The low quadrupole:
Theoretical issues and MCMC data analysis

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The Standard Model of the universe:
from Inflation to today Dark Energy

Outline

1. Is the low CMB TT quadruple too low?
   - Observational data
   - Cosmic variance
   - Independent random variables
2. Theoretical setup
   - EFT of Inflation
   - New inflation
   - Fluctuations and initial conditions
3. MCMC analysis
   - Cosmological MCMC
   - MCMC likelihoods
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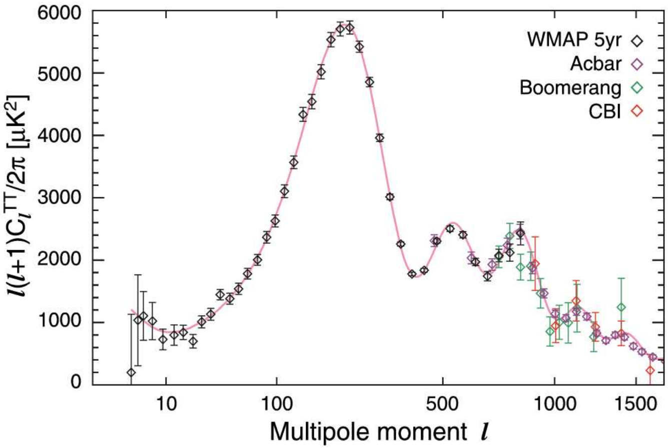
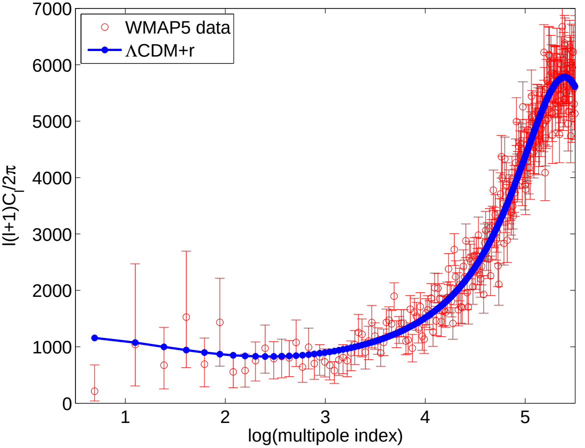
The WMAP+small scale TT multipoles (binned)

from "M. R. Nolta et al.", arXiv:0803.0593 [astro-ph] 5 Mar 2008



$$C_2 = 223\ \mu K^2 \ \text{(WMAP5 ML value)}\ , \qquad C_2 \simeq 1200\ \mu K^2\ (\Lambda\text{CDM})$$

WMAP5 unbinned C_ℓ for $\ell \leq 250$



(experimental error)/(cosmic variance) $\leq$ 20% for $\ell \leq 250$

Other analysis of WMAP5 data

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$$C_2 = 557\ \mu K^2\ \ (\text{WMAP5}+150\%)\ , \qquad C_3 = 306\ \mu K^2\ (\text{WMAP5}-40\%)$$

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$$C_2, C_3, C_4 \to 0\ , \qquad C_2, C_3, C_4, C_5, C_6 \to (\text{WMAP5}-50\%)$$

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Neglecting all uncertainities but cosmic variance:

Let $X_\ell = C_\ell^{(data)}/C_\ell^{(model)}$; then

$Pr(X_\ell = x|model) \propto \frac{1}{x}\left(xe^{-x}\right)^{\ell+1/2}$ (reduced chi-square distribution) is the probability density for $C_\ell^{(data)}$ given the model, with

$$\langle X_\ell \rangle = 1 \text{ and } (X_\ell)_{ML} = \frac{2\ell-1}{2\ell+1}$$

At the same time, if $Y_\ell = 1/X_\ell = C_\ell^{(model)}/C_\ell^{(data)}$, then

$Pr(Y_\ell = y|data) \propto \left(e^{-1/y}/y\right)^{\ell+1/2}$ is the probability density for $C_\ell^{(model)}$ given the data (assuming flat priors), with

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An example: lowest 9 TT multipoles

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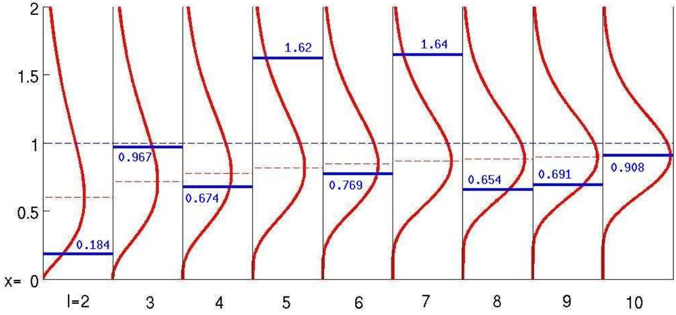
probability curves from best fit ΛCDM
WMAP5 data



$$\text{Prob}[C_2^{(data)} < 0.184\, C_2^{(model)}] \simeq 0.031\ldots$$

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Let $p_\ell = Pr(X_\ell \leq x|model)$ (recall $X_\ell = C_\ell^{(data)}/C_\ell^{(model)}$), then

all p_ℓ are independent random numbers flatly distributed in $(0,1)$

$Pr[\text{there are } k \text{ of the first } n\ p_\ell \text{ in } (0,p)] = \binom{n}{k} p^k (1-p)^{n-k}$

$$\langle k \rangle = p\,n \qquad (\Delta k)^2 = p(1-p)n$$

In the first 250 multipoles we expect (to 1σ) up to 15 $C_\ell^{(data)}$ so low w.r.t. $C_\ell^{(model)}$ to have a probability less than 0.031

$p_\ell < 0.031$

2 22 48 54 72 84 98 105 113 114
120 124 149 181 195 209 228 234 249

$p_\ell > 1 - 0.031$

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Early accelerated cosmic expansion, $ds^2 = dt^2 - a(t)dx^2\ , \quad \ddot{a} > 0$

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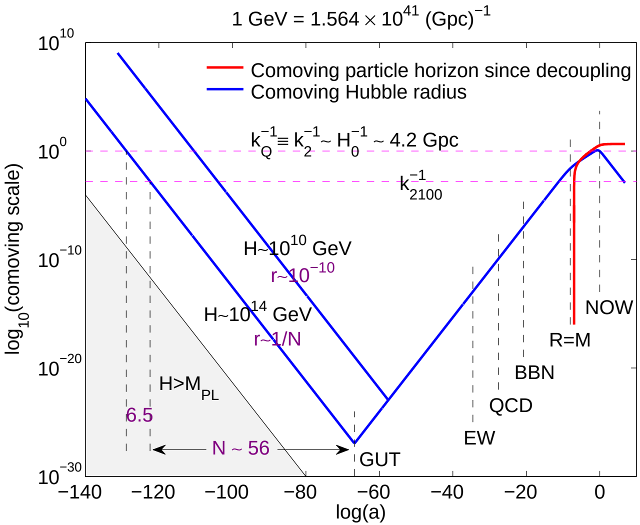

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MD stage:

$\boxed{\frac{1}{aH} \sim \sqrt{a}}$

RD stage:

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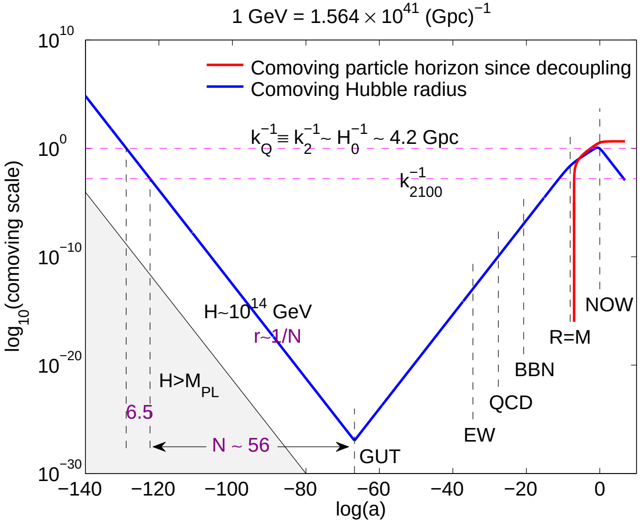

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Fundamental bounds

CMB isotropy or the *horizon problem* (with $\Delta H \sim \sqrt{N}$)

$$N_Q \geq 63 + \frac{1}{2} \log \frac{H}{10^{-4} M_{PL}}$$

Entropy of the Universe (dominated by photon and neutrinos)

$$N_{tot} \geq 63 + \frac{1}{2} \log \frac{H}{10^{-4} M_{PL}} - \frac{1}{12} \log \frac{g_{reh}}{1000}$$

tensor–scalar ratio in *generic* single-field new inflation

$$r = \frac{2}{\pi^2 A_s^2} \left(\frac{H}{M_{PL}} \right)^2 \sim 0.8 \left(\frac{H}{10^{-4} M_{PL}} \right)^2 \gtrsim \frac{1}{N}$$

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EFT of (single field) inflation à la Ginsburg-Landau

D. Boyanowski, C.D., H.J. de Vega, N. Sanchez, arXiv:0901.0549, to appear on IJMP

Inflaton potential ($\hbar = 1$, $c = 1$, $M_{PL} = 2.4 \times 10^{18}$ GeV)

$$V(\varphi) = M^4 v(\phi)\,, \quad \phi = \frac{\varphi}{M_{PL}}$$

Energy scale of inflation and inflaton mass

$$M \simeq 0.57 \times 10^{16}\ \text{GeV} \sim M_{\text{GUT}}, \quad m = M^2/M_{PL} \sim 1.3 \times 10^{13}\ \text{GeV}$$

Hubble parameter and quantum corrections

$$H \sim 7m \ll M_{PL}\,, \quad \text{loops} \to (H/M_{PL})^2 \sim 10^{-9}$$

Number of inflation efolds since horizon exit

$$N = \log \frac{a(t_{end})}{a(t_{exit})}\,, \quad v(\phi_{end}) = v'(\phi_{end}) = 0$$

t_{exit}: the mode with comoving k_0 becomes superhorizon ($\to N = N(k_0)$)

WMAP: $k_0 = 2\ \text{Gpc}^{-1}$, $N \simeq 61$
CosmoMC: $k_0 = 50\ \text{Gpc}^{-1}$, $N \simeq 57$

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$$M \simeq 0.57 \times 10^{16} \text{ GeV} \sim M_{\rm GUT}, \quad m = M^2/M_{PL} \sim 1.3 \times 10^{13} \text{ GeV}$$

Hubble parameter and quantum corrections

$$H \sim 7m \ll M_{PL} , \quad \text{loops} \rightarrow (H/M_{PL})^2 \sim 10^{-9}$$

Number of inflation efolds since horizon exit

$$N = \log \frac{a(t_{end})}{a(t_{exit})} , \quad v(\phi_{end}) = v'(\phi_{end}) = 0$$

t_{exit}: the mode with comoving k_0 becomes superhorizon ($\rightarrow N = N(k_0)$)

WMAP: $k_0 = 2 \text{ Gpc}^{-1}$, $N \simeq 61$
CosmoMC: $k_0 = 50 \text{ Gpc}^{-1}$, $N \simeq 57$

Dimensionless setup: t in units of m^{-1}, $H = h\,m$

Equations of motion

$$h^2 = \frac{1}{3}\left[\frac{1}{2}\dot{\phi}^2 + v(\phi)\right] , \quad \ddot{\phi} + 3h\,\dot{\phi} + v'(\phi) = 0 , \quad \dot{h} = -\frac{1}{2}\dot{\phi}^2$$

Energy density and pressure

$$\varepsilon = M^4\left[\frac{1}{2}\dot{\phi}^2 + v(\phi)\right] , \quad p = M^4\left[\frac{1}{2}\dot{\phi}^2 - v(\phi)\right]$$

Pre-inflation vs. fast-roll vs. slow-roll

$$\frac{1}{2}\dot{\phi}^2 > \frac{1}{2}v(\phi) , \quad \frac{1}{2}\dot{\phi}^2 \sim v(\phi) , \quad \frac{1}{2}\dot{\phi}^2 \lesssim \frac{1}{3N}v(\phi)$$

which potential $v(\phi)$?

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Outline

MCMC analysis of current data plus Ginsburg-Landau stability arguments point to double–well type potentials with the inflaton ϕ rolling from a region of negative curvature near $\phi = 0$ (the "false vacuum") toward the true absolute minimum ϕ_{min} of the potential where $v(\phi_{min}) = v'(\phi_{min}) = 0$.

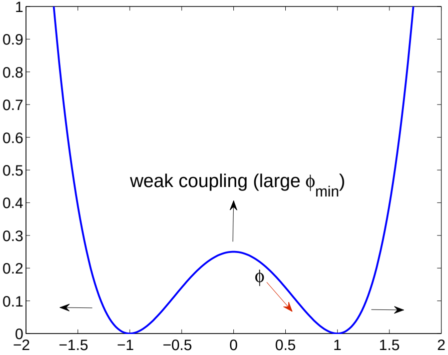
In general

$$v(\phi) = \phi_{min}^2 F(\phi/\phi_{min})$$

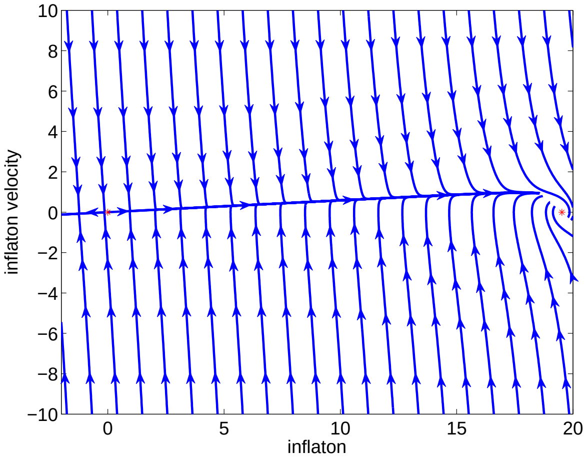
with $F(x) \simeq F_0 - \frac{1}{2}x^2$ as $x \to 0$.

For instance BNI
(Binomial New Inflation)

$$F(x) = \frac{1}{4}(x^2 - 1)^2$$



Inflaton flow in phase space



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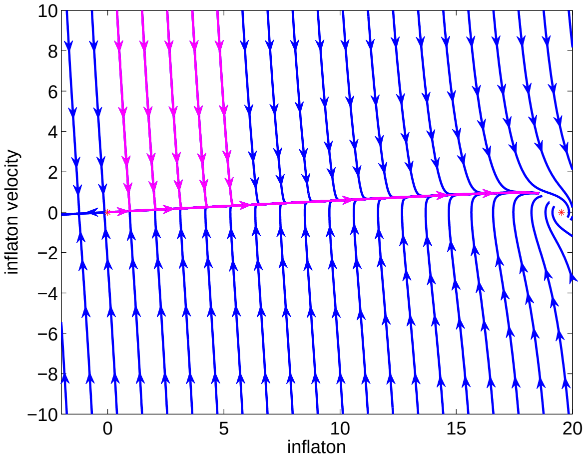

Generic inflaton trajectories are singular as $t \to t_*^+$

$$\phi \simeq \sqrt{2/3}\,\log\left(\frac{t-t_*}{b}\right) , \quad \dot\phi \simeq \frac{\sqrt{2/3}}{t-t_*} , \quad h \simeq \frac{1}{3(t-t_*)} , \quad a \simeq (t-t_*)^{1/3} , \quad \eta \to \eta_*$$

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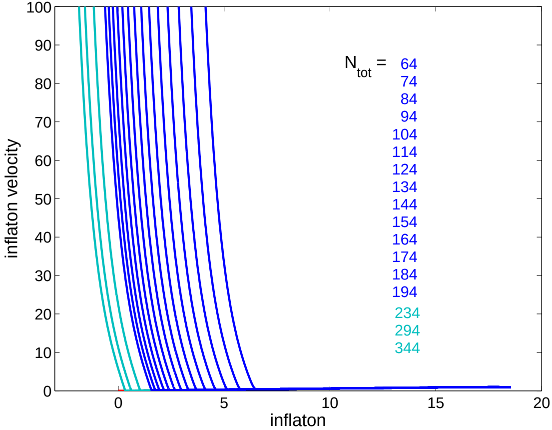

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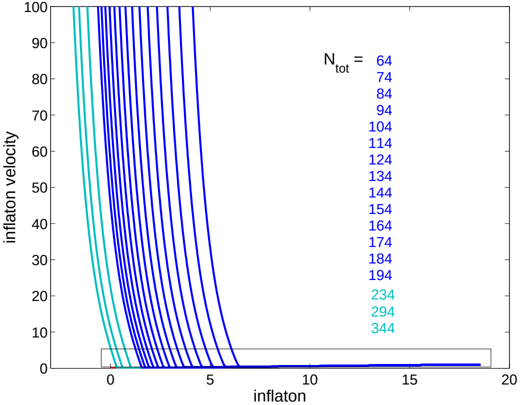

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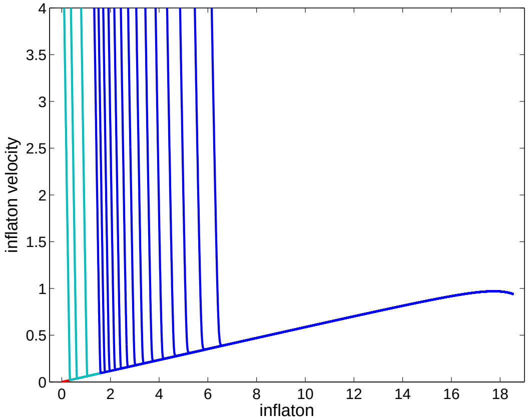

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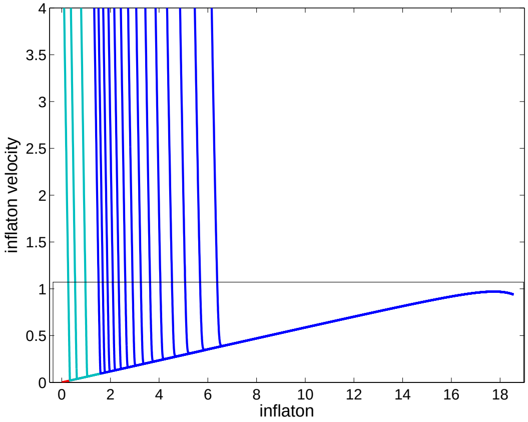

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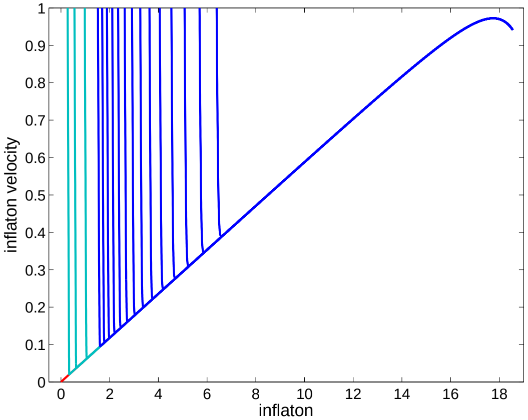

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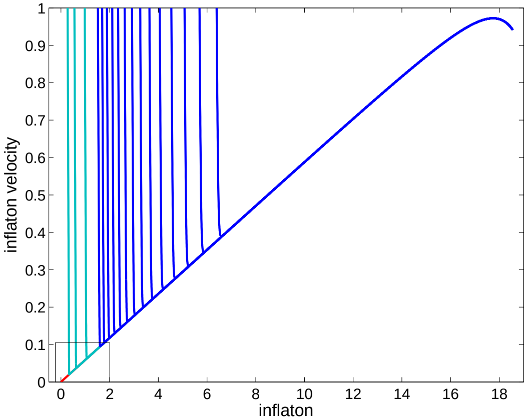

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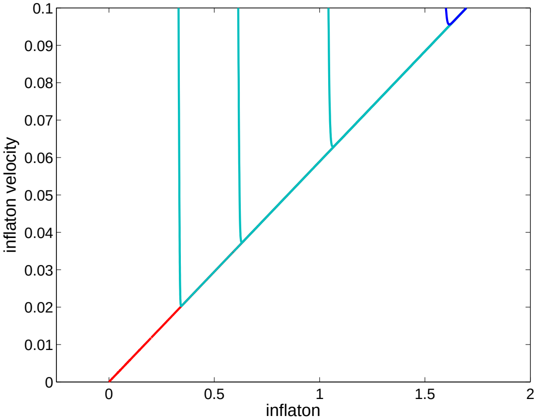

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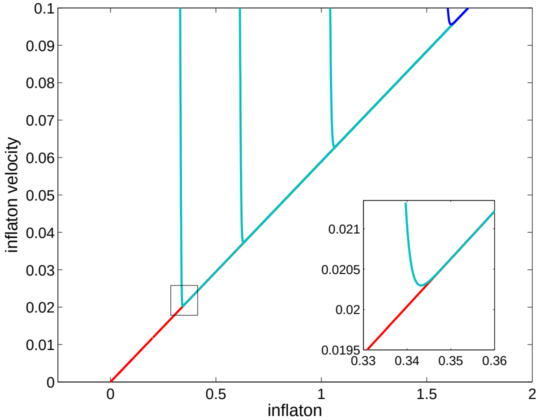
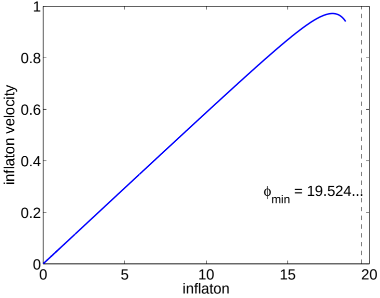


The extreme slow–roll solution (a sort of half de Sitter)

$$\ddot{\phi} + 3h\dot{\phi} + \phi = 0$$

$$\phi \propto \exp(\alpha t)\,, \quad t \to -\infty$$

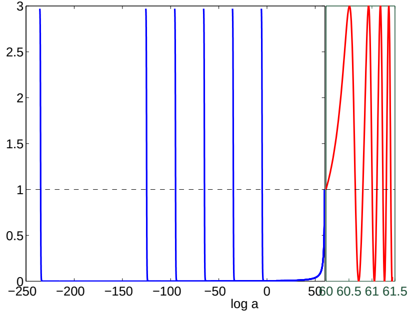
$$\alpha = \tfrac{1}{2}\left[(\sqrt{3v(0)+4} - \sqrt{3v(0)}\right]$$

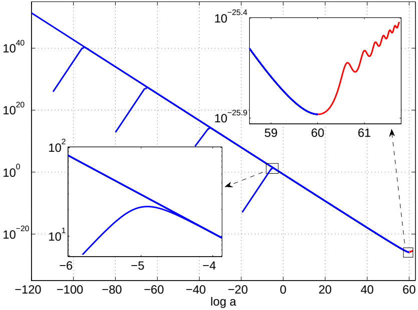


	start	$a=1$	end: $\ddot{a}=0^+$
t	$-344.9514017\ldots$	0	$17.40482446\ldots$
ϕ	10^{-8}	$6.7484118\ldots$	$18.5586530\ldots$
$\dot{\phi}$	$\alpha\,10^{-8} = 5.89371084\ldots10^{-10}$	$0.3973384\ldots$	$0.94150557\ldots$
$\log a$	$-1938.4867948\ldots$	0	60
h	$(12g)^{-1/2} = 5.6361006\ldots$	$4.9653973\ldots$	$0.6657449\ldots$
η	$-\infty$ (f.a.p.p)	$-0.2020610\ldots$	0

$$\epsilon_V = -\frac{\dot{h}}{h^2} = \frac{3\dot{\phi}^2}{\dot{\phi}^2 + 2v(\phi)}$$

$$\text{comoving Hubble radius} = \frac{1}{ah}$$

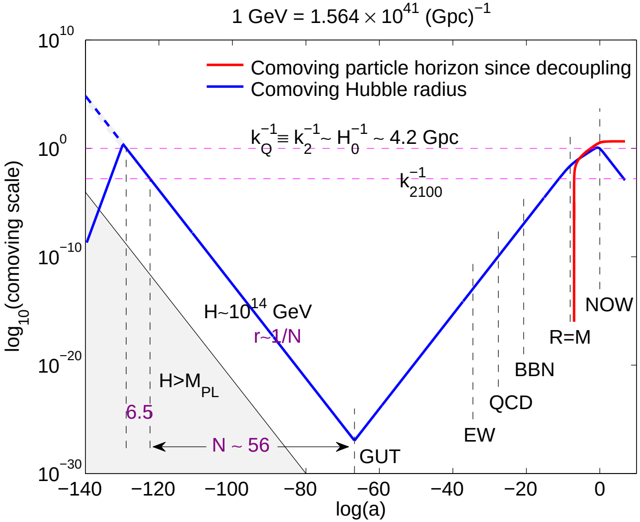




$N_{slowroll}$	63	93	123	153...	233
$N_{fastroll}$	0.917...	0.855...	0.819...	0.797...	0.773...

Inflation essentials [units: $c = \hbar = 1$]

Early accelerated cosmic expansion, $ds^2 = dt^2 - a(t)dx^2$, $\ddot{a} > 0$



MD stage:

$$\boxed{\frac{1}{aH} \sim \sqrt{a}}$$

RD stage:

$$\boxed{\frac{1}{aH} \sim a}$$

inflation:

$$\boxed{\frac{1}{aH} \sim \frac{1}{a}}$$

pre-inflation:

$$\boxed{\frac{1}{aH} \sim a^2}$$

Outline

Scalar fluctuations

Gauge–invariant quantum perturbation field

$$u(x,t) = -\xi(t) R(x,t) = \int \frac{d^3k}{(2\pi)^{3/2}} \left[\alpha_k S_k(\eta) e^{ik\cdot x} + \alpha_k^\dagger S_k^*(\eta) e^{-ik\cdot x} \right]$$

$$[\alpha_k, \alpha_{k'}^\dagger] = \delta^{(3)}(k-k') , \quad \xi(t) = \frac{a(t)}{H(t)} \dot\phi(t) , \quad \eta = \int \frac{dt}{a(t)}$$

Schroedinger–like dynamics

$$\left[\frac{d^2}{d\eta^2} + k^2 - W(\eta) \right] S_k = 0 , \quad W(\eta) = \frac{1}{\xi} \frac{d^2\xi}{d\eta^2}$$

$$\left[\frac{d^2}{dt^2} + h\frac{d}{dt} + \frac{k^2}{a^2} - U(t) \right] S_k = 0$$

Standard parametrization in dimensionless setup

$$U(t) = h^2(2 - 7\,\varepsilon_v + 2\,\varepsilon_v^2) - 2\,\dot\phi\,\frac{v'(\phi)}{h} - \eta_v\, v(\phi) , \quad \varepsilon_v = \frac{\dot\phi^2}{2\,h^2} , \quad \eta_v = \frac{v''(\phi)}{v(\phi)}$$

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$$\epsilon_v \simeq \frac{1}{2}\left[\frac{v'(\phi_{exit})}{v(\phi_{exit})}\right]^2 , \quad \eta_v \simeq \frac{v''(\phi_{exit})}{v(\phi_{exit})} , \quad W(\eta) = \frac{\nu^2 - 1/4}{\eta^2} , \quad \nu = \frac{3}{2} + 3\epsilon_v - \eta_v$$

Power spectrum

$$P(k) = \lim_{\eta\to 0}\left(\frac{m}{M_{PL}}\right)^2 \frac{k^3}{2\pi^2}\left|\frac{S_k \eta)}{\xi(\eta)}\right|^2$$

Bunch–Davies vacuum at $t \to -\infty$ in extreme slow–roll

$$S_k(\eta \to -\infty) = \frac{e^{ik\eta}}{\sqrt{2k}} , \quad P_\infty = A_s\left(\frac{k}{k_0}\right)^{n_s-1} , \quad A_s = \left(\frac{m}{M_{PL}}\right)^2 \frac{N^2}{12\pi^2}\mathcal{O}(1)$$

Bunch–Davies vacuum at finite times?

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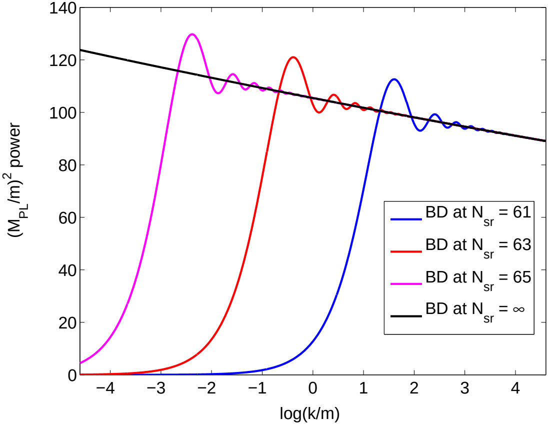
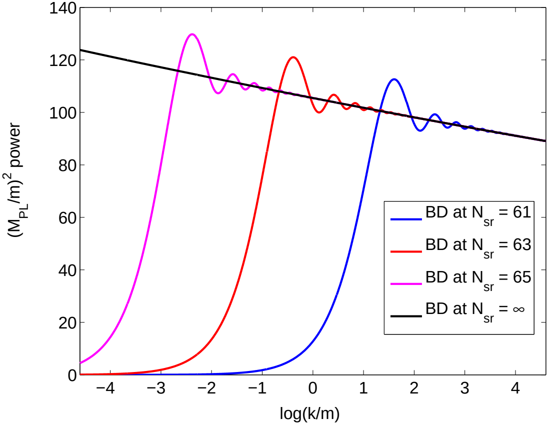


Compare the small $k-$ behavior of BD and quasi-De Sitter modes

$$S_k(\eta_0) = \frac{e^{ik\eta_0}}{\sqrt{2k}} \quad , \qquad \frac{1}{2} i^{\nu+\frac{1}{2}} \sqrt{-\pi\eta_0}\, H_\nu^{(1)}(-k\,\eta_0) \simeq \frac{\Gamma(\nu)}{\sqrt{2\pi k}} \left(\frac{2}{i\,k\,\eta_0}\right)^{\nu-\frac{1}{2}}$$

Bunch–Davies vacuum at finite times



Compare the small $k-$ behavior of BD and quasi-De Sitter modes

$$S_k(\eta_0) = \frac{e^{ik\eta_0}}{\sqrt{2k}} \quad , \qquad \frac{1}{2}\, i^{\nu+\frac{1}{2}} \sqrt{-\pi\eta_0}\, H_\nu^{(1)}(-k\,\eta_0) \simeq \frac{\Gamma(\nu)}{\sqrt{2\pi k}} \left(\frac{2}{i\,k\,\eta_0}\right)^{\nu-\frac{1}{2}}$$

The transfer function of initial conditions

$$P(k) = P_\infty(k)\left[1 + D(k)\right]$$

more formally ...

Effect on quadratic observables due to making linear combinations of solutions of second order linear differential equations, or Bogoliubov transformations on free-field creation–annihilation operators.

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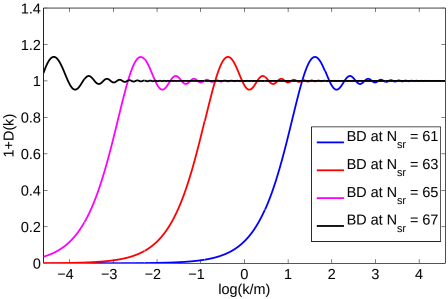
Effect on quadratic observables due to making linear combinations of solutions of second order linear differential equations, or Bogoliubov transformations on free-field creation–annihilation operators.

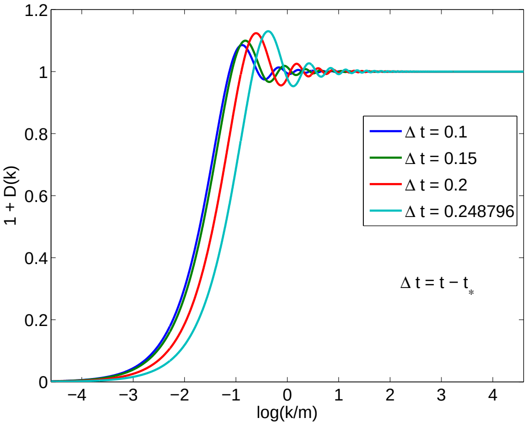
$$D(k) \simeq D(k\eta_0)$$

$$D(k) \sim k^{-2}\ , \quad k \to \infty$$

to have a negligible back–reaction on the metric



Transfer function for fast–roll trajectories
C.D., H.J. de Vega and N. Sanchez, in preparation



$\Longrightarrow$ depression of lowest multipoles

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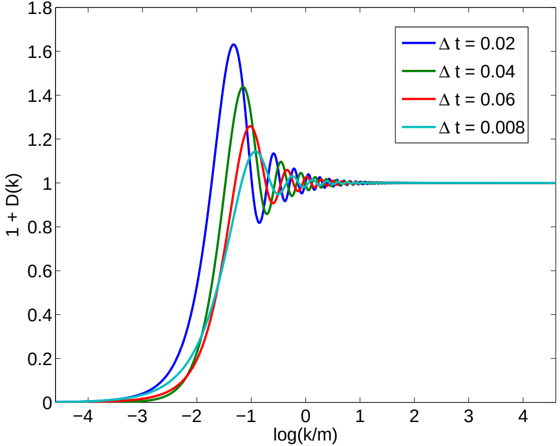
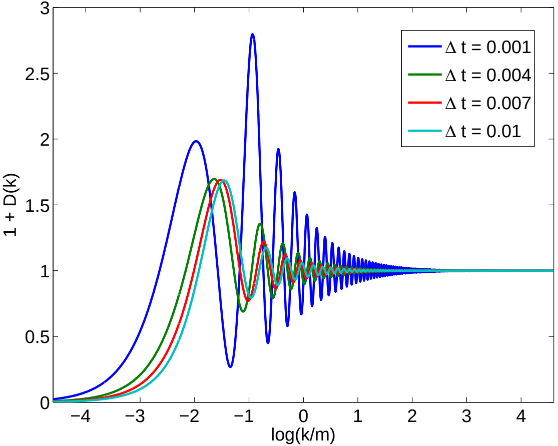


$\Longrightarrow$ up and down with little change on average

Transfer function for fast–roll trajectories
C.D., H.J. de Vega and N. Sanchez, in preparation



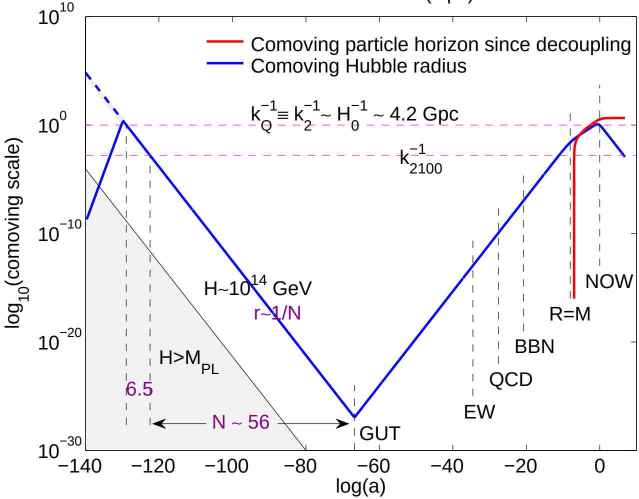
$\Longrightarrow$ up and down with net overall enhancement

Inflation essentials

[units: $c = \hbar = 1$]

Early accelerated cosmic expansion, $ds^2 = dt^2 - a(t)dx^2$, $\ddot{a} > 0$

1 GeV = 1.564×10^{41} (Gpc)$^{-1}$



MD stage:

$$\boxed{\frac{1}{aH} \sim \sqrt{a}}$$

RD stage:

$$\boxed{\frac{1}{aH} \sim a}$$

inflation:

$$\boxed{\frac{1}{aH} \sim \frac{1}{a}}$$

pre-inflation:

$$\boxed{\frac{1}{aH} \sim a^2}$$

Once upon a time ...

in the matter dominated era there was a very low quadrupole that would later (now) become a very low $\ell = 22$ multipole.

The argument based on fastroll to explain such a low quadrupole would have given $N_{tot} = 61$ but would have been proven wrong later on, when more superhorizon modes reentered. Except that ...

The entropy lower bound

$$N_{tot} \geq 63 + \frac{1}{2} \log \frac{H}{10^{-4} M_{PL}} - \frac{1}{12} \log \frac{g_{reh}}{1000} \simeq 63$$

We live when the homogeneity and entropy lower bound coincide!

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Let $p_\ell = Pr(X_\ell \leq x | model)$ (recall $X_\ell = C_\ell^{(data)} / C_\ell^{(model)}$), then

all p_ℓ are independent random numbers flatly distributed in $(0, 1)$

$$Pr\,[\text{there are } k \text{ of the first } n\ p_\ell \text{ in } (0, p)] = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\langle k \rangle = p\,n \qquad (\Delta k)^2 = p(1-p)n$$

In the first 250 multipoles we expect (to 1σ) up to 15 $C_\ell^{(data)}$ so low w.r.t. $C_\ell^{(model)}$ to have a probability less than 0.031

$p_\ell < 0.031$

2 22 48 54 72 84 98 105 113 114
120 124 149 181 195 209 228 234 249

$p_\ell > 1 - 0.031$

69 73 83 117

Outline

1. Is the low CMB TT quadruple too low?
   - Observational data
   - Cosmic variance
   - Independent random variables
2. Theoretical setup
   - EFT of Inflation
   - New inflation
   - Fluctuations and initial conditions
3. MCMC analysis
   - Cosmological MCMC
   - MCMC likelihoods
   - Best fit comparisons

The setup

- Observational CMB data $\Longrightarrow$ likelihood on $C_\ell^{(model)}$;
- Model with cosmological parameters $\lambda = \{\lambda_1, \lambda_2, \ldots \lambda_n\} \Longrightarrow C_\ell^{(model)}(\lambda)$ (CMBFAST, CAMB,...);

$\Longrightarrow$ likelihood $L(\lambda) = \exp[-\chi^2(\lambda)/2]$

The **MCMC** method produces sequences distributed as $L(\lambda)$ ($\times$ the **prior probability**), through an acceptance/rejection one-step algorithm (*e.g.* **Metropolis**)

$$W(\lambda^{(k+1)}, \lambda^{(k)}) = g(\lambda^{(k+1)}, \lambda^{(k)}) \min\left\{1, \frac{L(\lambda^{(k+1)})g(\lambda^{(k+1)}, \lambda^{(k)})}{L(\lambda^{(k)})g(\lambda^{(k)}, \lambda^{(k+1)})}\right\}$$

runs made with CosmoMC on a Linux cluster (Turing) with 8 to 16 parallel chains, repeated up to 4 times for each setup, with $R - 1 < 0.03$

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Binomial New Inflation with sharpcut or (simplified) fastroll
C.D., H.J. de Vega, N. Sanchez, Phys. Rev. D78

Simplification

- Born's approximation for k not too small.
- $k_{tran} = -1/\eta_0$ is the comoving wavenumber that exits the horizon when fast-roll ends and slow-roll starts.

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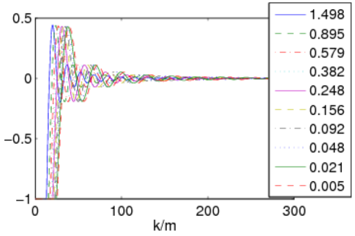
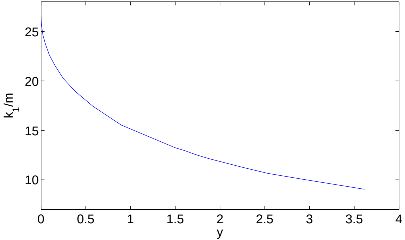
For BNI, $v(\phi) = \frac{1}{4}g(\phi^2 - 1/g)^2$, $g = y/(8N)$, $y = z - 1 - log(z)$





Outline

BNI+sharpcut vs. BNI+fastroll

MCMC parameters:	ω_b, ω_c, θ, τ, (slow), A_s, z, k_{tran} (fast)
Context:	$N = 60$, $\Omega_\nu = 0$, ... ; standard priors, no SZ, lensed CMB, linear mpk, ...
Datasets:	WMAP5, SDSS, ACBAR08

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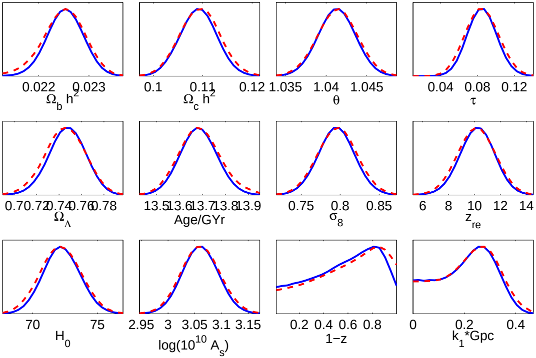
Context: $N = 60$, $\Omega_\nu = 0$, … ; standard priors, no SZ, lensed CMB, linear mpk, …
Datasets: WMAP5, SDSS, ACBAR08

param	best fit
$100\Omega_b h^2$	2.256
$\Omega_c h^2$	0.110
θ	1.041
100τ	8.83
H_0	71.82
σ_8	0.803
$\log[10^{10} A_s]$	0.307
z	0.162
k_1	0.260
$-\log(L)$	1253.96

sharp-cut   flat $0 < z < 1$ prior



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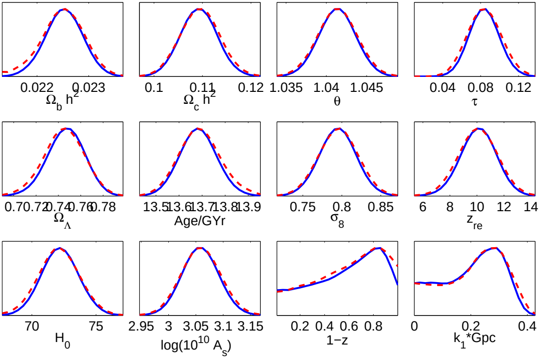
Datasets: WMAP5, SDSS, ACBAR08

param	best fit
$100\Omega_b h^2$	2.253
$\Omega_c h^2$	0.109
θ	1.041
100τ	8.42
H_0	72.00
σ_8	0.794
$\log[10^{10} A_s]$	0.306
z	0.102
k_1	0.284
$-\log(L)$	1253.82

fast-roll flat $0 < z < 1$ prior



Outline

$\Delta\chi^2$ w.r.t. ΛCDM+r

	WMAP5	+SDSS+ACBAR08	+SDSS+SN
BNI+sharpcut	-1.07	-0.71	-1.02
BNI+fastroll	-1.15	-0.99	-1.45

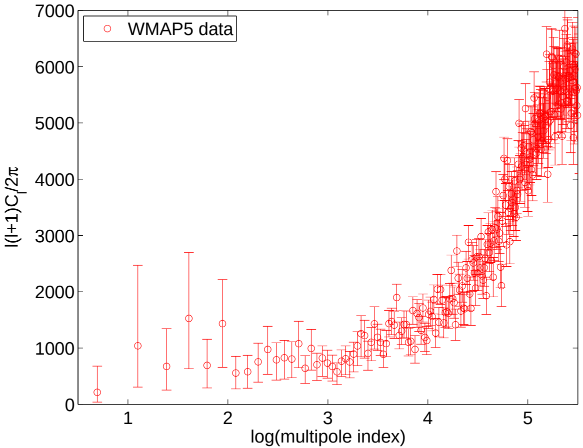
95% lower bound on r

	WMAP5	+SDSS+ACBAR08	+SDSS+SN
BNI+sharpcut	0.025	0.033	0.022
BNI+fastroll	0.024	0.032	0.023

most likely value of k_{tran} (in Gpc^{-1})

	WMAP5	+SDSS+ACBAR08	+SDSS+SN
BNI+sharpcut	0.258	0.260	0.244
BNI+fastroll	0.298	0.284	0.291

Comparing TT multipoles



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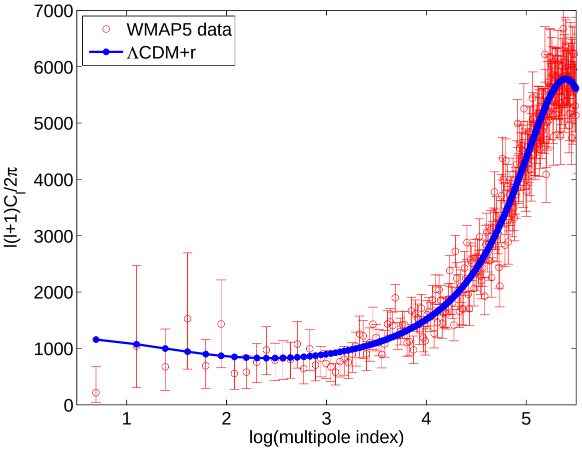

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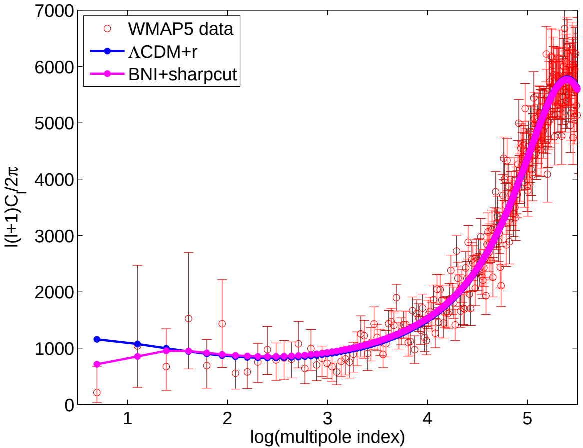

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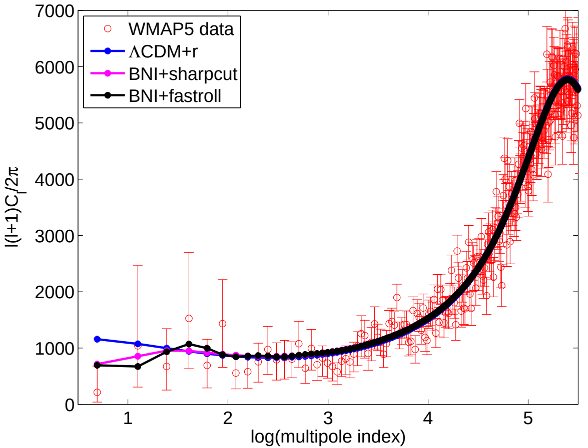

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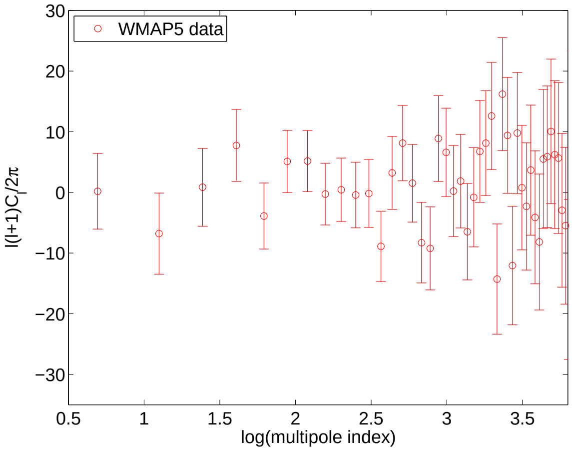

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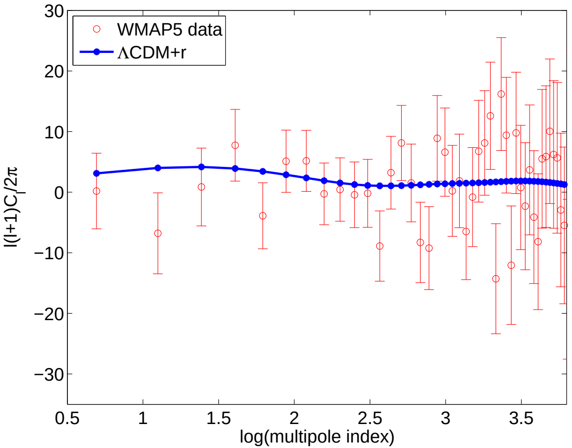

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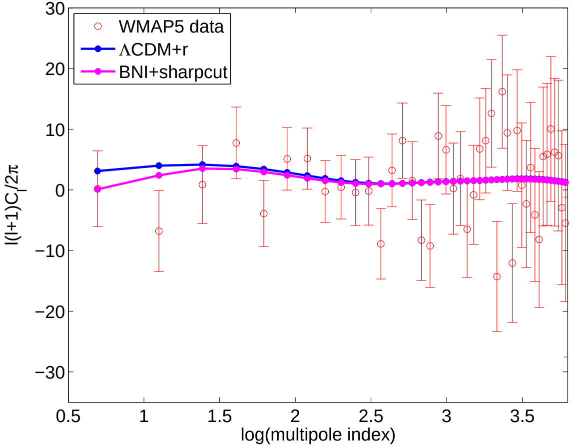

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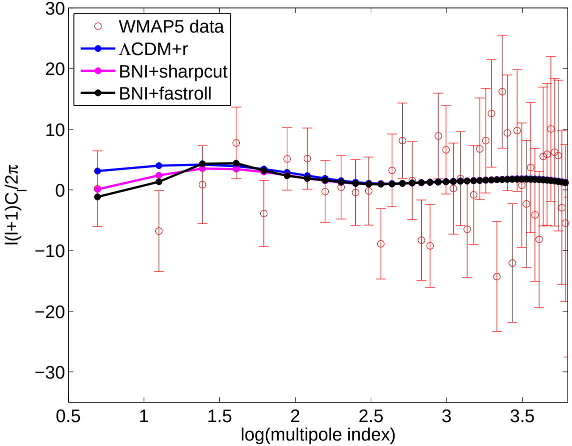
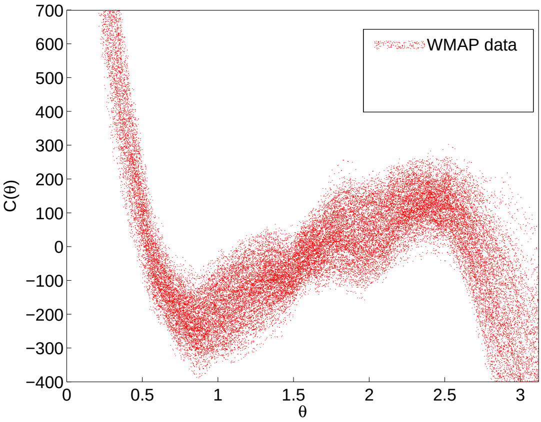


Comparing real-space TT correlations



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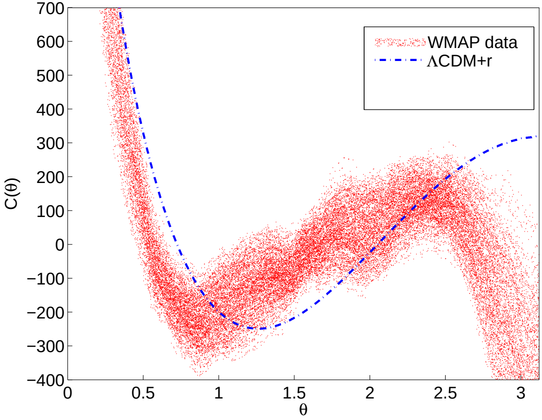

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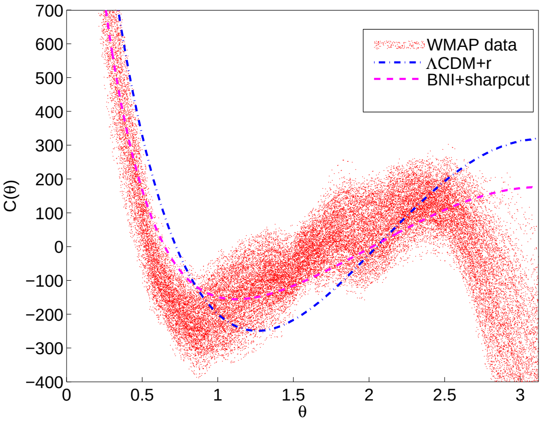

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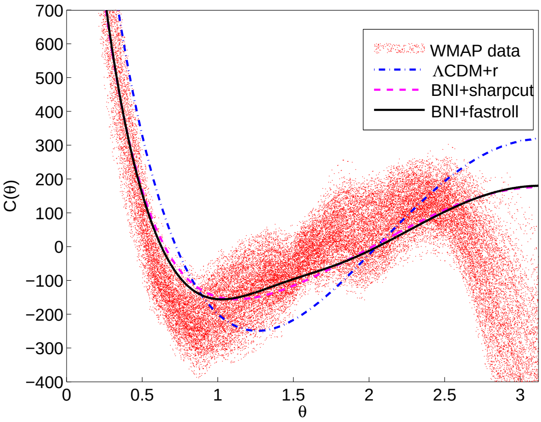

Highlight

The quadrupole wavenumber

$k_Q \simeq 0.83\, k_{tran}$ and exits roughly 1/10 of an efold before k_{tran}

The number of inflation efolds

$$N_{slowroll} \simeq 63 \quad , \qquad N_{tot} \simeq 64$$

Summary

- Large scale CMB anisotropies provide information on the beginning of inflation.
- Early fast-roll inflation is generic and provides a mechanism for lowest multipoles depression.
- BNI+fastroll significantly improves the fit w.r.t. ΛCDM$+r$.
- BNI+fastroll improves the fits also w.r.t. BNI+sharpcut.
- Fast–roll depression of the quadrupole sets to $\sim$ 64 the total number of inflation efolds.

- Outlook
  - Improve the EFT of inflations (entropy, reheating, ...)
  - Wait for better data (Plank, ACT, ...)
  - Refine, refine, refine

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Summary

- Large scale CMB anisotropies provide information on the beginning of inflation.
- Early fast-roll inflation is generic and provides a mechanism for lowest multipoles depression.
- BNI+fastroll significantly improves the fit w.r.t. ΛCDM$+r$.
- BNI+fastroll improves the fits also w.r.t. BNI+sharpcut.
- Fast–roll depression of the quadrupole sets to $\sim$ 64 the total number of inflation efolds.

- Outlook
  - Improve the EFT of inflations (entropy, reheating, ...)
  - Wait for better data (Plank, ACT, ...)
  - Refine, refine, refine