How WMAP Helps Constrain the Nature of Dark Energy

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Need For Dark "Energy"

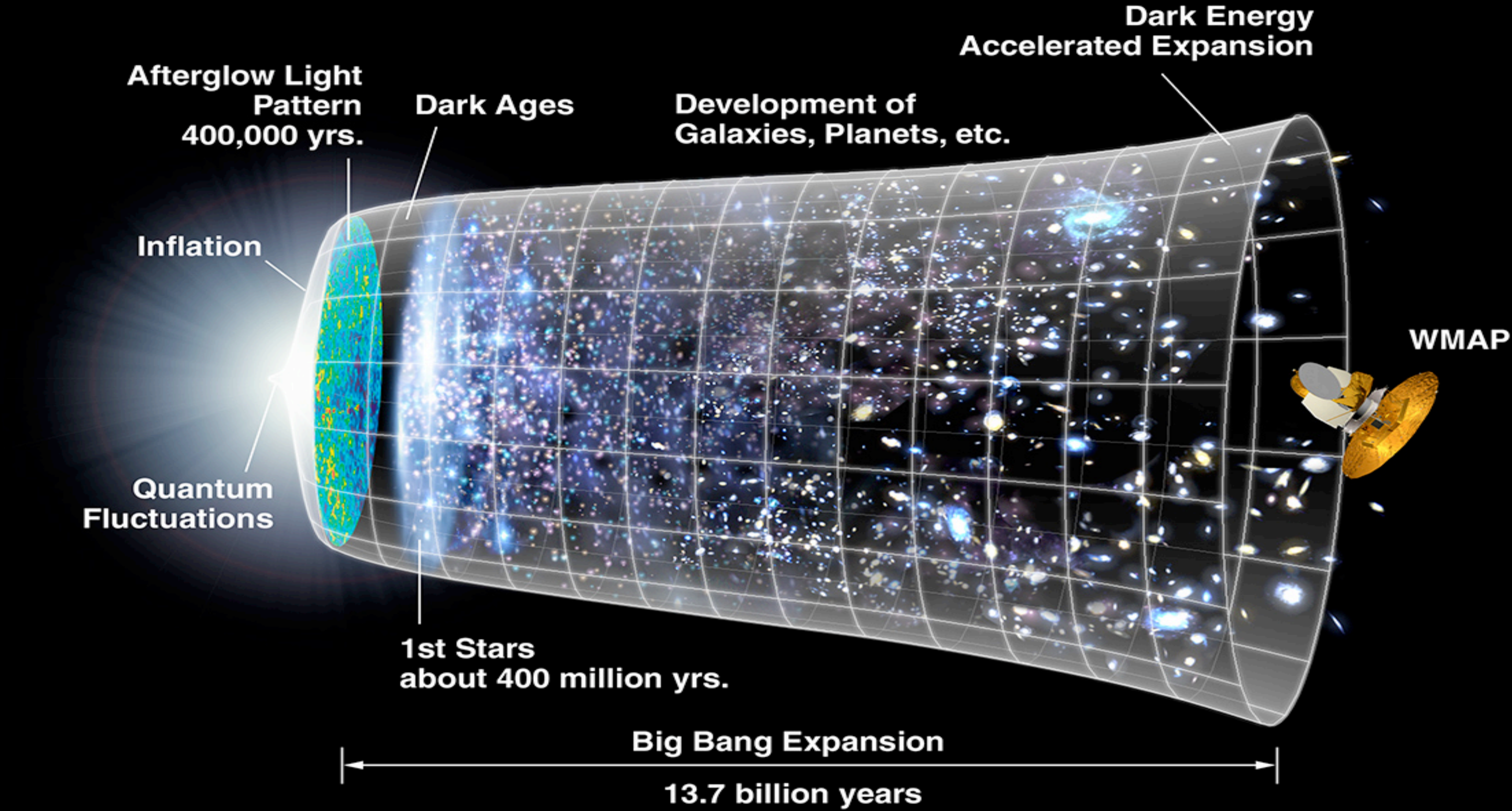
- First of all, DE does not even need to be an energy.
- At present, *anything* that can explain the observed

  (1) **Luminosity Distances** (Type Ia supernovae)

  (2) **Angular Diameter Distances** (BAO, CMB)

  simultaneously is qualified for being called "Dark Energy."
- The candidates in the literature include: (a) energy, (b) modified gravity, and (c) extreme inhomogeneity.
- Measurements of the (3) **growth of structure** break degeneracy. (The best data right now is the X-ray clusters.)

Measuring Distances, H(z) & Growth of Structure



NASA/WMAP Science Team

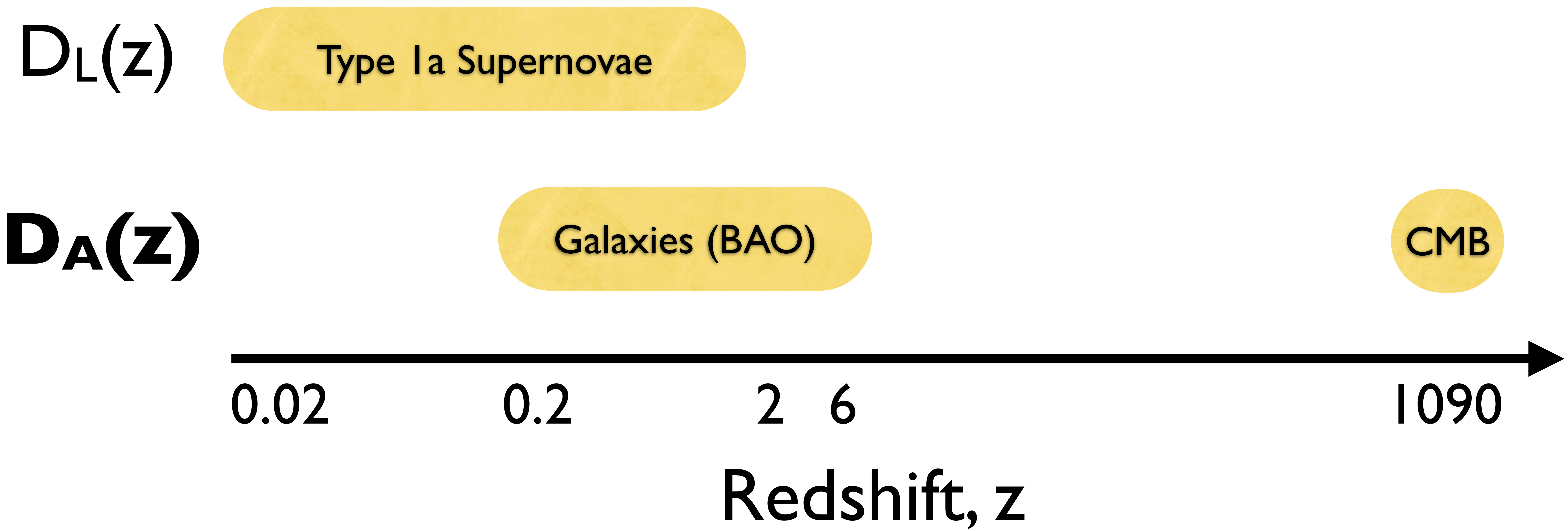
H(z): Current Knowledge

- $H^2(z) = H^2(0)[\Omega_r(1+z)^4+\Omega_m(1+z)^3+\Omega_k(1+z)^2+\Omega_{de}(1+z)^{3(1+w)}]$
  - (expansion rate) $H(0) = 70.5 \pm 1.3$ km/s/Mpc
  - (radiation) $\Omega_r = (8.4\pm0.3)\times10^{-5}$
  - (matter) $\Omega_m = 0.274\pm0.015$
  - (curvature) $\Omega_k < 0.008$ (95%CL)
  - (dark energy) $\Omega_{de} = 0.726\pm0.015$
  - (DE equation of state) $1+w = -0.006\pm0.068$

H(z) to Distances

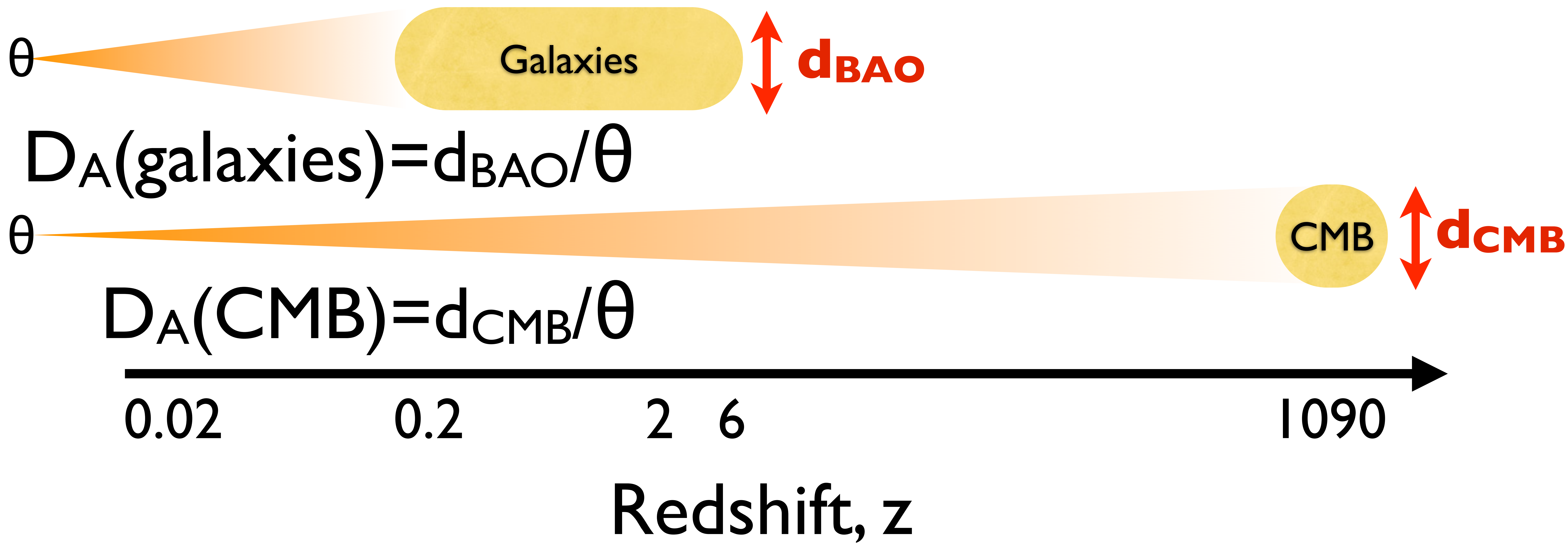
- Comoving Distance
  - $\chi(z) = c\int^{z}[dz'/H(z')]$
- Luminosity Distance
  - $D_L(z) = (1+z)\chi(z)[1-(k/6)\chi^2(z)/R^2+...]$
  - R=(curvature radius of the universe); k=(sign of curvature)
  - WMAP 5-year limit: $R>2\chi(\infty)$; justify the Taylor expansion
- Angular Diameter Distance
  - $D_A(z) = [\chi(z)/(1+z)][1-(k/6)\chi^2(z)/R^2+...]$

$$D_A(z) = (1+z)^{-2}\, D_L(z)$$



- To measure $D_A(z)$, we need to know the intrinsic size.
- What can we use as the *standard ruler*?

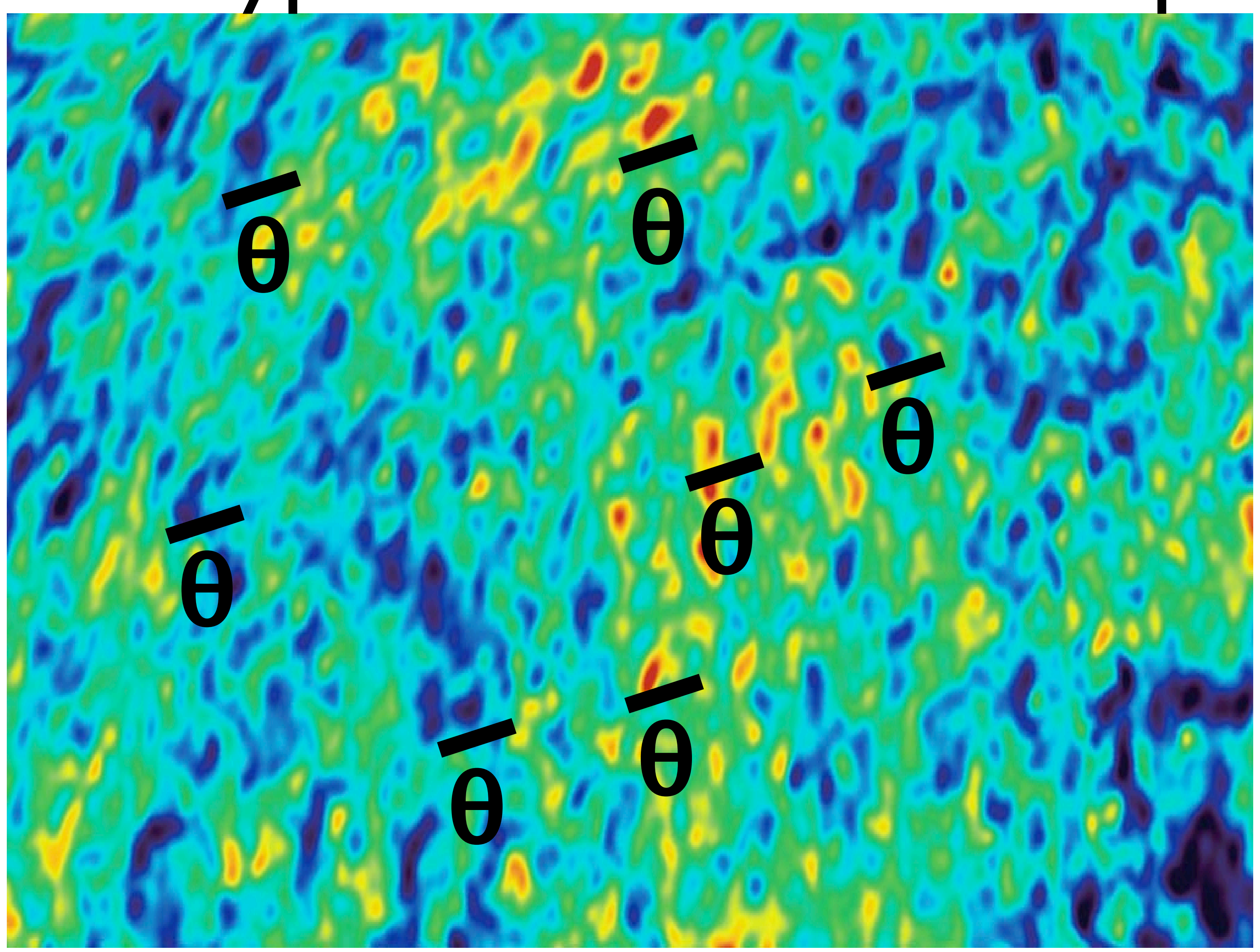
How Do We Measure $D_A(z)$?

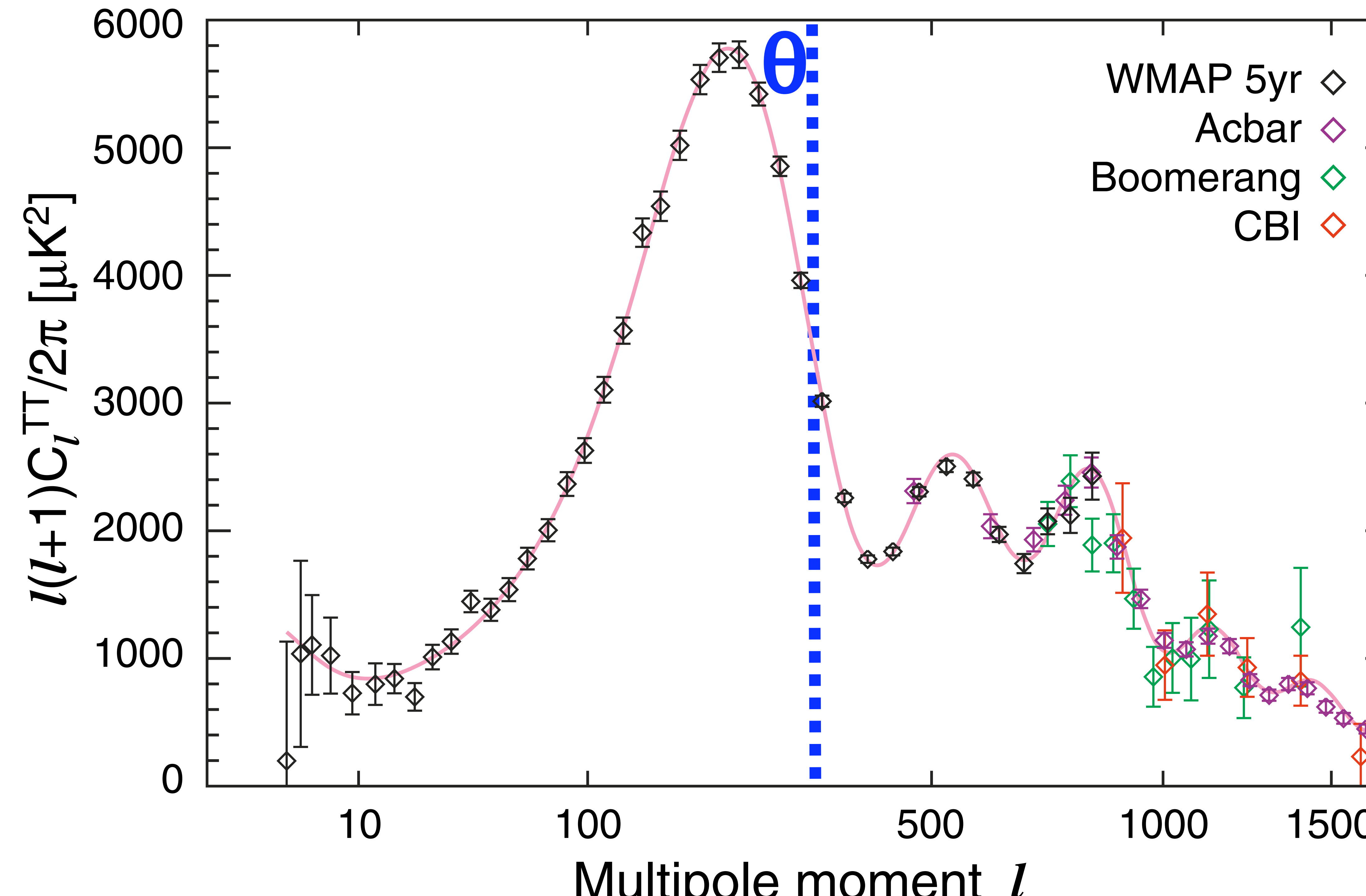


- If we know the intrinsic physical sizes, d, we can measure D_A. What determines d?

CMB as a Standard Ruler

θ~the typical size of hot/cold spots

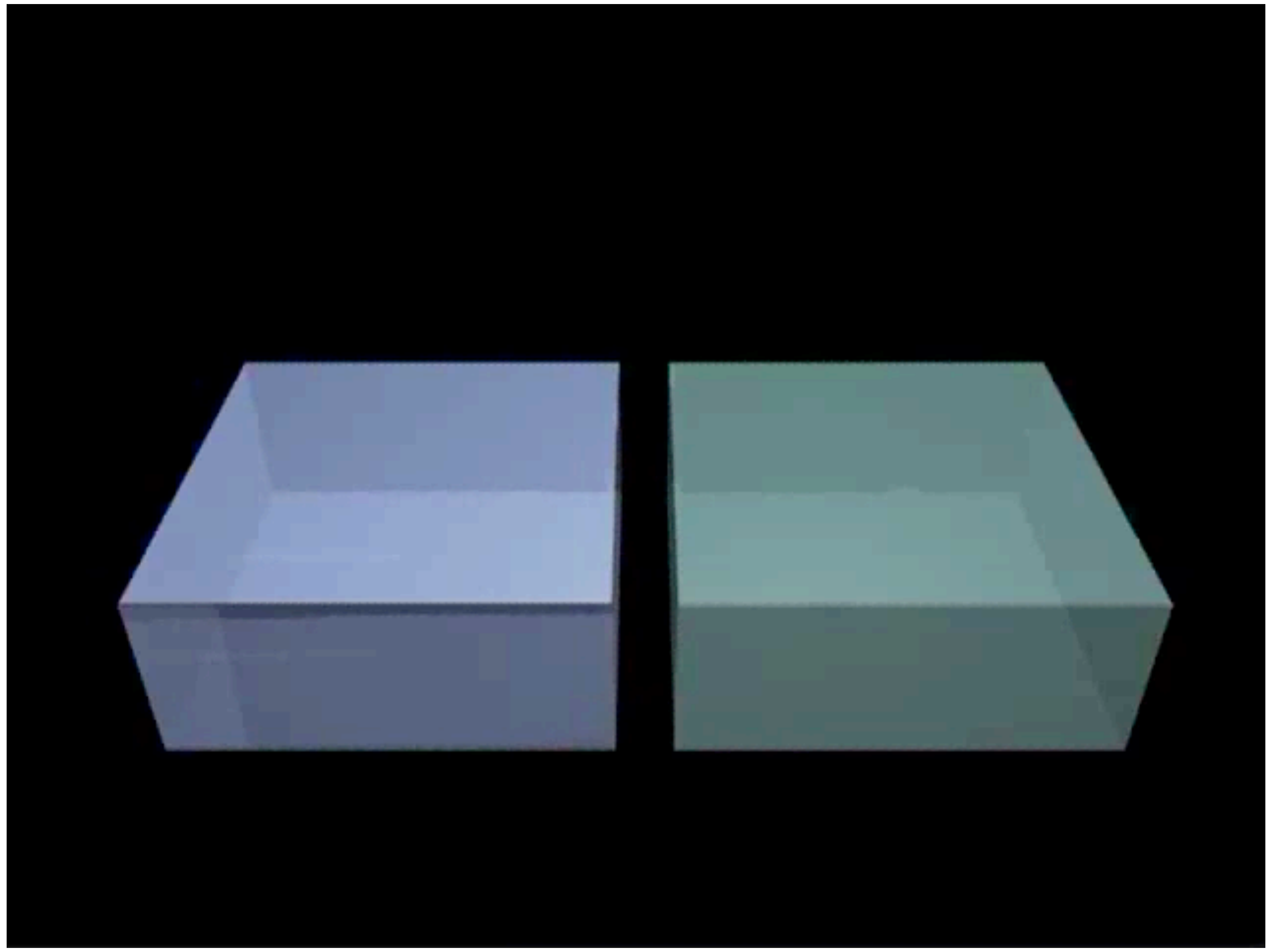


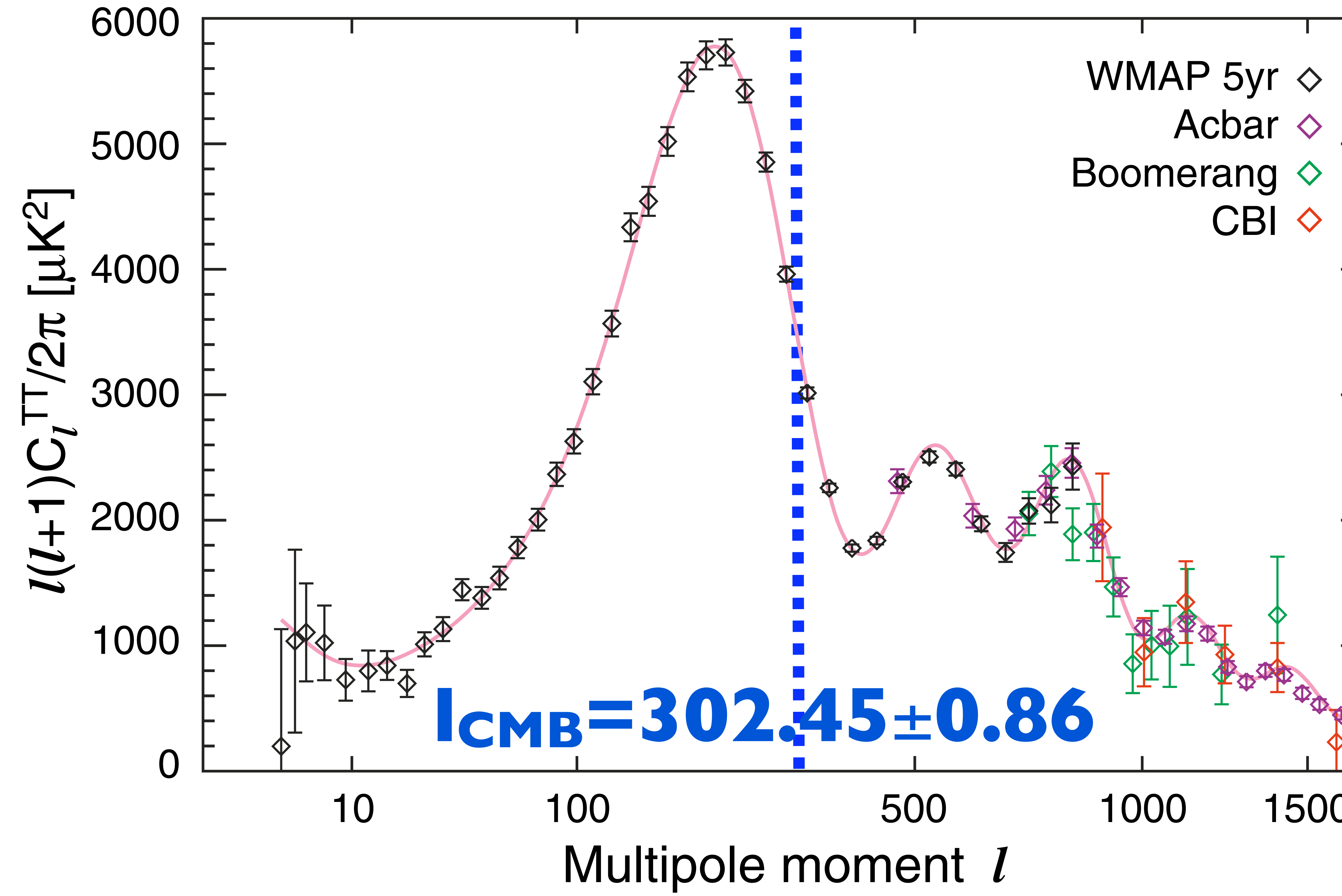


- The existence of typical spot size in image space yields oscillations in harmonic (Fourier) space. What determines the physical size of typical spots, **d_{CMB}**?

Sound Horizon

- The typical spot size, **d_{CMB}**, is determined by the **physical distance traveled by the sound wave** from the Big Bang to the decoupling of photons at z_{CMB}~1090 (t_{CMB}~380,000 years).
- The causal horizon (photon horizon) at t_{CMB} is given by
  - $d_H(t_{CMB}) = a(t_{CMB})$*Integrate[**c** dt/$a$(t), {t,0,$t_{CMB}$}].
- The **sound** horizon at t_{CMB} is given by
  - $d_s(t_{CMB}) = a(t_{CMB})$*Integrate[**$c_s$(t)** dt/$a$(t), {t,0,$t_{CMB}$}], where c_s(t) is the time-dependent **speed of sound of photon-baryon fluid**.

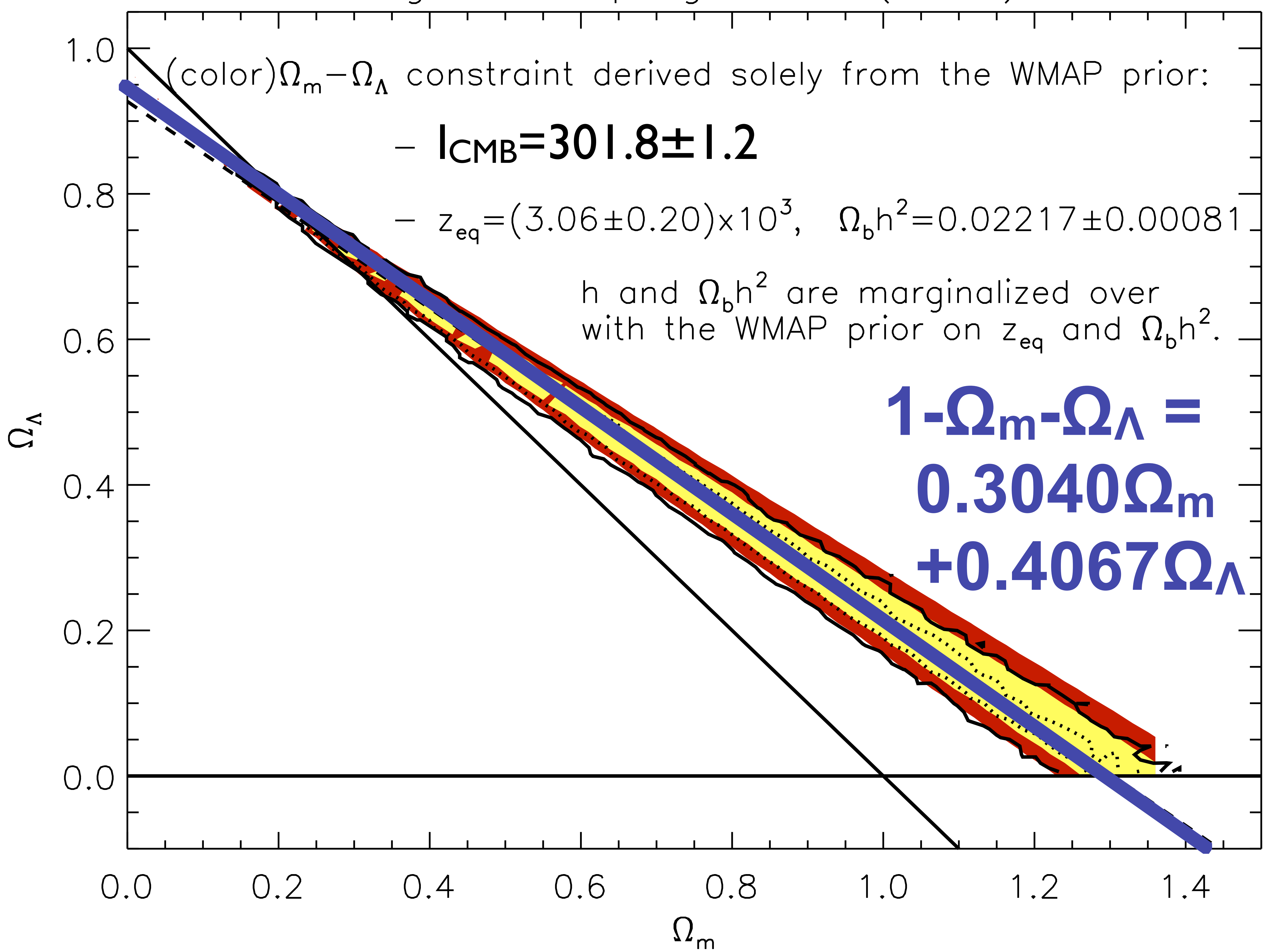




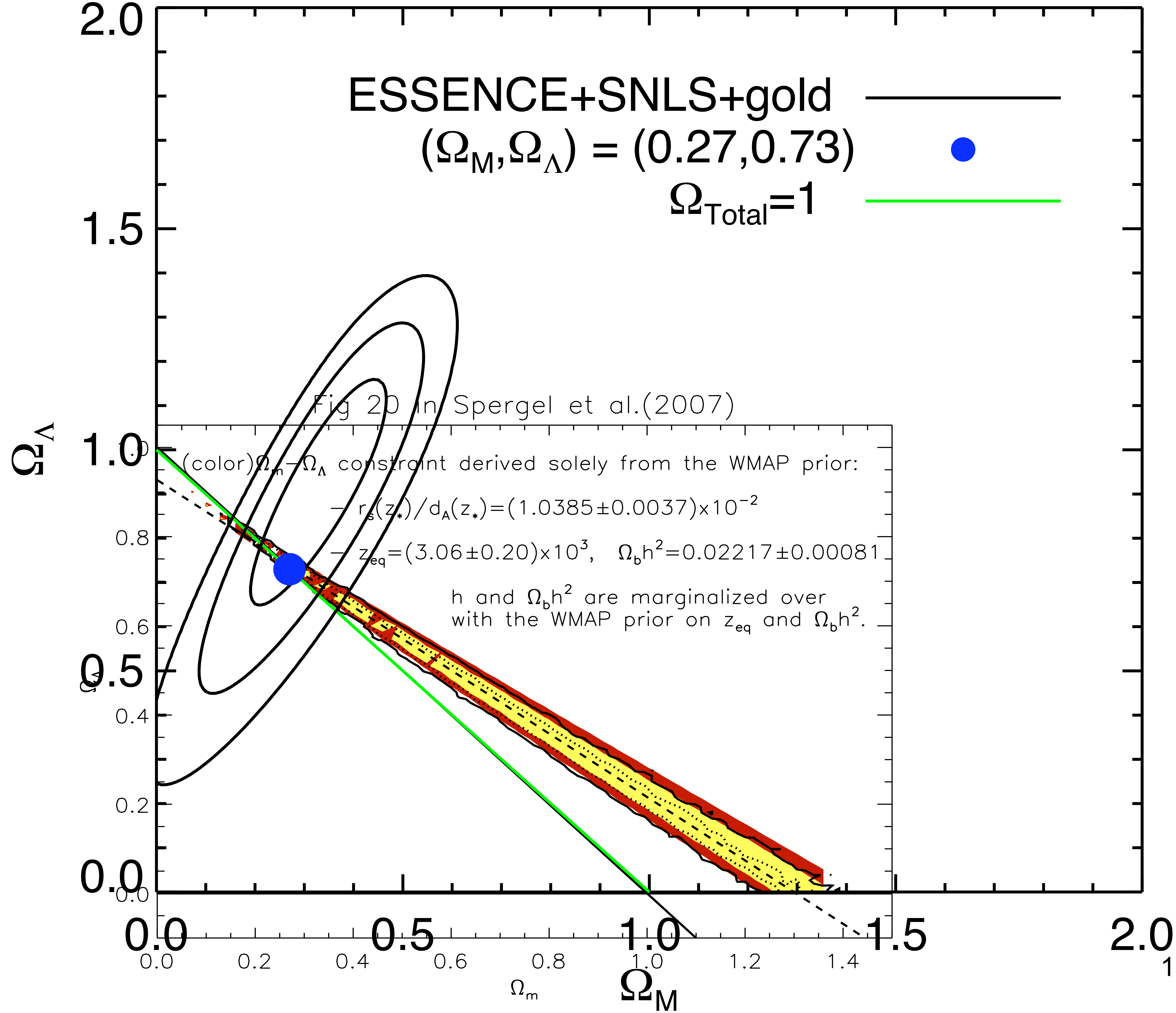
- The WMAP 5-year values:
  - $l_{CMB} = \pi/\theta = \pi D_A(z_{CMB})/d_s(z_{CMB}) = 302.45 \pm 0.86$
  - CMB data constrain the ratio, $\mathbf{D_A(z_{CMB})/d_s(z_{CMB})}$.
  - $r_s(z_{CMB}) = (1+z_{CMB})d_s(z_{CMB}) = 146.8 \pm 1.8$ Mpc (comoving)

What $D_A(z_{CMB})/d_s(z_{CMB})$ Gives You (3-year example)

Fig 20 in Spergel et al.(2007)

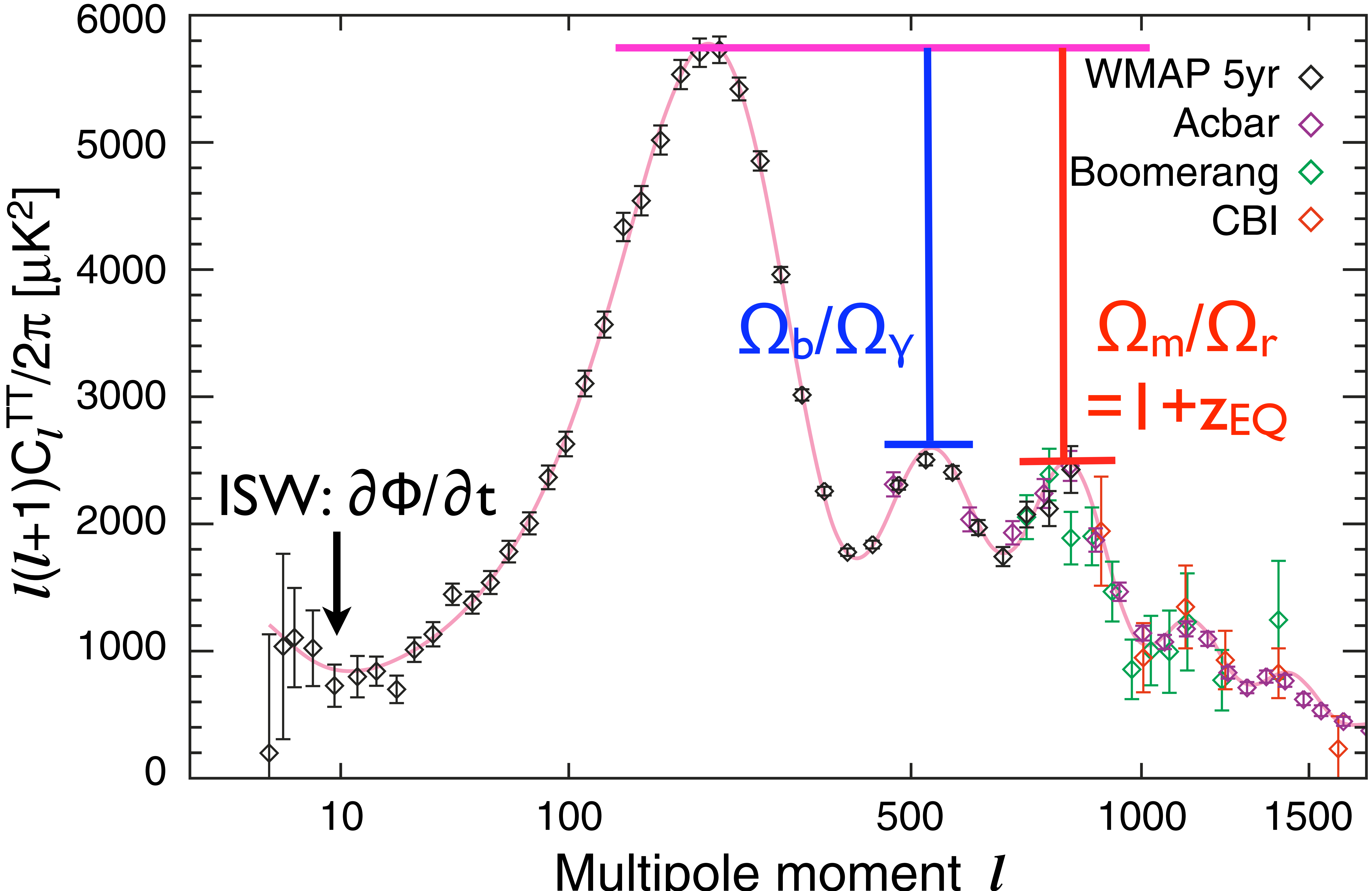


- Color: constraint from $l_{CMB}=\pi D_A(z_{CMB})/d_s(z_{CMB})$ with z_{EQ} & $\Omega_b h^2$.
- Black contours: Markov Chain from WMAP 3yr (Spergel et al. 2007)



ESSENCE+SNLS+gold
$(\Omega_M,\Omega_\Lambda) = (0.27,0.73)$
$\Omega_{Total}=1$
Fig 20 in Spergel et al.(2007)
(color) Ω_m–Ω_Λ constraint derived solely from the WMAP prior:
— $r_s(z_*)/d_A(z_*)=(1.0385\pm0.0037)\times10^{-2}$
— $z_{eq}=(3.06\pm0.20)\times10^3$, $\Omega_bh^2=0.02217\pm0.00081$
h and Ω_bh^2 are marginalized over with the WMAP prior on z_{eq} and Ω_bh^2.
Ω_Λ
Ω_M
Ω_m

Other Observables



- 1-to-2: baryon-to-photon; 1-to-3: matter-to-radiation ratio
- Low-l: Integrated Sachs Wolfe Effect

Dark Energy From Distance Information Alone

- We provide a set of "WMAP distance priors" for testing various dark energy models.

Ω_b/Ω_γ
- Redshift of decoupling, $z_*=1091.13$ (Err=0.93)
- Acoustic scale, $l_A=\pi d_A(z_*)/r_s(z_*)=302.45$ (Err=0.86)

Ω_m/Ω_r
- Shift parameter, $R=\mathrm{sqrt}(\Omega_m H_0^2) d_A(z_*)=1.721$(Err=0.019)
- Full covariance between these three quantities are also provided.

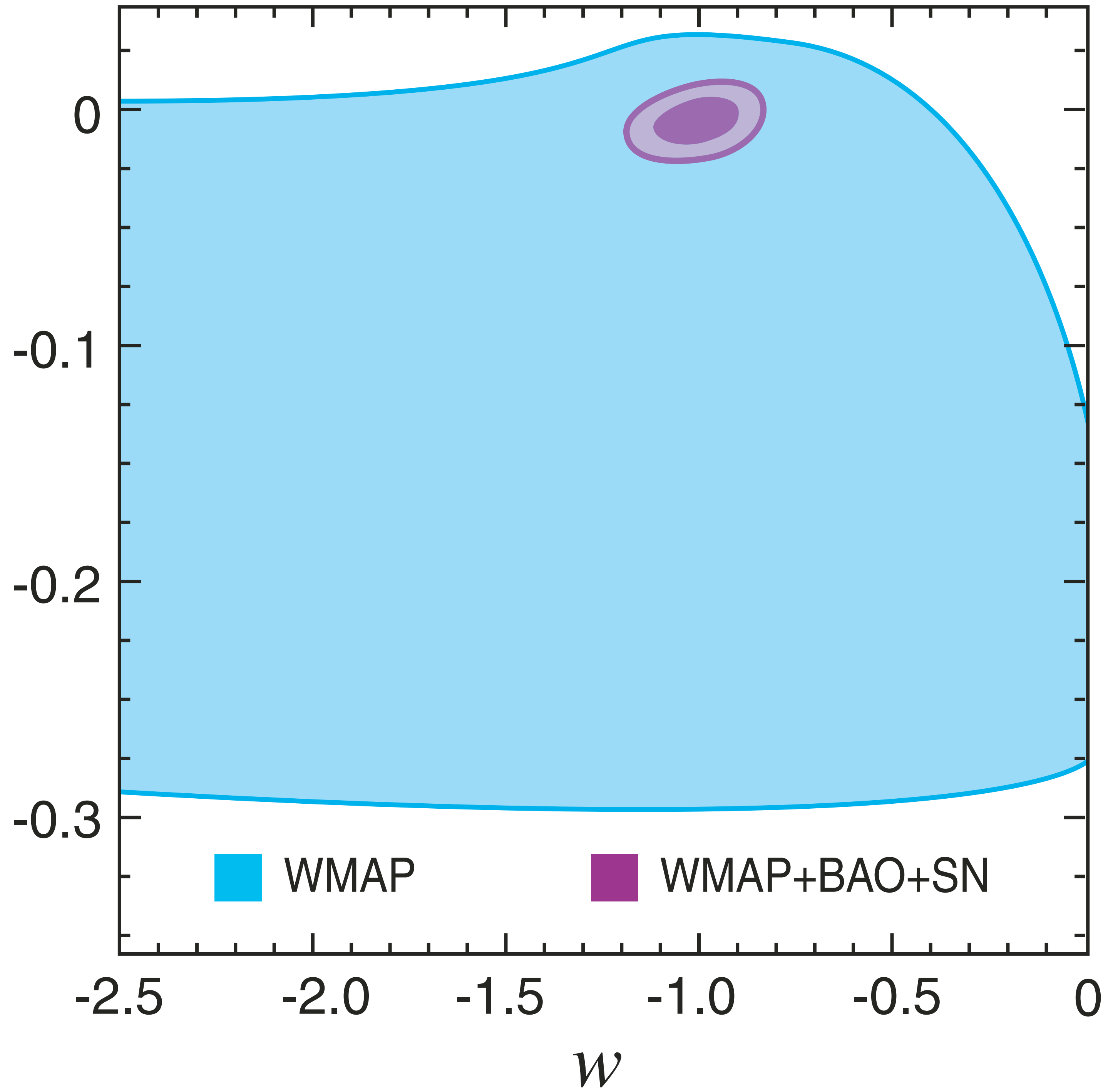
Inverse covariance matrix for the *WMAP* distance priors

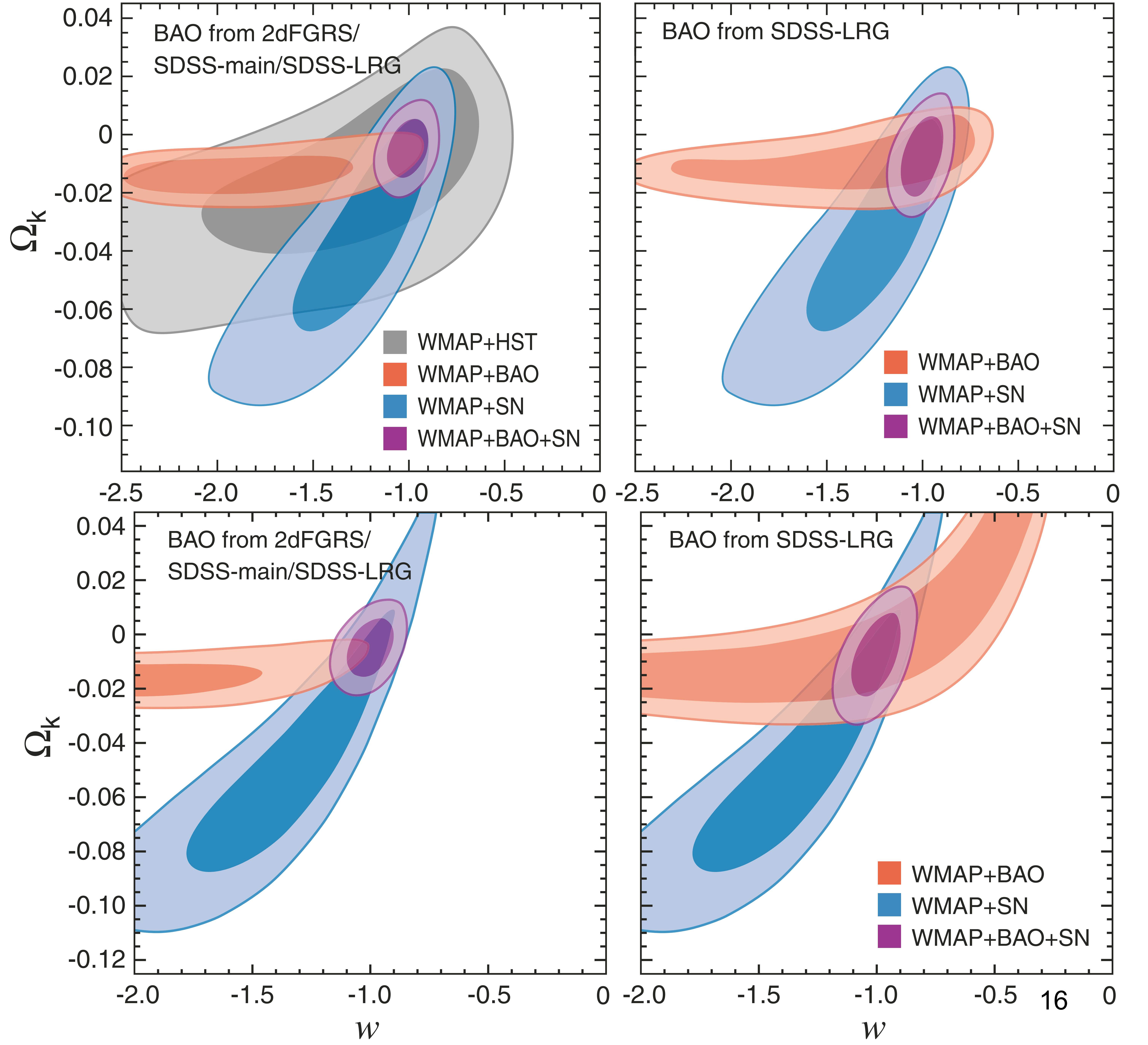
	$l_A(z_*)$	$R(z_*)$	z_*
$l_A(z_*)$	1.800	27.968	−1.103
$R(z_*)$		5667.577	−92.263
z_*			2.923

- WMAP 5-Year ML
  - z_*=1091.13
  - l_A=302.45
  - R=1.721
  - $100\Omega_b h^2$=2.2765

Inverse covariance matrix for the extended *WMAP* distance priors. The maximum likelihood value of $\Omega_b h^2$ is $100\Omega_b h^2 = 2.2765$.

	$l_A(z_*)$	$R(z_*)$	z_*	$100\Omega_b h^2$
$l_A(z_*)$	31.001	−5015.642	183.903	2337.977
$R(z_*)$		876807.166	−32046.750	−403818.837
z_*			1175.054	14812.579
$100\Omega_b h^2$				187191.186

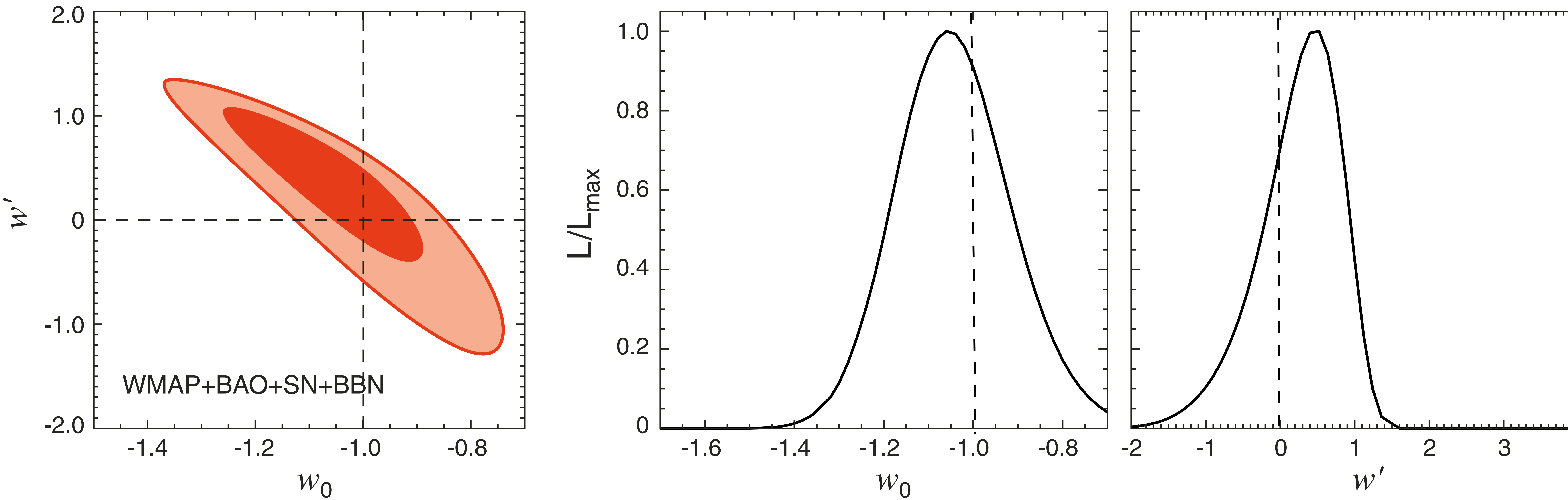




- Top
  - Full WMAP Data
- Bottom
  - WMAP Distance Priors

Dark Energy EOS: $w(z)=w_0+w'z/(1+z)$

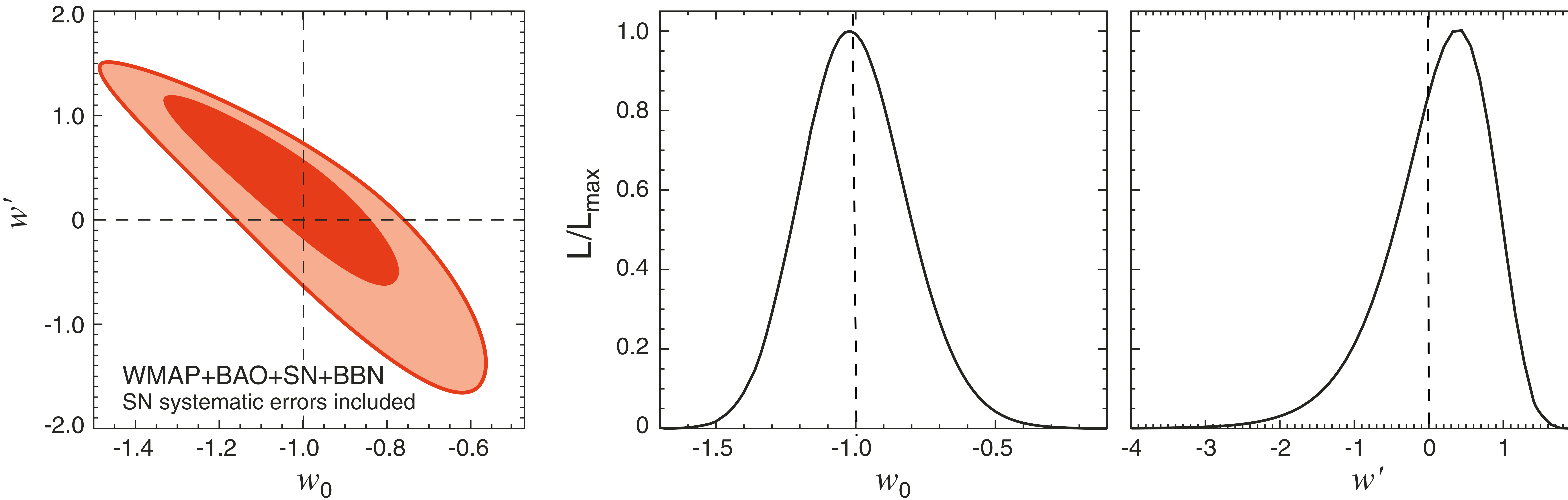
WMAP5+BAO+SN



- Dark energy is pretty consistent with cosmological constant: $w_0=-1.04\pm0.13$ & $w'=0.24\pm0.55$ (68%CL)

Dark Energy EOS: Including Sys. Err. in SN 1a

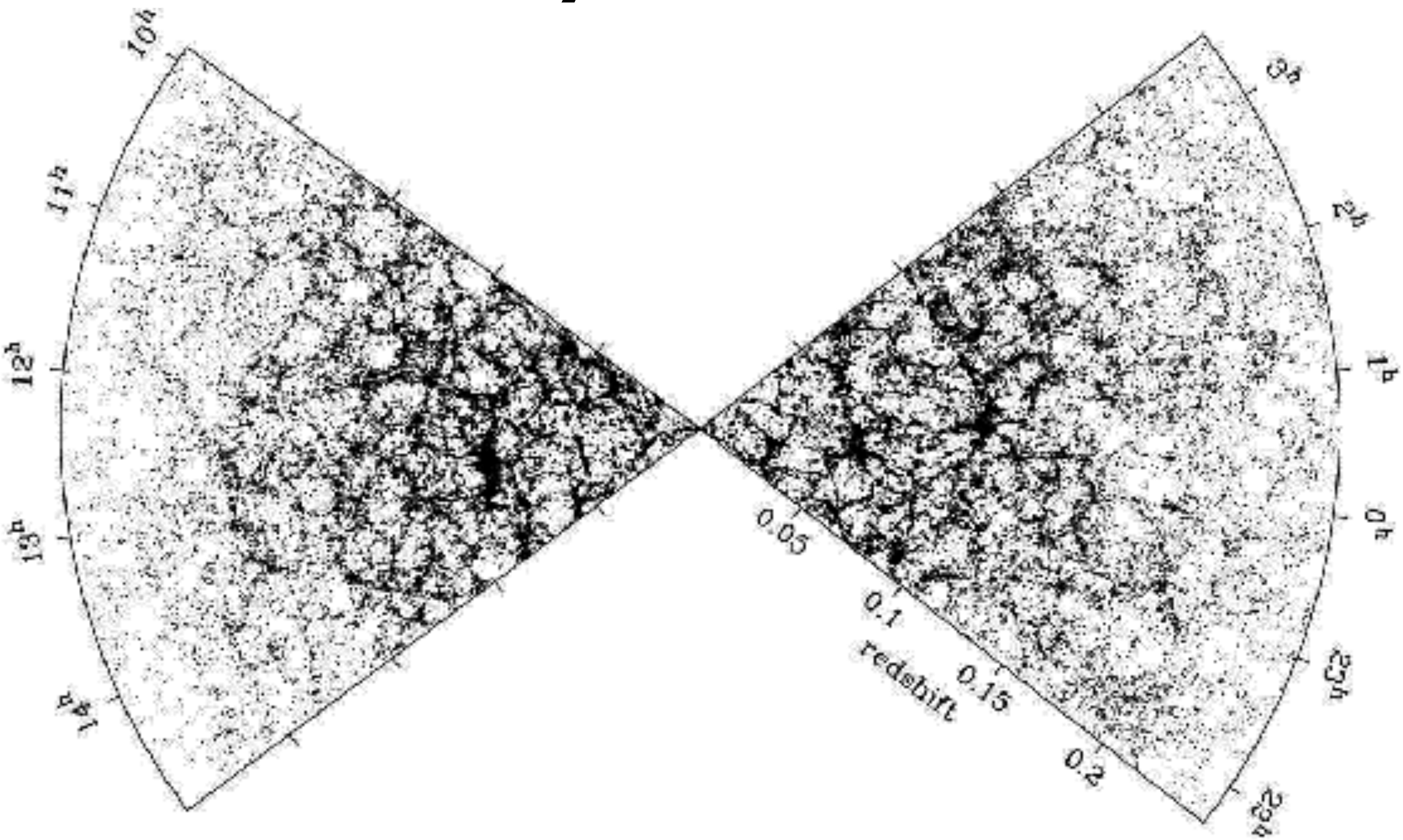
WMAP5+BAO+SN



- Dark energy is pretty consistent with cosmological constant: w_0=–1.00±0.19 & w'=0.11±0.70 (68%CL)

BAO in Galaxy Distribution

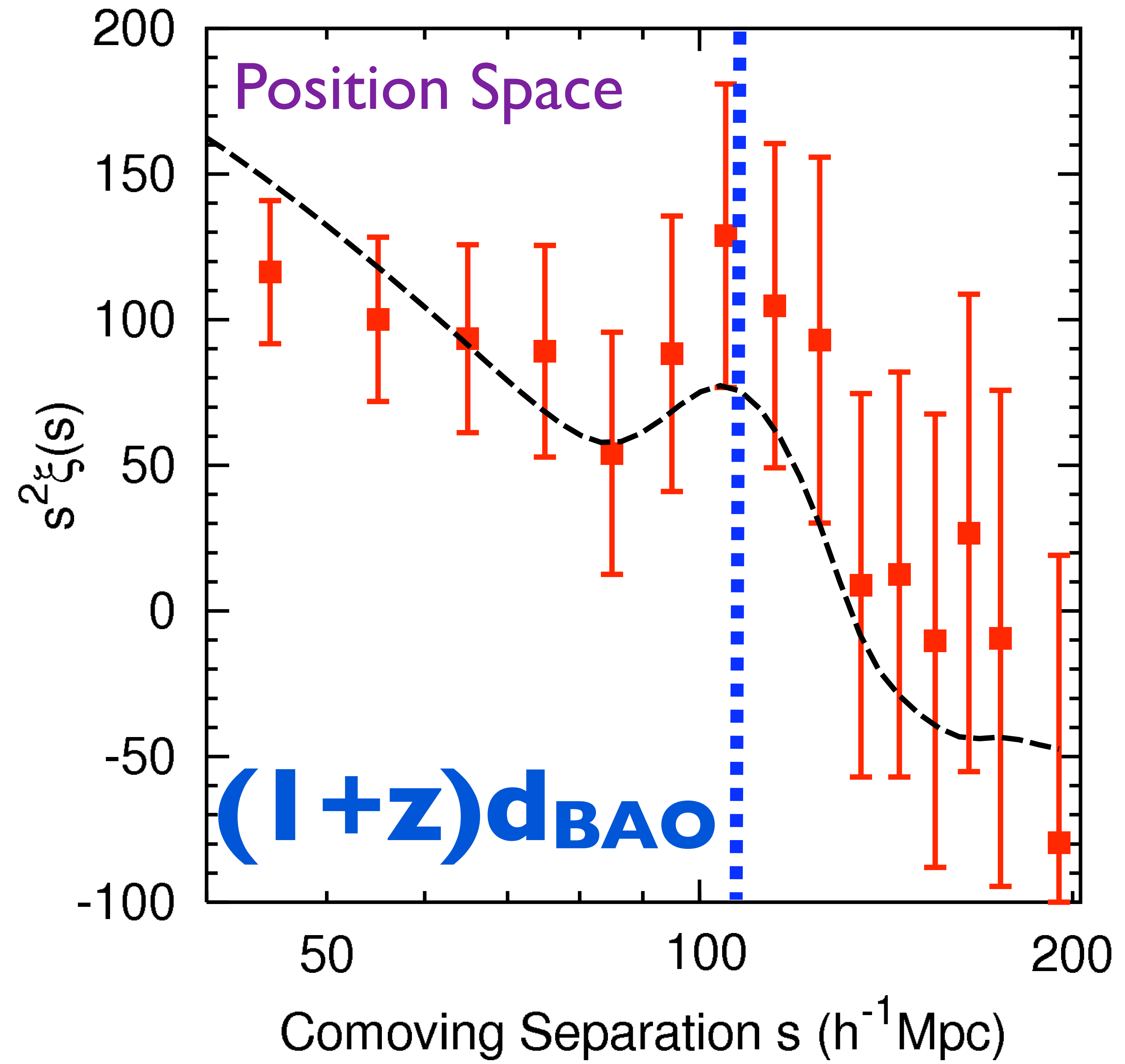
2dFGRS

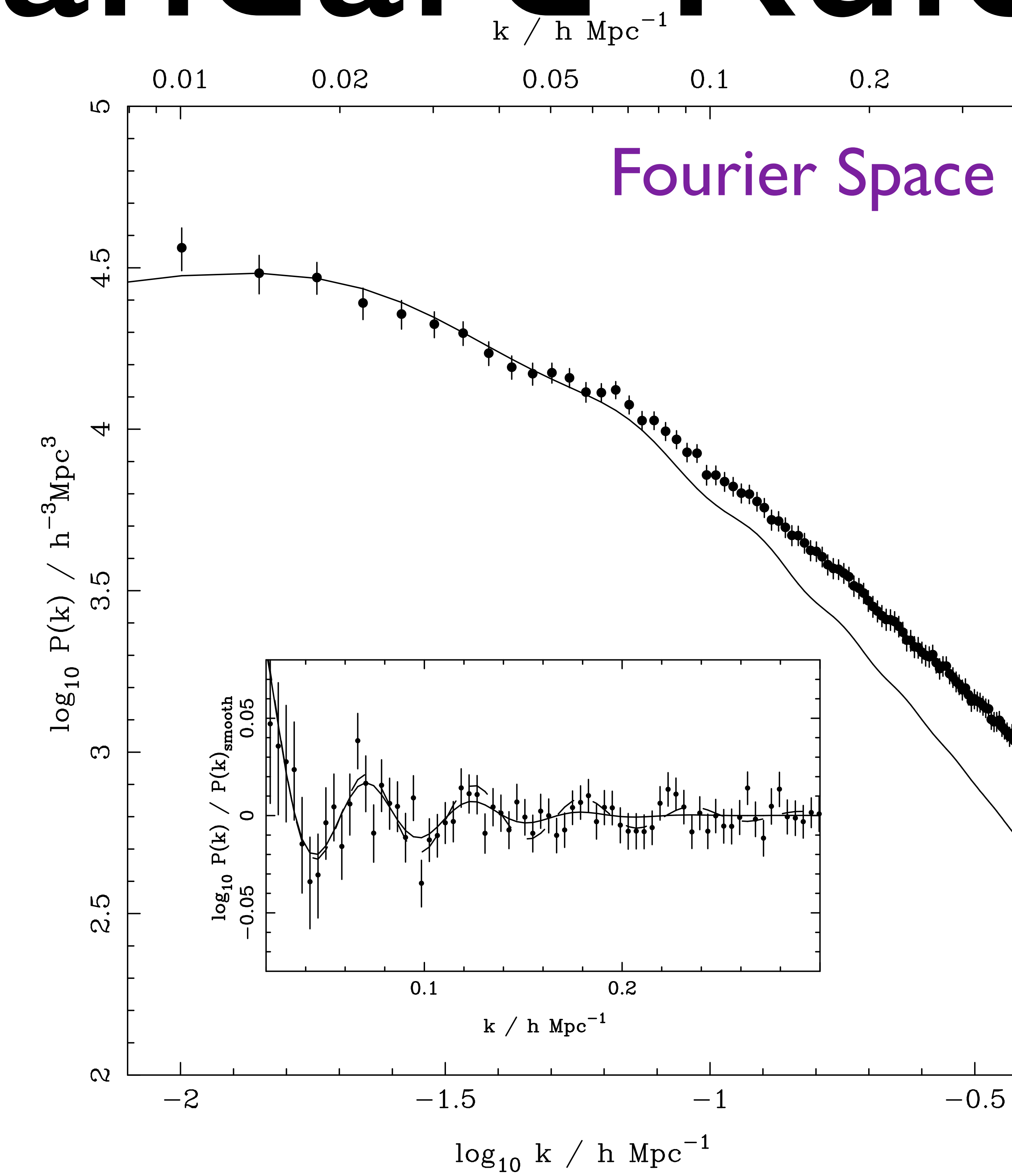


- The same acoustic oscillations should be hidden in this galaxy distribution...

BAO as a Standard Ruler

Okumura et al. (2007)





Percival et al. (2006)

- The existence of a localized clustering scale in the 2-point function yields oscillations in Fourier space. What determines the physical size of clustering, d_{BAO}?

Sound Horizon Again

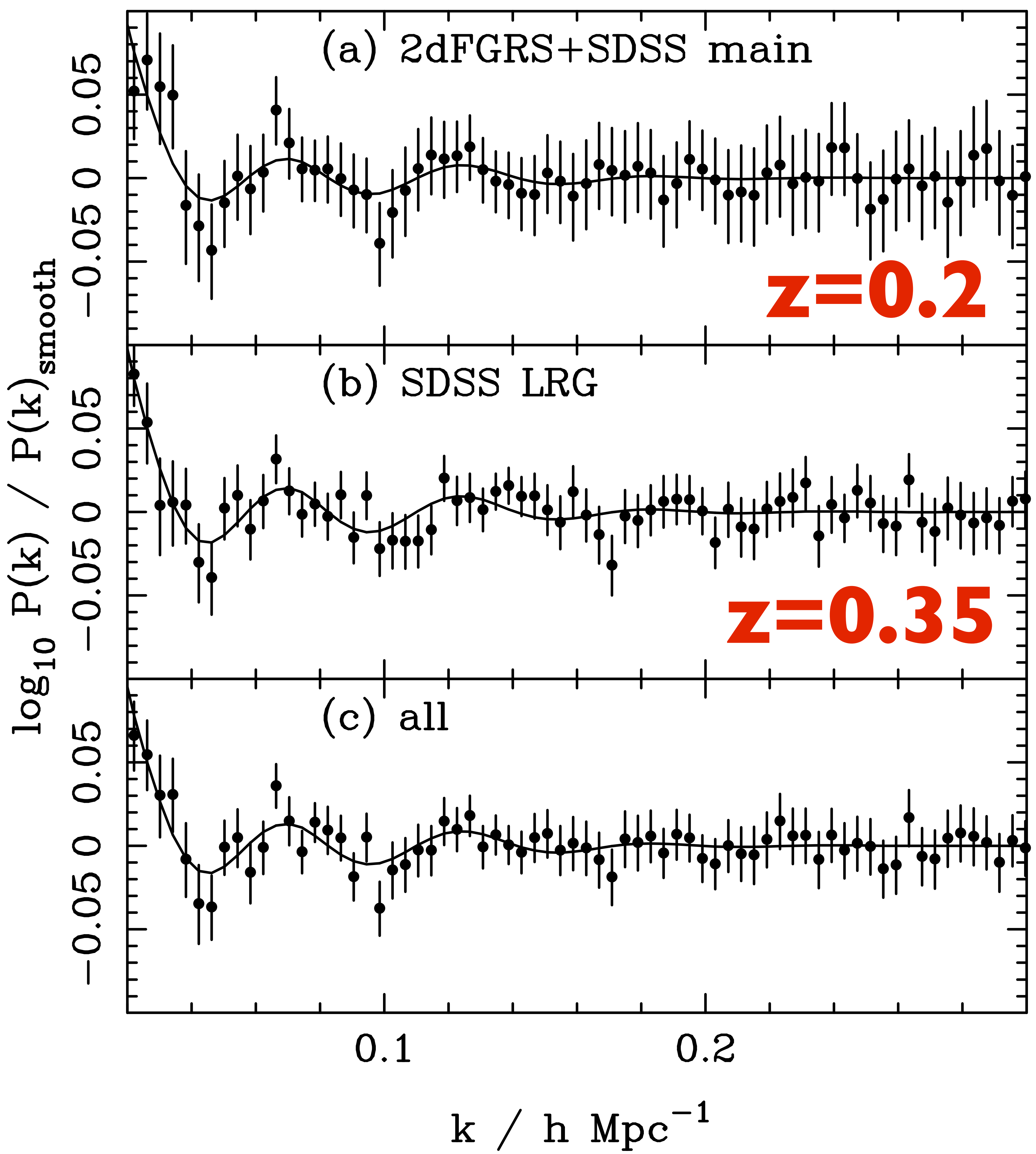
- The clustering scale, **d_{BAO}**, is given by the physical distance traveled by the sound wave from the Big Bang to the **decoupling of baryons** at z_{BAO}=1020.5±1.6 (c.f., z_{CMB}=1091±1).
- The baryons decoupled slightly later than CMB.
  - By the way, this is not universal in cosmology, but *accidentally* happens to be the case for our Universe.
  - If $3\rho_{baryon}/(4\rho_{photon}) = 0.64(\Omega_b h^2/0.022)(1090/(1+z_{CMB}))$ is greater than unity, $z_{BAO}>z_{CMB}$. Since our Universe happens to have $\Omega_b h^2=0.022$, $z_{BAO}<z_{CMB}$. (ie, $d_{BAO}>d_{CMB}$)

Standard Rulers in CMB & Matter

	Quantity	Eq.	5-year WMAP
CMB	z_*	(66)	1090.51 ± 0.95
CMB	$r_s(z_*)$	(6)	146.8 ± 1.8 Mpc
Matter	z_d	(3)	1020.5 ± 1.6
Matter	$r_s(z_d)$	(6)	153.3 ± 2.0 Mpc

- For flat LCDM, but very similar results for $w \neq -1$ and curvature$\neq 0$!

BAO Measurements



- 2dFGRS and SDSS main samples at z=0.2
- SDSS LRG samples at z=0.35
- These measurements constrain the ratio, $\mathbf{D_A(z)/d_s(z_{BAO})}$.

Percival et al. (2007)

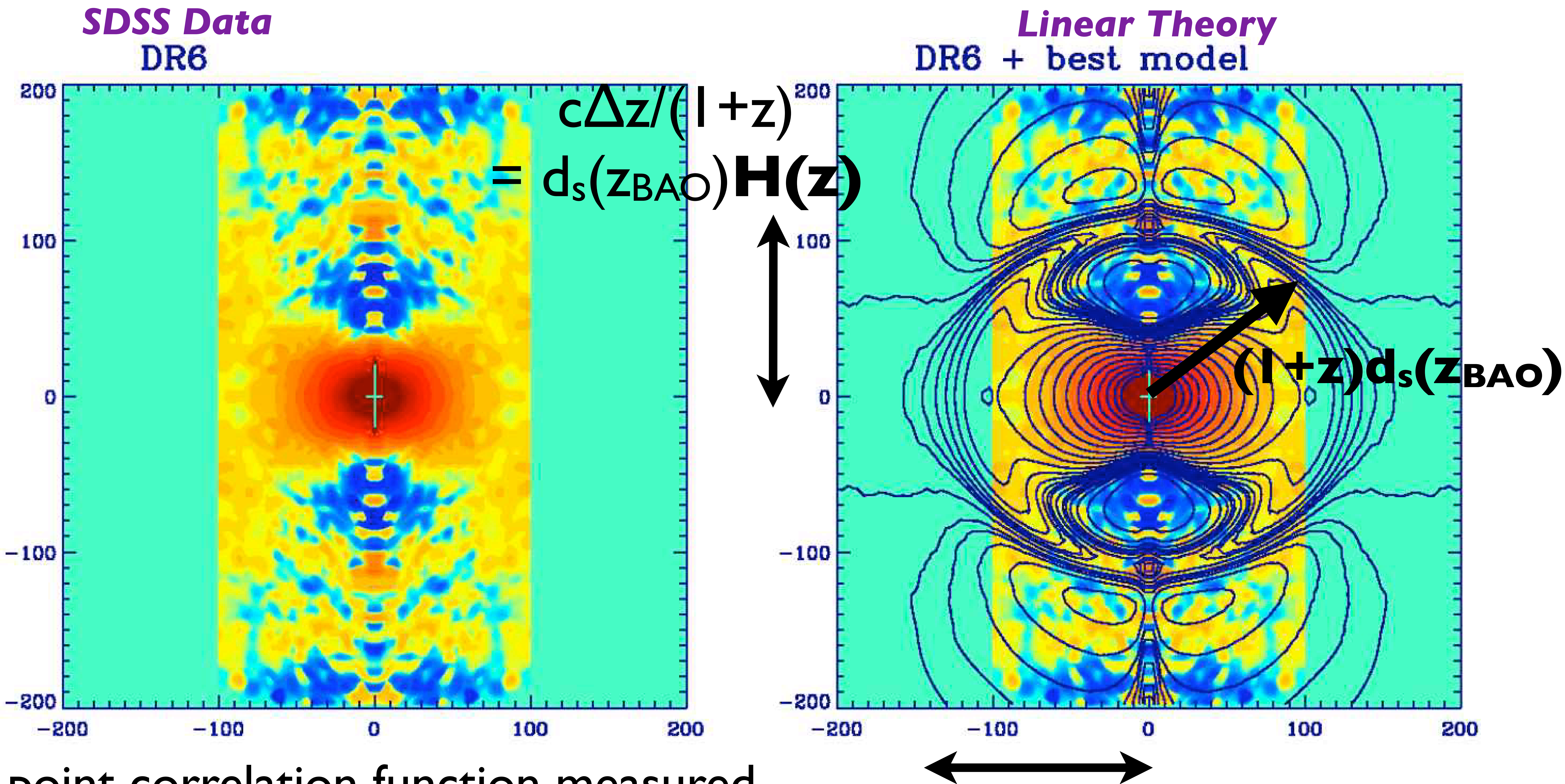
Not Just $D_A(z)$...

- A really nice thing about BAO at a given redshift is that it can be used to measure not only $D_A(z)$, but also the expansion rate, H(z), directly, at **that** redshift.
  - BAO perpendicular to l.o.s

    => $D_A(z) = d_s(z_{BAO})/\theta$
  - BAO parallel to l.o.s

    => $\mathbf{H(z) = c\Delta z/[(1+z)d_s(z_{BAO})]}$

Transverse=$D_A(z)$; Radial=$H(z)$

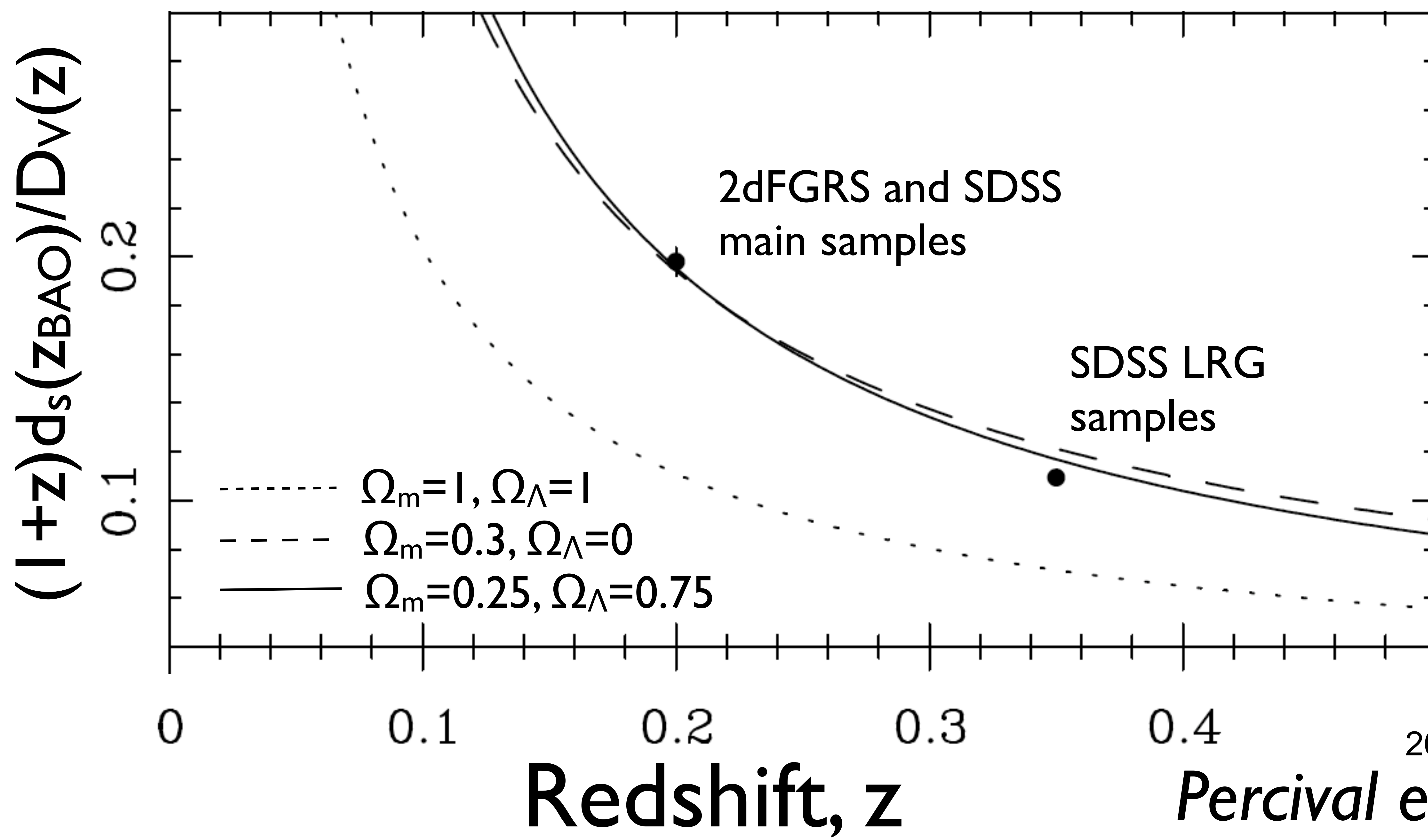


$\theta = d_s(z_{BAO})/\mathbf{D_A(z)}$

Two-point correlation function measured from the SDSS Luminous Red Galaxies (Gaztanaga, Cabre & Hui 2008)

$$D_V(z) = \{(1+z)^2 D_A^2(z)[cz/H(z)]\}^{1/3}$$

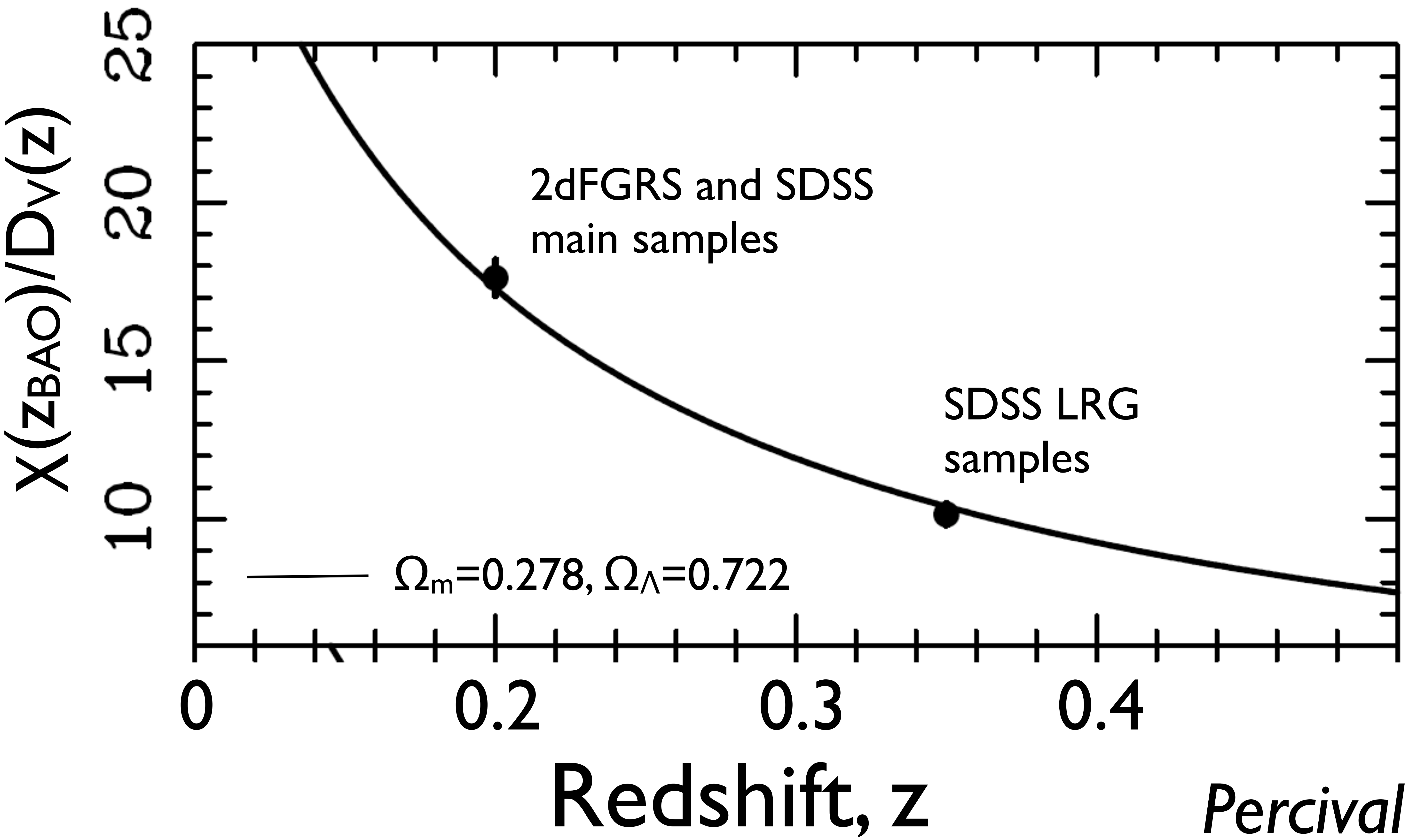
Since the current data are not good enough to constrain $D_A(z)$ and $H(z)$ separately, a combination distance, **$D_V(z)$**, has been constrained.



Percival et al. (2007)

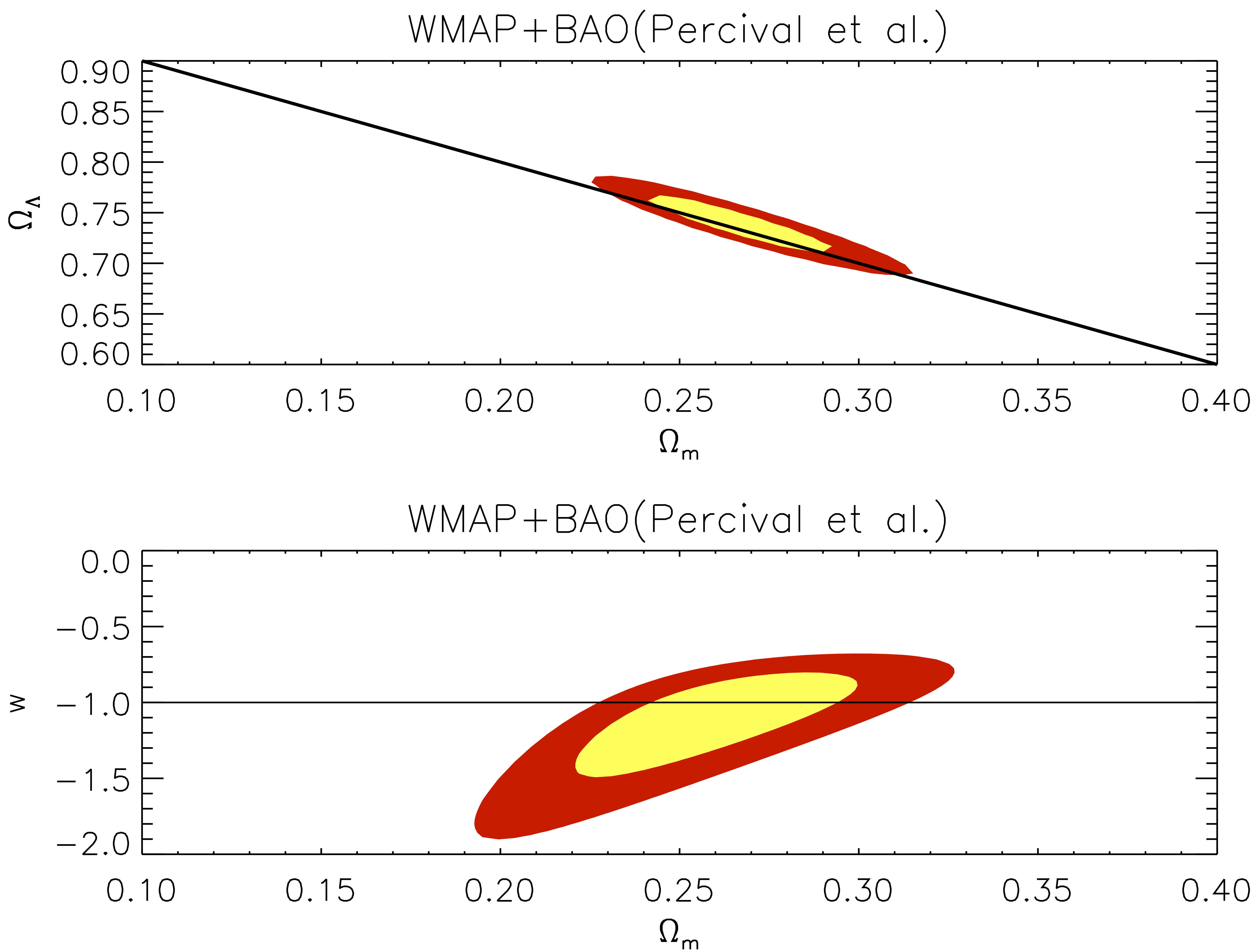
The Latest Data from SDSS DR7

z=0.2 and z=0.35 data are now more consistent with the best-fitting ΛCDM model.



Percival et al. (2009)

CMB + BAO => Curvature

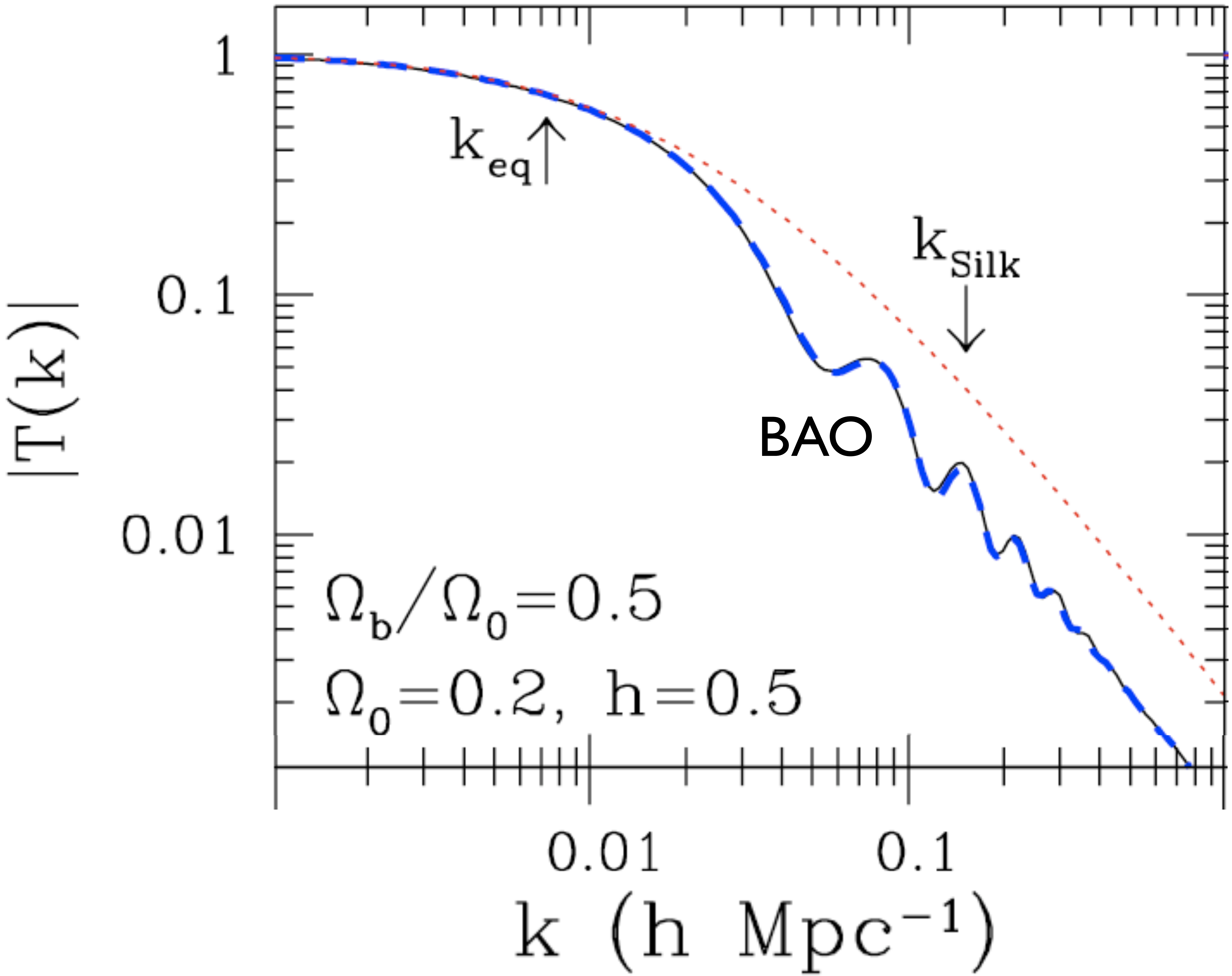


- Both CMB and BAO are **absolute** distance indicators.
- Type Ia supernovae only measure relative distances.
- CMB+BAO is the winner for measuring spatial curvature.

Beyond BAO

- BAOs capture only a **fraction** of the information contained in the galaxy power spectrum!
- BAOs use the sound horizon size at z~1020 as the standard ruler.
- However, there are other standard rulers:
  - Horizon size at the matter-radiation equality epoch (z~3200)
  - Silk damping scale

Eisenstein & Hu (1998)



...and, these are all well known

- Cosmologists have been measuring k_{eq} over the last three decades.
- This was usually called the "Shape Parameter," denoted as Γ.
- Γ is proportional to k_{eq}/h, and:
    - The effect of the Silk damping is contained in the constant of proportionality.
    - Easier to measure than BAOs: the signal is much stronger.

WMAP & Standard Ruler

- **With WMAP 5-year data only,** the scales of the standard rulers have been determined accurately.
- Even when $w \neq -1$, $\Omega_k \neq 0$,

1.3%
  - $d_s(z_{BAO}) = 153.4^{+1.9}_{-2.0}$ Mpc (z_{BAO}=1019.8 ± 1.5)

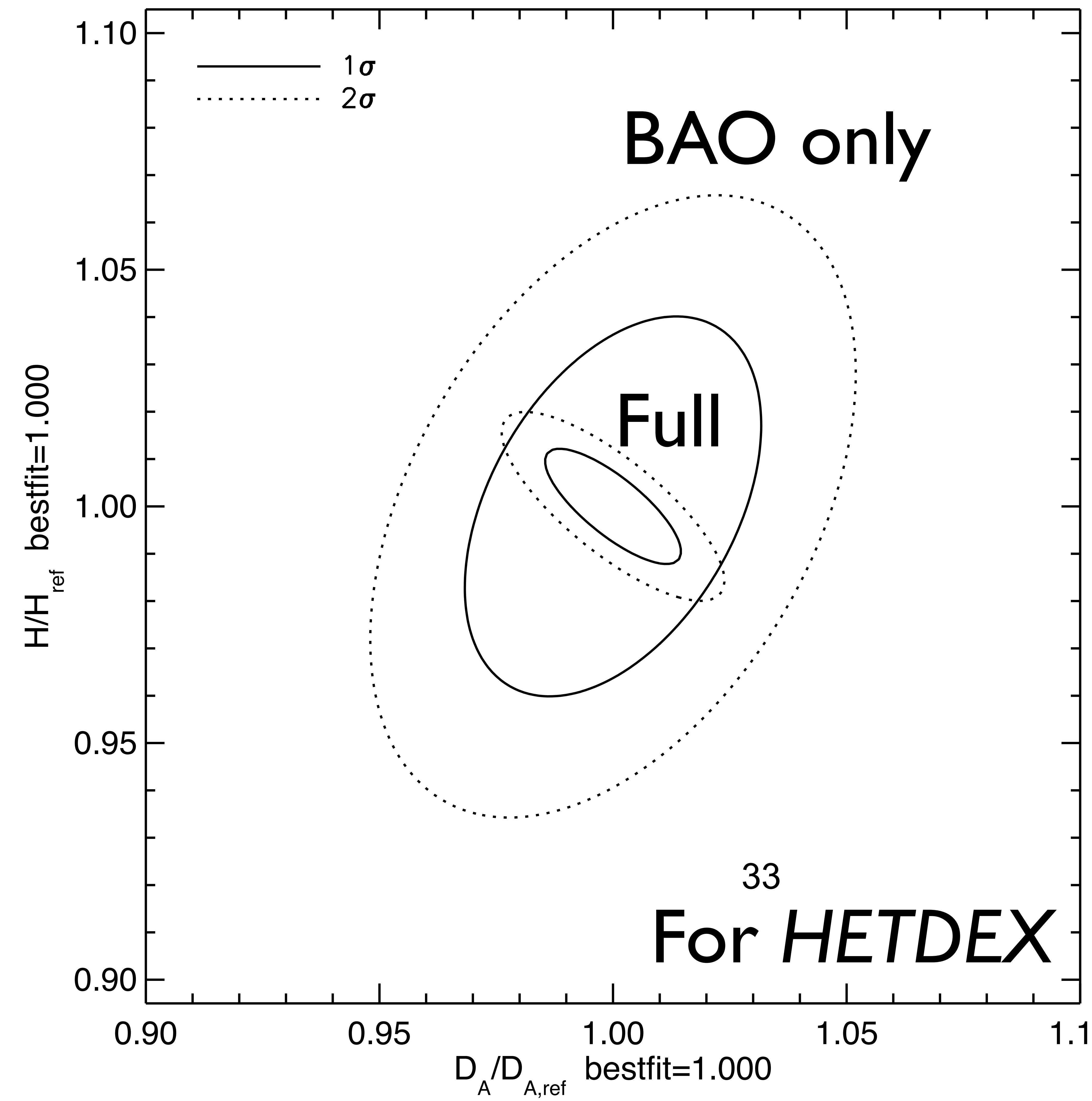
4.6%
  - $k_{eq}=(0.975^{+0.044}_{-0.045})\times10^{-2}$ Mpc^{-1} ($z_{eq}=3198^{+145}_{-146}$)

2.3%
  - $k_{silk}=(8.83 \pm 0.20)\times10^{-2}$ Mpc^{-1}

With Planck, they will be determined to higher precision.

BAO vs Full Modeling

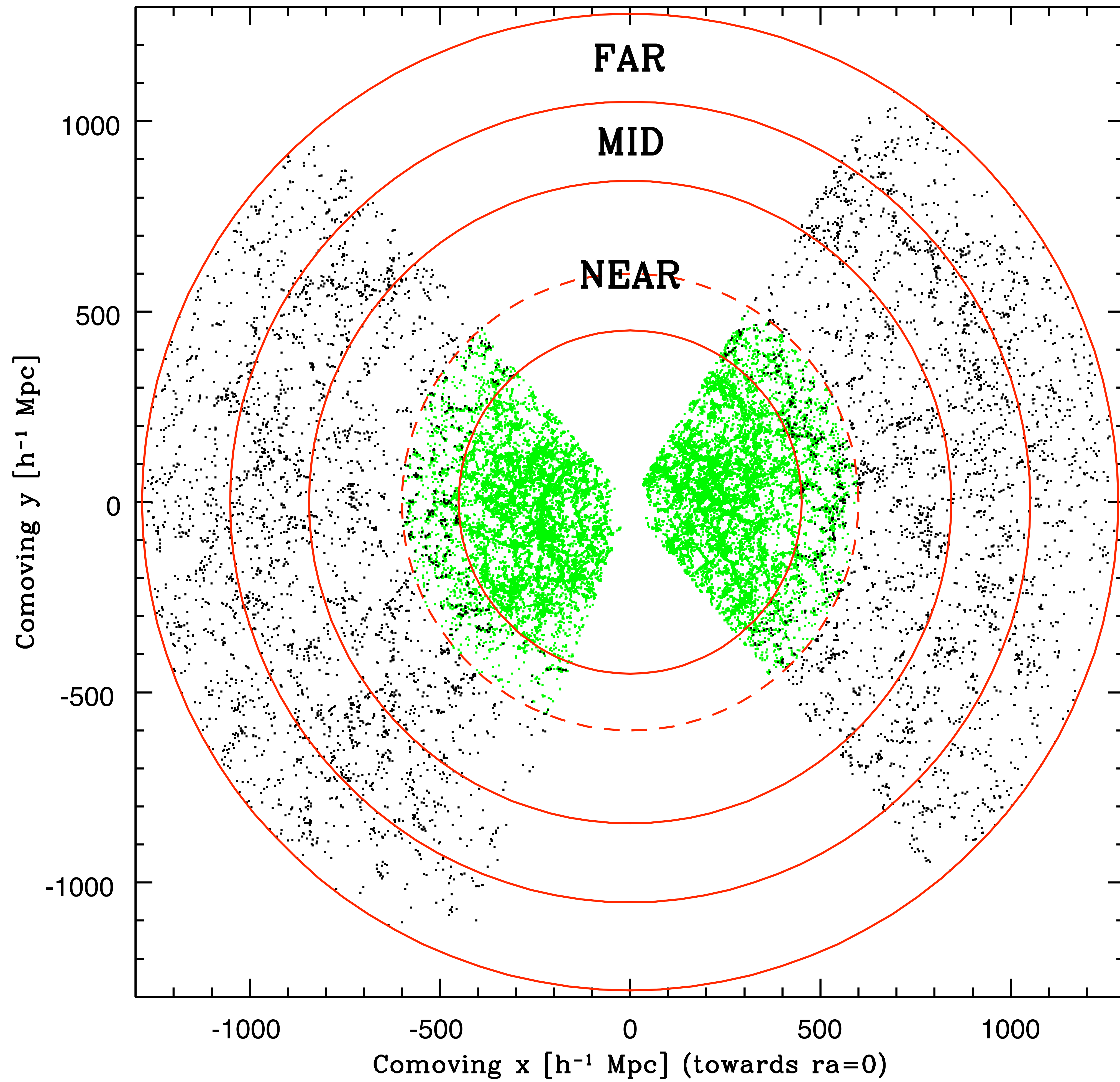
- Full modeling improves upon the determinations of D_A & H by more than a factor of two.
- On the D_A-H plane, the size of the ellipse shrinks by more than a factor of four.

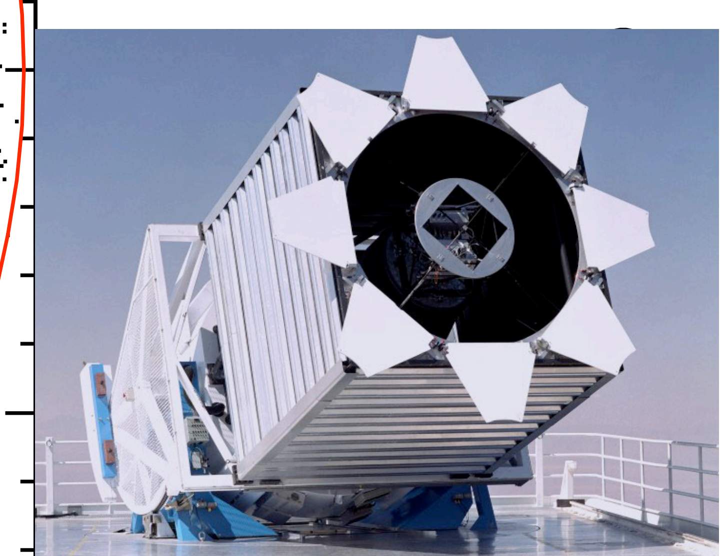


Why Not "GPS," Instead of "BAO"?

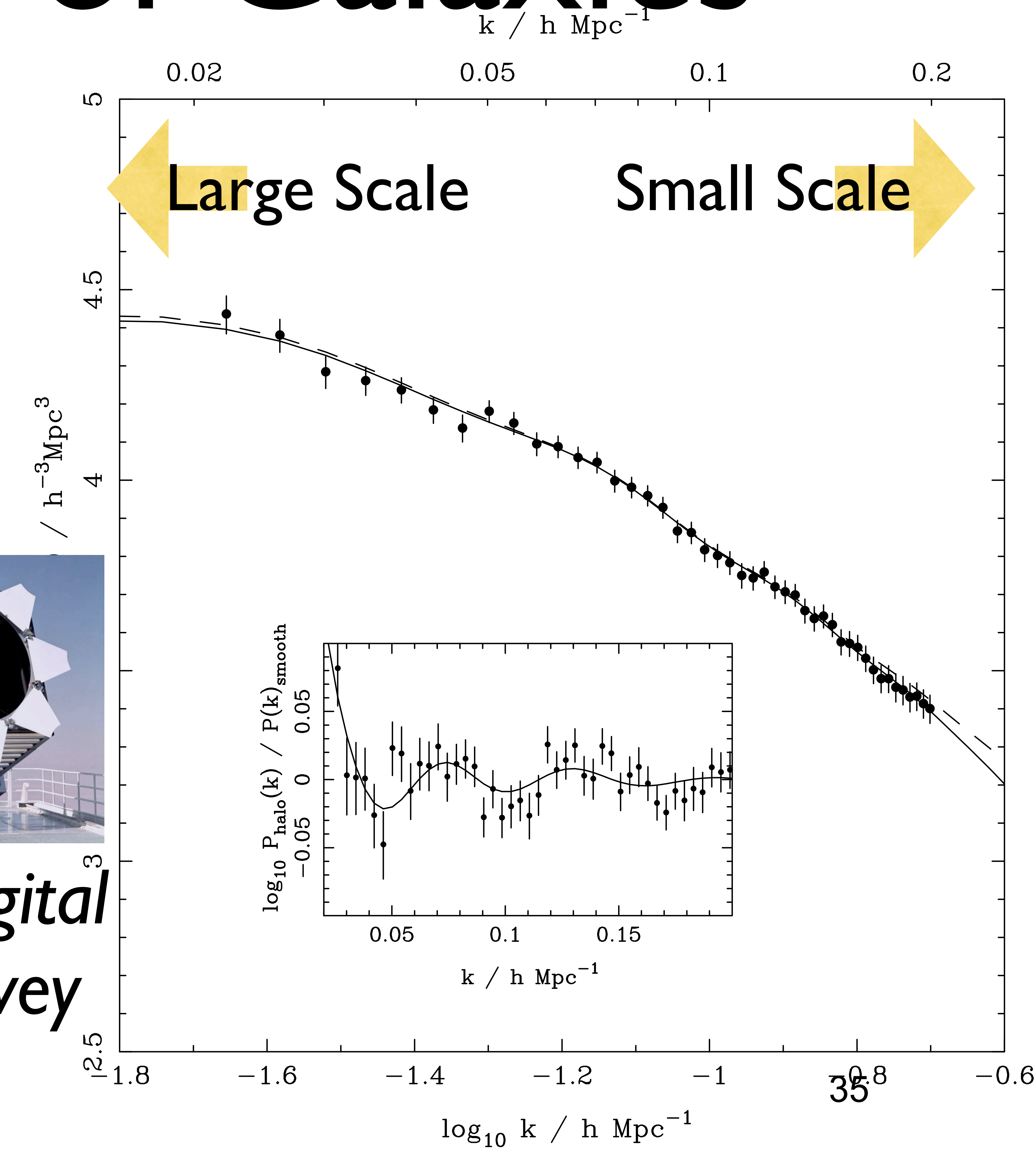
- JDEM says, "SN, WL, or BAO at minimum."
- It does not make sense to single out "BAO": the observable is the **galaxy power spectrum (GPS)**.
- To get BAO, we need to measure the galaxy power spectrum anyway.
- If we measure the galaxy power spectrum, why just focus on BAO? There is much more information!

HETDEX: Sound Waves in the Distribution of Galaxies

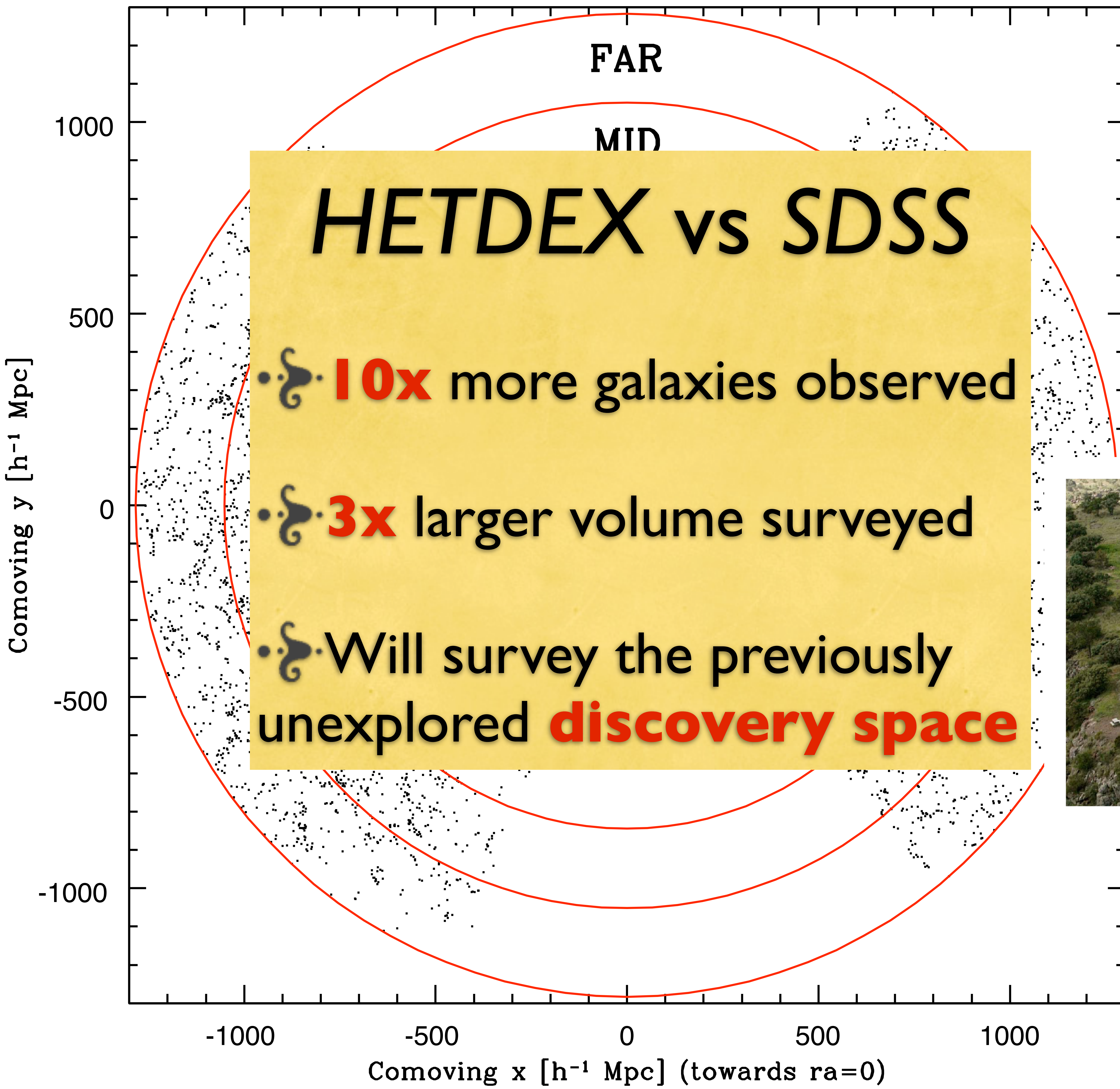




Sloan Digital Sky Survey



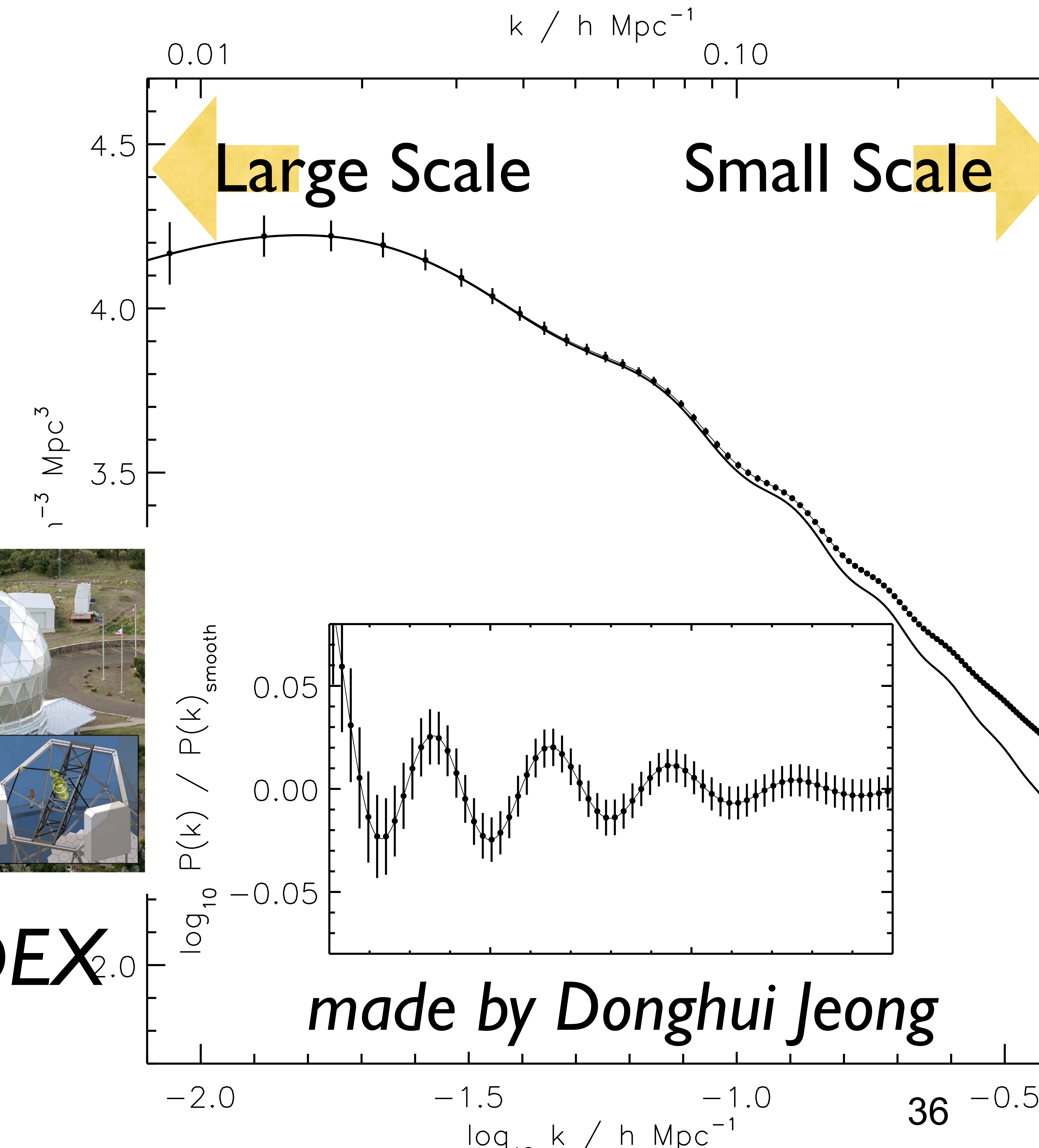
HETDEX: Sound Waves in the Distribution of Galaxies



HETDEX vs SDSS

- **10x** more galaxies observed
- **3x** larger volume surveyed
- Will survey the previously unexplored **discovery space**

HETDEX



WMAP Amplitude Prior

- WMAP measures the amplitude of curvature perturbations at z~1090. Let's call that R_k. The relation to the density fluctuation is

$$\delta_{m,\mathbf{k}}(z) = \frac{2k^3}{5H_0^2\Omega_m}\mathcal{R}_{\mathbf{k}}T(k)D(k,z)$$

- Variance of R_k has been constrained as:

AMPLITUDE OF CURVATURE PERTURBATIONS, $\mathcal{R}$, MEASURED BY *WMAP* AT $k_{WMAP} = 0.02$ Mpc^{-1}

Model	$10^9 \times \Delta^2_{\mathcal{R}}(k_{WMAP})$
$\Omega_k = 0$ and $w = -1$	2.211 ± 0.083
$\Omega_k \neq 0$ and $w = -1$	2.212 ± 0.084
$\Omega_k = 0$ and $w \neq -1$	2.208 ± 0.087
$\Omega_k \neq 0$ and $w \neq -1$	2.210 ± 0.084
$\Omega_k = 0$, $w = -1$ and $m_\nu > 0$	2.212 ± 0.083
$\Omega_k = 0$, $w \neq -1$ and $m_\nu > 0$	2.218 ± 0.085
WMAP Normalization Prior	2.21 ± 0.09

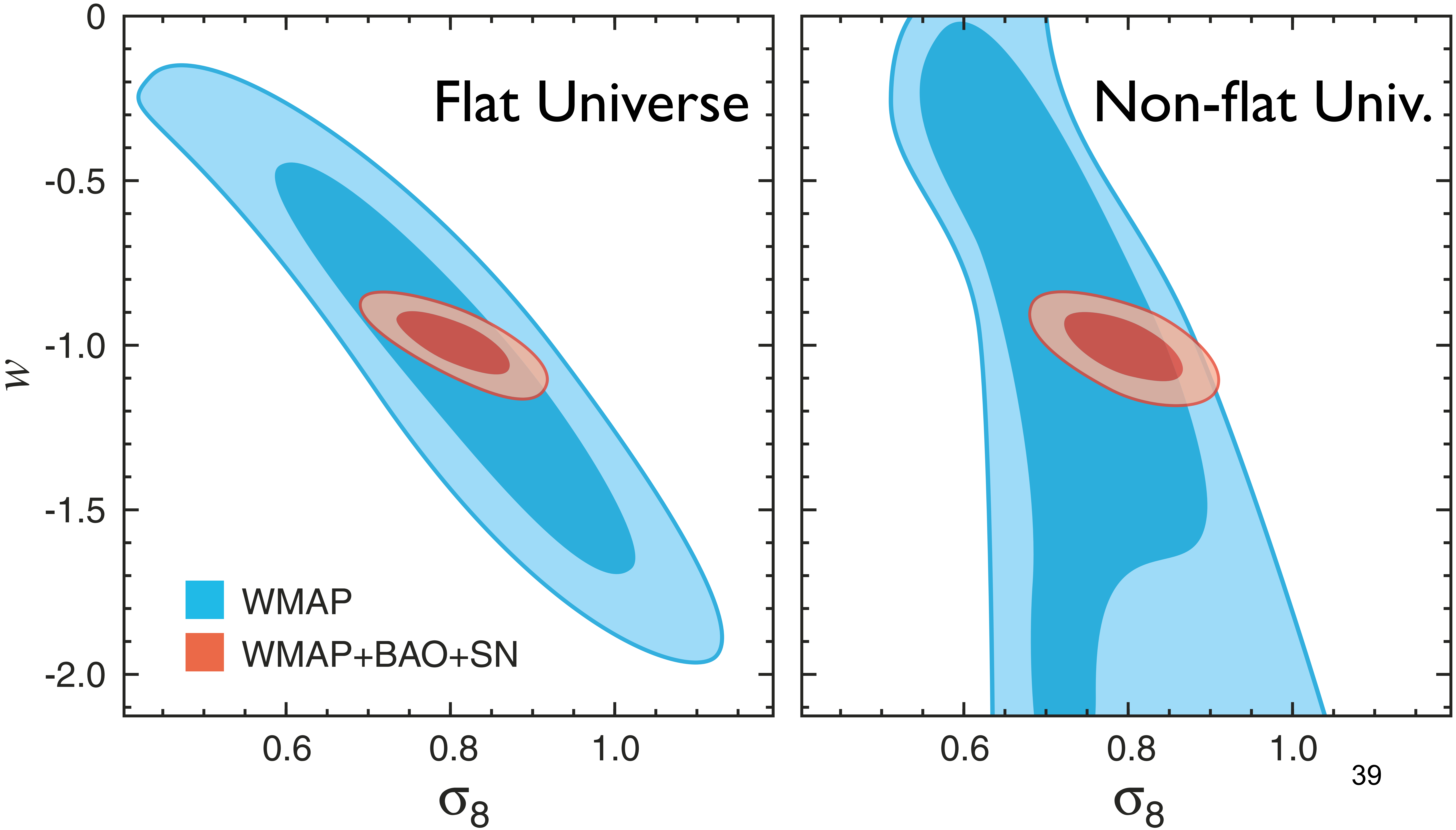
Then Solve This Diff. Equation...

Ignoring the mass of neutrinos and modifications to gravity, one can obtain the growth rate by solving the following differential equation (Wang & Steinhardt 1998; Linder & Jenkins 2003): **g(z)=(1+z)D(z)**

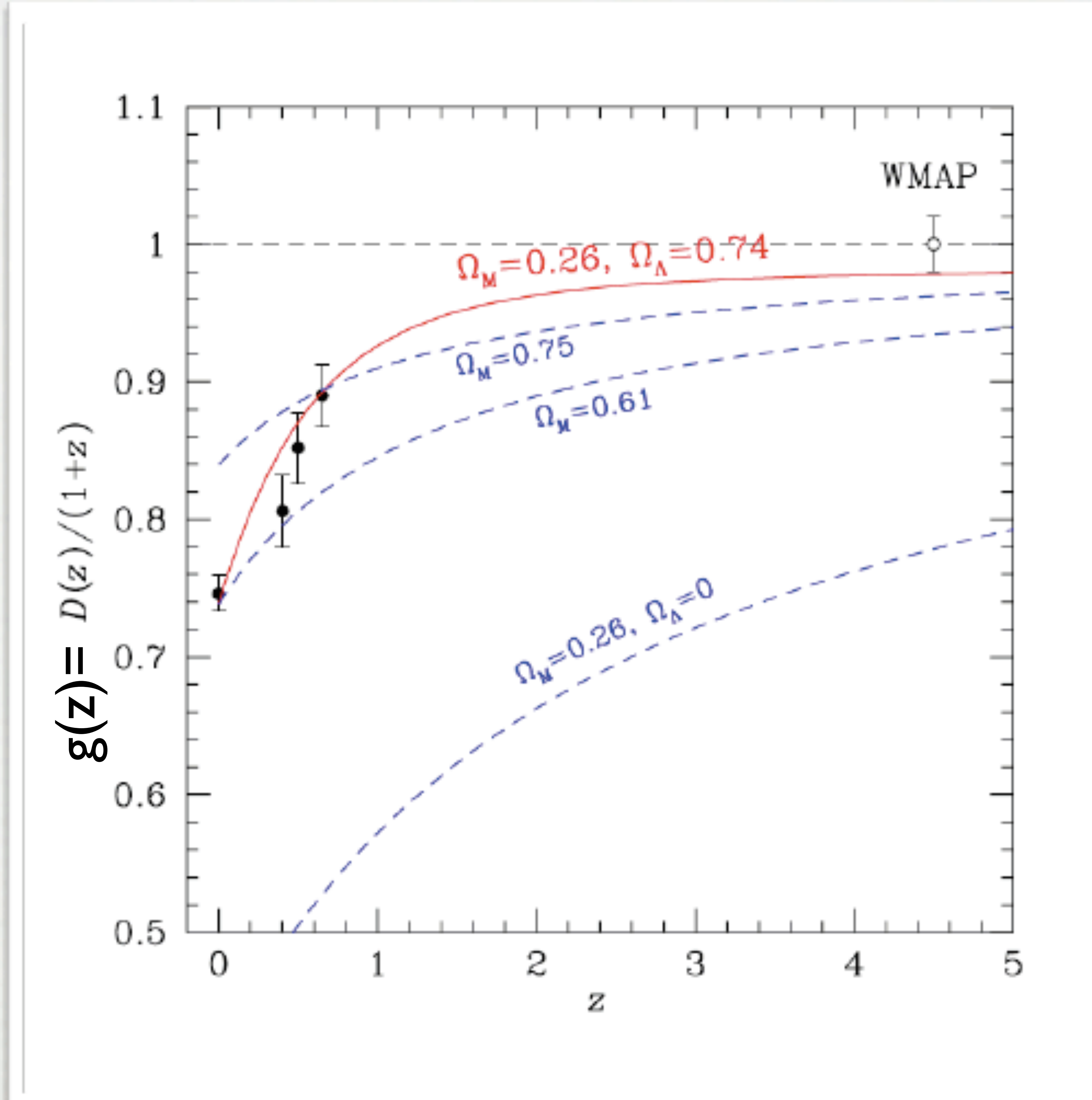
$$\frac{d^2 g}{d\ln a^2} + \left[\frac{5}{2} + \frac{1}{2}\left(\Omega_k(a) - 3\boldsymbol{W}(a)\Omega_{de}(a)\right)\right]\frac{dg}{d\ln a} + \left[2\Omega_k(a) + \frac{3}{2}(1 - \boldsymbol{W}(a))\Omega_{de}(a)\right]g(a) = 0, \qquad (76)$$

- If you need a code for doing this, search for "**Cosmology Routine Library**" on Google

Degeneracy Between Amplitude at z=0 (σ_8) and w



DETECTION OF Λ BY GROWTH HISTORY



Alexey Vikhlinin, from a slide presented at the IPMU Dark Energy Conference in Japan, June 2009

Λ>0 at ~3.5σ from perturbations growth only

at ~5σ from growth + geometry

ISW Effect

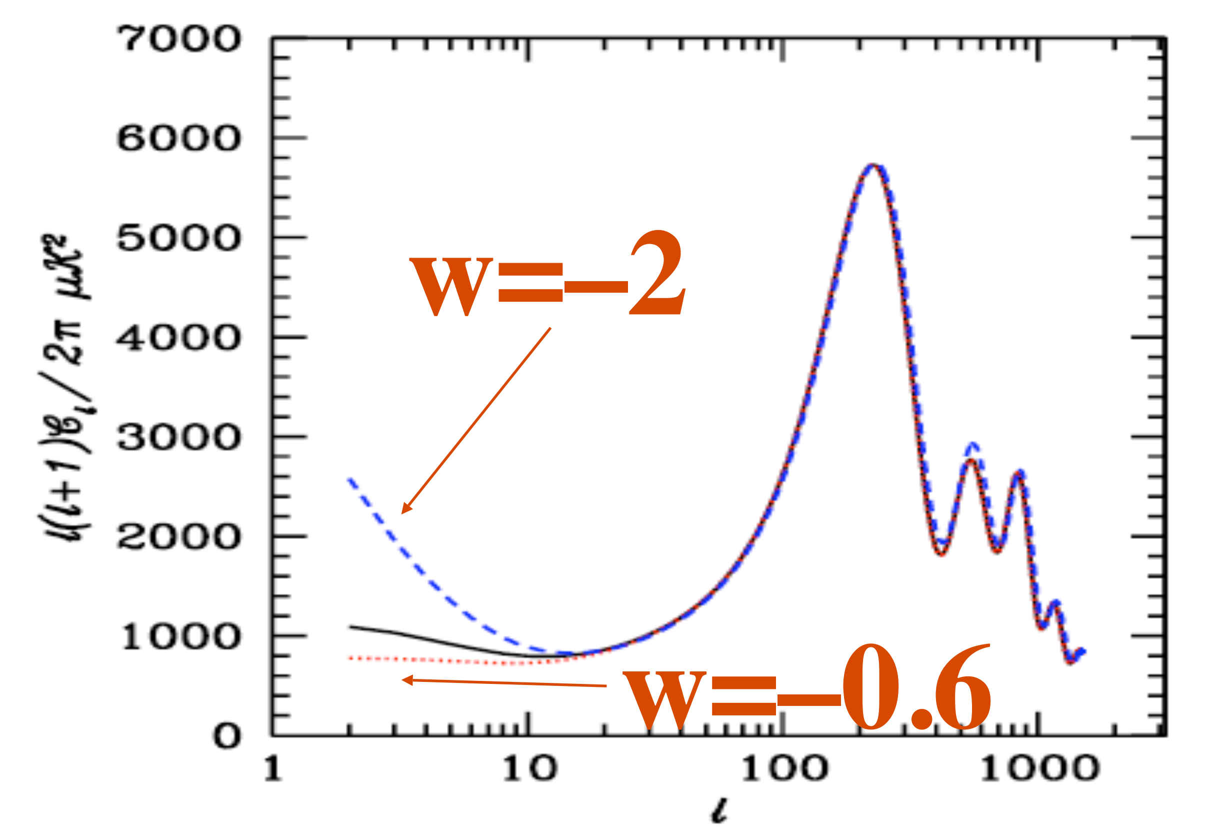
$$\frac{\Delta T}{T} = -2\int d\tau \frac{\partial \Phi}{\partial \tau}$$

Temperature anisotropy is sensitive to the growth rate of structure. In a ΛCDM universe, Φ evolves as

$$\Phi \propto \frac{H(a)}{a}\int \frac{da'}{[a'H(a')]^3}.$$

No ISW would arise during the matter dominated era during which Φ is independent of time; however, the effects of Λ or curvature would cause Φ to decay, yielding significant ISW effects at $l < 10$.

One should always keep in mind that for $w \neq -1$ the evolution equation for Φ above is not valid, and one has to solve a differential equation for the correct evolution. Nevertheless, a rough trend may still be obtained, which is that models with $w > -1$ yield less ISW than $w = -1$, while $w < -1$ yield more ISW than $w = -1$.



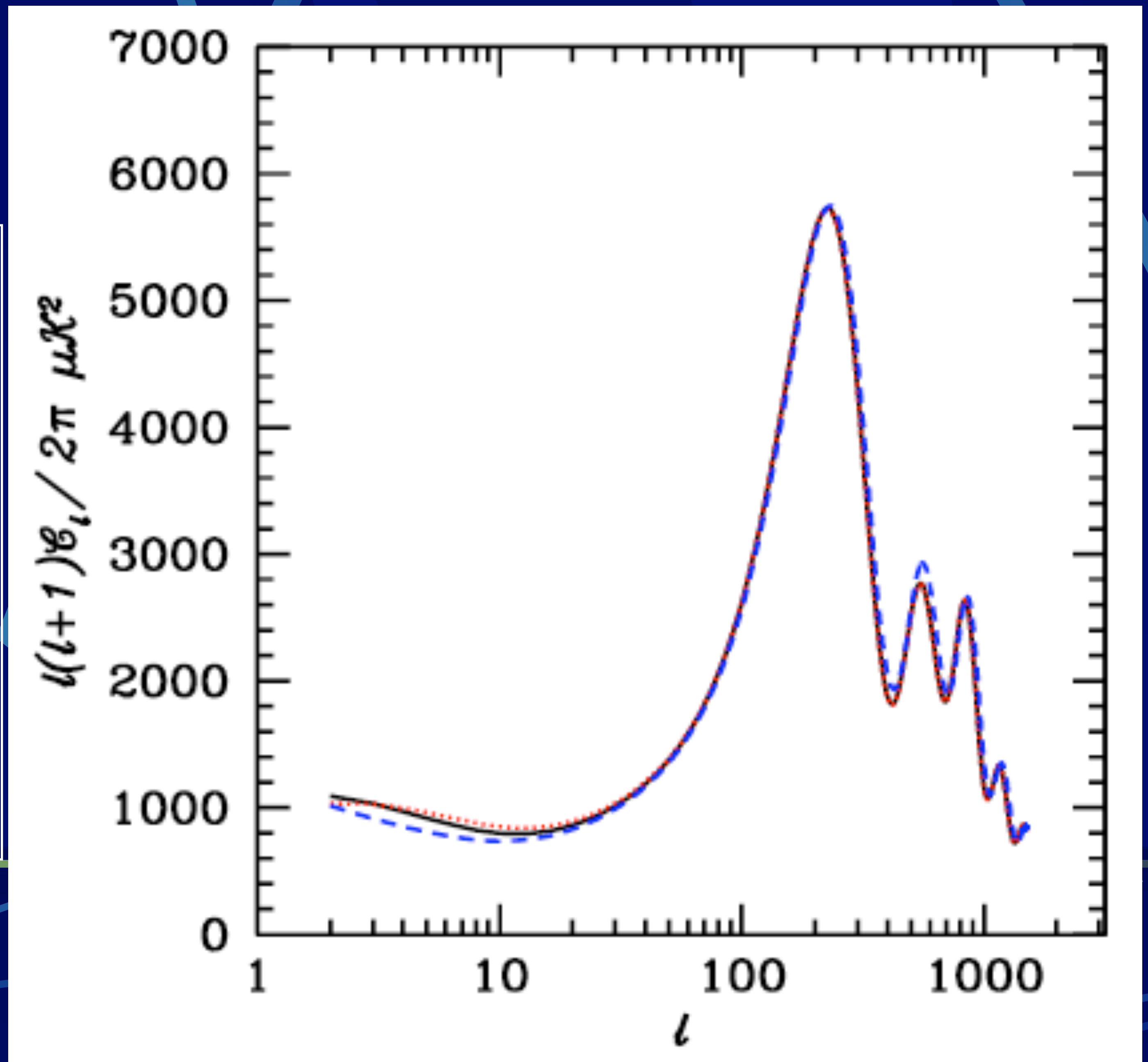
Therefore, one might hope that the ISW would help break degeneracy between w and the other parameters. However…

Weller & Lewis (2003)

Perturbations in DE

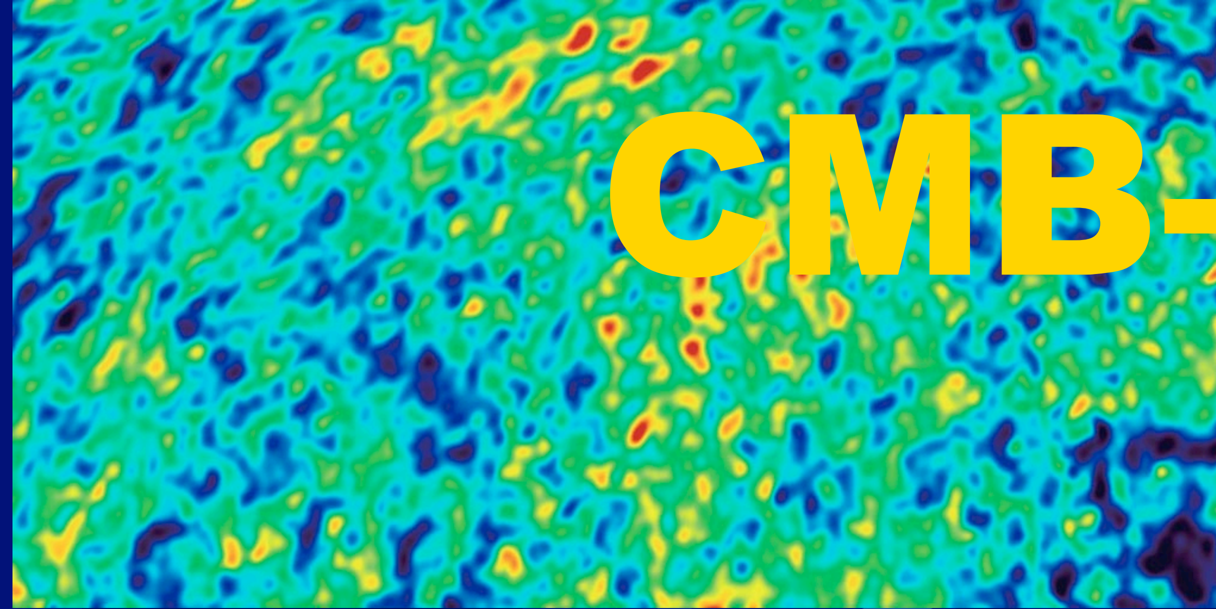
- Dark energy is required to be uniform in space (I.e., no fluctuations) if it is a cosmological constant (w=–1).
- However, in general dark energy can fluctuate and cluster on large scales when w is not –1.
- The clustering of DE can…
  - source the growth of potential,
  - compensate the suppression of growth due to a faster expansion rate, and
  - lower the ISW effect.

- This property makes it **absolutely** impossible to constrain *w* with the ISW in CMB data alone, no matter how good the CMB data would be.



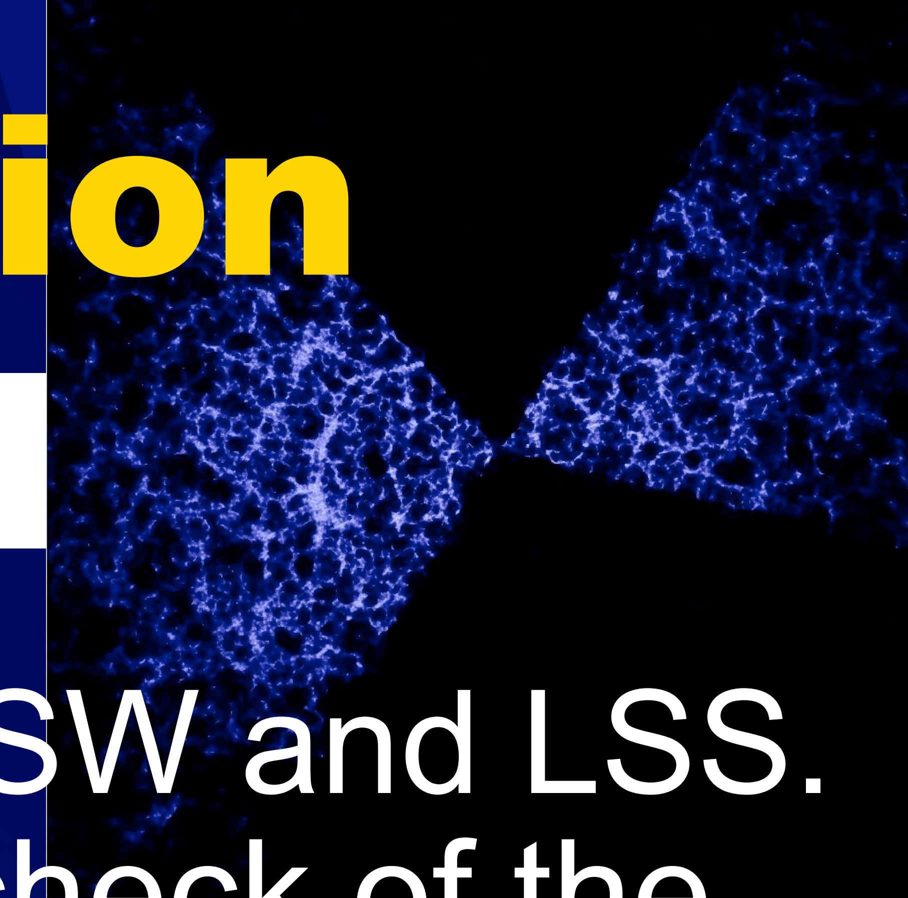
Weller & Lewis (2003)

CMB-LSS Correlation



$$\frac{\Delta T}{T} = -2\int d\tau \frac{\partial \Phi}{\partial \tau}$$

$$k^2\Phi = 4\pi G\rho a^2\delta$$



- The same gravitational potential would cause ISW and LSS. Cross-correlation signal is an important cross-check of the existence of dark energy. There are ~2-sigma detections of various correlations:
  - Boughn and Crittenden (2004): WMAP x Radio & X-ray sources
  - Nolta et al. (2004): WMAP x NVSS radio sources
  - Scranton et al. (2003): WMAP x LRGs in SDSS
  - Afshordi et al. (2004): WMAP x 2MASS galaxies

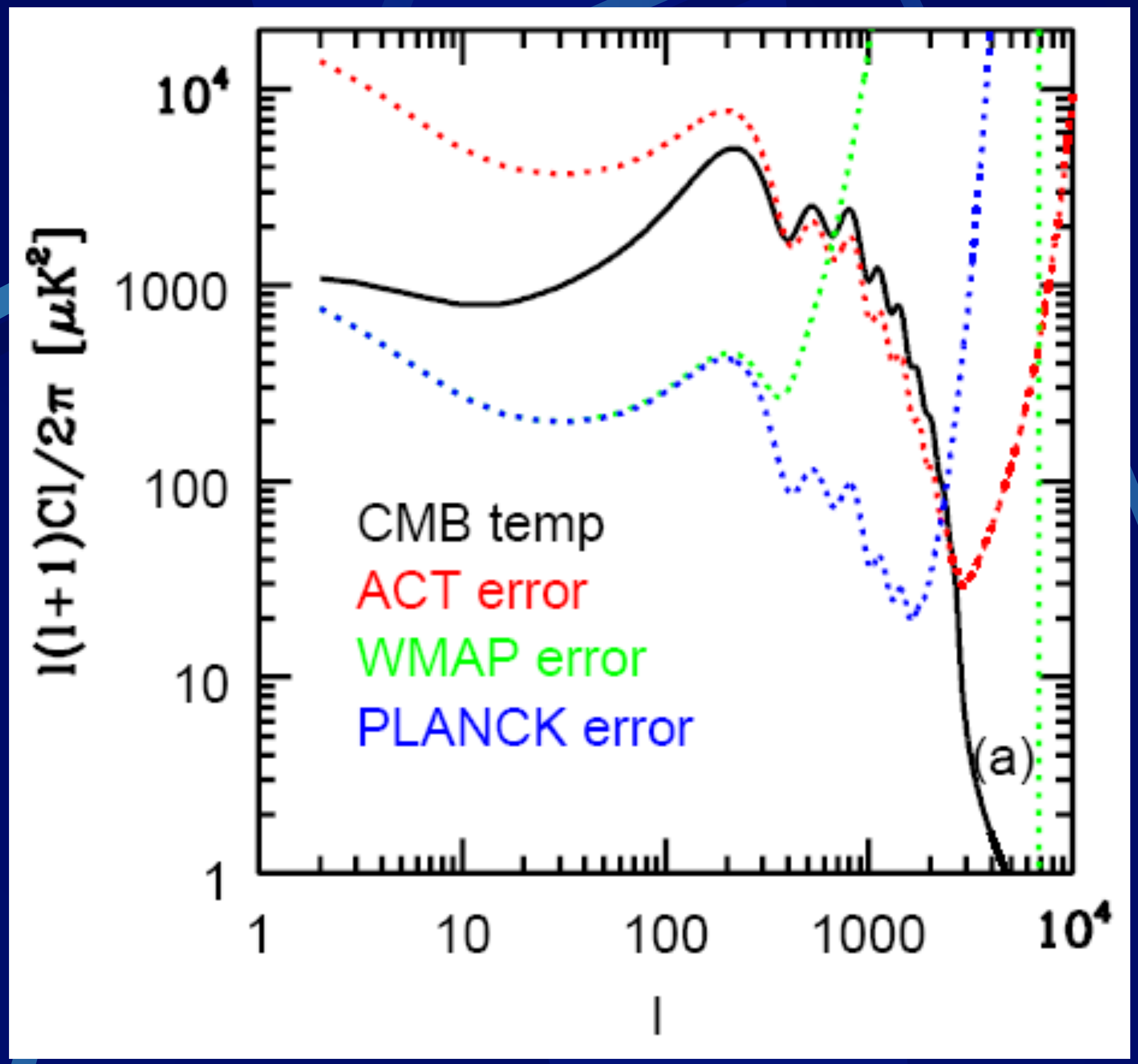
More recent compilation:
Ho et al. (2008);
Giannantonio et al. (2008)

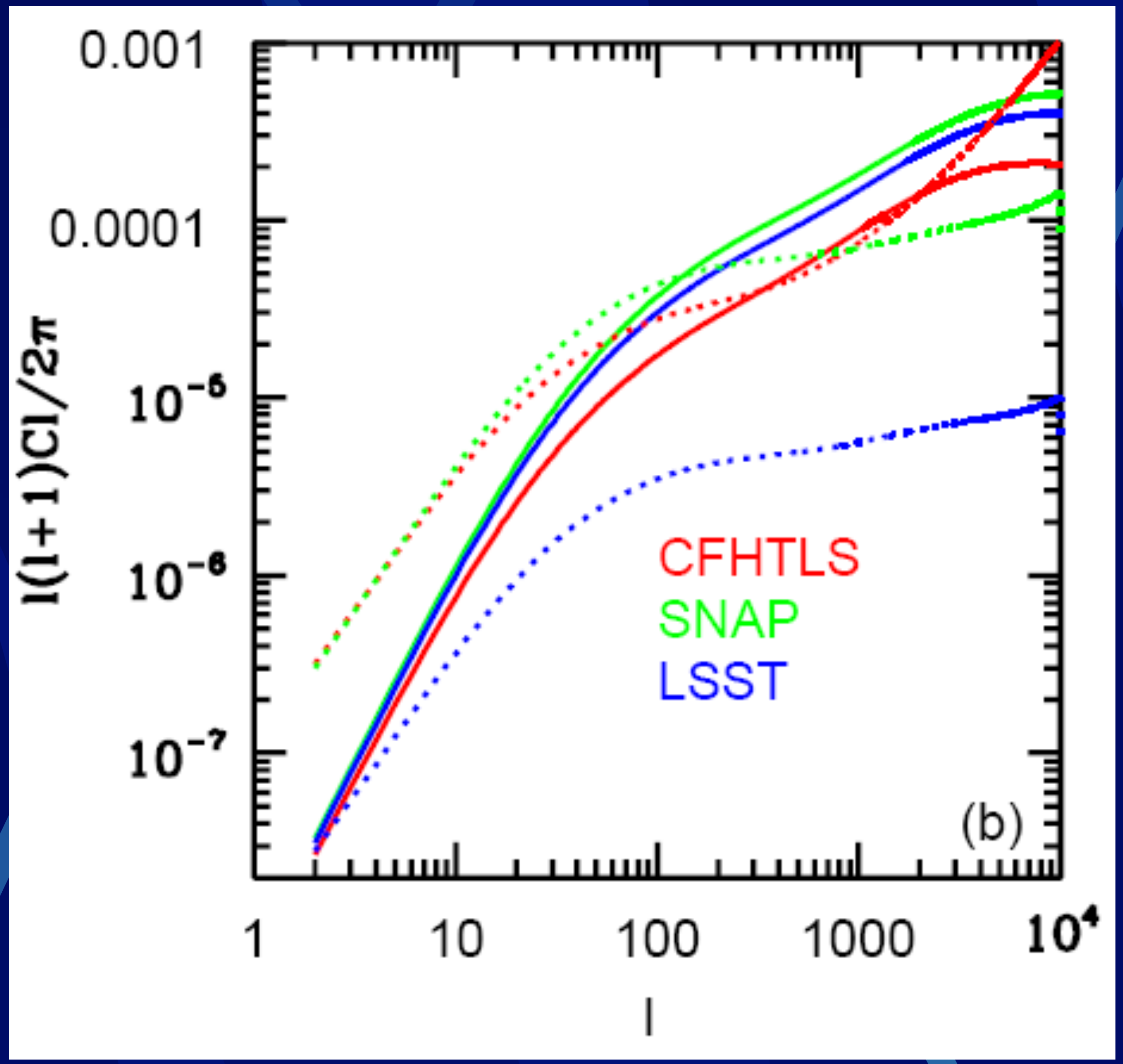
- But it's hard!
  - CMB is already signal-dominated on large scales, so nothing to be improved on the CMB side.
  - An all-sky galaxy survey observing 10 million galaxies at 0<z<1 gives only 5-sigma detection (Afshordi 2004).

CMB-WL Correlation

$$\frac{\Delta T(\hat{n})}{T} = -2\int_{\eta_*}^{\eta_0} d\eta \frac{\partial \Phi}{\partial \eta}[\hat{n}(\eta_0 - \eta), \eta]$$

$$\kappa(\hat{n}; z_s) = \frac{3\Omega_m H_0^2}{2}\int_0^{r(z_s)} dr \frac{r[r(z_s) - r]}{a(r) r(z_s)} \delta(\hat{n}r, r)$$





- Non-linear growth of structure at small scales also provides the ISW signal (a.k.a. RS effect)
- Would that be observable (ever)?
  - The future lensing experiments would be signal-dominated.
  - A lot of room for CMB experiments to improve at small scales.

$$a_{lm}^{RS} = -8\pi(-i)^l \int \frac{d^3k}{(2\pi)^3} Y_{lm}^*(\hat{k}) \int_0^{r_*} dr \frac{\partial \Phi_{\mathbf{k}}}{\partial r} j_l(kr),$$

$$a_{lm}^{\kappa}(z_s) = 6\pi(-i)^l \Omega_m H_0^2 \int \frac{d^3k}{(2\pi)^3} Y_{lm}^*(\hat{k}) \int_0^{r_s} dr \frac{r(r_s - r)}{a(r) r_s} \delta_{\mathbf{k}}(r) j_l(kr)$$

$$= 4\pi(-i)^l \int \frac{d^3k}{(2\pi)^3} k^2 Y_{lm}^*(\hat{k}) \int_0^{r_s} dr \frac{r(r_s - r)}{r_s} \Phi_{\mathbf{k}}(r) j_l(kr),$$

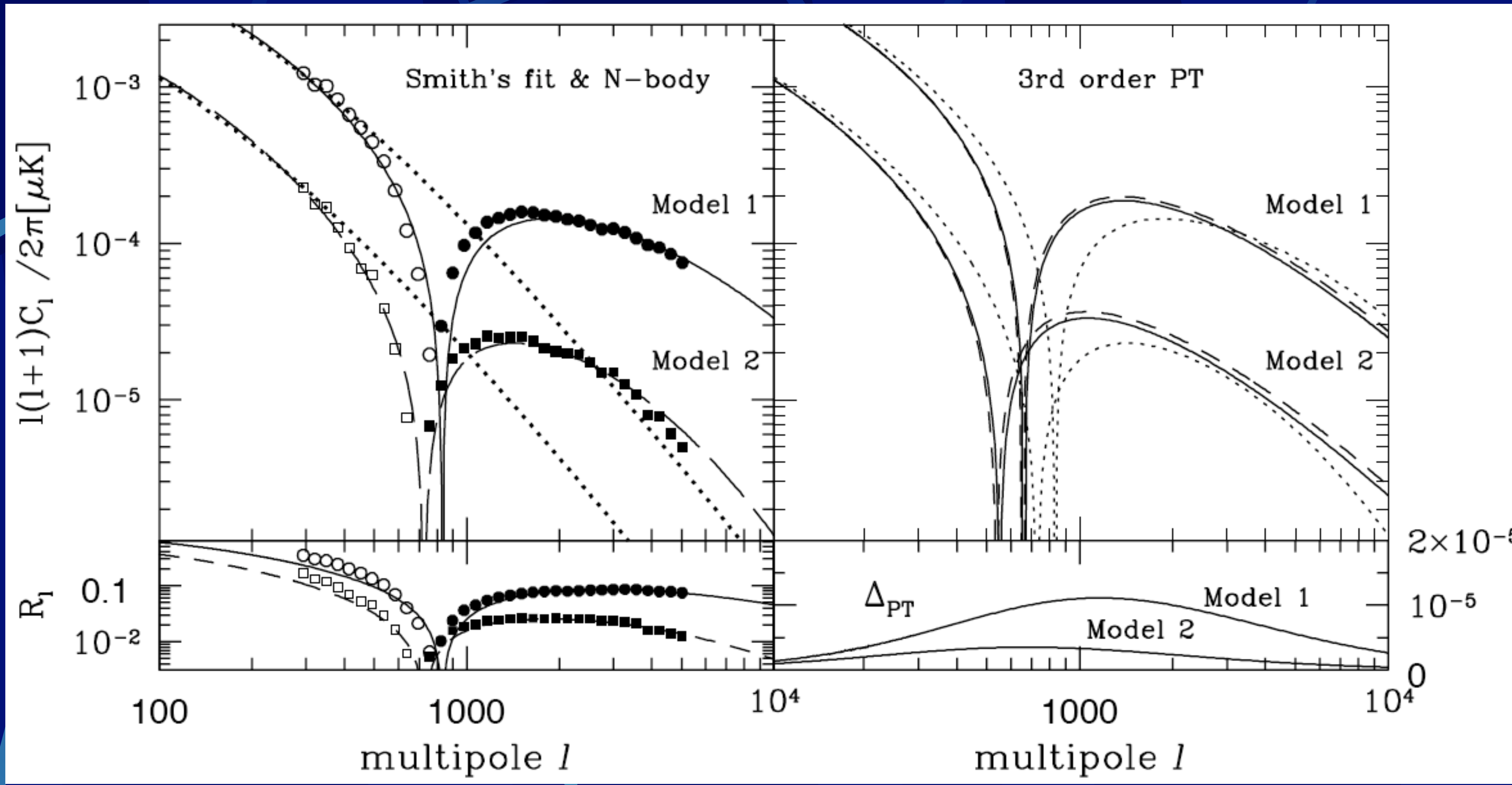
$$C_l^{RS-\kappa}(z_s) = \langle a_{lm}^{RS} a_{lm}^{\kappa *}(z_s) \rangle = -\frac{4}{\pi} \int dk\, k^4 \int_0^{r_*} dr \int_0^{r_s} dr' \frac{r'(r_s - r')}{r_s} P_{\dot{\Phi}\Phi}(k; r, r') j_l(kr) j_l(kr')$$

$$C_l^{RS-\kappa}(z_s) = -2l^2 \int_0^{r_s} dr \frac{r_s - r}{r^3 r_s} P_{\dot{\Phi}\Phi}(k; r)\Big|_{k=l/r} = -l^2 \int_0^{r_s} dr \frac{r_s - r}{r^3 r_s} \left. \frac{\partial P_{\Phi}(k; r)}{\partial r} \right|_{k=l/r}$$

- The RS-WL correlation picks up a time-derivative of the growth rate of structure: **a potential probe of *w***
- Several different source redshifts allow us to do tomography on the time derivatives.

Nishizawa, Komatsu et al. (2008)

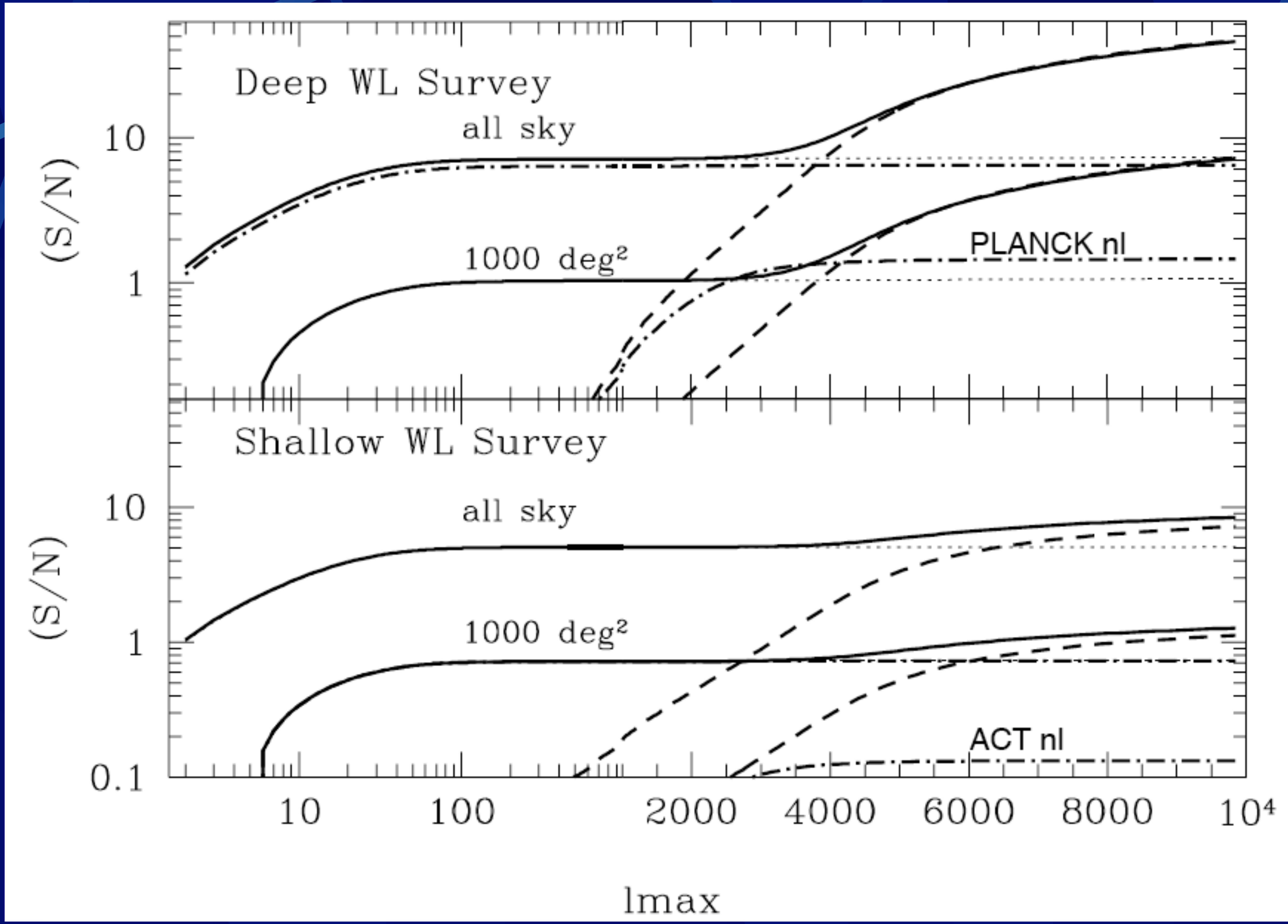
CMB(RS)-Lensing Correlation



- Model 1: Deep Lens Survey
- Model 2: Shallow Lens Survey

Nishizawa, Komatsu et al. (2008)

Signal-to-Noise Calculation



- Cosmic-variance dominated CMB data would yield a lot of S/N; Planck gives S/N~1.5.

Summary

- WMAP helps constrain the nature of DE by providing:
  - Angular diameter distance to $z_* \sim 1090$,
  - Amplitude of fluctuations at $z_* \sim 1090$, and
  - $\partial\Phi/\partial t$ at $z<1$ via the Integrated Sachs-Wolfe effect.
- WMAP also measures the sound horizon size for baryons, d_{BAO}, which is used by BAO experiments to constrain $D_A(z)$ and $H(z)$.
- Not just BAO! WMAP also provides the other standard rulers, k_{eq} and k_{silk}, with which the accuracy of $D_A(z)$ and $H(z)$ from galaxy surveys can be improved greatly.