Fast–roll eras, primordial fluctuations and the lowest CMB multipoles: theory and observations

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The Standard Model of the universe:
Theory and Observations

Outline

1 Theory

- The inflation paradigm
- EFT of (single field) inflation à la Ginsburg-Landau
- Fast–roll and initial conditions on fluctuations

2 Observations

- Is the low CMB TT quadruple too low?
- Probabilities and likelihoods
- MCMC analysis

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Basics [units: $c = \hbar = 1$]

Early accelerated cosmic expansion, $ds^2 = dt^2 - a(t) dx^2 \,, \quad \ddot{a} > 0$

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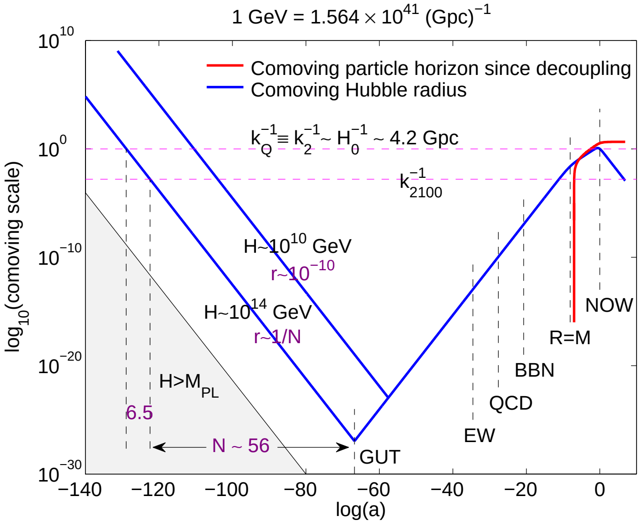

$\boxed{T \sim a^{-1}}$

MD stage:

$$\boxed{\frac{1}{aH} \sim \sqrt{a}}$$

RD stage:

$$\boxed{\frac{1}{aH} \sim a}$$

inflation:

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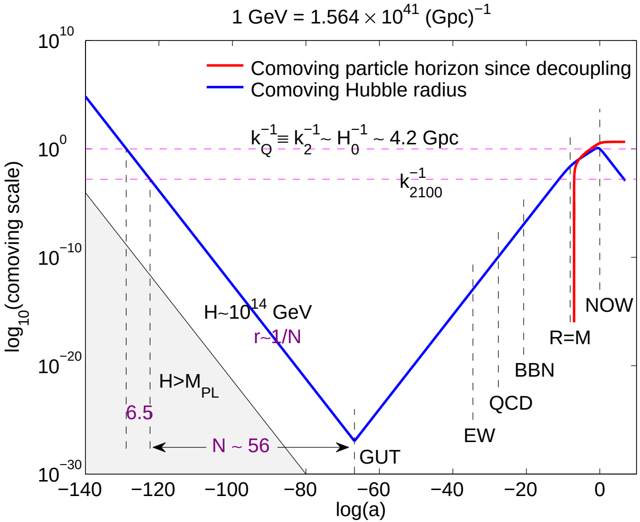

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Single-field inflation

Inflaton lagrangian

$$\mathscr{L} = a^3(t)\left[\frac{\dot\varphi^2}{2} - \frac{(\nabla\varphi)^2}{2a^2(t)} - V(\varphi)\right]$$

Fast expansion $\longrightarrow$ classical uniform field

$$H^2 = \frac{\rho}{3M_{Pl}^2}\ , \quad H \equiv \frac{\dot a}{a}\ , \quad \rho = \frac{\dot\varphi^2}{2} + V(\varphi)$$

$$\ddot\varphi + 3H\dot\varphi + V'(\varphi) = 0$$

H decreases monotonically

$$\dot H = -\frac{\dot\varphi^2}{2M_{Pl}^2}$$

Fundamental bounds

CMB isotropy or the *horizon problem* (with $\Delta H \sim \sqrt{N}$)

$$N_Q \geq 63 + \tfrac{1}{2} \log \frac{H}{10^{-4} M_{PL}}$$

Entropy of the Universe (dominated by photon and neutrinos)

$$N_{tot} \geq 63 + \tfrac{1}{2} \log \frac{H}{10^{-4} M_{PL}} - \tfrac{1}{12} \log \frac{g_{reh}}{1000}$$

tensor–scalar ratio in *generic* single-field new inflation

$$r = \frac{2}{\pi^2 A_s^2} \left(\frac{H}{M_{PL}} \right)^2 \sim 0.8 \left(\frac{H}{10^{-4} M_{PL}} \right)^2 \gtrsim \frac{1}{N} \,, \quad N \sim 60$$

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D. Boyanowski, C.D., H.J. de Vega, N. Sanchez, IJMP 24, no. 20-21, 3669 (2009)

Inflaton potential ($\hbar = 1$, $c = 1$, $M_{PL} = 2.4 \times 10^{18}$ GeV)

$$V(\varphi) = M^4 v(\phi) , \quad \phi = \varphi / M_{PL}$$

Energy scale of inflation and inflaton mass

$$M \sim M_{\rm GUT} \sim 10^{16} \text{ GeV}, \quad m = M^2 / M_{PL} \sim 10^{13} \text{ GeV}$$

Hubble parameter and quantum corrections

$$H \sim \sqrt{N}\, m \ll M_{PL} , \quad \text{loops} \to (H / M_{PL})^2 \sim 10^{-9}$$

Number of inflation efolds since horizon exit

$$N = \log \frac{a(t_{end})}{a(t_{exit})} , \quad v(\phi_{end}) = v'(\phi_{end}) = 0$$

t_{exit}: the mode with comoving k_0 becomes superhorizon ($\to N = N(k_0)$)

WMAP: $k_0 = 2 \text{ Gpc}^{-1}$, $N \simeq 61$
CosmoMC: $k_0 = 50 \text{ Gpc}^{-1}$, $N \simeq 57$

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Dimensionless setup: t in units of m^{-1}, H in units of m

Equations of motion

$$H^2 = \frac{1}{3}\left[\frac{1}{2}\dot{\phi}^2 + v(\phi)\right] , \quad \ddot{\phi} + 3H\,\dot{\phi} + v'(\phi) = 0 , \quad \dot{H} = -\frac{1}{2}\dot{\phi}^2$$

Energy density and pressure

$$\varepsilon = M^4\left[\frac{1}{2}\dot{\phi}^2 + v(\phi)\right] , \quad p = M^4\left[\frac{1}{2}\dot{\phi}^2 - v(\phi)\right]$$

Pre–inflation vs. fast–roll vs. slow–roll (see later)

$$\frac{1}{2}\dot{\phi}^2 > \frac{1}{2}v(\phi) , \quad \frac{1}{2}\dot{\phi}^2 \sim v(\phi) , \quad \frac{1}{2}\dot{\phi}^2 \lesssim \frac{1}{3N}v(\phi)$$

Slow–roll $(\phi = \phi_{exit} \sim N)$

$$n_s - 1 = -3\left[\frac{v'(\phi)}{v(\phi)}\right]^2 + 2\frac{v''(\phi)}{v(\phi)} \sim \frac{1}{N} , \quad r = 8\left[\frac{v'(\phi)}{v(\phi)}\right]^2 \sim \frac{1}{N}$$

which potential $v(\phi)$?

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Ginsburg-Landau effective approach

Polynomial inflation

$$V(\varphi) = V(0) \pm \tfrac{1}{2} m^2 \varphi^2 - \tfrac{1}{3} g \varphi^3 + \tfrac{1}{4} \lambda \varphi^4 + \ldots$$

Trinomial inflation (rescaled)

$$v(\phi) = v(0) \pm \tfrac{1}{2} \phi^2 + \tfrac{1}{3} h \sqrt{\tfrac{y}{2N}}\, \phi^3 + \tfrac{1}{32N} y \phi^4$$

with h and y of order 1

$$g = -h \sqrt{\tfrac{y}{2N}} \left(\frac{M}{M_{Pl}}\right)^2 \sim 10^{-9} , \quad \lambda = \frac{y}{8N} \left(\frac{M}{M_{Pl}}\right)^4 \sim 10^{-12}$$

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Trinomial chaotic (= large field) inflation

$$v(\phi) = \frac{1}{2}\phi^2 + \dots \,, \qquad -1 < h < 0$$

Fixing N $\quad (\Delta \equiv \sqrt{1-h^2})$

$$y = z + \frac{4}{3}h\sqrt{z} + \left(1 - \frac{4}{3}h^2\right)\log\left(1 + 2h\sqrt{z} + z\right)$$
$$-\frac{4h}{3\Delta}\left(\frac{5}{2} - 2h^2\right)\left[\arctan\left(\frac{h+\sqrt{z}}{\Delta}\right) - \arctan\left(\frac{h}{\Delta}\right)\right]$$

Effective coupling

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$$y = z - 2h^2 - 1 - 2h\Delta + \tfrac{4}{3}h(h+\Delta-\sqrt{z})$$
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Trinomial chaotic inflation

Scalar spectral index and tensor–scalar ratio

$$n_s = 1 - \frac{y}{2Nz}\left[3\,\frac{\left(1+2h\sqrt{z}+z\right)^2}{\left(1+\frac{4}{3}h\sqrt{z}+\frac{1}{2}z\right)^2} - \frac{1+4h\sqrt{z}+3z}{1+\frac{4}{3}h\sqrt{z}+\frac{1}{2}\,z}\right]$$

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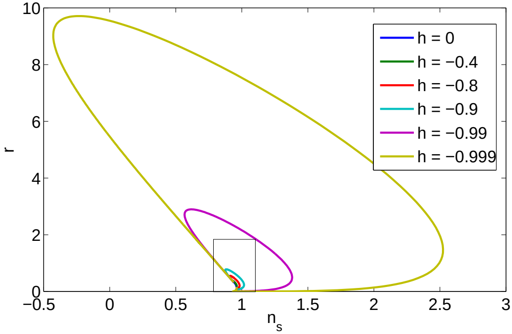

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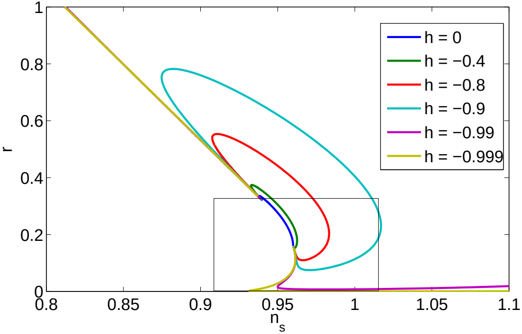

Trinomial chaotic inflation

Scalar spectral index and tensor–scalar ratio

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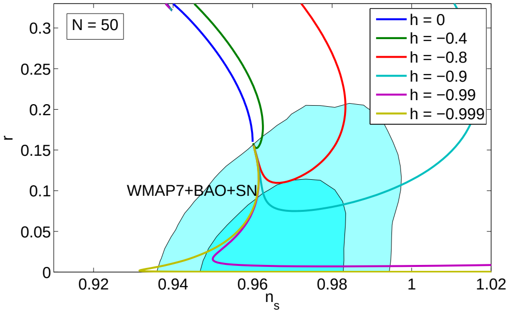

Trinomial new inflation

Scalar spectral index and tensor–scalar ratio

$$n_s = 1 - \frac{6y}{N} \frac{z(z+2h\sqrt{z}-1)^2}{\left[F(h)-2z+\frac{8}{3}hz^{3/2}+z^2\right]^2} + \frac{y}{N} \frac{3z+4h\sqrt{z}-1}{F(h)-2z+\frac{8}{3}hz^{3/2}+z^2}$$

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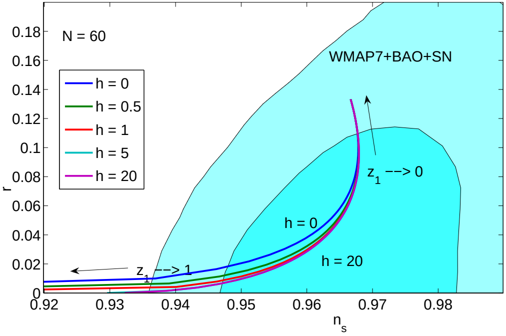

Binomial New Inflation

Setting the asymmentry $h = 0$ in TNI

$$v(\phi) = \frac{y}{32N}\left(\phi^2 - \frac{8N}{y}\right)^2$$

Scalar spectral index and tensor–scalar ratio

$$n_s = 1 - \frac{y}{N}\,\frac{3z+1}{(1-z)^2}\,, \quad r = \frac{16\,y}{N}\,\frac{z}{(1-z)^2}\,, \quad y = z - 1 - \log z\,, \quad 0 < z < 1$$

MCMC analysis of WMAP5 + small scale + LSS + SN

best fit: $y \simeq 1.26$, $r \simeq 0.05$
95% lower bound: $r \gtrsim 0.025$

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More generally

MCMC analysis of current data plus Ginsburg-Landau stability arguments point to double–well type potentials with the inflaton ϕ rolling from a region of negative curvature near $\phi = 0$ (the "false vacuum") toward the true absolute minimum ϕ_{min} of the potential where $v(\phi_{min}) = v'(\phi_{min}) = 0$.

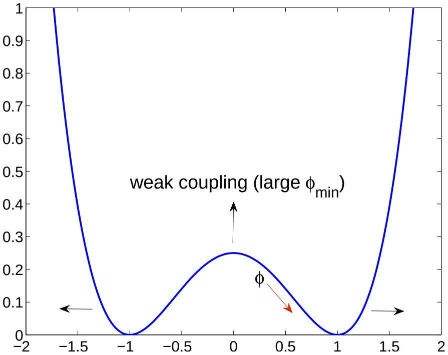
In general

$$v(\phi) = \phi_{min}^2 F(\phi/\phi_{min})$$

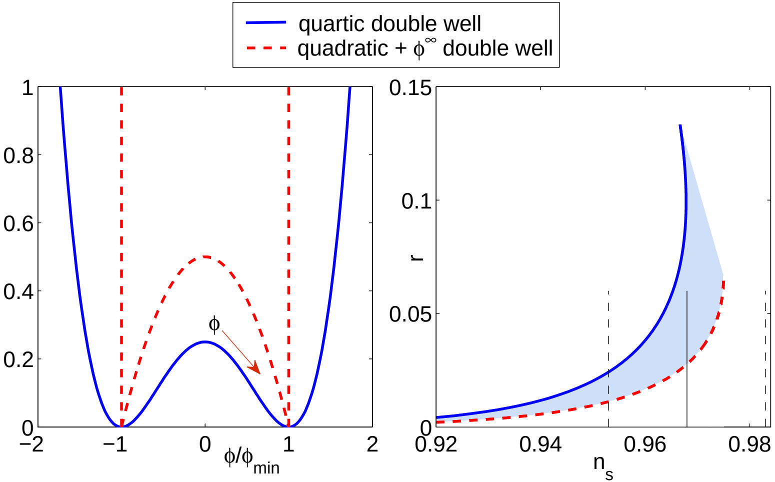
with $F(x) \simeq F_0 - \frac{1}{2}x^2$ as $x \to 0$.

For instance BNI
(Binomial New Inflation)

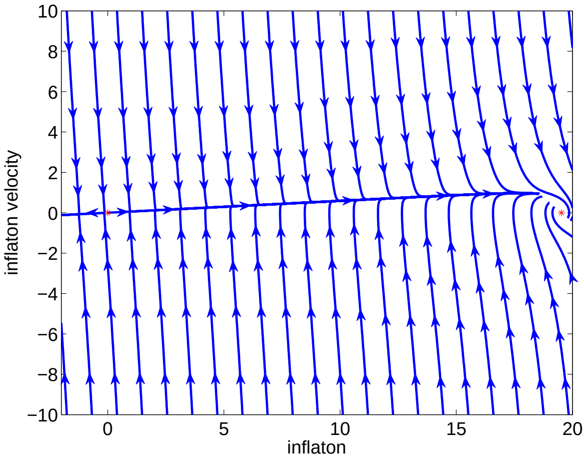
$$F(x) = \frac{1}{4}(x^2 - 1)^2$$



Higher order terms and the universal banana
C.D., H.J. de Vega, N. Sanchez, arXiv:0906.4102



BNI: Inflaton flow in phase space



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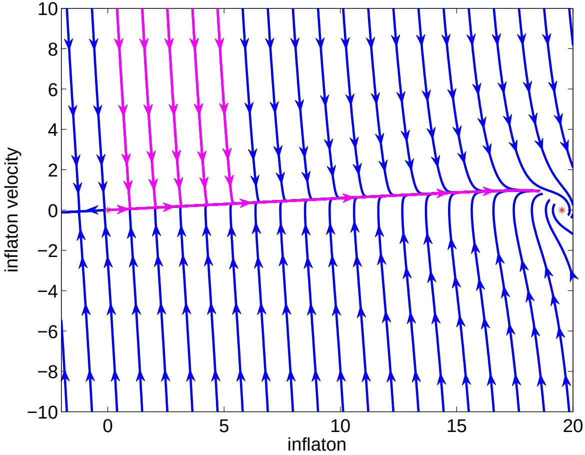

Generic inflaton trajectories are singular as $t \to t_*^+$

$$\phi \simeq \sqrt{2/3}\,\log\left(\frac{t-t_*}{b}\right) \;,\quad \dot\phi \simeq \frac{\sqrt{2/3}}{t-t_*} \;,\quad H \simeq \frac{1}{3(t-t_*)} \;,\quad a \simeq (t-t_*)^{1/3} \;,\quad \eta \to \eta_*$$

Pre-inflationary ($\ddot{a} < 0$!) $\longrightarrow$ fast–roll $\longrightarrow$ slow–roll

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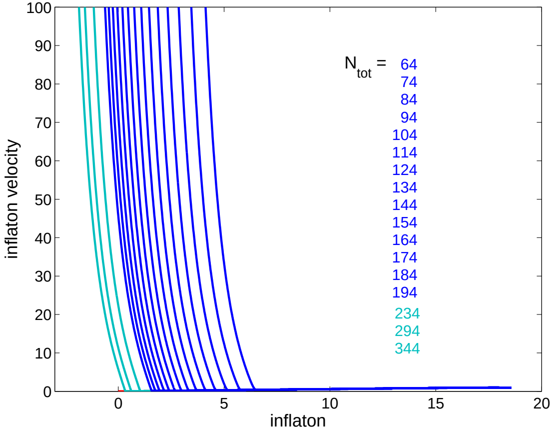

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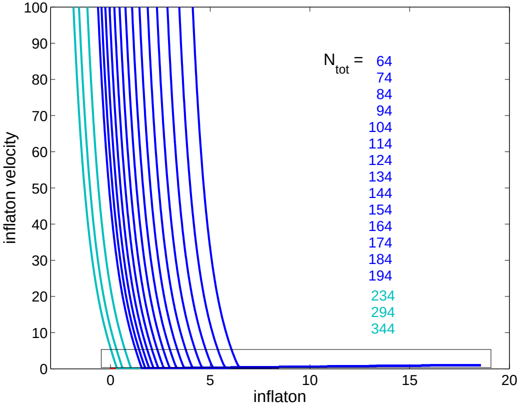

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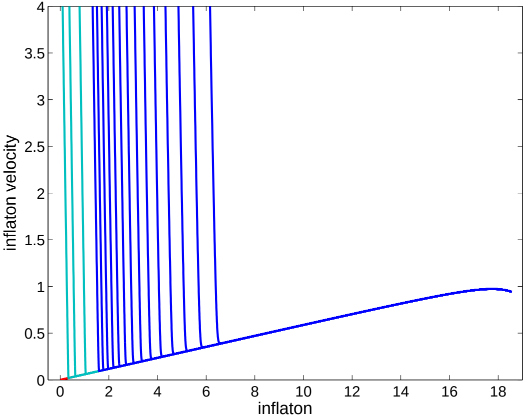

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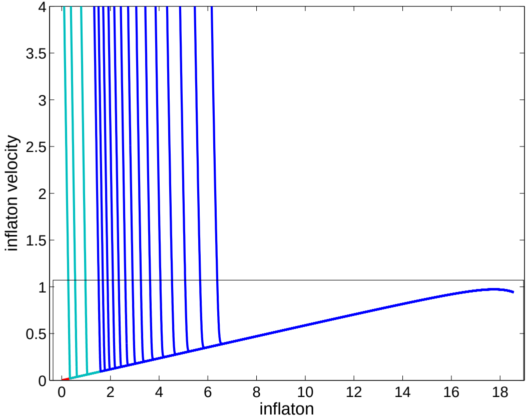

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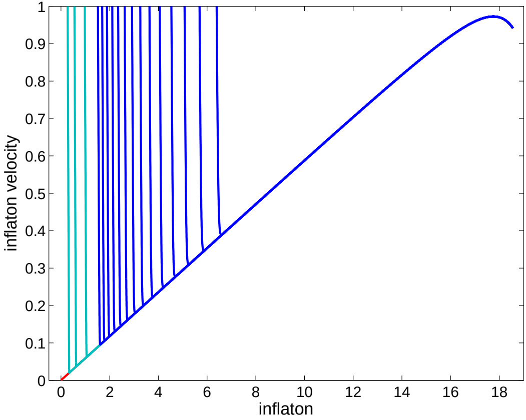

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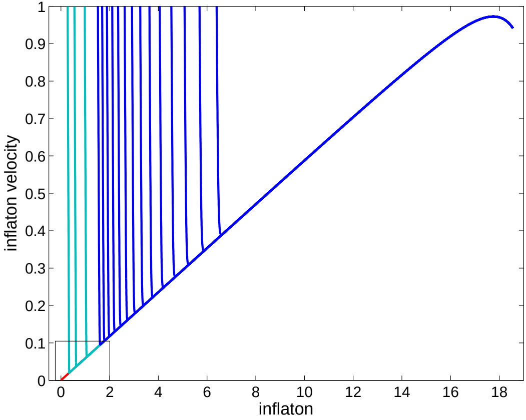

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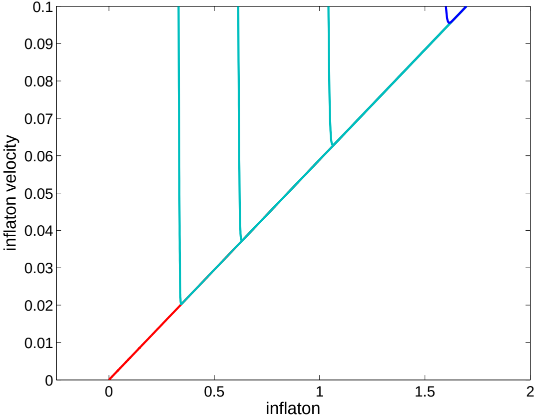

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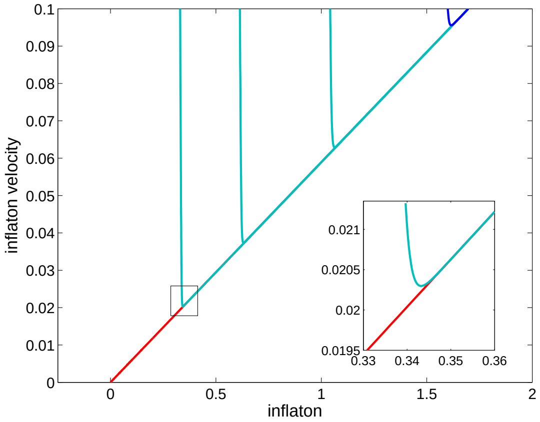
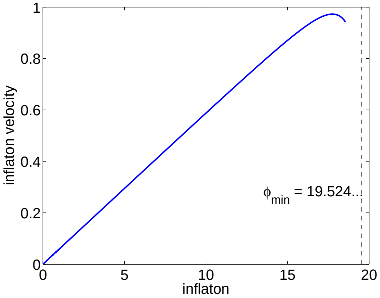


The extreme slow–roll solution (sort of half de Sitter)

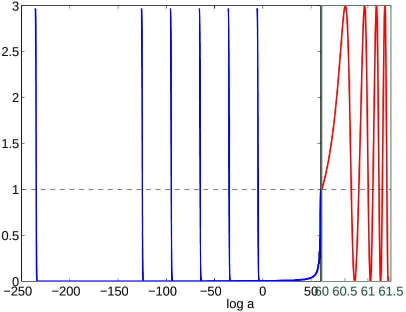
$$\ddot{\phi} + 3h\,\dot{\phi} + \phi = 0$$
$$\phi \propto \exp(\alpha t)\ , \quad t \to -\infty$$
$$\alpha = \tfrac{1}{2}\left[(\sqrt{3v(0)+4} - \sqrt{3v(0)}\right]$$

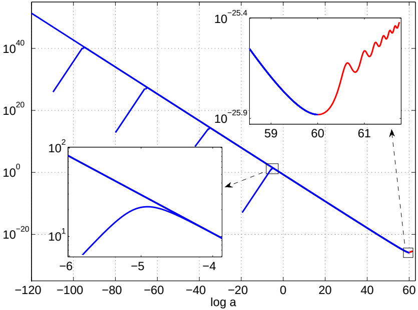


	start	$a = 1$	end: $\ddot{a} = 0^+$
t	$-344.9514017\ldots$	0	$17.40482446\ldots$
ϕ	10^{-8}	$6.7484118\ldots$	$18.5586530\ldots$
$\dot{\phi}$	$\alpha\,10^{-8} = 5.89371084\ldots 10^{-10}$	$0.3973384\ldots$	$0.94150557\ldots$
$\log a$	$-1938.4867948\ldots$	0	60
h	$(12g)^{-1/2} = 5.6361006\ldots$	$4.9653973\ldots$	$0.6657449\ldots$
η	$-\infty$ (f.a.p.p)	$-0.2020610\ldots$	0

$$\epsilon_v = -\frac{\dot{h}}{h^2} = \frac{3\dot{\phi}^2}{\dot{\phi}^2 + 2v(\phi)}$$

comoving Hubble radius $= \dfrac{1}{ah}$





$N_{slowroll}$	63	93	123	153...	233
$N_{fastroll}$	0.917...	0.855...	0.819...	0.797...	0.773...

Validity of the classical inflaton picture

Quantum loop corrections as $t \to t_*$

$$\left(\frac{H}{M_{Pl}}\right)^2 \sim \left[\frac{m}{3(t-t_*)M_{Pl}}\right]^2 = \left(\frac{1.66\ldots \times 10^{-6}}{t-t_*}\right)^2$$

are less than 1% as long as

$$t - t_* > 1.66\ldots \times 10^{-5}$$

Positivity of ϕ if a condensate

$$\phi \simeq \frac{\sqrt{2}}{\sqrt{3}} \log\left(\frac{t-t_*}{b}\right) > 0 \Longrightarrow t - t_* > b = 4.7452\ldots\, 10^{-5} \,, \quad N_{slowroll} = 63$$

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Outline

1. Theory
   - The inflation paradigm
   - EFT of (single field) inflation à la Ginsburg-Landau
   - Fast–roll and initial conditions on fluctuations

2. Observations
   - Is the low CMB TT quadruple too low?
   - Probabilities and likelihoods
   - MCMC analysis

Scalar fluctuations

Gauge–invariant quantum perturbation field

$$u(x,t) = -\xi(t) R(x,t) = \int \frac{d^3k}{(2\pi)^{3/2}} \left[\alpha_k S_k(\eta) e^{ik\cdot x} + \alpha_k^\dagger S_k^*(\eta) e^{-ik\cdot x}\right]$$

$$[\alpha_k, \alpha_{k'}^\dagger] = \delta^{(3)}(k-k') \,, \quad \xi(t) = \frac{a(t)}{H(t)} \dot\phi(t) \,, \quad \eta = \int \frac{dt}{a(t)}$$

Schroedinger–like dynamics

$$\left[\frac{d^2}{d\eta^2} + k^2 - W(\eta)\right] S_k = 0 \,, \quad W(\eta) = \frac{1}{\xi}\frac{d^2\xi}{d\eta^2}$$

$$\left[\frac{d^2}{dt^2} + H\frac{d}{dt} + \frac{k^2}{a^2} - U(t)\right] S_k = 0$$

Standard parametrization in dimensionless setup

$$U(t) = H^2(2 - 7\,\varepsilon_V + 2\,\varepsilon_V^2) - 2\,\dot\phi\,\frac{v'(\phi)}{H} - \eta_V\, v(\phi) \,, \quad \varepsilon_V = \frac{\dot\phi^2}{2\,H^2} \,, \quad \eta_V = \frac{v''(\phi)}{v(\phi)}$$

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$$\epsilon_v \simeq \frac{1}{2}\left[\frac{v'(\phi_{exit})}{v(\phi_{exit})}\right]^2 , \quad \eta_v \simeq \frac{v''(\phi_{exit})}{v(\phi_{exit})} , \quad W(\eta) = \frac{\nu^2 - 1/4}{\eta^2} , \quad \nu = \frac{3}{2} + 3\epsilon_v - \eta_v$$

Power spectrum

$$P(k) = \lim_{\eta\to 0}\left(\frac{m}{M_{PL}}\right)^2 \frac{k^3}{2\pi^2}\left|\frac{S_k \eta)}{\xi(\eta)}\right|^2$$

Bunch–Davies vacuum at $t \to -\infty$ in extreme slow–roll

$$S_k(\eta \to -\infty) = \frac{e^{ik\eta}}{\sqrt{2k}} , \quad P_\infty = A_s\left(\frac{k}{k_0}\right)^{n_s - 1} , \quad A_s = \left(\frac{m}{M_{PL}}\right)^2 \frac{N^2}{12\pi^2}\mathscr{O}(1)$$

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Bunch–Davies vacuum at finite times?

Slow–roll approximation

$$\epsilon_v \simeq \frac{1}{2}\left[\frac{v'(\phi_{exit})}{v(\phi_{exit})}\right]^2 , \quad \eta_v \simeq \frac{v''(\phi_{exit})}{v(\phi_{exit})} , \quad W(\eta) = \frac{\nu^2 - 1/4}{\eta^2} , \quad \nu = \frac{3}{2} + 3\epsilon_v - \eta_v$$

Power spectrum

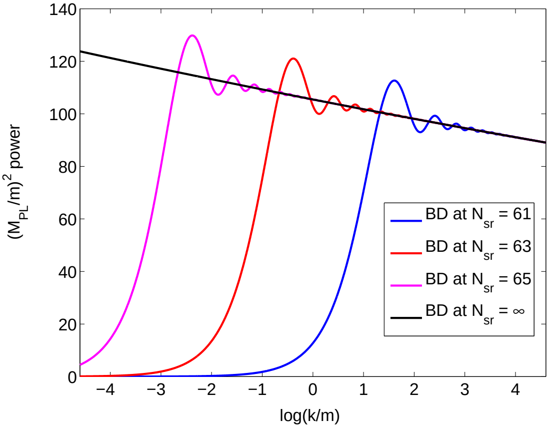
$$P(k) = \lim_{\eta \to 0} \left(\frac{m}{M_{PL}}\right)^2 \frac{k^3}{2\pi^2} \left|\frac{S_k \eta)}{\xi(\eta)}\right|^2$$

Bunch–Davies vacuum at $t \to -\infty$ in extreme slow–roll

$$S_k(\eta \to -\infty) = \frac{e^{ik\eta}}{\sqrt{2k}} , \quad P_\infty = A_s \left(\frac{k}{k_0}\right)^{n_s - 1} , \quad A_s = \left(\frac{m}{M_{PL}}\right)^2 \frac{N^2}{12\pi^2} \mathscr{O}(1)$$

Bunch–Davies vacuum at finite times?

Bunch–Davies vacuum at finite times



Compare the small $k-$ behavior of BD and quasi-De Sitter modes

$$S_k(\eta_0) = \frac{e^{i k \eta_0}}{\sqrt{2k}} \quad , \qquad \frac{1}{2} i^{\nu+\frac{1}{2}} \sqrt{-\pi \eta_0} H_\nu^{(1)}(-k\,\eta_0) \simeq \frac{\Gamma(\nu)}{\sqrt{2\pi k}} \left(\frac{2}{i k \eta_0}\right)^{\nu-\frac{1}{2}}$$

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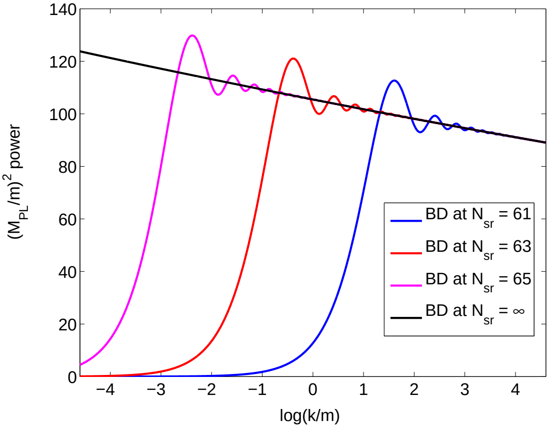

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The transfer function of initial conditions

$$P(k) = P_\infty(k)\Big[1 + D(k)\Big]$$

more formally ...

Effect on quadratic observables due to making linear combinations of solutions of second order linear differential equations, or Bogoliubov transformations on free-field creation–annihilation operators.

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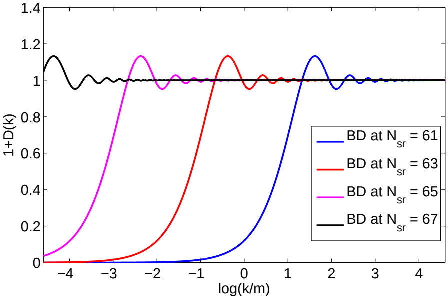
Effect on quadratic observables due to making linear combinations of solutions of second order linear differential equations, or Bogoliubov transformations on free-field creation–annihilation operators.

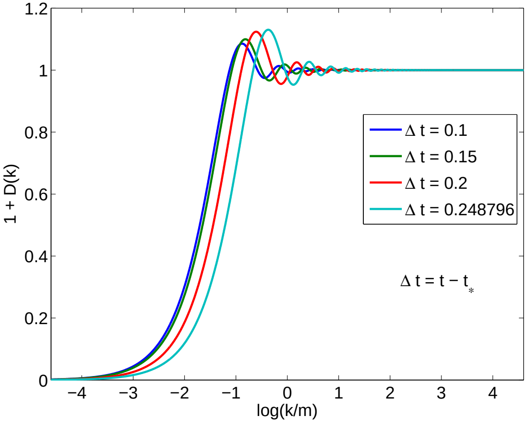
$$D(k) \simeq D(k\eta_0)$$

$$D(k) \sim k^{-2}\,, \quad k \to \infty$$

to have a negligible back–reaction on the metric



Transfer function for fast–roll trajectories
C.D., H.J. de Vega and N. Sanchez, Phys. Rev. D 81, 063520 (2010)



$\Longrightarrow$ depression of lowest multipoles

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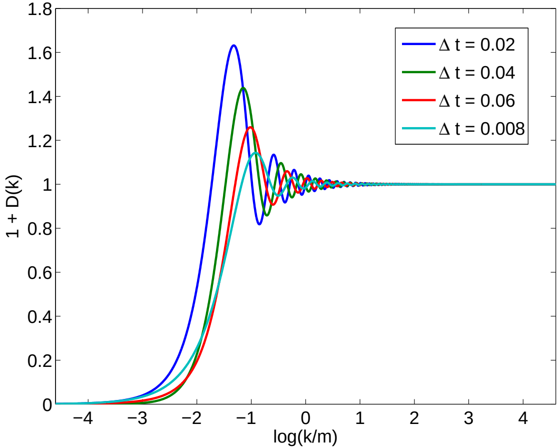

$\Longrightarrow$ up and down with little change on average

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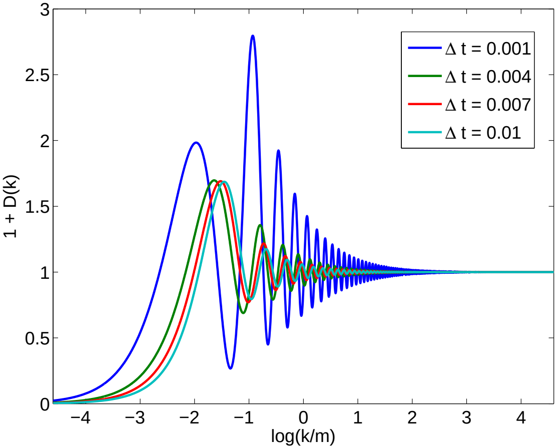
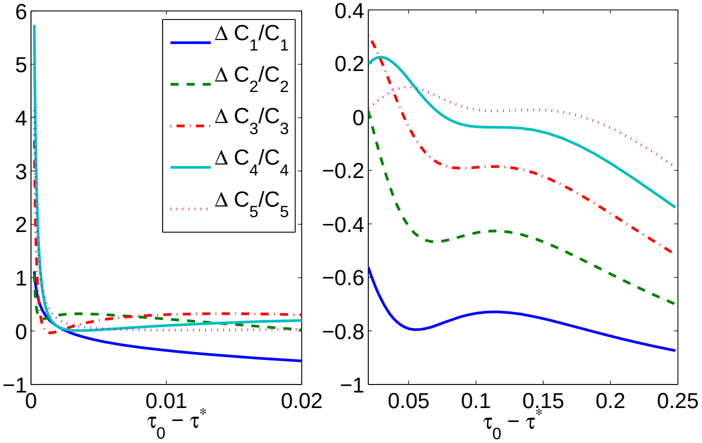


$\Longrightarrow$ up and down with net overall enhancement

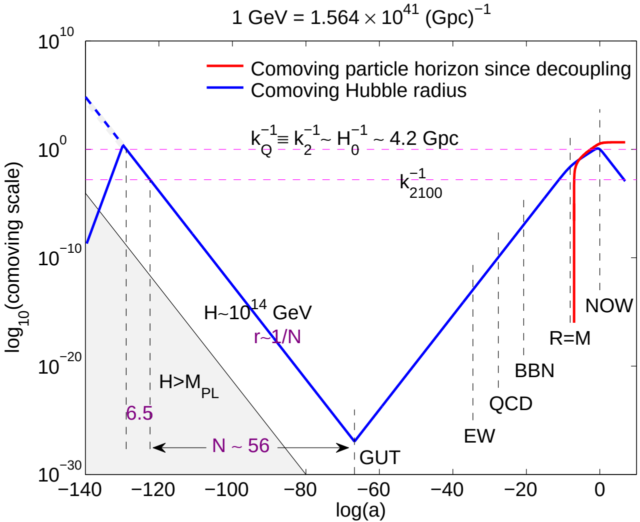
$$\frac{\Delta C_l}{C_l} = \frac{\int_0^\infty dx\, D(0.303\ldots H_0 x) x^{n_s-2}[j_l(x)]^2}{\int_0^\infty dx\, x^{n_s-2}[j_l(x)]^2}$$



Basics with fast–roll

[units: $c = \hbar = 1$]

$$ds^2 = dt^2 - a(t)dx^2\ , \quad H = \dot{a}/a$$



MD stage:

$$\boxed{\frac{1}{aH} \sim \sqrt{a}}$$

RD stage:

$$\boxed{\frac{1}{aH} \sim a}$$

inflation:

$$\boxed{\frac{1}{aH} \sim \frac{1}{a}}$$

pre-inflation:

$$\boxed{\frac{1}{aH} \sim a^2}$$

A summarizing comparison

Extreme slow–roll

- It's unique.
- Has adiabatic Bunch–Davies vacuum of de Sitter spacetime.
- Gravity always semiclassical ($H \ll M_{PL}$ at any time).
- All quantum modes were once trans–planckian, including cosmological relevant ones.

Fast–roll

- It's generic.
- No "natural" initial conditions for quantum amplitudes.
- Needs quantum gravity when $t \to t_*$;
- CMB-relevant quantum modes are not be trans–planckian for $N_{slowroll} \lesssim 70$.
- **Provides a simple mechanism for suppression of low multipoles if $N_{slowroll} = 63$.**

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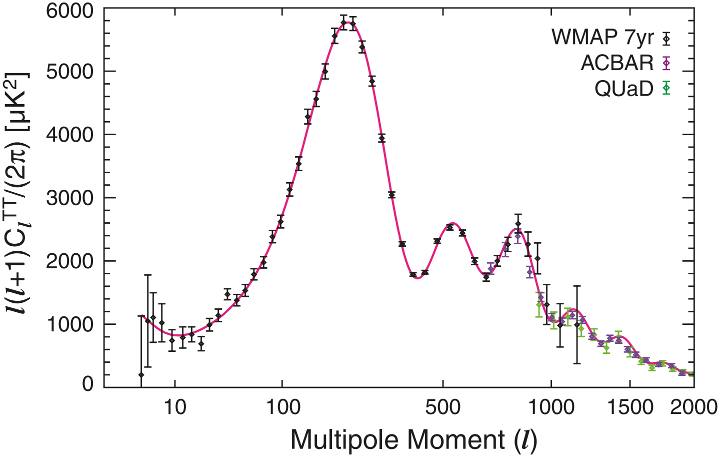
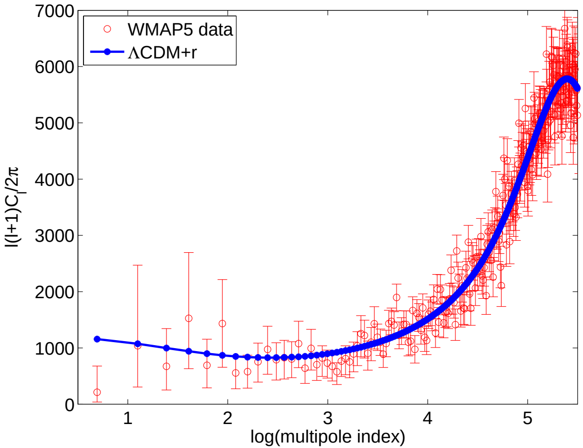
The WMAP+small scale TT multipoles (binned)

from *Komatsu, E., et.al.*, arXiv:1001.475 [astro-ph] 26 Jan 2010



$$C_2 = 200\ \mu K^2 \text{ (WMAP7 ML value)} \ , \qquad C_2 \simeq 1200\ \mu K^2 \ (\Lambda\text{CDM})$$

WMAP5 unbinned C_ℓ for $\ell \leq 250$



(experimental error)/(cosmic variance) $\leq$ 20% for $\ell \leq 250$

Other analysis of WMAP5 data

- ...
- P.K. Samal, R. Saha, J. Delabrouille, S. Prunet, P. Jain, T. Souradeep, *"CMB Polarization and Temperature Power Spectra Estimation using Linear Combination of WMAP 5-year Maps"*, Astrophysical Journal714:840-851, 2010

$$C_2 = 557\ \mu K^2 \quad (\text{WMAP5}+150\%)\ , \qquad C_3 = 306\ \mu K^2\ (\text{WMAP5}-40\%)$$

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From T anisotropy maps to TT multipoles

Statistically isotropic fluctuation maps $t = t(n)$

$$\langle t(n) \rangle = 0 \quad , \quad \langle t(n)\, t(n') \rangle = C(n \cdot n')$$

Spherical harmonic decomposition $\longrightarrow$ angular power spectrum

$$t(n) = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{m=\ell} a_{\ell m}\, Y_{\ell m}(n) \quad , \quad C(n \cdot n') = \frac{1}{4\pi} \sum_{\ell} (2\ell+1)\, C_\ell\, P_l(\cos\theta)$$

$$\langle a_{\ell m} \rangle = \langle a^*_{\ell m} \rangle = 0 \quad , \quad \langle a_{\ell m}\, a^*_{\ell' m'} \rangle = \delta_{\ell\ell'}\, \delta_{mm'}\, C_\ell$$

Gaussian distribution

$$\Pr(t\,|\,C) = [\mathrm{Det}(2\pi C)]^{-1/2} \exp\left[-\frac{1}{2} \int d^2n \int d^2n'\, t(n)\, (C^{-1})(n \cdot n')\, t(n') \right]$$

$$= \prod_{\ell} \left[(2\pi C_\ell)^{-1/2} \exp\left(-\frac{\overline{C}_\ell}{2\, C_\ell} \right) \right]^{(2\ell+1)} \quad , \quad \overline{C}_\ell = \frac{1}{2\,\ell+1} \sum_{m=-\ell}^{m=\ell} |a_{\ell m}|^2$$

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Bayesian inference

Cosmic variance + finite resolution + detector noise

- $a_{\ell m} \Longrightarrow a_{\ell m}^{(data)}$, $\overline{C}_\ell \Longrightarrow C_\ell^{(data)}$ (pseudo–C_ℓ)
- $C_\ell \Longrightarrow w_\ell C_\ell^{(model)} + N$ (white noise summed in quadrature)
- window function in ℓ–space: $w_\ell = \exp[-b\,\ell(\ell+1)]$, $b = \theta_{\rm pix}^2/(8\ln 2)$

The likelihood function L (only from TT, assuming flat priors)

$$\log L = -\sum_\ell \left(\ell + \frac{1}{2}\right)(x_\ell - \log x_\ell - 1) \quad , \quad x_\ell = \frac{C_\ell^{(data)}}{w_\ell\, C_\ell^{(model)} + N}$$

In simulations, assuming no bias

$$C_\ell^{(data)} \Longrightarrow w_\ell C_\ell^{(fiducial)} + N$$

Noise / signal $\lesssim 1$

WMAP7: $\ell \lesssim 500$, PLANK (two surveys): $\ell \lesssim 1500$

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- $a_{\ell m} \Longrightarrow a_{\ell m}^{(data)}$, $\overline{C}_\ell \Longrightarrow C_\ell^{(data)}$ (pseudo–C_ℓ)
- $C_\ell \Longrightarrow w_\ell C_\ell^{(model)} + N$ (white noise summed in quadrature)
- window function in ℓ–space: $w_\ell = \exp[-b\,\ell(\ell+1)]$, $b = \theta_{\rm pix}^2/(8\ln 2)$

The likelihood function L (only from TT, assuming flat priors)

$$\log L = -\sum_\ell \left(\ell + \frac{1}{2}\right)(x_\ell - \log x_\ell - 1) \quad , \quad x_\ell = \frac{C_\ell^{(data)}}{w_\ell\, C_\ell^{(model)} + N}$$

In simulations, assuming no bias

$$C_\ell^{(data)} \Longrightarrow w_\ell C_\ell^{(fiducial)} + N$$

Noise / signal $\lesssim 1$

WMAP7: $\ell \lesssim 500$, PLANK (two surveys): $\ell \lesssim 1500$

Probabilities and normalized likelihoods

Recall $X_\ell = C_\ell^{(data)}/(w_\ell C_\ell^{(model)} + N)$; then

$\Pr(X_\ell = x|model) \propto \frac{1}{x}\left(x e^{-x}\right)^{\ell+1/2}$ (reduced chi-square distribution) is the probability density for $C_\ell^{(data)}$ given the model, with

$$\langle X_\ell \rangle = 1 \text{ and } (X_\ell)_{ML} = \frac{2\ell-1}{2\ell+1}$$

At the same time, if $Y_\ell = 1/X_\ell = (w_\ell C_\ell^{(model)} + N)/C_\ell^{(data)}$, then

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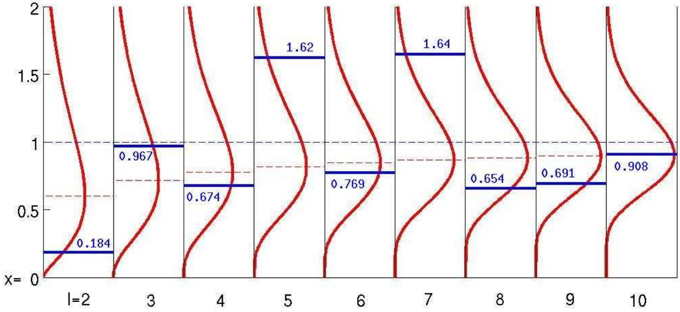
An example: lowest 9 TT multipoles

probability curves for $X_\ell = C_\ell^{(data)} / C_\ell^{(model)}$ from best fit ΛCDM WMAP5 data



$$p_2(x) \equiv \Pr[C_2^{(data)} < x\, C_2^{(model)}] \quad , \qquad p_2(0.184) \simeq 0.031 \ldots$$

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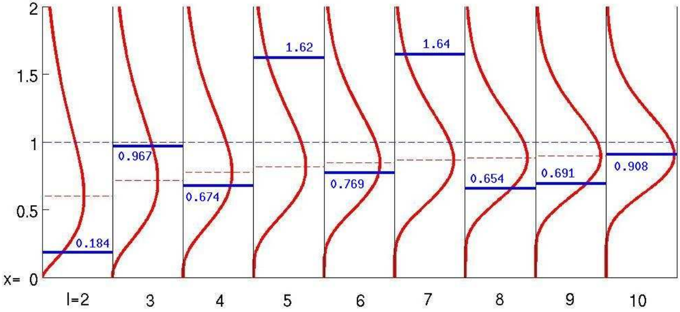

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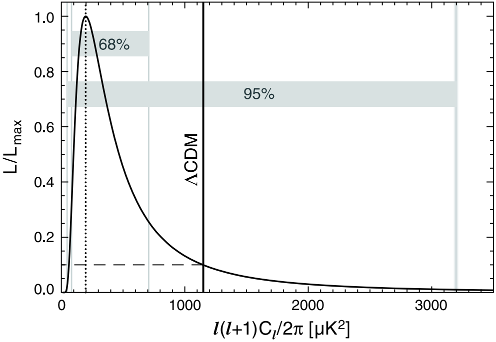
WMAP7 data: $p_2(0.169) \simeq 0.026\ldots$
(including T-E coupling and with detector noise added in quadrature)

C. L. Bennett et. al., arXiv:1001.4758

Seven-year WMAP: are there CMB anomalies?



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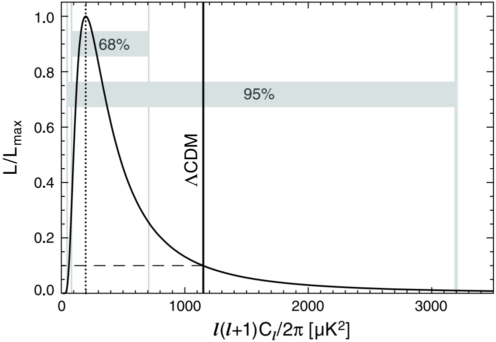

$\Pr(X_\ell = x | model)$ or $\Pr(Y_\ell = y | data)$ to compute $p_2(x)$?

$\Pr(X_2 = x | model) \propto x^{3/2} e^{-5x/2} \longrightarrow p_2(0.169) \simeq 0.026\ldots$

$\Pr(1/X_2 = 1/x | data) \propto x^{1/2} e^{-5x/2} \longrightarrow p_2(0.169) \simeq 0.159\ldots$

[N.B.: $p_2(0.167) \simeq 0.176\ldots$ in *Bennett et. al.*]

Probabilities as i.i.d. random variables

Recall $p_\ell(x) = Pr(X_\ell \leq x|model)$ with $X_\ell = C_\ell^{(data)}/(w_\ell C_\ell^{(model)} + N)$, then

A simple exercise

if numbers x_ℓ are extracted from the distributions $\Pr(X_\ell = x|model)$ then the $p_\ell = p_\ell(x_\ell)$ are independent random numbers flatly distributed in $(0, 1)$

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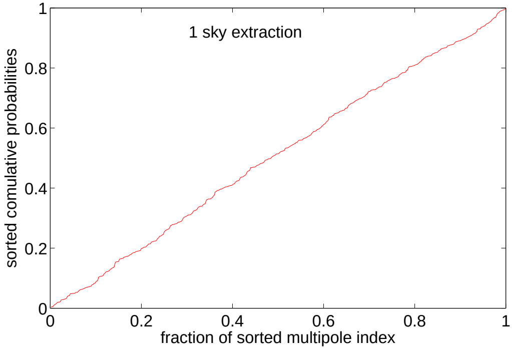

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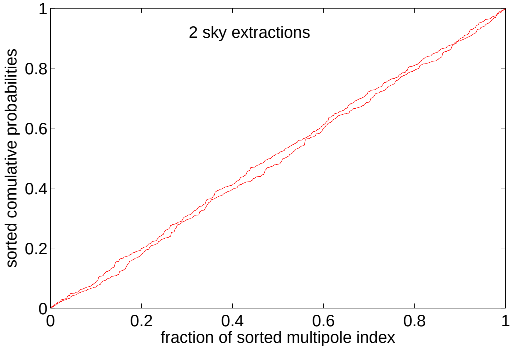

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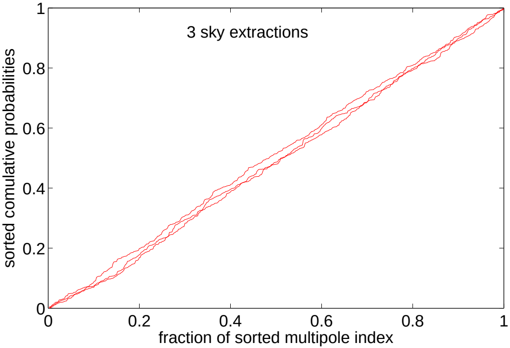

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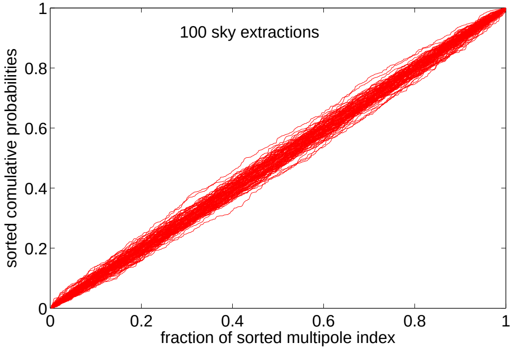

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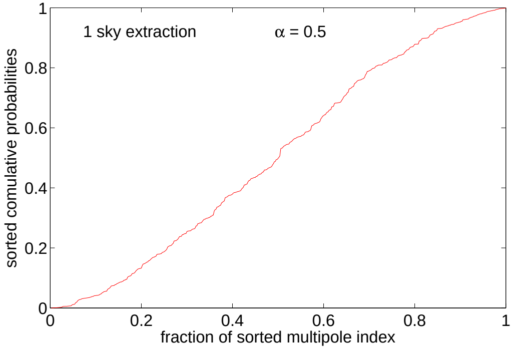

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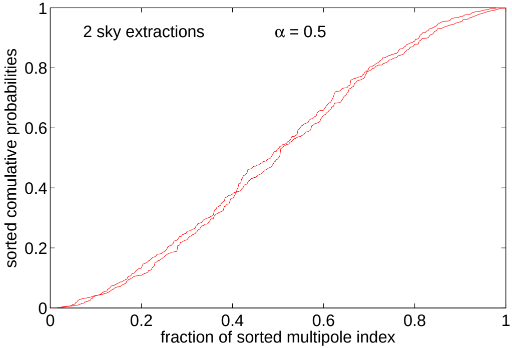

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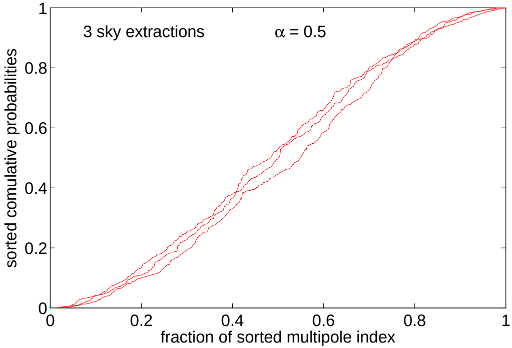

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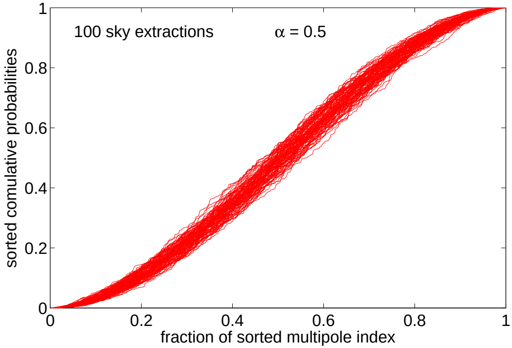

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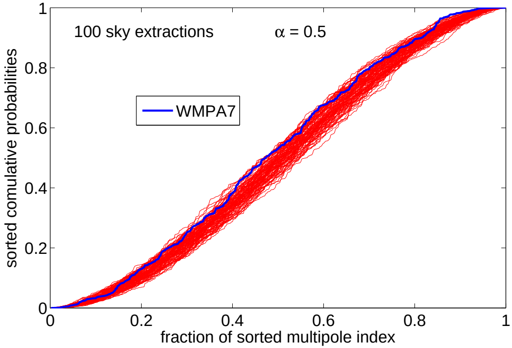

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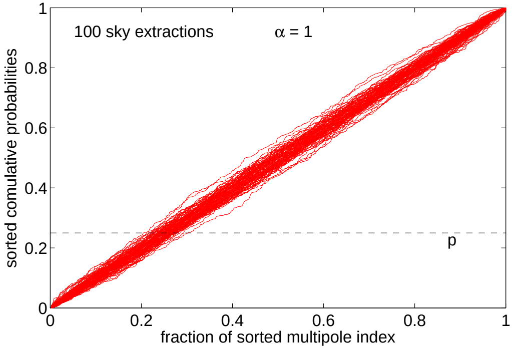

$N=500$

Pr($p_\ell \leq p$ is true k times as $\ell=2,3,\ldots N+1$) = $\binom{N}{k} p^k (1-p)^{N-k}$

$\langle k\rangle_N = pN = 13 \qquad (\Delta k)^2_N = p(1-p)N = 12.66$ for $p=p_2=0.026$

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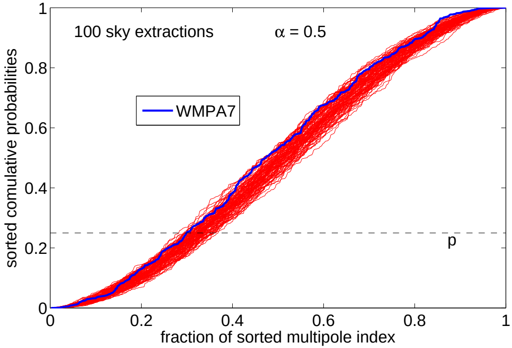

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$k = 39$ in WMAP7$(2 \to 501)$; $(k = 7$ in $2 \to 100)$; $p_{114} = 1.4636 \times 10^{-5}$

Outline

ΛCDM+BNI with sharpcut or (simplified) fastroll
C.D., H.J. de Vega, N. Sanchez, Phys. Rev. D78 023013 (2008)

Simplification

- Born's approximation for k not too small.
- k_{tran} is the comoving wavenumber (used as MCMC parameter) that exits the horizon when fast-roll ends and slow-roll starts.

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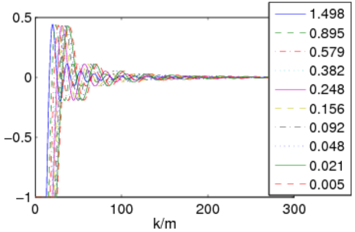
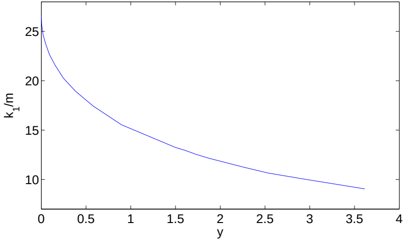
For BNI, $v(\phi) = \frac{1}{4} g(\phi^2 - 1/g)^2$, $g = y/(8N)$, $y = z - 1 - log(z)$





BNI+sharpcut vs. BNI+fastroll

MCMC parameters:	ω_b, ω_c, θ, τ, (slow), A_s, z, k_{tran} (fast)
Context:	$N = 60$, $\Omega_\nu = 0$, ... ; standard priors, no SZ, lensed CMB, linear mpk, ...
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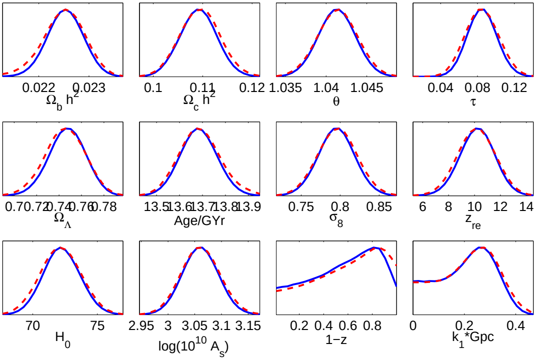
param	best fit
$100\Omega_b h^2$	2.256
$\Omega_c h^2$	0.110
θ	1.041
100τ	8.83
H_0	71.82
σ_8	0.803
$\log[10^{10} A_s]$	0.307
z	0.162
k_{tran}	0.260
$-\log(L)$	1253.96

sharp-cut   flat $0 < z < 1$ prior



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100τ	8.42
H_0	72.00
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z	0.102
k_{tran}	0.284
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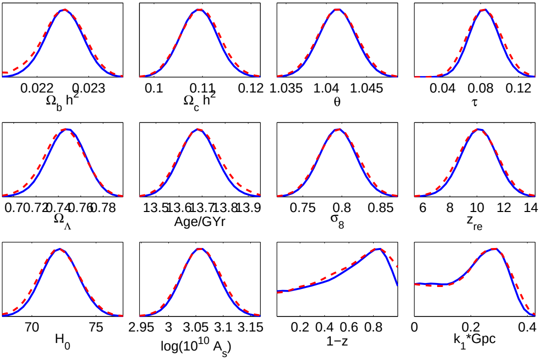
fast-roll   flat $0 < z < 1$ prior



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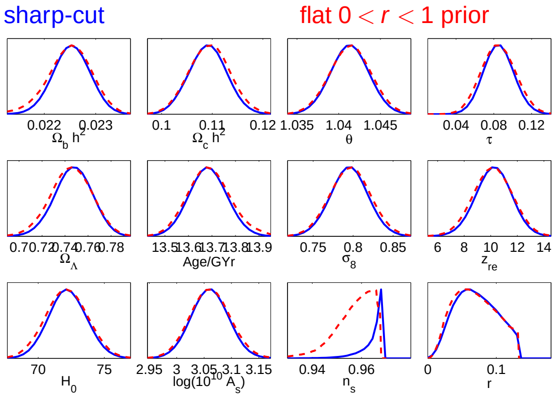

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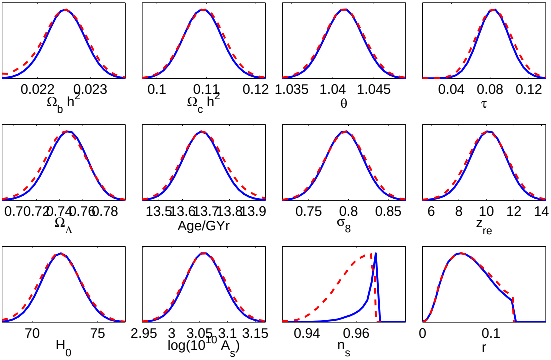
fast-roll   flat $0 < r < 1$ prior



$\Delta\chi^2$ w.r.t. ΛCDM$+r$

	WMAP5	+SDSS+ACBAR08	+SDSS+SN
BNI+sharpcut	-1.07	-0.71	-1.02
BNI+fastroll	-1.15	-0.99	-1.45

95% lower bound on r

	WMAP5	+SDSS+ACBAR08	+SDSS+SN
BNI+sharpcut	0.025	0.033	0.022
BNI+fastroll	0.024	0.032	0.023

most likely value of k_{tran} (in Gpc^{-1})

	WMAP5	+SDSS+ACBAR08	+SDSS+SN
BNI+sharpcut	0.258	0.260	0.244
BNI+fastroll	0.298	0.284	0.291

Summary

- Large scale CMB anisotropies provide information on the beginning of inflation.
- Early fast-roll inflation is generic and provides a mechanism for lowest multipoles depression.
- BNI+fastroll improves the fit w.r.t. ΛCDM$+r$.
- Fast–roll depression of the quadrupole sets to ~ 64 the total number of inflation efolds.

- Outlook
  - Improve the MCMC analysis using more accurate $D(k)$ and newer data (WMAP7, QUAD).
  - Why do the homogeneity and entropy bounds coincide?
  - Prepare for better data (Plank, ACT, ...) as in *C. Burigana, C. D., H. J. de Vega, A. Gruppuso, N. Mandolesi, P. Natoli, N. G. Sanchez* arXiv:1003.6108.

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    - Improve the MCMC analysis using more accurate $D(k)$ and newer data (WMAP7, QUAD).
    - Why do the homogeneity and entropy bounds coincide?
    - Prepare for better data (Plank, ACT, ...) as in *C. Burigana, C. D., H. J. de Vega, A. Gruppuso, N. Mandolesi, P. Natoli, N. G. Sanchez* arXiv:1003.6108.