Inflation in the Standard Model of the Universe: New understanging after WMAP

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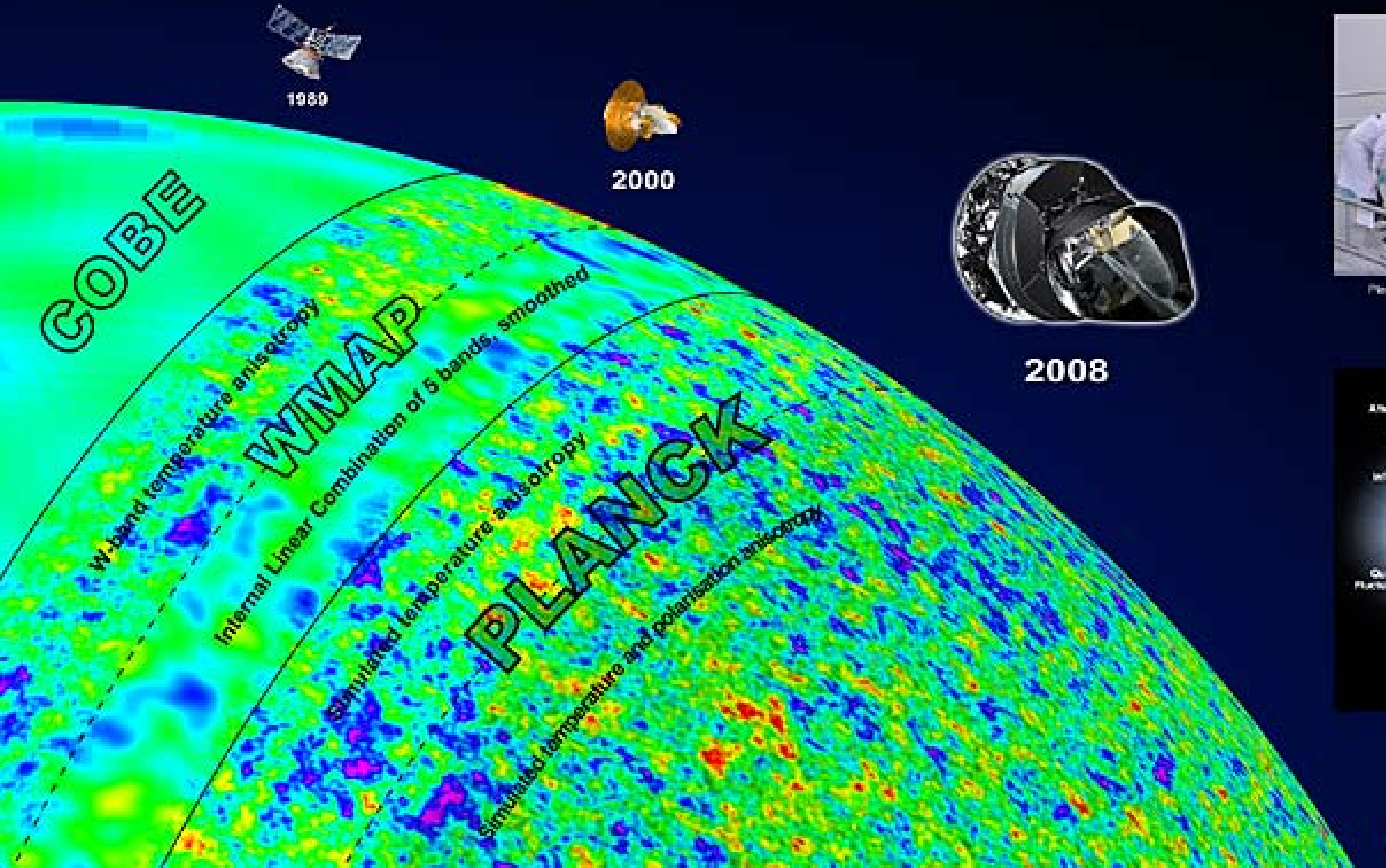
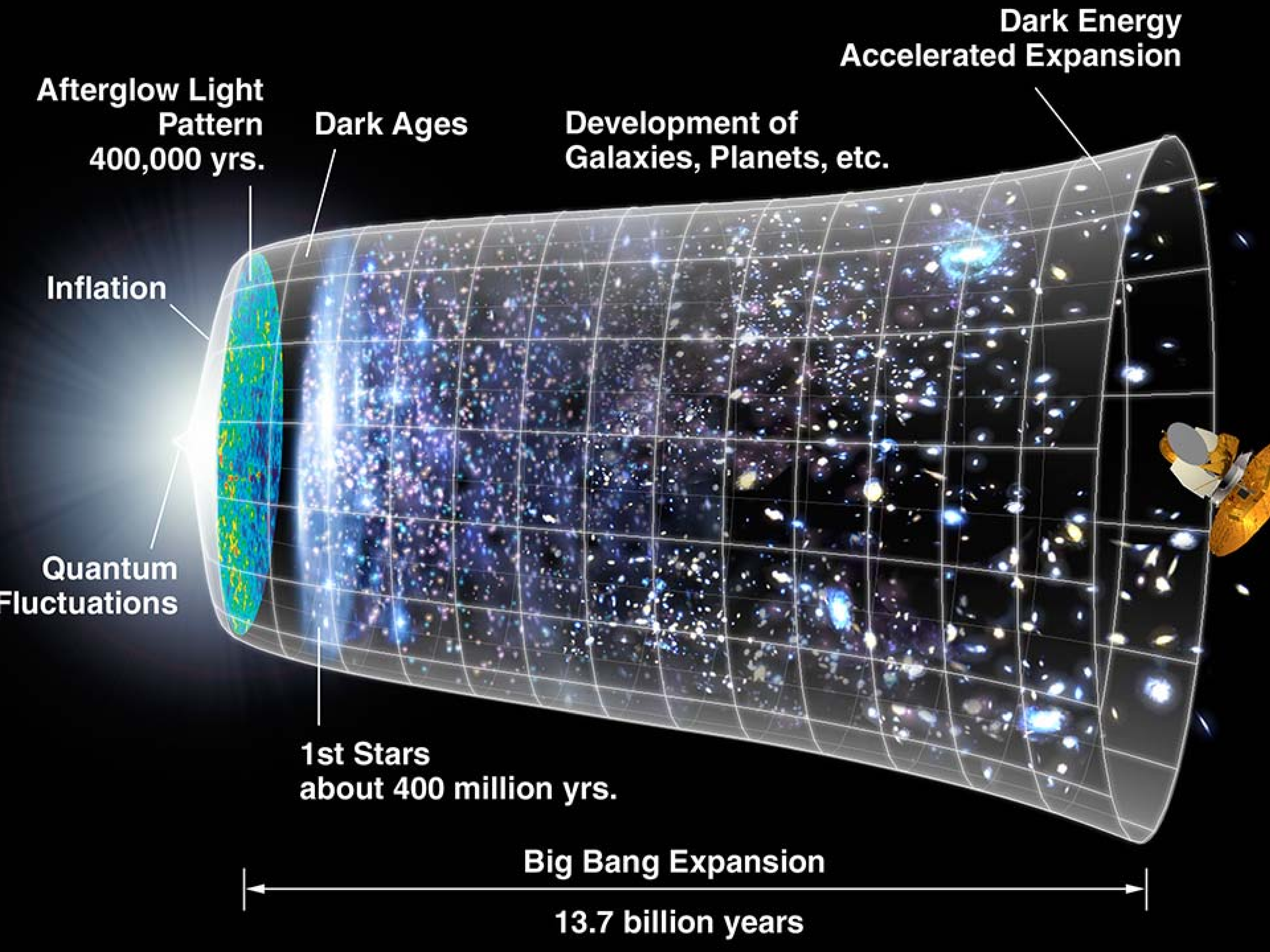
CMB Missions Revolutionise Our Understanding of the Universe




Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
400,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years

Standard Cosmological Model: ΛCDM

ΛCDM = Cold Dark Matter + Cosmological Constant

Explains the Observations:

- 5 years WMAP data and previous CMB data
- Light Elements Abundances
- Large Scale Structures (LSS) Observations
- Supernova Luminosity/Distance Relations (Acceleration of the Universe expansion)
- Gravitational Lensing Observations
- Baryonic Acoustic Oscillations
- Hubble Constant (H_0) Measurements
- Properties of Clusters of Galaxies
- ....

Standard Cosmological Model: Concordance Model

$ds^2 = dt^2 - a^2(t)\, d\vec{x}^{\,2}$: spatially flat geometry.

The Universe starts by an **INFLATIONARY ERA**.

Inflation = Accelerated Expansion: $\frac{d^2 a}{dt^2} > 0$.
During inflation the universe expands by at least sixty efolds: $e^{60} \simeq 10^{26}$. Inflation lasts $\simeq 10^{-34}$ sec and ends by $z \sim 10^{28}$ followed by a radiation dominated era.
Energy scale when inflation starts $\sim 10^{16}$ GeV.
This energy scale **coincides** with the GUT scale ($\Longleftarrow$ CMB anisotropies).

Matter can be effectively described during inflation by an Scalar Field $\phi(t, x)$: the Inflaton.

Lagrangean: $\mathcal{L} = a^3(t)\left[\frac{\dot{\phi}^2}{2} - \frac{(\nabla\phi)^2}{2\, a^2(t)} - V(\phi)\right]$.

Friedmann eq.: $H^2(t) = \frac{1}{3\, M_{Pl}^2}\left[\frac{\dot{\phi}^2}{2} + V(\phi)\right]$, $H(t) \equiv \dot{a}(t)/a(t)$

Expected CMB constraints on Δ_T *should still improve* this support.

THE SCALE OF SEMICLASSICAL GRAVITY

Δ_T and Δ_R expressed in terms of the semiclassical and quantum Gravity Temperature scales

$$T_{sem} = h\, H /(2\pi\, k_B) \qquad , \quad T_{Pl} = M_{Pl}\, c^2/ (2\pi\, k_B)$$

T_{sem} is the semiclassical or Hawking-Gibbons temperature of the initial state (or Bunch-Davies vacuum) of inflation. T_{Pl} is the Planck temparature $10^{32°}$ K.

$$T_{sem} / T_{Pl} = 2\pi\, (2\, \varepsilon_V)^{1/2}\, \Delta_R, \qquad T_{sem} / T_{Pl} = \pi\, (2)^{-1/2}\, \Delta_T$$

Therefore, WMAP data yield for the Hawking-Gibbons Temperature of Inflation:

→ → → $T_{sem} \sim (\varepsilon_V)^{1/2}\, 10^{28\,°}$ K.

LOWER BOUND on r

(ON THE PRIMORDIAL GRAVITONS

Our approach (our theory input in the MCMC data analysis of WMAP5+LSS+SN data). [C. Destri, H J de Vega, N G Sanchez, Phys Rev D77, 043509 (2008)].

Besides the upper bound for r (tensor to scalar ratio) $r < 0.22$, we find a clear peak in the r distribution and we obtain a lower bound

$$r > 0.016 \quad \text{at } 95\% \text{ CL} \quad \text{and}$$

$$r > 0.049 \quad \text{at } 68\% \text{ CL.}$$

Moreover, we find $r = 0.055$ as the most probable value.

For the other cosmological parameters, both analysis agree.

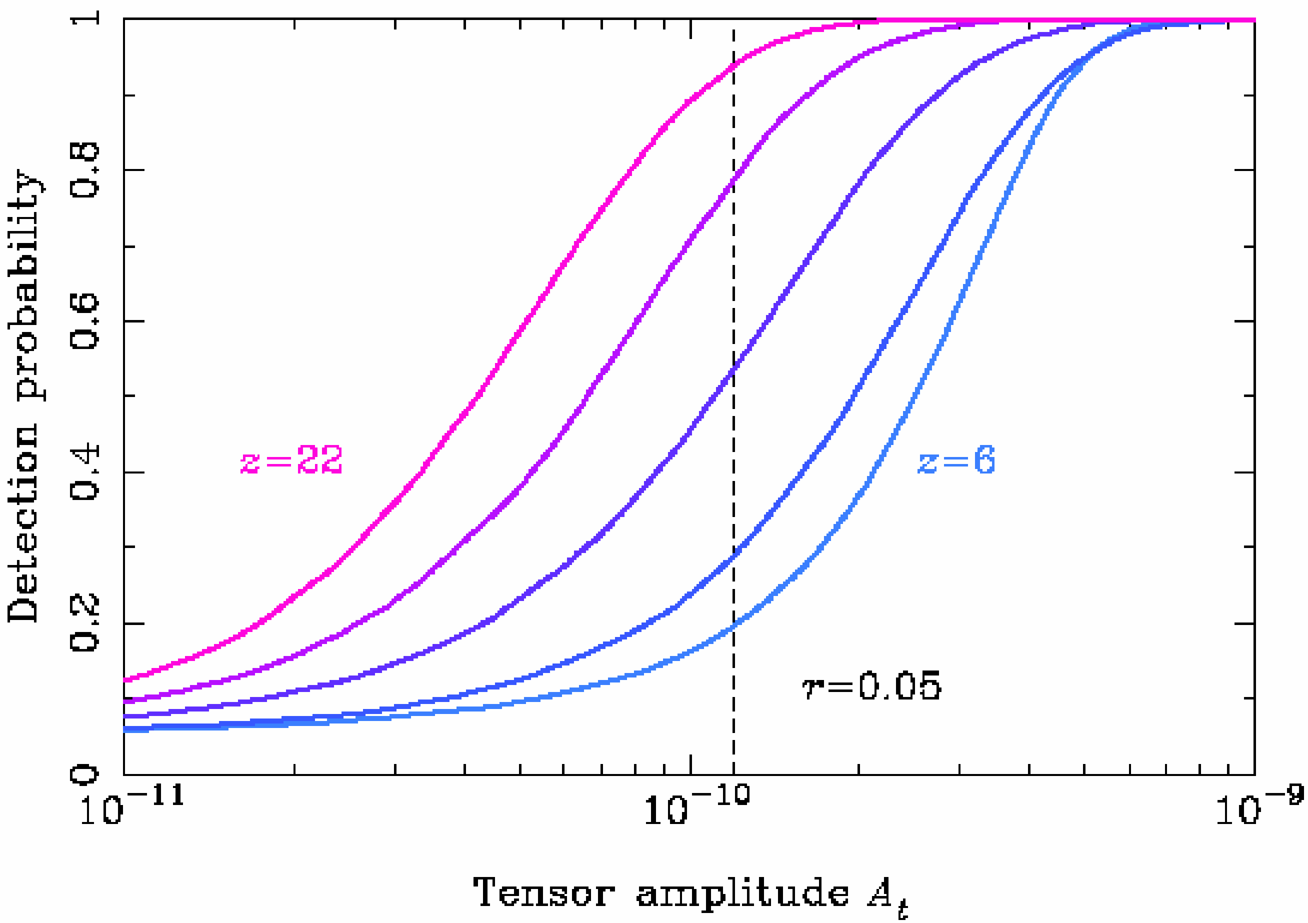
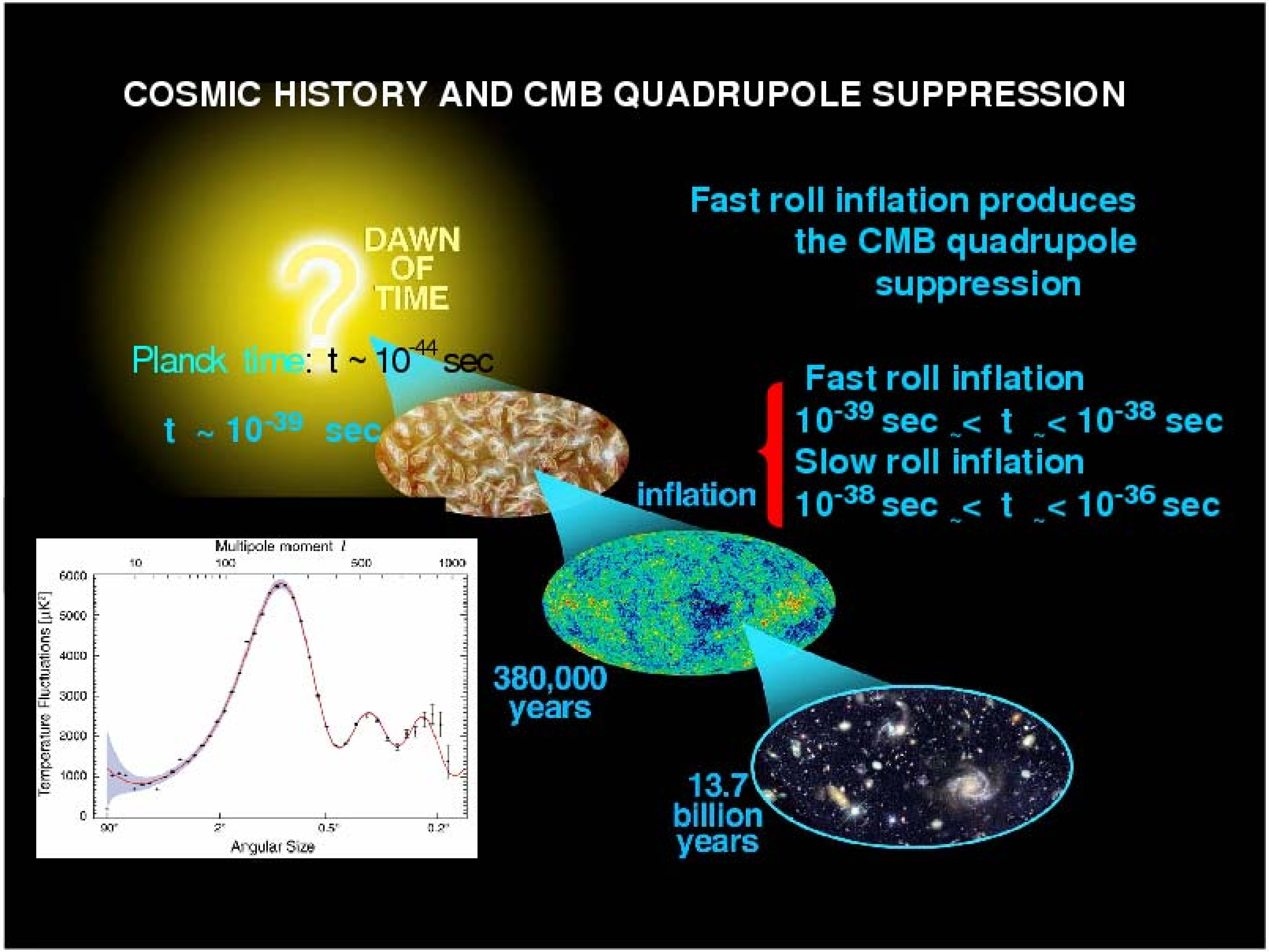
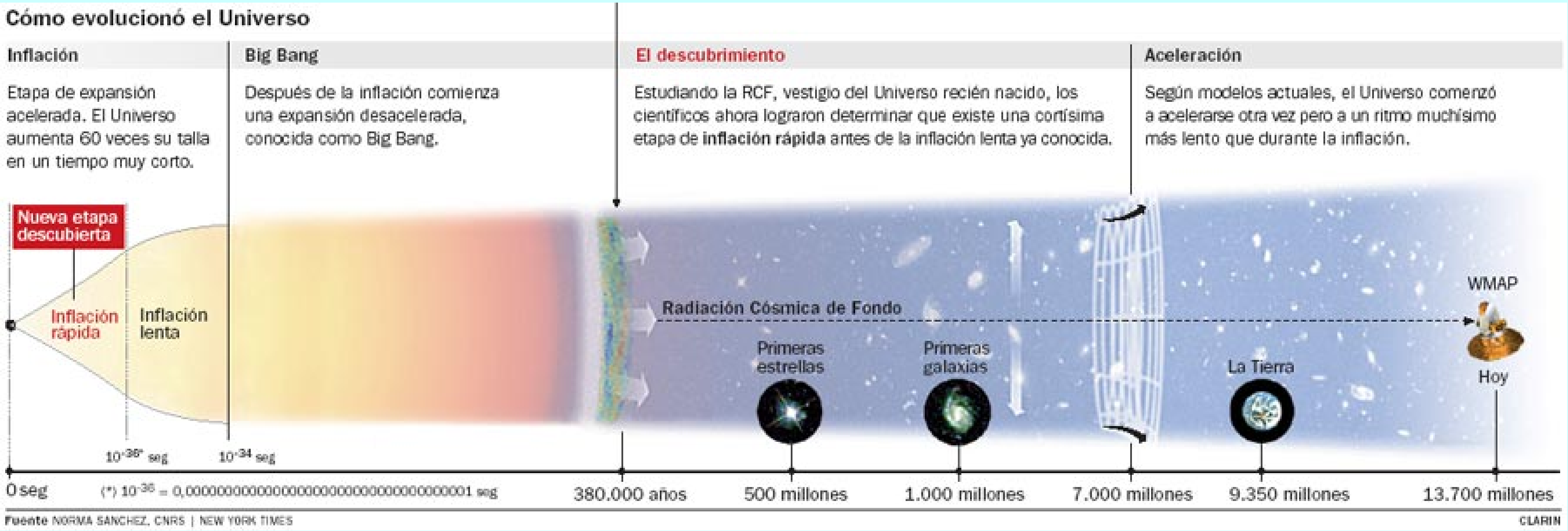


FIG 2.16.—The probability of detecting B-mode polarization at 95% confidence as a function of A_T, the amplitude of the primordial tensor power spectrum (assumed scale-invariant), for *Planck* observations using 65% of the sky. The curves correspond to different assumed epochs of (instantaneous) reionization: $z = 6$, 10, 14, 18 and 22. The dashed line corresponds to a tensor-to-scalar ratio $r = 0.05$ for the best-fit scalar normalisation, $A_S = 2.7 \times 10^{-9}$, from the one-year *WMAP* observations.


COSMIC HISTORY AND CMB QUADRUPOLE SUPPRESSION
Fast roll inflation produces the CMB quadrupole suppression
DAWN OF TIME
Planck time: t ~ 10^{-44} sec
t ~ 10^{-39} sec
Fast roll inflation
10^{-39} sec $\lesssim$ t $\lesssim 10^{-38}$ sec
Slow roll inflation
10^{-38} sec $\lesssim$ t $\lesssim 10^{-36}$ sec
inflation
380,000 years
13.7 billion years
Multipole moment l
10
100
500
1000
6000
5000
4000
3000
2000
1000
0
Temperature Fluctuations [μK^2]
90°
2°
0.5°
0.2°
Angular Size

Fast roll Inflation produces the Observed Quadrupole CMB Suppression



D. Boyanovsky, H. J de Vega and N. G. Sanchez,

” CMB quadrupole suppression II : The early fast roll stage ”

Phys. Rev. D74 , 123006 (2006)

Quadrupole suppression and Fast-roll Inflation

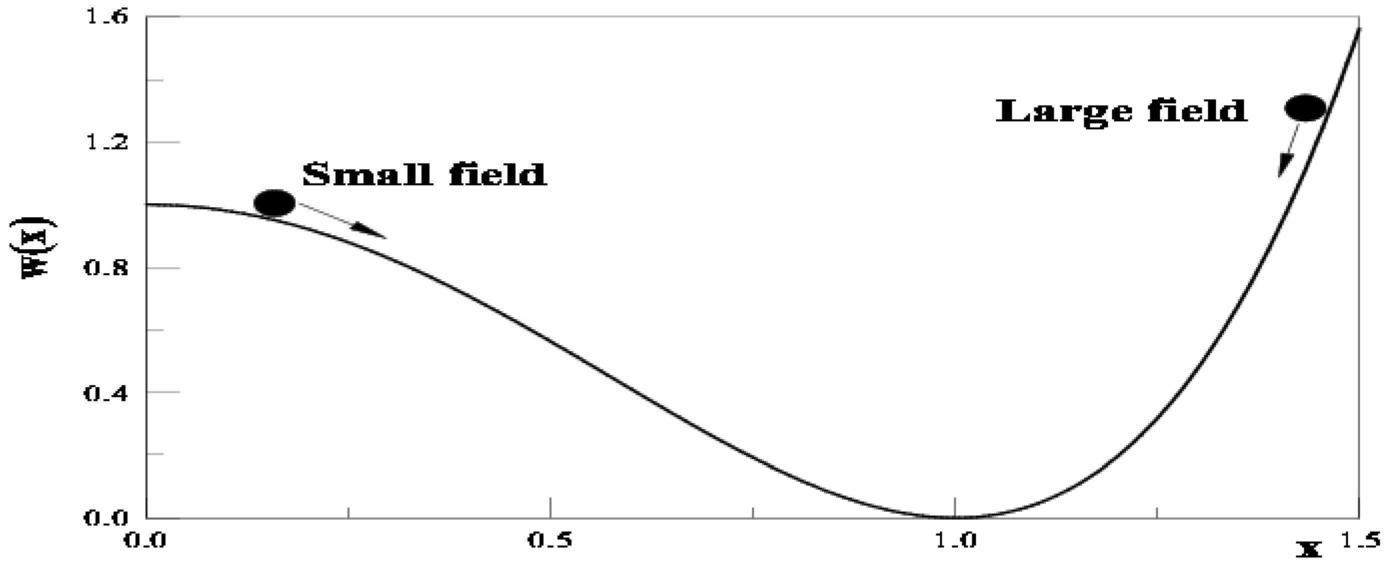
The observed CMB-quadrupole (COBE,WMAP5) is six times smaller than the ΛCDM model value.
In the best ΛCDM fit the probability that the quadrupole is as low or lower than the observed value is 3%.
It is hence relevant to find a cosmological explanation of the quadrupole supression.

Generically, the classical evolution of the inflaton has a brief **fast-roll stage** that precedes the slow-roll regime.
In case the quadrupole CMB mode leaves the horizon during fast-roll, before slow-roll starts, we find that the quadrupole mode gets suppressed.

$$P(k) = |\Delta_{k\ ad}^{(S)}|^2\ (k/k_0)^{n_s-1}[1 + D(k)]$$

The transfer function $D(k)$ changes the primordial power.
$1 + D(0) = 0, \quad D(\infty) = 0$

Slow Roll Inflaton Models



$V(\mathrm{Min}) = V'(\mathrm{Min}) = 0$: inflation ends after a finite number of efolds. Universal form of the slow-roll inflaton potential:

$V(\varphi) = N\ M^4\ w(\chi)\ ,\ \chi \equiv \frac{\varphi}{\sqrt{N}\ M_{Pl}} \quad , \quad \chi$ and $w(\chi) = \mathcal{O}(1)$

$N \sim 60$ number of efolds since horizon exit till end of inflation. $M =$ energy scale of inflation.

Slow-roll is needed to produce enough efolds of inflation.

Slow Roll expansion: a hierarchy of dimensionless parameters:

$$\epsilon_v = \frac{M_P^2}{2} \left[\frac{V^{'}(\phi)}{V(\phi)}\right]^2 \quad , \quad \eta_v = M_P^2 \, \frac{V^{''}(\phi)}{V(\phi)} \cdots$$

as a 1/Ne expansion:

$$N[\phi(t)] = -\frac{1}{M_P^2} \int_{\phi(t)}^{\phi_{end}} V(\phi) \, \frac{d\phi}{dV} \, d\phi$$

$$\phi = \sqrt{N_e} M_P \, \chi$$ ← **Rescale field**

$$V(\phi) = N_e \, M^4 \, w(\chi)$$

$w(\chi) \rightarrow \sim \mathcal{O}(1)$

$M^4 \rightarrow$ **energy scale of inflation**

$$1 = - \int_{\chi_c}^{\chi_{end}} \frac{w(\chi)}{w'(\chi)}\, d\chi$$

value of χ at the END of inflation

value of χ Ne e-folds before END of inflation

Independent of Ne: an implicit relation between χ and couplings

$$\epsilon_v = \frac{1}{2\, N_e} \left[\frac{w'(\chi_c)}{w(\chi_c)} \right]^2 \quad , \quad \eta_v = \frac{1}{N_e} \, \frac{w''(\chi_c)}{w(\chi_c)}$$

Explicit dependence on Ne: SIMPLE RESCALING

spectral index n_s and the ratio r

$r \equiv$ ratio of tensor to scalar fluctuations.
tensor fluctuations = primordial gravitons.

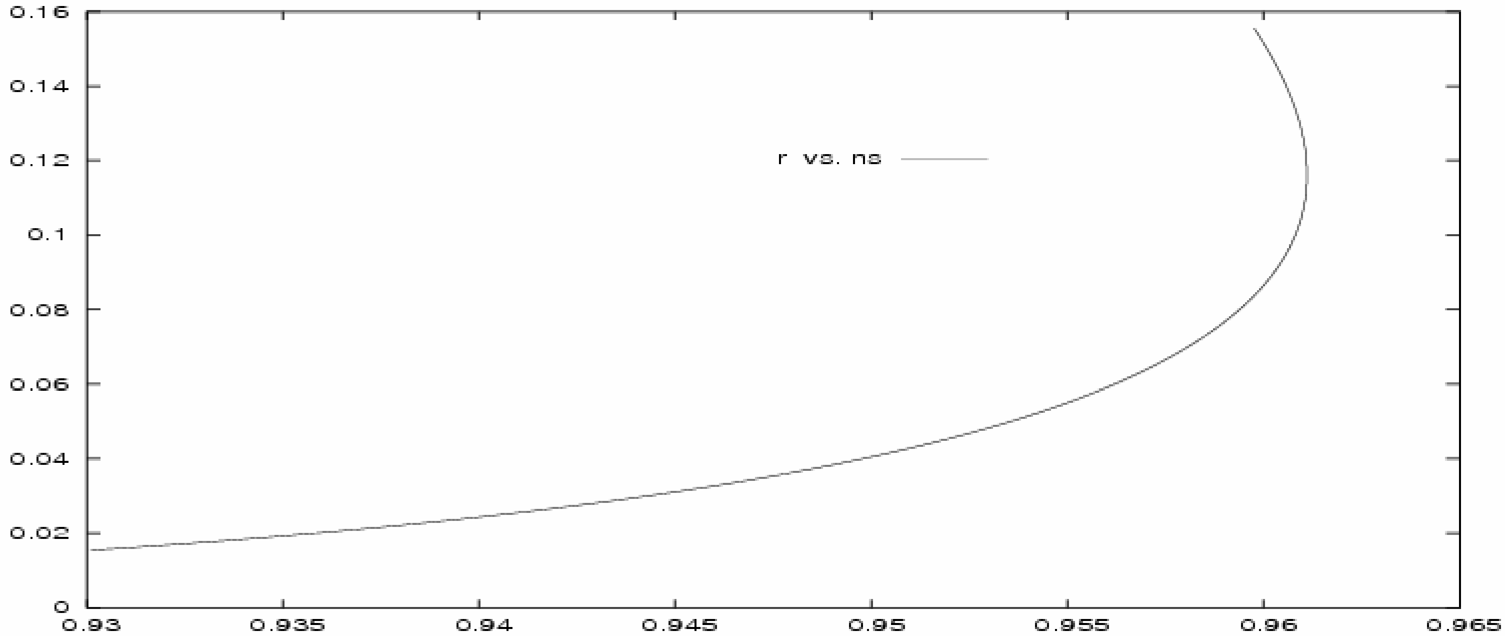
$$n_s - 1 = -\frac{3}{N}\left[\frac{w'(\chi)}{w(\chi)}\right]^2 + \frac{2}{N}\,\frac{w''(\chi)}{w(\chi)} \quad , \quad r = \frac{8}{N}\left[\frac{w'(\chi)}{w(\chi)}\right]^2$$

$$\frac{dn_s}{d\ln k} = -\frac{2}{N^2}\,\frac{w'(\chi)\,w'''(\chi)}{w^2(\chi)} - \frac{6}{N^2}\,\frac{[w'(\chi)]^4}{w^4(\chi)} + \frac{8}{N^2}\,\frac{[w'(\chi)]^2\,w''(\chi)}{w^3(\chi)} ,$$

χ is the inflaton field at horizon exit.
$n_s - 1$ and r are always of order $1/N \sim 0.02$ (model indep.)
Running of n_s of order $1/N^2 \sim 0.0003$ (model independent).

D. Boyanovsky, H. J. de Vega, N. G. Sanchez,
Phys. Rev. D 73, 023008 (2006), astro-ph/0507595.

Binomial New Inflation



$r = \frac{8}{N} = 0.16$ and $n_s = 1 - \frac{2}{N} = 0.96$ at $y = 0$.

r is a double valued function of n_s.

Primordial Power Spectrum

Adiabatic Scalar Perturbations: $P(k) = |\Delta_{k\ ad}^{(S)}|^2\ (k/k_0)^{n_s-1}$.
To dominant order in slow-roll:

$$|\Delta_{k\ ad}^{(S)}|^2 = \frac{N^2}{12\,\pi^2}\ \left(\frac{M}{M_{Pl}}\right)^4\ \frac{w^3(\chi)}{w'^2(\chi)}\ .$$

Hence, for all slow-roll inflation models:

$$|\Delta_{k\ ad}^{(S)}| \sim \frac{N}{2\,\pi\sqrt{3}}\left(\frac{M}{M_{Pl}}\right)^2$$

The WMAP5 result: $|\Delta_{k\ ad}^{(S)}| = (0.494 \pm 0.1) \times 10^{-4}$
determines the scale of inflation M (using $N \simeq 60$)

$$\left(\frac{M}{M_{Pl}}\right)^2 = 0.85 \times 10^{-5}\ \longrightarrow\ M = 0.70 \times 10^{16}\ \text{GeV}$$

The inflation energy scale turns to be the grand unification energy scale !! We find the scale of inflation without knowing the tensor/scalar ratio r !!
The scale M is independent of the shape of $w(\chi)$.

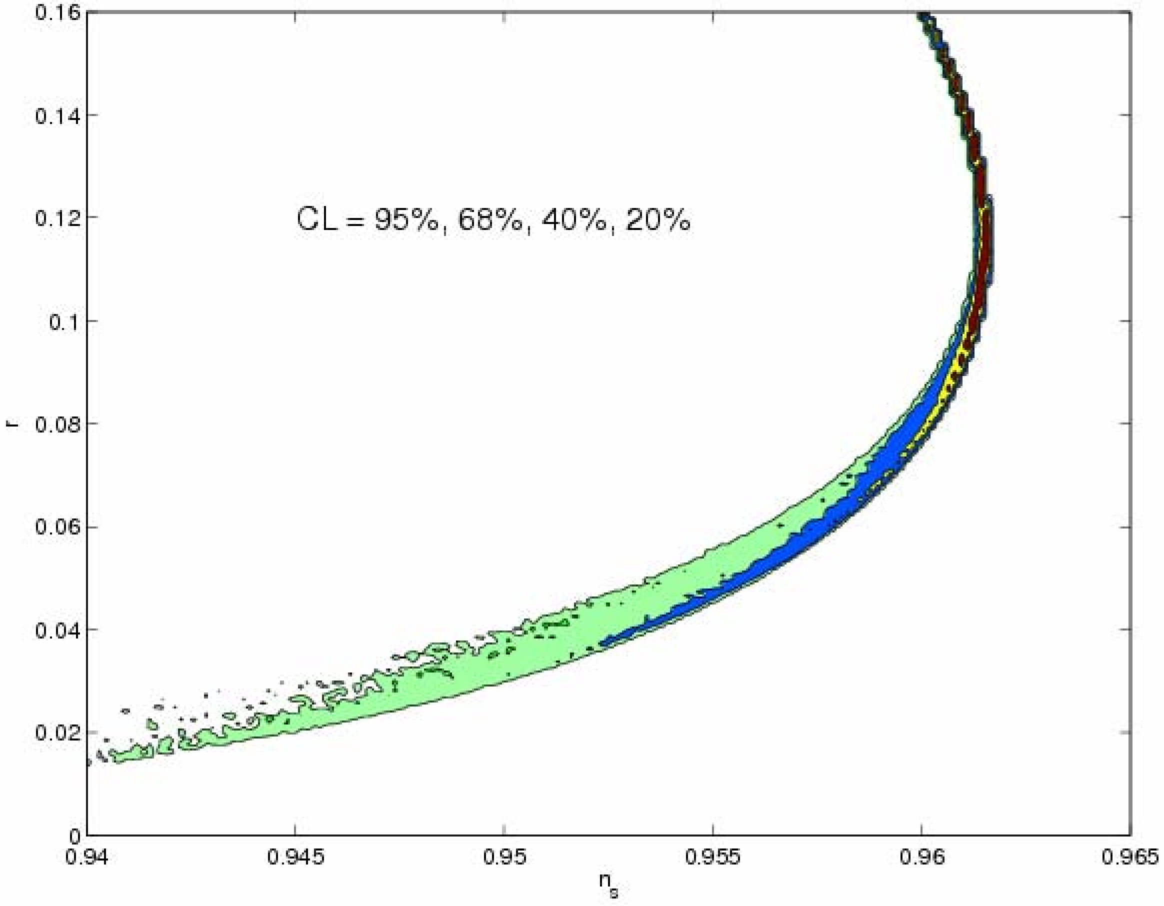
Trinomial Inflationary Models

- Trinomial Chaotic inflation:
  $w(\chi) = \frac{1}{2}\,\chi^2 + \frac{h}{3}\,\sqrt{\frac{y}{2}}\,\chi^3 + \frac{y}{32}\,\chi^4\;.$
- Trinomial New inflation:
  $w(\chi) = -\frac{1}{2}\,\chi^2 + \frac{h}{3}\,\sqrt{\frac{y}{2}}\,\chi^3 + \frac{y}{32}\,\chi^4 + \frac{2}{y}\;F(h)\;.$

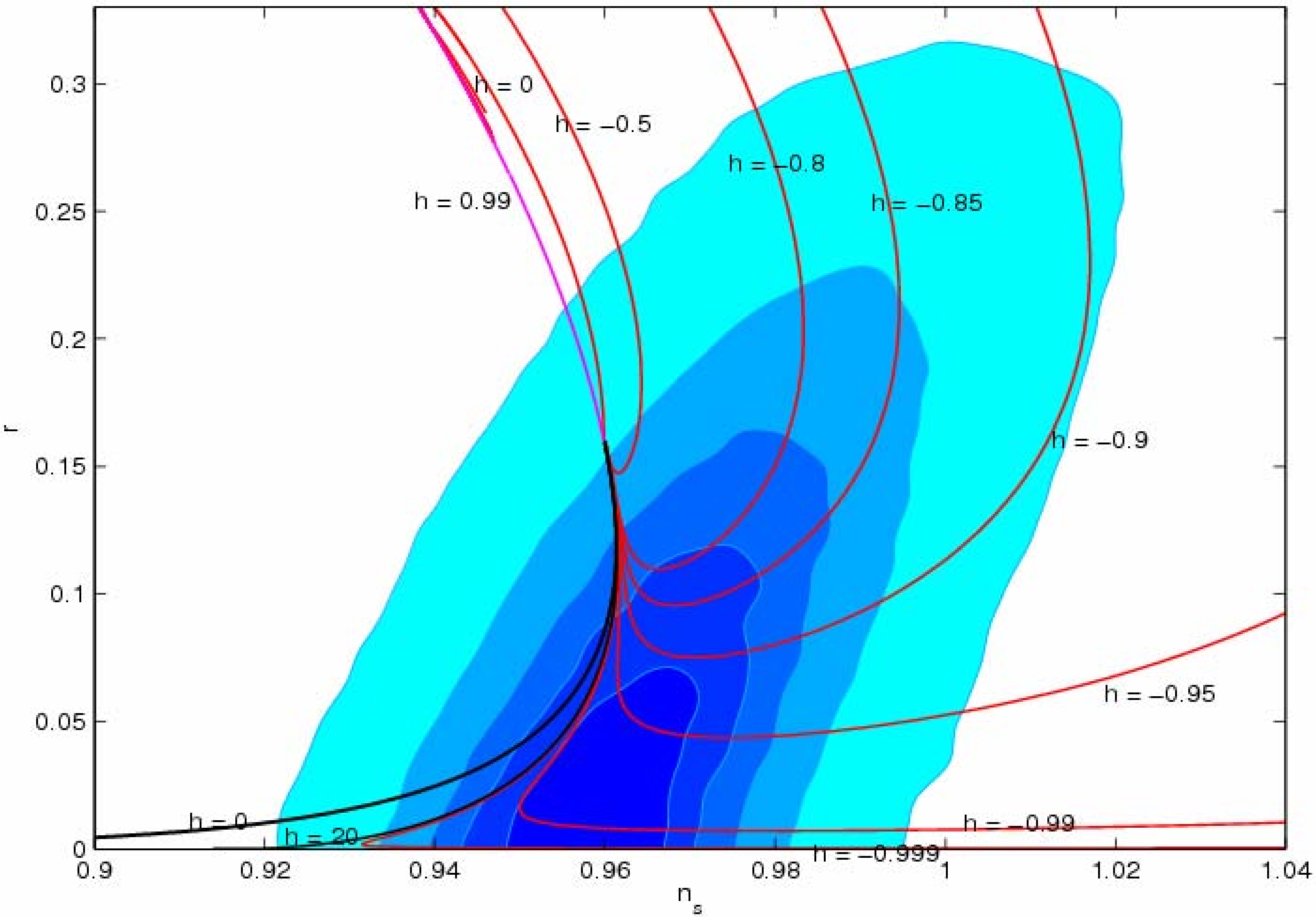
h = asymmetry parameter. $w(\text{min}) = w'(\text{min}) = 0,$
$y =$ quartic coupling, $F(h) = \frac{8}{3}\,h^4 + 4\,h^2 + 1 + \frac{8}{3}\,|h|\;(h^2+1)^{\frac{3}{2}}\;.$

H. J. de Vega, N. G. Sanchez, Single Field Inflation models allowed and ruled out by the three years WMAP data. Phys. Rev. D 74, 063519 (2006), astro-ph/0604136.

r vs. n_s data within the Trinomial New Inflation Region.



MCMC Results for Trinomial New Inflation.



Monte Carlo Markov Chains Analysis of Data: MCMC.

MCMC is an efficient stochastic numerical method to find the probability distribution of the theoretical parameters that describe a set of empirical data. We found n_s, the ratio r of tensor to scalar fluctuations and the couplings by MCMC.

NEW: We imposed as a hard constraint that r and n_s are given by the trinomial potential.

Our analysis differs in this crucial aspect from previous MCMC studies.

MCMC Results for Trinomial New Inflation:

Bounds: $r > 0.016$ (95% CL) , $r > 0.049$ (68% CL)

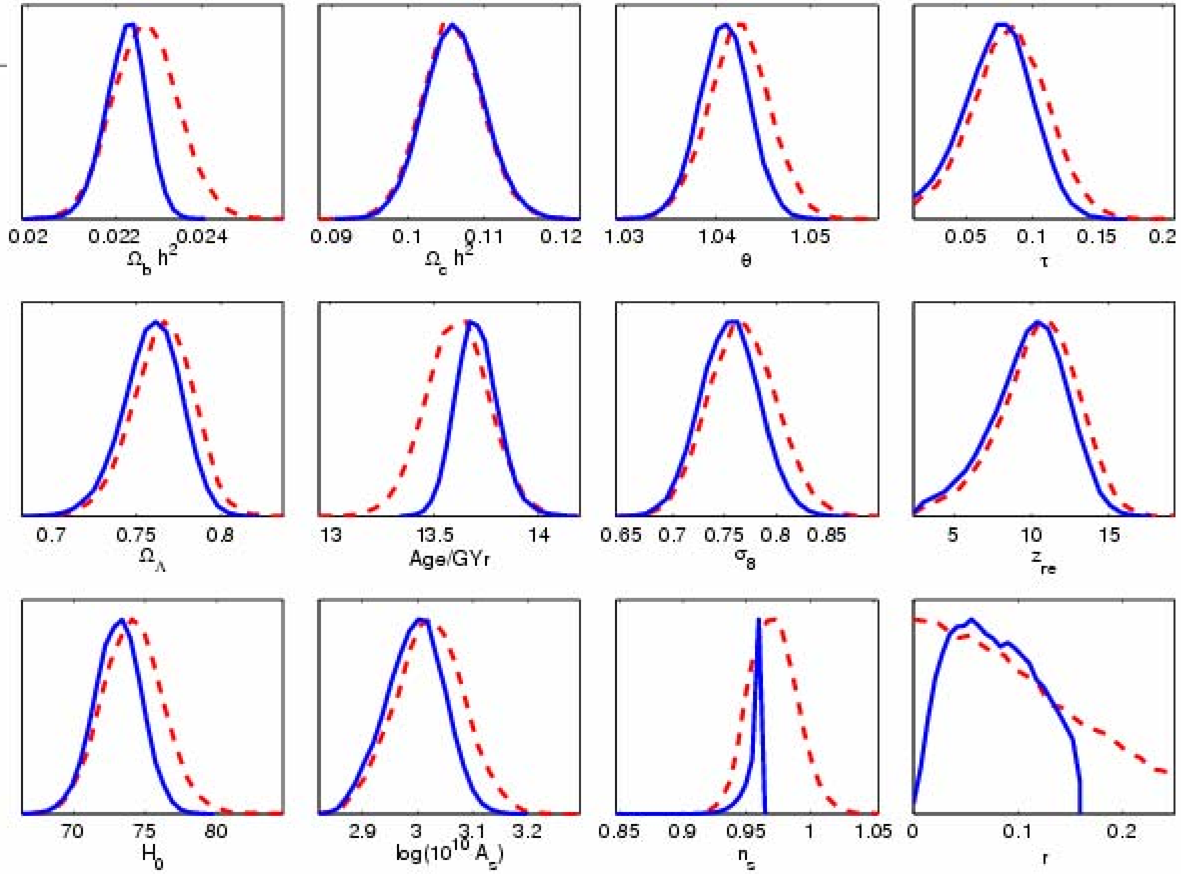
Most probable values: $n_s \simeq 0.956$, $r \simeq 0.055$.

The most probable potential is symmetric and has a moderate nonlinearity:

$$w(\chi) = \frac{y}{32}\left(\chi^2 - \frac{8}{y}\right)^2 \quad , \quad y \simeq 2$$

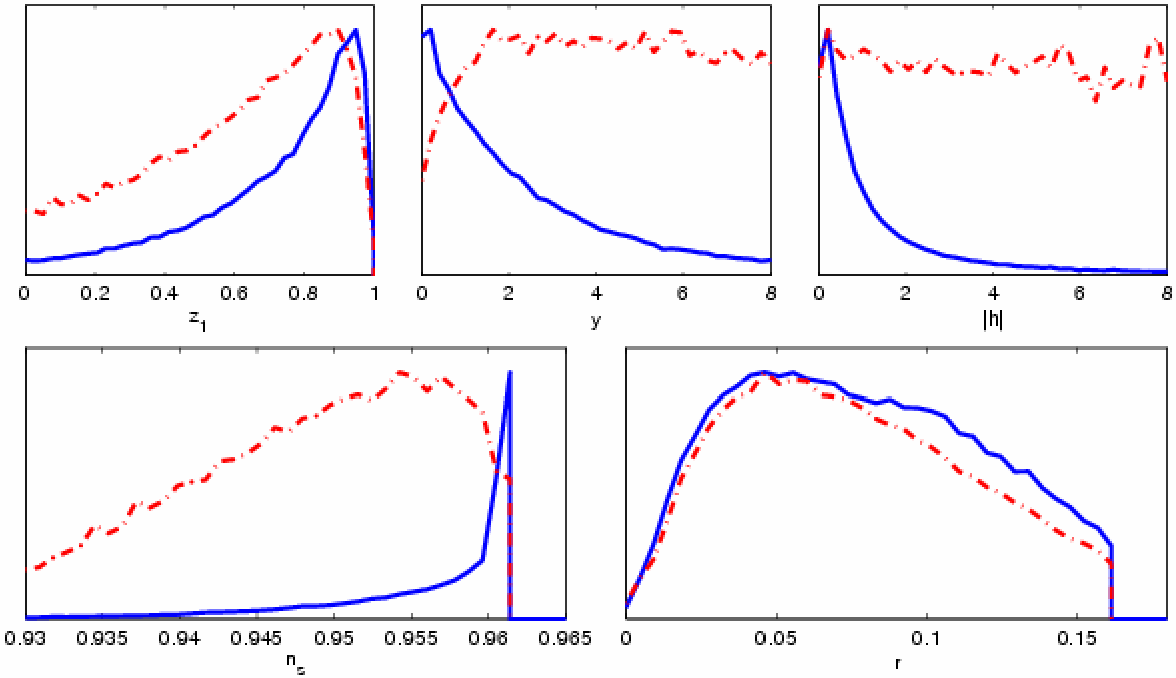
C. Destri, H. J. de Vega, N. Sanchez, astro-ph/0703417, Phys. Rev. D77, 043509 (2008).

Marginalized probability distributions. New Inflation.



Imposing the trinomial potential (solid blue curves) and just the ΛCDM+r model (dashed red curves).
(curves normalized to have the maxima equal to one).

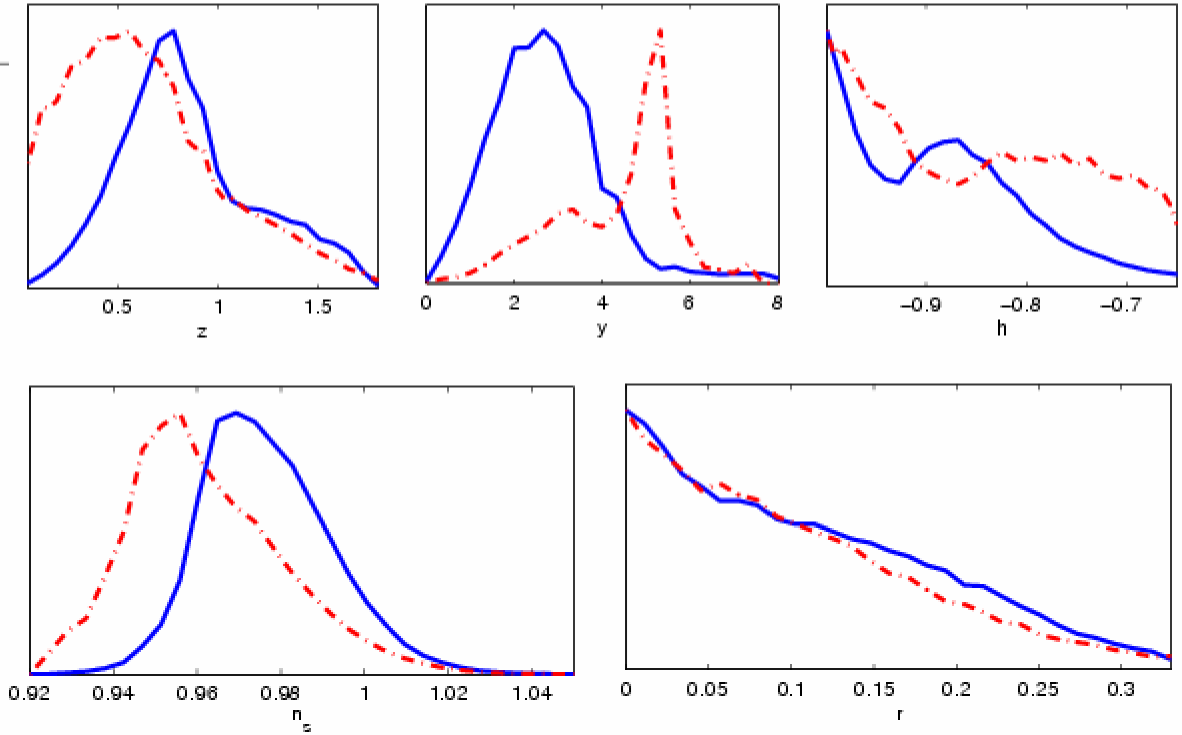
Probability Distributions. Trinomial New Inflation.



Probability distributions: solid blue curves
Mean likelihoods: dot-dashed red curves.

$$z_1 = 1 - \frac{y}{8\left(|h| + \sqrt{h^2+1}\right)^2}\ \chi^2 \ .$$

Probability Distributions. Trinomial Chaotic Inflation.

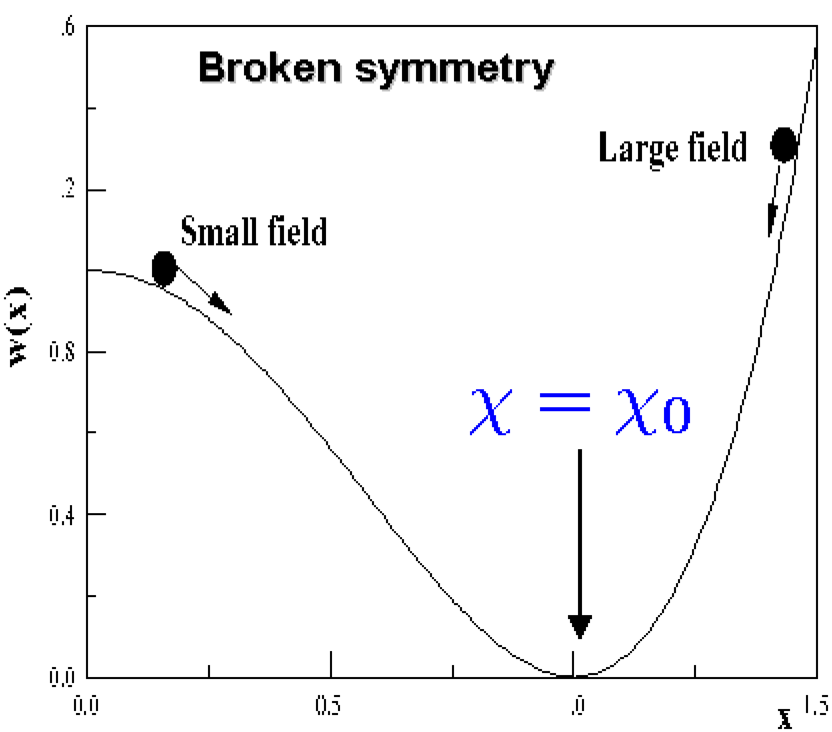


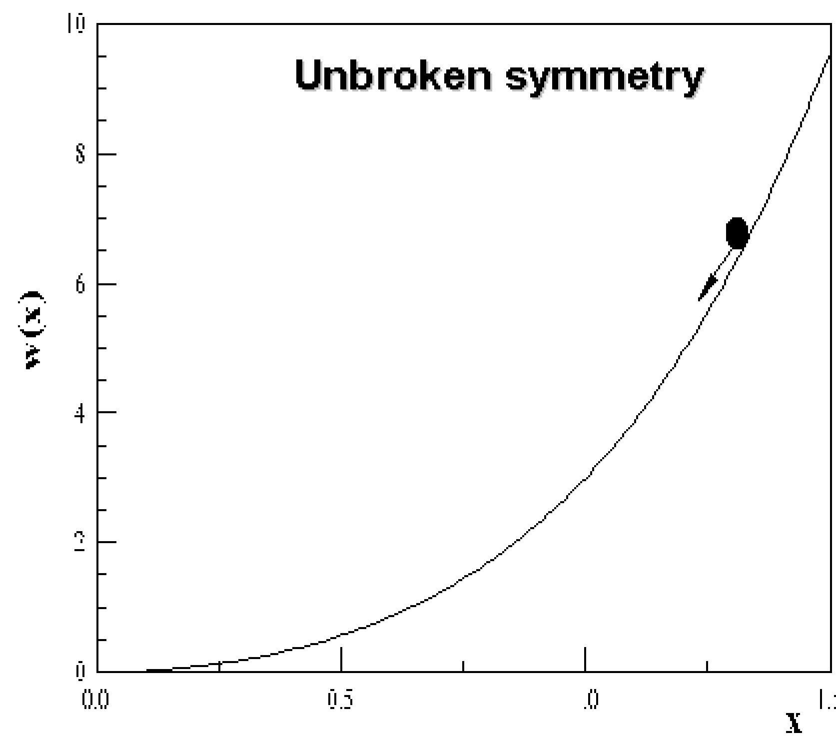
Probability distributions (solid blue curves) and mean likelihoods (dot-dashed red curves).
The data request a strongly asymmetric potential in chaotic inflation almost having two minima. That is, a strong breakdown of the $\chi \to -\chi$ symmetry.

Rescale fields and couplings:

$$\lambda = \frac{m^2 \; g}{M_{Pl}^{2\,n-2} \; N_e^{n-1}} \; ; \; g = \frac{1}{\chi_0^{2\,n-2}} \; ; \; x = \frac{\chi}{\chi_0}$$

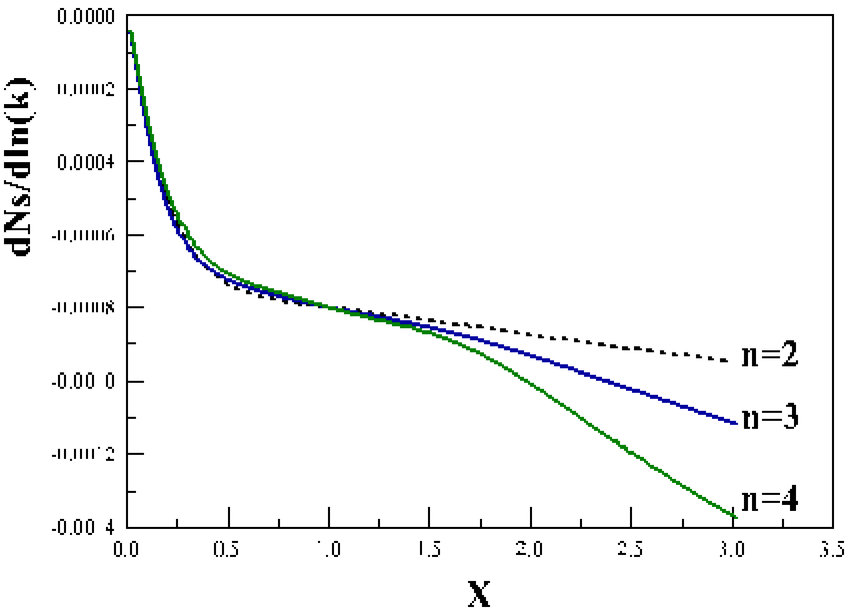
$$w(\chi) = \frac{\chi_0^2}{2\,n} \; \left[n \; (1 - x^2) + x^{2\,n} - 1 \right] \qquad w(\chi) = \frac{\chi_0^2}{2\,n} \; \left[n \, x^2 + x^{2\,n} \right]$$

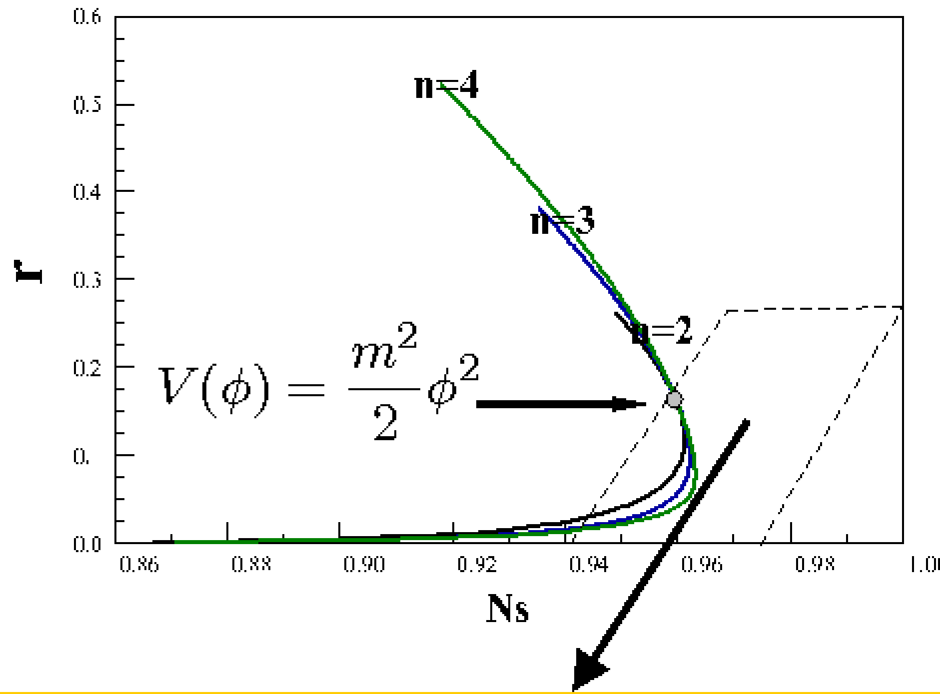




RESULTS

1) New inflation (B.S.) Ne= 50 (change accordingly)

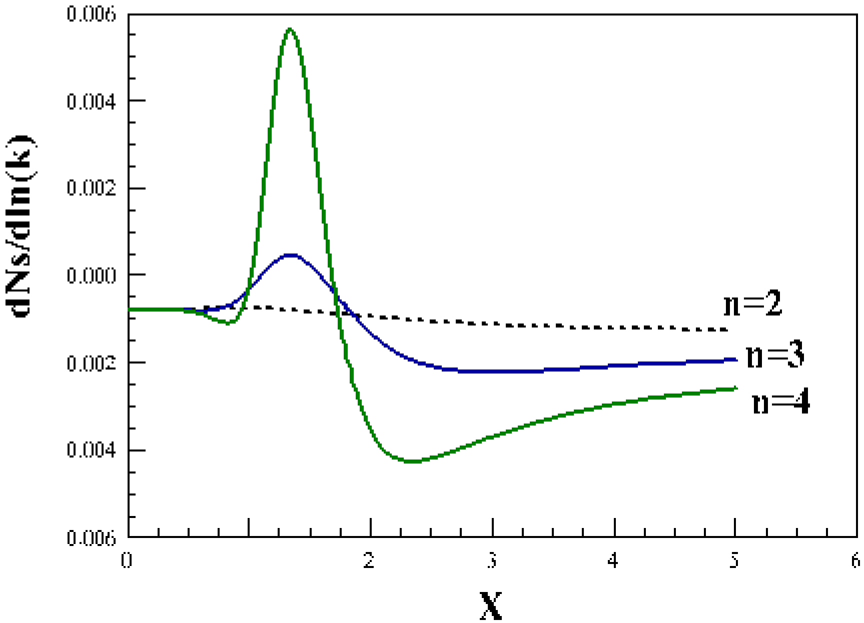


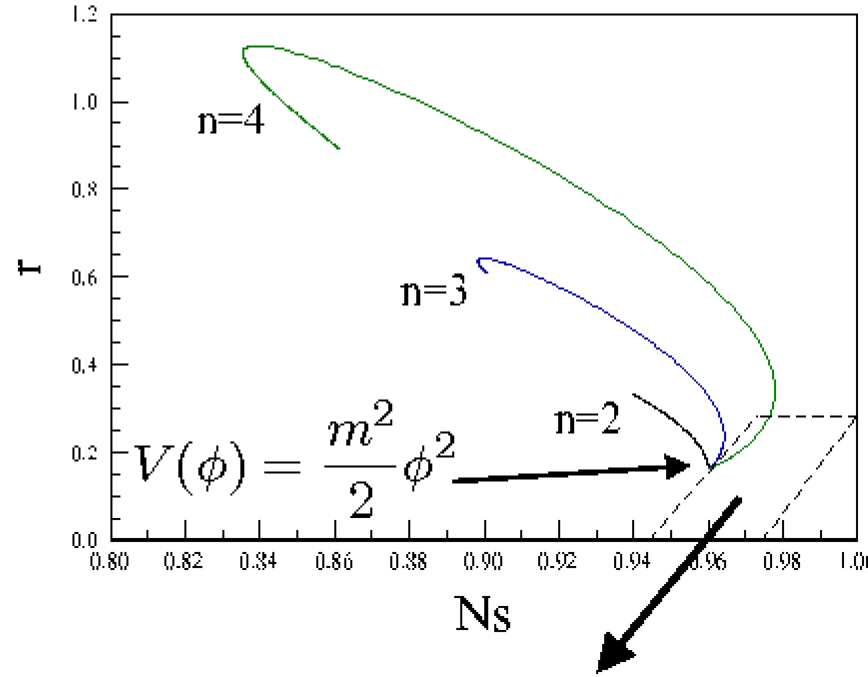


WMAP 3 marginalized region of r- n_s (95% CL)

* *Large region of consistency for small field New Inflation*

2) Chaotic inflation





WMAP 3 marginalized region of r- ns (95% CL)

*** Small region of consistency with WMAP 3**

Higher Order Inflaton Potentials

Till here we considered fourth degree inflaton potentials. Can higher order terms modify the physical results and the observable predictions?

We systematically study the effects produced by higher order terms ($n > 4$) in the inflationary potential on the observables n_s and r.

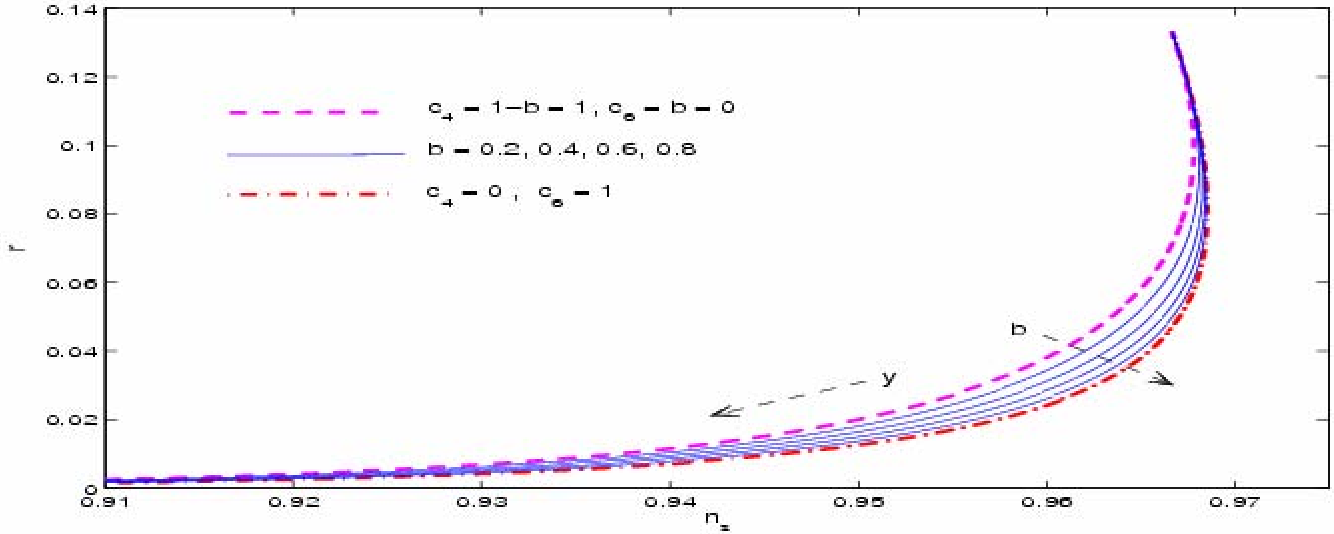
All coefficients in the potential w become order one using the field χ within the Ginsburg-Landau approach:

$$w(\chi) = c_0 - \frac{1}{2}\,\chi^2 + \sum_{n=3}^{\infty} \frac{c_n}{n}\,\chi^n \quad , \quad c_n = \mathcal{O}(1) \; .$$

All $r = r(n_s)$ curves for double–well potentials of arbitrary high order fall **inside** a universal banana-shaped region $\mathcal{B}$.

Moreover, the $r = r(n_s)$ curves for double–well potentials even for arbitrary positive higher order terms lie inside the banana region $\mathcal{B}$.

The sextic double–well inflaton potential



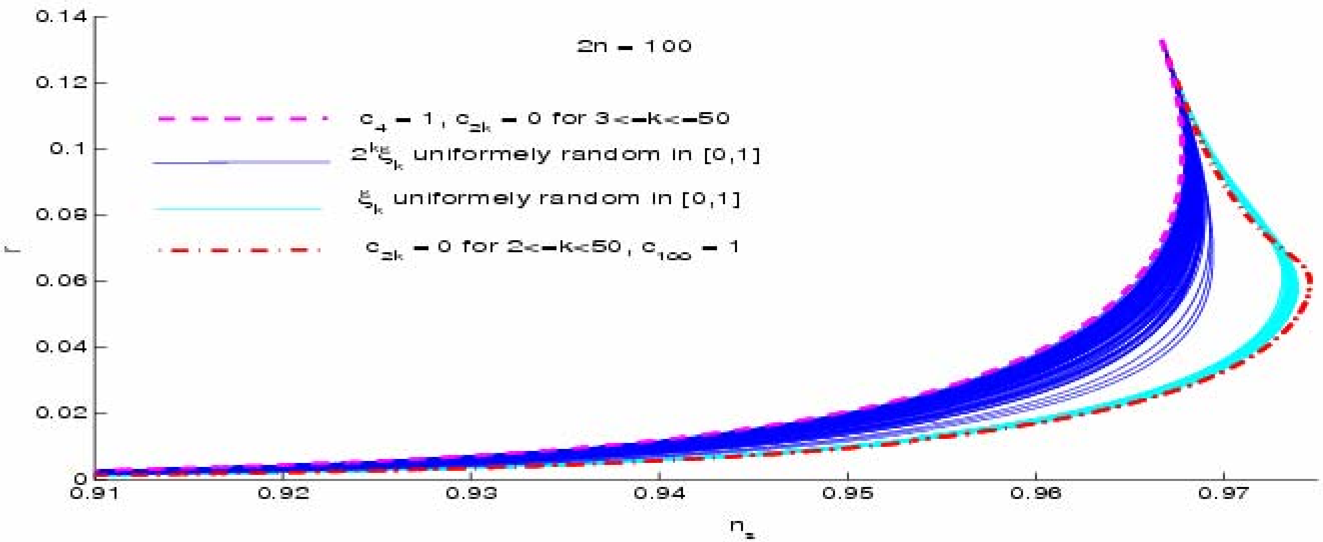
$$w_b(\chi) = \frac{y}{96}\left(\chi^2 - \frac{8}{y}\right)^2 \left(3 + b + \frac{1}{4}\, y\, b\, \chi^2\right) \; .$$

$0 < y < \infty$ coupling. $0 < b < 1$ shape-parameter.

$w_{b=0}(\chi) = \frac{y}{32}\left(\chi^2 - \frac{8}{y}\right)^2$ fourth order double–well.

$w_{b=1}(\chi) = \frac{8}{3\,y} - \frac{1}{2}\,\chi^2 + \frac{y^2}{384}\,\chi^6$ sixth order double–well.

The 100th degree polynomial inflaton potential

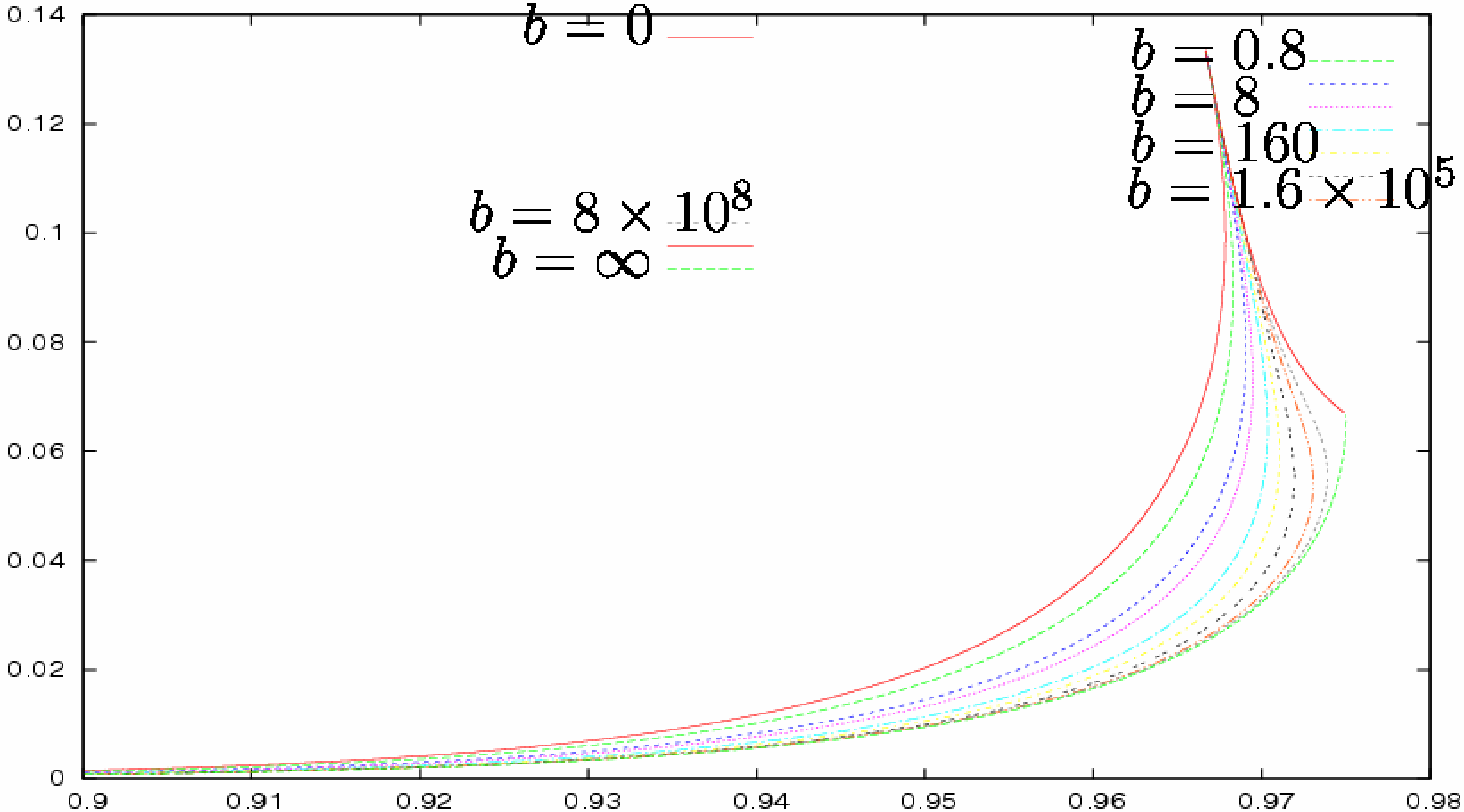


$$w(\chi) = \frac{4}{y} - \frac{1}{2}\chi^2 + \frac{4}{y}\sum_{k=2}^{n} \frac{c_{2k}}{k} \left(\frac{y^k}{8^k} u^{2k} - 1\right)$$

The coefficients c_{2k} were extracted at random.
The lower border of the banana-shaped region is given by the potential:

$$w(\chi) = \frac{4}{y} - \frac{1}{2}\chi^2 + \frac{4}{n\,y} \left(\frac{y^n}{8^n} u^{2n} - 1\right) \text{ with } n = 50.$$

The exponential inflaton potential



$$w(\chi) = \frac{2}{y\, b(e^{2\,b} - 1)} \left[e^{\frac{1}{4}\, y\, b\, \chi^2} - e^{2\,b} \left(1 - 2\,b + \frac{1}{4}\, y\, b\, \chi^2 \right) \right]$$

The upper border of the banana region is obtained for $b = 0$ and the lower border for $b \to \infty$.

The inflaton potential from a fermion condensate

Inflaton coupled to Dirac fermions Ψ during inflation:

$$\mathcal{L} = \overline{\Psi} \left[i \, \gamma^\mu \, \mathcal{D}_\mu - m_f - g_Y \, \phi \right] \Psi$$

$g_Y =$ Yukawa coupling, $\gamma^\mu =$ curved space-time γ-matrices.
Hubble parameter $H =$ constant. Effective potential $\equiv$ fermions energy for a constant inflaton ϕ during inflation.
Dynamically generated inflaton potential:

$$V_f(\phi) = V_0 + \frac{1}{2} \, \mu^2 \, \phi^2 + \frac{1}{4} \, \lambda \, \phi^4 + H^4 \, Q \left(g_Y \, \frac{\phi}{H} \right) \text{, where}$$

$\mu^2 = -m^2 < 0$ mass squared, $\lambda =$ quartic coupling,

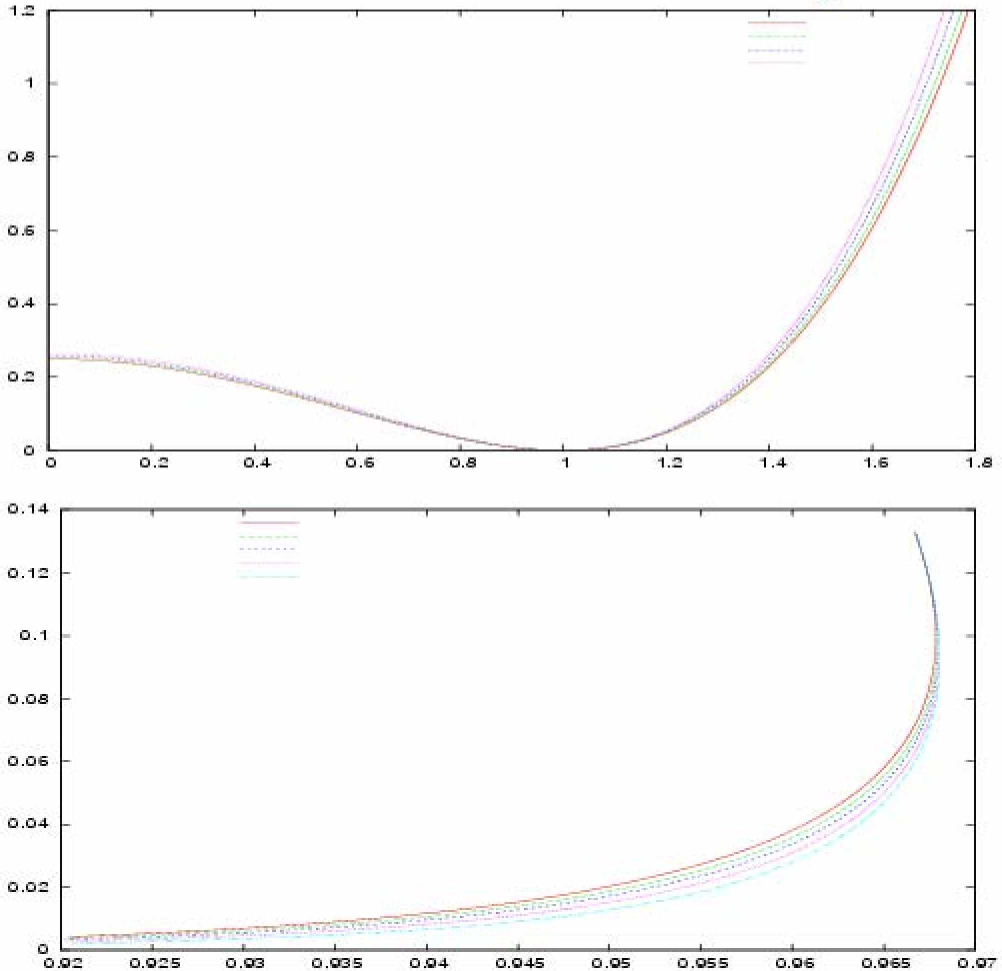
$$Q(x) = \frac{x^2}{8\,\pi^2} \left\{ (1 + x^2) \left[\gamma + \mathrm{Re}\, \psi(1 + i\, x) \right] - \zeta(3) \, x^2 \right\} =$$

$$= \frac{x^4}{8\,\pi^2} \left[(1 + x^2) \sum_{n=1}^{\infty} \frac{1}{n\,(n^2 + x^2)} - \zeta(3) \right] \; , \quad x \equiv g_Y \, \frac{\phi}{H}$$

$$Q(x) \overset{x \to \infty}{=} \frac{x^4}{8\,\pi^2} \left[\log x + \gamma - \zeta(3) + \mathcal{O}\left(\frac{1}{x}\right) \right]$$

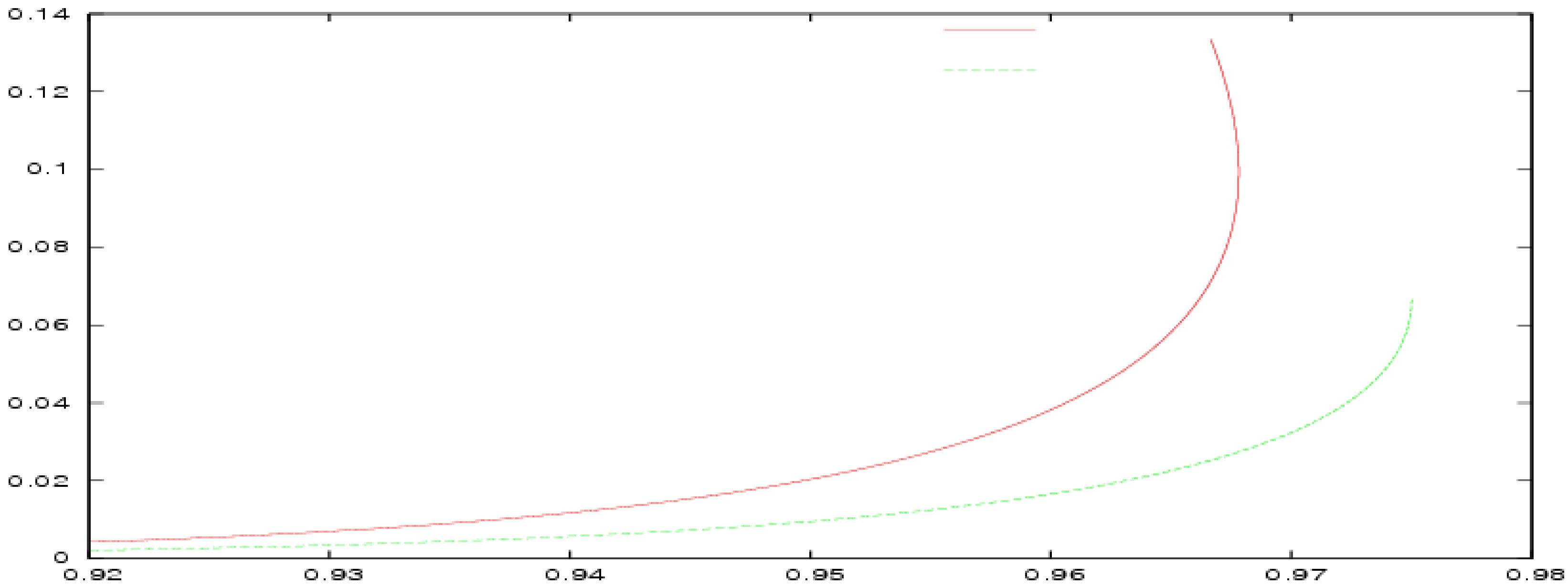
Minkowski limit (Coleman-Weinberg potential)

Effective fermionic inflaton potential and r vs. n_s



$y\, V(\phi)/[8\, N\ M^4]$ vs. ϕ/ϕ_{min} for $0 < g_Y < 500\ H/\phi_{min}$
r vs. n_s for $0 < g_Y < 800\ H/\phi_{min}$

The universal banana region



We find that all $r = r(n_s)$ curves for double–well inflaton potentials in the Ginsburg-Landau spirit fall **inside** the universal banana region $\mathcal{B}$.

The lower border of $\mathcal{B}$ corresponds to the limiting potential:

$$w(\chi) = \frac{4}{y} - \frac{1}{2}\chi^2 \quad \text{for } \chi < \sqrt{\frac{8}{y}} \quad , \quad w(\chi) = +\infty \quad \text{for } \chi > \sqrt{\frac{8}{y}}$$

This gives a lower bound for r for **all** potentials in the Ginsburg-Landau class: $r > 0.021$ for the current best value of the spectral index $n_s = 0.964$.

CONCLUSIONS

Most probable values with the fourth degree double–well inflaton potential: $n_s \simeq 0.964, \; r \simeq 0.051$

Lower bound for r for **all** potentials in the Ginsburg-Landau class: $r > 0.021$ for the current best value $n_s = 0.964$.

Notice that at $n_s = 0.964$:

$\frac{dr}{dn_s} = 4.9$ on the upper border of $\mathcal{B}$ (fourth degree double–well).

$\frac{dr}{dn_s} = 1.35$ on the lower border of $\mathcal{B}$.

Notice that an improvement δ on the precision of n_s implies an improvement $\simeq 5 \; \delta$ on the precision of r for the fourth degree double–well potential.

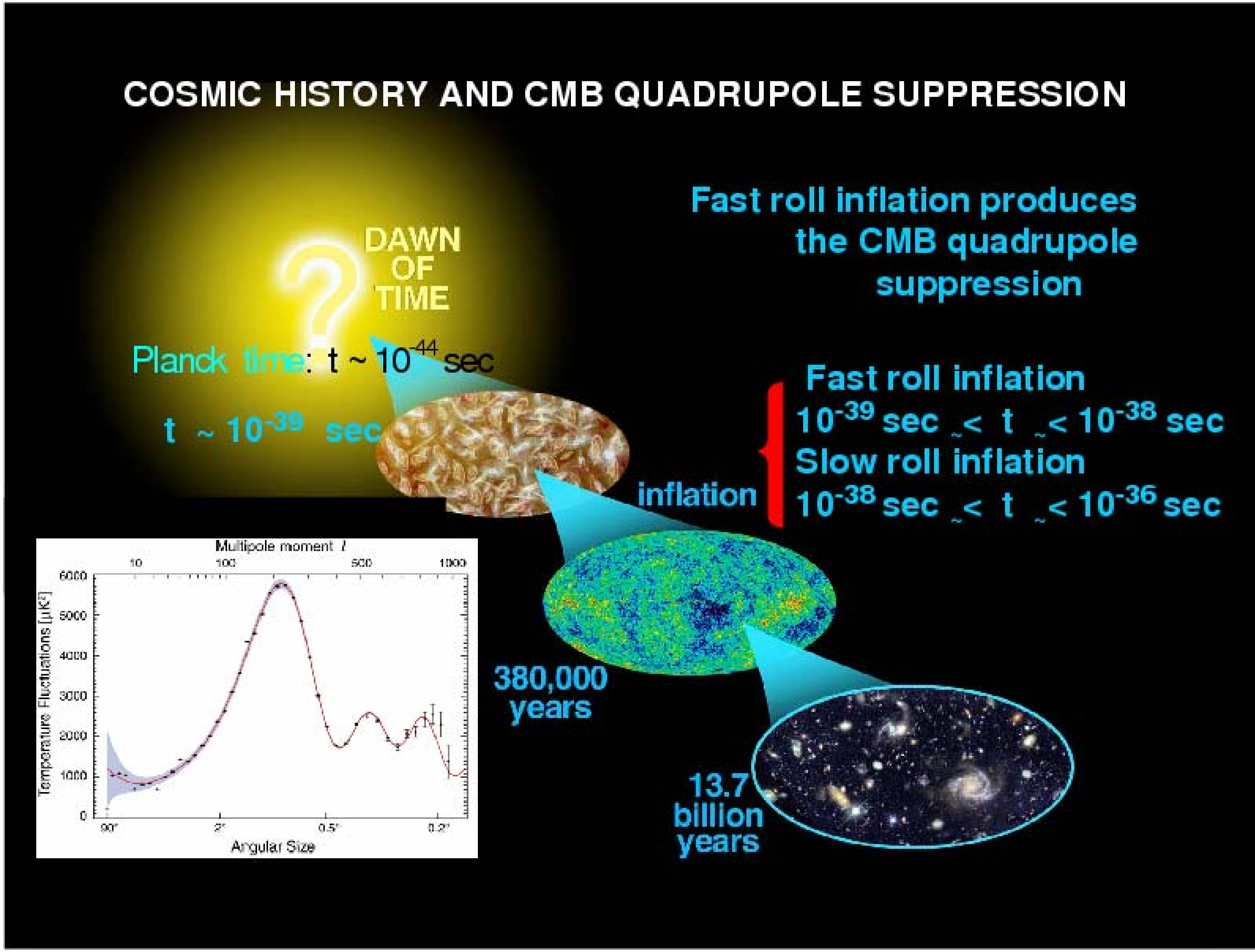
PREDICTIONS

From the upper universal curve:
UPPER BOUND $r < 0.053$

From the lower universal curve:
LOWER BOUND $r > 0.021$

$$0.021 < r < 0.053$$

Most probable value: $r \sim 0.051$


COSMIC HISTORY AND CMB QUADRUPOLE SUPPRESSION
Fast roll inflation produces the CMB quadrupole suppression
DAWN OF TIME
Planck time: t ~ 10^-44 sec
t ~ 10^-39 sec
Fast roll inflation
10^-39 sec ≲ t ≲ 10^-38 sec
Slow roll inflation
10^-38 sec ≲ t ≲ 10^-36 sec
inflation
380,000 years
13.7 billion years
Multipole moment l
10
100
500
1000
Temperature Fluctuations [μK²]
6000
5000
4000
3000
2000
1000
0
90°
2°
0.5°
0.2°
Angular Size

Fast and Slow Roll Inflation

$$H^2 = \frac{1}{3\, M_{PL}^2} \left[\frac{1}{2}\, \dot{\Phi}^2 + V(\Phi)\right] \ ,$$

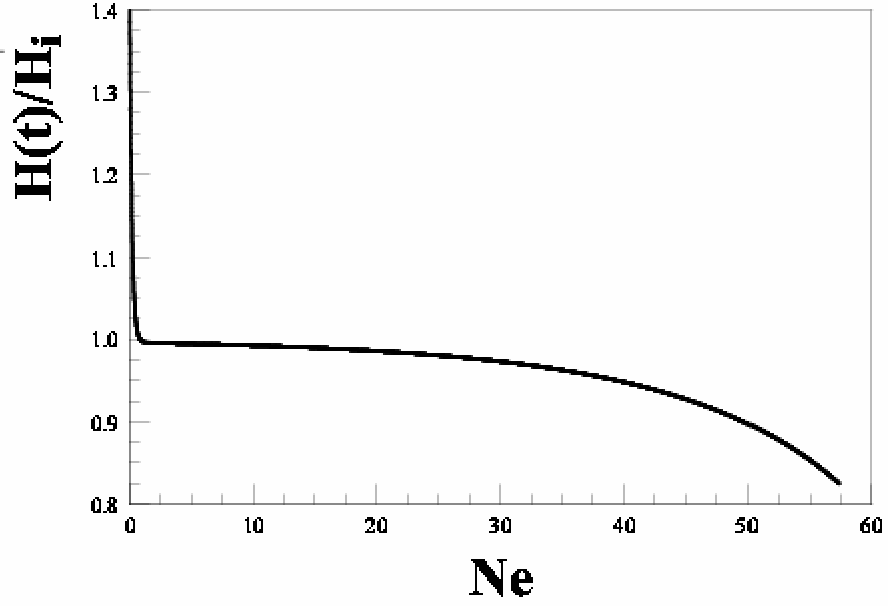
$$\ddot{\Phi} + 3\, H\, \dot{\Phi} + V'(\Phi) = 0\ .$$

Slow-roll corresponds to $\dot{\Phi}^2 \ll V(\Phi)$.

Generically, we can have $\dot{\Phi}^2 \sim V(\Phi)$ to start.
That is, FAST ROLL inflation.

However, slow-roll is an attractor with a large basin.

Hubble vs. number of efolds



$H_i =$ Hubble at the beginning of slow-roll.
Fast-roll lasts about one-efold.

Extreme fast roll solution ($y^2 = 3$) in cosmic time:

$$H = \frac{1}{3\,t} \quad , \quad a(t) = a_0 \, t^{\frac{1}{3}} \quad , \quad \Phi = -M_{Pl} \sqrt{\frac{2}{3}} \, \log(m\,t) \; .$$

A: a fast-roll stage PRIOR to slow roll

Allowing for ***RAPID*** variation of the condensate ϕ

$$\left[\frac{d^2}{d\eta^2} + k^2 - \frac{\nu^2 - \frac{1}{4}}{\eta^2} - \mathcal{V}(\eta)\right] v_k = 0$$

↓

Depends on high(er) derivatives of ϕ : ***neglible in slow roll***, ***large for fast roll***

WHEN? large INITIAL $\dot{\phi}$ but large FRICTION term → short fast roll stage

→ $\mathcal{V}(\eta) =$ ***LOCALIZED POTENTIAL***

$D(k) \propto T(k)$ = Transmission coeff. of SCATTERING PBM!!

General solution: $S(k;\eta) = A(k)\, g_\nu(k;\eta) + B(k)\, [g_\nu(k;\eta)]^*$

With normalization condition $|A(k)|^2 - |B(k)|^2 = 1$

Quantization:

$$v_k(\eta) = a_k S(k;\eta) + a_k^\dagger S^*(k;\eta)$$

B(k)=0 → B.D. vacuum

Power spectrum

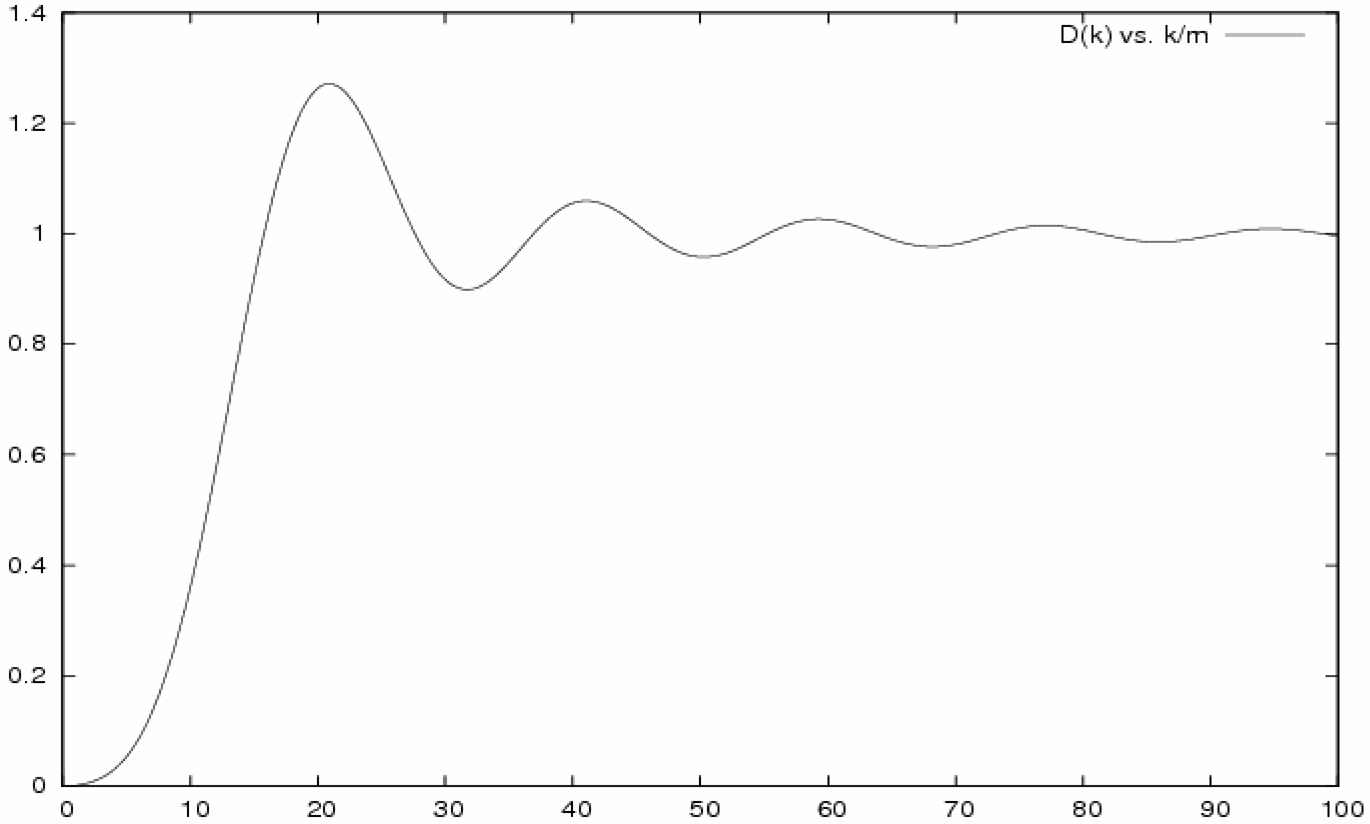
$$\mathcal{P}(k) = \langle 0 | |\delta\Psi_k|^2 | 0 \rangle = \mathcal{P}_{BD}(k)\,[1 + D(k)]$$

$$\mathcal{P}_{BD}(k) = \frac{H^2}{\epsilon_v M_P^2} \left(\frac{k}{k_0} \right)^{n_s - 1}$$

$D(k)$: **Transfer function for boundary conditions**

$$n_s = 1 - 6\epsilon_v + 2\eta_v$$

The Fast-Roll Transfer Function



$k_Q = 11.5\ m,\ k_{fastroll \to slowroll} = 14\ m,\ k_{pivot} = 96.7\ m,$

$m = 1.21\,10^{13}\text{GeV}, k_Q^{today} = 0.238\ \text{Gpc}^{-1}$ $\Longrightarrow$ redshift at the beginning of inflation $= 0.9 \times 10^{56} \simeq e^{129}$.

Quadrupole suppression and Fast-roll Inflation

The observed CMB-quadrupole (COBE,WMAP5) is six times smaller than the ΛCDM model value.
In the best ΛCDM fit the probability that the quadrupole is as low or lower than the observed value is 3%.
It is hence relevant to find a cosmological explanation of the quadrupole supression.

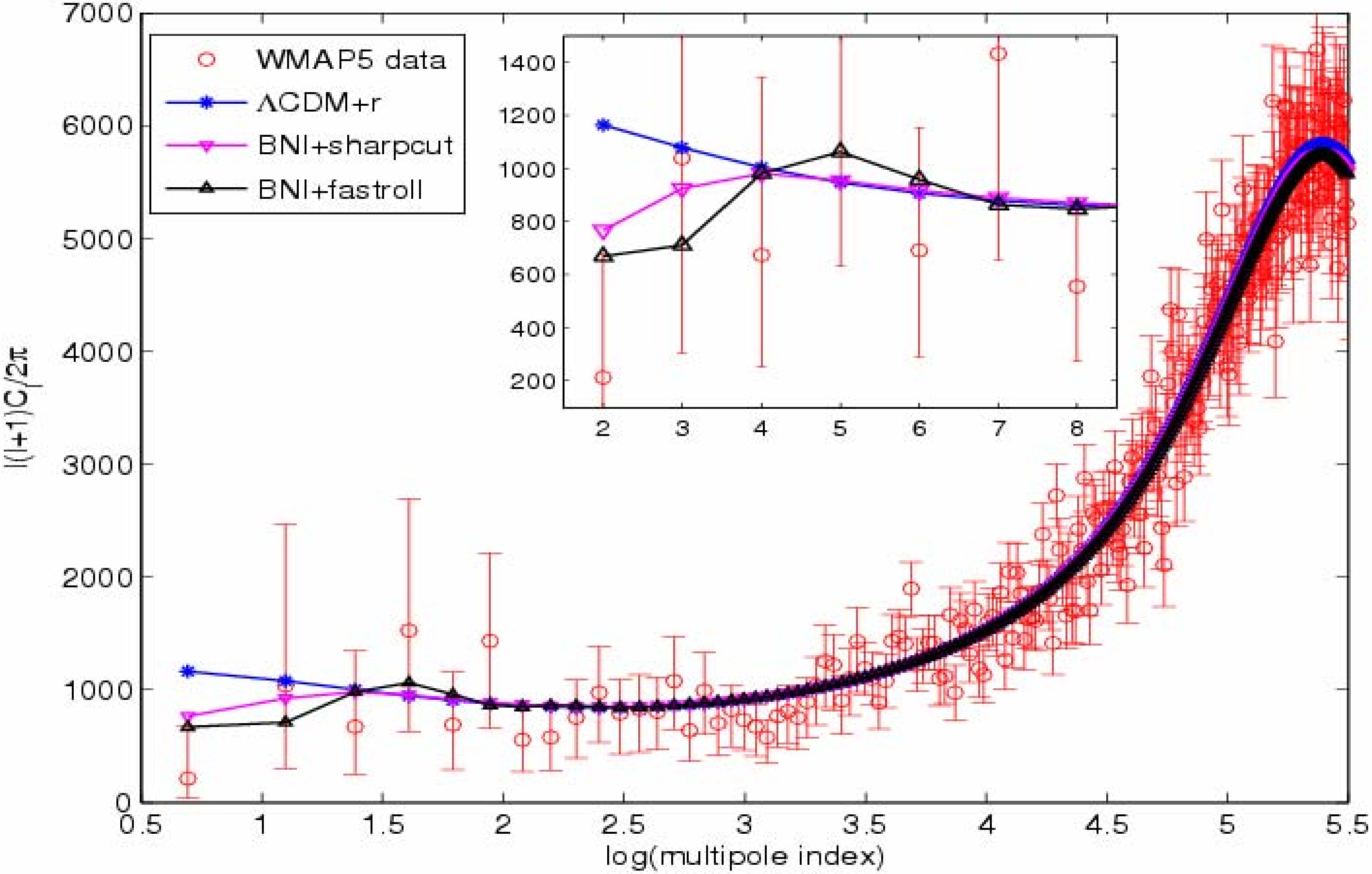
Generically, the classical evolution of the inflaton has a brief **fast-roll stage** that precedes the slow-roll regime.
In case the quadrupole CMB mode leaves the horizon during fast-roll, before slow-roll starts, we find that the quadrupole mode gets suppressed.

$$P(k) = |\Delta_{k\ ad}^{(S)}|^2 \ (k/k_0)^{n_s - 1}[1 + D(k)]$$

The transfer function $D(k)$ changes the primordial power.
$1 + D(0) = 0, \quad D(\infty) = 0$

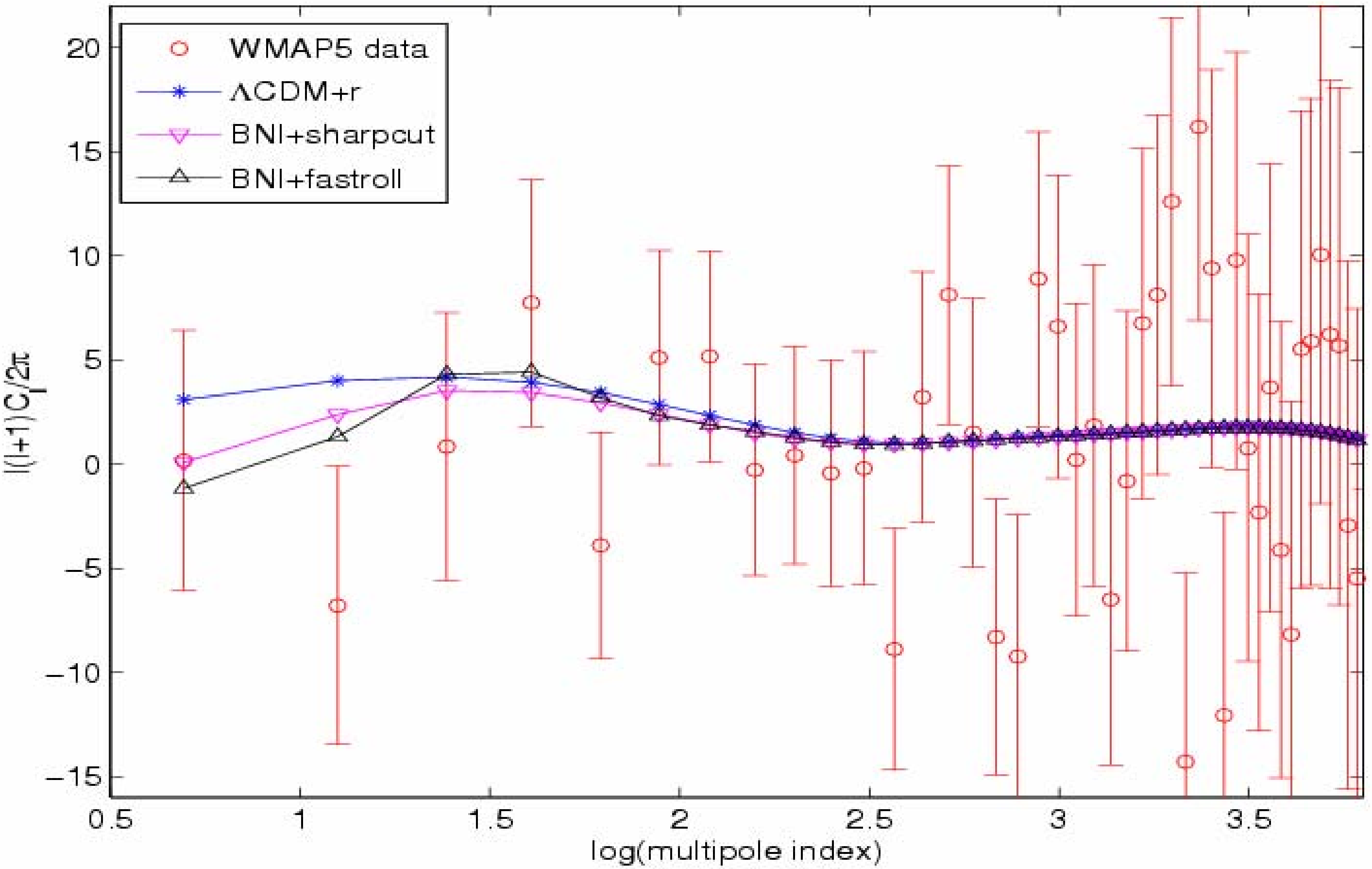
Comparison, with the experimental WMAP5 data

of the theoretical C_ℓ^{TT} multipoles



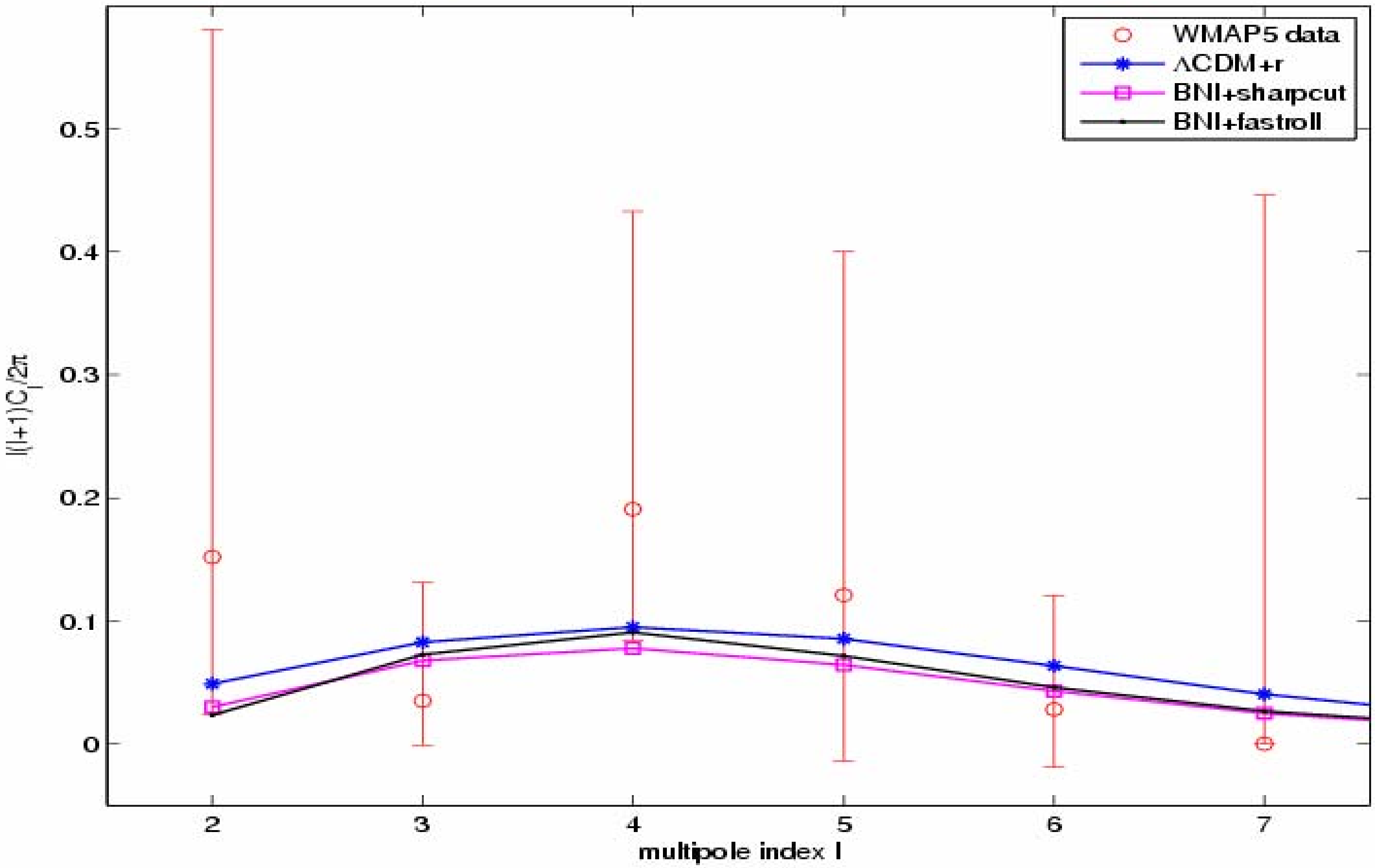
Comparison, with the experimental WMAP5 data

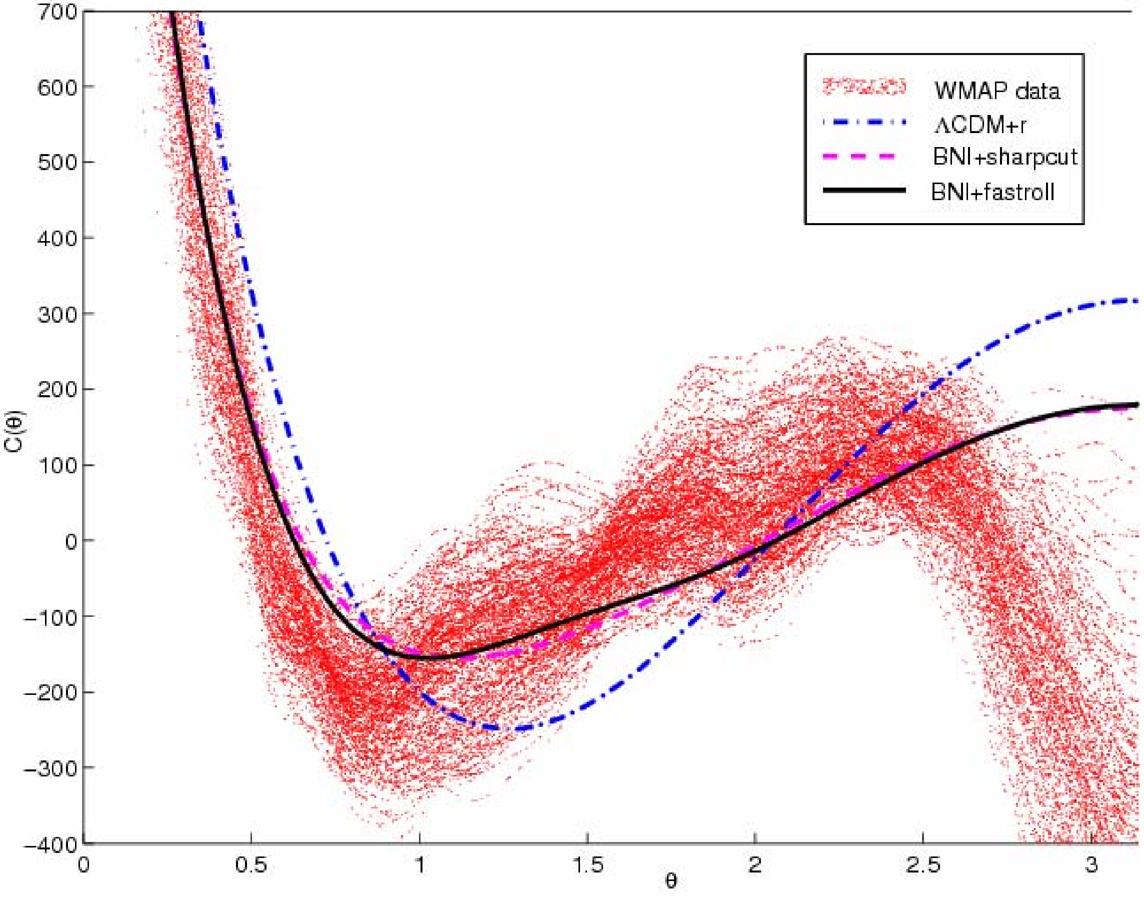
of the theoretical C_ℓ^{TE} multipoles



Comparison, with the experimental WMAP-5 data

of the theoretical $C_\ell^{\rm EE}$ multipoles





WMAP data
ΛCDM+r
BNI+sharpcut
BNI+fastroll
C(θ)
θ
700
600
500
400
300
200
100
0
−100
−200
−300
−400
0
0.5
1
1.5
2
2.5
3

Summary and Conclusions

- Inflation can be formulated as an effective field theory in the Ginsburg-Landau spirit with energy scale $M \sim M_{GUT} \sim 10^{16}$ GeV $\ll M_{Pl}$.
  Inflaton mass small: $m \sim H/\sqrt{N} \sim M^2/M_{Pl} \ll M$.
  Infrared regime !!
- The slow-roll approximation is a $1/N$ expansion, $N \sim 60$
- MCMC analysis of WMAP+LSS data plus the Trinomial Inflation potential indicates a spontaneously symmetry breaking potential (new inflation): $w(\chi) = \frac{y}{32}\left(\chi^2 - \frac{8}{y}\right)^2$.
- Lower Bounds: $r > 0.016$ (95% CL) , $r > 0.049$ (68% CL).
  The most probable values are $n_s \simeq 0.956$, $r \simeq 0.055$ with a quartic coupling $y \simeq 1.3$.

Summary and Conclusions 2

- CMB quadrupole suppression may be explained by the effect of fast-roll inflation provided the today's horizon size modes exited by the end of fast-roll inflation.
- Quantum (loop) corrections in the effective theory are of the order $(H/M_{Pl})^2 \sim 10^{-9}$. Same order of magnitude as loop graviton corrections.

D. Boyanovsky, H. J. de Vega, N. G. Sanchez,

Quantum corrections to the inflaton potential and the power spectra from superhorizon modes and trace anomalies, Phys. Rev. D 72, 103006 (2005), astro-ph/0507596.

Quantum corrections to slow roll inflation and new scaling of superhorizon fluctuations. Nucl. Phys. B 747, 25 (2006), astro-ph/0503669.

Quantum Corrections to the Inflaton Potential

Upon UV renormalization the Friedmann equation results

$$H^2 = \frac{1}{3\,M_{Pl}^2}\left[\frac{1}{2}\,\dot{\Phi_0}^2 + V_R(\Phi_0) + \left(\frac{H_0}{4\,\pi}\right)^2 \frac{V_R''(\Phi_0)}{\Delta} + \mathcal{O}\left(\frac{1}{N}\right)\right]$$

Quantum corrections are proportional to $\left(\frac{H}{M_{Pl}}\right)^2 \sim 10^{-9}$!!

The Friedmann equation gives for the effective potential:

$$V_{eff}(\Phi_0) = V_R(\Phi_0) + \left(\frac{H_0}{4\,\pi}\right)^2 \frac{V_R''(\Phi_0)}{\Delta}$$

$$V_{eff}(\Phi_0) = V_R(\Phi_0)\left[1 + \left(\frac{H_0}{4\,\pi\,M_{Pl}}\right)^2 \frac{\eta_V}{\eta_V - \epsilon_V}\right]$$

in terms of slow-roll parameters

Very DIFFERENT from the one-loop effective potential in Minkowski space-time:

$$V_{eff}(\Phi_0) = V_R(\Phi_0) + \frac{[V_R''(\Phi_0)]^2}{64\,\pi^2}\ln\frac{V_R''(\Phi_0)}{M^2}$$

Quantum Fluctuations:

Scalar Curvature, Tensor, Fermion, Light Scalar.

All these quantum fluctuations contribute to the inflaton potential and to the primordial power spectra.

In de Sitter space-time: $< T_{\mu\nu} >= \frac{1}{4} \, g_{\mu\nu} \;\; < T^{\alpha}_{\alpha} >$

Hence, $V_{eff} = V_R + < T^0_0 >= V_R + \frac{1}{4} < T^{\alpha}_{\alpha} >$

Sub-horizon (Ultraviolet) contributions appear through the trace anomaly and only depend on the spin of the particle.

Superhorizon (Infrared) contributions are of the order N^0 and can be expressed in terms of the slow-roll parameters.

$$V_{eff}(\Phi_0) = V(\Phi_0) \left[1 + \frac{H_0^2}{3 \, (4\pi)^2 \, M_{Pl}^2} \left(\frac{\eta_v - 4 \, \epsilon_v}{\eta_v - 3 \, \epsilon_v} + \frac{3 \, \eta_\sigma}{\eta_\sigma - \epsilon_v} + \mathcal{T} \right) \right]$$

$\mathcal{T} = \mathcal{T}_\Phi + \mathcal{T}_s + \mathcal{T}_t + \mathcal{T}_F = -\frac{2903}{20}$ is the total trace anomaly.

$\mathcal{T}_\Phi = \mathcal{T}_s = -\frac{29}{30}, \; \mathcal{T}_t = -\frac{717}{5}, \; \mathcal{T}_F = \frac{11}{60}$

$\longrightarrow$ the graviton (t) dominates.

Corrections to the Primordial Scalar and Tensor Power

$$|\Delta^{(S)}_{k,eff}|^2 = |\Delta^{(S)}_{k}|^2 \{1+$$
$$+\frac{2}{3}\left(\frac{H_0}{4\,\pi\,M_{Pl}}\right)^2 \left[1+\frac{\frac{3}{8}\,r\,(n_s-1)+2\,\frac{dn_s}{d\ln k}}{(n_s-1)^2}+\frac{2903}{40}\right]\Bigg\}$$
$$|\Delta^{(T)}_{k,eff}|^2 = |\Delta^{(T)}_{k}|^2 \left\{1-\frac{1}{3}\left(\frac{H_0}{4\,\pi\,M_{Pl}}\right)^2 \left[-1+\frac{1}{8}\,\frac{r}{n_s-1}+\frac{2903}{20}\right]\right\}.$$

The anomaly contribution $-\frac{2903}{20} = -145.15$ DOMINATES as long as the number of fermions less than 783.

The scalar curvature fluctuations $|\Delta^{(S)}_{k}|^2$ are ENHANCED and the tensor fluctuations $|\Delta^{(T)}_{k}|^2$ REDUCED.

However, $\left(\frac{H}{M_{Pl}}\right)^2 \sim 10^{-9}$.

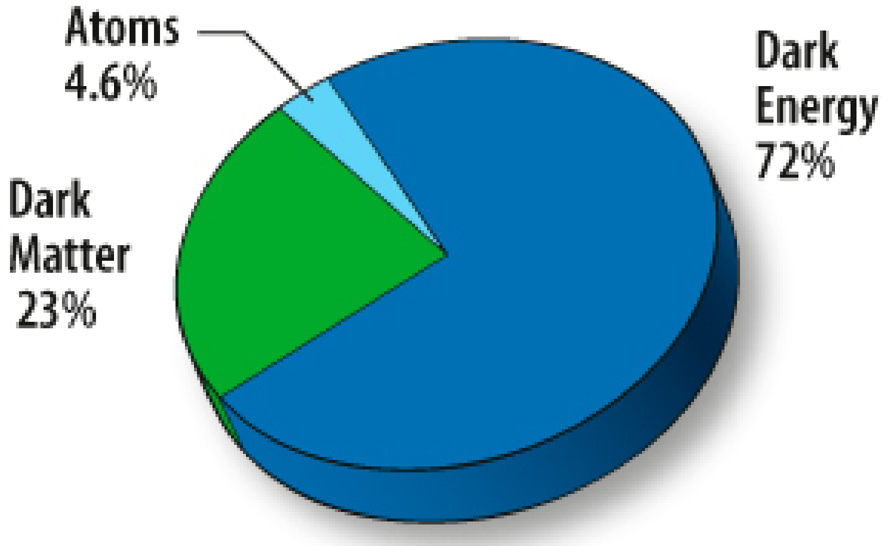
D. Boyanovsky, H. J. de Vega, N. G. Sanchez, Phys. Rev. D 72, 103006 (2005), astro-ph/0507596.

CONTENT OF THE UNIVERSE

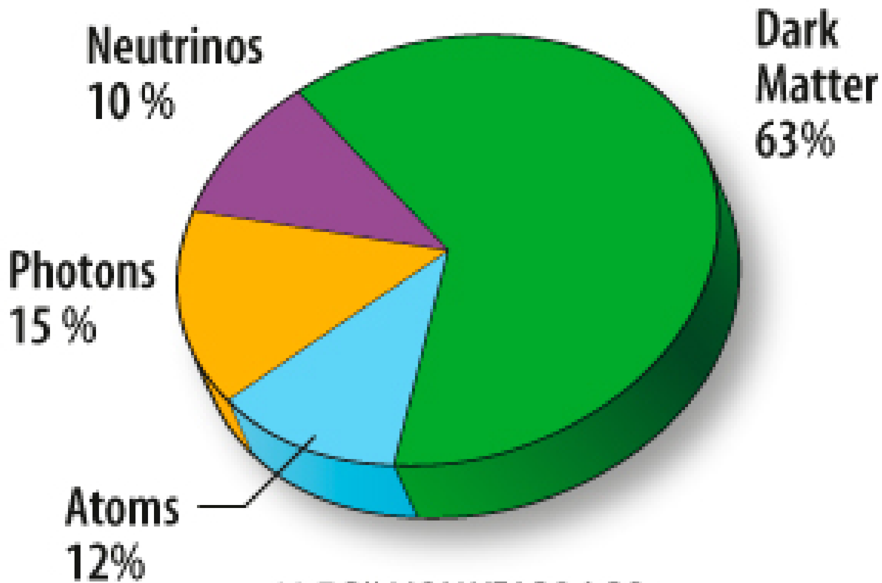
WMAP data reveals that its contents include 4.6% atoms, the building blocks of stars and planets.

Dark matter comprises 23% of the universe. This matter, different from atoms, does not emit or absorb light. It has only been detected indirectly by its gravity.

72% of the Universe, is composed of "dark energy", that acts as a sort of an anti-gravity. This energy, distinct from dark matter, is responsible for the present-day acceleration of the universal expansion.



TODAY



13.7 BILLION YEARS AGO
(Universe 380,000 years old)

Dark Matter

DM must be non-relativistic by structure formation ($z < 30$) in order to reproduce the observed small structures at $\sim 2-3$ kpc. DM particles can decouple being ultrarelativistic (UR) at $T_d \gg m$ or non-relativistic $T_d \ll m$.

Consider particles that decouple at or out of LTE (LTE = local thermal equilibrium).

Distribution function:

$f_d[a(t)\ P_f(t)] = f_d[p_c]$ freezes out at decoupling.

$P_f(t) = p_c/a(t) =$ Physical momentum.

$p_c =$ comoving momentum.

Velocity fluctuations: $y = P_f(t)/T_d(t) = p_c/T_d$

$$\langle \vec{V}^2(t) \rangle = \langle \frac{\vec{P}_f^2(t)}{m^2} \rangle = \frac{\int \frac{d^3 P_f}{(2\pi)^3} \frac{\vec{P}_f^2}{m^2} f_d[a(t)\ P_f]}{\int \frac{d^3 P_f}{(2\pi)^3} f_d[a(t)\ P_f]} = \left[\frac{T_d}{m\ a(t)}\right]^2 \frac{\int_0^\infty y^4 f_d(y) dy}{\int_0^\infty y^2 f_d(y) dy} .$$

Dark Matter density and DM velocity dispersion

Energy Density: $\rho_{DM}(t) = g \int \frac{d^3 P_f}{(2\pi)^3} \sqrt{m^2 + P_f^2} \, F_d[a(t) \, P_f]$

g : # of internal degrees of freedom of the DM particle, $1 \leq g \leq 4$. For $z \lesssim 30 \Rightarrow$ DM particles are non-relativistic:

$$\rho_{DM}(t) = \frac{m \, g}{2\pi^2} \frac{T_d^3}{a^3(t)} \int_0^\infty y^2 \, F_d(y) \, dy \; ,$$

Using entropy conservation: $T_d = \left(\frac{2}{g_d}\right)^{\frac{1}{3}} T_{CMB}$,

$g_d =$ effective # of UR degrees of freedom at decoupling, $T_{CMB} = 0.2348 \, 10^{-3}$ eV, and

$$\rho_{DM}(\text{today}) = \frac{m \, g}{\pi^2 \, g_d} T_{CMB}^3 \int_0^\infty y^2 \, F_d(y) \, dy = 1.107 \, \frac{\text{keV}}{\text{cm}^3} \quad (1)$$

We obtain for the primordial velocity dispersion:

$$\sigma_{DM}(z) = \sqrt{\frac{1}{3} \langle \vec{V}^2 \rangle(z)} = 0.05124 \, \frac{1+z}{g_d^{\frac{1}{3}}} \left[\frac{\int_0^\infty y^4 \, F_d(y) \, dy}{\int_0^\infty y^2 \, F_d(y) \, dy}\right]^{\frac{1}{2}} \frac{\text{keV}}{m} \frac{\text{km}}{\text{s}}$$

Goal: determine m and g_d. We need TWO constraints.

The Phase-space density ρ/σ^3 and its decrease factor Z

The phase-space density $\frac{\rho}{\sigma^3}$ is invariant under the cosmological expansion and can only decrease under self-gravity interactions (gravitational clustering).

The phase-space density today follows observing dwarf spheroidal satellite galaxies in the Milky Way (dSphs)

$\frac{\rho_s}{\sigma_s^3} \sim 5 \times 10^3 \, \frac{\text{keV}/\text{cm}^3}{(\text{km}/\text{s})^3} = (0.18 \text{ keV})^4$ Gilmore et al. 07 and 08.

During structure formation $(z \lesssim 30)$, ρ/σ^3 decreases by a factor that we call Z.

$$\frac{\rho_s}{\sigma_s^3} = \frac{1}{Z} \, \frac{\rho_{DM}}{\sigma_{DM}^3} \quad (2)$$

N-body simulations results: $1000 > Z > 1$.

Constraints: First $\rho_{DM}(\text{today})$, Second $\rho/\sigma^3(\text{today}) = \rho_s/\sigma_s^3$

Mass Estimates for DM particles

Combining the previous expressions lead to general formulas for m and g_d:

$$m = 0.2504\,\mathrm{keV} \left(\frac{Z}{g}\right)^{\frac{1}{4}} \frac{\left[\int_0^\infty y^4 \, F_d(y) \, dy\right]^{\frac{3}{8}}}{\left[\int_0^\infty y^2 \, F_d(y) \, dy\right]^{\frac{5}{8}}}$$

$$g_d = 35.96\, Z^{\frac{1}{4}} \, g^{\frac{3}{4}} \left[\int_0^\infty y^4 \, F_d(y) \, dy \int_0^\infty y^2 \, F_d(y) \, dy\right]^{\frac{3}{8}}$$

These formulas yield for relics decoupling UR at LTE:

$$m = \left(\frac{Z}{g}\right)^{\frac{1}{4}} \mathrm{keV} \left\{ \begin{array}{l} 0.568 \\ 0.484 \end{array} \right. , \; g_d = g^{\frac{3}{4}} \, Z^{\frac{1}{4}} \left\{ \begin{array}{ll} 155 & \text{Fermions} \\ 180 & \text{Bosons} \end{array} \right. .$$

Since $g = 1 - 4$, we see that $g_d > 100 \Rightarrow T_d > 100$ GeV.

$1 < Z^{\frac{1}{4}} < 5.6$ for $1 < Z < 1000$. Example: for DM Majorana fermions $(g = 2)$ $m \simeq 0.85$ keV.

The formula for the Mass of the Dark Matter particles

Energy Density: $\rho_{DM}(t) = g \int \frac{d^3P_f}{(2\pi)^3} \sqrt{m^2 + P_f^2}\; f_d[a(t)\, P_f]$

g : # of internal degrees of freedom of the DM particle, $1 \leq g \leq 4$. For $z \lesssim 30 \;\Rightarrow$ DM particles are non-relativistic:

$\rho_{DM}(t) = m\; g\; \frac{T_d^3}{a^3(t)} \int_0^\infty y^2\; f_d(y)\; \frac{dy}{2\pi^2}$.

Using entropy conservation: $T_d = \left(\frac{2}{g_d}\right)^{\frac{1}{3}}\; T_\gamma\; (1+z_d)$,

$g_d =$ effective # of UR degrees of freedom at decoupling, $T_\gamma = 0.2348$ meV , $1\text{ meV} = 10^{-3}$ eV.

Today $\Omega_{DM} = \rho_{DM}(0)/\rho_c = 0.105/h^2$ and we obtain for the mass of the DM particle:

$$m = 6.986\text{ eV}\; \frac{g_d}{g \displaystyle\int_0^\infty y^2\; f_d(y)\; dy}$$

. Goal: determine m and g_d

Phase-space density invariant under universe expansion

Using again entropy conservation to replace T_d yields for the one-dimensional velocity dispersion,

$$\sigma_{DM}(z) = \sqrt{\frac{1}{3}\,\langle \vec{V}^2\rangle(z)} = \frac{2^{\frac{1}{3}}}{\sqrt{3}}\,\frac{1+z}{g_d^{\frac{1}{3}}}\,\frac{T_\gamma}{m}\,\sqrt{\frac{\int_0^\infty y^4\,F_d(y)\,dy}{\int_0^\infty y^2\,F_d(y)\,dy}} =$$

$$= 0.05124\,\frac{1+z}{g_d^{\frac{1}{3}}}\,\frac{\mathrm{keV}}{m}\,\left[\frac{\int_0^\infty y^4\,F_d(y)\,dy}{\int_0^\infty y^2\,F_d(y)\,dy}\right]^{\frac{1}{2}}\,\frac{\mathrm{km}}{\mathrm{s}}.$$

Phase-space density: $\mathcal{D} \equiv \frac{n(t)}{\langle \vec{P}^2_{phys}(t)\rangle^{\frac{3}{2}}} \overset{\mathrm{non-rel}}{=} \frac{\rho_{DM}}{3\sqrt{3}\,m^4\,\sigma^3_{DM}}$

$\mathcal{D}$ is computed theoretically from freezed-out distributions:

$$\mathcal{D} = \frac{g}{2\,\pi^2}\,\frac{\left[\int_0^\infty y^2 F_d(y)dy\right]^{\frac{5}{2}}}{\left[\int_0^\infty y^4 F_d(y)dy\right]^{\frac{3}{2}}}$$

Theorem: The phase-space density $\mathcal{D}$ can only decrease under self-gravity interactions (gravitational clustering) [Lynden-Bell, Tremaine, Henon, 1986].

Relics decoupling non-relativistic

$$F_d^{NR}(p_c) = \frac{2^{\frac{5}{2}}\,\pi^{\frac{7}{2}}}{45}\; g_d\, Y_\infty\; \left(\frac{T_d}{m}\right)^{\frac{3}{2}}\; e^{-\frac{p_c^2}{2\,m\,T_d}} = \frac{2^{\frac{5}{2}}\,\pi^{\frac{7}{2}}}{45}\; \frac{g_d\, Y_\infty}{x^{\frac{3}{2}}}\; e^{-\frac{y^2}{2\,x}}$$

$Y(t) = n(t)/s(t),\ n(t)$ number of DM particles per unit volume, $s(t)$ entropy per unit volume, $x \equiv m/T_d,\ T_d < m$.

$Y_\infty = \frac{1}{\pi}\sqrt{\frac{45}{8}}\;\frac{1}{\sqrt{g_d}\;T_d\;\sigma_0\;M_{Pl}}$ late time limit of Boltzmann.

σ_0: thermally averaged total annihilation cross-section times the velocity.

From our general equations for m and g_d:

$$m = \frac{45}{4\,\pi^2}\;\frac{\Omega_{DM}\;\rho_c}{g\;T_\gamma^3\;Y_\infty} = \frac{0.748}{g\;Y_\infty}\ \text{eV} \quad \text{and} \quad m^{\frac{5}{2}}\;T_d^{\frac{3}{2}} = \frac{45}{2\,\pi^2}\;\frac{1}{g\;g_d\;Y_\infty}\;Z\;\frac{\rho_s}{\sigma_s^3}$$

Finally:

$$\sqrt{m\,T_d} = 1.47\;\left(\frac{Z}{g_d}\right)^{\frac{1}{3}}\ \text{keV}. \quad m = 3.67\ \text{keV}\ Z^{\frac{1}{3}}\;\frac{g_d^{\frac{1}{12}}}{\sqrt{g}}\;\sqrt{\frac{\sigma_0}{\text{pb}}}$$

We used ρ_{DM} today and the decrease of the phase space density by a factor Z. 1 pb $= 10^{-36}\ \text{cm}^2 = 0.257\ /(10^5\ \text{GeV}^2)$.

Relics decoupling non-relativistic 2

Allowed ranges for m and T_d.

$m > T_d > b$ eV where $b > 1$ or $b \gg 1$ for DM decoupling in the RD era

$$\left(\frac{Z}{g_d}\right)^{\frac{1}{3}} \; 1.47 \text{ keV} < m < \frac{2.16}{b} \text{ MeV} \; \left(\frac{Z}{g_d}\right)^{\frac{2}{3}}$$

$g_d \simeq 3 \quad \text{for} \quad 1 \text{ eV} < T_d < 100 \text{ keV}$ and $1 < Z < 10^3$

$1.02 \text{ keV} < m < \frac{104}{b} \text{ MeV} \quad , \quad T_d < 10.2$ keV.

D. Boyanovsky, H. J. de Vega, N. Sanchez,
Phys. Rev. D 77, 043518 (2008), arXiv:0710.5180.
H. J. de Vega, N. G. Sanchez, arXiv:0901.0922.

Only using ρ_{DM} today (ignoring the phase space density information) gives:

$$\sigma_0 = 0.16 \text{ pbarn} \, \frac{g}{\sqrt{g_d}} \, \frac{m}{T_d}$$

http://pdg.lbl.gov

The constant surface density in dark matter galaxies

Surface density of dark matter (DM) halos $\mu_{0D} \equiv r_0\ \rho_0$,
$r_0 =$ halo core radius, $\rho_0 =$ central density

$$\mu_{0D} \simeq 140\ \frac{M_\odot}{\mathrm{pc}^2} = 6400\ \mathrm{MeV}^3 = (18.6\ \mathrm{Mev})^3$$

Universal value for μ_{0D}: independent of galaxy luminosity for a large number of galactic systems (spirals, dwarf irregular and spheroidals, elliptics) spanning over 14 magnitudes in luminosity and of different Hubble types.

Similar values $\mu_{0D} \simeq 80\ \frac{M_\odot}{\mathrm{pc}^2}$ in interstellar molecular clouds of size r_0 of different type and composition over scales $0.001\,\mathrm{pc} < r_0 < 100$ pc (Larson laws, 1981).

Density profile in Galaxies: $\rho(r) = \rho_0\ F\left(\frac{r}{r_0}\right)\ ,\ F(0) = 1.$

Profiles: $F_{Burkert}(x) = \frac{1}{(1+x)(1+x^2)}\ ,\ F_{Sersic}(x) = e^{-x^{\frac{1}{n}}}\ ,\ x \equiv \frac{r}{r_0}$

Same $1/r^3$ tail as cuspy NFW profile $F_{NFW}(x) = \frac{4}{x\,(1+x)^2}$

Virial theorem plus extensivity of energy $\Longrightarrow \mu_{0D} =$ constant

Virial theorem for self-gravitating systems:

$E = \frac{1}{2} \langle U \rangle = -\langle K \rangle$, $\quad E =$ total energy,

$U =$ potential energy, $K =$ kinetic energy. Therefore,

$$E = -\frac{G}{4} \int \frac{d^3r \, d^3r'}{|\vec{r}-\vec{r}'|} \langle \rho(r) \, \rho(r') \rangle = -\frac{G}{4} \rho_0^2 \, r_0^5 \int \frac{d^3x \, d^3x'}{|\boldsymbol{x}-\boldsymbol{x}'|} \langle F(x) \, F(x') \rangle$$

Energy divided by the characteristic volume r_0^3 goes as

$$\frac{-E}{r_0^3} \sim G \, \rho_0^2 \, r_0^2 = G \, \mu_{0D}^2$$

Energy extensivity requires E/r_0^3 fixed for large values of r_0
$\Longrightarrow \mu_{0D}$ must take the same constant value for all r_0

Estimating $\langle K \rangle$ yields $\langle K \rangle = \frac{1}{2} \int d^3r \, \langle \rho(r) \rangle \, \langle v^2 \rangle =$

$$= \frac{1}{2} \, \rho_0 \, r_0^3 \, \langle v^2 \rangle \int d^3x \, \langle F(x) \rangle \sim \rho_0 \, r_0^3 \, \langle v^2 \rangle \Longrightarrow \langle v^2 \rangle \sim G \, \mu_{0D} \, r_0$$

This is true both for molecular clouds and for galaxies.

DM surface density from linear Boltzmann-Vlasov eq

The distribution function of the decoupled DM particles:

$$f(\vec{x},\vec{p};t) = g\ f_0(p) + F_1(\vec{x},\vec{p};t)$$

$f_0(p) =$ thermal equilibrium function at temperature T_d

$$m\ g \int \frac{d^3p}{(2\pi)^3}\ f_0(p) = \rho_{DM} = \Omega_M\ \rho_c = 3\,\Omega_M\ M_{Pl}^2\ H_0^2$$

The linearized Boltzmann-Vlasov equation in the MD era can be recasted as the Gilbert integral equation (Volterra equation of 2nd kind) for

$$\Delta(k,t) \equiv m \int \frac{d^3p}{(2\pi)^3} \int d^3x\ e^{-i\,\vec{x}\cdot\vec{k}}\ F_1(\vec{x},\vec{p};t)$$

We evolve the fluctuations during the MD era using as initial conditions the density fluctuations by the end of the RD era,

$\Delta(k,t_{eq}) = \Omega_M\ \rho_c\ V\ \delta(k,t_{eq})\ ,\ t_{eq} =$ equilibration time,

$$V \sim 1/k_{eq}^3 \simeq \frac{f}{H_0^3},\ k_{eq} \simeq 42.04\ H_0 = 9.88\ \mathrm{Gpc}^{-1},\ f \simeq 1.35\,10^{-5}$$

Fluctuations $k > k_{eq}$ inside the horizon by t_{eq} are relevant

Density Profiles from the Gilbert equation

At the end of the RD era $t = t_{eq}$:

$$\delta(k, t_{eq}) = 24\ |\phi_k|\ \log\left(0.116\ \frac{k}{k_{eq}}\right)$$

[W. Hu and N. Sugiyama (1996).]

$|\phi_k| =$ primordial inflationary fluctuations:

$$|\phi_k| = \sqrt{2}\ \pi\ |\Delta_0|\ \left(\frac{k}{k_0}\right)^{n_s/2-2} ,$$

where $|\Delta_0| \simeq 4.94\ 10^{-5},\ n_s \simeq 0.964,\ k_0 = 2\ \mathrm{Gpc}^{-1}$.

Density profile today in the linear approximation:

$$\rho_{lin}(r) = \frac{1}{2\,\pi^2\,r}\ \int_0^\infty k\ dk\ \sin(k\,r)\ \Delta(k, t_{\rm today})$$

H. J. de Vega, N. G. Sanchez,
On the constant surface density in dark matter galaxies and interstellar molecular clouds, arXiv:0907.0006

The Gilbert equation

Define: $\widehat{\Delta}(k,t) \equiv \Delta(k,t)/\Delta(k,t_{eq})$.

The Gilbert equation takes the form:

$$\widehat{\Delta}(k,u) - \frac{6}{\alpha}\int_0^u \Pi[\alpha\,(u-u')]\,\frac{\widehat{\Delta}(k,u')}{[1-u']^2}\,du' = I[\alpha\,u]$$

where,

$$\Pi[z] = \frac{1}{I_2}\int_0^\infty dy\; y\; f_0(y)\;\sin(y\,z),\;\; I[z] = \frac{1}{I_2}\int_0^\infty dy\; y\; f_0(y)\;\frac{\sin(y\,z)}{z}$$

$$y \equiv \frac{p}{T_d}, \quad z \equiv \alpha\,u, \quad \alpha \equiv \frac{2\,k}{H_0}\sqrt{\frac{1+z_{eq}}{\Omega_M}}\,\frac{T_d}{m},$$

$$I_2 = \int_0^\infty dy\; y^2\; f_0(y), \quad 1+z_{eq} = \frac{1}{a_{eq}} \simeq 3200,$$

$u =$ dimensionless time variable,

$$u = 1 - \sqrt{\frac{a_{eq}}{a}}\;,\; 0 \leq u \leq u_{\text{today}} = 1 - \sqrt{a_{eq}} \simeq 0.982$$

$$a(u) = \frac{a_{eq}}{(1-u)^2}\;,\; a(\text{today}) = 1\;.$$

$\widehat{\Delta}(k,t) \overset{t\to t_{\text{today}}}{=} \frac{3}{5}\,T(k)\,(1+z_{eq}), \quad T(k) =$ transfer function.

The solution of the Gilbert equation today

Transfer function: $T(0) = 1$ and $T(k \to \infty) = 0$.

The solution of the Gilbert equation $\widehat{\Delta}(k,t)$ for $k < k_{fs}$ grows proportional to the scale factor.

k_{fs} = free-streaming (Jeans) comoving wavenumber.

k_{fs} = characteristic scale for the decreasing of $T(k)$ with k
$\Longrightarrow$ the natural variable here is $\gamma \equiv k \, r_{lin}$

$r_{lin} \equiv \frac{\sqrt{2}}{k_{fs}} = \frac{2}{H_0} \, \sigma_{DM} \, \sqrt{\frac{1+z_{eq}}{\Omega_M}}$ and

$$\sigma_{DM} = \left(3 \; M_{Pl}^2 \; H_0^2 \, \Omega_{DM} \; \frac{1}{Z} \; \frac{\sigma_s^3}{\rho_s}\right)^{\frac{1}{3}} \Longrightarrow r_{lin} = 125.1 \; \left(\frac{10}{Z}\right)^{\frac{1}{3}} \text{ kpc}$$

Collecting all formulas we obtain for the fluctuations today

$$\Delta(k, t_{\text{today}}) = 1.926 \, \frac{M_{Pl}^2}{H_0} \, |\Delta_0| \; T(k) \; \left(\frac{k}{k_0}\right)^{n_s/2-2} \; \log\left(0.116 \, \frac{k}{k_{eq}}\right)$$

Density profiles in the linear approximation

The Fourier transform of the fluctuations today yield

$$\rho_{lin}(r) = (5.826 \text{ Mev})^3 \; \frac{Z^{n_s/6}}{r} \times$$
$$\times \int_0^\infty \gamma^{n_s/2-1} \; \log\left(\widehat{c}\, Z^{\frac{1}{3}}\, \gamma\right) \; \sin\left(\gamma \, \frac{r}{r_{lin}}\right) \; T(\gamma)\, d\gamma \; ,$$

$$\mu_{0D} = r_{lin}\; \rho_{lin}(0) =$$
$$= (5.826 \text{ Mev})^3 \; Z^{n_s/6} \; \int_0^\infty \gamma^{n_s/2} \; \log\left(\widehat{c}\, Z^{\frac{1}{3}}\, \gamma\right) \; T(\gamma)\, d\gamma \; ,$$

where:
$n_s/2 - 1 = -0.518, \; n_s/2 = 0.482, \; n_s/6 = 0.160$ and $\widehat{c} = 43.6$

Particle Statistics	$\mu_{0D} = r_{lin}\, \rho_{lin}(0)$
Bose-Einstein	$(16.71 \text{ Mev})^3 \; (Z/10)^{0.16}$
Fermi-Dirac	$(15.65 \text{ Mev})^3 \; (Z/10)^{0.16}$
Maxwell-Boltzmann	$(14.73 \text{ Mev})^3 \; (Z/10)^{0.16}$

Observed value: $\mu_{0D} \simeq (18.6 \text{ Mev})^3 \Rightarrow Z \sim 10 - 100$

Linear results for μ_{0D} and the profile vs. observations

Since the surface density $r_0 \, \rho(0)$ should be universal, we can identify $r_{lin} \, \rho_{lin}(0)$ from a spherically symmetric solution of the linearized Boltzmann-Vlasov equation.

The linear profiles obtained are cored since $T(k)$ decays for $k > k_{fs} \sim 1/r_{lin} \sim 0.008 \; (Z/10)^{\frac{1}{3}} \; (\text{kpc})^{-1}$.

$\rho_{lin}(r)$ scales with the primordial spectral index n_s:

$\rho_{lin}(r) \overset{r \gg r_{lin}}{=} r^{-1-n_s/2} = r^{-1.482}$,

in agreement with the universal empirical behaviour $r^{-1.6 \pm 0.4}$, M. G. Walker et al., (2009).

For larger scales nonlinear effects from small k should give the customary r^{-3} tail.

The agreement between the linear theory and the observations is remarkable.

The comparison of our theoretical values for μ_{0D} and the observational value indicates that $Z \sim 10 - 100$.

This implies that the DM particle mass is in the keV range.

Dark Energy

$76 \pm 5\%$ of the present energy of the Universe is Dark !
Current observed value:
$\rho_\Lambda = \Omega_\Lambda \; \rho_c = (2.39 \text{ meV})^4 \quad , \quad 1 \text{ meV} = 10^{-3} \text{ eV}.$
Equation of state $p_\Lambda = -\rho_\Lambda$ within observational errors.
Quantum zero point energy. Renormalized value is finite.
Bosons (fermions) give positive (negative) contributions.
Mass of the lightest particles ~ 1 meV is in the right scale.
Spontaneous symmetry breaking of continuous symmetries produces massless scalars as Goldstone bosons. A small symmetry breaking provide light scalars: axions,majorons...
Observational Axion window $10^{-3} \text{ meV} \lesssim M_{\text{axion}} \lesssim 10 \text{ meV}$.
Dark energy can be a cosmological zero point effect. (As the Casimir effect in Minkowski with non-trivial boundaries).
We need to learn the physics of light particles (< 1 MeV), also to understand dark matter !!

Summary and Conclusions

- We formulate inflation as an effective field theory in the Ginsburg-Landau spirit with energy scale $M \sim M_{GUT} \sim 10^{16}$ GeV $\ll M_{Pl}$. Inflaton mass small: $m \sim H/\sqrt{N} \sim M^2/M_{Pl} \ll M$. Infrared regime !!
- For all slow-roll models $n_s - 1$ and r are $1/N, \; N \sim 60$.
- MCMC analysis of WMAP+LSS data plus this theory input indicates a spontaneously broken inflaton potential: $w(\chi) = \frac{y}{32}\left(\chi^2 - \frac{8}{y}\right)^2 \;, \; y \simeq 1.26$.
- Lower Bounds: $r > 0.023 \; (95\% \; \text{CL}) \;, \; r > 0.046 \; (68\% \; \text{CL})$. The most probable values are $r \simeq 0.051 (\Leftarrow$ measurable !!) $n_s \simeq 0.964$.
- Model independent analysis of dark matter points to a particle mass at the keV scale. T_d may be > 100 GeV. DM is cold.

Summary and Conclusions 2

- CMB quadrupole suppression may be explained by the effect of fast-roll inflation provided the today's horizon size modes exited by the end of fast-roll inflation.
- Quantum (loop) corrections in the effective theory are of the order $(H/M_{Pl})^2 \sim 10^{-9}$. Same order of magnitude as loop graviton corrections.

D. Boyanovsky, H. J. de Vega, N. G. Sanchez,

Quantum corrections to the inflaton potential and the power spectra from superhorizon modes and trace anomalies, Phys. Rev. D 72, 103006 (2005), astro-ph/0507596.

Quantum corrections to slow roll inflation and new scaling of superhorizon fluctuations. Nucl. Phys. B 747, 25 (2006), astro-ph/0503669.

Future Perspectives

The Golden Age of Cosmology and Astrophysics continues.

A wealth of data from WMAP (7 yr), Planck, Atacama Cosmology Tel and further experiments are coming.

Galaxy and Star formation. DM properties from astronomical observations. Better bounds on DM cross-sections.

DM in planets and the earth. Flyby and Pioneer anomalies?

The Dark Ages...Reionisation...the 21cm line...

Nature of Dark Energy? 76% of the energy of the universe.

Nature of Dark Matter? 83% of the matter in the universe.

Light DM particles are strongly favoured $m_{DM} \sim$ keV.

Sterile neutrinos? Some unknown light particle ??

Need to learn about the physics of light particles (< 1 MeV).

END

THANK YOU FOR YOUR ATTENTION