Primordial non-Gaussianity and the CMB (Part I)

Nicola Bartolo
Dipartimento di Fisica *Galileo Galilei*, Padova, Italy

The standard model of the Universe: from Inflation to dark energy
Paris, 23-25 July 2009

Plan of the talk

What is (primordial) non-Gaussianity and why it is important ?

Primordial non-Gaussianity from Inflation and alternative scenarios

Primordial non-Gaussianity vs secondary effects

Contamination from CMB anisotropies at second-order in perturbation theory

Conclusions

Primordial NG

$\Phi(\mathbf{x})$: primordial gravitational potential

If the fluctuations $\Phi(\mathbf{x})$ are Gaussian distributed then their statistical properties are completely characterized by the two-point correlation function, $<\Phi(\mathbf{x}_1)\Phi(\mathbf{x}_2)>$ or its Fourier transform, the power-spectrum.

Thus a non-vanishing **three point function**, or its Fourier transform, the **bispectrum is an indicator of non-Gaussianity**

$$\left\langle \Phi(\vec{k}_1)\Phi(\vec{k}_2)\Phi(\vec{k}_3) \right\rangle = (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) B(k_1, k_2, k_3) \rightarrow \left\langle \frac{\Delta T}{T}(n_1) \frac{\Delta T}{T}(n_2) \frac{\Delta T}{T}(n_3) \right\rangle$$

$$\boxed{B(k_1, k_2, k_3) = f_{NL} \; F(k_1, k_2, k_3)}$$

Amplitude

Shape

$$ds^2 = a^2(\tau)[-(1+2\Phi)d\tau^2 + \omega_i\, d\tau\, dx^i + ((1-2\Psi)\delta_{ij} + h_{ij})dx^i dx^j]$$

Gravitational potential perturbation in the perturbed Friedmann-Robertson-Walker metric
(Newtonian or Poisson gauge)

Why primordial NG is important?

- ✓ In general different models for the generation of the primordial perturbations predict (standard inflation vs alternatives):

  - some departure from scale invariance $P_{\zeta} = A\, k^{n-1}$
  - some amount of tensor perturbations $P_T \propto H_*^2$
  - some amount of non-Gaussianity f_{NL}

- ✓ In fact many models predict negligible amount of tensor perturbation and negligible departure from scale-invariance

Even a precise measurement of the spectral index might not allow to discriminate among competing scenarios; if no G.W. is detected (e.g. via CMB polarization) no efficient discrimination

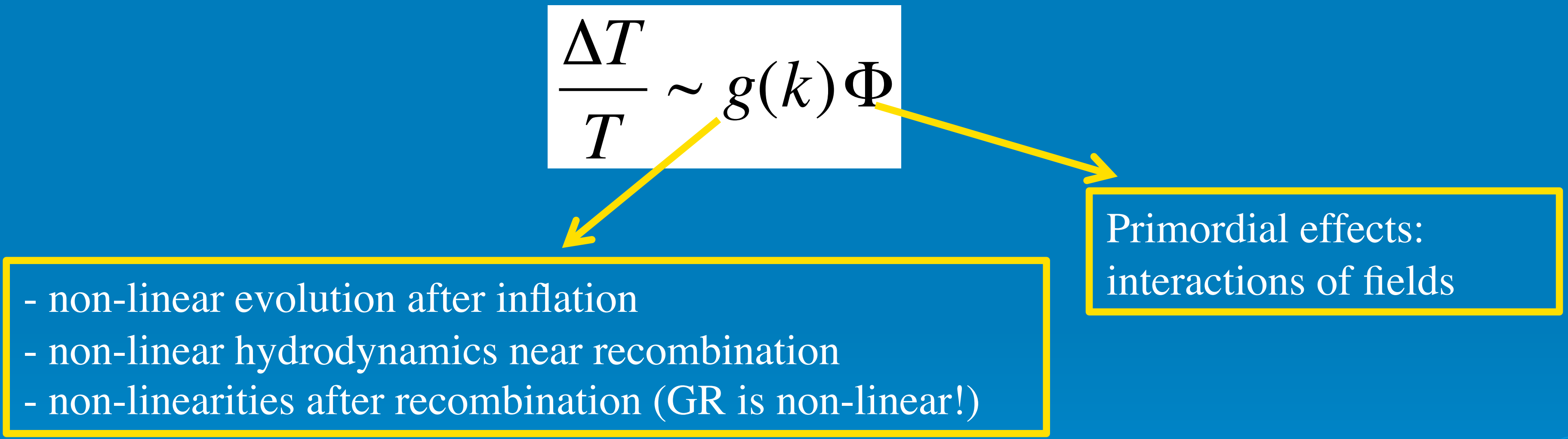
However these models predict different NG

✓A crucial example:

a detection of a primordial f_{NL}~10 would rule out standard single-field models of slow-roll inflation.

Going beyond linear order

- NG is generated by non-linear physics producing mode-mode couplings: break linear approximation



$$\frac{\Delta T}{T} \sim g(k)\Phi$$

Primordial effects:
interactions of fields

- non-linear evolution after inflation
- non-linear hydrodynamics near recombination
- non-linearities after recombination (GR is non-linear!)

- NG requires more than linear theory

  Various techniques available: second-order perturbation theory, fully non-linear approach a la Salopek/Bond '91 (or the so called δN formalism), quantum field theory (Schwinger-Keldish formalism)

Primordial non-Gaussianity: predictions from standard Inflation and alternative scenarios

Primordial non-Gaussianity

It is useful to characterize it in terms of the non-linearities present in the ***curvature perturbation*** ζ (in the uniform energy density slice)

$$g_{ij} = a^2(t)e^{-2\zeta}\gamma_{ij} = a^2(t,\mathbf{x})\,\gamma_{ij}$$

Think of $\zeta \approx \frac{\delta\rho}{\rho}$

Note that

a) this is a ***fully non-linear definition*** for the curvature perturbation *(Salopek/Bond'91; Kolb et al. '05; Lyth at al. '05)* and at linear order it reduces to the usual definition

$$\zeta^{(1)} = -\Psi - H\frac{\delta\rho}{\dot\rho}, \qquad \text{where } \zeta = \zeta^{(1)} + \frac{1}{2}\zeta^{(2)} + ...$$

$$\zeta^{(1)} = -3/5\Phi^{(1)}$$

The key point is that ζ remains ***constant*** on superhorizon scales after it has been generated and possible isocurvature (entropy) perturbations are no longer present.

$\zeta^{(2)}$ provides all the information about the primodial level of the NG generated during inflation, as in the standard scenario, or after inflation, as in the curvaton scenario.

The value of $\zeta^{(2)}$ is different for different scenarios

$$\zeta^{(2)} = \frac{3}{5} f_{NL} \left(\zeta^{(1)}\right)^2$$

➡ It is not directly connected to the measurable quantity, the CMB anisotrooy

Case 1:

Standard single-field models of inflation

Acquaviva, N.B, Matarrese and Riotto (2002); Maldacena (2002);
Lidsey and Seery (2004)

Primordial non-Gaussianity in standard single field models of Inflation

- non-Gaussianity generated during inflation:

Accounting for the inflaton self-interactions *and metric fluctuations at second-order in the perturbations* brings

$$\boxed{f_{NL} = -\frac{5}{12}(n-1) + n_T f(\mathbf{k}_1, \mathbf{k}_2)}$$

Acquaviva, N.B., Matarrese, Riotto (2002); Maldacena (2002)

Primordial non-Gaussianity for single-field models of slow-roll inflation is tiny ~ O(ε,η)

$$|n-1| = |2\eta - 6\varepsilon| << 1; \quad n_T = 2\varepsilon \qquad \varepsilon \equiv \frac{1}{16\pi \mathrm{G}}\left(\frac{V'}{V}\right)^2, \ \eta \equiv \frac{1}{8\pi G}\frac{V''}{V}$$

- Notice that in the **squeezed limit $k_1 << k_2 \approx k_3$** $f(k_1,k_2)$ goes to zero (Maldacena 2001)

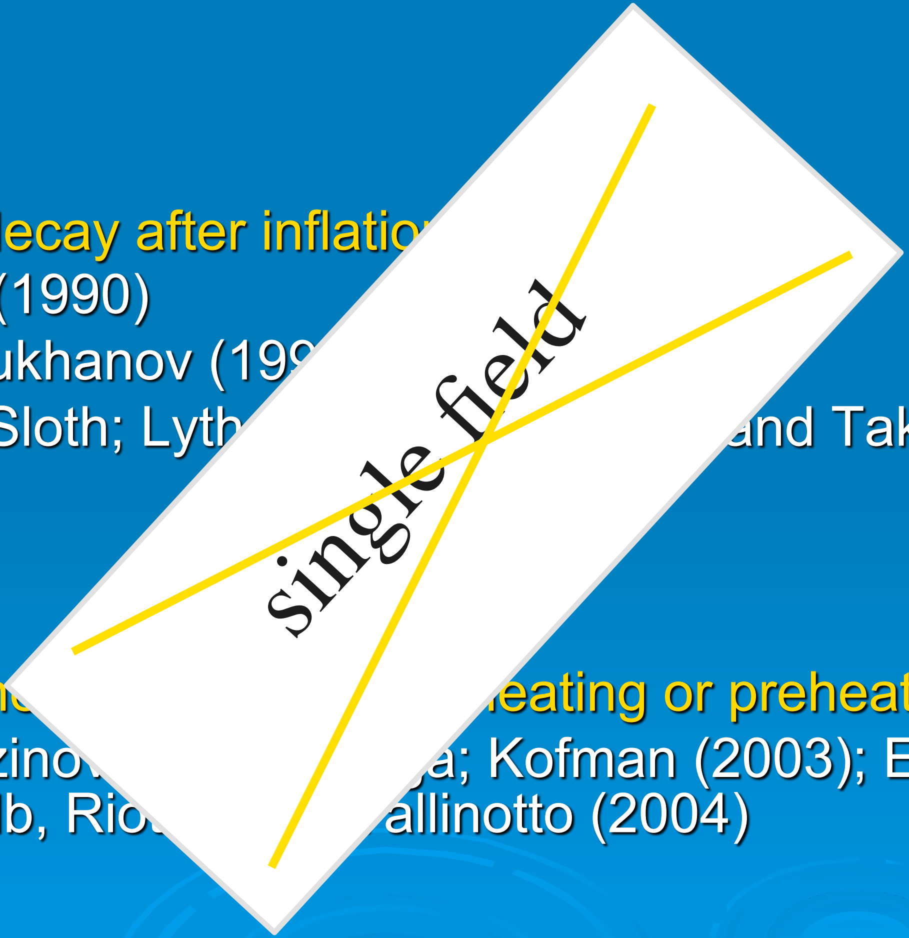
Case 2: Local models

f_{NL}=constant

NG local in real space: non-linearities develop outside the horizon

$$\Phi(\mathbf{x}) = \Phi_{\rm L}(\mathbf{x}) + f_{\rm NL}^{\rm loc} \left(\Phi_{\rm L}^2(\mathbf{x}) - \langle\Phi_{\rm L}^2(\mathbf{x})\rangle\right)$$

Radiation density can be perturbed by other scalar fields during or after inflation



- Curvaton decay after inflatio
  Mollerach (1990)
  Linde & Mukhanov (199
  Enqvist & Sloth; Lyth and Takahashi (2001)

- Inhomogene eating or preheating
  Dvali, Gruzino a; Kofman (2003); Enqvist et al.
  (2003); Kolb, Rio allinotto (2004)

NG from the curvaton decay

$$\rho_\sigma = \frac{m^2}{2}\sigma^2 = \frac{m^2}{2}\left(\sigma_0^2 + 2\sigma_0\delta\sigma + \delta\sigma^2\right)$$

$$\zeta_{\textbf{late times}} = \sum_i \frac{\dot\rho_i}{\dot\rho}\zeta_i \approx (1-r)\cancel{\zeta_\gamma} + r\zeta_\sigma$$

$$r = \left(\frac{\rho_\sigma}{\rho}\right)_{\textbf{decay}}$$

$$\zeta_\sigma \approx \left(\frac{\delta\sigma}{\sigma}\right) + \left(\frac{\delta\sigma}{\sigma}\right)^2$$

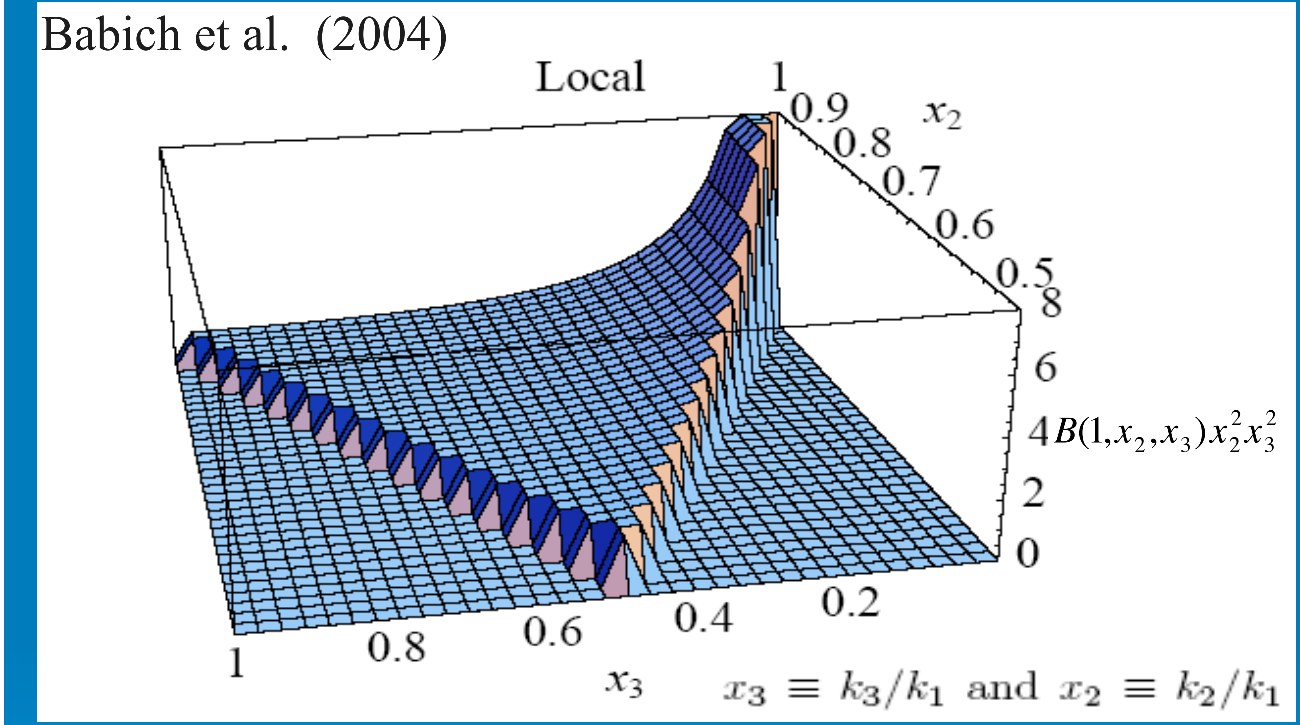
$$\zeta_1 \approx r\left(\frac{\delta\sigma}{\sigma}\right)$$

$$\boxed{f_{NL} = \frac{5}{4r} - \frac{5r}{6} - \frac{5}{3} \quad \left(= \frac{5}{4r} \quad if\ \ r << 1\right)}$$

Lyth, Ungarelli and Wands (2002);
N.B, Matarrese and Riotto (2004);
Lyth, Rodriguez, Malik (2005+)

Non-Gaussianity shape

(see Babich et al 04; for other shapes see Fergusson and Shellard 08)



Babich et al. (2004)

$x_3 \equiv k_3/k_1$ and $x_2 \equiv k_2/k_1$

Bispectrum peaks for squeezed triangles: $k_1 << k_2 \sim k_3$

$$\langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_2)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)B(k_1, k_2, k_3)$$

$$B(k_1, k_2, k_3) = 2f_{\rm NL}^{\rm loc}P_\Phi(k_1)P_\Phi(k_2) + \text{cycl.}$$

$$P_\Phi(k) = \frac{A}{k^{4-n_s}}$$

$$\zeta^{(1)} = -\frac{5}{3}\Phi^{(1)}$$

Case 3: equilateral models

NG non-local in real space: non-linearities generated by operators with gradients

Such as	Ghost inflation, N. Arkani-Hamed et al., 2003
	DBI inflation, E. Silverstein and D. Tong, 2003

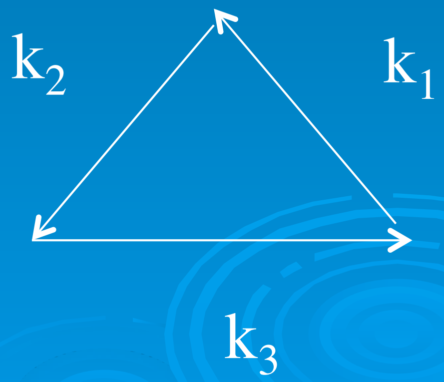
‘Equilateral’ models:

- Single field models of inflation with **non-canonical kinetic term** L=P(φ, X) where $X=(\partial\varphi)^2$ (DBI or K-inflation)
  models where NG comes from higher derivative interactions of the inflaton field

$$\frac{1}{8\Lambda^4}(\nabla\phi)^2(\nabla\phi)^2$$

- The bispectrum peaks for equilateral triangles: $k_1=k_2=k_3$
  since in this case the correlation is among modes that exit the horizon at the same time

$$B^{\mathrm{eq}}(k_1,k_2,k_3)=f_{NL}^{\mathrm{eq}}\left[-3\frac{A^2}{k_1^{4-n_s}k_2^{4-n_s}}\right.$$
$$-2\frac{A^2}{k_1^{2(4-n_s)/3}k_2^{2(4-n_s)/3}k_3^{2(4-n_s)/3}}$$
$$\left.+6\frac{A^2}{k_1^{(4-n_s)/3}k_2^{2(4-n_s)/3}k_3^{4-n_s}}\right]+(\mathrm{symm.})$$

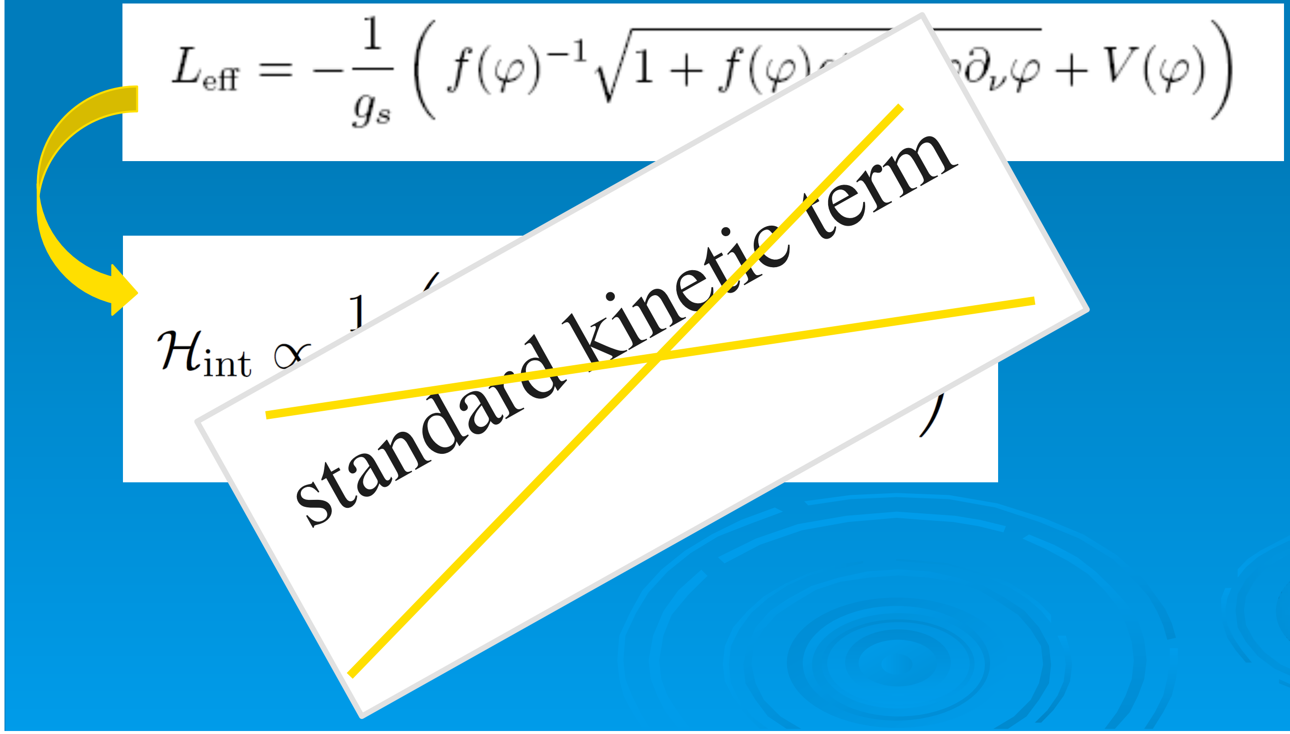


- In this case an equilateral non-linearity parameter $\boxed{f_{NL}^{\text{equil}}}$

  is used to characterize the amplitude of the bispectra. It is defined

  so that $F(k,k,k) = 6 f_{NL}^{\text{equil}} P_\Phi(k)$

- **Ex: DBI inflation**, E. Silverstein and D. Tong, 2003



$L_{\text{eff}} = -\frac{1}{g_s}\Big(f(\varphi)^{-1}\sqrt{1+f(\varphi)\ldots\partial_\nu\varphi} + V(\varphi)\Big)$

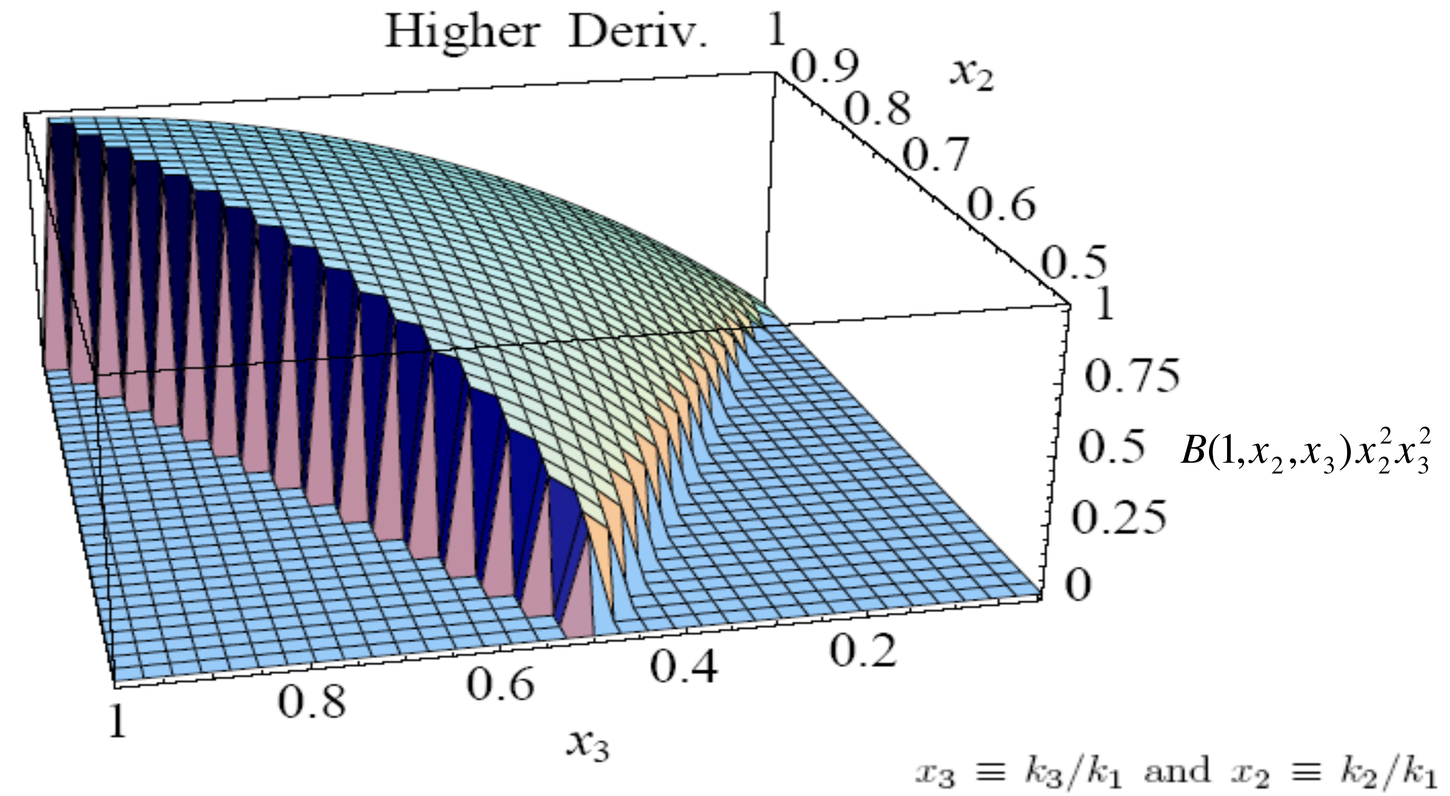
$\mathcal{H}_{\text{int}} \propto \ldots$

~~standard kinetic term~~

$$\boxed{f_{NL}^{\text{equil}} = -\frac{0.32}{c_s^2}}$$

with sound speed $c_s^2 < 1$

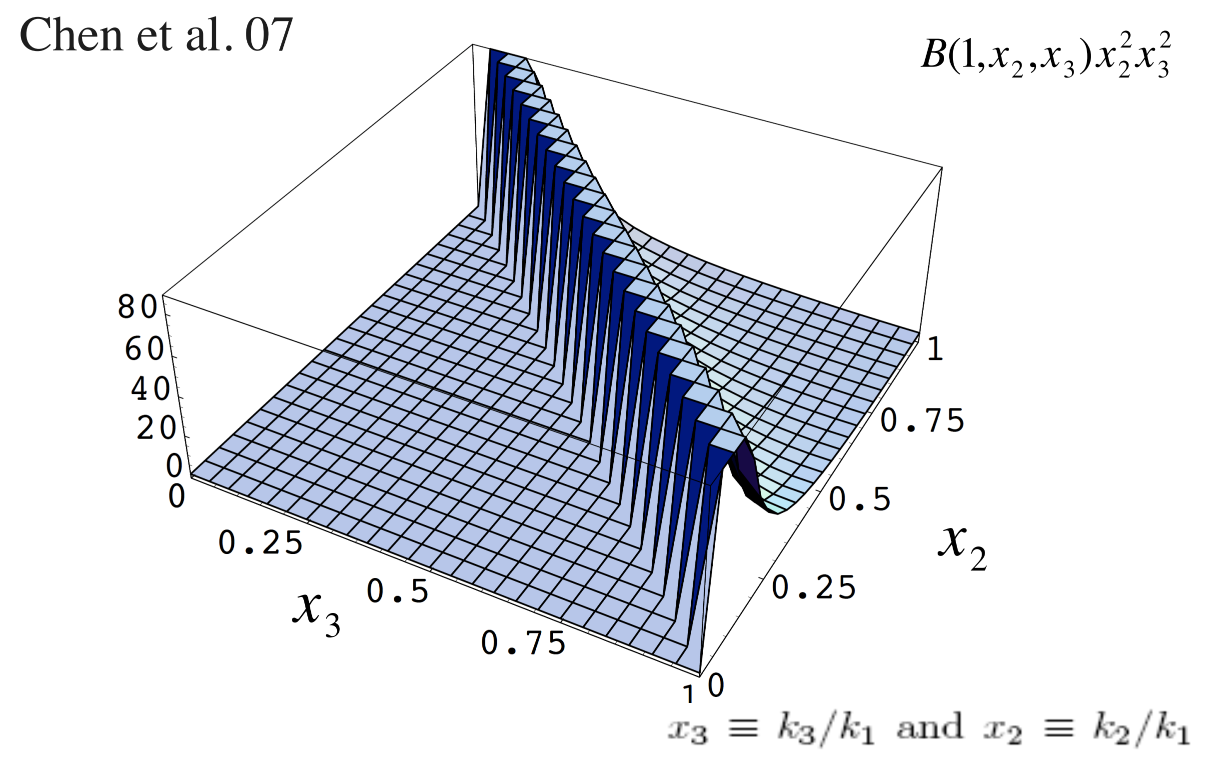
Babich et al. (2004)



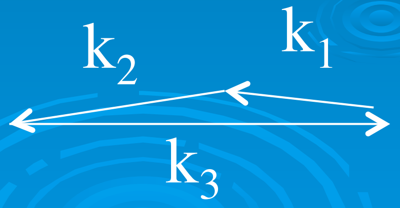
$x_3 \equiv k_3/k_1$ and $x_2 \equiv k_2/k_1$

Most signal in the equilateral triangles $\mathbf{k_1=k_2=k_3}$

Another interesting example……



Most signal in the flattened triangles **$k_1=k_2+k_3$**



(typical of NG from excited initial states, see Chen et al 07;
Holman & Tolley 08)

Inflation models and f_{NL}

Updated table from N. B., E. Komatsu, S. Matarrese and A. Riotto., Phys. Rept. 2004

model	$f_{NL}(k_1,k_2)$	comments
Standard inflation	$O(\varepsilon,\eta)$	
curvaton	$5/4r - 5r/6 - 5/3$	$r \sim (\rho_\sigma/\rho)_{decay}$
modulated reheating	$-5/4 - I$	$I = - 5/2 + 5\Gamma / (12\,\alpha\Gamma_1)$ $I = 0$ (*minimal case*)
multi-field inflation	may be large ?	
Feature in the inflaton potential	$\mid f_{NL} \mid > 1$	
"unconventional" inflation set-ups		
DBI	$- 0.32 / c_s^2$	*equilateral configuration*
Generalized single-field inflation (e.g:k-and brane inflation)	$-(35/108)\,(1/c_s^2-1)$ $+(5/81)\,(1/c_s^2-1-2\lambda/\Sigma)$	High when the sound of speed c_s <<1 or λ/Σ>>1
ghost inflation	$- 85\,\beta\,\alpha^{-3/5}$	*equilateral configuration*

model	$f_{NL}(k_1,k_2)$	comments
Excited initial states+derivative interactions	$\sim(6.3\ 10^{-4}\ M_{Pl}/M)$ $\sim(1\text{-}100)$	Flatten configuration M: cut-off scale
Preheating scenarios	e.g. $(M_{Pl}/\varphi_0)\ e^{Nq/2} \sim 50$	N:n° of inflaton oscillations
Inhomogeneous preheating And inhom. hybrid inflation	e.g. $(5/6)\ \lambda_\varphi\ (M_{Pl}/m_\chi)$	λ_φ: inflaton coupling to the waterfall field χ
Warm inflation	typically 10^{-1}	
Warm Inflation (II)	$-15L(r) < f_{NL} < L(r)$	$L(r) \approx \ln(1+r/14)$; $r=\Gamma/3H >> 1$
Ekpyrotic models	$-50 < f_{NL} < 200$	depends on the sharpness of conversion from isocurvature to curvature perturbations
Generalized slow-roll inflation (higher-order kinetic terms)	$f_{NL} >> +1$	equilateral configuration
Multi-DBI inflation	$f_{NL} \sim c_s^{-2}\ 1/(1+T^2{}_{RS})$	both local and equil. shapes

Observational constraints on NG

WMAP 5yr limits on local models

✓ From an analysis of the bispectrum of CMB temperature anisotropies

$$-9 < f_{\rm NL}^{\rm loc} < 111 \ (95\%)$$

- from the template-cleaned V+W channel;
- accounting for a bias from unresolved point sources
- for $l_{\rm max} = 500$

✓ See also Senatore et al. (2009)

$$-4 < f_{\rm NL}^{\rm loc} < 80 \ (95\%)$$

✓ and claimed evidence from Yadav & Wandelt 08, WMAP3

$$27 < f_{NL} < 147 \ (95\%)$$

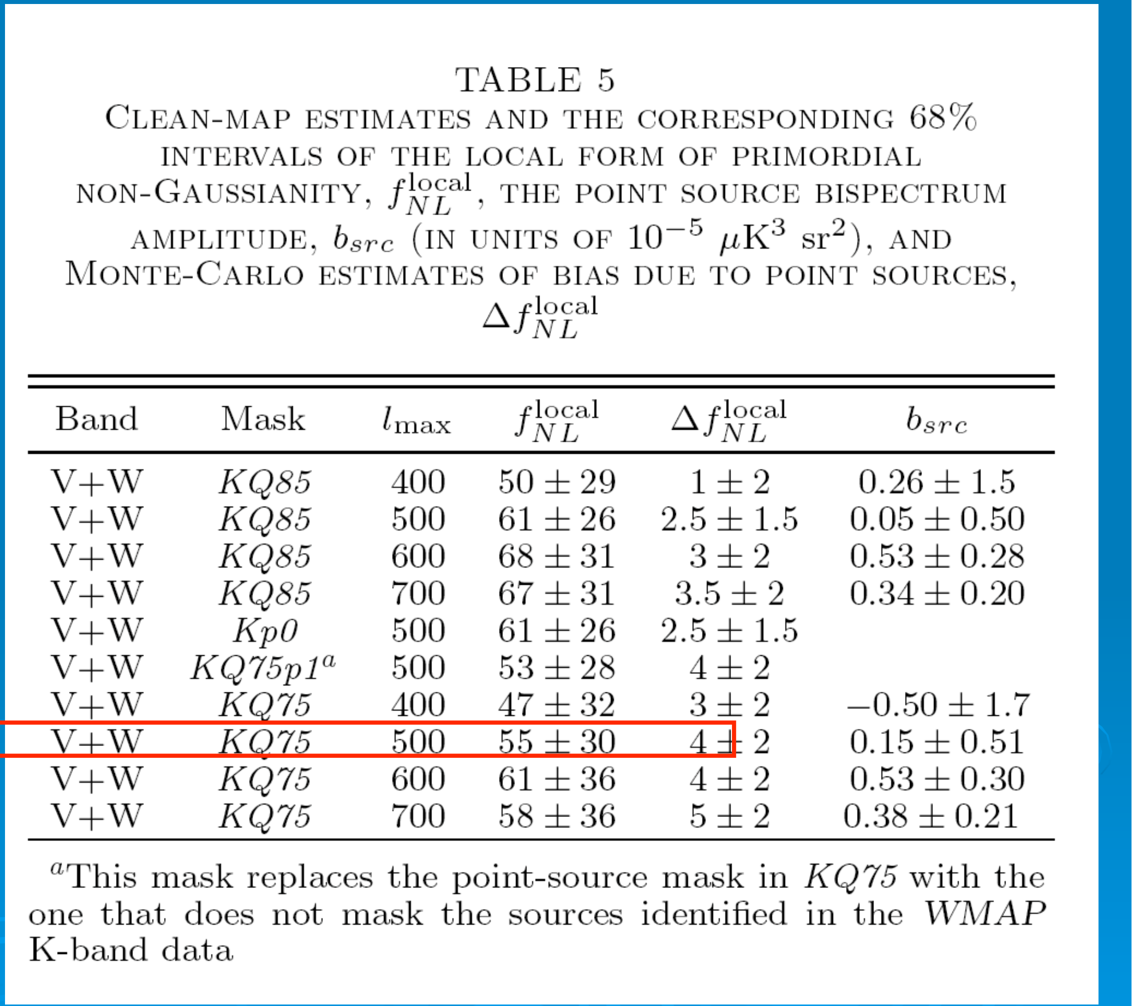


TABLE 5

CLEAN-MAP ESTIMATES AND THE CORRESPONDING 68% INTERVALS OF THE LOCAL FORM OF PRIMORDIAL NON-GAUSSIANITY, $f_{NL}^{\rm local}$, THE POINT SOURCE BISPECTRUM AMPLITUDE, b_{src} (IN UNITS OF 10^{-5} μK^3 sr^2), AND MONTE-CARLO ESTIMATES OF BIAS DUE TO POINT SOURCES, $\Delta f_{NL}^{\rm local}$

Band	Mask	$l_{\rm max}$	$f_{NL}^{\rm local}$	$\Delta f_{NL}^{\rm local}$	b_{src}
V+W	*KQ85*	400	50 ± 29	1 ± 2	0.26 ± 1.5
V+W	*KQ85*	500	61 ± 26	2.5 ± 1.5	0.05 ± 0.50
V+W	*KQ85*	600	68 ± 31	3 ± 2	0.53 ± 0.28
V+W	*KQ85*	700	67 ± 31	3.5 ± 2	0.34 ± 0.20
V+W	*Kp0*	500	61 ± 26	2.5 ± 1.5	
V+W	*KQ75p1*[a]	500	53 ± 28	4 ± 2	
V+W	*KQ75*	400	47 ± 32	3 ± 2	-0.50 ± 1.7
V+W	*KQ75*	500	55 ± 30	4 ± 2	0.15 ± 0.51
V+W	*KQ75*	600	61 ± 36	4 ± 2	0.53 ± 0.30
V+W	*KQ75*	700	58 ± 36	5 ± 2	0.38 ± 0.21

[a] This mask replaces the point-source mask in *KQ75* with the one that does not mask the sources identified in the *WMAP* K-band data

Komatsu et al. 2008

WMAP 5yr limits on Equilateral models

From an analysis of the bispectrum of CMB temperature anisotropies

$$-151 < f_{NL}^{\mathrm{equil}} < 253 \ (95\%)$$

for $l_{\max} = 700$

See also Senatore et al. (2009)

$$-125 < f_{\mathrm{NL}}^{\mathrm{equil}} < 435 \ (95\%)$$

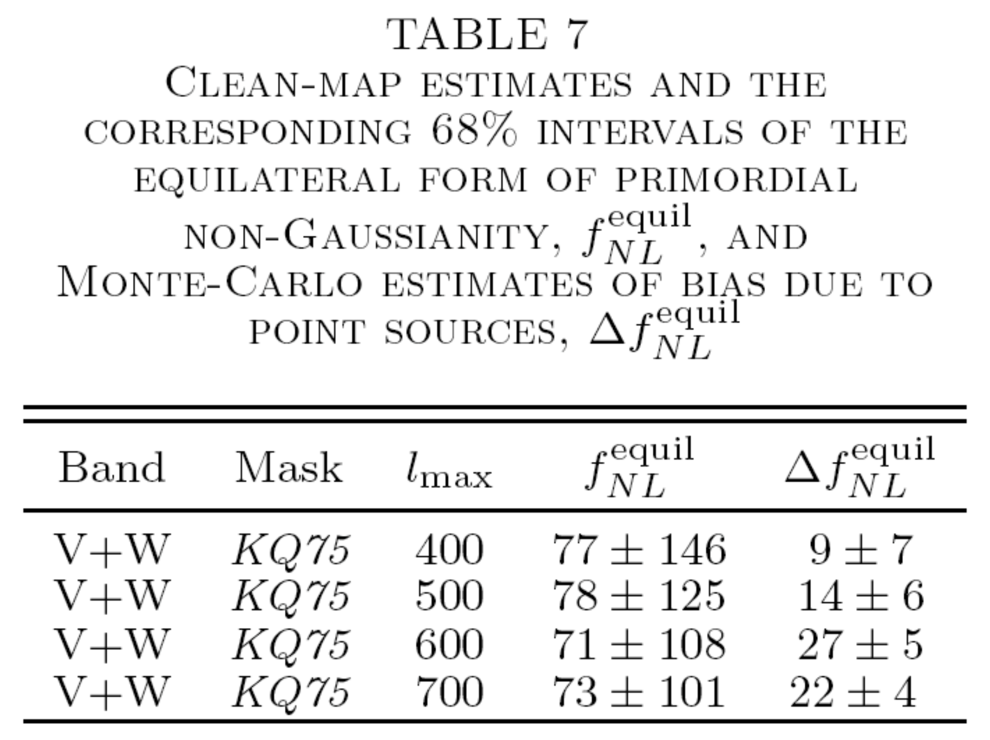


TABLE 7

CLEAN-MAP ESTIMATES AND THE CORRESPONDING 68% INTERVALS OF THE EQUILATERAL FORM OF PRIMORDIAL NON-GAUSSIANITY, f_{NL}^{equil}, AND MONTE-CARLO ESTIMATES OF BIAS DUE TO POINT SOURCES, $\Delta f_{NL}^{\mathrm{equil}}$

Band	Mask	$l_{\max}$	f_{NL}^{equil}	$\Delta f_{NL}^{\mathrm{equil}}$
V+W	*KQ75*	400	77 ± 146	9 ± 7
V+W	*KQ75*	500	78 ± 125	14 ± 6
V+W	*KQ75*	600	71 ± 108	27 ± 5
V+W	*KQ75*	700	73 ± 101	22 ± 4

Komatsu et al. 2008

N.B. 1)

Use of other statistical tools can be very useful to have an independent test of consistency

e.g.: $-8 < f_{NL}^{\mathrm{loc}} < 111 \ (95\%)$ applying wavelets to WMAP 5yr, a result very similar to the limits of the WMAP team (Curto et al. 08)

N.B. 2)

An experiment like Planck can reach a minimum detectable value

$f_{\mathrm{NL}}^{\mathrm{loc}} \sim 3$ and $f_{\mathrm{NL}}^{\mathrm{equil}} \sim 67$

(Komatsu & Spergel 01; Babich & Zaldarriaga 04; Smith & Zaldarriaga 06; Yadav, Komatsu, Wandelt, Hansen, Liguori, Matarrese 08)

N.B. 3):

A lot of information can come from other observables as, e.g, CMB polarization and Large Scale-Structures → See Sabino's Talk

Primordial NG vs secondary effects

Non-primordial sources of NG

✓Any non-linearity can contribute to NG and this can contaminate the extraction of the primordial signal:

NG=NG primordial+non-linear effects

✓ Planck and future experiments will be sensitive to non-Gaussianity at the level of second- or higher order perturbation theory:
need to keep under control all possible sources of contamination to the primordial non-Gaussianity

✓ Need to specify of which primordial non-Gaussianity we study the contamination from the non-primordial sources (local, equilateral..)

Non-primordial sources of NG

There are mainly three classes:

I) (residual) foregrounds and unresolved point sources

II) 'known' secondaries such as
Sunyaev-Zel'dovich effect,
gravitational lensing,
Rees-Sciama effect,……..

III) Less known effects:
non-linearities from Boltzmann equations for
photons, baryons and CDM at second-order in perturbation theory
i.e.: non-Gaussianities related to the non-linear nature of General Relativity
and to the non-linear dynamics of the photon-baryon fluid

Non-primordial sources of NG

What is the impact of non-primordial sources of NG on the extraction of a primordial non-Gaussian signal?

Mainly they act in two ways:

1) They might 'mimic' a three-point function similar in shape (and amplitude) to the primordial one in the CMB →

   bias or contamination to primordial NG

2) They might increase the variance of the estimator of primordial NG
   (in other words they degrade the S/N of primordial NG)

Mimicking a primordial f_{NL}?

Yes: it is possible,and such effects can be important for a local NG in a intermediate region $1< |f^{loc}_{NL}| <10$

For an experiment like Planck (minimum detectable value $f^{loc}_{NL}\sim5$)

- e.g. ISW-lensing bispectrum can contribute a bias $f_{NL}\sim9$ to the local primordial NG; negligible bias to equilateral NG
  (Goldberg & Spergel 99; Smith & Zaldarriaga 06; Serra & Cooray 08; Hanson et al. 09)
- Lensing-Rees Sciama bispectrum contribute a bias $f^{loc}_{NL}\sim10$ to the local primordial NG (Mangilli & Verde 09)

- Lensing can also modify the shape of the primordial bispectrum by smoothing its acoustic features: negligible effect
  (Hanson, Smith, Challinor, Liguori 09; but see also Cooray et al. 09)

- Bispectrum from correlations of number density and lensing magnification of radio and SZ point sources with ISW give a bias of $f^{loc}_{NL}\sim 1.3$
  (Babich & Pierpaoli 08)

From perturbations at second-order

- e.g.: inhomogenous recombination due to electron density perturbations $\delta_e \sim 5\ \delta_b$ give a bias $|f^{loc}_{NL}| \leq 1$
  (Khatri & Wandelt 09; Senatore, Tassev, Zaldarriaga 08)

CMB anisotropies at second-order

Aim:Second-order transfer function

First step: calculation of the full 2-nd order radiation transfer function on large scales (low-l), which includes:

- ✓ **NG initial conditions**
- ✓ non-linear evolution of gravitational potentials on large scales
- ✓ second-order SW effect (and second-order temperature fluctuations on last-scattering surface)
- ✓ second-order ISW effect, both early and late
- ✓ ISW from second-order tensor modes (unavoidably arising from non-linear evolution of scalar modes), also accounting for second-order tensor modes produced during inflation

(N.B, Matarrese & Riotto ’04-’06;
see Boubekeur et al. 09 for lensing)

Second step: **Boltzmann equation at 2-nd order for the photon, baryon and CDM which allows to follow CMB anisotropies at 2-nd order at all scales**

- ✓ **NG from gravity and non-linear dynamics of the photon-baryon fluid at recombination (acoustic oscillations at second order)**
  Crucial to extract information from the CMB bispectrum are the scales of acoustic oscillations according to the analysis of Komatsu and Spergel 2001

(N.B, Matarrese & Riotto JCAP’06-’07)

This also includes both scattering and gravitational secondaries, like:

- ✓ Thermal and Kinetic Sunyaev-Zel’dovich effect
- ✓ Ostriker-Vishniac effect
- ✓ Inhomogeneous reionization
- ✓ Further gravitational terms, including gravitational lensing (both by scalar and tensor modes), Rees-Sciama effect, Shapiro time-delay, effects from second-order vector (i.e. rotational) modes, etc. …

Non-Linear Sachs-Wolfe effect (large scales)

√ Non-linear perturbations $ds^2 = -e^{2\Phi}dt^2 + a^2(t)e^{-2\Psi}\delta_{ij}dx^i dx^j$

a la Salopek-Bond (1990) definition of the non-linear ζ quantity

√ $T_O = \frac{\omega_O}{\omega_\varepsilon} T_\varepsilon$

$$\boxed{\frac{\delta_{\mathbf{np}} T}{T} = e^{\Phi/3} - 1}$$

N.B., S. Matarrese, A. Riotto JCAP 2005

Linear order $\frac{\delta^{(1)}T}{T} = \frac{1}{3}\Phi^{(1)}$

Second order $\boxed{\frac{\delta^{(2)}T}{T} = \frac{1}{3}\Phi^{(2)} + \frac{1}{9}\left(\Phi^{(1)}\right)^2}$

see expression in *N.B., Matarrese, Riotto, Phys. Rev. Lett. (2004)*

It includes primordial NG

Post-inflation non-linear gravity common to all scenarios

Second-order CMB Anisotropies

$$\frac{df}{d\eta} = a\,C[f]$$

Collision term

$$\frac{df}{d\eta} = \underbrace{\frac{\partial f}{\partial \eta} + \frac{\partial f}{\partial x^i}\frac{dx^i}{d\eta} + \frac{\partial f}{\partial p}\frac{dp}{d\eta} + \frac{\partial f}{\partial n^i}\frac{dn^i}{d\eta}}_{\text{Gravity effects}}$$

Metric perturbations: Poisson gauge

$$\boxed{ds^2 = a^2(\eta)\left[-e^{2\Phi}d\eta^2 + 2\omega_i dx^i d\eta + (e^{-2\Psi}\delta_{ij} + \chi_{ij})dx^i dx^j\right]}$$

$$\Phi = \Phi^{(1)} + \frac{1}{2}\Phi^{(2)}, \qquad \psi = \psi^{(1)} + \frac{1}{2}\psi^{(2)}$$

Example: using the geodesic equation for the photons

$$\frac{1}{p}\frac{dp}{d\eta} = -\mathcal{H} + \Psi' - \Phi_{,i}\,n^i e^{\Phi+\Psi} - \omega_i' n^i - \frac{1}{2}\chi_{ij}' n^i n^j$$

→ Redshift of the photon (*Sachs-Wolfe and ISW effects*)

PS: Here the photon momentum is **p=** pn^i with $p^2 = g_{ij} P^i P^j$

($P^\mu = dx^\mu(\lambda)/d\lambda$ quadri-momentum vector)

The 2nd-order photon Boltzmann equation

$$\Delta^{(2)\prime} + n^i \frac{\partial \Delta^{(2)}}{\partial x^i} - \tau' \Delta^{(2)} = S$$

$$\Delta^{(2)}(x^i, n^i, \eta) = \frac{\int dp p^3 f^{(2)}}{\int dp p^3 f^{(0)}}$$

N.B: for a derivation of the Boltzmann equations see also
C. Pitrou CQG 09 (includes polarization);
Senatore, Tassev, Zaldarriaga, arXiv:0812.36523

with $\tau' = -n_e \sigma_T a$ optical depth

Source term $S = S^{(2)} + S^{(I\times I)}$

Second-order baryon velocity

Sachs-Wolfe effect

$$S^{(2)} = -\tau'\underline{(\Delta^{(2)}_{00} + 4\Phi^{(2)})} + 4(\Phi^{(2)} + \Psi^{(2)})' - 8\omega_i' n^i - 4\chi_{ij}' n^i n^j - \tau'\left[4\mathbf{v}^{(2)}\cdot\mathbf{n} - \frac{1}{2}\sum_{m=-2}^{2}\frac{\sqrt{4\pi}}{5^{3/2}}\Delta^{(2)}_{2m} Y_{2m}(\mathbf{n})\right]$$

$$\begin{aligned} S^{(I\times I)} &= -8\Delta^{(1)}\left(\Psi^{(1)\prime} - \Phi^{(1)}_{,i} n^i\right) + 2n^i(\Phi^{(1)} + \Psi^{(1)})\partial_i(\Delta^{(1)} + 4\Phi^{(1)}) \\ &+ \left[(\Phi^{(1)}_{,j} + \Psi^{(1)}_{,j}) n^i n^j + (\Phi^{,i} - \Psi^{,i})\right]\frac{\partial \Delta^{(1)}}{\partial n^i} \xrightarrow{\qquad} \text{Gravitational lensing} \\ &- \tau'\left[2\delta^{(1)}_e\left(4\mathbf{v}\cdot\mathbf{n} + \Delta^{(1)}_0 - \Delta^{(1)} + \frac{1}{2}\Delta^{(1)}_2 P_2(\hat{\mathbf{v}}\cdot\mathbf{n})\right)\right. \\ &+ \left. 2(\mathbf{v}\cdot\mathbf{n})\left[\Delta^{(1)}_0 + 3\Delta^{(1)}_0 - \Delta^{(1)}_2\left(1 - \frac{5}{2}P_2(\hat{\mathbf{v}}\cdot\mathbf{n})\right)\right] - v\Delta^{(1)}_1(5 + 4P_2(\hat{\mathbf{v}}\cdot\mathbf{n})) + 14(\mathbf{v}\cdot\mathbf{n})^2 - 2v^2\right] \end{aligned}$$

Quadratic-Doppler effect

Coupling velocity and linear photon anisotropies

CMB angular bispectrum

$$B^{m_1 m_2 m_3}_{l_1 l_2 l_3} \equiv \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle \,,$$

$$B_{l_1 l_2 l_3} = \sum_{\text{all } m} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} B^{m_1 m_2 m_3}_{l_1 l_2 l_3} \,,$$

$$\Delta^{(1)} + ik\mu\Delta^{(1)} - \tau'\Delta^{(1)} = S^{(1)}(\mathbf{k}, \hat{\mathbf{n}}, \eta)$$
$$\Delta^{(2)} + ik\mu\Delta^{(2)} - \tau'\Delta^{(2)} = S^{(2)}(\mathbf{k}, \hat{\mathbf{n}}, \eta)$$

$$S^{(2)}_{lm}(\mathbf{k}) = \int \frac{d^3k'}{(2\pi)^3} \int d^3k'' \delta^3(\mathbf{k}' + \mathbf{k}'' - \mathbf{k}) \times \mathcal{S}^{(2)}_{lm}(\mathbf{k}', \mathbf{k}'', \mathbf{k}) \zeta(\mathbf{k}') \zeta(\mathbf{k}'')$$

Harmonic components of the CMB source function

CMB angular bispectrum (II)

$$a^{(1)}_{lm} = 4\pi(-i)^l \int \frac{d^3k}{(2\pi)^3} g_l(k) Y^*_{lm} \zeta(\mathbf{k})$$

$$\Phi(\mathbf{x}) = \Phi_L(\mathbf{x}) + f_{NL}\left(\Phi_L^2(\mathbf{x}) - \langle\Phi_L^2(\mathbf{x})\rangle\right)$$

$$\begin{aligned} a^{(2)}_{lm} &= \frac{4\pi}{8}(-i)^l \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3k'}{(2\pi)^3} \int d^3k'' \delta^3(\mathbf{k}' + \mathbf{k}'' - \mathbf{k}) \\ &\times \sum_{l'm'} \underline{F^{l'm'}_{lm}(\mathbf{k}', \mathbf{k}'', \mathbf{k})} Y^*_{l'm'}(\hat{\mathbf{k}}) \zeta(\mathbf{k}') \zeta(\mathbf{k}'') \end{aligned}$$

Second-order radiation transfer function

Primordial curvature perturbation

Nitta, Komatsu, N.B, Matarrese, Riotto 09

$$\begin{aligned} F^{l'm'}_{lm}(\mathbf{k}', \mathbf{k}'', \mathbf{k}) &= i^l \sum_{\lambda\mu} (-1)^m (-i)^{\lambda - l'} \mathcal{G}^{-mm'\mu}_{ll'\lambda} \\ &\times \sqrt{\frac{4\pi}{2\lambda + 1}} \int_0^{\eta_0} d\eta e^{-\tau} \underline{\mathcal{S}^{(2)}_{\lambda\mu}(\mathbf{k}', \mathbf{k}'', \mathbf{k})} j_{l'}[k(\eta - \eta_0)]. \end{aligned}$$

CMB source function

A closer look at the CMB source function

Two types of contributions:

$$\Theta^{(2)} = (\Delta^{(2)}_{00}/4 + \Phi^{(2)})$$

1) Intrinsically second-order term

on large scales it gives rise to the Sachs-Wolfe effect;
on small scales it grows as η^2
(pointed out by Pitrou, Uzan, Bernadeau 08)

Contamination to equilateral NG

2) (first-order)^2-terms (oscillating/constant in time)

Contamination to local NG

Acoustic oscillations at second-order

In the tight coupling limit the energy and momentum continuity equations for photons and baryons at second-order describes the ***non-linear dynamics of the photon-baryon fluid at recombination***

$$\left(\Delta_{00}^{(2)\prime\prime} - 4\Psi^{(2)\prime\prime}\right) + H\frac{R}{1+R}\left(\Delta_{00}^{(2)\prime} - 4\Psi^{(2)\prime}\right) - c_s^2\nabla^2\left(\Delta_{00}^{(2)} - 4\Psi^{(2)}\right) = \frac{4}{3}\nabla^2\left(\Phi^{(2)} + \frac{\Psi^{(2)}}{1+R}\right) + \boxed{S'_\Delta + H\frac{R}{1+R}S'_\Delta - \frac{4}{3}\partial_i S_V^i}$$

Source made by (first-order)^2 terms.
For expressions and analytical solutions see
N.B, Matarrese, Riotto JCAP 07

$$\Delta_{00}^{(2)} = \delta_\gamma^{(2)}$$

$$c_s = \frac{1}{\sqrt{3(1+R)}}, \qquad R = \frac{3}{4}\frac{\rho_b}{\rho\gamma}$$

Non-linear dynamics at recombination

On small scales, i.e. modes k >> k_{eq}, the second-order anisotropies at recombination are dominated by the 2nd-oder gravitational potential sourced by dark matter perturbations

$$\Phi^{(2)} \simeq \Psi^{(2)} = \Psi^{(2)}(0) - \frac{1}{14}\left(\partial_k\Phi^{(1)}\partial^k\Phi^{(1)} - \frac{10}{3}\frac{\partial_i\partial^j}{\nabla^2}\left(\partial_i\Phi^{(1)}\partial_j\Phi^{(1)}\right)\right)\eta^2$$

Initial conditions that contain the primordial NG

in Fourier space gives the convolution kernel

$$G(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k})\eta^2 = \left[\mathbf{k}_1 \bullet \mathbf{k}_2 - \frac{10}{3}\frac{(\mathbf{k}\bullet\mathbf{k}_2)(\mathbf{k}\bullet\mathbf{k}_1)}{k^2}\right]\eta^2$$

As a generalization to the well known expression at linear-order

$$\Theta^{(2)} = \frac{1}{4}\Delta_{00}^{(2)} + \Phi^{(2)} \sim A\cos[kc_s\eta]\, e^{-(k/k_D)^2} - R\Phi^{(2)} + S$$

(For details see Pitrou et al. 08; see also N.B, Matarrese, Riotto 07)

On small scales the combination of the damping effects AND the growth of the potential as η^2 make dominant the term

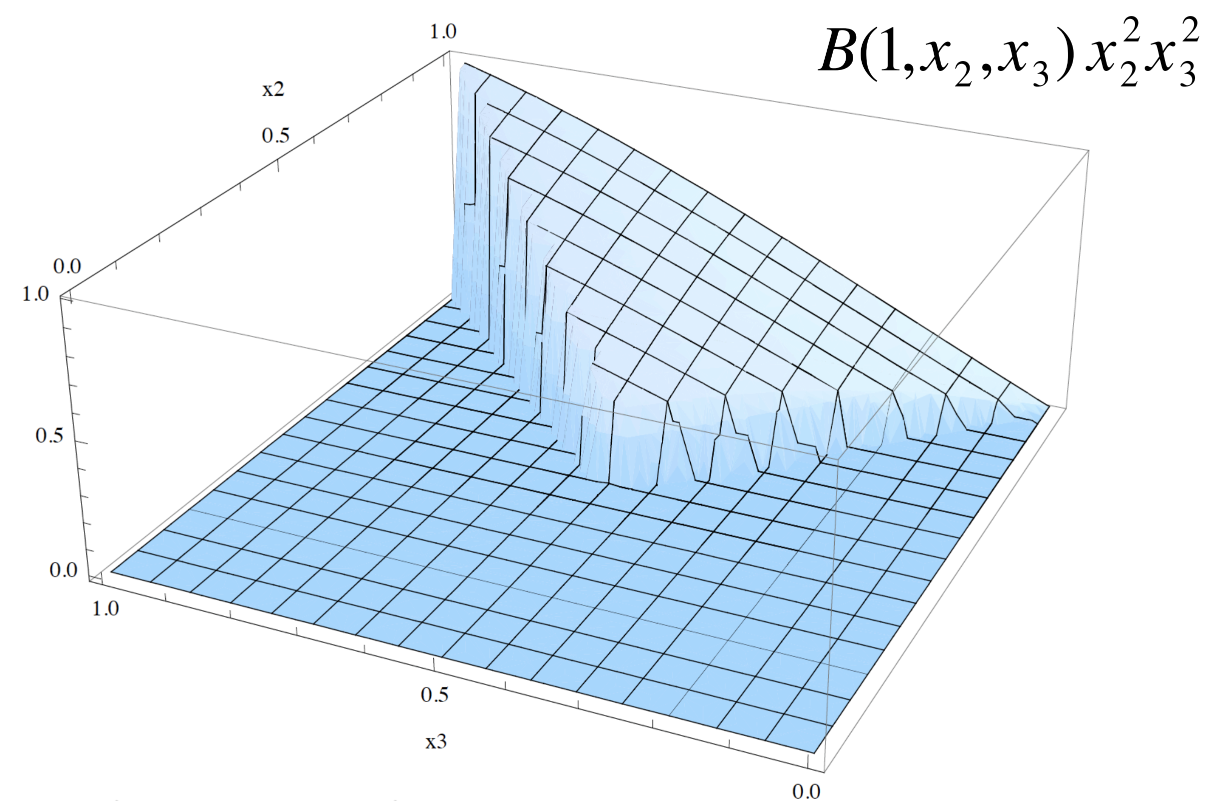
$$-R\Phi^{(2)} = -\frac{R}{14}G(\mathrm{k}_1,\mathrm{k}_2,\mathrm{k})\eta^2\Phi^{(1)}(\mathrm{k}_1)\Phi^{(1)}(\mathrm{k}_2)$$

Non-Gaussianity from 2nd-order gravitational potential

$$\Theta^{(2)} \cong -R\Phi^{(2)} \cong -\frac{R}{14} G(\mathrm{k}_1,\mathrm{k}_2,\mathrm{k})\eta^2\Phi^{(1)}(\mathrm{k}_1)\Phi^{(1)}(\mathrm{k}_2)$$

This effect is a causal one, i.e. developing on small scales; its origin is gravitational (due to the non-linear growth sourced by dark matter perturbations)

We expect the corresponding CMB bispectrum will be of the equilateral type.



$x_3 \equiv k_3/k_1$ and $x_2 \equiv k_2/k_1$

Important comment:

so, even if the NG at recombination is dominated by this effect, it will have a minimal contamination to the local primordial NG

Signal to Noise ratio and correlation

See Spergel and Goldberg '99; Cooray and Hu 200; Komatsu and Spergel 2001

$$\chi^2 \equiv \sum_{2\leq l_1\leq l_2\leq l_3} \frac{\left(B^{obs}_{l_1l_2l_3} - \sum_i A_i B^{(i)}_{l_1l_2l_3}\right)^2}{\sigma^2_{l_1l_2l_3}}$$

$$\sigma^2_{l_1l_2l_3} \equiv \langle B^2_{l_1l_2l_3}\rangle - \langle B_{l_1l_2l_3}\rangle^2 \approx \mathcal{C}_{l_1}\mathcal{C}_{l_2}\mathcal{C}_{l_3}\Delta_{l_1l_2l_3}$$

$B^{(i)}_{l_1l_2l_3}$: primordial or secondary bispectra

Fisher matrix

$$\boxed{F_{ij} \equiv \sum_{2\leq l_1\leq l_2\leq l_3} \frac{B^{(i)}_{l_1l_2l_3} B^{(j)}_{l_1l_2l_3}}{\sigma^2_{l_1l_2l_3}}}$$

$$\boxed{\left(\frac{S}{N}\right)_i \equiv \frac{1}{\sqrt{F^{-1}_{ii}}},}$$

$$\boxed{r_{ij} = \frac{F^{-1}_{ij}}{\sqrt{F^{-1}_{ii}F^{-1}_{jj}}}} \rightarrow$$

How similar are two bispectra?

Contamination to primordial f_NL

We fit the primordial bispectrum template to th 2nd-order bispectrum:

define the best-fitting ‘contamination’ f^{cont}_{NL} by minimizing

$$\chi^2 = \sum_{2\leq l_1\leq l_2\leq l_3} \frac{\left(f^{\mathrm{cont}}_{NL} B^{prim}_{l_1 l_2 l_3} - B^{2nd}_{l_1 l_2 l_3}\right)^2}{\sigma^2_{l_1 l_2 l_3}}$$

↓

$$f^{\mathrm{cont}}_{NL} = \frac{F_{2nd,prim}}{F_{prim,prim}}\bigg|_{f_{NL}=1} = \frac{1}{N} \sum_{2\leq l_1\leq l_2\leq l_3} \frac{B^{2nd}_{l_1 l_2 l_3} B^{prim}_{l_1 l_2 l_3}}{\sigma^2_{l_1 l_2 l_3}}$$

$$N = \sum_{2\leq l_1\leq l_2\leq l_3} \frac{{B^{prim}_{l_1 l_2 l_3}}^2}{\sigma^2_{l_1 l_2 l_3}}$$

Correlation to primordial f_{NL} of the intrinsically second-order term $\theta^{(2)}=-R\Phi^{(2)}$

Fisher matrix

$$F_{ij} = \frac{f_{\rm sky}}{\pi} \frac{1}{(2\pi)^2} \int d^2\ell_1 d^2\ell_2 d^2\ell_3 \, \delta^{(2)}(\vec{\ell}_{123}) \frac{B^i(\ell_1,\ell_2,\ell_3)\, B^j(\ell_1,\ell_2,\ell_3)}{6\, C(\ell_1)\, C(\ell_2)\, C(\ell_3)}$$

For EQUILATERAL primordial f^{eq}_{NL}

$$\begin{aligned}
\left(\frac{S}{N}\right)_{\rm equil} &= \frac{1}{\sqrt{F^{-1}_{\rm equil,equil}}} \simeq 12.6 \times 10^{-3} f_{\rm NL}^{\rm equil} \\
\left(\frac{S}{N}\right)_{\rm rec} &= \frac{1}{\sqrt{F^{-1}_{\rm rec,rec}}} \simeq 0.1 \\
r_{\rm rec,equil} &= \frac{F^{-1}_{\rm rec,equil}}{\sqrt{F^{-1}_{\rm equil,equil} F^{-1}_{\rm rec,rec}}} \simeq -0.53 \\
d_{\rm rec} &= F_{\rm rec,rec} F^{-1}_{\rm rec,rec} \simeq 1.4 \\
d_{\rm equil} &= F_{\rm equil,equil} F^{-1}_{\rm equil,equil} \simeq 1.4
\end{aligned}$$

As a confirmation of our expectations the NG from recombination (governed by the non-linear evolution of the 2nd-order gravitational potential) shows a quite high correlation with an equilateral primordial bispectrum

N.B, Riotto JCAP 09

✓ Degradation of the 1-σ uncertainty of f^{eq}_{NL}

$r_{rec,equil}$= –0.53 translates into an increase of the minimum detectable value for f^{eq}_{NL}. We find a mimimum value of

$$\boxed{f_{\rm NL}^{\rm equil} \simeq 79} \quad \longleftrightarrow \quad \boxed{\Delta f_{\rm NL}^{\rm equil} = \mathcal{O}(10)}$$

It corresponds to a an increase of O(10): recall: f^{eq}_{NL}=67, not accounting for the cross-correlation (i.e. not marginalized over the signal from recombination).

✓ Contamination to primordial f_{NL}

Contamination to equilateral	$f_{NL}^{cont} = O(10)$
Contamination to local	$f_{NL}^{cont} \approx 0.3$

Given the 1-σ uncertainty $f^{loc}_{NL} \sim 5$ for local, and $f^{eq}_{NL} \sim 67$ for equilateral, these numbers are not relevant.

So the contamination from the intrinsically second-order term $\theta^{(2)}=-R\Phi^{(2)}$ to a primordial local NG is minimal;
what is relevant is the degradation in the minimum detectable value of f^{eq}_{NL} for primordial equilateral NG

A check for our model:
S/N for the primordial equilateral bispectrum

Use the flat sky approximation

$$a(\vec{\ell}) = \int \frac{dk^z}{2\pi} e^{ik^z(\eta_0-\eta_r)} \Phi(\mathbf{k}') \tilde{\Delta}^T(\ell, k^z)$$

transfer function on small scales
(l >>l$_*$ ~ 750; a ~3)

$$\tilde{\Delta}^T(\ell, k^z) = a\,(\eta_0 - \eta_r)^{-2} e^{-1/2(\ell/\ell_*)^{1.2}} e^{-1/2(|k_z|/k_*)^{1.2}}$$

power spectrum
(see also Babich & Zaldarriaga 04)

$$C(\ell) \simeq a^2 \frac{A}{\pi} \frac{\ell_*}{\ell^3}\, e^{-(\ell/\ell_*)^{1.2}}$$

Bispectrum

$$B_{\rm equil}(\ell_1, \ell_2, \ell_3) = \frac{(\eta_0 - \eta_r)^2}{(2\pi)^2} \int dk_1^z dk_2^z dk_3^z \delta^{(1)}(k_{123}^z) B_{\rm equil}(k'_1, k'_2, k'_3) \tilde{\Delta}^T(\ell_1, k_1^z) \tilde{\Delta}^T(\ell_2, k_2^z) \tilde{\Delta}^T(\ell_3, k_3^z)$$

(5

$$B_{\rm equil}(k_1, k_2, k_3) = f_{\rm NL}^{\rm equil} \cdot 6A^2 \cdot \left(-\frac{1}{k_1^3 k_2^3} - \frac{1}{k_1^3 k_3^3} - \frac{1}{k_2^3 k_3^3} - \frac{2}{k_1^2 k_2^2 k_3^2} + \frac{1}{k_1 k_2^2 k_3^3} + (5\ \text{perm.}) \right)$$

$$B_{\rm equil}(\ell_1, \ell_2, \ell_3) = \frac{24 f_1}{(2\pi)^2} f_{\rm NL}^{\rm equil} a^3\, A^2\, e^{-\ell_T^{1.2}/2\ell_*^{1.2}}\, \ell_*^2 \left(-\frac{1}{\ell_1^3 \ell_2^3} - \frac{1}{\ell_1^3 \ell_3^3} - \frac{1}{\ell_2^3 \ell_3^3} - \frac{2}{\ell_1^2 \ell_2^2 \ell_3^2} + \frac{1}{\ell_1 \ell_2^2 \ell_3^3} + (5\ \text{perm.}) \right)$$

S/N for the primordial equilateral bispectrum

Signal-to-noise ratio

$$\left(\frac{S}{N}\right)^2_{\mathrm{equil}} \simeq 8\, f_{\mathrm{sky}}\, A\, (f_{\mathrm{NL}}^{\mathrm{equil}})^2\, \ell_{\mathrm{max}}$$

N.B. & Riotto JCAP 09

For f_{sky}=0.8 and l_{max}=2000, for an experiment like Planck gives a minimum detectable

$$f_{\mathrm{NL}}^{\mathrm{equil}} \simeq 66$$

very good agreement with the numerical results of Smith and Zaldarriaga 06, and Liguori 09

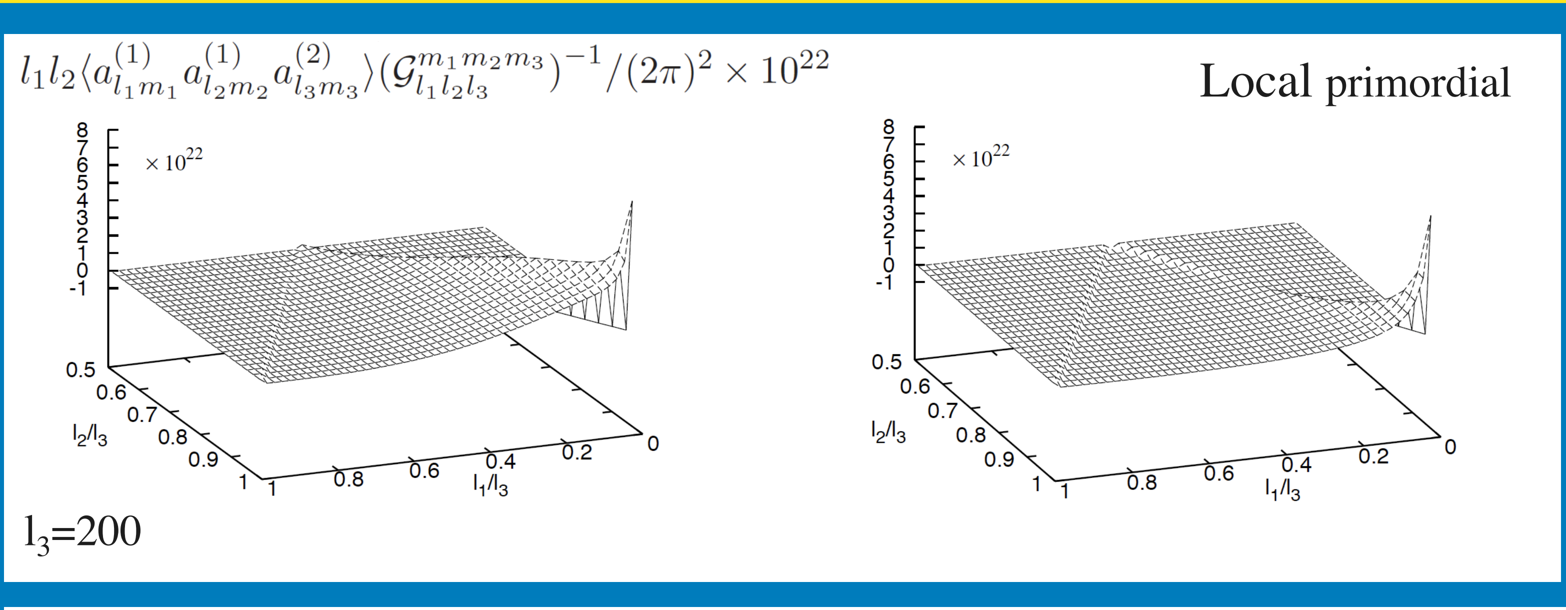
N.B. The S/N scale as $(l_{max})^{1/2}$, not as l_{max} as in the local case;

$(S/N)^2$ receive contribution from the configuration which is peaked at l_1~l_2~l_3,

$$(S/N)^2 \propto \int d^2l_1\, d^2l_2\, \delta(l_1 - l_2)/l_1^2 \propto l_{\max}$$

Shape of the second-order bispectrum from products of 1st-order terms

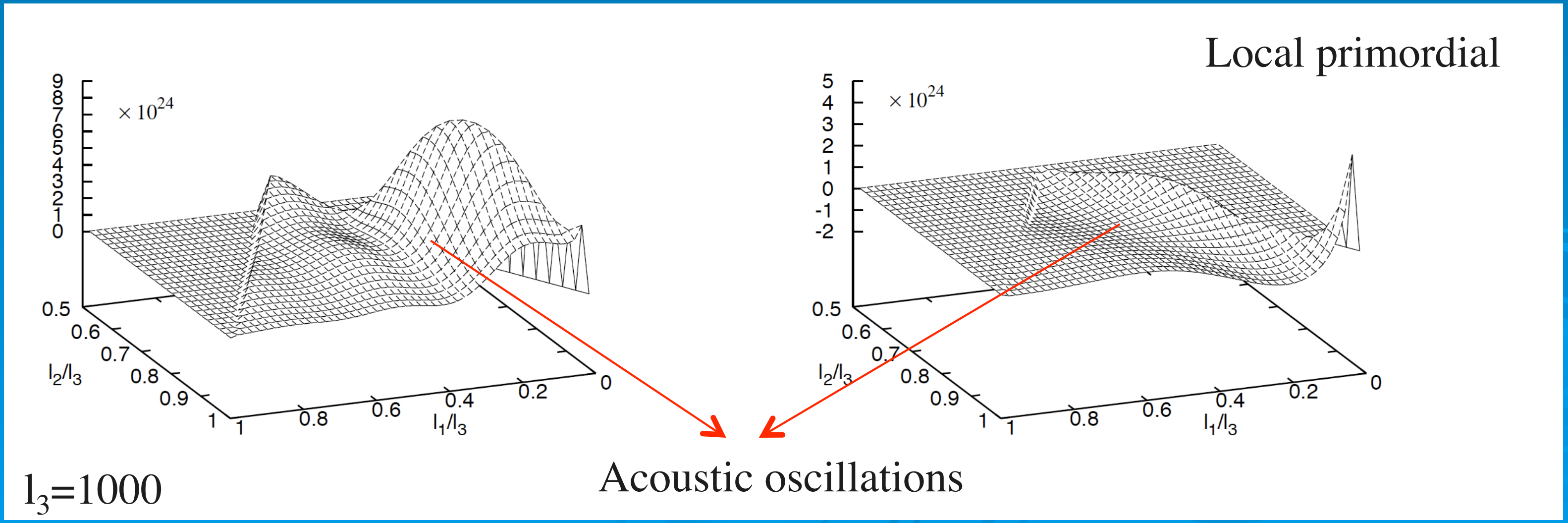
Maximum signal in the squeezed triangles, $l_1 \ll l_2 \sim l_3$, similar to the local primordial bispectrum



$l_1 l_2 \langle a^{(1)}_{l_1 m_1} a^{(1)}_{l_2 m_2} a^{(2)}_{l_3 m_3} \rangle (\mathcal{G}^{m_1 m_2 m_3}_{l_1 l_2 l_3})^{-1}/(2\pi)^2 \times 10^{22}$

Local primordial

l_3=200



Local primordial

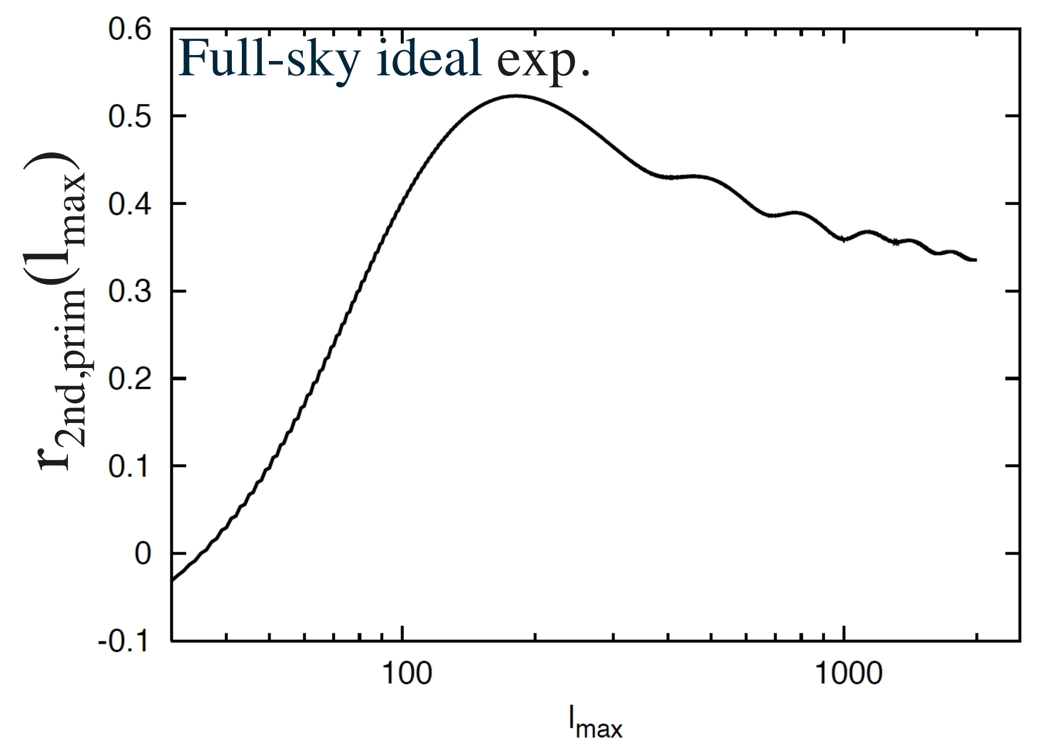
Acoustic oscillations

l_3=1000

Cross-correlation

$$\boxed{r_{ij} = \frac{F_{ij}^{-1}}{\sqrt{F_{ii}F_{ij}}}}$$

→ How similar are 2 bispectra?

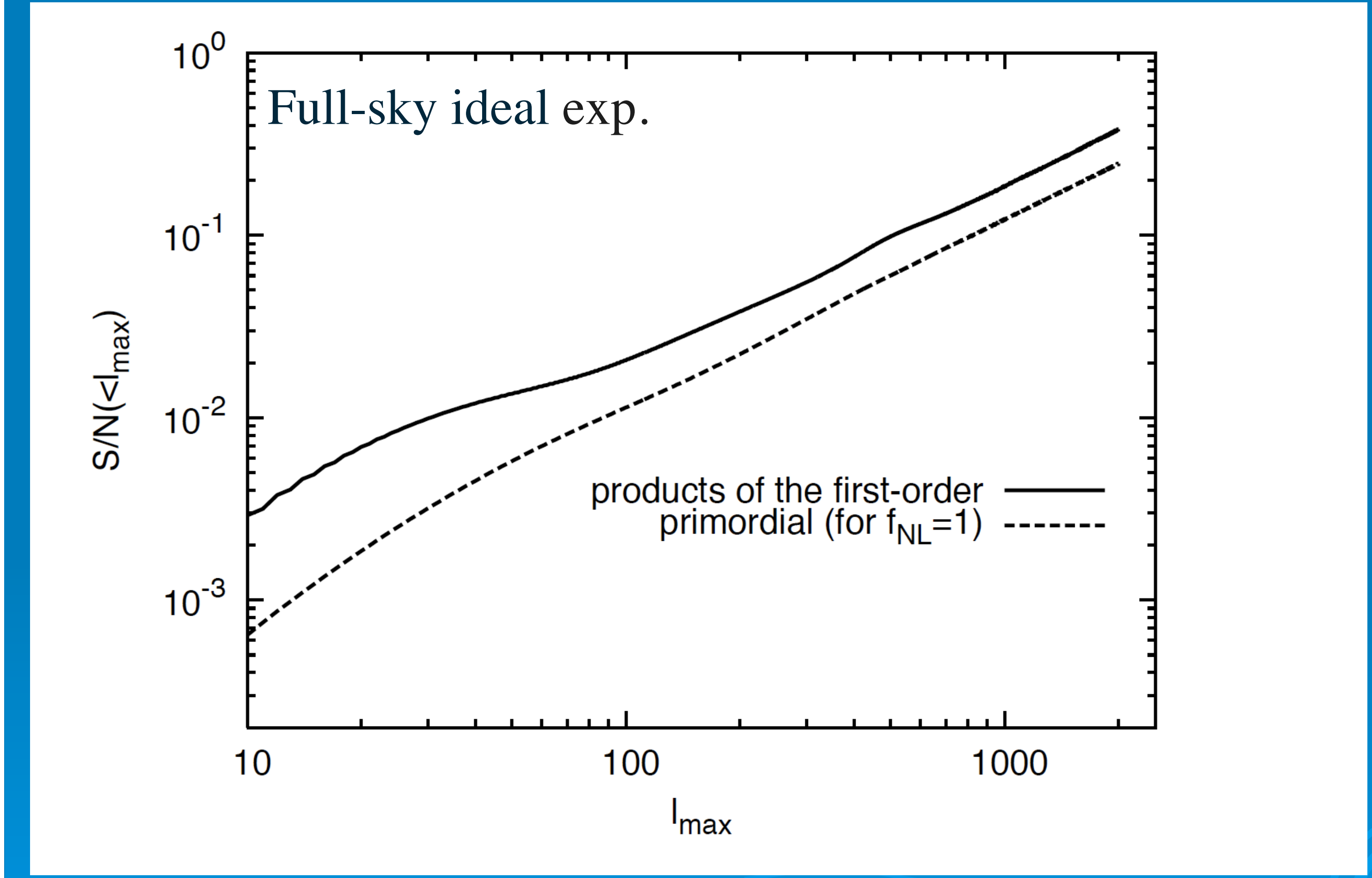


2^nd^-order bispectrum and local primordial are fairly similar

$r_{2nd,prim} \sim 0.5$ at lmax~200

$r_{2nd,prim} \sim 0.3$ at lmax~2000

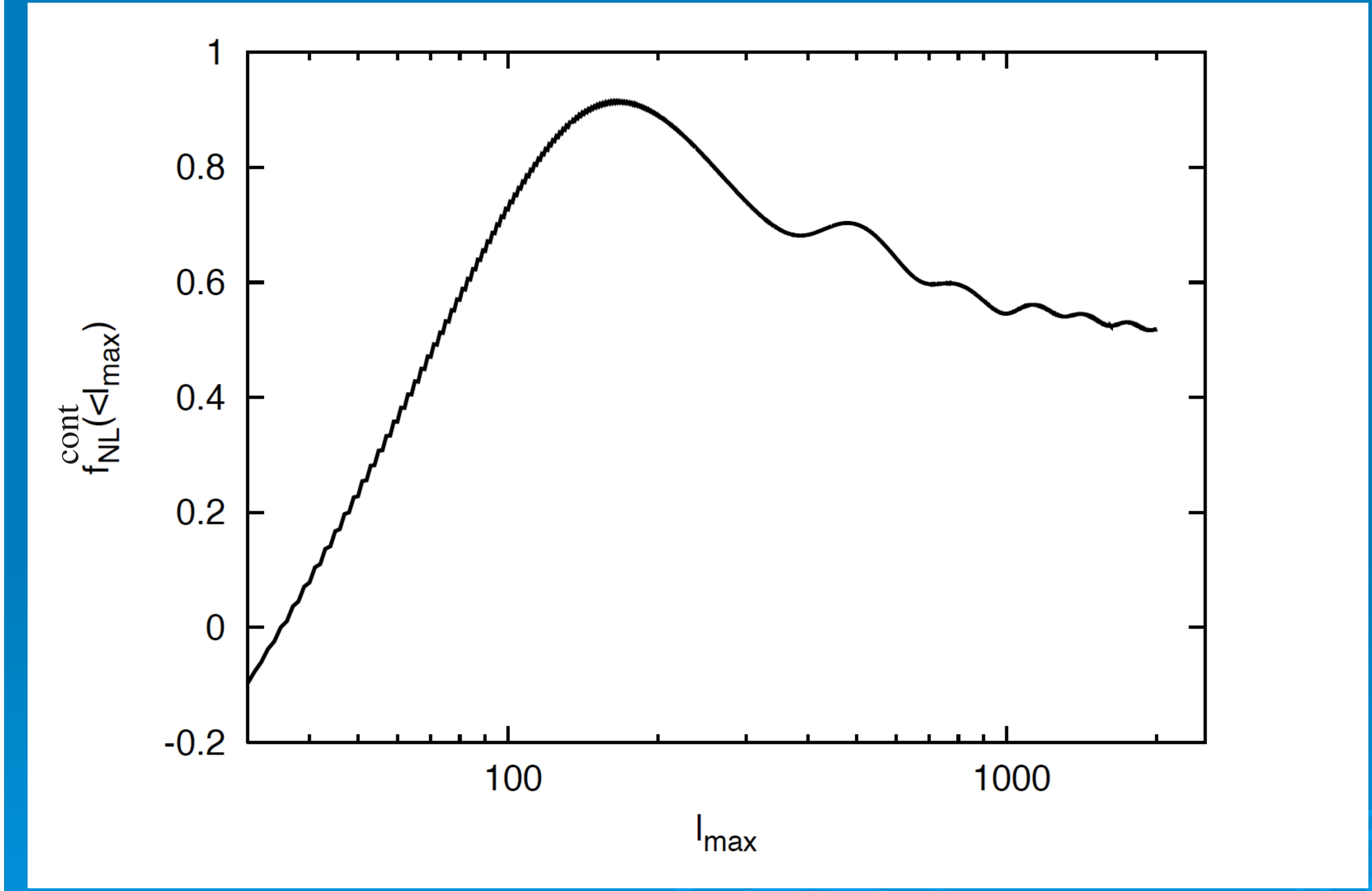
Signal to Noise ratio: numerical results for (first-order)^2-terms



(Nitta, Komatsu, N.B., Matarrese, Riotto, JCAP 09)

(S/N) from the (first-order×first-order) terms is about 0.4 at $l_{max}\approx 2000$ for an ideal full-sky experiment

Contamination to primordial f_NL of the local type from (first-order)^2-terms



(Nitta, Komatsu, N.B., Matarrese, Riotto, JCAP 09)

Contamination is 0.9 at l_{max}≈200 and 0.5 at l_{max}≈2000 vs 5 which is the minimum detectable value for Planck

FURTHER CONSIDERATIONS

Anisotropic non-Gaussianity

✓ Growing interest in models which can produce some level of statistical anisotropy in the CMB

✓ There exist constraints on a preferred direction and the level of statistical anisotropy (Eriksen et al. 09)

$$P(\mathbf{k}) = P(k)(1 + g(k)(\hat{\mathbf{k}} \cdot \hat{\mathbf{n}})^2) \qquad |g| < 0.3$$

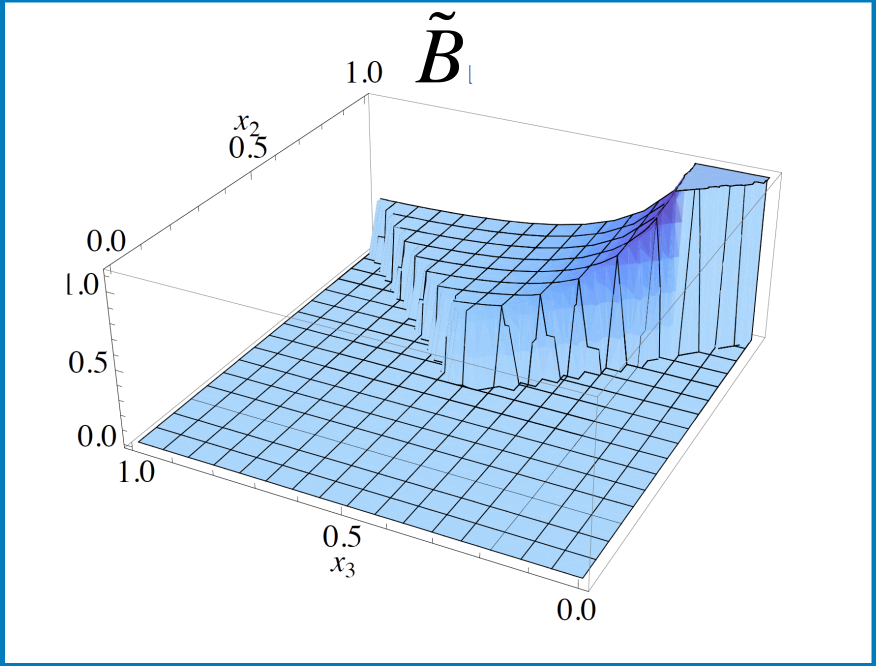
✓**<u>Anisotropic non-Gaussianity</u>** predicted in models where vector fields play a role during inflation (e.g. Karciauskas, Dimopolous, Lyth 09)

$$\langle \Phi(\vec{k}_1)\Phi(\vec{k}_2)\Phi(\vec{k}_2)\rangle = (2\pi)^3\delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)B(\vec{k}_1, \vec{k}_2, \vec{k}_3)$$

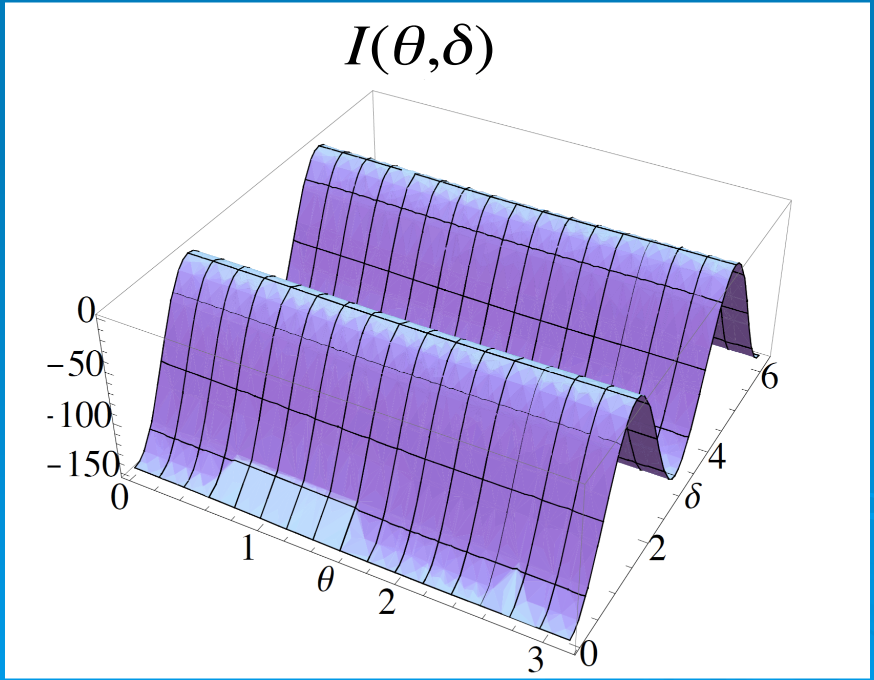
violation of rotational invariance

We considered a SU(2)-vector multiplet: NG peaks in the local configuration with an amplitude that is modulated by the preferred directions that break rotational invariance
(Dimastrogiovanni, N.B., Matarrese & Riotto 09)

$$B(\vec{\mathrm{k}}_1,\vec{\mathrm{k}}_2,\vec{\mathrm{k}}_1) \sim \mathrm{I}(\vec{n}_i \bullet \vec{k}_i)\ \tilde{B}(\mathrm{k}_1,\mathrm{k}_2,\mathrm{k}_3)$$



Isotropic contribution to the bispectrum



Modulation dependent on the preferred directions

If the primordial f_{NL} is compatible with zero, are primordial pertb.ns Gaussian distributed ?

NO!!!

- One can have models with zero bispectrum and non-vanishing *trispectrum*

$$\left\langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3)\Phi(\mathbf{k}_4) \right\rangle$$

- e.g., this is what happens in some configurations of the curvaton models

TRISPECTRUM

$$\boxed{\Phi = \Phi_L + f_{NL} * \left(\Phi_L^2 - \left\langle \Phi_L^2 \right\rangle \right) + g_{NL} * \Phi_L^3}$$

(e.g., Okamoto & Hu, 2002)

Also in this case primordial cubic non-linearities + non-linearities coming from the post-inflationary evolution

Ex.: see expressions in D'Amico, N.B., Matarrese, Riotto (JCAP 08) for cubic non-linearities in the CMB anisotropies on all scales from gravitational effects

Theoretical Predictions for the trispectrum

model	$g_{NL}(\mathbf{k}_1,\mathbf{k}_2)$	comments
Slow-roll inflation (including multiple fields)	$O(\varepsilon,\eta)$	ε,η : slow-roll parameters
Curvaton scenario	$(9/4\ r^2)\ (g^2\ g'''/g'^3+3g\ g''/g'^2)$ + - $(2/r)\ (1+\ 3g\ g''/g'^2)$	g'': deviation from a quadratic potential
Inhomogeneous reheating	$(5/3)\ f_{NL}^2+\ (25/24)\ (Q'''(x)/\ Q'^3(x))$	$X=\Gamma/H$ at the end of inflation
DBI inflation	$\sim 0.1\ c_s^{-4}$	C_s^2 : sound speed
Ekpyrotric models	$\lvert g_{NL}\rvert < 10^4$	depends on the parameter choice

CMB trispectrum on large scales

See N.B, Matarrese and Riotto (JCAP 05) for the expressions of the Measurable g_{NL} entering in the trispectrum of CMB anisotropies, allowing for generic NG initial conditions

$$\frac{\Delta T}{T} = \frac{1}{3}\Phi$$

$$\Phi = \Phi_L + f_{NL} * \left(\Phi_L\right)^2 + g_{NL} * \left(\Phi_L\right)^3$$

$$g_{NL} = \frac{25}{9} b_{NL} + \frac{5}{9} a_{NL}\left[5\mathbf{A}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) - 1\right] + \frac{1}{54} + \frac{25}{9}\mathbf{C}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$
$$- \frac{1}{3}\left[\frac{(\mathbf{k}_1 \cdot (\mathbf{k}_1 + \mathbf{k}_2))(\mathbf{k}_2 \cdot (\mathbf{k}_1 + \mathbf{k}_2))}{|\mathbf{k}_1 + \mathbf{k}_2|^2} - \frac{1}{3}\frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{|\mathbf{k}_1 + \mathbf{k}_2|^2} + cycl.\right]$$

N.B., Matarrese & Riotto 2005
D'Amico, N.B., Matarrese & Riotto 2008

$$\zeta = \zeta_L + a_{NL}\zeta_L^2 + b_{NL}\zeta_L^3 + \ldots.$$

Conclusions

- √ Both a positive measurement of the non-Gaussainity or an upper limit on its amplitude will represent a crucial observational discriminant between competing models for the primordial perturbation generation

- √ A detection of f_{NL}~10 would rule out all standard single-field models of inflation

- √ To asses a detection of primordial NG: take care of foregrounds and any secondary signal that can mimick a primordial NG signal

- √ We are assessing precisely the level and shape of NG from 2nd-order perturbations and their contamination to primordial NG

- √ Future techniques exploiting the CMB polarization and additional independent statistical estimators might help NG detection down to f_{NL}~3: need to compute exactly the ***predicted amplitude and shape of NG from the post-inflationary evolution of perturbations***.