The case for a sterile neutrino as warm dark matter

- Hey! ΛCDM do you have a ***small scale*** problem?
- WDM: Sterile neutrinos come to the rescue!!
- Any sightings of (El) ν_s lately?

- Properties:
  - ***abundance***
  - ***phase space density***
  - ***small scales: cutoff in P(k)+features***

- Constraints and bounds
- Production and decoupling: models
- Small scale transfer function
- A cosmic roadmap

- Cold (CDM): small velocity dispersion: small structure forms first, ***bottom-up*** hierarchical merger –***WIMPs***
- Hot (HDM) : large velocity dispersion: big structure forms first, ***top-down***, fragmentation—light neutrinos $m_\nu \ll \mathrm{eV}$
- Warm (WDM): ``in between'' –sterile neutrinos $m_s \sim \text{few keV}$

ΛCDM Concordance Model:

CMB+ LSS + N-body:

DM is COLD and COLLISIONLESS

N-body:

- ``clumpy halo'', large number of satellite galaxies
- $\rho(r) \sim 1/r^\beta$, $1 \lesssim \beta \lesssim 1.5$ (NFW)

Observational Evidence:

❑ ***Too few ``satellite'' galaxies***
❑ ***dSphS: cores instead of cusps ~ 1 kpc***

Cusps vs Cores ??

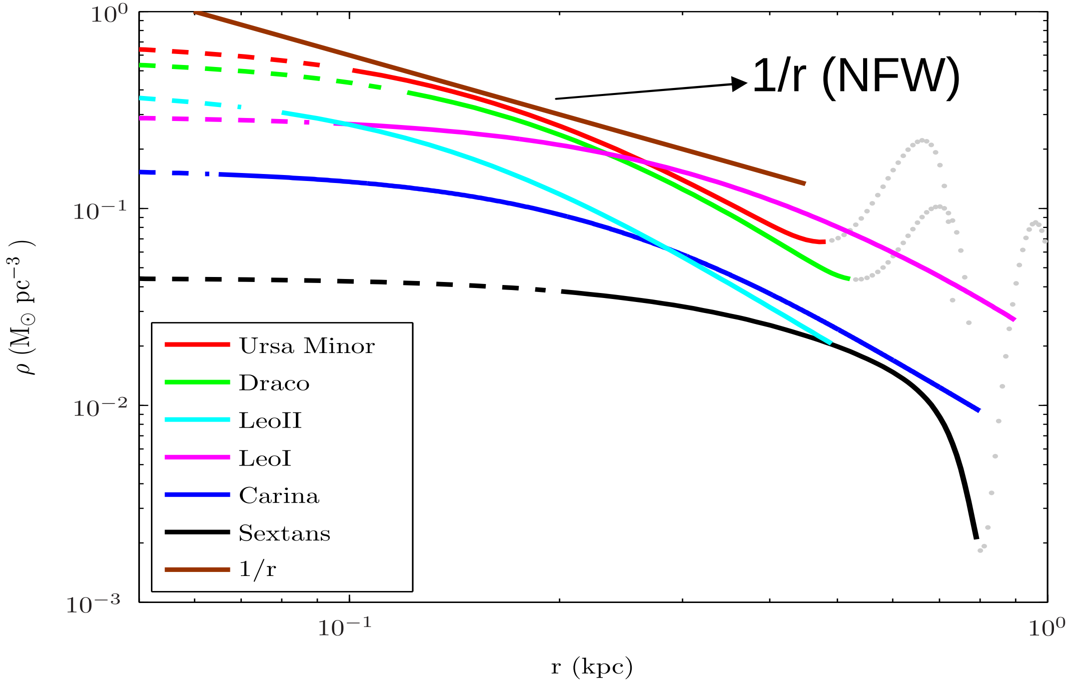


Fig. 4.— Derived inner mass distributions from isotropic Jeans' equation analyses for six dSph galaxies. The modelling is reliable in each case out to radii of log (r)kpc∼ 0.5. The unphysical behaviour at larger radii is explained in the text. The general similarity of the inner mass profiles is striking, as is their shallow profile, and their similar central mass densities. Also shown is an r^{-1} density profile, predicted by many CDM numerical simulations (eg Navarro, Frenk & White 1997). The individual dynamical analyses are described in full as follows: Ursa Minor (Wilkinson et al. (2004)); Draco (Wilkinson et al. (2004)); LeoII (Koch et al. (2007)); LeoI (Koch et al. (2006)); Carina (Wilkinson et al. (2006), and Wilkinson et al in preparation); Sextans (Kleyna et al. (2004)).

A common mass for Dwarf Spheroidals (dSphs)??

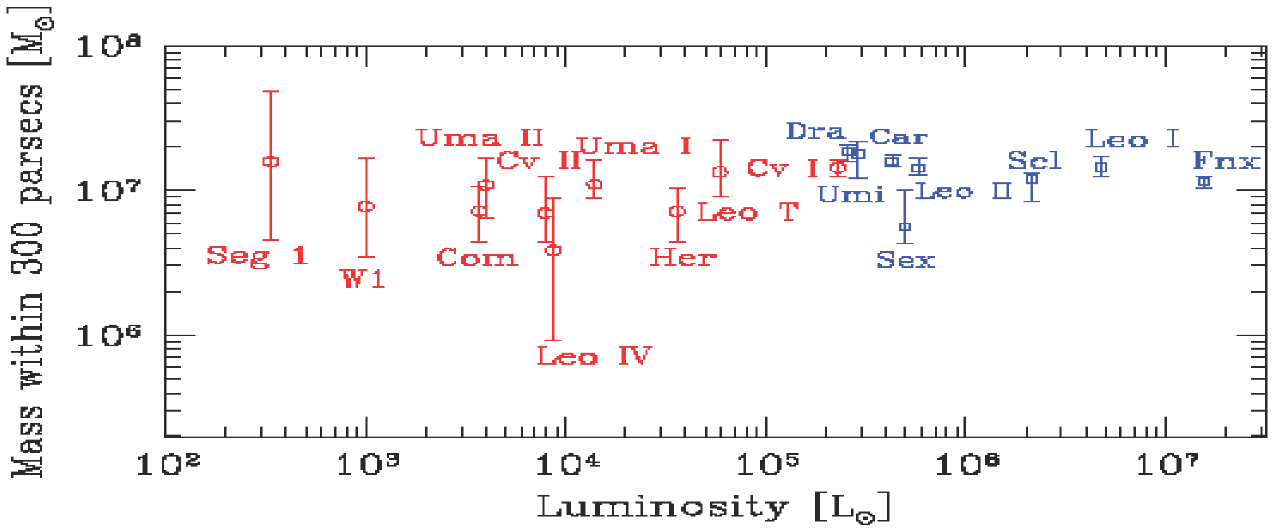


Figure 1: The integrated mass of the Milky Way dwarf satellites, in units of solar masses, within their inner 0.3 kpc as a function of their total luminosity, in units of solar luminosities. The circle (red) points on the left refer to the newly-discovered SDSS satellites, while the square (blue) points refer to the classical dwarf satellites discovered pre-SDSS. The error bars reflect the points where the likelihood function falls off to 60.6% of its peak value.

From Strigari et.al.

``A warm DM candidate with m > 1 keV would yield a DM halo consistent with observations..'' (Strigari. Et.al.)

And the plot thickens…..

- ***Another over abundance problem--``mini voids'': too many small haloes in*** ΛCDM

$\longrightarrow$ ***voids are too small compared to observations: Tikhonov, Klypin, Tikhonov, et. al.***

ΛWDM with $m \sim \mathrm{keV}$ ***may be a possible solution (Tikhonov et.al)***

- ΛCDM ***overpredicts the number of DM haloes with masses >*** $10^{10}\,M_{\odot}$ ***Sawala et.al.***

WDM ``may offer a viable possibility [of resolution..]'' (Sawala et. al.) (Millenium II)

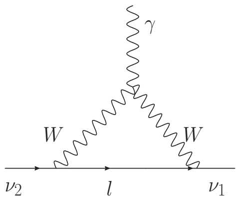
A keV Sterile Neutrino fits the bill…

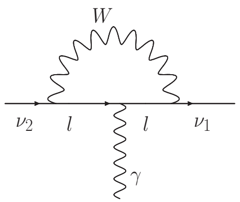
Q: What IS a Sterile Neutrino?
A: A massive neutrino without electroweak interactions, couples to active neutrinos via a (seesaw) mass matrix.
Q: Why/how does it help?
A: Larger velocity dispersion, larger free-streaming length $\lambda_{fs} \propto \left[\frac{\langle V^2 \rangle}{H_o \Omega_m}\right]^{\frac{1}{2}}$
cuts-off power spectrum at small scales < λ_{fs}

Is there ***any*** evidence? May be..

Sterile neutrinos and the X-ray background: the evidence

$$\nu_2 \rightarrow \nu_1 \gamma$$





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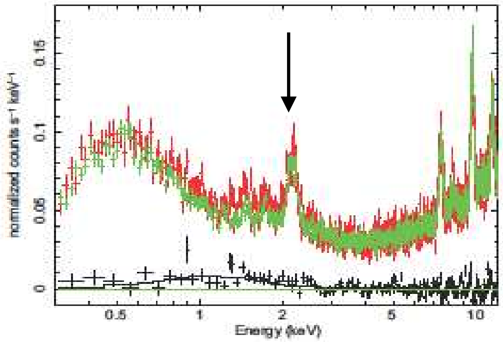
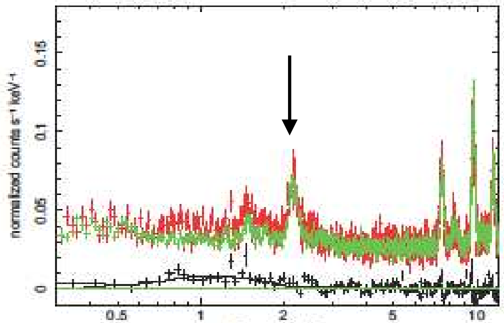
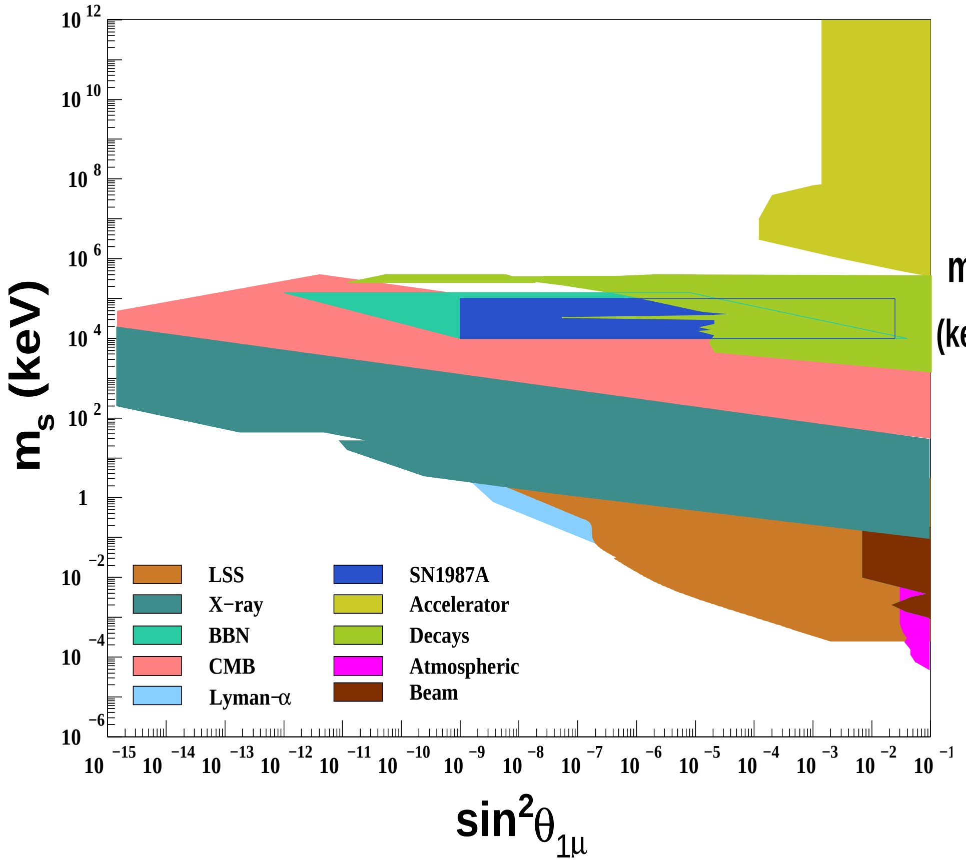


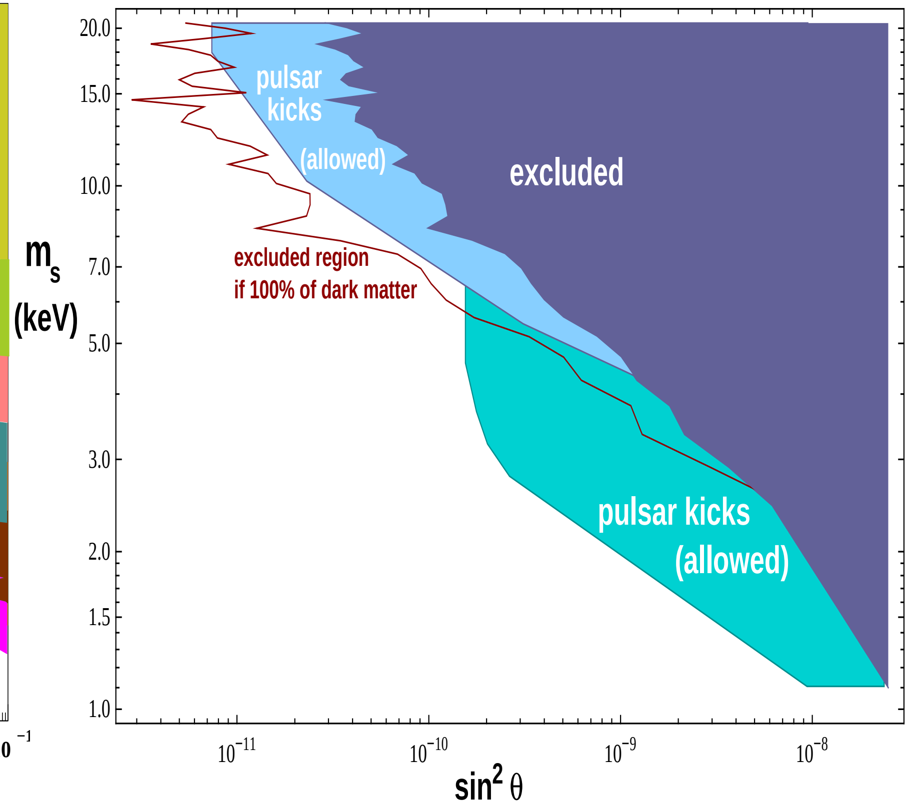
Fig. 1.— **(a) top:** Total (red), PB (green), and source (*i.e.* PB-subtracted; black) spectra. The histogram is the best-fit GXB+CXB model to the source spectrum. **(b) bottom:** Same as **top** with the quiescent-particle-background reduced in the total and PB spectra as explained in the text.



From Kusenko and Lowenstein

Constraints from sterile neutrino decays (see Kusenko)





Microphysics:

Collisionless DM= decoupled particles

$$\Gamma > H \longrightarrow$$ LTE: Eq. distribution, FD, BE,MB

$$\Gamma < H \longrightarrow$$ Out of LTE: frozen distribution

Frozen distribution obeys collisionless BE:

$$\{\frac{\partial}{\partial t} - Hp_f \frac{\partial}{\partial p_f}\} f(p_f;t) = 0$$

$$f(p_f;t) = f(a(t)p_f) = f(p_c) \longrightarrow f(y, x_1, x_2, \cdots)$$

Distribution function
Of decoupled particle

$$y = \frac{p_c}{T_d}$$

Dimensionless const.(microphysics)

Non-relativistic particles: m > T(t) $\Longrightarrow$

$$\rho(t) = mn(t) = mg\int \frac{d^3P_f}{(2\pi)^3} f(P_f, T) = mg\,\frac{T_d^3}{2\pi^2\, a^3(t)} \int_0^\infty y^2 f(y)dy$$

$$\langle \vec{V}^2 \rangle = \left\langle \frac{\vec{P}_f^2}{m^2} \right\rangle = \frac{\int \frac{d^3P_f}{(2\pi)^3} \frac{\vec{P}_f^2}{m^2} f\,[a(t)P_f\,]}{\int \frac{d^3P_f}{(2\pi)^3} f\,[a(t)P_f\,]} = \left(\frac{T_d}{ma(t)}\right)^2 \frac{\int_0^\infty y^4 f\,(y)dy}{\int_0^\infty y^2 f\,(y)dy}$$

Constraints:

1) ABUNDANCE: Upper bound

$$\Omega_{DM} h^2 = 0.105$$

# of Relativistic d.o.f at decoupling

$$m_a \leq 2.695 \frac{2 g_d\, \xi(3)}{g \int_0^\infty y^2 f(y)dy} \text{ (eV)}$$

2) Phase space density: LOWER BOUND

For decoupled non-relativistic particles the phase space density is **CONSERVED**

$$\mathcal{D} = \frac{n(t)}{\langle p_f^2 \rangle^{\frac{3}{2}}} = \frac{g}{2\pi^2} \frac{\left[\int_0^\infty y^2 f(y) dy\right]^{\frac{5}{2}}}{\left[\int_0^\infty y^4 f(y) dy\right]^{\frac{3}{2}}}$$

$$\frac{\rho_{DM}}{\sigma_{DM}^3} = 6.611 \times 10^8 [\mathcal{D}] [\frac{m}{\text{keV}}]^4 \frac{M_\odot}{(\text{kpc}\,\text{km}/\text{s})^3}$$

NR-DM phase space density

one dimensional velocity dispersion

OBSERVATIONS: dSphs:

$$0.9 \leq \left[10^{-4} \frac{\rho}{\sigma^3} \frac{(\text{km}/\text{s})^3}{M_\odot / (\text{kpc})^3}\right] \leq 20$$

Thm: phase space density diminishes in ``violent'' relaxation (mergers)

$$\underbrace{\frac{[62.36\,\mathrm{eV}]}{\mathcal{D}^{\frac{1}{4}}}\left[10^{-4}\,\frac{\rho}{\sigma^3}\,\frac{(\mathrm{km/s})^3}{M_\odot/(\mathrm{kpc})^3}\right]^{\frac{1}{4}}}_{\text{Lower bound from phase space density of dSphs}} < m \leq \underbrace{2.695\,\frac{2g_d\,\xi(3)}{g\int_0^\infty y^2 f(y)\,dy}\,(\mathrm{eV})}_{\text{Upper bound from abundance}}$$

BOUNDS FOR ***THERMAL*** RELICS

1) Relativistic at decoupling: Fermions or Bosons

m ~ 1 keV consistent with CORED profiles for $T_d \leq$ 100-300 GeV

for cusps $g_d \geq$ 2000 (beyond SM!!)

2) Non-relativistic at decoupling : Wimps

M ~ 100 GeV, $T_d \sim$ 10 MeV

$$\left.\frac{\rho}{\sigma^3}\right)_{Wimp} = \left\{ \begin{array}{l} \left.\frac{\rho}{\sigma^3}\right)_{cusp} \times 10^{15} \\ \left.\frac{\rho}{\sigma^3}\right)_{cored} \times 10^{18} \end{array} \right\}$$

CAN THIS RELAXATION BE POSSIBLE??

CONSTRAINTS: SUMMARY

- ARBITRARY DECOUPLED DISTRIBUTION FUNCTION
- ABUNDANCE $\Rightarrow$ UPPER BOUND
- dSphs (DM dominated) PHASE SPACE $\Rightarrow$ LOWER BOUND
- m ~ keV THERMAL RELICS decoupled when relativistic $\lesssim$ 100-300 GeV consistent with CORES
- Wimps with m ~ 100 GeV, T_d ~ 10 MeV PSD ~ 10^{18}-10^{15} x (dSphs)!!

Power spectrum of density perturbations during matter era: P(k) features a cutoff determined by the free streaming wave vector: $k_{fs}(t_{eq}) = \frac{2\pi}{\lambda_{fs}}$

$$\lambda_{fs}(t_{eq}) = 616\left(\frac{2}{g_d}\right)^{\frac{1}{3}}\left(\frac{\text{keV}}{m}\right) \frac{\int_0^\infty y^4 f(y)dy}{\int_0^\infty y^2 f(y)dy} \text{ (kpc)}$$

<u>smaller for</u> $\left\{ \begin{array}{l} \text{larger } g_d \text{ (\textbf{\textit{colder species}})} \\ f(y) \text{ larger for small } y \end{array} \right.$

Transfer function and power spectrum:

- ❑ Boltzmann-Einstein eqn for (DM+rad.) density + gravitational perturbation
- ❑ Radiation-matter dominated cosmology (z > 2)
- ❑ All scales relevant for structure formation $k \gg k_{eq} \sim 0.01(Mpc)^{-1}$

What's out?

❖ Baryons: modify T(k) ~ few %. BAO on scales ~ 150 Mpc (acoustic horizon) (interested in MUCH smaller scales!!)

❖ Anisotropic stresses

Why?

✓ Study arbitrary distribution functions, couplings, masses

✓ Semi-analytical understanding of small scale properties

✓No tinkering with codes

$$f(p,\vec{x},\eta) = f_0(p) + F_1(p,\vec{x},\eta)$$

Unperturbed decoupled distribution

(DM) perturbation

No anisotropic stresses

$$\phi(\vec{k},\eta) = \psi(\vec{k},\eta)$$

Newtonian potential

Spatial curvature

Solution of Linearized Boltzmann eqn.

$$F_1(\vec{k},\vec{p};\eta) = F_1(\vec{k},\vec{p};\eta_i)e^{-ik\mu l(p,\eta,\eta_i)} - p\left(\frac{d\,f_0(p)}{dp}\right)\int_{\eta_i}^{\eta} d\tau\, e^{-ik\mu l(p,\eta,\tau)}\left[\frac{d\phi(\vec{k},\tau)}{d\tau} - i\frac{k\mu}{v(p,\tau)}\phi(\vec{k},\tau)\right]$$

$$l(p,\eta,\eta') = \int_{\eta'}^{\eta} d\tau\, v(p,\tau) \;\; ; \; v(p,\tau) = \frac{p}{\sqrt{p^2 + m^2a^2(\eta)}}$$

Free streaming distance between η, η'

$$\mu = \hat{\mathbf{k}} \cdot \hat{\mathbf{p}}$$

ϕ from 00-Einstein equation

keV DM: Three stages:

- i) R.D;relativistic: relativistic free streaming $l \propto \eta$
- ii) R.D; no-relativistic: N.R. free streaming $l \propto \ln(\eta)$
- iii) M.D: no-relativistic.

In i) + ii) gravitational perturbations determined by ***radiation fluid***

$$\phi(z) = -3\,\phi_i(k)\left[\frac{\left(\frac{z}{\sqrt{3}}\right)\cos\left(\frac{z}{\sqrt{3}}\right) - \sin\left(\frac{z}{\sqrt{3}}\right)}{\left(\frac{z}{\sqrt{3}}\right)^3}\right] \; ; \; z = k\,\eta$$

acoustic oscillations of radiation fluid

In iii) 00-Einstein at small scales $\longrightarrow$ ***Poisson's eqn***.

gravitational potential $\longrightarrow$

$$\phi_m(k,\eta) = -\frac{3}{4}\frac{k_{eq}^2 a_{eq}}{k^2 a(\eta)}\delta(k,\eta)$$

$\longleftarrow$ DM density perturbation

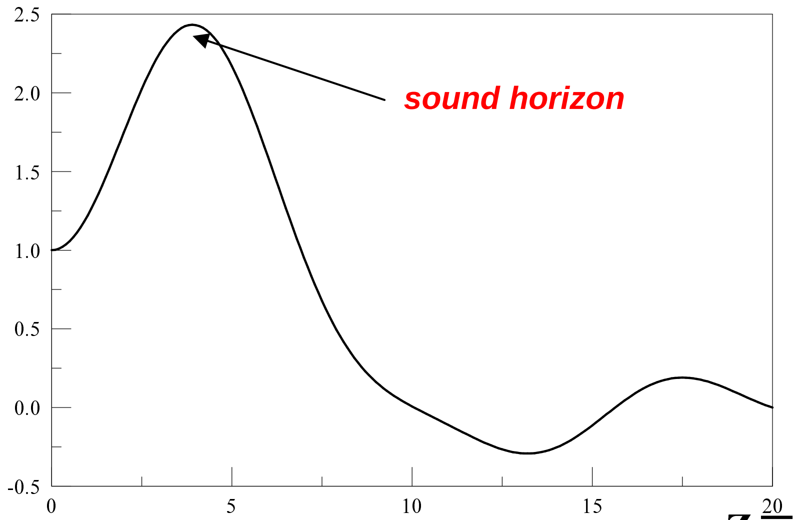
Strategy

a) Initial conditions: adiabatic: $F_1(\vec{k},\vec{p};\eta_i)=\frac{1}{2}\phi_i(k)\,p\left(\frac{df_0(p)}{dp}\right)\;;\;k\eta_i \ll 1$

b) Integrate BE during stage i) R.D.—rel. use the result as initial condition for stage ii) R.D.—non. rel. repeat for stage iii).

Stage i): R.D. relativistic: time dependence of ϕ from acoustic oscillations of radiation fluid → ***ISW effect: enhancement of perturbation***

monopole of DM density perts. → $\frac{\theta_0(z)}{\theta_0(0)}$



$z \doteq k\eta$

Wavelengths larger than sound horizon amplified by ISW.

Stage iii): Boltzmann-Poisson $\longrightarrow$ inhomogeneous differential integral equation for density ***perturbations,initial conditions + inhomogeneity determined by history during stages i) + ii).*** Fredholm solution, leading order (in free streaming): ***Born approximation***: ***WDM fluid description.***

$$\frac{d^2\delta}{d\tilde{a}^2}+\frac{(2+3\tilde{a})}{2\tilde{a}(1+\tilde{a})}\frac{d\delta}{d\tilde{a}}-\frac{3\delta}{2\tilde{a}(1+\tilde{a})}+\frac{\kappa^2\delta}{4\tilde{a}^2(1+\tilde{a})}=I[k,\eta]$$

I + initial conditions determined by past history during stages i + ii) : R.D.

$$\kappa\equiv\frac{\sqrt{6}\,k}{k_{fs}}$$

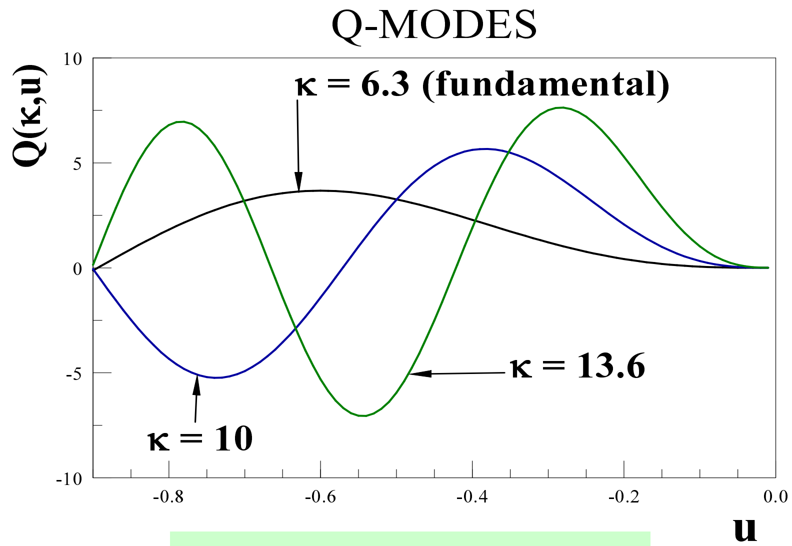
$$k_{fs}=\frac{11.17}{\sqrt{\overline{y^2}}}\left(\frac{m}{\mathrm{keV}}\right)\left(\frac{g_d}{2}\right)^{\frac{1}{3}}(\mathrm{Mpc})^{-1}$$

$$\tilde{a}=\frac{a}{a_{eq}}\equiv-\frac{1}{\sinh[u]}$$

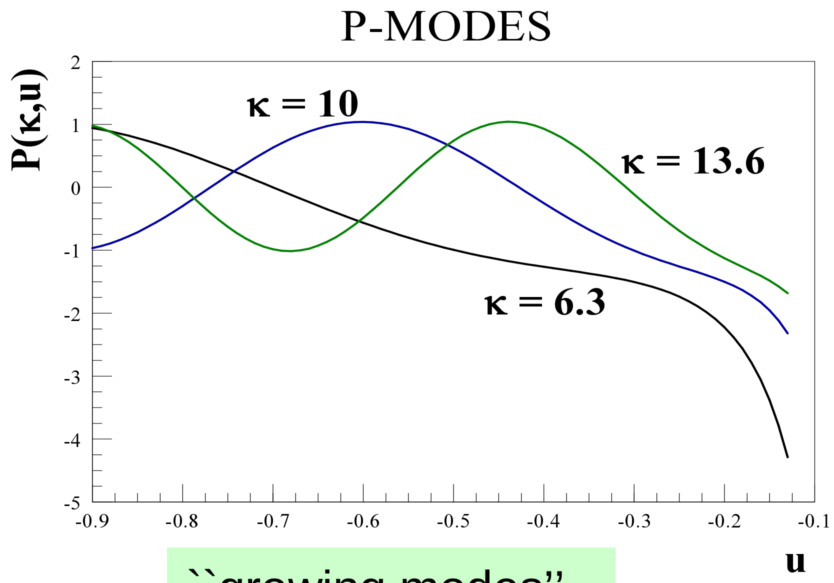
$$\kappa=0=\mathrm{CDM}$$

For $I[k,\eta]=0$ ***Meszaro's equation for WDM: can be solved EXACTLY***:

WDM acoustic oscillations



``decaying modes''



``growing modes''

Transfer function in Born approximation

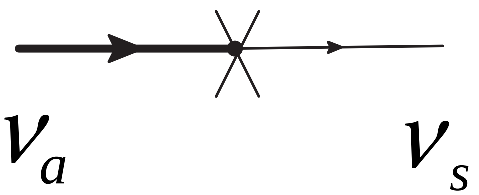
$$T(k;\kappa) = \frac{30\,k_{eq}^2}{k^2(1+\kappa^2)(4+\kappa^2)\,\phi_i(k)} \int_{u_{eq}}^{0} Q(\kappa,u')\tilde{a}(u')I[k;\kappa;u']du'$$

$$T_{CDM}(k) = \frac{45}{4}\frac{k_{eq}^2}{k^2}\ln\left[\frac{4\sqrt{2}\,k\,e^{\gamma_E-\frac{7}{2}}}{\sqrt{3}\,k_{eq}}\right]$$

Main ingredient: distribution function at decoupling of DM particle:

Sterile neutrinos: a NON-Thermal DM candidate

Non resonant Dodelson Widrow: production via active-sterile mixing



$$f(y) = \frac{\beta}{e^y + 1}$$

$$T_d \simeq 150\,\mathrm{MeV}$$

$$\beta \sim 10^{-2}$$

Constrained from abundance for $m_s \sim \mathrm{keV}$

$$\lambda_{fs} \simeq 1\,\mathrm{Mpc}$$

independent of β

Sterile neutrinos II: a different non-resonant production mechanism : Boson decay

$$\mathcal{L} = \mathcal{L}_{SM} + \overline{\nu_s}\, i\partial\, \nu_s - Y_1 \overline{\nu_s} \tilde{H}^{\dagger} l - Y_2 \overline{\nu_s} \Phi\, \nu_s + \mathcal{L}[\Phi] + \text{h.c}$$

ν_s = ``sterile neutrino'' SU(2) singlet

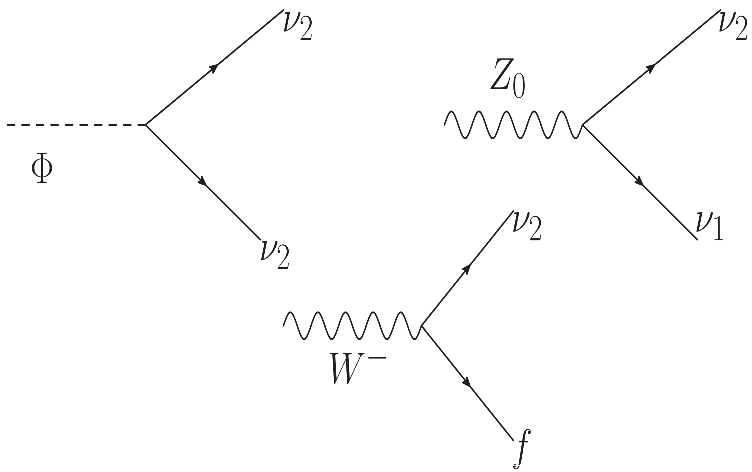
$$\tilde{H} = \begin{pmatrix} H^0 \\ H^- \end{pmatrix} \quad l = \begin{pmatrix} \nu_a \\ f \end{pmatrix}$$

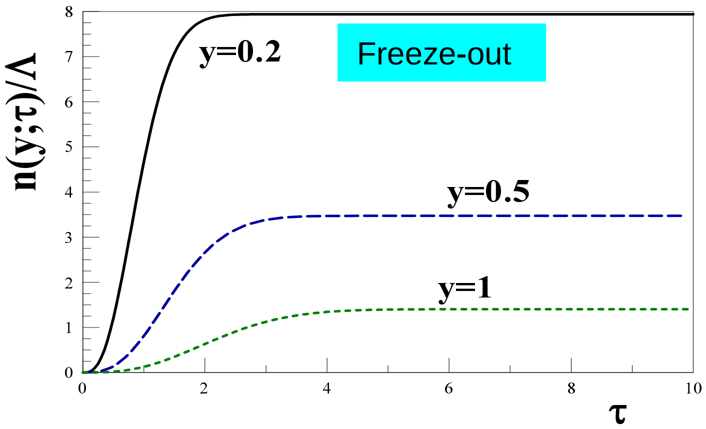
Φ = Scalar, gauge singlet,could be Higgs

$$\langle \Phi \rangle \sim \langle H^0 \rangle \sim 100\,\text{GeV}$$

$$Y_2 \sim 10^{-8} \rightarrow m_s \sim \text{keV} \; ; \; \frac{Y_1}{Y_2} = \sin\theta \sim 10^{-5}$$

Production: Scalar + Vector boson decay, all with thermal abundance at 100 GeV





$$\Lambda \sim 10^{-2}$$

$$\tau = \frac{M}{T(t)}$$

Frozen distribution

$$f_0(y) = 2\Lambda\sqrt{\pi}\,\frac{g_{\frac{5}{2}}(y)}{y^{\frac{1}{2}}}$$

y = p/T_d

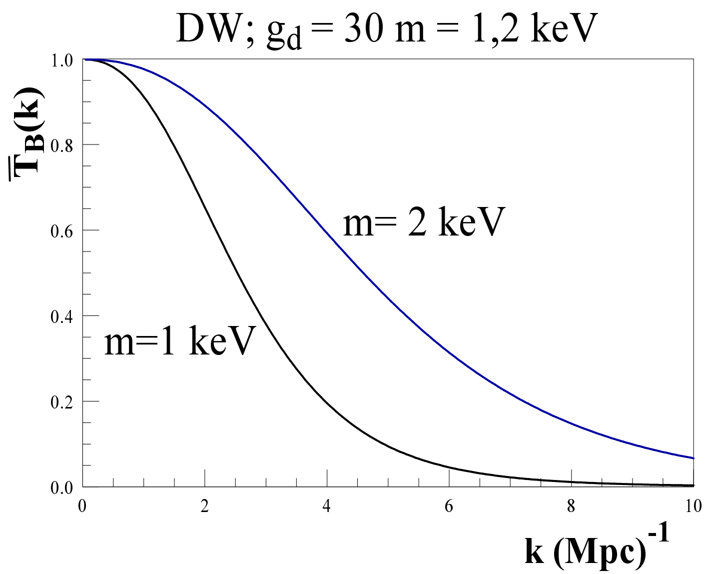
Decoupling at ~ 100 GeV

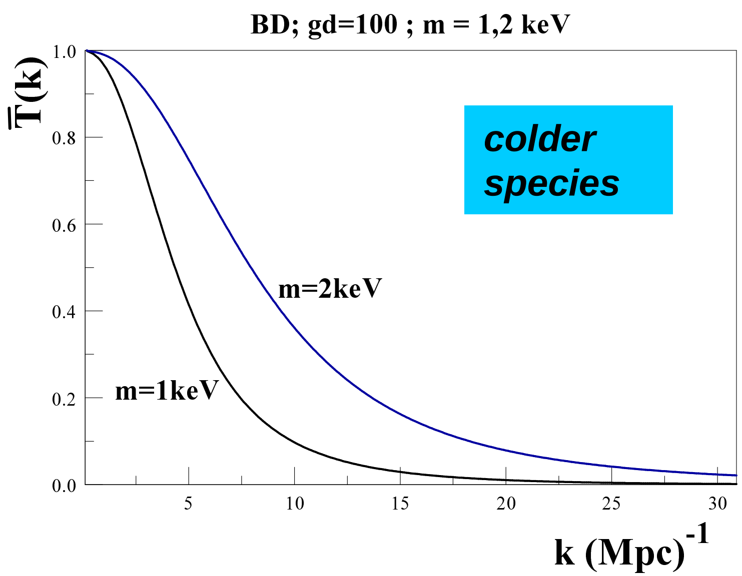
Strong enhancement of small momentum

Abundance + phase space density constraints:

$$560\,\mathrm{eV} \lesssim m \lesssim 1330\,\mathrm{eV}$$

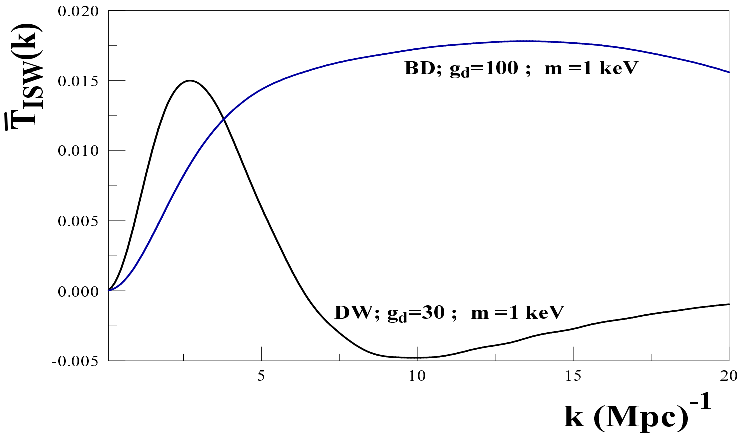
Consistent with model ``beyond SM''





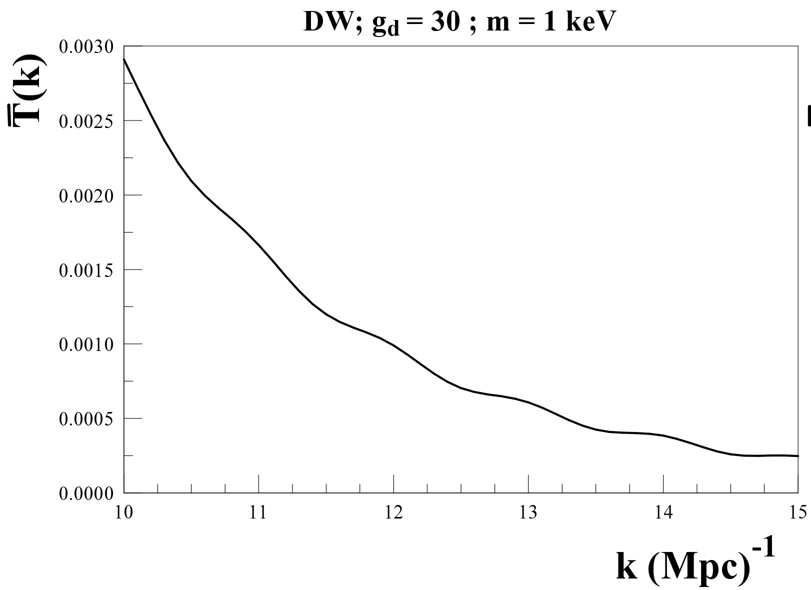
$$\bar{T}(k) = \frac{T(k)}{T_{cdm}(k)}$$

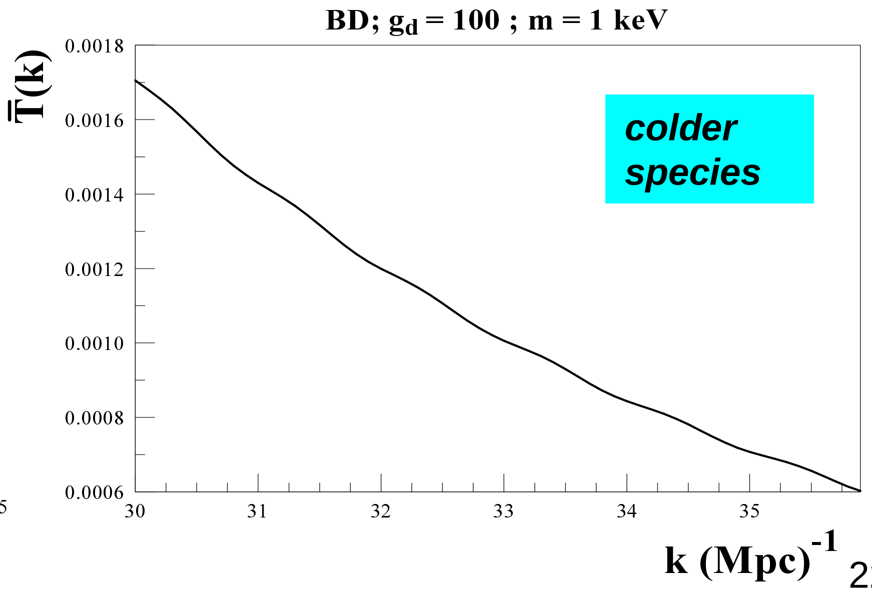
Enhancement at small k by ISW: remnant from early RD-rel. stage i)



WDM acoustic oscillations at small scales:

scale determined by ``fundamental'' Q-mode.





colder species

Roadmap: from micro to macro

1) ***Microphysics:*** Particle physics model, kinetics of production, decoupling

→ $f(y)$ = decoupled distribution function, $y=p/T_{0,d}$

2) ***Constrain*** mass, couplings, $T_{0,d}$ from abundance + phase space density

$$\underbrace{\frac{100\,\mathrm{eV}}{\mathcal{D}^{\frac{1}{4}}}}_{\text{Lower bound from phase Space density of dSphs}} \leq m \leq 6.5\,\mathrm{eV}\underbrace{\frac{g_d}{g\int_0^\infty y^2\, f(y)\, dy}}_{\text{Upper bound from abundance}} \quad ; \quad \mathcal{D} = \frac{g}{2\pi^2}\frac{\left[\int_0^\infty y^2 f(y)dy\right]^{\frac{5}{2}}}{\left[\int_0^\infty y^4 f(y)dy\right]^{\frac{3}{2}}}$$

Thermal relics or non-thermal sterile neutrinos that decouple relativistically

$\mathcal{D} \sim 2\times10^{-3}$ → m ~ keV

3) ***T(k):*** Solve BE-equation—cutoff on scale λ_{fs}

Born approximation: easily implementable for generic distribution function: scales, WDMAO's

Executive Summary

- LCDM ***May*** ***have problems at small scales: cores vs cusps, over abundance of satellites, minivoids and massive haloes.***

- ***A keV particle May provide a solution.***

- ***A few keV sterile neutrino is a suitable candidate: direct detection unlikely but indirect: X-ray background may already hint.***

- ***Microscopic physics (beyond standard model): production and freeze out ⟶ distribution function ⟶ constraints from abundances, phase space densities and power spectrum (free streaming scale). Several possible mechanisms: DW, boson decay at EW scales…***

- ***B-E: obtain T(k) for generic distribution. Assess small scale properties, cutoff and WDMAO's scale.***
- ***Systematic program: from micro to macro.***