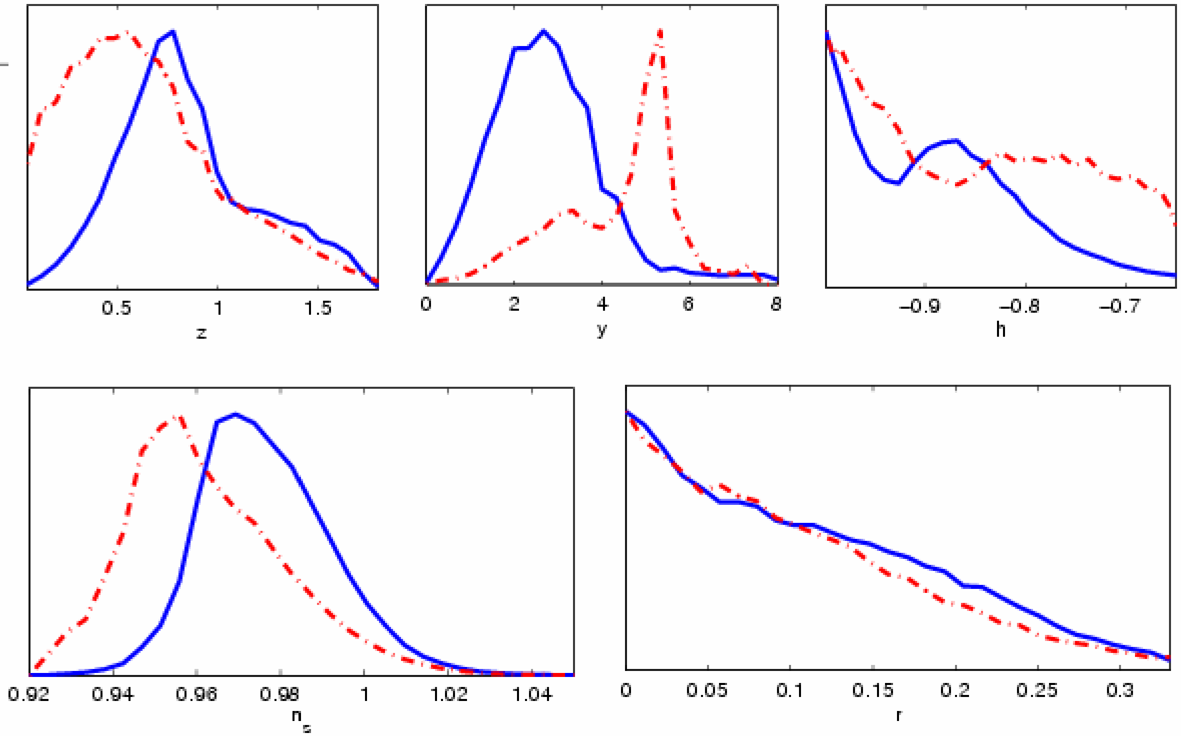
Probability distributions: solid blue curves
Mean likelihoods: dot-dashed red curves.

$$z_1 = 1 - \frac{y}{8\left(|h|+\sqrt{h^2+1}\right)^2}\ \chi^2 \ .$$

r vs. n_s data within the Trinomial New Inflation Region.



Probability Distributions. Trinomial Chaotic Inflation.



Probability distributions (solid blue curves) and mean likelihoods (dot-dashed red curves).
The data request a strongly asymmetric potential in chaotic inflation almost having two minima. That is, a strong breakdown of the $\chi \to -\chi$ symmetry.

$$n_s - 1 = -6\epsilon_v + 2\eta_v \sim 1/N_e$$

$$r = 16\epsilon_v \sim 1/N_e$$

$$\frac{dn_s}{d\ln k} = -\frac{2}{N_e^2}\left\{\frac{w'(\chi_c)w'''(\chi_c)}{w^2(\chi_c)} + 3\left[\frac{w'(\chi_c)}{w(\chi_c)}\right]^4 - 4\,\frac{[w'(\chi_c)]^2\,w''(\chi_c)}{w^3(\chi_c)}\right\}$$

Simple scaling with 1/N_e, choose N = 50 as representative

Family of potentials

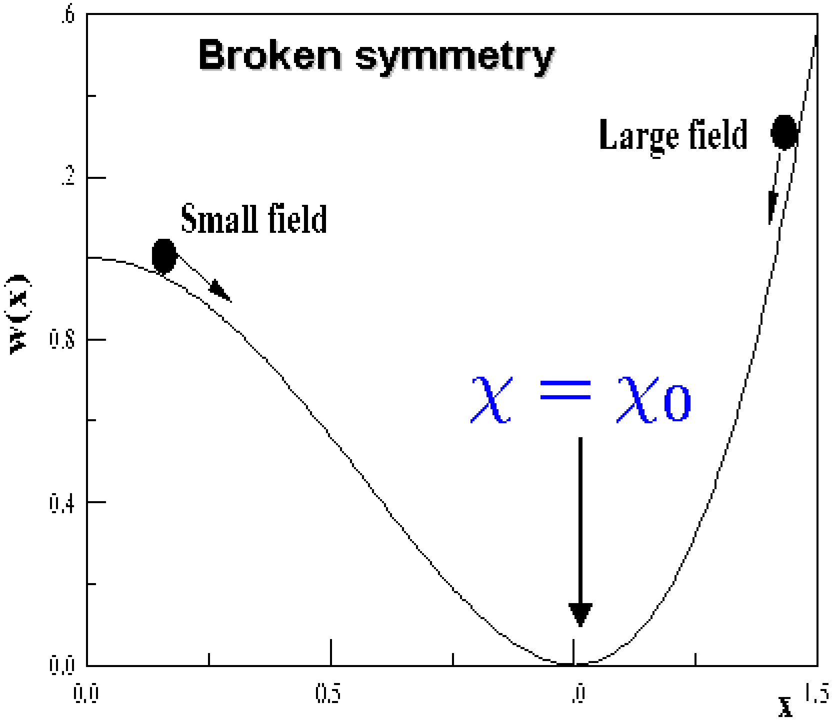
$$V(\phi) = V_0 - \frac{1}{2}\,m^2\,\phi^2 + \frac{\lambda}{2\,n}\,\phi^{2\,n} \quad \text{, broken symmetry}$$

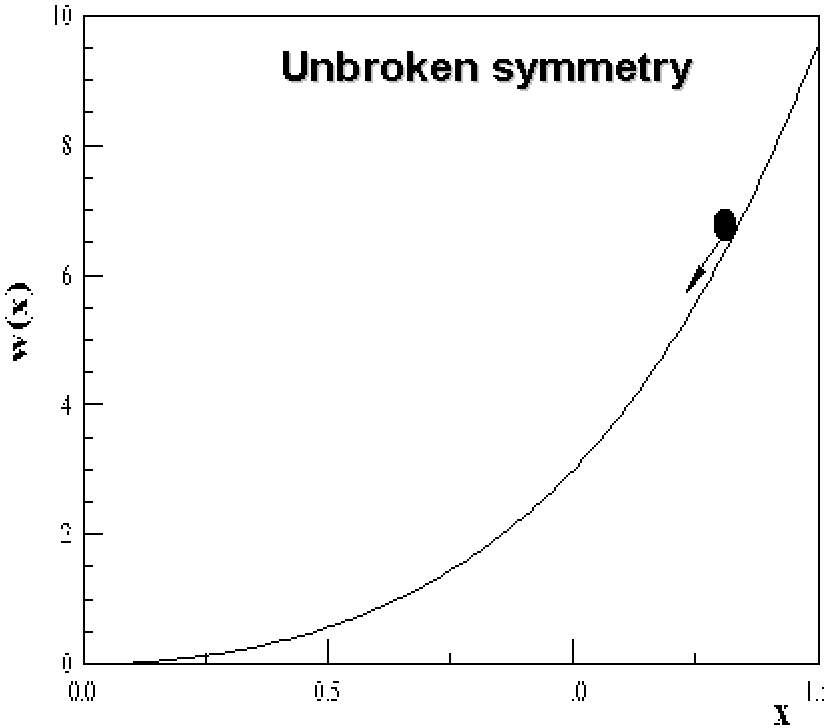
$$V(\phi) = \frac{1}{2}\,m^2\,\phi^2 + \frac{\lambda}{2\,n}\,\phi^{2\,n} \quad \text{, unbroken symmetry}$$

Rescale fields and couplings:

$$\lambda = \frac{m^2\, g}{M_{Pl}^{2\,n-2}\; N_e^{n-1}} \; ; \; g = \frac{1}{\chi_0^{2\,n-2}} \; ; \; x = \frac{\chi}{\chi_0}$$

$$w(\chi) = \frac{\chi_0^2}{2\,n} \left[n\,(1-x^2) + x^{2\,n} - 1 \right] \qquad w(\chi) = \frac{\chi_0^2}{2\,n} \left[n\,x^2 + x^{2\,n} \right]$$





Broken Symmetry

$$\frac{2\,n}{\chi_0^2} = \int_X^1 \frac{dx}{x}\,\frac{n\,(1-x^2)+x^{2\,n}-1}{1-x^{2\,n-2}}$$

Unbroken symmetry

$$\frac{2\,n}{\chi_0^2} = \int_0^X \frac{n+x^{2\,n-2}}{1+x^{2\,n-2}}\;x\;dx$$

$$X = \frac{\chi_c}{\chi_0}$$

Conditions for number of e-folds

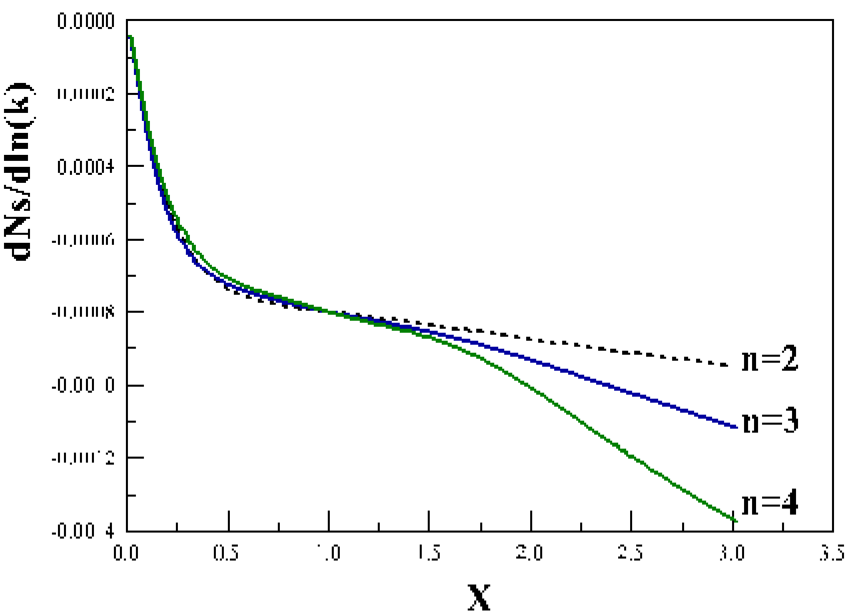
- X determines the value of the field Ne e-folds before the end of inflation
- χ_0 determines the value of the coupling for which the value of X is Ne before the end of inflation

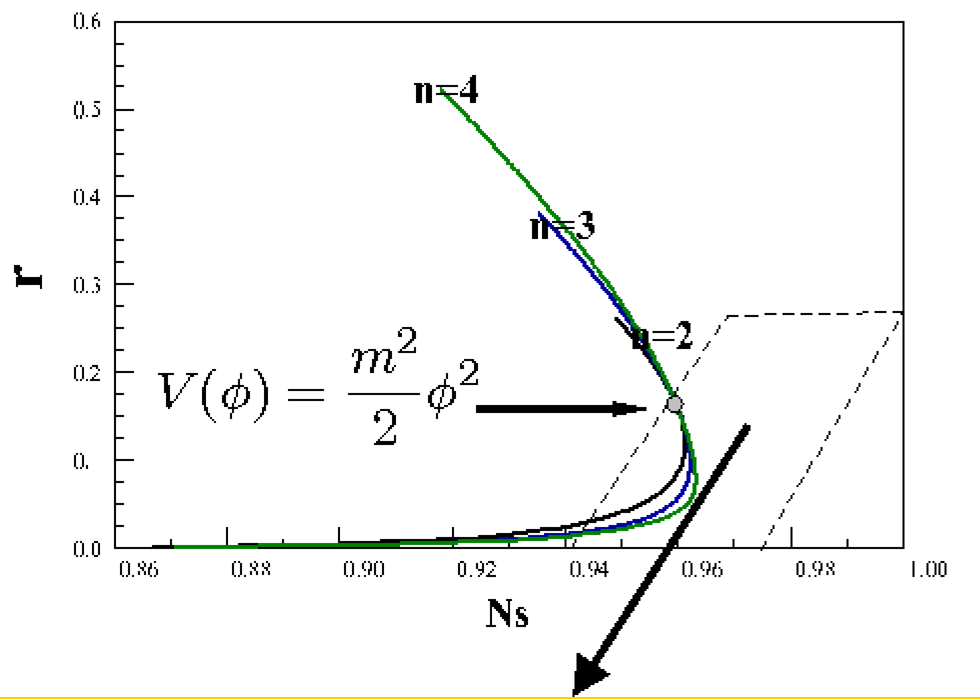
STRATEGY

1) Vary X, find χ_0 construct w[x] and derivatives
2) Find ϵ_v, η_v, $n_s, r, dn_s/d\ln k$ as a function of X
3) Plot parametrically

RESULTS

1) New inflation (B.S.) Ne= 50 (change accordingly)

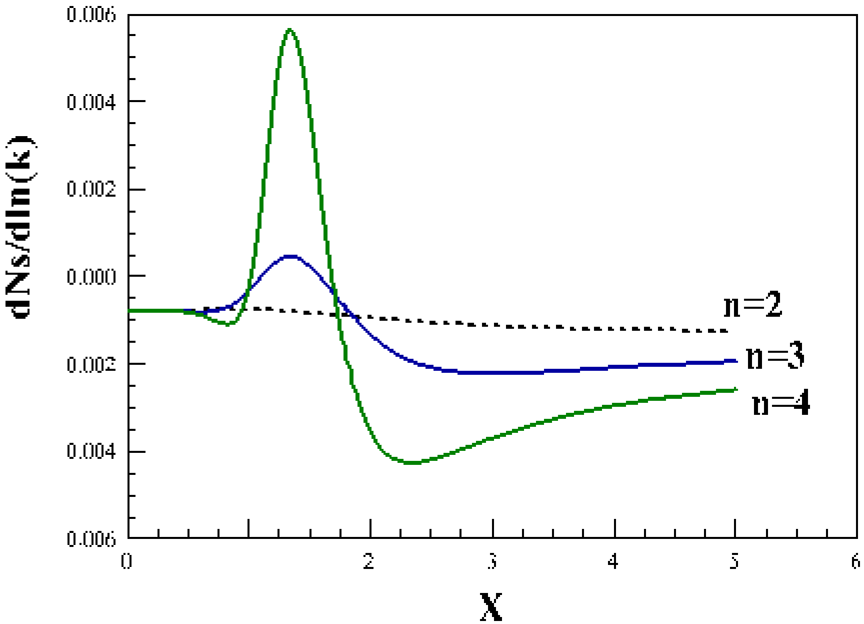


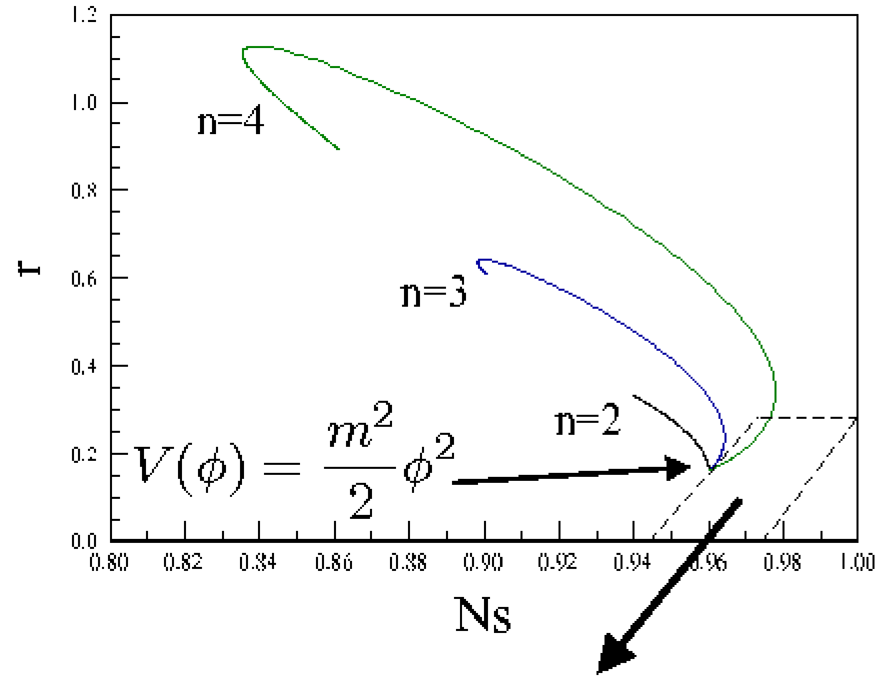


WMAP 3 marginalized region of r- n_s (95% CL)

* *Large region of consistency for small field New Inflation*

2) Chaotic inflation





WMAP 3 marginalized region of r- ns (95% CL)

*** Small region of consistency with WMAP 3**

The Energy Scale of Inflation

Grand Unification Idea (GUT)

- Renormalization group running of electromagnetic, weak and strong couplings shows that they all meet at $E_{GUT} \simeq 2 \times 10^{16}$ GeV
- Neutrino masses are explained by the see-saw mechanism: $m_\nu \sim \frac{M^2_{\rm Fermi}}{M_R}$ with $M_R \sim 10^{16}$ GeV.
- Inflation energy scale: $M \simeq 10^{16}$ GeV.

Conclusion: the GUT energy scale appears in at least three independent ways.

Moreover, moduli potentials: $V_{moduli} = M^4_{\rm SUSY} \; v\left(\frac{\phi}{M_{Pl}}\right)$ ressemble inflation potentials provided $M_{\rm SUSY} \sim 10^{16}$ GeV.
First observation of SUSY in nature??

De Sitter Geometry and Scale Invariance

The De Sitter metric is scale invariant:

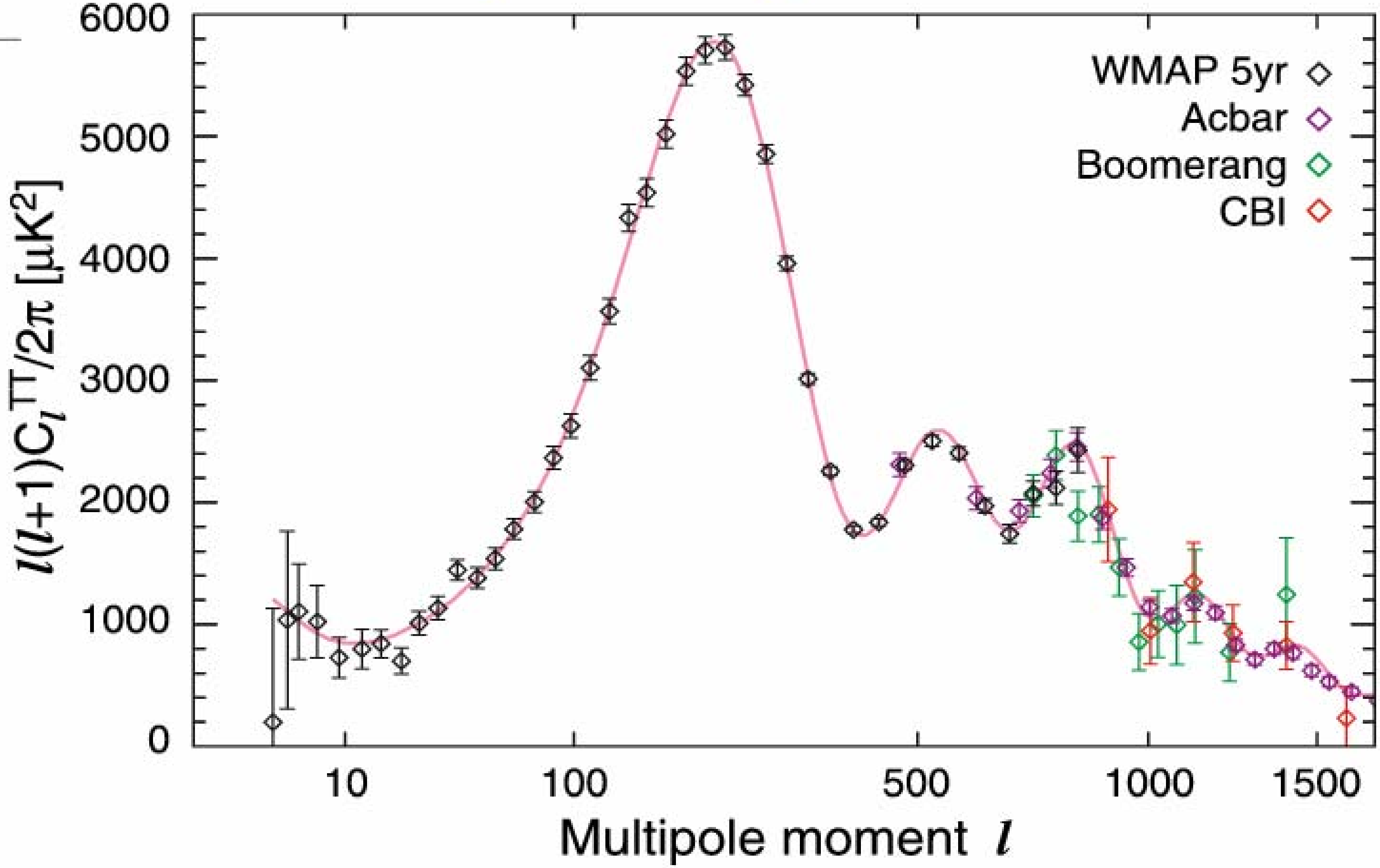
$$ds^2 = \frac{1}{(H\,\eta)^2}\left[(d\eta)^2 - (d\vec{x})^2\right] \ .$$

$\eta =$ conformal time.
But inflation only lasts for N efolds !
Corrections to scale invariance:
$|n_s - 1|$ as well as the ratio r are of order $\sim 1/N$
$n_s = 1$ and $r = 0$ correspond to a critical point.
It is a gaussian fixed point around which the inflation model hovers in the renormalization group (RG) sense with an almost scale invariant spectrum during the slow roll stage.
The quartic coupling:

$$\lambda = \frac{G_4}{N}\left(\frac{M}{M_{Pl}}\right)^4 \quad , \quad N = \log\frac{a(\text{inflation end})}{a(\text{horizon exit})}$$

runs like in four dimensional RG in flat euclidean space.

WMAP 5 years data plus further data



Theory (ΛCDM) and observations nicely agree except for the lowest multipoles: the quadrupole suppression.

Quadrupole Suppression and Fast-Roll

Slow-roll inflation is generically preceded by a fast-roll stage where $\dot\phi^2 \sim V(\phi)$. Fast-Roll typically lasts 1 efold.
The slow-roll regime is an attractor with a large basin of attraction.
During fast-roll curvature and tensor perturbations feel a potential equal to the slow-roll potential plus an extra attractive piece. This new piece suppresses the low multipoles as $1/\ell^2$.
If the quadrupole modes ($\sim$ Hubble radius today) exited the horizon about the end of fast-roll, then the quadrupole modes get suppresed $\sim 20\%$ in agreement with the observations. Upper bound on the total number of inflation efolds: $N_{total} < 82$. Favoured value: $N_{total} \simeq 66$.

C. Destri, H. J. de Vega, N. G. Sanchez, arXiv:0804.2387, to appear in PRD. D. Boyanovsky, H. J. de Vega, N. G. Sanchez, Phys. Rev. D74, 123006 and 123007 (2006).

Fast and Slow Roll Inflation

$$H^2 = \frac{1}{3\, M_{PL}^2} \left[\frac{1}{2}\, \dot{\Phi}^2 + V(\Phi)\right] \ ,$$

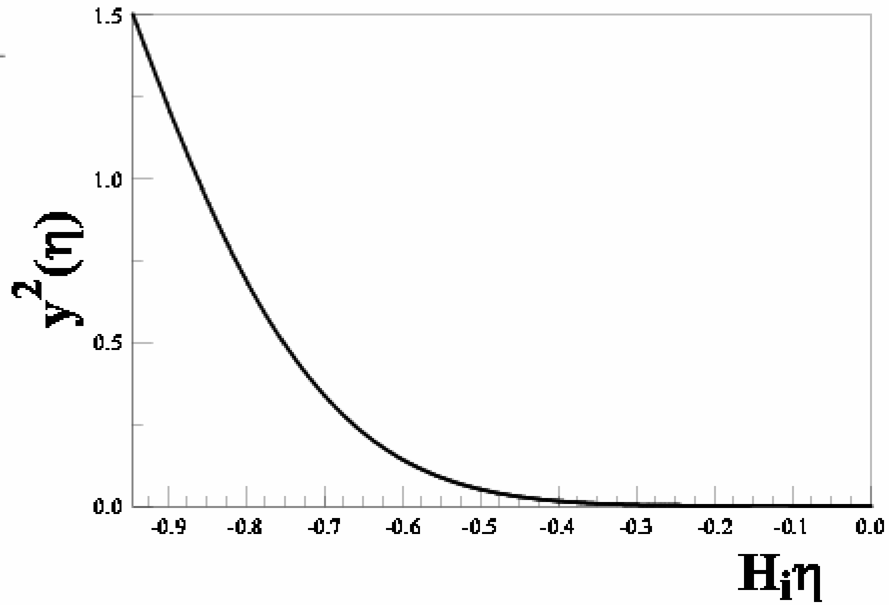
$$\ddot{\Phi} + 3\, H\, \dot{\Phi} + V'(\Phi) = 0 \ .$$

Slow-roll corresponds to $\dot{\Phi}^2 \ll V(\Phi)$.

Generically, we can have $\dot{\Phi}^2 \sim V(\Phi)$ to start.
That is, FAST ROLL inflation.

However, slow-roll is an attractor with a large basin.

Fast roll for new inflation



$$y^2 = \frac{\dot\Phi^2}{2\, M_{Pl}^2\; H^2} = 3\left[1 - \frac{V(\Phi)}{3\, M_{Pl}^2\;\, H^2}\right]\;,\; 0 \leq y^2 \leq 3\;, \qquad N \sim 60.$$

$\eta =$ conformal time.

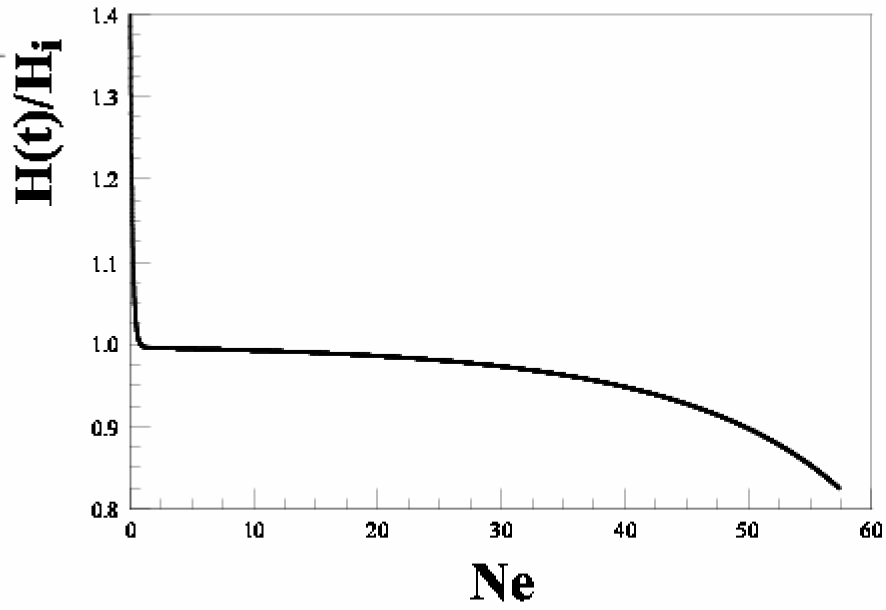
$H_i =$ Hubble at the beginning of slow-roll.

$y^2 \sim 1 =$ Fast-roll for $H_i\,\eta < -0.5$.

$y^2 = \frac{1}{N} \ll 1 =$ slow-roll for $H_i\,\eta > -0.5$.

[$y^2 = \epsilon_V$ during slow-roll.]

Hubble vs. number of efolds



$H_i =$ Hubble at the beginning of slow-roll.
Fast-roll lasts about one-efold.

Extreme fast roll solution ($y^2 = 3$) in cosmic time:

$$H = \frac{1}{3\,t} \quad , \quad a(t) = a_0 \; t^{\frac{1}{3}} \quad , \quad \Phi = -M_{Pl} \sqrt{\frac{2}{3}} \; \log(m\;t) \; .$$

Gauge Invariant Curvature Perturbations

$\mathcal{R}(\boldsymbol{x},t) = -\psi(\boldsymbol{x},t) - \frac{H(t)}{\dot{\Phi}(t)}\,\phi(\boldsymbol{x},t)$

$\phi(\boldsymbol{x},t) =$ inflaton fluctuations. $\psi(\boldsymbol{x},t) =$ newtonian potential.

These fluctuations around the FRW geometry are responsible of the CMB anisotropies and the LSS formation.

Gauge invariant potential

$u(\boldsymbol{x},t) \equiv -z(t)\,\mathcal{R}(\boldsymbol{x},t)\;,\; z(t) \equiv a(t)\,\frac{\dot{\Phi}(t)}{H(t)}$

In Fourier space: $u(\boldsymbol{k},\eta) = \alpha_{\mathcal{R}}(\boldsymbol{k})\,S_{\mathcal{R}}(k;\eta) + \alpha^{\dagger}_{\mathcal{R}}(\boldsymbol{k})\,S^{*}_{\mathcal{R}}(k;\eta)$

$\alpha^{\dagger}_{\mathcal{R}}(\boldsymbol{k})$ and $\alpha_{\mathcal{R}}(\boldsymbol{k})$ are creation and annihilation operators.

The mode functions obey a Schrödinger-like equation,

$$\left[\frac{d^2}{d\eta^2} + k^2 - W_{\mathcal{R},\mathcal{T}}(\eta)\right] S_{\mathcal{R},\mathcal{T}}(k;\eta) = 0\;.$$

A: a fast-roll stage PRIOR to slow roll

Allowing for ***RAPID*** variation of the condensate ϕ

$$\left[\frac{d^2}{d\eta^2} + k^2 - \frac{\nu^2 - \frac{1}{4}}{\eta^2} - \mathcal{V}(\eta)\right] v_k = 0$$

↓

Depends on high(er) derivatives of ϕ : ***neglible in slow roll***, ***large for fast roll***

WHEN? large INITIAL $\dot{\phi}$ but large FRICTION term → **short fast roll stage**

→ $\mathcal{V}(\eta) =$ ***LOCALIZED POTENTIAL***

$D(k) \propto T(k)$ = Transmission coeff. of SCATTERING PBM!!

Scalar Curvature and tensor fluctuations

$W_{\mathcal{R}}(\eta) = \frac{1}{z}\frac{d^2 z}{d\eta^2}$ for scalar, $W_{\mathcal{T}}(\eta) = \frac{1}{a}\frac{d^2 a}{d\eta^2}$ for tensor.

$W_{\mathcal{R},\mathcal{T}}(\eta) = \frac{\nu^2_{\mathcal{R},\mathcal{T}} - \frac{1}{4}}{\eta^2} + \mathcal{V}_{\mathcal{R},\mathcal{T}}(\eta)$.

Like a centrifugal barrier plus $\mathcal{V}_{\mathcal{R},\mathcal{T}}(\eta)$.

scalar: $\nu_{\mathcal{R}} = \frac{3}{2} + 3\,\epsilon_V - \eta_V$, tensor: $\nu_{\mathcal{T}} = \frac{3}{2} + \epsilon_V$

$$\epsilon_V = \frac{1}{2N}\left[\frac{w'(\chi)}{w(\chi)}\right]^2 \quad , \quad \eta_V = \frac{1}{N}\,\frac{w''(\chi)}{w(\chi)} \; .$$

$\mathcal{V}(\eta) = 0$ during slow-roll, $\mathcal{V}(\eta) \neq 0$ during fast-roll.

During slow-roll: $S(k;\eta) = A(k)\, g_\nu(k;\eta) + B(k)\, f_\nu(k;\eta)$

$g_\nu(k;\eta) = \frac{1}{2}\, i^{\nu+\frac{1}{2}}\, \sqrt{-\pi\eta}\, H^{(1)}_\nu(-k\eta)$, $f_\nu(k;\eta) = [g_\nu(k;\eta)]^*$

$H^{(1)}_\nu(z)$: Hankel function.

Scale invariant limit: $g_{\frac{3}{2}}(k;\eta) = \frac{e^{-ik\eta}}{\sqrt{2k}}\left[1 - \frac{i}{k\eta}\right]$.

The effect of $\mathcal{V}_{\mathcal{R},\mathcal{T}}(\eta)$ during the fast roll

The initial conditions on the modes $S(k;\eta)$ plus $\mathcal{V}_{\mathcal{R},\mathcal{T}}(\eta)$ determine the coefficients $A_{\mathcal{R},\mathcal{T}}(k)$ and $B_{\mathcal{R},\mathcal{T}}(k)$.

We choose Bunch-Davies initial conditions:

$$S_\nu(k;\eta) \overset{\eta\to-\infty}{=} \frac{1}{\sqrt{2\,k}}\, e^{-ik\eta}$$

$$\mathcal{V}_{\mathcal{R},\mathcal{T}}(\eta) = 0 \longrightarrow A(k) = 1,\ B(k) = 0$$

$\mathcal{V}_{\mathcal{R},\mathcal{T}}(\eta) \neq 0$ is analogous to a one dimensional scattering problem in the η-axis.

D. Boyanovsky, H. J. de Vega, N. Sanchez,
CMB quadrupole suppression:
I. Initial conditions of inflationary perturbations,
II. The early fast roll stage,
Phys.Rev. D74 (2006) 123006 and 123007,
astro-ph/0607508 and astro-ph/0607487.

The Evolution of Perturbations as a Scattering Problem.

Fluctuations	Scattering Problem
$-\infty < \eta < 0$	$0 < r < \infty$
Bunch-Davies init. conditions: $S(k;\eta) = \frac{e^{-ik\eta}}{\sqrt{2k}}$ for $\eta \to -\infty$	Jost solutions: $f_\nu(k,r) = e^{ik\,r}$ for $r \to \infty$
Superhorizon modes: $S(k;\eta) \overset{\eta\to 0^-}{\sim} (-\eta)^{\frac{1}{2}-\nu}$	Jost Function: $F_\nu(k) \equiv \frac{\sqrt{\pi}}{\Gamma(\nu)} \lim_{r\to 0} \left(\frac{k\,r}{2\,i}\right)^{\nu-\frac{1}{2}} f_\nu(k,r$
Power spectra $\frac{P_\nu(k)}{P^{sr}(k)}$	Modulus Sq. of the Jost Function $= |F_\nu(k)|^2$

TABLE 1. Correspondence between the scalar fluctuations as functions of the conformal time $\eta < 0$ and the radial wave functions, of $r > 0$ and angular momentum $L \equiv \nu - \frac{1}{2}$.

Primordial Power Spectrum

$$P_{\mathcal{R}}(k) \overset{\eta\to 0^-}{=} \frac{k^3}{2\,\pi^2}\left|\frac{S_{\mathcal{R}}(k;\eta)}{z(\eta)}\right|^2 = P^{sr}_{\mathcal{R}}(k)\Big[1 + D_{\mathcal{R}}(k)\Big] \; ,$$

$$P_T(k) \overset{\eta\to 0^-}{=} \frac{k^3}{2\,\pi^2}\left|\frac{S_T(k;\eta)}{a(\eta)}\right|^2 = P^{sr}_T(k)\Big[1 + D_T(k)\Big] \; .$$

Standard slow roll power spectrum:

$$P^{sr}_{\mathcal{R}}(k) = \mathcal{A}^2_{\mathcal{R}}\;\left(\frac{k}{k_0}\right)^{n_s-1} \; , \; P^{sr}_T(k) = \mathcal{A}^2_T\;\left(\frac{k}{k_0}\right)^{n_T}$$

$$D(k) = 2\,|B(k)|^2 - 2\;\mathrm{Re}\left[A(k)\;B^*(k)\,i^{2\nu-3}\right]$$

$D_{\mathcal{R}}(k)$ and $D_T(k)$ are the transfer functions of curvature and tensor perturbations taking into account the effect of the fast-roll stage.

General solution: $S(k;\eta) = A(k)\, g_\nu(k;\eta) + B(k)\, [g_\nu(k;\eta)]^*$

With normalization condition $|A(k)|^2 - |B(k)|^2 = 1$

Quantization:

$$v_k(\eta) = a_k S(k;\eta) + a_k^\dagger S^*(k;\eta)$$

B(k)=0 → B.D. vacuum

Power spectrum

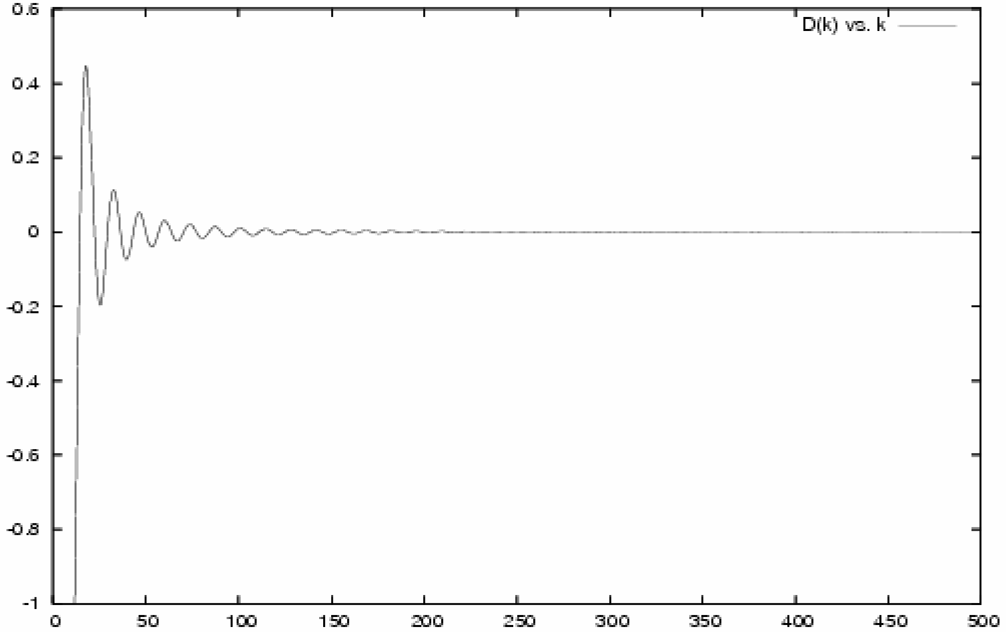
$$\mathcal{P}(k) = \langle 0 | |\delta\Psi_k|^2 | 0 \rangle = \mathcal{P}_{BD}(k)\,[1 + D(k)]$$

$$\mathcal{P}_{BD}(k) = \frac{H^2}{\epsilon_v M_P^2}\left(\frac{k}{k_0}\right)^{n_s - 1}$$

$D(k)$: **Transfer function for boundary conditions**

$$n_s = 1 - 6\epsilon_v + 2\eta_v$$

The Transfer Function $D(k)$ for the scalar fluctuations.



The transfer function $D_{\mathcal{R}}(k)$ computed in the Born approximation for trinomial new inflation $y \simeq 2, \; h = 0$.

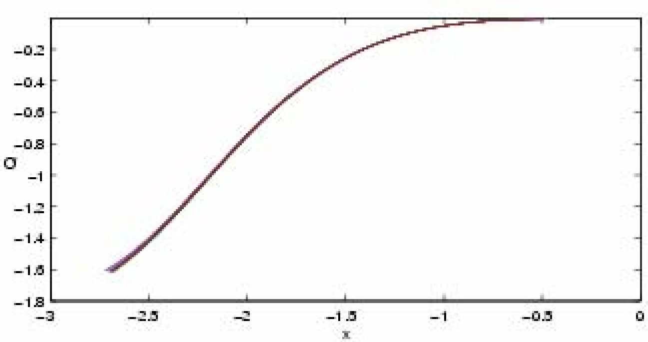
$$P_{\mathcal{R}}(k) = P_{\mathcal{R}}^{sr}(k)\Big[1 + D_{\mathcal{R}}(k)\Big]$$

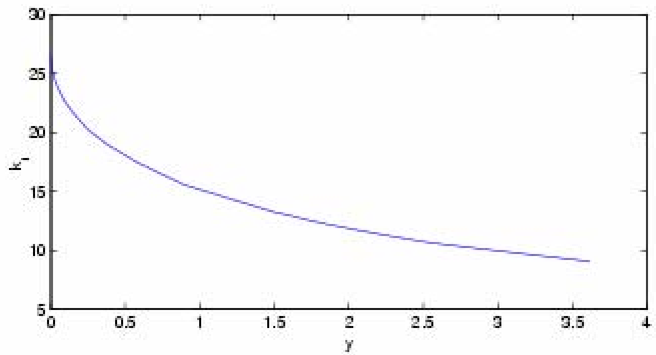
Properties of the Transfer Function $D_{\mathcal{R}}(k)$

$\mathcal{V}_{\mathcal{R}}(\eta)$ and $D_{\mathcal{R}}(k)$ **scale** with k_1:

$\mathcal{V}_{\mathcal{R}}(\eta) = k_1^2 \, Q(k_1 \, \eta) \quad , \quad D_{\mathcal{R}}(k) = \Psi\left(\frac{k}{k_1}\right)$

$Q(x)$ and $\Psi(x)$ are universal functions. y independent, while $k_1 = k_1(y)$.





C. Destri, H. J. de Vega, N. G. Sánchez,
The CMB Quadrupole depression produced by early fast-roll inflation: MCMC analysis of WMAP and SDSS data, arXiv:0804.2387.

Conditions on D(k):

- **Negligible backreaction on Einstein's eqns.**
- **Finite** $T_{\mu\nu}$
- **Renormalization of** $T_{\mu\nu}$ **with counterterms indep. of b.c**

→ **D(k) falls faster than 1/k⁴**

Change in b.c. → change in C_l

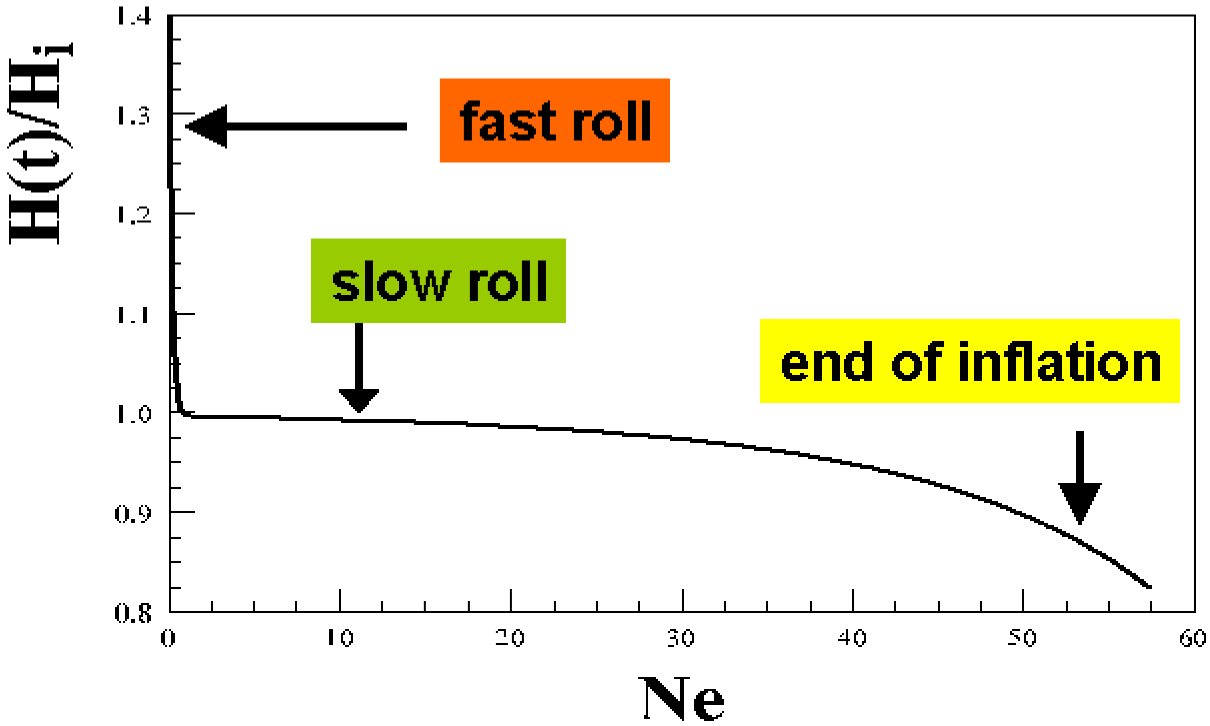
$$\frac{\Delta C_l}{C_l} = \frac{\int_0^\infty D(\kappa\, x)\;\; f_l(x)\, dx}{\int_0^\infty f_l(x)\, dx}$$

$$\kappa = \frac{a_0 H_0}{3.3}$$

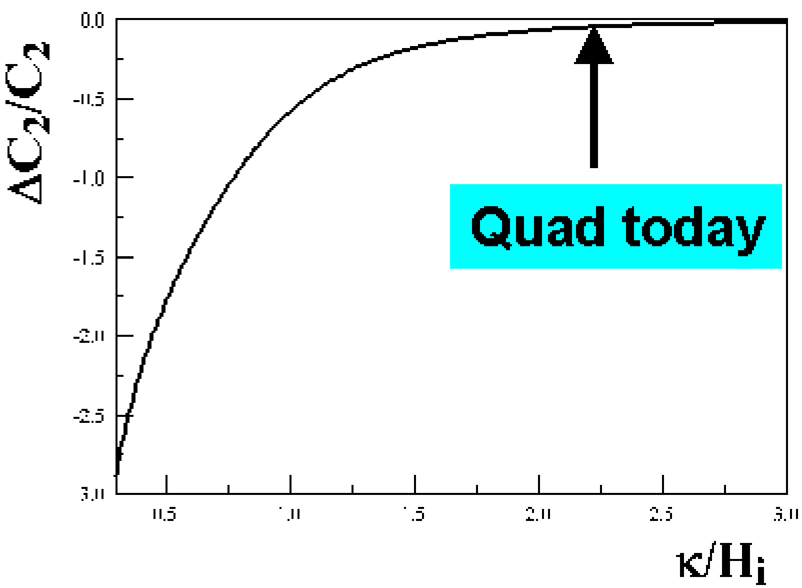
$$f_l(x) = x^{n_s - 2}[j_l(x)]^2$$

Rapid fall-off of D(k) → ***ONLY LOW MULTIPOLES ARE AFFECTED***

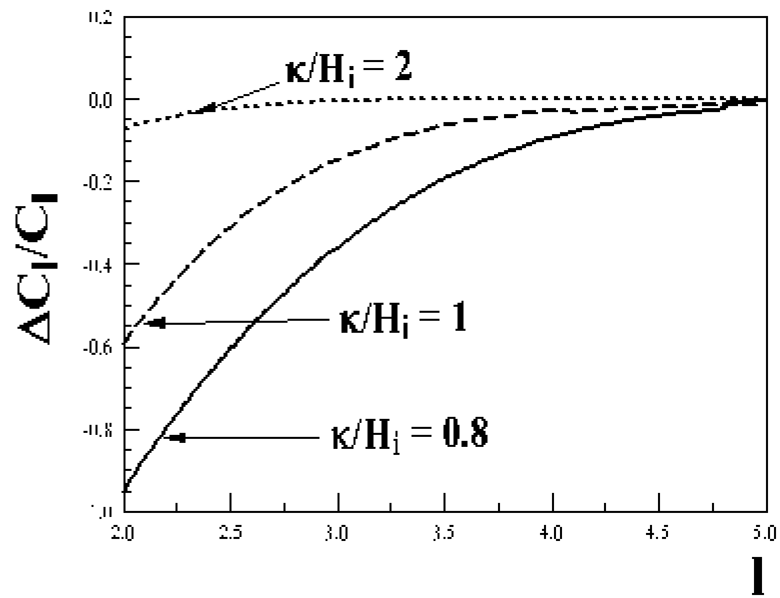
Q: What is the origin of D(k)?? (what determines the b.c.??)



H(t)/H$_i$
fast roll
slow roll
end of inflation
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0
10
20
30
40
50
60
Ne

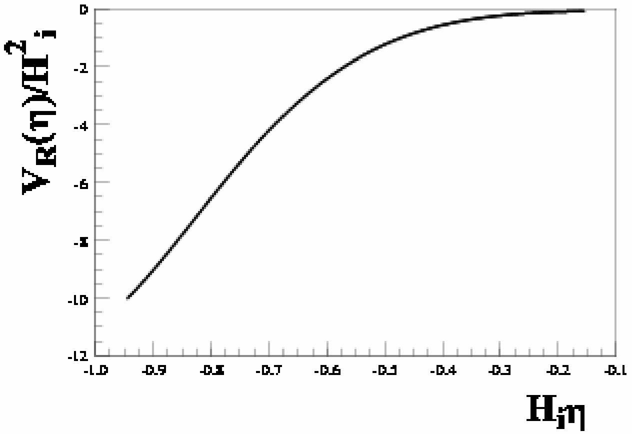


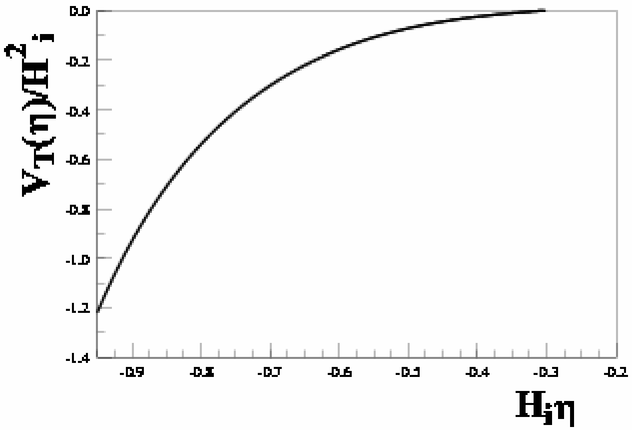
ΔC$_2$/C$_2$
Quad today
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
0.5
2.0
2.5
κ/H$_i$



ΔC$_l$/C$_l$
κ/H$_i$ = 2
κ/H$_i$ = 1
κ/H$_i$ = 0.8
0.0
-0.2
-0.4
-0.6
2.0
2.5
3.0
3.5
4.0
4.5
5.0
l

Potential felt by the Scalar and by the Tensor Fluctuations





$H_i =$ Hubble at the beginning of slow-roll.

Both $\mathcal{V}_\mathcal{R}(\eta)$ and $\mathcal{V}_\mathcal{T}(\eta)$ are ATTRACTIVE potentials.

Potential felt by tensor fluctuations much smaller:

$$\mathcal{V}_\mathcal{T}(\eta) \sim \frac{1}{10} \ \mathcal{V}_\mathcal{R}(\eta)$$

Change in the C_l due to fast roll

$$C_l \equiv C_l^{sr} + \Delta C_l \quad , \quad \frac{\Delta C_l}{C_l} = \frac{\int_0^\infty D_{\mathcal{R},\mathcal{T}}(\kappa\, x)\ f_l(x)\, dx}{\int_0^\infty f_l(x)\, dx}$$

$$\kappa \equiv H_0/3.29 = a_{sr}\ H_i/3.29 \quad , \quad f_l(x) \equiv x^{n_s-2}[j_l(x)]^2 \ .$$

Since $\mathcal{V}_{\mathcal{R},\mathcal{T}}(\eta)$ are quite small we can compute the transfer functions in the Born approximation:

$$D_{\mathcal{R},\mathcal{T}}(k) = \int_{-\infty}^0 d\eta \mathcal{V}_{\mathcal{R},\mathcal{T}}(\eta)[\sin(2k\eta)\left(1 - \frac{1}{k^2\eta^2}\right) + \frac{2}{k\eta}\ \cos(2k\eta)]/k$$

and then, $\frac{\Delta C_2}{C_2} = \frac{1}{\kappa}\int_{-\infty}^0 d\eta\ \mathcal{V}_{\mathcal{R},\mathcal{T}}(\eta)\ \Psi(\kappa\ \eta)$

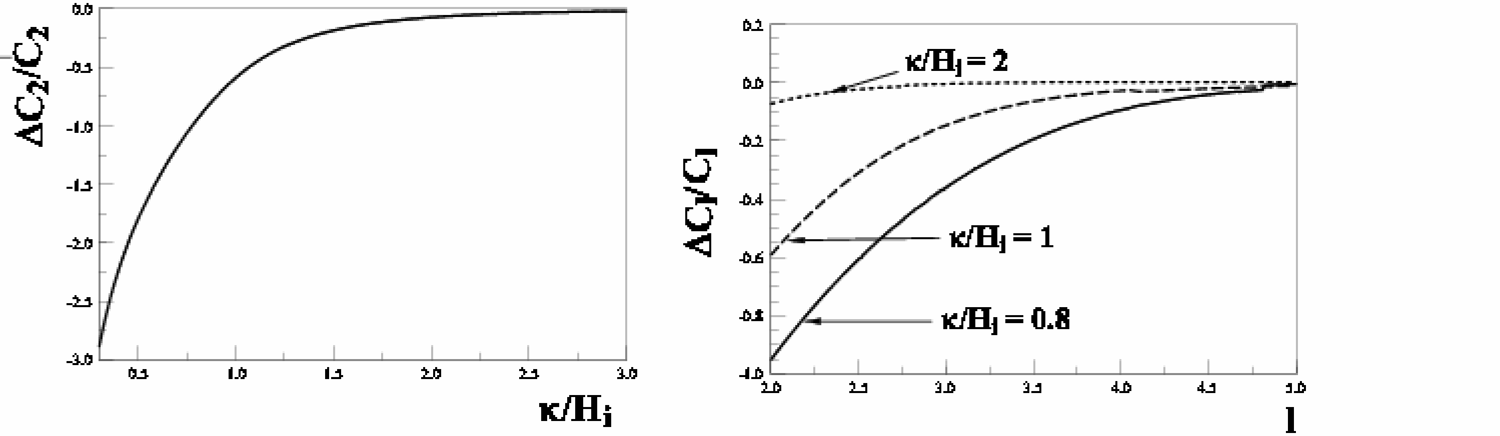
where $\Psi(\kappa\ \eta) > 0$ for $\eta < 0$.

ATTRACTIVE $\mathcal{V}_{\mathcal{R},\mathcal{T}}(\eta) < 0$ implies $\Delta C_2 < 0$.

$\longrightarrow$ QUADRUPOLE SUPPRESSION.

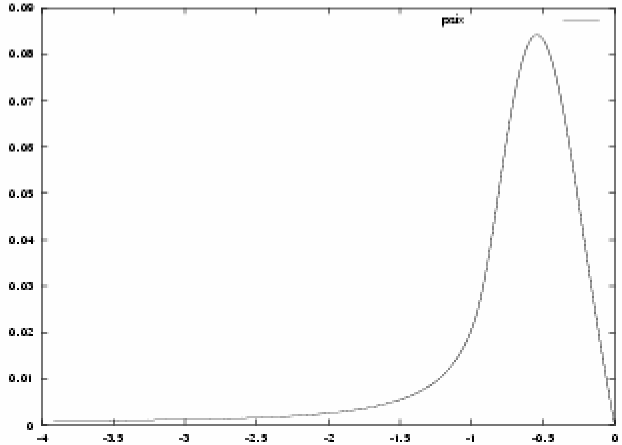
In general, $0 > \frac{\Delta C_l}{C_l} = \mathcal{O}\left(\frac{1}{l^2}\right)$.

Quadrupole Suppression vs. Fast Roll



Quadrupole Suppression Explanation:
Inflation starts with fast roll: 0 efolds.
Horizon size modes (quadrupole) exit the horizon by 0.4 efolds where $k_Q = 0.145 \text{ Gpc}^{-1}$.
Fast-roll ends and slow-roll begins: 1 efold. The mode $k_1 = 0.266 \text{ Gpc}^{-1}$ then exits the horizon. End of fast-roll.
Inflation ends at ~ 65 efolds plus $\simeq 1 = 66$ efolds.
$k_Q^{com} = 10^{14}$GeV and $k_1^{com} = 1.9 \times 10^{14}$GeV with a total redshift of $\sim 10^{56}$.

$\Psi(x)$ is an odd function.



$$\Psi(x) \equiv 3 \int_0^\infty \frac{dy}{y^4} [j_2(y)]^2 \left[(y^2 - \frac{1}{x^2}) \sin(2\,y\,x) + \frac{2\,y}{x} \cos(2\,y\,x) \right] =$$

$$= \frac{1}{105\;x^2} \left[p(x)\;(1-x)^3\;\log\left|1 - \frac{1}{x}\right| - (x \to -x) \right] + \frac{2}{105\,x} - \frac{13\,x}{126} + \frac{22\,x^3}{105} - \frac{2\,x^5}{21} \,,$$

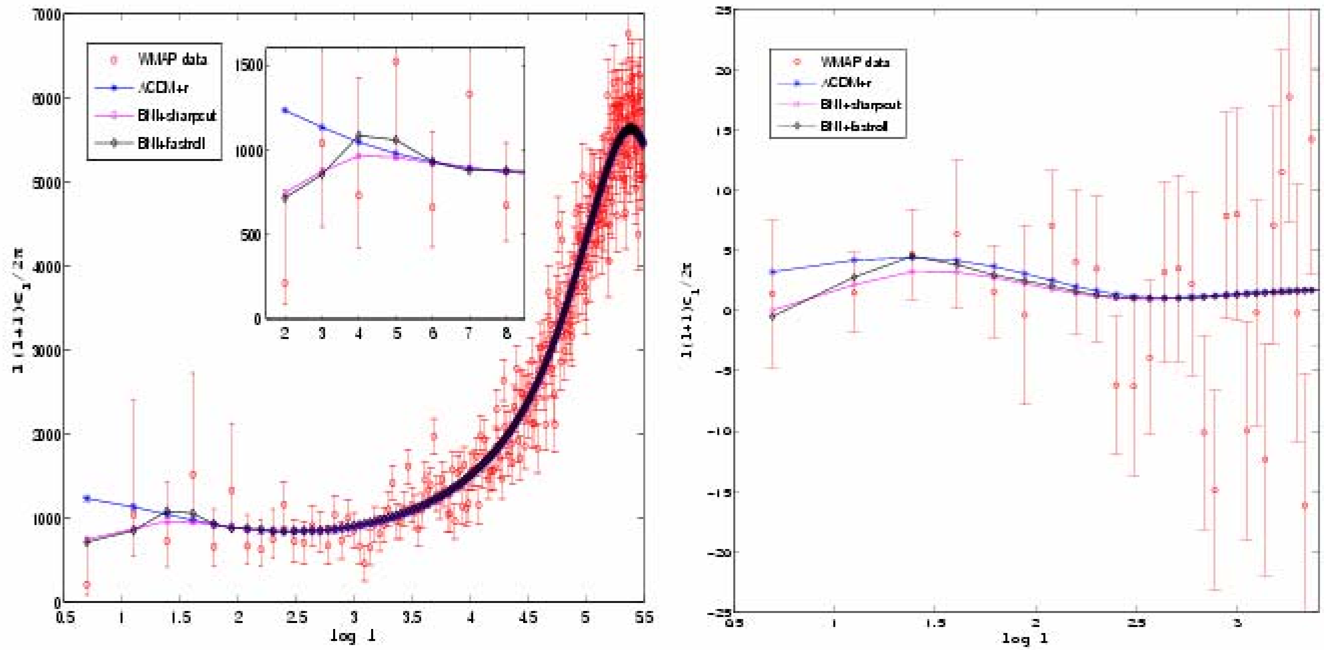
$p(x)$ is the sixth order polynomial:

$$p(x) \equiv 10\,x^6 + 30\,x^5 + 33\,x^4 + 19\,x^3 + 9\,x^2 + 3\,x + 1\,.$$

$\Psi(x) < 0$ for $x > 0$.

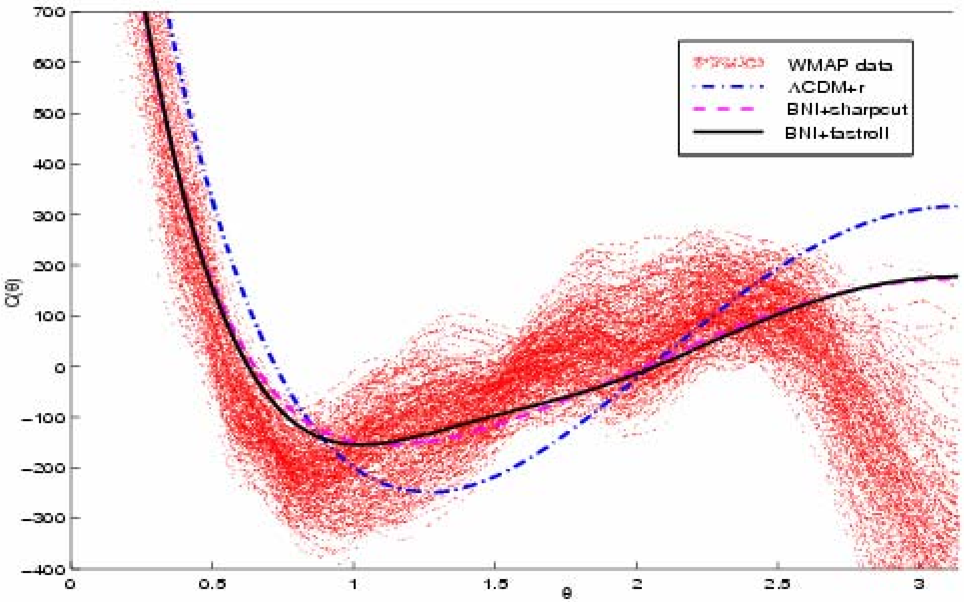
$$\Psi(x) \overset{x \to 0}{=} -\frac{x}{6} + \mathcal{O}(x^3) \quad , \quad \Psi(x) \overset{x\to\infty}{=} -\frac{1}{60\;x^3} + \mathcal{O}\left(\frac{1}{x^5}\right)\,.$$

TT and TE CMB multipoles compared with the WMAP3 data.



Comparison, with the experimental WMAP3 data, of the theoretical $C_\ell^{\rm TT}$ and $C_\ell^{\rm TE}$ for ΛCDM, fast-roll and sharpcut.

The Real Space Two Point TT Correlation Function.



The real space two point TT correlation function $C^{TT}(\theta)$ for ΛCDM, sharpcut and fast-roll models vs. the angle θ.
This shows how important are the low multipoles in the large angle correlations.
The ΛCDM correlator differs from the two others only for large angles $\theta \gtrsim 1$.

Quantum Fluctuations:

Scalar Curvature, Tensor, Fermion, Light Scalar.

All these quantum fluctuations contribute to the inflaton potential and to the primordial power spectra.

In de Sitter space-time: $< T_{\mu\nu} >= \frac{1}{4}\, g_{\mu\nu} \;\; < T^{\alpha}_{\alpha} >$

Hence, $V_{eff} = V_R + < T^0_0 >= V_R + \frac{1}{4} < T^{\alpha}_{\alpha} >$

Sub-horizon (Ultraviolet) contributions appear through the trace anomaly and only depend on the spin of the particle. Superhorizon (Infrared) contributions are of the order N^0 and can be expressed in terms of the slow-roll parameters.

$$V_{eff}(\Phi_0) = V(\Phi_0) \left[1 + \frac{H_0^2}{3\,(4\pi)^2\, M_{Pl}^2} \left(\frac{\eta_v - 4\,\epsilon_v}{\eta_v - 3\,\epsilon_v} + \frac{3\,\eta_\sigma}{\eta_\sigma - \epsilon_v} + \mathcal{T} \right) \right]$$

$\mathcal{T} = \mathcal{T}_\Phi + \mathcal{T}_s + \mathcal{T}_t + \mathcal{T}_F = -\frac{2903}{20}$ is the total trace anomaly.

$\mathcal{T}_\Phi = \mathcal{T}_s = -\frac{29}{30}, \;\; \mathcal{T}_t = -\frac{717}{5}, \;\; \mathcal{T}_F = \frac{11}{60}$

$\longrightarrow$ the graviton (t) dominates.

Corrections to the Primordial Scalar and Tensor Power

$$|\Delta^{(S)}_{k,eff}|^2 = |\Delta^{(S)}_k|^2 \{1+$$

$$+\frac{2}{3}\left(\frac{H_0}{4\,\pi\,M_{Pl}}\right)^2\left[1+\frac{\frac{3}{8}\,r\,(n_s-1)+2\,\frac{dn_s}{d\ln k}}{(n_s-1)^2}+\frac{2903}{40}\right]\Bigg\}$$

$$|\Delta^{(T)}_{k,eff}|^2 = |\Delta^{(T)}_k|^2\left\{1-\frac{1}{3}\left(\frac{H_0}{4\,\pi\,M_{Pl}}\right)^2\left[-1+\frac{1}{8}\,\frac{r}{n_s-1}+\frac{2903}{20}\right]\right\}.$$

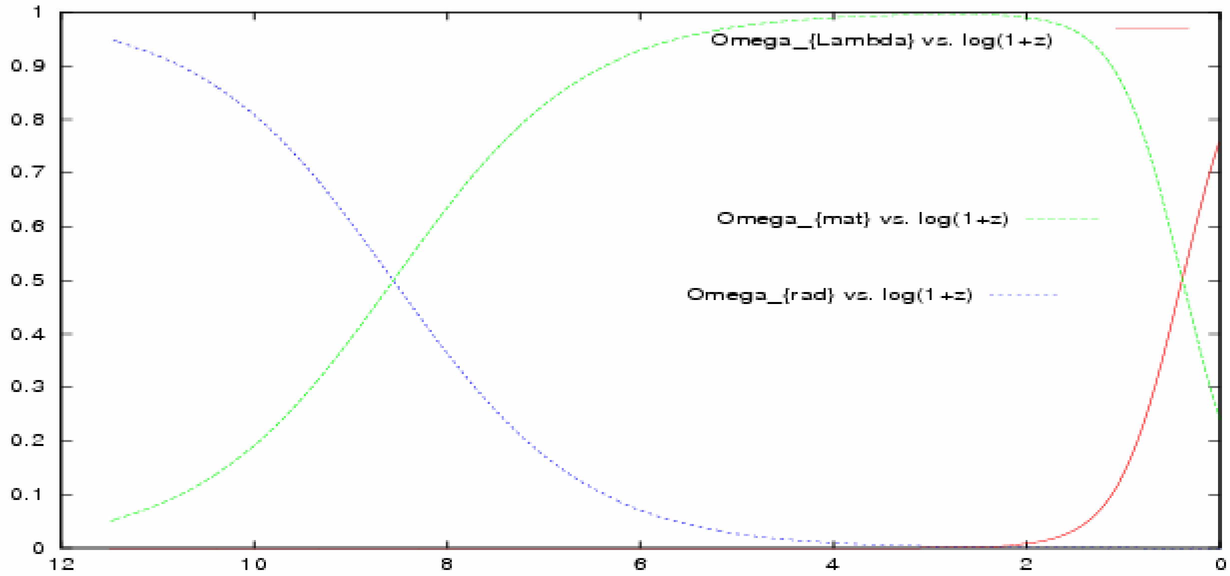
The anomaly contribution $-\frac{2903}{20} = -145.15$ DOMINATES as long as the number of fermions less than 783.

The scalar curvature fluctuations $|\Delta^{(S)}_k|^2$ are ENHANCED and the tensor fluctuations $|\Delta^{(T)}_k|^2$ REDUCED.

However, $\left(\frac{H}{M_{Pl}}\right)^2 \sim 10^{-9}$.

D. Boyanovsky, H. J. de Vega, N. G. Sanchez, Phys. Rev. D 72, 103006 (2005), astro-ph/0507596.

The Universe is made of radiation, matter and dark energy



Electro-Weak phase transition: $z \sim 10^{15}$, $T_{\rm EW} \sim 100$ GeV.
QCD phase transition (conf.): $z \sim 10^{12}$, $T_{\rm QCD} \sim 170$ MeV.
BBN: $z \sim 10^9$, $\ln(1+z) \sim 21$, $T \simeq 0.1$ MeV.
Rad-Mat equality: $z \simeq 3050$, $\ln(1+z) \simeq 8$, $T \simeq 0.7$ eV.
CMB last scattering: $z \simeq 1100$, $\ln(1+z) \simeq 7$, $T \simeq 0.25$ eV.
Mat-DE equality: $z \simeq 0.47$, $\ln(1+z) \simeq 0.38$, $T \simeq 0.345$ meV.
Today: $z = 0$, $\ln(1+z) = 0$, $T = 2.725$K $= 0.2348$ meV.

Dark Energy

$76 \pm 5\%$ of the present energy of the Universe is Dark!
Current observed value:
$\rho_\Lambda = \Omega_\Lambda \; \rho_c = (2.39 \text{ meV})^4 \;\; , \;\; 1 \text{ meV} = 10^{-3} \text{ eV}.$
Equation of state $p_\Lambda = -\rho_\Lambda$ within observational errors.
Quantum zero point energy. Renormalized value is finite.
Bosons (fermions) give positive (negative) contributions.
Mass of the lightest particles ~ 1 meV is in the right scale.
Spontaneous symmetry breaking of continuous symmetries produces massless scalars as Goldstone bosons. A small symmetry breaking provide light scalars: axions,majorons...
Observational Axion window $10^{-3} \text{ meV} \lesssim M_{\text{axion}} \lesssim 10 \text{ meV}$.
Dark energy can be a cosmological zero point effect. (As the Casimir effect in Minkowski with non-trivial boundaries).
We need to learn the physics of light particles (< 1 MeV), also to understand dark matter !!

Dark Matter

DM must be non-relativistic by structure formation ($z < 30$) in order to reproduce the observed small structure at $\sim 2-3$ kpc. DM particles can decouple being ultrarelativistic (UR) at $T_d \gg m$.
Consider particles that decouple at or out of LTE (LTE = local thermal equilibrium).
Distribution function: $f_d[a(t)\; P_f(t)] = f_d[p_c]$.
$P_f(t) = p_c/a(t) =$ Physical momentum.
$p_c =$ comoving momentum.
DM decoupling at LTE: $f_d(p_c) = 1/[\exp[\sqrt{m^2+p_c^2}/T_d] \pm 1]$
In general (out of equilibrium): $f_d(p_c) = f_d\left(\frac{p_c}{T_d}; \frac{m}{T_d}; \ldots\right)$
Velocity fluctuations: $y = P_f(t)/T_d(t) = p_c/T_d$

$$\langle \vec{V}^2(t) \rangle = \langle \frac{\vec{P}_f^2(t)}{m^2} \rangle = \frac{\int \frac{d^3P_f}{(2\pi)^3} \frac{\vec{P}_f^2}{m^2}\; f_d[a(t)\; P_f]}{\int \frac{d^3P_f}{(2\pi)^3}\; f_d[a(t)\; P_f]} = \left[\frac{T_d(t)}{m}\right]^2 \frac{\int_0^\infty y^4 f_d(y) dy}{\int_0^\infty y^2 f_d(y) dy}\,.$$

Velocity Dispersion of Dark Matter particles

Using entropy conservation: $T_d(t) = \left(\frac{2}{g_d}\right)^{\frac{1}{3}} \; T_\gamma \; [1 + z(t)]$

$g_d =$ effective # of UR degrees of freedom at decoupling,

$$\sqrt{\langle \vec{V}^2 \rangle(z)} = 0.08875 \; \frac{1+z}{g_d^{\frac{1}{3}}} \; \frac{\mathrm{keV}}{m} \; \left[\frac{\int_0^\infty y^4 \; f_d(y) \; dy}{\int_0^\infty y^2 \; f_d(y) \; dy}\right]^{\frac{1}{2}} \; \frac{\mathrm{km}}{\mathrm{s}}$$

Energy Density: $\rho_{DM}(t) = g \int \frac{d^3 P_f}{(2\pi)^3} \sqrt{m^2 + P_f^2} \; f_d[a(t) \, P_f]$

$\rho_{DM}(t) = m \; g \; T_d^3(t) \; \int_0^\infty y^2 \; f_d(y) \; \frac{dy}{2\pi^2}$ for $m \gg T_d(t)$.

Today $\Omega_{DM} = \rho_{DM}(0)/\rho_c = 0.105/h^2$ and therefore:

$$m = 6.46 \text{ eV} \; g_d/[g \; \int_0^\infty y^2 \; f_d(y) \; dy]$$

For Fermions decoupling at LTE:

$f_d(y) = 1/[e^y + 1]$ and $m = 3.593 \text{ eV} \; g_d/g$.

The formula for m

m increases:

a) if the DM particle decouples earlier because g_d increases.
b) if it decouples out of LTE, $f_d(y)$ can favour small momenta and increase $1/[\int_0^\infty y^2\, f_d(y)\, dy]$.

Special Cases of the formula for m :

Particles decoupling non-relativistically $\rightarrow$ Lee-Weinberg (1977) lower bound.

Particles decoupling ultrarelativistically $\rightarrow$ Cowsik-McClelland (1972) upper bound.

Out of equilibrium Decoupling

Thermalization mechanism: k-modes cascade towards the UV till the thermal distribution is attained.
[D. Boyanovsky, C. Destri, H. J. de Vega, PRD69, 045003 (2004). C. Destri, H. J. de Vega, PRD73, 025014 (2006)]

Hence, before LTE is reached: lower momenta are more populated than at LTE.
An approximate description:

$$f_d(y) = f_{equil}(y/\xi)\,\theta(y_0 - y), \quad \xi < 1 \text{ out of equilibrium}$$

Modes with $p_c > y_0\,T_d$ are empty. [$y = p_c/T_d$].

For fermions: $m = 6.46 \text{ eV } (g_d/g)\; F(\infty)/[\xi^3\; F(y_0/\xi)]$

$F(s) \equiv \int_0^s f_{equil}(w)\; w^2\; dw \quad , \quad F(\infty)/[\xi^3\; F(y_0/\xi)] > 1.$

Phase-space density invariant under universe expansion

$$\mathcal{D} \equiv \frac{n(t)}{\langle \vec{P}^2_{phys}(t)\rangle^{\frac{3}{2}}} \overset{\text{non-rel}}{=} \frac{\rho_{DM}}{m^4\,\sigma^3_{DM}} \quad , \quad \sigma_{DM} \equiv \sqrt{\langle \vec{V}^2\rangle} =$$

computed theoretically from equilibrium distributions.

$\rho_{DM} = 1.107 \times \mathrm{keV}/\mathrm{cm}^3 =$ observed value today.

$$\frac{\rho_{DM}}{\sigma^3_{DM}} \sim 10^3\,\frac{\mathrm{keV}/\mathrm{cm}^3}{(\mathrm{km/s})^3}\,\left(\frac{m}{\mathrm{keV}}\right)^3\,g_d \begin{cases} 0.177 & \text{Fermions} \\ 0.247 & \text{Bosons} \end{cases} .$$

$g_d = \#$ of UR degrees of freedom at decoupling.

Observing dwarf spheroidal satellite galaxies in the Milky Way (dSphs) yields: $\frac{\rho_s}{\sigma_s^3} \sim 5 \times 10^3\,\frac{\mathrm{keV}/\mathrm{cm}^3}{(\mathrm{km/s})^3}$.

Theorem: The phase-space density $\mathcal{D}$ can only decrease under self-gravity interactions (Lynden-Bell, Tremaine, Henon).

N-body simulations results: $\frac{\rho_s}{\sigma_s^3} \sim 10^{-2}\,\frac{\rho_{DM}}{\sigma^3_{DM}}$.

Mass Estimates of DM particles

Collecting all formulas yields for relics decoupling at LTE:

$m \sim \frac{2}{g^{\frac{1}{4}}}$ keV , $\quad g_d \geq 500 \; g^{\frac{3}{4}}$,

Hence, $T_d > 100$ GeV. $[g = 1 - 4]$.

g_d can be smaller for relics decoupling out of LTE

Let us consider now WIMPS (weakly interactive massive particles): $m \sim 100$ GeV, $T_d \sim 10$ MeV. We find:

$$\frac{\rho_{wimp}}{\sigma^3_{wimp}} \sim 10^{21} \; \frac{\mathrm{keV/cm^3}}{(\mathrm{km/s})^3} \; \left(\frac{\sqrt{m \; T_d}}{1 \; \mathrm{GeV}}\right)^3 \; g_d \; .$$

Eighteen orders of magnitude larger than the observations in dShps.

D. Boyanovsky, H. J. de Vega, N. Sanchez, Phys. Rev. D 77, 043518 (2008), arXiv:0710.5180.

Summary and Conclusions

- Inflation can be formulated as an effective field theory in the Ginsburg-Landau spirit with energy scale $M \sim M_{GUT} \sim 10^{16}$ GeV $\ll M_{Pl}$.
  Inflaton mass small: $m \sim H/\sqrt{N} \sim M^2/M_{Pl} \ll M$.
  Infrared regime !!
- The slow-roll approximation is a $1/N$ expansion, $N \sim 60$
- MCMC analysis of WMAP+LSS data plus the Trinomial Inflation potential indicates a spontaneously symmetry breaking potential (new inflation): $w(\chi) = \frac{y}{32}\left(\chi^2 - \frac{8}{y}\right)^2$.
- Lower Bounds: $r > 0.016$ (95% CL) , $r > 0.049$ (68% CL).
  The most probable values are $n_s \simeq 0.956$, $r \simeq 0.055$ with a quartic coupling $y \simeq 1.3$.

Executive summary:

- Initial conditions: set by a *fast roll* stage prior to slow roll
- Fast roll ~ *GENERIC* initial condition with kin. ~pot. energy
- Fast roll: localized *attractive* potential in mode equations
- D(k) = transmission coeff. = transfer function
- Suppression of low multipoles

Analysis: fast roll ~ 2-3 e-folds, IF k_Q crosses horizon ~ 2-3 e-folds after beginning of slow roll ~ 15-20% suppression!

Quad. suppression mechanism *WITHIN* eff. field theory

FINAL—FINAL SUMMARY AND CONCLUSIONS

✓ Effective field theory H/Mp <<1, 1/Ne-slow roll expansion robust, systematic, predictive

✓ Quantum corrections suppressed by (H/Mp)^2

✓ Fast roll stage prior to slow roll ⟶ modifies b.c. for scalar perturbations ⟶ quadrupole suppression ~ 15-20% for total Ne ~ 55.

✓ 1/Ne expansion ⟶ systematic exploration of family of inflaton potentials+ field reconstruction .

✓ Small field New Inflation larger region of consistency with WMAP3+LSS data.

✓ Potentials with larger overlap with marginalized WMAP 3 data symmetry breaking scale ~ 10 Mp, crossing scale ~ Mp.

Summary and Conclusions 2

- The quadrupole suppression may be explained by the effect of fast-roll inflation provided the today's horizon size modes exited 0.1 efolds before the end of fast-roll inflation.
- Quantum (loop) corrections in the effective theory are of the order $(H/M_{Pl})^2 \sim 10^{-9}$.

D. Boyanovsky, H. J. de Vega, N. G. Sanchez,

Quantum corrections to the inflaton potential and the power spectra from superhorizon modes and trace anomalies, Phys. Rev. D 72, 103006 (2005), astro-ph/0507596.

Quantum corrections to slow roll inflation and new scaling of superhorizon fluctuations. Nucl. Phys. B 747, 25 (2006), astro-ph/0503669.

Future Perspectives

The Golden Age of Cosmology and Astrophysics continues.

A wealth of data from WMAP (6 yr), Planck, Atacama Cosmology Tel and further experiments are coming.

Galaxy formation. Gigantic black-holes ($M \sim 10^9 \; M_\odot$) as galaxy nuclei, early star formation...

The Dark Ages...Reionisation...the 21cm line...

Nature of Dark Energy? 76% of the energy of the universe.

Nature of Dark Matter? 83% of the matter in the universe.

Light DM particles are strongly favoured $m_{DM} \sim 2$ keV.

Sterile neutrinos? Some unknown light particle ??

Need to learn about the physics of light particles (< 1 MeV).

THANK YOU VERY MUCH
FOR YOUR ATTENTION!!

17.00 – 18.00 **Norma G. SANCHEZ (CNRS LERMA Observatoire de Paris, France)** ***Understanding of Inflation, Dark Matter and Dark Energy in the Standard Model of the Universe***

18.00 - 19.00 **Tour to the historic Perrault building guided by Prof. Suzanne DEBARBAT (SYRTE-Observatoire de Paris) around the subject *"Mechain and the Meter"* and Exhibition**

19.00 - 20.30 **APERITIF/COCKTAIL at SALLE CASSINI**

in Perrault building, for all participants and accompanying persons

End of the Conference

THANK YOU VERY MUCH
FOR YOUR ATTENTION!!