The Effective Theory of Inflation and the Early Fast-Roll Stage, Dark Matter and Dark Energy in the Standard Model of the Universe

H. J. de Vega

LPTHE, CNRS/Université Paris VI

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The History of the Universe

It is a history of EXPANSION and cooling down.

EXPANSION: the space itself expands with the time.

$ds^2 = dt^2 - a^2(t)\, d\vec{x}^2 \quad , \quad a(t) =$ scale factor.

FRW: Homogeneous, isotropic and spatially flat geometry.

Cooling: temperature decreases as $1/a(t)$: $T(t) \sim 1/a(t)$.

The Universe underwent a succesion of phase transitions towards the less symmetric phases.

Wavelenghts redshift as $a(t) \quad : \quad \lambda(t) = a(t)\, \frac{\lambda(t_0)}{a(t_0)}$

Redshift z : $\quad z + 1 = \frac{a(\text{today})}{a(t)} \quad , \quad a(\text{today}) \equiv 1$

The deeper you go in the past, the larger is the redshift and the smaller is $a(t)$.

Standard Cosmological Model: ΛCDM

ΛCDM = Cold Dark Matter + Cosmological Constant begins by the Inflationary Era. Explains the Observations:

- 5 years WMAP data and previous CMB data
- Light Elements Abundances
- Large Scale Structures (LSS) Observations. BAO.
- Acceleration of the Universe expansion: Supernova Luminosity/Distance and Radio Galaxies.
- Gravitational Lensing Observations
- Lyman α Forest Observations
- Hubble Constant (H_0) Measurements
- Properties of Clusters of Galaxies
- ....

Standard Cosmological Model: Concordance Model

$ds^2 = dt^2 - a^2(t)\, d\vec{x}^{\,2}$: spatially flat geometry.

The Universe starts by an **INFLATIONARY ERA**.

Inflation = Accelerated Expansion: $\frac{d^2 a}{dt^2} > 0$.
During inflation the universe expands by at least sixty efolds: $e^{62} \simeq 10^{27}$. Inflation lasts $\simeq 10^{-36}$ sec and ends by $z \sim 10^{29}$ followed by a radiation dominated era.
Energy scale when inflation starts $\sim 10^{16}$ GeV ($\Longleftarrow$ CMB anisotropies) which **coincides** with the GUT scale.

Matter can be effectively described during inflation by a Scalar Field $\phi(t, \boldsymbol{x})$: the Inflaton.

Lagrangean: $\mathcal{L} = a^3(t) \left[\frac{\dot{\phi}^2}{2} - \frac{(\nabla\phi)^2}{2\, a^2(t)} - V(\phi) \right]$.

Friedmann eq.: $H^2(t) = \frac{1}{3\, M_{Pl}^2} \left[\frac{\dot{\phi}^2}{2} + V(\phi) \right]$, $H(t) \equiv \dot{a}(t)/a(t)$

Physics during Inflation

- Out of equilibrium evolution in a fastly expanding geometry. Vacuum energy DOMINATED (De Sitter) universe $a(t) \simeq e^{H\,t}$.
- Explosive particle production due to spinodal or parametric instabilities. Quantum non-linear phenomena eventually shut-off the instabilities and stop inflation. Radiation dominated era follows: $a(t) = \sqrt{t}$.
- Huge redshift classicalizes the dynamics: an assembly of (superhorizon) quantum modes behave as a classical and homogeneous inflaton field. Inflaton slow-roll.

D. Boyanovsky, C. Destri, H. J. de Vega, N. G. Sánchez, The Effective Theory of Inflation in the Standard Model of the Universe and the CMB+LSS data analysis (review article), arXiv:0901.0549, 135 pages, to appear in Int.J.Mod.Phys.A

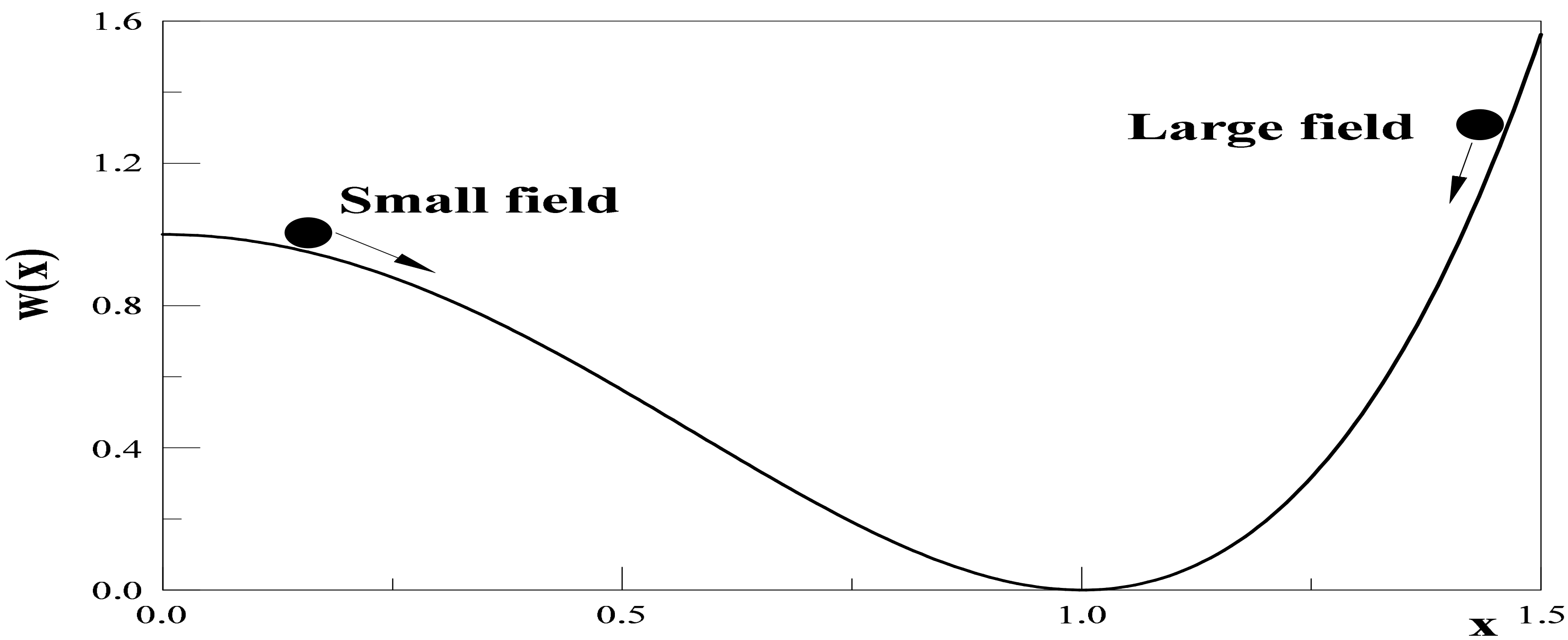
The Theory of Inflation

The inflaton is an effective field in the Ginsburg-Landau sense.

Relevant effective theories in physics:

- Ginsburg-Landau theory of superconductivity. It is an effective theory for Cooper pairs in the microscopic BCS theory of superconductivity.
- The $O(4)$ sigma model for pions, the sigma and photons at energies $\lesssim 1$ GeV. The microscopic theory is QCD: quarks and gluons. $\pi \simeq \bar{q}q$, $\sigma \simeq \bar{q}q$.
- The theory of second order phase transitions à la Landau-Kadanoff-Wilson... (ferromagnetic, antiferromagnetic, liquid-gas, Helium 3 and 4, ...)
- Fermi Theory of Weak Interactions (current-current).

Slow Roll Inflation



The field evolves towards the minimum of the potential.

$V(\mathrm{Min}) = V'(\mathrm{Min}) = 0$: inflation ends after a finite number of efolds.

Slow-roll is needed to produce enough efolds of inflation (≥ 62) to explain the entropy of the universe today
$\Longrightarrow$ the slope of the potential $V(\phi)$ must be small.

Slow-roll evolution of the Inflaton

During slow-roll the inflaton derivatives are small and the evolution equations (1) and (2) can be approximated by:

$$3\,H(t)\,\dot{\phi} + V'(\phi) = 0 \quad , \qquad H^2(t) = \frac{V(\phi)}{3M_{Pl}^2}$$

These first order equations can be solved in closed from as:

$$M_{Pl}^2\,N[\phi] = -\int_{\phi}^{\phi_{end}} V(\varphi)\,\frac{d\varphi}{dV}\,d\varphi \quad .$$

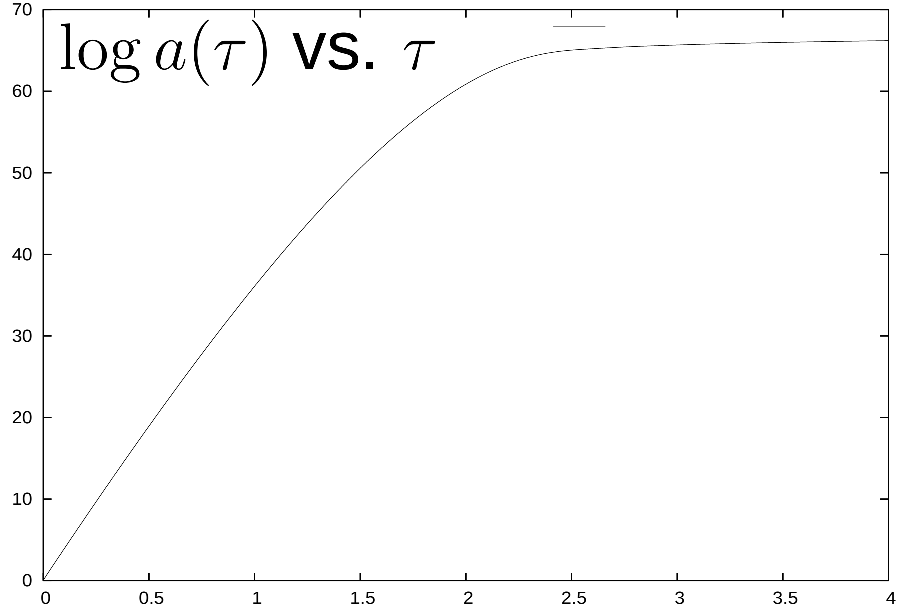
$N[\phi] =$ the number of e-folds since the field ϕ exits the horizon till the end of inflation. $N \sim 60$.
$\phi_{end} =$ absolute minimum of $V(\phi)$.

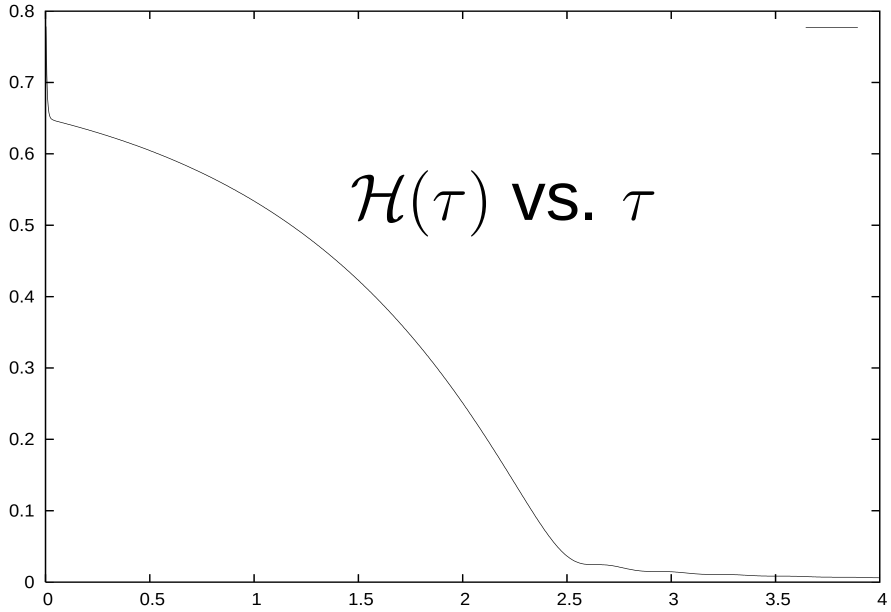
Therefore, $\phi^2 =$ scales as $N\;M_{Pl}^2$. We define:

$\chi \equiv \frac{\phi}{\sqrt{N}\;M_{Pl}}$ dimensionless and slow field.

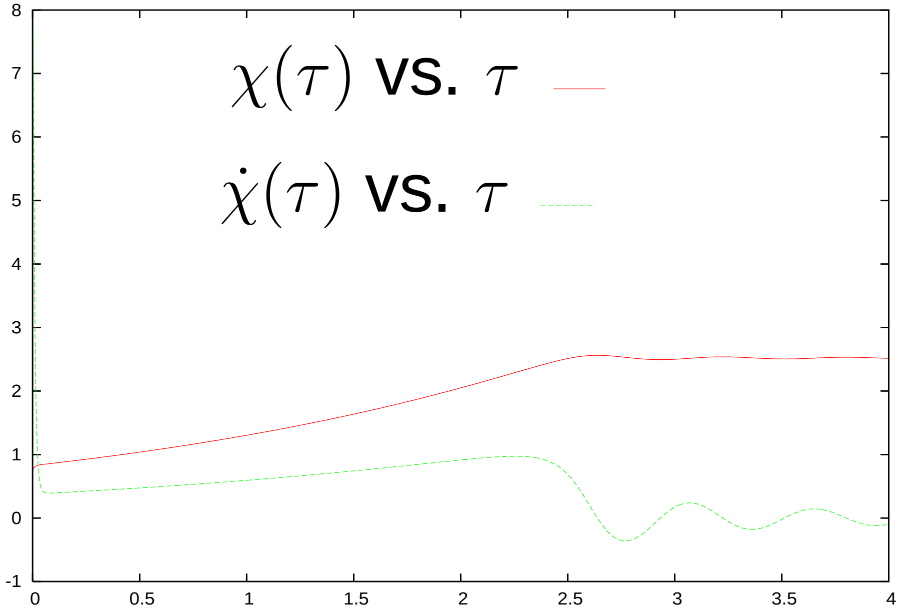
Universal form of the slow-roll inflaton potential:
$V(\phi) = N\;M^4\;w(\chi)\;,\;\;M =$ energy scale of inflation.

Exact Inflaton Dynamics: $w(\chi) = \frac{y}{32}(\chi^2 - \frac{8}{y})^2$





Exact Inflaton Dynamics: $w(\chi) = \frac{y}{32}(\chi^2 - \frac{8}{y})^2$



In these plots $y = 1.26$ and $\chi_{min} = \sqrt{\frac{8}{y}} = 2.52$.

We choose $\chi(0) = 0.73587, \quad \frac{1}{2\,N}\;\dot{\chi}(0)^2 = w(\chi(0))$,
$\Longrightarrow \dot{\chi}(0) = 12.624$ which ensure $N_{tot} \simeq 66$.

We have here neglected spatial gradient terms:

$\frac{(\nabla\phi)^2}{2\;a^2(t)}$ since $a(t)$ grows exponentially during inflation.

Primordial Power Spectrum

Adiabatic Scalar Perturbations: $P(k) = |\Delta_{k\ ad}^{(S)}|^2\ k^{n_s - 1}$.
To dominant order in slow-roll:

$$|\Delta_{k\ ad}^{(S)}|^2 = \frac{N^2}{12\,\pi^2}\ \left(\frac{M}{M_{Pl}}\right)^4\ \frac{w^3(\chi)}{w'^2(\chi)}\ .$$

Hence, for all slow-roll inflation models:

$$|\Delta_{k\ ad}^{(S)}| \sim \frac{N}{2\,\pi\sqrt{3}}\left(\frac{M}{M_{Pl}}\right)^2$$

The WMAP5 result: $|\Delta_{k\ ad}^{(S)}| = (0.494 \pm 0.1) \times 10^{-4}$
determines the scale of inflation M (using $N \simeq 60$)

$$\left(\frac{M}{M_{Pl}}\right)^2 = 0.85 \times 10^{-5} \longrightarrow M = 0.70 \times 10^{16} \text{ GeV}$$

The inflation energy scale turns to be the grand unification energy scale !! We find the scale of inflation without knowing the tensor/scalar ratio r !!
The scale M is independent of the shape of $w(\chi)$.

spectral index n_s and the ratio r

$r \equiv$ ratio of tensor to scalar fluctuations.
tensor fluctuations = primordial gravitons.

$$n_s - 1 = -\frac{3}{N} \left[\frac{w'(\chi)}{w(\chi)}\right]^2 + \frac{2}{N} \frac{w''(\chi)}{w(\chi)} \quad , \quad r = \frac{8}{N} \left[\frac{w'(\chi)}{w(\chi)}\right]^2$$

$$\frac{dn_s}{d\ln k} = -\frac{2}{N^2} \frac{w'(\chi)\, w'''(\chi)}{w^2(\chi)} - \frac{6}{N^2} \frac{[w'(\chi)]^4}{w^4(\chi)} + \frac{8}{N^2} \frac{[w'(\chi)]^2\, w''(\chi)}{w^3(\chi)} \, ,$$

χ is the inflaton field at horizon exit.
$n_s - 1$ and r are always of order $1/N \sim 0.02$ (model indep.)
Running of n_s of order $1/N^2 \sim 0.0003$ (model independent).

D. Boyanovsky, H. J. de Vega, N. G. Sanchez,
Phys. Rev. D 73, 023008 (2006), astro-ph/0507595.

Ginsburg-Landau Approach

Ginsburg-Landau potentials:
polynomials in the field starting by a constant term.

Linear terms can always be eliminated by a constant shift of the inflaton field.

The quadratic term can have a positive or a negative sign:

$$\begin{cases} w''(0) > 0 \to \text{single well potential} \to \text{large field (chaotic) inflation} \\ w''(0) < 0 \to \text{double well potential} \to \text{small field (new) inflation} \end{cases}$$

The inflaton potential must be bounded from below $\Longrightarrow$ highest order term must be even with a positive coefficient.

Renormalizability $\Longrightarrow$ degree of the inflaton potential ≤ 4.

The theory of inflation is an effective theory $\Longrightarrow$
higher degree potentials are acceptable

Fourth order Ginsburg-Landau inflationary models

$$w(\chi) = w_o \pm \frac{\chi^2}{2} + G_3\,\chi^3 + G_4\,\chi^4 \quad , \quad G_3 = \mathcal{O}(1) = G_4$$

$$V(\phi) = N\,M^4\,w\left(\frac{\phi}{\sqrt{N}\,M_{Pl}}\right) = V_o \pm \frac{m^2}{2}\,\phi^2 + g\,\phi^3 + \lambda\,\phi^4\,.$$

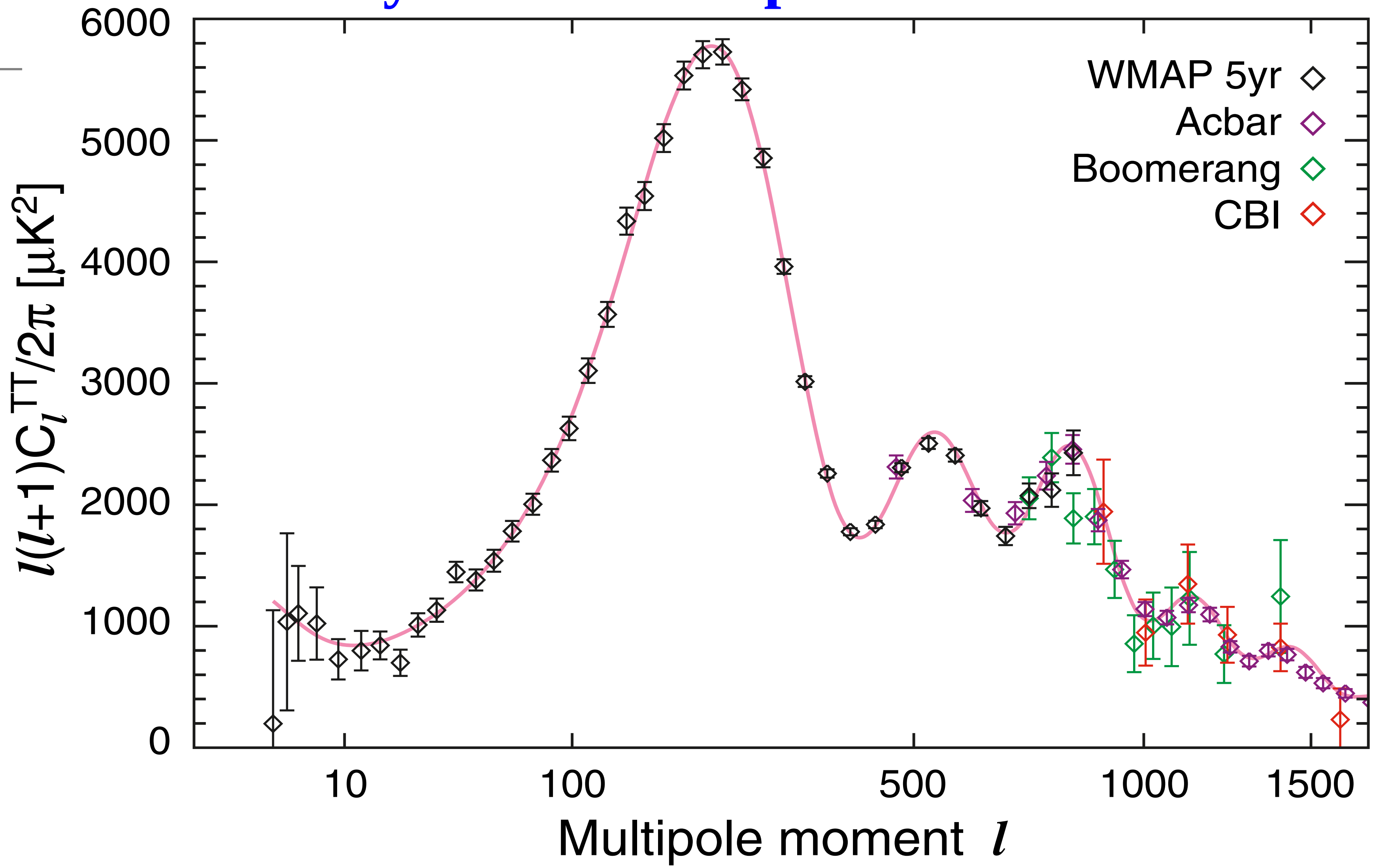
$$m = \frac{M^2}{M_{Pl}} \quad , \quad g = \frac{m}{\sqrt{N}}\left(\frac{M}{M_{Pl}}\right)^2 G_3 \quad , \quad \lambda = \frac{G_4}{N}\,\left(\frac{M}{M_{Pl}}\right)^4$$

Notice that

$$\left(\frac{M}{M_{Pl}}\right)^2 \simeq 10^{-5} \quad , \quad \left(\frac{M}{M_{Pl}}\right)^4 \simeq 10^{-10} \quad , \quad N \simeq 60\,.$$

- Small couplings arise naturally as ratio of two energy scales: inflation and Planck.
- The inflaton is a light particle:
  $m = \frac{M^2}{M_{Pl}} \simeq 0.003\,M\,, \quad m = 2.5 \times 10^{13}$ GeV
  $H \sim \sqrt{N}\,m \simeq 2 \times 10^{14}$ GeV.

WMAP 5 years data set plus other CMB data



Theory and observations nicely agree except for the lowest multipoles: the quadrupole suppression.

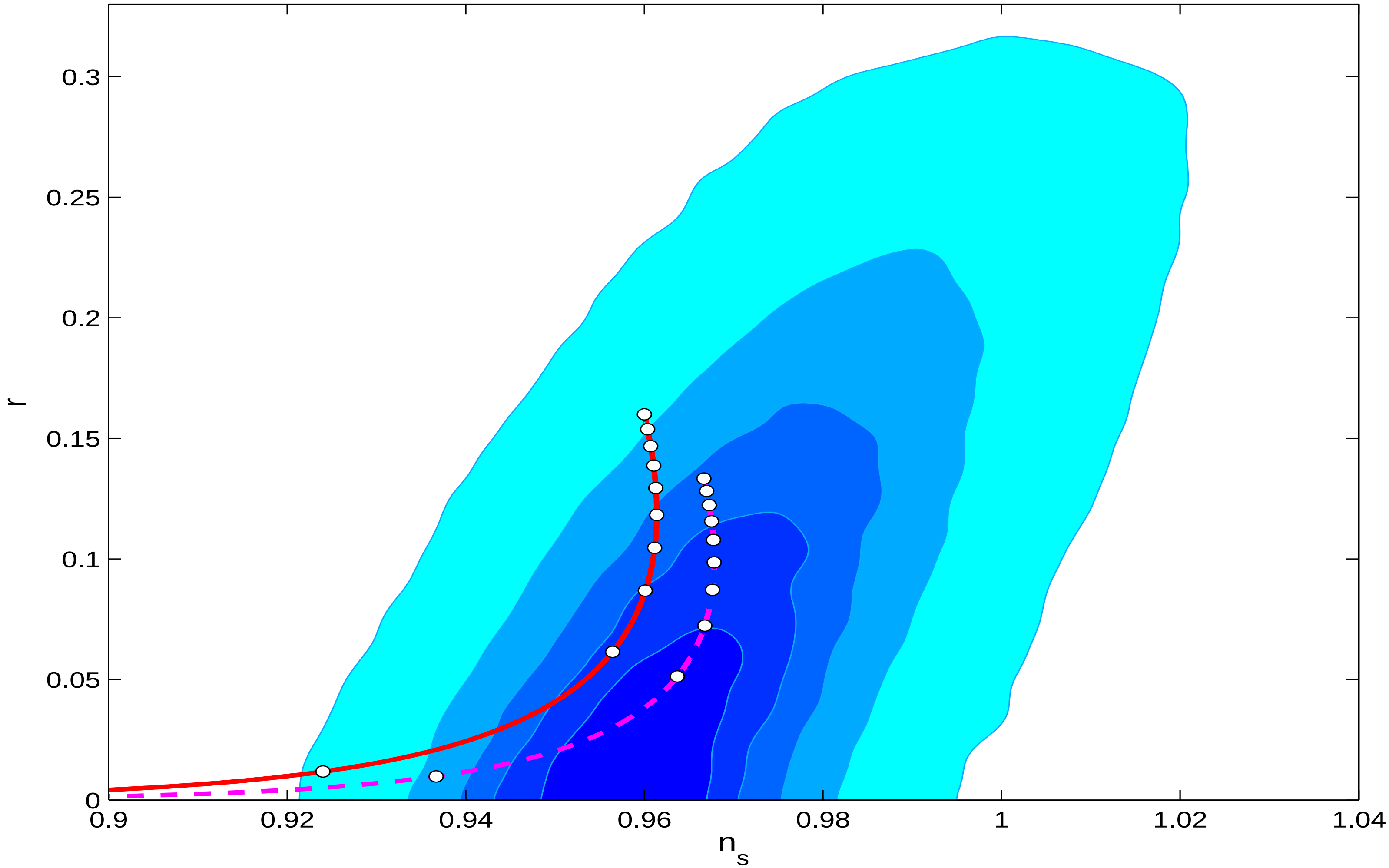
Monte Carlo Markov Chains Analysis of Data: MCMC.

MCMC is an efficient stochastic numerical method to find the probability distribution of the theoretical parameters that describe a set of empirical data.

We found n_s and r and the couplings y and h by MCMC.
NEW: We imposed as a hard constraint that r and n_s are given by the inflaton potential.
Our analysis differs in this crucial aspect from previous MCMC studies of the WMAP data.

The color–filled areas correspond to $12\%,\ 27\%,\ 45\%,\ 68\%$ and 95% confidence levels according to the WMAP3 and Sloan data.
The color of the areas goes from the darker to the lighter for increasing CL.

MCMC Results for the double–well inflaton potential



Solid line for $N = 50$ and dashed line for $N = 60$
White dots: $z = 0.01 + 0.11 * n, \ n = 0, 1, \ldots, 9$,
y increases from the uppermost dot $y = 0, \ z = 1$.

MCMC Results for double–well inflaton potential

Bounds: $r > 0.023$ (95% CL) , $r > 0.046$ (68% CL)
Most probable values: $n_s \simeq 0.964$, $r \simeq 0.051 \Leftarrow$measurable!!
The most probable double–well inflaton potential has a moderate nonlinearity with the quartic coupling $y \simeq 1.26 \ldots$.
The $\chi \to -\chi$ symmetry is here spontaneously broken since the absolute minimum of the potential is at $\chi \neq 0$

$$w(\chi) = \frac{y}{32} \left(\chi^2 - \frac{8}{y}\right)^2$$

MCMC analysis calls for $w''(\chi) < 0$ at horizon exit
$\Longrightarrow$ double well potential favoured.
C. Destri, H. J. de Vega, N. Sanchez, MCMC analysis of WMAP3 data points to broken symmetry inflaton potentials and provides a lower bound on the tensor to scalar ratio, Phys. Rev. D77, 043509 (2008), astro-ph/0703417.
Similar results from WMAP5 data.
Acbar08 data slightly increases $n_s < 1$ and r.

Dark Matter

DM must be non-relativistic by structure formation ($z < 30$) in order to reproduce the observed small structures at $\sim 2 - 3$ kpc.

DM particles can decouple being ultrarelativistic (UR) at $T_d \gg m$ or non-relativistic $T_d \ll m$.

Consider particles that decouple at or out of LTE (LTE = local thermal equilibrium).

Distribution function:

$F_d[a(t)\ P_f(t)] = F_d[p_c]$ freezes out at decoupling.
$P_f(t) = p_c/a(t) =$ Physical momentum.
$p_c =$ comoving momentum.

Velocity fluctuations: $y = P_f(t)/T_d(t) = p_c/T_d$

$$\langle \vec{V}^2(t) \rangle = \langle \frac{\vec{P}_f^2(t)}{m^2} \rangle = \frac{\int \frac{d^3 P_f}{(2\pi)^3} \frac{\vec{P}_f^2}{m^2} F_d[a(t)\ P_f]}{\int \frac{d^3 P_f}{(2\pi)^3} F_d[a(t)\ P_f]} = \left[\frac{T_d}{m\ a(t)}\right]^2 \frac{\int_0^\infty y^4 F_d(y) dy}{\int_0^\infty y^2 F_d(y) dy} \; .$$

Dark Matter density and DM velocity dispersion

Energy Density: $\rho_{DM}(t) = g \int \frac{d^3P_f}{(2\pi)^3} \sqrt{m^2 + P_f^2} \; F_d[a(t)\, P_f]$

g : $\#$ of internal degrees of freedom of the DM particle, $1 \leq g \leq 4$. For $z \lesssim 30 \;\Rightarrow$ DM particles are non-relativistic:

$$\rho_{DM}(t) = \frac{m\; g}{2\pi^2} \, \frac{T_d^3}{a^3(t)} \int_0^\infty y^2 \; F_d(y)\; dy \;,$$

Using entropy conservation: $\quad T_d = \left(\frac{2}{g_d}\right)^{\frac{1}{3}} \; T_{CMB}$,

$g_d =$ effective $\#$ of UR degrees of freedom at decoupling, $T_{CMB} = 0.2348\; 10^{-3}$ eV, and

$$\rho_{DM}(\text{today}) = \frac{m\; g}{\pi^2\; g_d} \, T_{CMB}^3 \int_0^\infty y^2 \; F_d(y)\; dy = 1.107 \, \frac{\text{keV}}{\text{cm}^3} \quad (1)$$

We obtain for the primordial velocity dispersion:

$$\sigma_{DM}(z) = \sqrt{\frac{1}{3} \, \langle \vec{V}^2 \rangle(z)} = 0.05124 \, \frac{1+z}{g_d^{\frac{1}{3}}} \left[\frac{\int_0^\infty y^4 \; F_d(y)\; dy}{\int_0^\infty y^2 \; F_d(y)\; dy} \right]^{\frac{1}{2}} \frac{\text{keV}}{m} \, \frac{\text{km}}{\text{s}}$$

Goal: determine m and g_d. We need TWO constraints.

The Phase-space density ρ/σ^3 and its decrease factor Z

The phase-space density $\frac{\rho}{\sigma^3}$ is invariant under the cosmological expansion and can only decrease under self-gravity interactions (gravitational clustering).

The phase-space density today follows observing dwarf spheroidal satellite galaxies of the Milky Way (dSphs)

$\frac{\rho_s}{\sigma_s^3} \sim 5 \times 10^3 \; \frac{\mathrm{keV/cm}^3}{(\mathrm{km/s})^3} = (0.18 \; \mathrm{keV})^4$ Gilmore et al. 07 and 08.

During structure formation $(z \lesssim 30)$, ρ/σ^3 decreases by a factor that we call Z.

$$\frac{\rho_s}{\sigma_s^3} = \frac{1}{Z} \; \frac{\rho_{DM}}{\sigma_{DM}^3} \quad (2)$$

N-body simulations results: $1000 > Z > 1$.

Constraints: First $\rho_{DM}(\mathrm{today})$, Second $\rho/\sigma^3(\mathrm{today}) = \rho_s/\sigma_s^3$

Mass Estimates for DM particles

Combining the previous expressions lead to general formulas for m and g_d:

$$m = 0.2504\,\mathrm{keV}\ \left(\frac{Z}{g}\right)^{\frac{1}{4}} \frac{\left[\int_0^\infty y^4\ F_d(y)\ dy\right]^{\frac{3}{8}}}{\left[\int_0^\infty y^2\ F_d(y)\ dy\right]^{\frac{5}{8}}}$$

$$g_d = 35.96\, Z^{\frac{1}{4}}\ g^{\frac{3}{4}}\ \left[\int_0^\infty y^4\ F_d(y)\ dy\ \int_0^\infty y^2\ F_d(y)\ dy\right]^{\frac{3}{8}}$$

These formulas yield for relics decoupling UR at LTE:

$$m = \left(\frac{Z}{g}\right)^{\frac{1}{4}}\ \mathrm{keV}\ \left\{\begin{array}{l} 0.568 \\ 0.484 \end{array}\right.\ ,\ g_d = g^{\frac{3}{4}}\ Z^{\frac{1}{4}}\ \left\{\begin{array}{ll} 155 & \mathrm{Fermions} \\ 180 & \mathrm{Bosons} \end{array}\right. .$$

Since $g = 1 - 4$, we see that $g_d > 100 \Rightarrow T_d > 100$ GeV.

$1 < Z^{\frac{1}{4}} < 5.6$ for $1 < Z < 1000$. Example: for DM Majorana fermions $(g = 2)$ $m \simeq 0.85$ keV.

Out of thermal equilibrium decoupling

Results for m and g_d on the same scales for DM particles decoupling UR out of thermal equilibrium.

Particle physics candidates for UR decoupling in the keV scale: sterile neutrinos, gravitinos, ...

D. Boyanovsky, H. J. de Vega, N. Sanchez,
Phys. Rev. D 77, 043518 (2008), arXiv:0710.5180.

H. J. de Vega, N. G. Sanchez,
arXiv:0901.0922 and arXiv:0907.0006

Relics decoupling non-relativistic

$$F_d^{NR}(p_c) = \frac{2^{\frac52}\,\pi^{\frac72}}{45}\; g_d\, Y_\infty\; \left(\frac{T_d}{m}\right)^{\frac32}\; e^{-\frac{p_c^2}{2\,m\,T_d}} = \frac{2^{\frac52}\,\pi^{\frac72}}{45}\; \frac{g_d\, Y_\infty}{x^{\frac32}}\; e^{-\frac{y^2}{2\,x}}$$

$Y(t) = n(t)/s(t),\; n(t)$ number of DM particles per unit volume, $s(t)$ entropy per unit volume, $x \equiv m/T_d,\; T_d < m$.

$Y_\infty = \frac{1}{\pi}\sqrt{\frac{45}{8}}\,\frac{1}{\sqrt{g_d}\,T_d\,\sigma_0\,M_{Pl}}$ late time limit of Boltzmann.

σ_0: thermally averaged total annihilation cross-section times the velocity.

From our previous general equations for m and g_d:

$m = \frac{45}{4\,\pi^2}\,\frac{\Omega_{DM}\,\rho_c}{g\,T_\gamma^3\,Y_\infty} = \frac{0.748}{g\,Y_\infty}$ eV   and $m^{\frac52}\,T_d^{\frac32} = \frac{45}{2\,\pi^2}\,\frac{1}{g\,g_d\,Y_\infty}\,Z\,\frac{\rho_s}{\sigma_s^3}$

Finally: $$\sqrt{m\,T_d} = 1.47\,\left(\frac{Z}{g_d}\right)^{\frac13}\,\mathrm{keV}$$

We used ρ_{DM} today and the decrease of the phase space density by a factor Z.

Relics decoupling non-relativistic 2

Allowed ranges for m and T_d.

$m > T_d > b$ eV where $b > 1$ or $b \gg 1$ for DM decoupling in the RD era

$$\left(\frac{Z}{g_d}\right)^{\frac{1}{3}} \; 1.47 \text{ keV} < m < \frac{2.16}{b} \text{ MeV } \left(\frac{Z}{g_d}\right)^{\frac{2}{3}}$$

$g_d \simeq 3 \quad \text{for} \quad 1 \text{ eV} < T_d < 100 \text{ keV}$ and $1 < Z < 10^3$

$1.02 \text{ keV} < m < \frac{104}{b} \text{ MeV} \quad , \quad T_d < 10.2$ keV.

Only using ρ_{DM} today (ignoring the phase space density information) gives one equation with three unknowns:
$m, \; T_d$ and σ_0,

$\sigma_0 = 0.16 \text{ pbarn } \dfrac{g}{\sqrt{g_d}} \, \dfrac{m}{T_d}$ http://pdg.lbl.gov

WIMPS with $m = 100$ GeV and $T_d = 5$ GeV require $Z \sim 10^{23}$.

The constant surface density in dark matter galaxies

Surface density of dark matter (DM) halos $\mu_{0D} \equiv r_0 \; \rho_0$,
$r_0 =$ halo core radius, $\rho_0 =$ central density

$\mu_{0D} \simeq 140 \; \frac{M_\odot}{\mathrm{pc}^2} = 6400 \; \mathrm{MeV}^3 = (18.6 \; \mathrm{Mev})^3$ Donato et al.09

Universal value for μ_{0D}: independent of galaxy luminosity for a large number of galactic systems (spirals, dwarf irregular and spheroidals, elliptics) spanning over 14 magnitudes in luminosity and of different Hubble types.

Similar values $\mu_{0D} \simeq 80 \; \frac{M_\odot}{\mathrm{pc}^2}$ in interstellar molecular clouds of size r_0 of different type and composition over scales $0.001 \, \mathrm{pc} < r_0 < 100$ pc (Larson laws, 1981).

Density profile in Galaxies: $\rho(r) = \rho_0 \; F\left(\frac{r}{r_0}\right) \; , \; F(0) = 1.$

Profiles: $F_{Burkert}(x) = \frac{1}{(1+x)(1+x^2)} \; , \; F_{Sersic}(x) = e^{-x^{\frac{1}{n}}} \; , \; x \equiv \frac{r}{r_0}$

Same $1/r^3$ tail as cuspy NFW profile $F_{NFW}(x) = \frac{4}{x \; (1+x)^2}$

Virial theorem plus extensivity of energy $\Longrightarrow \mu_{0D} =$ constan

Virial theorem for self-gravitating systems:

$E = \frac{1}{2}\,\langle U \rangle = -\langle K \rangle$, $\quad E =$ total energy,

$U =$ potential energy, $K =$ kinetic energy. Therefore,

$$E = -\frac{G}{4} \int \frac{d^3r\; d^3r'}{|\vec{r}-\vec{r}'|} \langle \rho(r)\; \rho(r') \rangle = -\frac{G}{4} \rho_0^2\; r_0^5 \int \frac{d^3x\; d^3x'}{|\boldsymbol{x}-\boldsymbol{x}'|} \langle F(x)\; F(x') \rangle$$

Energy divided by the characteristic volume r_0^3 goes as

$$\frac{-E}{r_0^3} \sim G\; \rho_0^2\; r_0^2 = G\; \mu_{0D}^2$$

Energy extensivity requires E/r_0^3 fixed for large values of r_0
$\Longrightarrow \mu_{0D}$ must take the same constant value for all r_0

Estimating $\langle K \rangle$ yields $\langle K \rangle = \frac{1}{2} \int d^3r\; \langle \rho(r) \rangle\; \langle v^2 \rangle =$

$$= \frac{1}{2}\; \rho_0\; r_0^3\; \langle v^2 \rangle \int d^3x\; \langle F(x) \rangle \sim \rho_0\; r_0^3\; \langle v^2 \rangle \Longrightarrow \langle v^2 \rangle \sim G\; \mu_{0D}\; r_0$$

This is true both for molecular clouds and for galaxies.

DM surface density from linear Boltzmann-Vlasov eq

The distribution function of the decoupled DM particles:

$$f(\vec{x}, \vec{p}; t) = g\ f_0(p) + F_1(\vec{x}, \vec{p}; t)$$

$f_0(p) =$ thermal equilibrium function at temperature T_d

$$m\ g \int \frac{d^3p}{(2\pi)^3}\ f_0(p) = \rho_{DM} = \Omega_M\ \rho_c = 3\,\Omega_M\ M_{Pl}^2\ H_0^2$$

The linearized Boltzmann-Vlasov equation in the MD era can be recasted as the Gilbert integral equation (Volterra equation of 2nd kind) for

$$\Delta(k, t) \equiv m\ \int \frac{d^3p}{(2\pi)^3}\ \int d^3x\ e^{-i\,\vec{x}\cdot\vec{k}}\ F_1(\vec{x}, \vec{p}; t)$$

We evolve the fluctuations during the MD era using as initial conditions the density fluctuations by the end of the RD era,

$\Delta(k, t_{eq}) = \Omega_M\ \rho_c\ V\ \delta(k, t_{eq})\ ,\ t_{eq} =$ equilibration time,

$$V \sim 1/k_{eq}^3 \simeq \frac{f}{H_0^3},\ k_{eq} \simeq 42.04\ H_0 = 9.88\ \mathrm{Gpc}^{-1},\ f \simeq 1.35\,10^{-5}$$

Fluctuations $k > k_{eq}$ inside the horizon by t_{eq} are relevant

Density Profiles from the Gilbert equation

At the end of the RD era $t = t_{eq}$:

$$\delta(k, t_{eq}) = 24 \, |\phi_k| \, \log\left(0.116 \, \frac{k}{k_{eq}}\right)$$

[W. Hu and N. Sugiyama (1996).]

$|\phi_k| =$ primordial inflationary fluctuations:

$$|\phi_k| = \sqrt{2} \, \pi \, |\Delta_0| \, \left(\frac{k}{k_0}\right)^{n_s/2-2} ,$$

where $|\Delta_0| \simeq 4.94 \; 10^{-5}, \; n_s \simeq 0.964, \; k_0 = 2 \; \mathrm{Gpc}^{-1}$.

Density profile today in the linear approximation:

$$\rho_{lin}(r) = \frac{1}{2 \, \pi^2 \, r} \int_0^\infty k \, dk \; \sin(k \, r) \; \Delta(k, t_{\mathrm{today}})$$

H. J. de Vega, N. G. Sanchez,
On the constant surface density in dark matter galaxies and interstellar molecular clouds, arXiv:0907.0006

The Gilbert equation

Define: $\widehat{\Delta}(k,t) \equiv \Delta(k,t)/\Delta(k,t_{eq})$.

The Gilbert equation takes the form:

$$\widehat{\Delta}(k,u) - \frac{6}{\alpha}\int_0^u \Pi[\alpha\,(u-u')]\,\frac{\widehat{\Delta}(k,u')}{[1-u']^2}\,du' = I[\alpha\,u]$$

where,

$$\Pi[z] = \frac{1}{I_2}\int_0^\infty dy\; y\; f_0(y)\;\sin(y\,z),\;\; I[z] = \frac{1}{I_2}\int_0^\infty dy\; y\; f_0(y)\;\frac{\sin(y\,z)}{z}$$

$$y \equiv \frac{p}{T_d}, \quad z \equiv \alpha\; u, \quad \alpha \equiv \frac{2\,k}{H_0}\sqrt{\frac{1+z_{eq}}{\Omega_M}}\,\frac{T_d}{m},$$

$$I_2 = \int_0^\infty dy\; y^2\; f_0(y), \quad 1+z_{eq} = \frac{1}{a_{eq}} \simeq 3200,$$

$u =$ dimensionless time variable,

$$u = 1 - \sqrt{\frac{a_{eq}}{a}}\;,\; 0 \leq u \leq u_{\text{today}} = 1 - \sqrt{a_{eq}} \simeq 0.982$$

$$a(u) = \frac{a_{eq}}{(1-u)^2}\;,\; a(\text{today}) = 1\;.$$

$$\widehat{\Delta}(k,t) \overset{t\to t_{\text{today}}}{=} \frac{3}{5}\,T(k)\,(1+z_{eq}), \quad T(k) = \text{transfer function}.$$

The solution of the Gilbert equation today

Transfer function: $T(0) = 1$ and $T(k \to \infty) = 0$.

The solution of the Gilbert equation $\widehat{\Delta}(k,t)$ for $k < k_{fs}$ grows proportional to the scale factor.

$k_{fs} =$ free-streaming (Jeans) comoving wavenumber.

$k_{fs} =$ characteristic scale for the decreasing of $T(k)$ with k
$\Longrightarrow$ the natural variable here is $\gamma \equiv k\, r_{lin}$

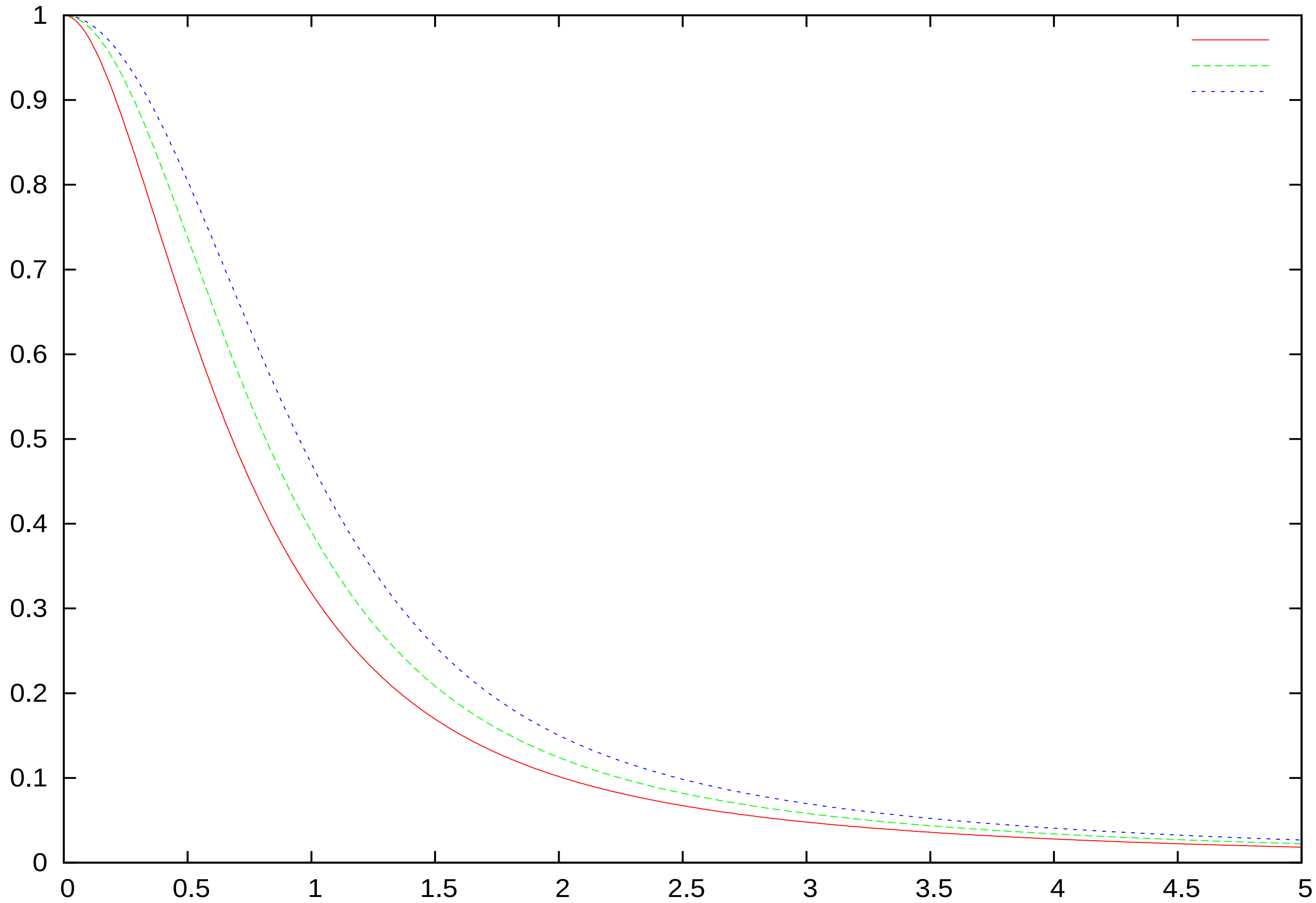
$r_{lin} \equiv \frac{\sqrt{2}}{k_{fs}} = \frac{2}{H_0}\, \sigma_{DM}\, \sqrt{\frac{1+z_{eq}}{\Omega_M}}$ and

$$\sigma_{DM} = \left(3\;\, M^2_{Pl}\, H_0^2\, \Omega_{DM}\, \frac{1}{Z}\, \frac{\sigma_s^3}{\rho_s}\right)^{\frac{1}{3}} \Longrightarrow r_{lin} = 125.1\;\; \left(\frac{10}{Z}\right)^{\frac{1}{3}} \text{ kpc}$$

Collecting all formulas we obtain for the fluctuations today

$$\Delta(k, t_{\text{today}}) = 1.926\, \frac{M^2_{Pl}}{H_0}\, |\Delta_0|\, T(k)\;\, \left(\frac{k}{k_0}\right)^{n_s/2-2}\; \log\left(0.116\, \frac{k}{k_{eq}}\right)$$

Density profiles in the linear approximation



Profiles $\rho_{lin}(r)/\rho_{lin}(0)$ vs. $x \equiv r/r_{lin}$

Fermions and Bosons decoupling ultrarelativistically and particles decoupling non-relativistically (Maxwell-Boltzmann statistics)

Density profiles in the linear approximation

The Fourier transform of the fluctuations today yield

$$\rho_{lin}(r) = (5.826 \text{ Mev})^3 \; \frac{Z^{n_s/6}}{r} \times$$
$$\times \int_0^\infty \gamma^{n_s/2-1} \; \log\left(\widehat{c}\, Z^{\frac{1}{3}}\, \gamma\right) \; \sin\left(\gamma \, \frac{r}{r_{lin}}\right) \; T(\gamma)\, d\gamma \; ,$$

$$\mu_{0D} = r_{lin}\; \rho_{lin}(0) =$$
$$= (5.826 \text{ Mev})^3 \; Z^{n_s/6} \; \int_0^\infty \gamma^{n_s/2} \; \log\left(\widehat{c}\, Z^{\frac{1}{3}}\, \gamma\right) \; T(\gamma)\, d\gamma \; ,$$

where:

$n_s/2 - 1 = -0.518, \; n_s/2 = 0.482, \; n_s/6 = 0.160$ and $\widehat{c} = 43.6$

Particle Statistics	$\mu_{0D} = r_{lin}\, \rho_{lin}(0)$
Bose-Einstein	$(16.71 \text{ Mev})^3 \; (Z/10)^{0.16}$
Fermi-Dirac	$(15.65 \text{ Mev})^3 \; (Z/10)^{0.16}$
Maxwell-Boltzmann	$(14.73 \text{ Mev})^3 \; (Z/10)^{0.16}$

Observed value: $\mu_{0D} \simeq (18.6 \text{ Mev})^3 \Rightarrow Z \sim 10 - 100$

Linear results for μ_{0D} and the profile vs. observations

Since the surface density $r_0\, \rho(0)$ should be universal, we can identify $r_{lin}\, \rho_{lin}(0)$ from a spherically symmetric solution of the linearized Boltzmann-Vlasov equation.

The linear profiles obtained are cored since $T(k)$ decays for $k > k_{fs} \sim 1/r_{lin} \sim \ 0.008\ (Z/10)^{\frac{1}{3}}\ (\text{kpc})^{-1}$.

$\rho_{lin}(r)$ scales with the primordial spectral index n_s:

$\rho_{lin}(r) \overset{r \gg r_{lin}}{=} \ r^{-1-n_s/2} \ = \ r^{-1.482}$,

in agreement with the universal empirical behaviour $r^{-1.6\pm0.4}$, M. G. Walker et al., I. M. Vass et al. (2009).

For larger scales nonlinear effects from small k should give the customary r^{-3} tail.

The agreement between the linear theory and the observations is remarkable.

The comparison of our theoretical values for μ_{0D} and the observational value indicates that $Z \sim 10 - 100$.

This implies that the DM particle mass is in the keV range.

Dark Energy

$76 \pm 5\%$ of the present energy of the Universe is Dark !
Current observed value:
$\rho_\Lambda = \Omega_\Lambda \, \rho_c = (2.39 \text{ meV})^4 \;\; , \;\; 1 \text{ meV} = 10^{-3} \text{ eV}$.
Equation of state $p_\Lambda = -\rho_\Lambda$ within observational errors.
Quantum zero point energy. Renormalized value is finite.
Bosons (fermions) give positive (negative) contributions.
Mass of the lightest particles ~ 1 meV is in the right scale.
Spontaneous symmetry breaking of continuous symmetries produces massless scalars as Goldstone bosons. A small symmetry breaking provide light scalars: axions,majorons...
Observational Axion window $10^{-3} \text{ meV} \lesssim M_{\text{axion}} \lesssim 10 \text{ meV}$.
Dark energy can be a cosmological zero point effect. (As the Casimir effect in Minkowski with non-trivial boundaries).
We need to learn the physics of light particles (< 1 MeV), also to understand dark matter !!

Summary and Conclusions

- We formulate inflation as an effective field theory in the Ginsburg-Landau spirit with energy scale $M \sim M_{GUT} \sim 10^{16}$ GeV $\ll M_{Pl}$. Inflaton mass small: $m \sim H/\sqrt{N} \sim M^2/M_{Pl} \ll M$. Infrared regime !!
- For all slow-roll models $n_s - 1$ and r are $1/N, \; N \sim 60$.
- MCMC analysis of WMAP+LSS data plus this theory input indicates a spontaneously broken inflaton potential: $w(\chi) = \frac{y}{32}\left(\chi^2 - \frac{8}{y}\right)^2 \;, \; y \simeq 1.26$.
- Lower Bounds: $r > 0.023 \; (95\% \text{ CL}) \;, \; r > 0.046 \; (68\% \text{ CL})$. The most probable values are $r \simeq 0.051 (\Leftarrow$ measurable !!) $n_s \simeq 0.964$.
- CMB quadrupole suppression may be explained by the effect of fast-roll inflation provided the today's horizon size modes exited by the end of fast-roll inflation.

Summary and Conclusions 2

- Model independent analysis of dark matter points to a particle mass at the keV scale. T_d may be > 100 GeV. DM is cold.
- Universal Surface density in DM galaxies [$\mu_{0D} \simeq (19\text{MeV})^3$] explained by keV mass scale DM. Density profile scales and decreases for intermediate scales with the spectral index $n_s: \;\; \rho(r) \sim r^{-1-n_s/2}$.
- Quantum (loop) corrections in the effective theory of inflation are of the order $(H/M_{Pl})^2 \sim 10^{-9}$. Same order of magnitude as loop graviton corrections.

D. Boyanovsky, H. J. de Vega, N. G. Sanchez, Quantum corrections to the inflaton potential and the power spectra from superhorizon modes and trace anomalies, PRD72, 103006 (2005), Quantum corrections to slow roll inflation and new scaling of superhorizon fluctuations. Nucl. Phys. B 747, 25 (2006), astro-ph/0503669.

Future Perspectives

The Golden Age of Cosmology and Astrophysics continues.

A wealth of data from WMAP (7 yr), Planck, Atacama Cosmology Tel and further experiments are coming.

Galaxy and Star formation. DM properties from astronomical observations. Better bounds on DM cross-sections.

DM in planets and the earth. Flyby and Pioneer anomalies?

The Dark Ages...Reionisation...the 21cm line...

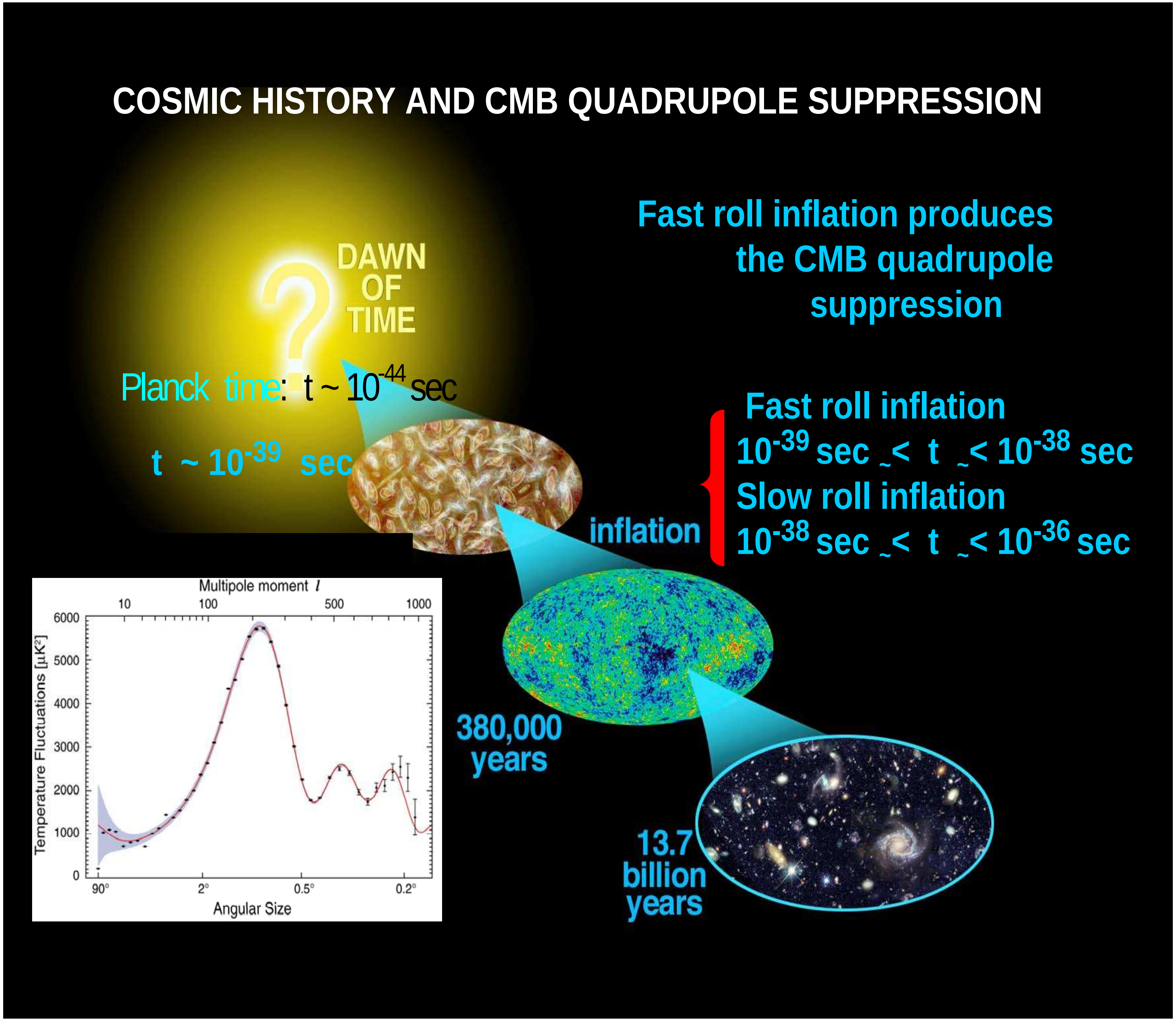
Nature of Dark Energy? 76% of the energy of the universe.

Nature of Dark Matter? 83% of the matter in the universe.

Light DM particles are strongly favoured $m_{DM} \sim$ keV.

Sterile neutrinos? Some unknown light particle ??

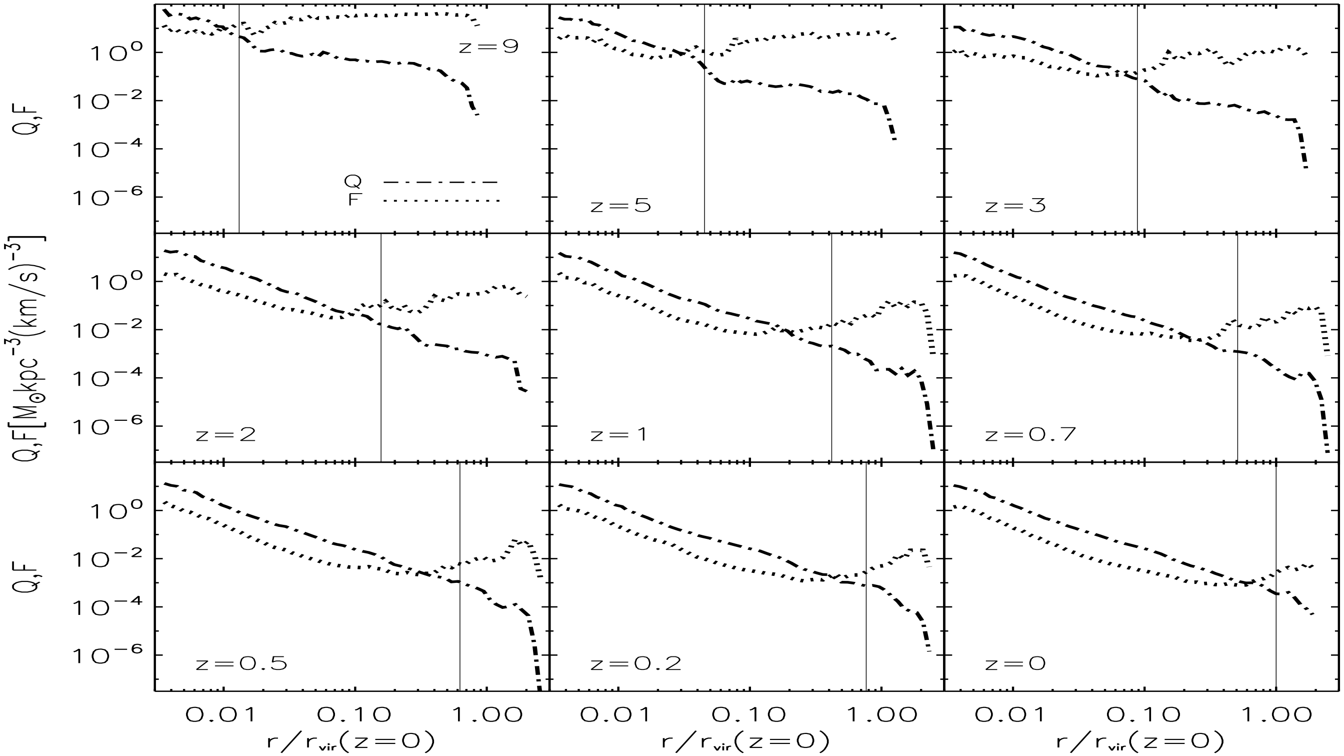
Need to learn about the physics of light particles (< 1 MeV).


COSMIC HISTORY AND CMB QUADRUPOLE SUPPRESSION
Fast roll inflation produces the CMB quadrupole suppression
DAWN OF TIME
Planck time: t ~ 10^-44 sec
t ~ 10^-39 sec
Fast roll inflation
10^-39 sec ≲ t ≲ 10^-38 sec
Slow roll inflation
10^-38 sec ≲ t ≲ 10^-36 sec
inflation
380,000 years
13.7 billion years
Multipole moment l
10
100
500
1000
Temperature Fluctuations [μK²]
6000
5000
4000
3000
2000
1000
0
90°
2°
0.5°
0.2°
Angular Size

THANK YOU VERY MUCH

FOR YOUR ATTENTION!!

ρ/σ^3 vs. r for different z from ΛCDM simulations



Phase-space density $Q \equiv \rho/\sigma^3$ vs. $r/r_{vir}(z=0)$ dot-dashed line for different redshifts: $0 \leq z \leq 9$.

We see that from $z = 9$ to $z = 0$ the r-average of ρ/σ^3 decreases by a factor $Z \sim 10$.

I. M. Vass et al. MNRAS, 395, 1225 (2009).

Higher Order Inflaton Potentials

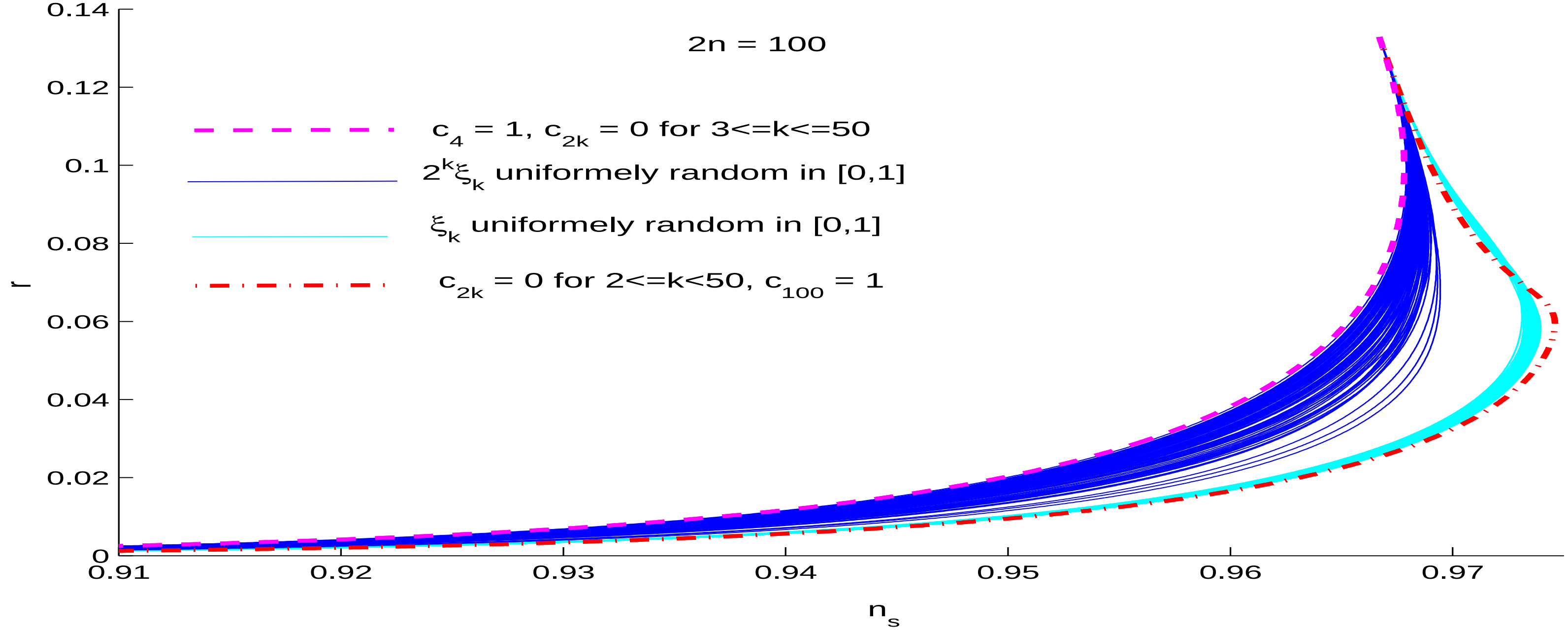
Till here we considered fourth degree inflaton potentials. Can higher order terms modify the physical results and the observable predictions?

We systematically study the effects produced by higher order terms ($n > 4$) in the inflationary potential on the observables n_s and r.
All coefficients in the potential w become order one using the field χ within the Ginsburg-Landau approach:
$$w(\chi) = c_0 - \frac{1}{2}\,\chi^2 + \sum_{n=2}^{\infty} \frac{c_n}{n}\,\chi^{2\,n} \quad , \quad c_n = \mathcal{O}(1) \; .$$

All $r = r(n_s)$ curves for double–well potentials of arbitrary high order fall **inside** a universal banana-shaped region $\mathcal{B}$.
Moreover, the $r = r(n_s)$ curves for double–well potentials even for arbitrary positive higher order terms lie inside the banana region $\mathcal{B}$.

C. Destri, H. J. de Vega, N. G. Sanchez, arXiv:0906.4102.

The 100th degree polynomial inflaton potential



$$w(\chi) = \frac{4}{y} - \frac{1}{2}\,\chi^2 + \frac{4}{y}\sum_{k=2}^{n}\frac{c_{2k}}{k}\;\left(\frac{y^k}{8^k}\;\chi^{2k} - 1\right)$$

The coefficients c_{2k} were extracted at random.
The lower border of the banana-shaped region is given by the potential:

$$w(\chi) = \frac{4}{y} - \frac{1}{2}\,\chi^2 + \frac{4}{n\,y}\;\left(\frac{y^n}{8^n}\;\chi^{2n} - 1\right)$$ with $n = 50$.

The inflaton potential from a fermion condensate

Inflaton coupled to Dirac fermions Ψ during inflation:

$$\mathcal{L} = \overline{\Psi} \left[i \, \gamma^\mu \, \mathcal{D}_\mu - m_f - g_Y \; \phi \right] \Psi$$

$g_Y =$ Yukawa coupling, $\gamma^\mu =$ curved space-time γ-matrices.

Hubble parameter $H =$ constant. Effective potential $\equiv$ fermions energy for a constant inflaton ϕ during inflation.

Dynamically generated inflaton potential:

$$V_f(\phi) = V_0 + \frac{1}{2} \, \mu^2 \, \phi^2 + \frac{1}{4} \, \lambda \, \phi^4 + H^4 \, Q\left(g_Y \, \frac{\phi}{H} \right)$$, where

$\mu^2 = -m^2 < 0$ mass squared, $\lambda =$ quartic coupling,

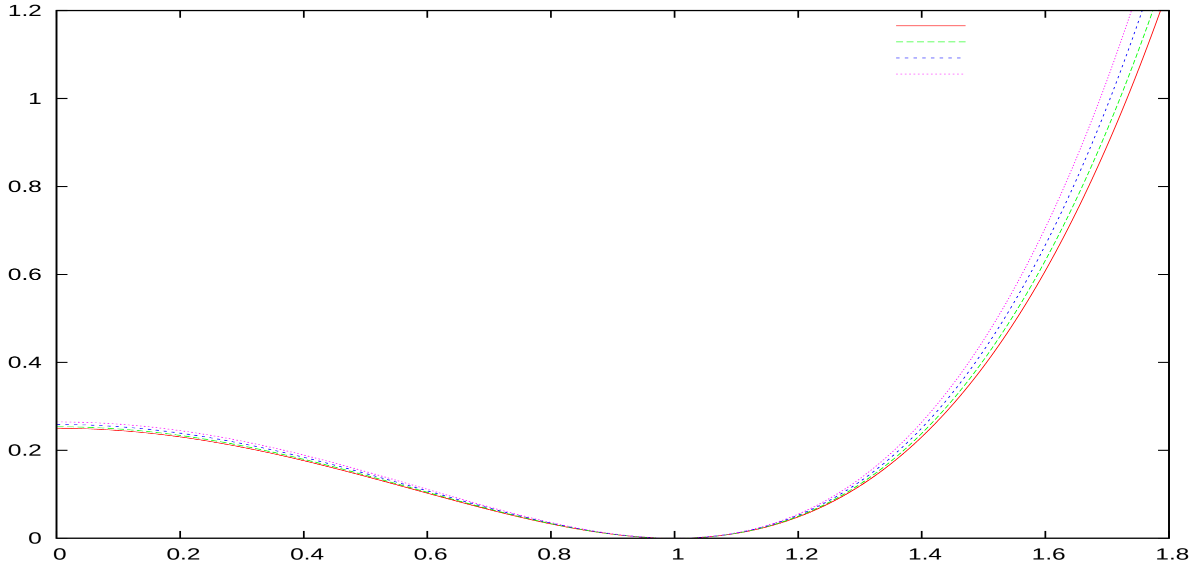
$$Q(x) = \frac{x^2}{8\,\pi^2} \left\{ (1+x^2) \left[\gamma + \mathrm{Re}\, \psi(1+i\,x) \right] - \zeta(3) \, x^2 \right\} =$$

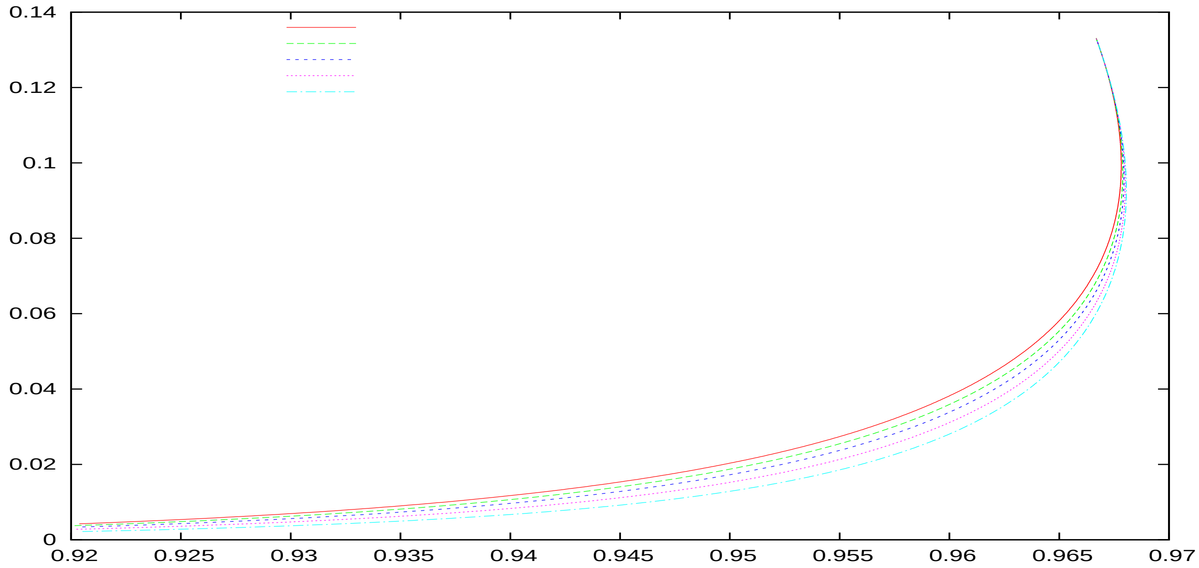
$$= \frac{x^4}{8\,\pi^2} \left[(1+x^2) \sum_{n=1}^\infty \frac{1}{n\,(n^2+x^2)} - \zeta(3) \right] \; , \quad x \equiv g_Y \; \frac{\phi}{H}$$

$$Q(x) \overset{x \to \infty}{=} \frac{x^4}{8\,\pi^2} \left[\log x + \gamma - \zeta(3) + \mathcal{O}\left(\frac{1}{x}\right) \right]$$

Minkowski limit (Coleman-Weinberg potential)

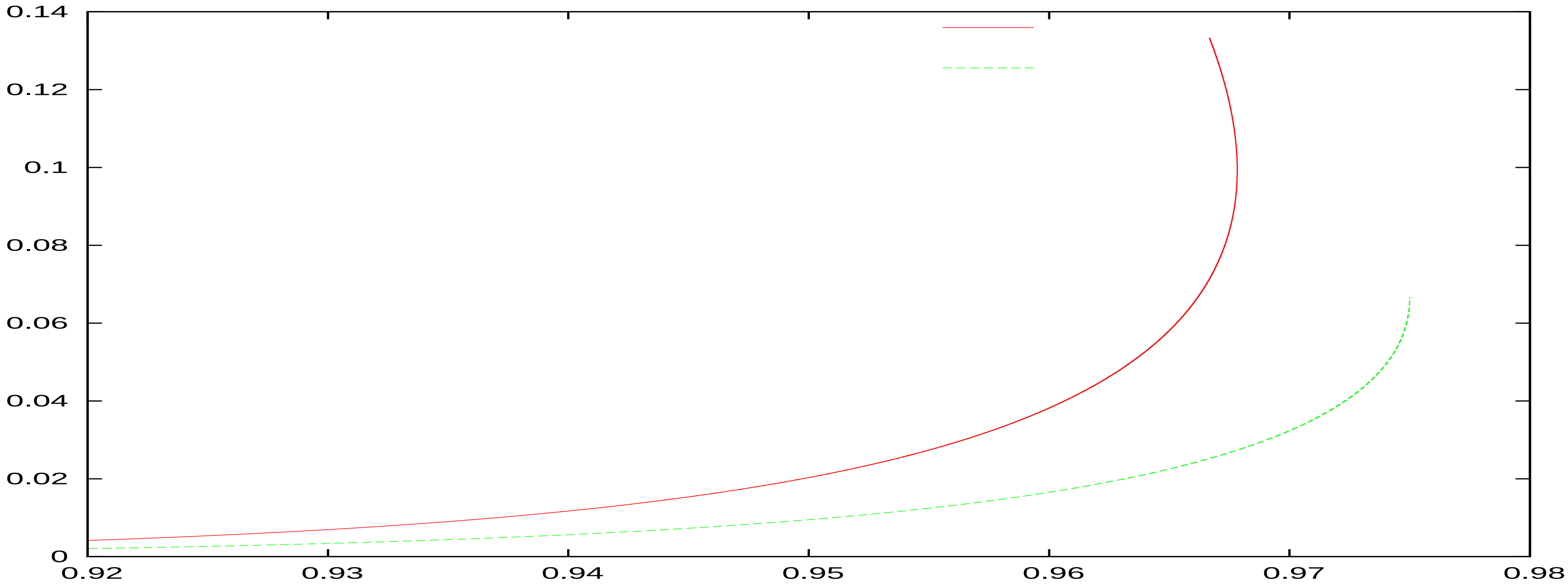
Effective fermionic inflaton potential and r vs. n_s





$y\, V(\phi)/[8\, N\ M^4]$ vs. ϕ/ϕ_{min} for $0 < g_Y < 500\ H/\phi_{min}$
r vs. n_s for $0 < g_Y < 800\ H/\phi_{min}$

The universal banana region



We find that all $r = r(n_s)$ curves for double–well inflaton potentials in the Ginsburg-Landau spirit fall inside the universal banana region $\mathcal{B}$.

The lower border of $\mathcal{B}$ corresponds to the limiting potential:

$$w(\chi) = \frac{4}{y} - \frac{1}{2}\,\chi^2 \quad \text{for } \chi < \sqrt{\frac{8}{y}} \quad , \quad w(\chi) = +\infty \quad \text{for } \chi > \sqrt{\frac{8}{y}}$$

This gives a lower bound for r for all potentials in the Ginsburg-Landau class: $r > 0.021$ for the current best value of the spectral index $n_s = 0.964$.

The Energy Scale of Inflation

Grand Unification Idea (GUT)

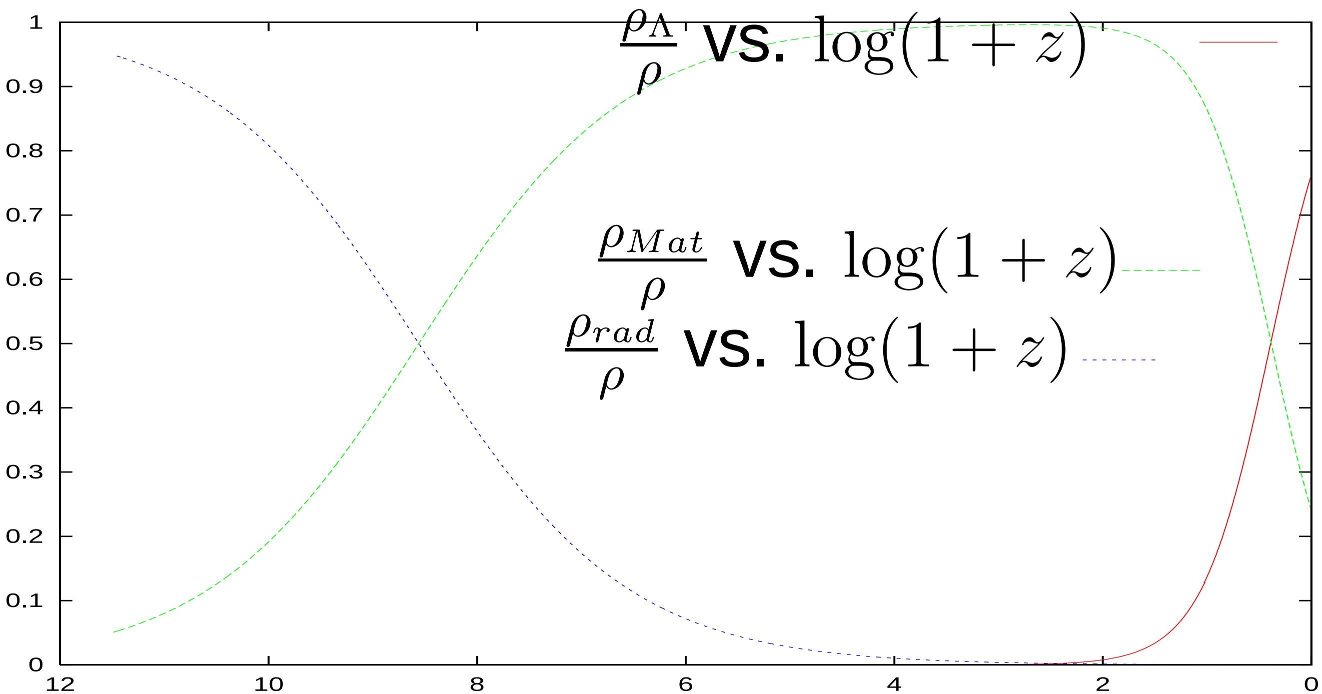
- Renormalization group running of electromagnetic, weak and strong couplings shows that they all meet at $E_{GUT} \simeq 2 \times 10^{16}$ GeV
- Neutrino masses are explained by the see-saw mechanism: $m_\nu \sim \frac{M^2_{\rm Fermi}}{M_R}$ with $M_R \sim 10^{16}$ GeV.
- Inflation energy scale: $M \simeq 10^{16}$ GeV.

Conclusion: the GUT energy scale appears in at least three independent ways.

Moreover, moduli potentials: $V_{moduli} = M^4_{\rm SUSY} \, v\left(\frac{\phi}{M_{Pl}}\right)$

ressemble inflation potentials provided $M_{\rm SUSY} \sim 10^{16}$ GeV.
First observation of SUSY in nature??

The Universe is made of radiation, matter and dark energy



End of inflation: $z \sim 10^{29}, \; T_{reh} \lesssim 10^{16}$ GeV, $t \sim 10^{-36}$ sec.
E-W phase transition: $z \sim 10^{15}, T_{\rm EW} \sim 100$ GeV, $t \sim 10^{-11}$ s.
QCD conf. transition: $z \sim 10^{12}, T_{\rm QCD} \sim 170$ MeV, $t \sim 10^{-5}$ s.
BBN: $z \sim 10^9 \; , \quad T \simeq 0.1$ MeV, $t \sim 20$ sec.
Rad-Mat equality: $z \simeq 3050, \; T \simeq 0.7$ eV, $t \sim 57000$ yr.
CMB last scattering: $z \simeq 1100, \; T \simeq 0.25$ eV , $t \sim 370000$ yr.
Mat-DE equality: $z \simeq 0.47, \; T \simeq 0.345$ meV , $t \sim 8.9$ Gyr.
Today: $z = 0, \; T = 2.725$K $= 0.2348$ meV $t = 13.72$ Gyr.

The number of efolds in Slow-roll

The number of e-folds $N[\chi]$ since the field χ exits the horizon till the end of inflation is:

$N[\chi] = N \int_{\chi_{end}}^{\chi} \frac{w(\chi)}{w'(\chi)} \, d\chi$. We choose then $N = N[\chi]$.

The spontaneously broken symmetric potential:

$$w(\chi) = \frac{y}{32} \left(\chi^2 - \frac{8}{y}\right)^2$$

produces inflation with $0 < \sqrt{y} \, \chi_{initial} \ll 1$ and $\chi_{end} = \sqrt{\frac{8}{y}}$.

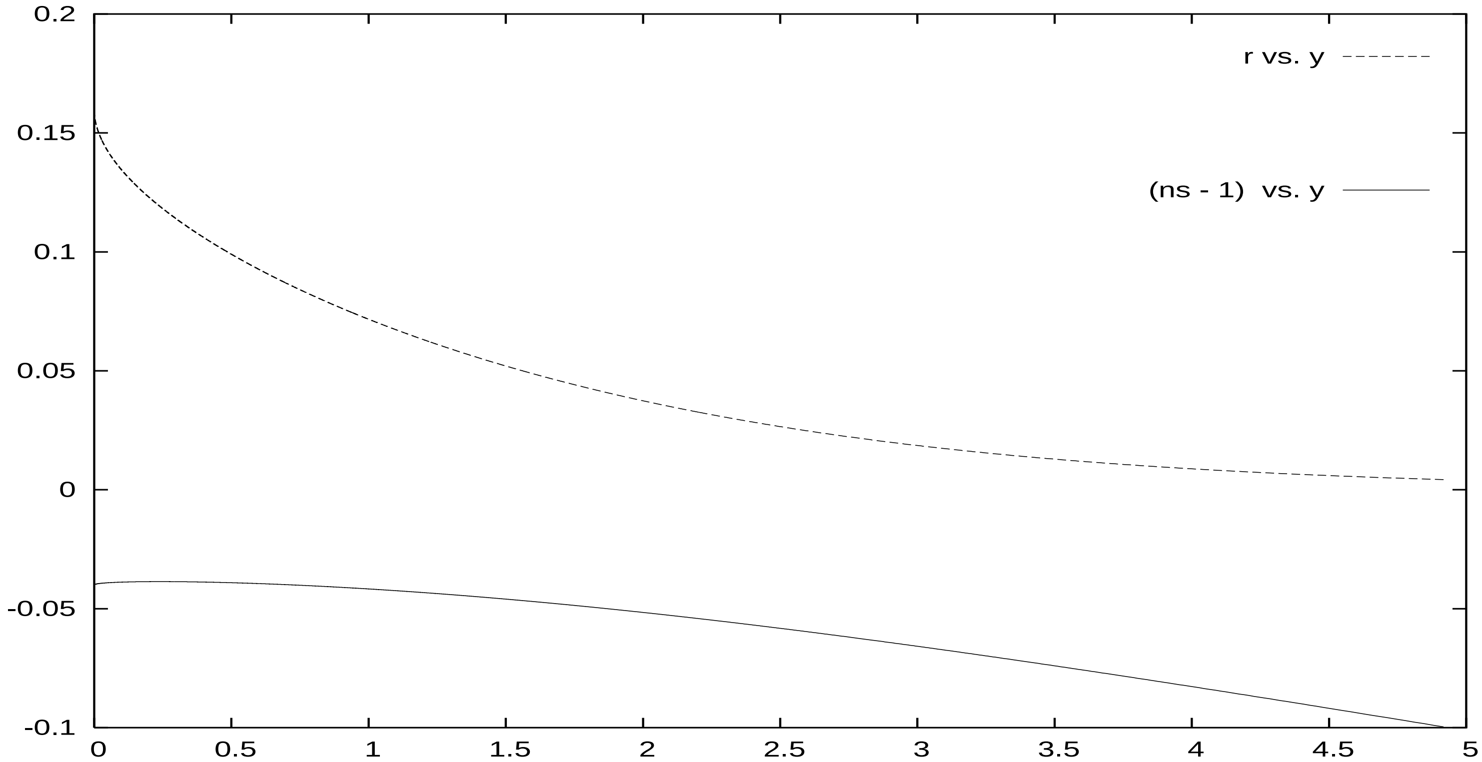
This is small field inflation.

From the above integral: $\quad y = z - 1 - \log z$
where $z \equiv y \, \chi^2/8$ and we have $0 < y < \infty$ for $1 > z > 0$.
Spectral index n_s and the ratio r as functions of y:

$n_s = 1 - \frac{y}{N} \frac{3\,z+1}{(z-1)^2} \quad , \quad r = \frac{16\, y}{N} \frac{z}{(z-1)^2}$

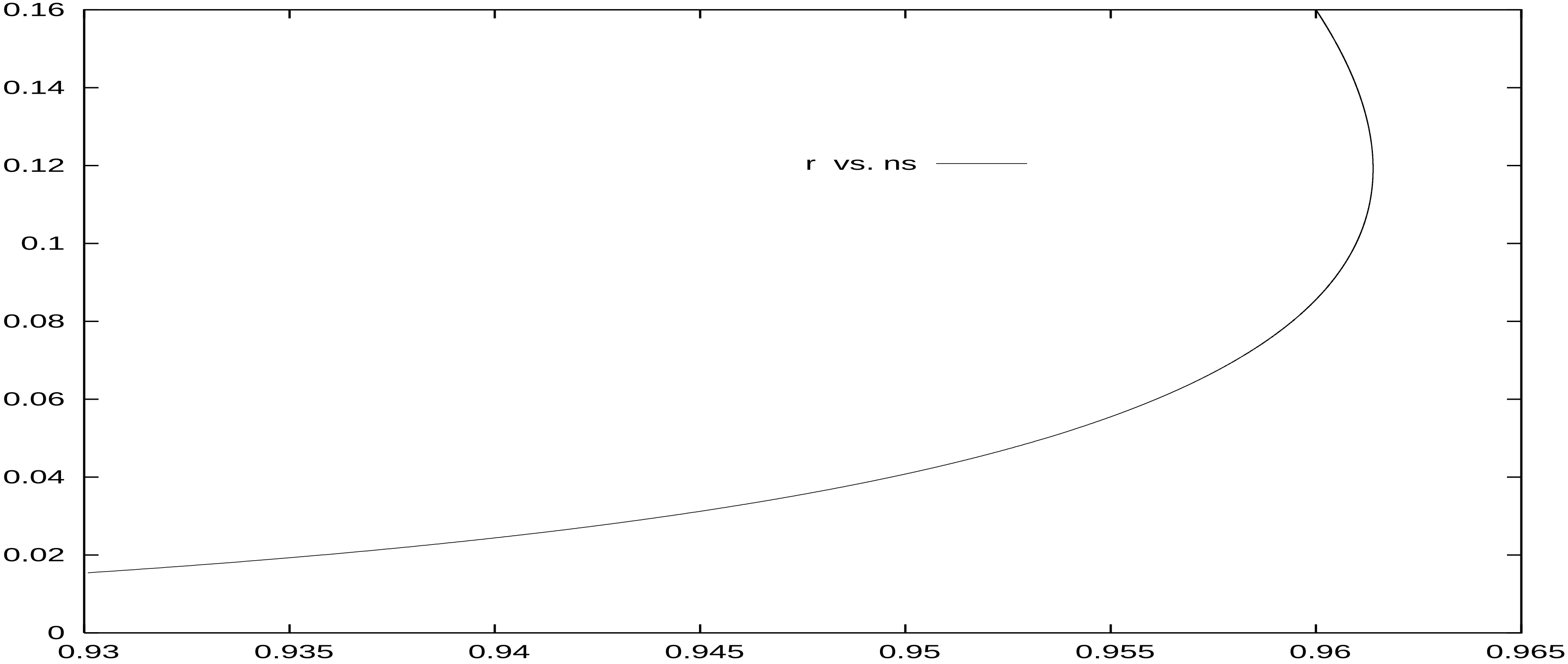
Binomial New Inflation: ($y =$ coupling).

r decreases monotonically with y :
(strong coupling) $0 < r < \frac{8}{N} = 0.16$ (zero coupling).



n_s first grows with y, reaches a maximum value
$n_{s,maximum} = 0.96139\ldots$ at $y = 0.2387\ldots$ and then n_s
decreases monotonically with y.

Binomial New Inflation



$r = \frac{8}{N} = 0.16$ and $n_s = 1 - \frac{2}{N} = 0.96$ at $y = 0$.

r is a double valued function of n_s.

Quadrupole suppression and Fast-roll Inflation

The observed CMB-quadrupole (COBE,WMAP5) is six times smaller than the ΛCDM model value.
In the best ΛCDM fit the probability that the quadrupole is as low or lower than the observed value is 3%.
It is hence relevant to find a cosmological explanation of the quadrupole suppression.

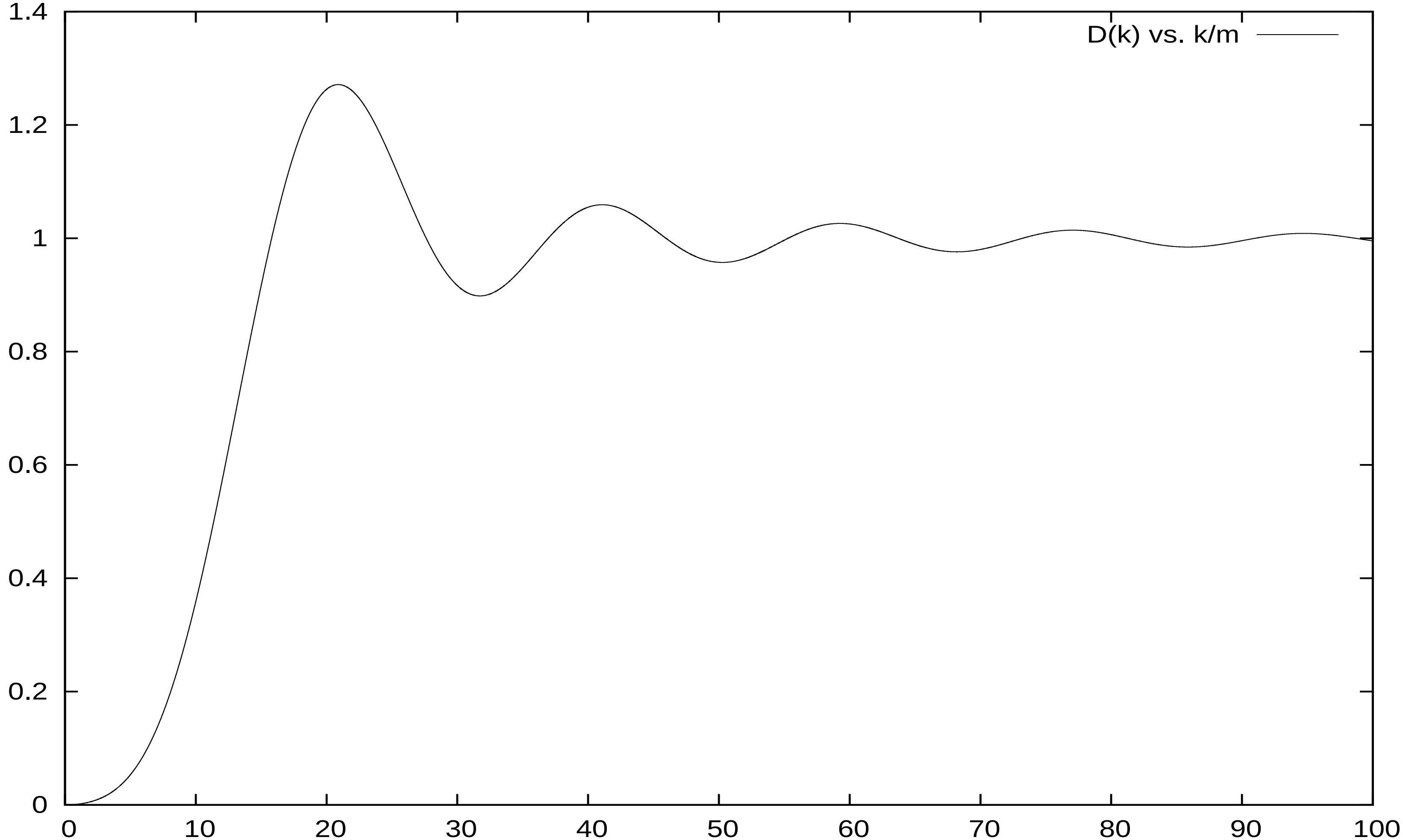
Generically, the classical evolution of the inflaton has a brief **fast-roll stage** that precedes the slow-roll regime.
In case the quadrupole CMB mode leaves the horizon during fast-roll, before slow-roll starts, we find that the quadrupole mode gets suppressed.

$$P(k) = |\Delta_{k\ ad}^{(S)}|^2\ (k/k_0)^{n_s-1}[1+D(k)]$$

The transfer function $D(k)$ changes the primordial power.

$$1+D(0)=0, \quad D(\infty)=0$$

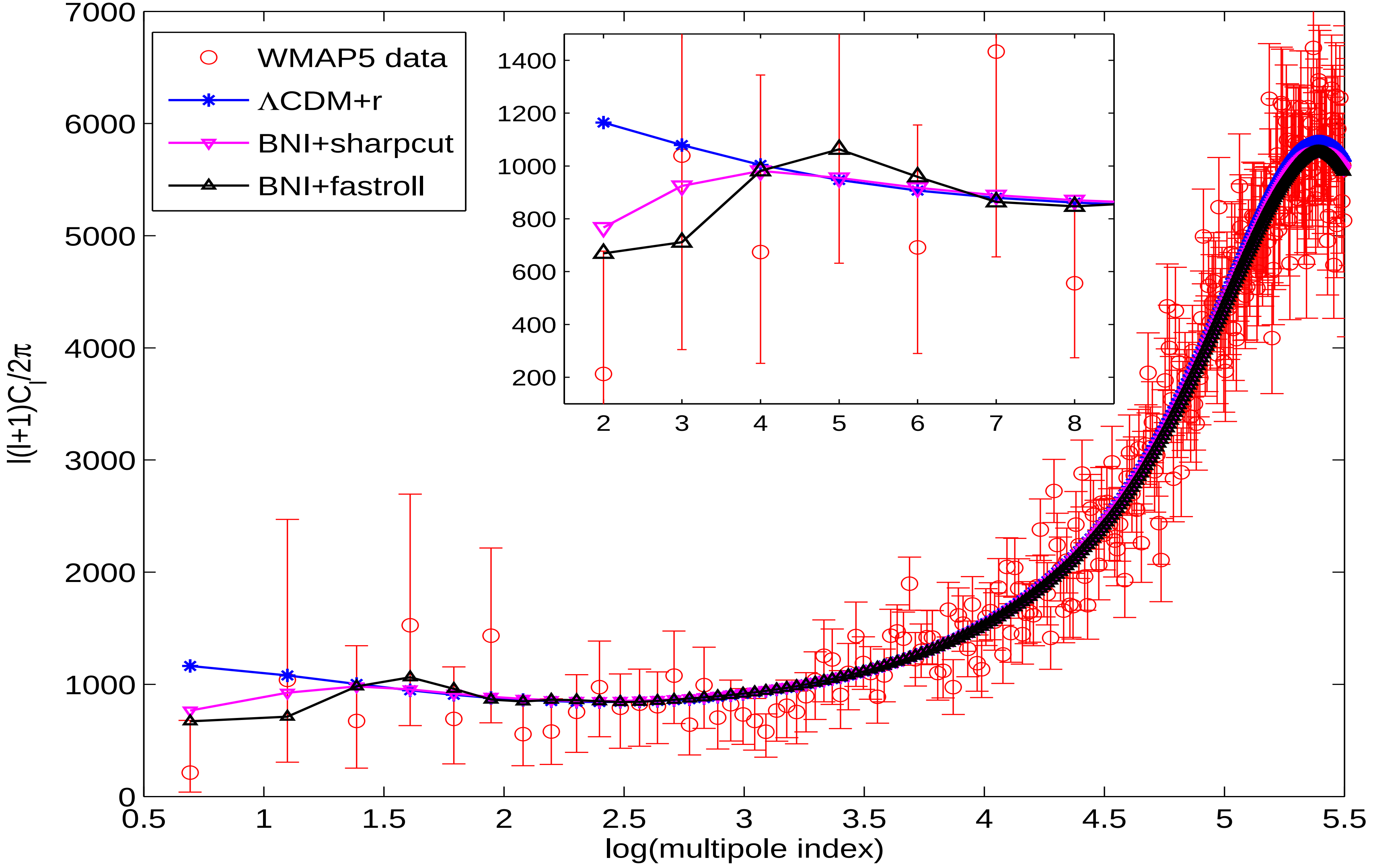
The Fast-Roll Transfer Function



$k_Q = 11.5\ m,\ k_{fastroll \to slowroll} = 14\ m,\ k_{pivot} = 96.7\ m,$
$m = 1.21\, 10^{13} \mathrm{GeV}, k_Q^{today} = 0.238\ \mathrm{Gpc}^{-1} \implies$ redshift at the beginning of inflation $= 0.9 \times 10^{56} \simeq e^{129}$.

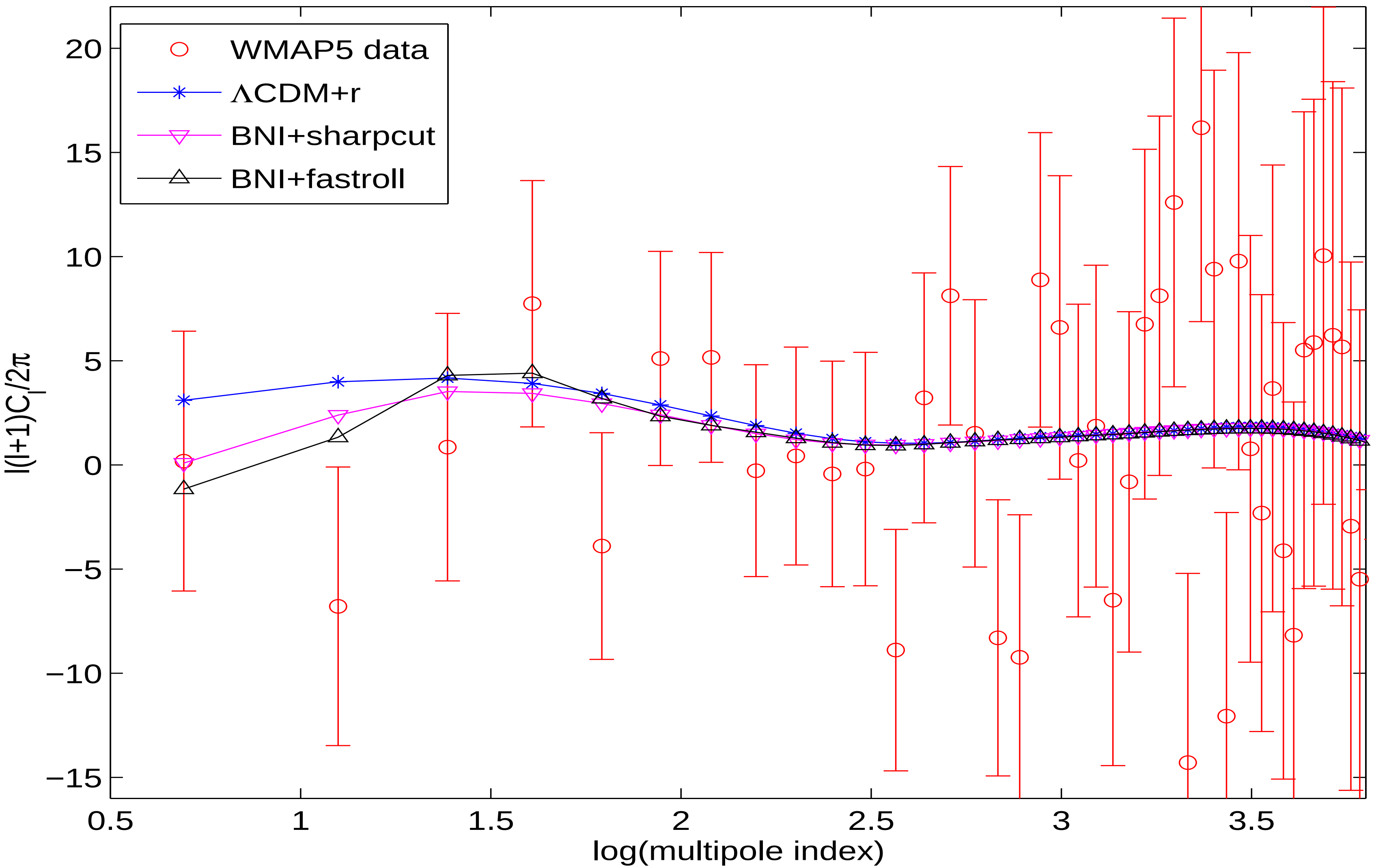
Comparison, with the experimental WMAP5 data

of the theoretical C_ℓ^{TT} multipoles



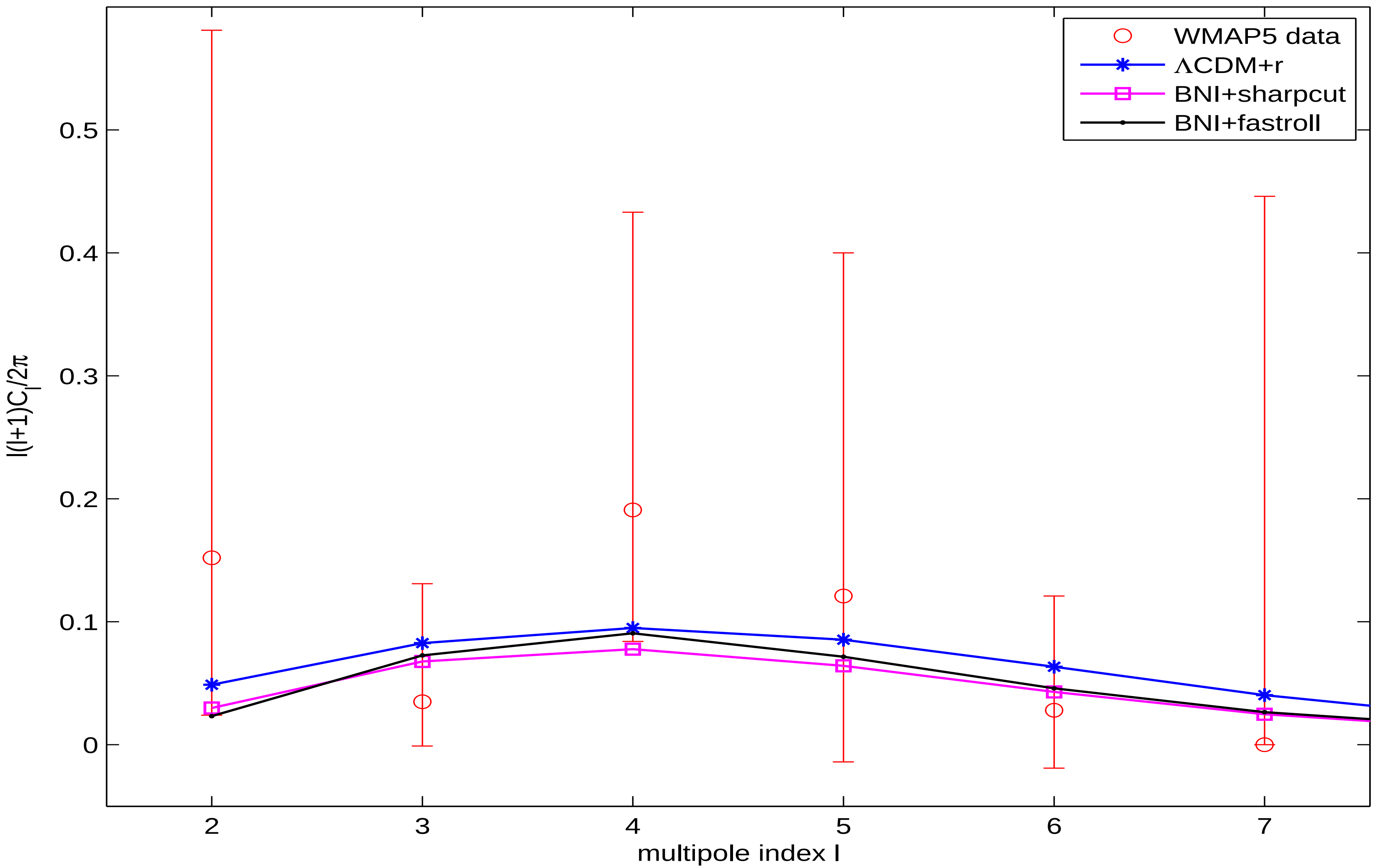
Comparison, with the experimental WMAP5 data

of the theoretical C_ℓ^{TE} multipoles

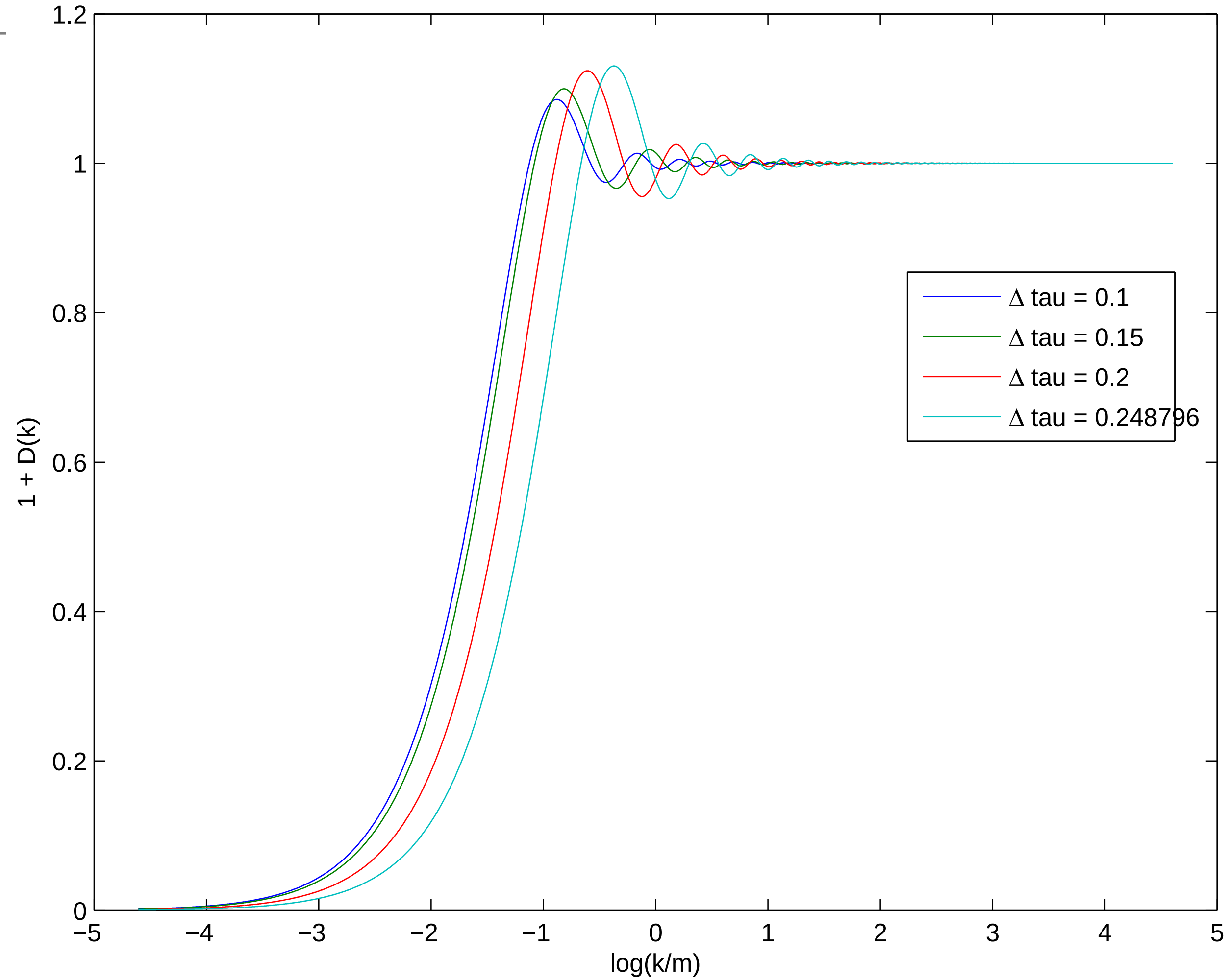


Comparison, with the experimental WMAP-5 data

of the theoretical C_ℓ^{EE} multipoles

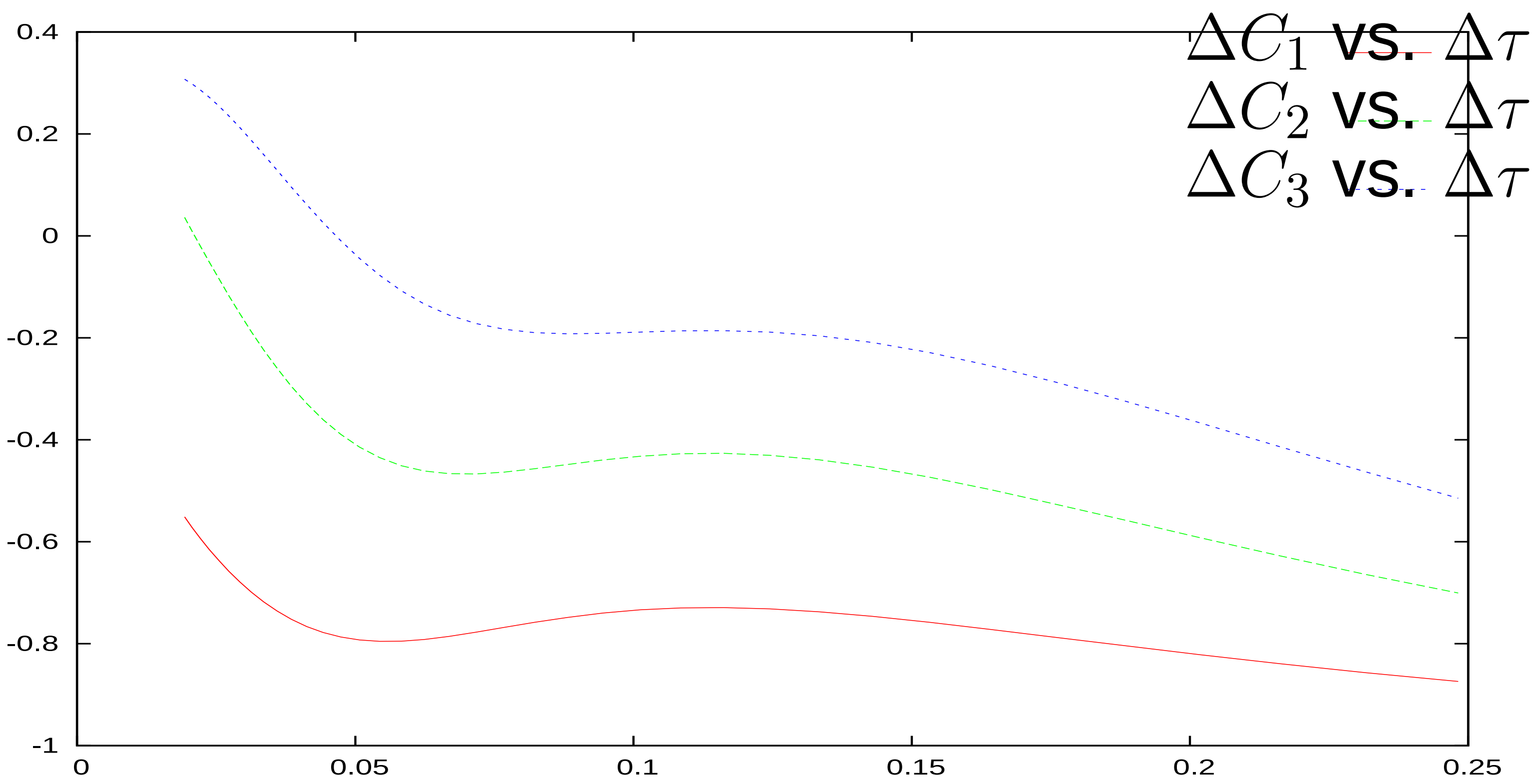


Transfer Function for different initial times of fluctuations



Transfer function $1 + D(k)$ for different initial times of fluctuations: $\Delta\tau$ from the begining of fast-roll. BD initial conditions. $\Delta\tau = 0.25$: begining of slow-roll.

ΔC_ℓ^{TT} vs. initial time of fluctuations



Changes on the dipole, quadrupole and octupole amplitudes according to the starting time $\Delta\tau$ chosen for the fluctuations from the begining of fast-roll. BD initial conditions. $\Delta\tau = 0.25$: begining of slow-roll.

Loop Quantum Corrections to Slow-Roll Inflation

$$\phi(\vec{x},t) = \Phi_0(t)+\varphi(\vec{x},t), \quad \Phi_0(t) \equiv < \phi(\vec{x},t) >, \quad < \varphi(\vec{x},t) >= 0$$

$$\varphi(\vec{x},t) = \frac{1}{a(\eta)} \int \frac{d^3k}{(2\,\pi)^3} \left[a_{\vec{k}}\; \chi_k(\eta)\; e^{i\vec{k}\cdot\vec{x}} + \text{h.c.}\right],$$

$a^\dagger_{\vec{k}},\; a_{\vec{k}}$ are creation/annihilation operators,
$\chi_k(\eta)$ are mode functions. $\eta =$ conformal time.
To one loop order the equation of motion for the inflaton is

$$\ddot{\Phi}_0(t) + 3\,H\;\dot{\Phi}_0(t) + V'(\Phi_0) + g(\Phi_0)\;\langle[\varphi(\boldsymbol{x},t)]^2\rangle = 0$$

where $g(\Phi_0) = \frac{1}{2}\,V^{'''}(\Phi_0)$.
The mode functions obey:

$$\chi^{''}_k(\eta) + \left[k^2 + M^2(\Phi_0)\;a^2(\eta) - \frac{a^{''}(\eta)}{a(\eta)}\right]\chi_k(\eta) = 0$$

where $M^2(\Phi_0) = V''(\Phi_0) = 3\;H_0^2\;\eta_V + \mathcal{O}(1/N^2)$

Quantum Corrections to the Friedmann Equation

The mode functions equations for slow-roll become,

$$\chi''_k(\eta) + \left[k^2 - \frac{\nu^2-\frac{1}{4}}{\eta^2}\right]\chi_k(\eta) = 0 \quad , \quad \nu = \frac{3}{2} + \epsilon_V - \eta_V + \mathcal{O}(1/N^2).$$

The scale factor during slow roll is $a(\eta) = -\frac{1}{H_0\,\eta\,(1-\epsilon_V)}$.

Scale invariant case: $\nu = \frac{3}{2}$. $\Delta \equiv \frac{3}{2} - \nu = \eta_V - \epsilon_V$ controls the departure from scale invariance.

Explicit solutions in slow-roll:

$$\chi_k(\eta) = \frac{1}{2}\sqrt{-\pi\eta}\; i^{\nu+\frac{1}{2}}\; H^{(1)}_\nu(-k\eta), \quad H^{(1)}_\nu(z) = \text{Hankel function}$$

Quantum fluctuations: $\langle[\varphi(\boldsymbol{x},t)]^2\rangle = \frac{1}{a^2(\eta)}\int\frac{d^3k}{(2\pi)^3}\;|\chi_k(\eta)|^2$

$$\frac{1}{2}\langle[\varphi(\boldsymbol{x},t)]^2\rangle = \left(\frac{H_0}{4\,\pi}\right)^2\left[{\Lambda_p}^2 + \ln\Lambda_p^2 + \frac{1}{\Delta} + 2\,\gamma - 4 + \mathcal{O}(\Delta)\right]$$

UV cutoff $\Lambda_p = \text{physical cutoff}/H$, $\quad \frac{1}{\Delta} =$ infrared pole.

$\langle\dot{\varphi}^2\rangle \quad , \quad \left\langle(\nabla\varphi)^2\right\rangle$ are infrared finite

Quantum Corrections to the Inflaton Potential

Upon UV renormalization the Friedmann equation results

$$H^2 = \frac{1}{3\,M_{Pl}^2}\left[\frac{1}{2}\,\dot{\Phi_0}^2 + V_R(\Phi_0) + \left(\frac{H_0}{4\,\pi}\right)^2 \frac{V_R^{''}(\Phi_0)}{\Delta} + \mathcal{O}\left(\frac{1}{N}\right)\right]$$

Quantum corrections are proportional to $\left(\frac{H}{M_{Pl}}\right)^2 \sim 10^{-9}$!!

The Friedmann equation gives for the effective potential:

$$V_{eff}(\Phi_0) = V_R(\Phi_0) + \left(\frac{H_0}{4\,\pi}\right)^2 \frac{V_R^{''}(\Phi_0)}{\Delta}$$

$$V_{eff}(\Phi_0) = V_R(\Phi_0)\left[1 + \left(\frac{H_0}{4\,\pi\,M_{Pl}}\right)^2 \frac{\eta_V}{\eta_V - \epsilon_V}\right]$$

in terms of slow-roll parameters

Very DIFFERENT from the one-loop effective potential in Minkowski space-time:

$$V_{eff}(\Phi_0) = V_R(\Phi_0) + \frac{[V_R^{''}(\Phi_0)]^2}{64\,\pi^2}\ln\frac{V_R^{''}(\Phi_0)}{M^2}$$

Quantum Fluctuations:

Scalar Curvature, Tensor, Fermion, Light Scalar.

All these quantum fluctuations contribute to the inflaton potential and to the primordial power spectra.

In de Sitter space-time: $< T_{\mu\nu} >= \frac{1}{4}\, g_{\mu\nu} \;\; < T_\alpha^\alpha >$

Hence, $V_{eff} = V_R + < T_0^0 >= V_R + \frac{1}{4} < T_\alpha^\alpha >$

Sub-horizon (Ultraviolet) contributions appear through the trace anomaly and only depend on the spin of the particle.

Superhorizon (Infrared) contributions are of the order N^0 and can be expressed in terms of the slow-roll parameters.

$$V_{eff}(\Phi_0) = V(\Phi_0) \left[1 + \frac{H_0^2}{3\,(4\pi)^2\, M_{Pl}^2} \left(\frac{\eta_v - 4\,\epsilon_v}{\eta_v - 3\,\epsilon_v} + \frac{3\,\eta_\sigma}{\eta_\sigma - \epsilon_v} + \mathcal{T} \right) \right]$$

$\mathcal{T} = \mathcal{T}_\Phi + \mathcal{T}_s + \mathcal{T}_t + \mathcal{T}_F = -\frac{2903}{20}$ is the total trace anomaly.

$\mathcal{T}_\Phi = \mathcal{T}_s = -\frac{29}{30}, \;\; \mathcal{T}_t = -\frac{717}{5}, \;\; \mathcal{T}_F = \frac{11}{60}$

$\longrightarrow$ the graviton (t) dominates.

Corrections to the Primordial Scalar and Tensor Power

$$|\Delta^{(S)}_{k,eff}|^2 = |\Delta^{(S)}_{k}|^2 \{1+$$

$$+\frac{2}{3}\left(\frac{H_0}{4\,\pi\,M_{Pl}}\right)^2\left[1+\frac{\frac{3}{8}\,r\,(n_s-1)+2\,\frac{dn_s}{d\ln k}}{(n_s-1)^2}+\frac{2903}{40}\right]\Bigg\}$$

$$|\Delta^{(T)}_{k,eff}|^2 = |\Delta^{(T)}_{k}|^2\left\{1-\frac{1}{3}\left(\frac{H_0}{4\,\pi\,M_{Pl}}\right)^2\left[-1+\frac{1}{8}\,\frac{r}{n_s-1}+\frac{2903}{20}\right]\right\}.$$

The anomaly contribution $-\frac{2903}{20}=-145.15$ DOMINATES as long as the number of fermions less than 783.

The scalar curvature fluctuations $|\Delta^{(S)}_{k}|^2$ are ENHANCED and the tensor fluctuations $|\Delta^{(T)}_{k}|^2$ REDUCED.

However, $\left(\frac{H}{M_{Pl}}\right)^2\sim 10^{-9}$.

D. Boyanovsky, H. J. de Vega, N. G. Sanchez, Phys. Rev. D 72, 103006 (2005), astro-ph/0507596.