keV Sterile Neutrinos as Dark Matter

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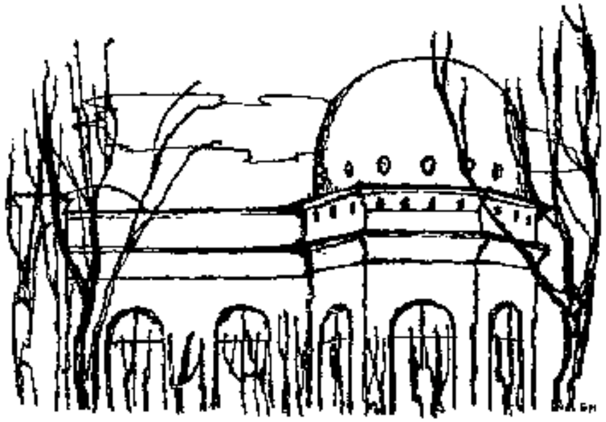
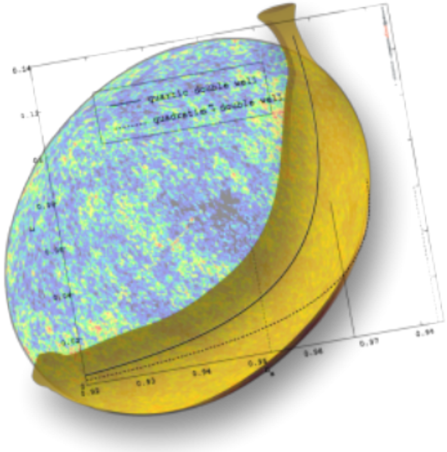


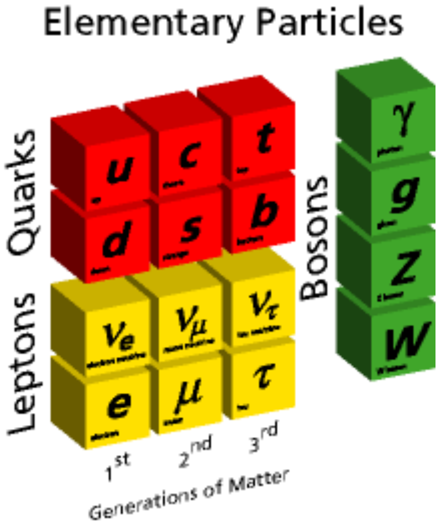


15th Paris Cosmology Cooloquium 2011, Observatoire de Paris, July 20-22, 2011

Physics Beyond the Standard Model

QED ➔QCD ➔SM
$U(1)_{em}$ $SU(3)_C$ $SU(3)_C \times SU(2)_L \times U(1)_Y$

Success story of d=4 renormalizable QFTs



TOE solving all problems does not (yet) exist ➔ solve some problems ➔ increasing levels of speculation:

1) new fields
2) extend gauge group
3) new concepts (SUSY, ...)
4) wild speculations

Theoretical reasons for BSM:

SM does not exist without cutoff (triviality)
Higgs-doublett = only simplest extension
Gauge hierarchy problem
Gauge unification, charge quantization
Strong CP problem
Unification with gravity
Why: 3 generations , which representations
Many parameters (9+? masses, 4+? mixings)

Experimental BSM facts:

- Electro weak scale << Planck scale
- Gauge couplings almost unify
- Neutrinos have masses & large mixings
- Baryon asymetry of the Universe
- Dark Matter,Dark Energy, few $\geq 2\sigma$ hints?

➔ BSM from a neutrino perspective

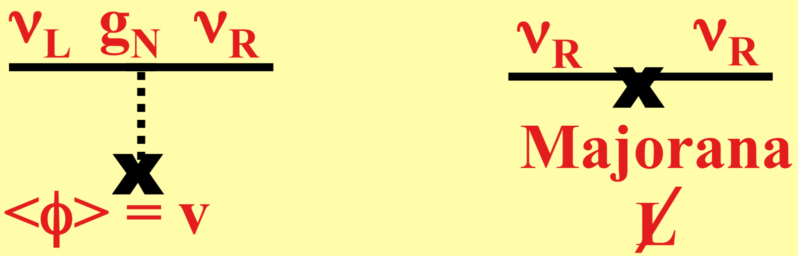
Adding neutrino masses to the SM ➔ SM+

New Physics: Neutrino Mass Terms

Mass terms ~ $\overline{m}LR = (2,1)$

1) Simplest possibility: add 3 right handed neutrino fields

Field	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$L_Q = \begin{pmatrix} l_u \\ l_d \end{pmatrix}$	3	2	1/3
r_u	3	1	4/3
r_d	3	1	-2/3
$L_L = \begin{pmatrix} l_\nu \\ l_e \end{pmatrix}$	1	2	-1
r_ν ???	1	1	0
r_e	1	1	-2



$$\rightarrow \quad \begin{pmatrix} \bar{\nu}_L & \bar{\nu}_R^c \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix}$$

like quarks and charged leptons ➔ Dirac mass terms (including NMS mixing)

New ingredients:
1) Majorana mass (explicit)
2) lepton number violation

6x6 block mass matrix block diagonalization M_R heavy ➔ 3 light ν's

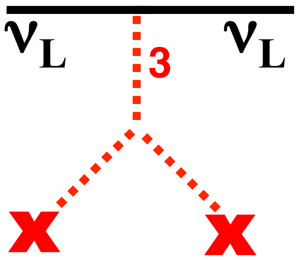
NEW ingredients, 9 parameters ➔ SM+

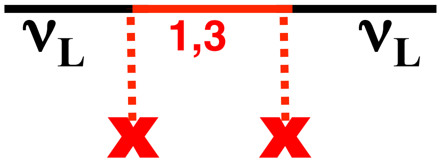
2) Maybe 3+N right handed neutrino fields

➔ (6+N) x (6+N) mass matrix

➔ how many of the 6+N eigenvalues are light (also for N=0)

3) new: scalar tripelts (3_L) or fermionic 1_L or 3_L





➔left-handed Majorana mass term:

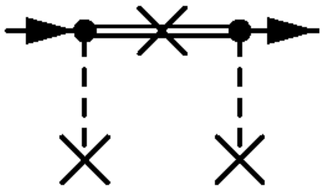
➔ $M_L \overline{L} L^c$

4) Both ν_R and new singlets / triplets:

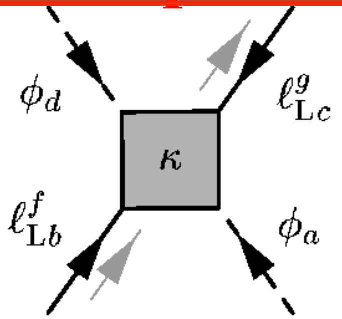
➔ see-saw type II, III

$$m_\nu = M_L - m_D M_R^{-1} m_D^T$$

5) Higher dimensional operators: d=5, …



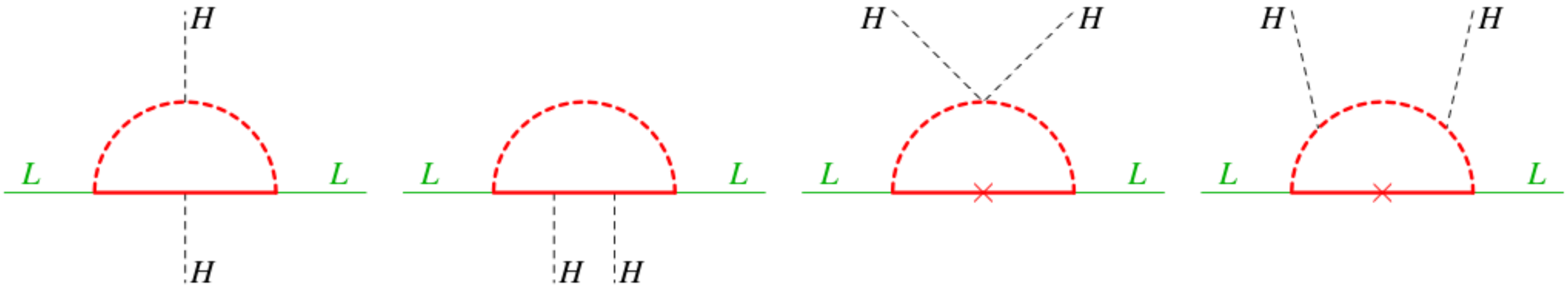
$\Rightarrow$



$\Leftrightarrow$ $\mathcal{L}_{mass} = \kappa \cdot \overline{\nu}_L^C \nu_L \Phi^T \Phi$

➔ $M_L \overline{L} L^c$

6) Radiative neutrino mass generation



7-N) SUSY, extra dimensions, …

Other effective Operators Beyond the SM

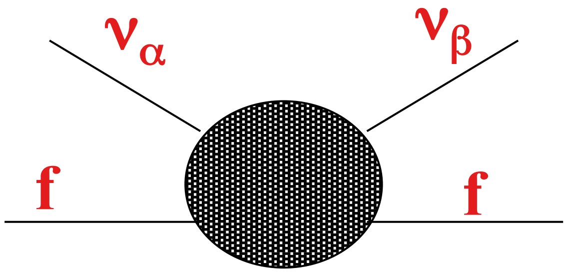
➔ effects beyond 3 flavours

➔ Non Standard Interactions = NSIs ➔ effective 4f opersators

$$\mathcal{L}_{NSI} \simeq \epsilon_{\alpha\beta} 2\sqrt{2} G_F (\bar{\nu}_{L\beta}\ \gamma^\rho\ \nu_{L\alpha})(\bar{f}_L \gamma_\rho f_L)$$

- integrating out heavy physics (c.f. G_F ⇔ M_W)

$$|\epsilon| \simeq \frac{M_W^2}{M_{NSI}^2}$$



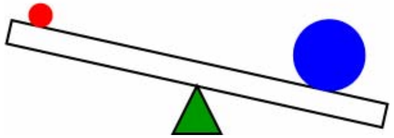
Grossman, Bergmann+Grossman, Ota+Sato, Honda et al., Friedland+Lunardini, Blennlow+Ohlsson+Skrotzki, Huber+Valle, Huber+Schwetz+Valle, Campanelli +Romanino, Bueno et al., Barranco+Miranda+Rashba, Kopp+ML+Ota, …

Suggestive Seesaw Features

QFT: natural value of mass operators ⇔ scale of symmetry

$m_D \sim$ electro-weak scale
$M_R \sim$ L violation scale ←?→ embedding (GUTs, …)

See-saw mechanism (type I)



$$m_\nu = m_D M_R^{-1} m_D^T$$

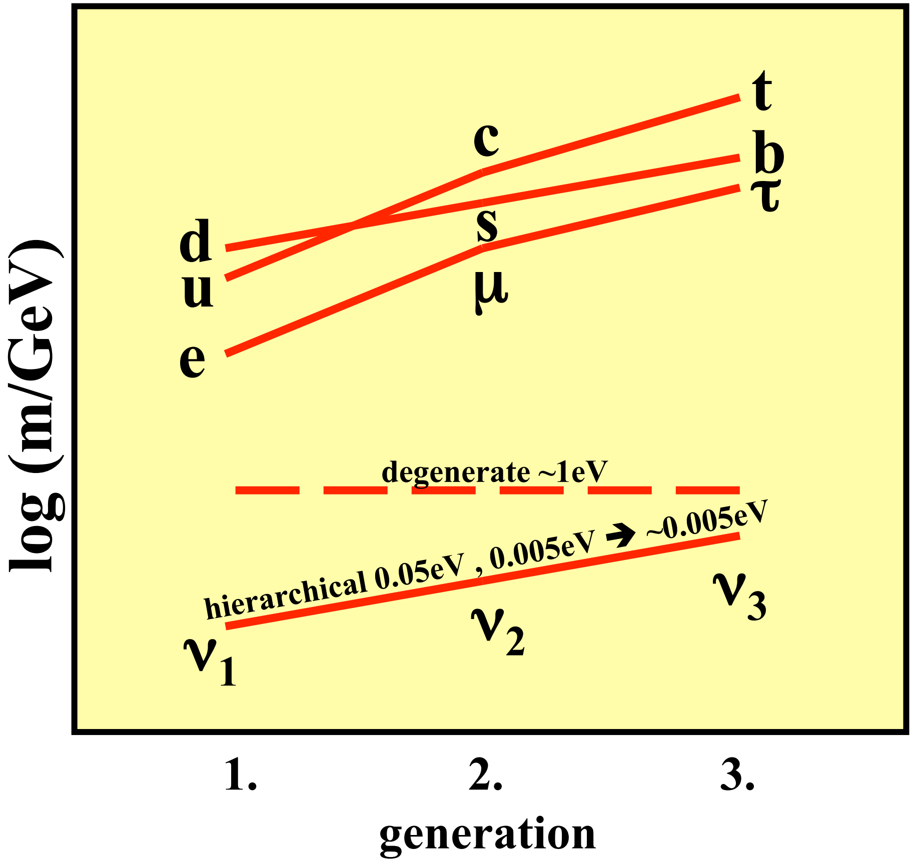
$$m_h = M_R$$

Numerical hints:

For $m_3 \sim (\Delta m^2_{atm})^{1/2}$, $m_D \sim$ leptons → $M_R \sim 10^{11} - 10^{16}$GeV
→ν's are Majorana particles , m_ν probes ~ GUT scale physics!
→smallness of m_ν ←→ high scale of $\not{L}$, symmetries of m_D ,M_R

2nd Look Questions

Quarks & charged leptons → hierarchical masses → neutrinos?



Quarks and charged leptons:

$m_D \sim H^n$; $n = 0,1,2$ → $H \geq 20...200$

Neutrinos: $m_\nu \sim H^n$ → $H \leq \sim 10$

See-saw:

$$m_\nu = -m_D^T \; M_R^{-1} \, m_D$$

↕ ↕ ↕ ↕

H $\simeq 10$ ≥ 20 ? ≥ 20

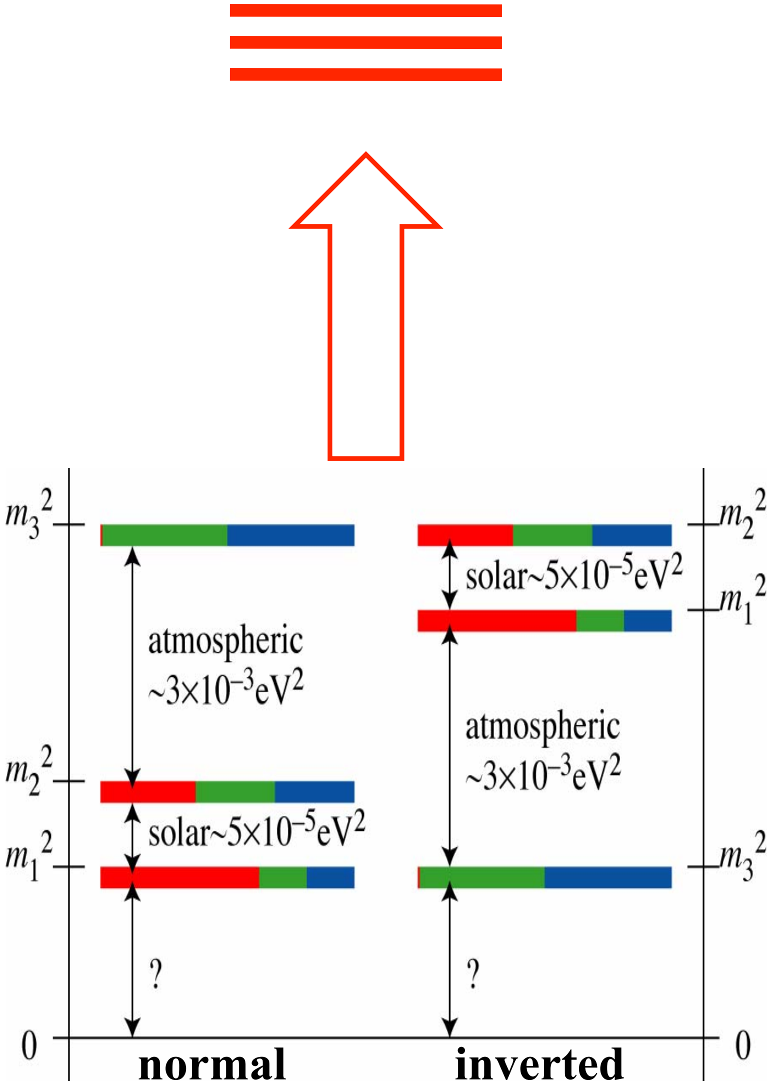
» less hierarchy in m_D or corr. hierarchy in M_R? → theoretically not connected!

» other version of see-saw? → type II, III, …?

» Dirac masses?

Standard Expectation

Heavy sterile neutrinos
←→natural scale in QFT = highest unless protected by some symmetry
←→ L-violation
→typical ≥ 10^{13} GeV
→tiny heavy – light mixings
role in leptogenesis, GUTs, …



hierarchical or degenerate

<u>light neutrinos & questions:</u> ν_e ν_μ ν_τ

- **→ Dirac / Majorana**
- **→ scale: m_1, ordering: sgn(Δm^2_{31})**
- **→ how small is θ_{13}, θ_{23} maximal?**
- **→ leptonic CP violation**
- **→ 3 flavour unitarity?**
- **→ why 3 generations, d=4, …**

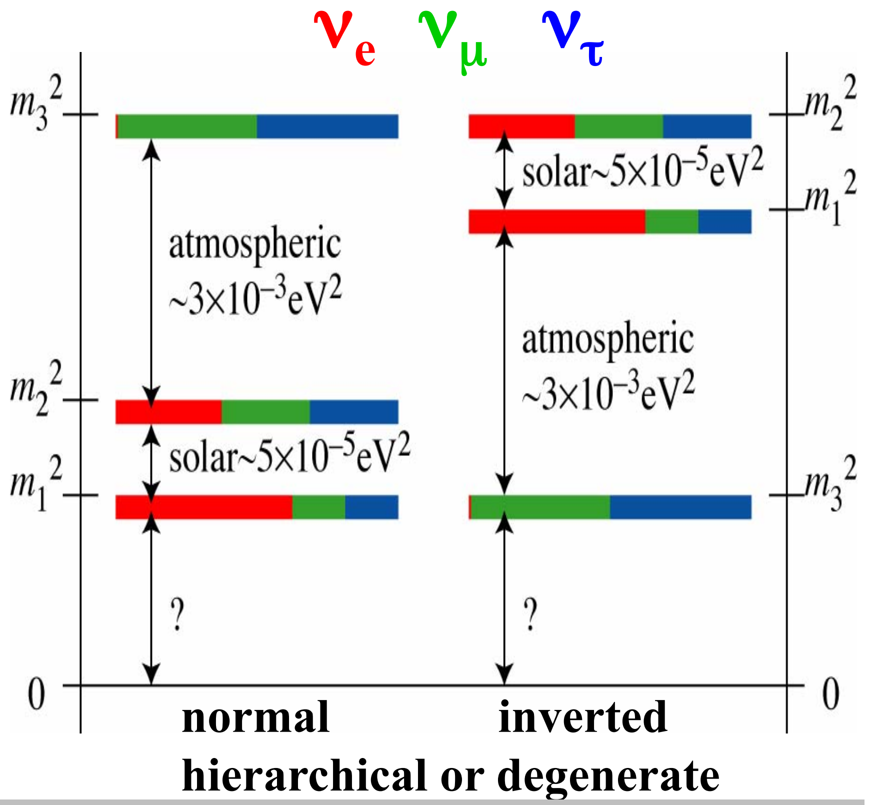
3 Light Neutrinos (...assumed)

Mass & mixing parameters: m_1 , Δm^2_{21}, $|\Delta m^2_{31}|$, sign(Δm^2_{31})

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \text{diag}(e^{i\alpha}, e^{i\beta}, 1)$$

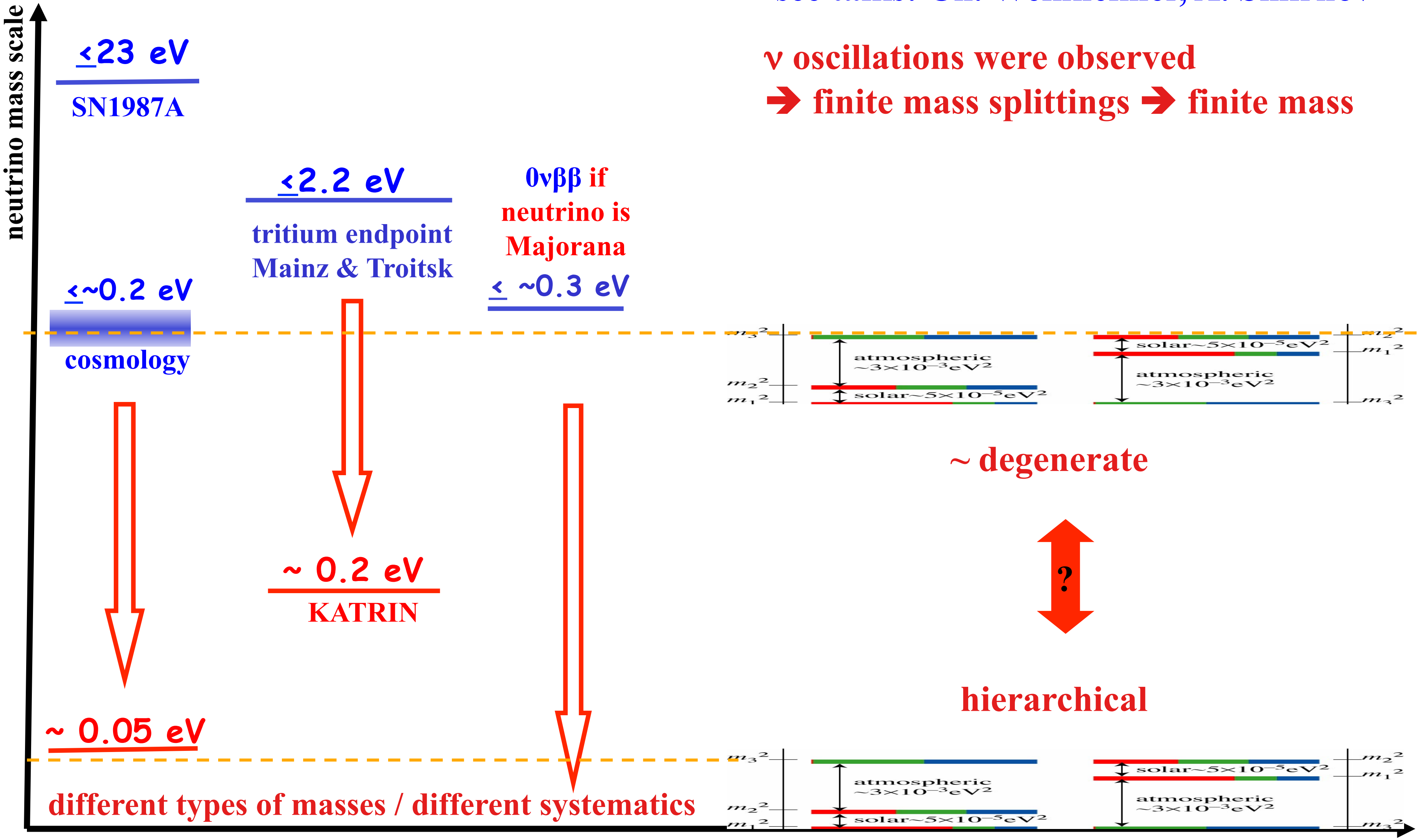
questions:

- ➔ Dirac / Majorana
- ➔ mass scale: m_1
- ➔ mass ordering: sgn(Δm^2_{31})
- ➔ how small is θ_{13}, θ_{23} maximal?
- ➔ leptonic CP violation
- ➔ 3 flavour unitarity?
- ⇔ indications for sterile neutrinos!?



Overview of Neutrino Mass Knowledge

see talks: Ch. Weinheimer, A. Smirnov



ν oscillations were observed
➔ finite mass splittings ➔ finite mass

neutrino mass scale

≤23 eV
SN1987A

≤2.2 eV
tritium endpoint Mainz & Troitsk

0νββ if neutrino is Majorana
≤ ~0.3 eV

≤~0.2 eV
cosmology

~ 0.2 eV
KATRIN

~ 0.05 eV

~ degenerate

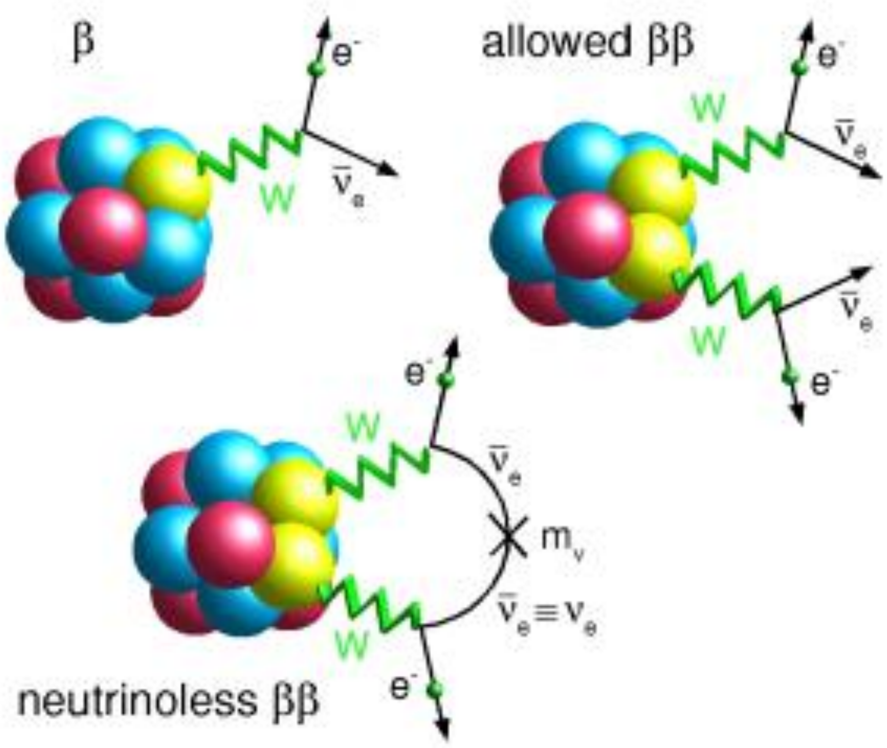
?

hierarchical

different types of masses / different systematics

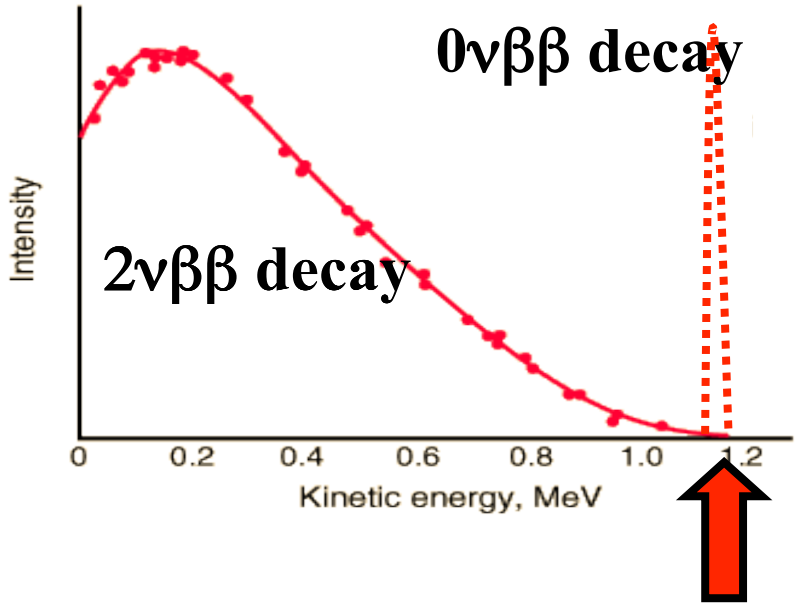
Could there be Surprises?

Neutrino-less Double Beta Decay



2νββ decay of ^{76}Ge observed:

$\tau = 1.5 \times 10^{21}$ y

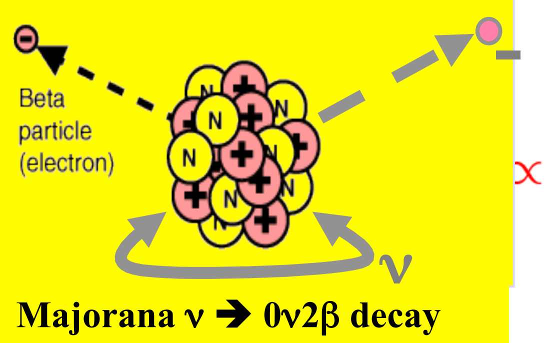


Majorana ν ➔ 0νββ decay

warning: other lepton number violating processes may exist…

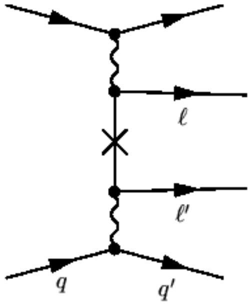
- **signal at known Q-value**
- **2νββ background (resulution)**
- **nuclear backgrounds**
  ➔ use different nuclei

Neutrino Masses from Double β-Decay



Majorana ν ➔ 0ν2β decay

$\propto$

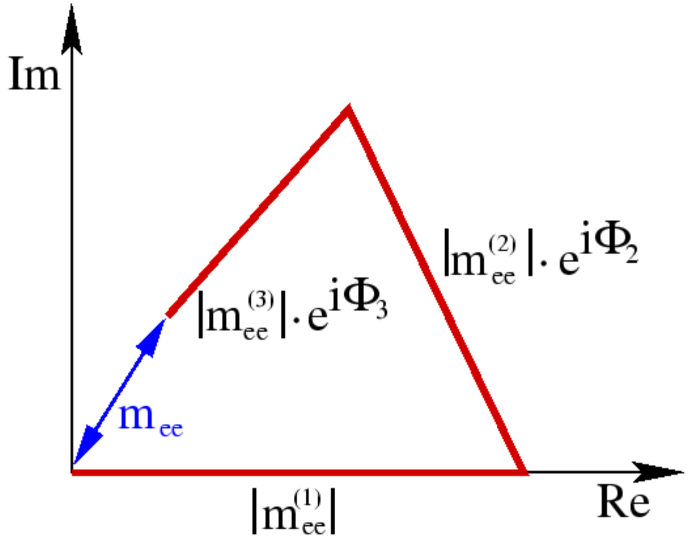


$\propto |\langle m_{ee}\rangle| = |\sum m_i U_{ei}^2|$ **$\leq$ 0.35 eV ?**

Heidelberg-Moscow experiment

$$m_{ee} = |m_{ee}^{(1)}| + |m_{ee}^{(2)}| \cdot e^{i\Phi_2} + |m_{ee}^{(3)}| \cdot e^{i\Phi_3}$$

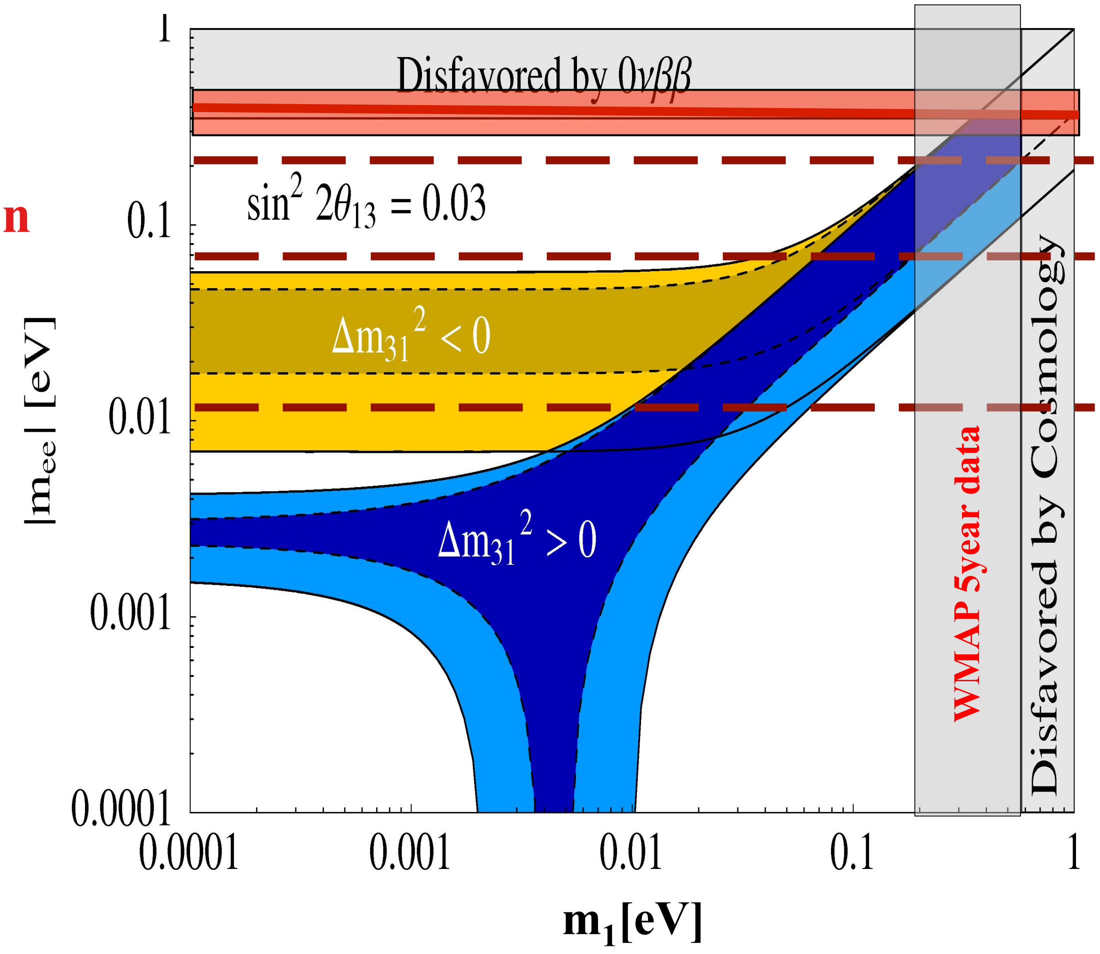
$$\begin{aligned} |m_{ee}^{(1)}| &= |U_{e1}|^2 m_1 \\ |m_{ee}^{(2)}| &= |U_{e2}|^2 \sqrt{m_1^2 + \Delta m_{21}^2} \\ |m_{ee}^{(3)}| &= |U_{e3}|^2 \sqrt{m_1^2 + \Delta m_{31}^2} \end{aligned}$$



solar $\Rightarrow$ $|U_{e1}|^2, |U_{e2}|^2, \Delta m_{21}^2$ atmosph. $\Rightarrow$ $|\Delta m_{31}^2|$ CHOOZ $\Rightarrow$ $|U_{e3}|^2 < 0.05$

➔ **free parameters: m_1 , sign(Δm_{31}^2) , CP-phases Φ_2, Φ_3**

Claim of part of the original Heidelberg-Moscow collaboration
⇔ cosmology ➔ ‚tension‘



aims of new experiments:

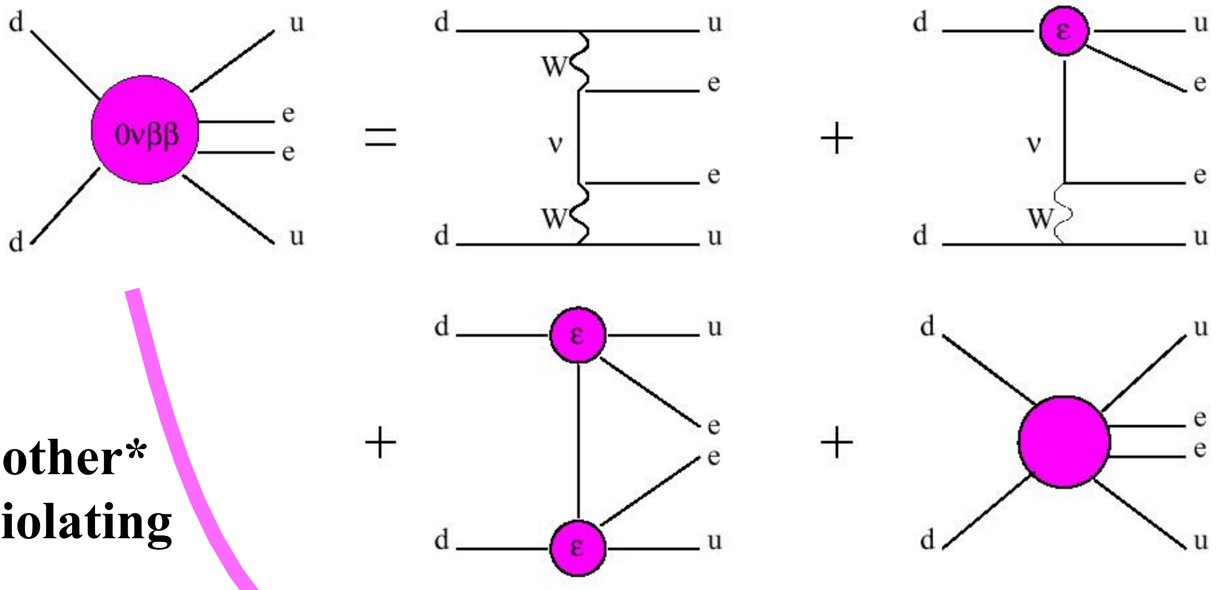
- **test HM claim**
- **$(\Delta m_{31}^2)^{1/2} \simeq 0.05\text{eV} \pm$ errors**
  - ➔ **reach 0.01eV**
  - ➔ **CUORE**
  - ➔ **GERDA phases I, II, (III)**

Comments:

- **cosmology: limitation by systematical errors ➔ ~another factor 5?**
- **$0\nu\beta\beta$ nuclear matrix elements ~factor 1.3-2 theoretical uncertainty in m_{ee}**
- **$\Delta m^2 > 0$ allows complete cancellation**
  - ➔ **$0\nu\beta\beta$ signal not guaranteed, but cancelation appears unlikely**

0νββ from Alternative ΔL=2 Operators

Various possibilities:
- LR symmetry
- SUSY (RPV)
- …



➔0νββ signal from *some other* new BSM lepton number violating operator

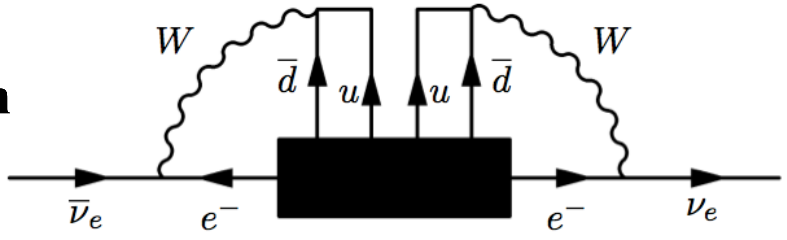
➔ very promising interplay of neutrino mass determinations, cosmology, LHC, LVF experiments and theory

Schechter + Valle: **Any ΔL=2 violating operator**

➔ radiative generation of Majorana mass term

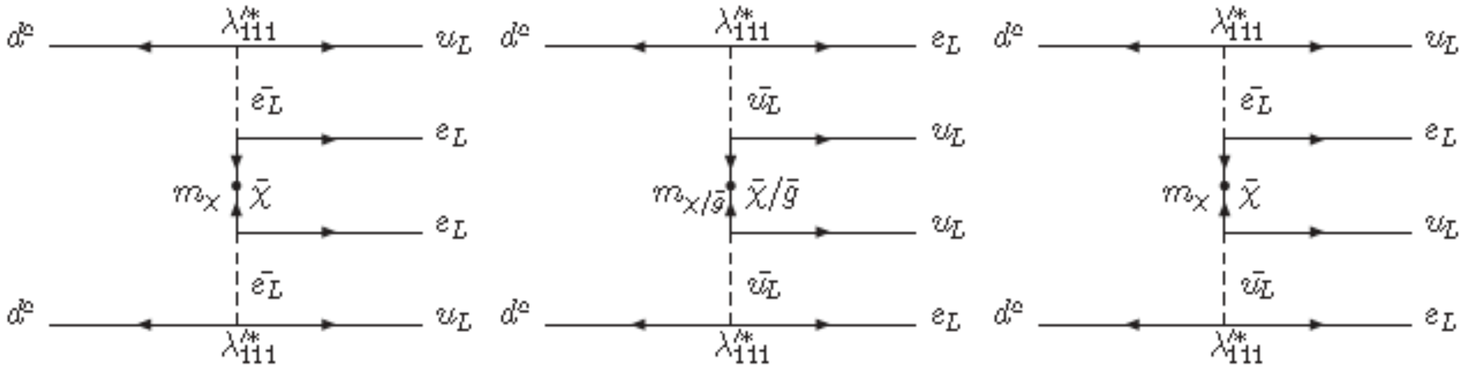
➔ Majorana nature of ν's guaranteed

➔ but how big is the mass?



A SUSY Example

Direct, TeV scale short range mediation w/o intermediate light ν, e.g.



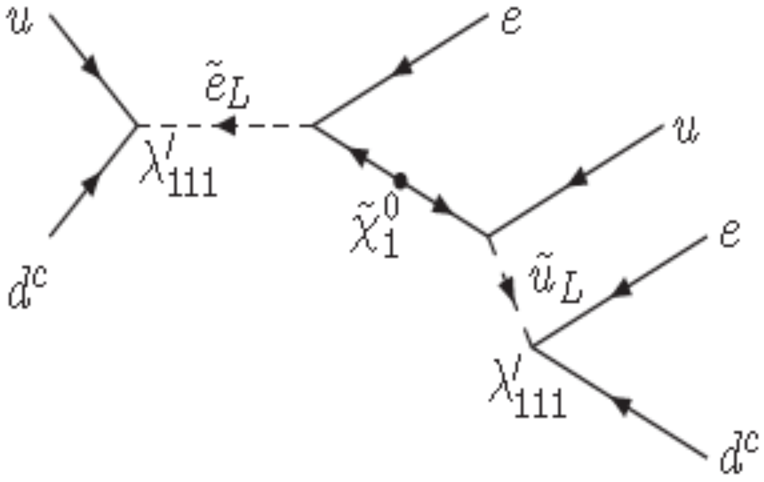
$$\mathcal{L}^{eff,\,\Delta L_e=2}_{\lambda'_{111}\lambda'_{111}}(x) = \frac{G_F^2}{2} m_p^{-1}[\bar{e}(1+\gamma_5)e^c]$$

$$\times\left[(\epsilon_{\tilde{g}}+\epsilon_\chi)\left(J_{PS}J_{PS}-\frac{1}{4}J_T^{\mu\nu}J_{T\mu\nu}\right)+(\epsilon_{\chi\tilde{e}}+\epsilon'_{\tilde{g}}+\epsilon_{\chi\tilde{f}})J_{PS}J_{PS}\right]$$

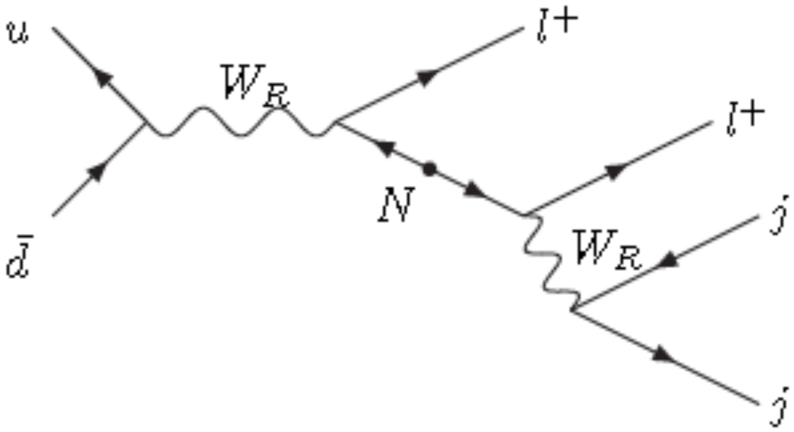
$$\epsilon_i \sim \pi\alpha_{(\mathrm{Strong,EW})}\frac{\lambda'^2_{111}}{G_F^2}\frac{m_P}{m_{(\tilde{g},\tilde{\chi})}}\frac{1}{m^4_{(\tilde{u},\tilde{d},\tilde{e})}}.$$

ΔL=2 Operators and TeV Scale Physics

SUSY: direct test of λ'_{111}

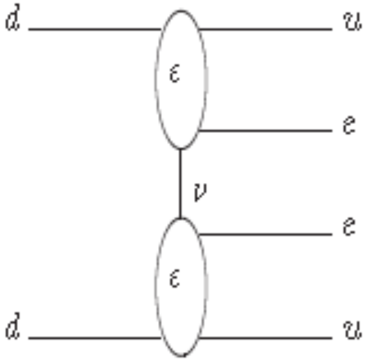


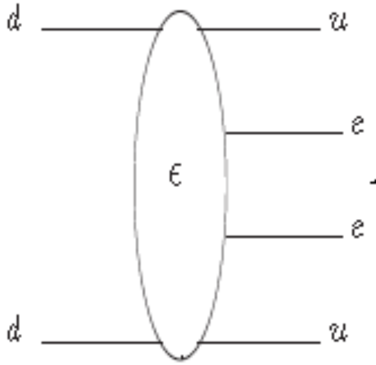
L-R symmetry: heavy N's



Relative strength of 'light' and 'heavy' $0\nu\beta\beta$ amplitudes:

$$M_{\mathrm{light}} \sim G_F^2 \frac{m_{\beta\beta}}{\langle k^2 \rangle}$$





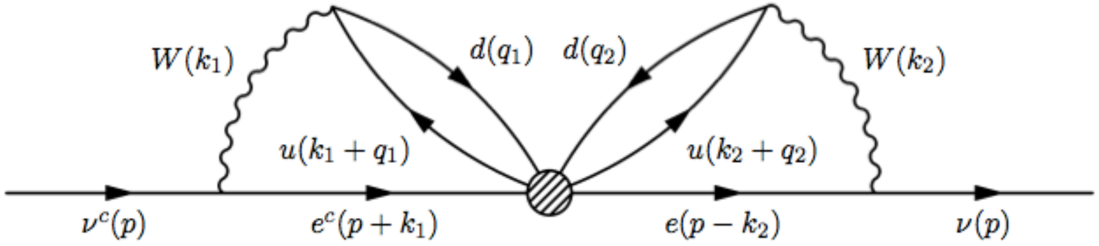
$$M_{\mathrm{heavy}} \sim G_F^2 \left(\frac{\lambda}{g_2}\right)^4 \frac{M_W^4}{\Lambda^5}$$

SV-induced Neutrino Masses

General Lorentz-invariant Lagrangian for 0νββ (point operator)

$$\mathcal{L} = \frac{G_F^2}{2} m_p^{-1} \left(\epsilon_1 J J j + \epsilon_2 J^{\mu\nu} J_{\mu\nu} j + \epsilon_3 J^\mu J_\mu j + \epsilon_4 J^\mu J_{\mu\nu} j^\nu + \epsilon_5 J^\mu J j_\mu \right)$$

$$J = \overline{u}\,(1 \pm \gamma_5)\, d, \;\; J^\mu = \overline{u} \gamma^\mu (1 \pm \gamma_5)\, d \text{ etc.}$$



Outcome: **M. Dürr, ML, A. Merle**

If other ΔL=2 physics drives 0νββ ➔ SV gives $\delta m_\nu = 10^{-24}$ eV

➔ mass correction too small to explain observed masses and splittings

➔ explicit neutrino mass operators required

Dirac: 0νββ essentially unrelated to neutrino masses ←→ other BSM

Majorana: dominates over SV contribution

0νββ may be a mixture of Majorana mass and other ΔL=2 physics

➔ mimics higher Majorana neutrino mass

Neutrinos as Dark Matter?

Could Neutrinos be Dark Matter?

- **Active neutrinos would be perfect Hot Dark Matter** ➔ ruled out:
  - **destroys small scale structures in cosmological evolution**
  - **required neutrino masses much too small → maybe HDM component**

- **keV sterile neutrinos: Warm Dark Matter** ➔ workes very well:
  - ➔ relativistic at decoupling
  - ➔ non-relativistic at radiation to matter dominance transition
  - **OK for $M_X \simeq$ few keV with very tiny mixing**
  - **reduced small scale structure ➔ smoother profile, less dwarf satellites**
  - **➔ scenario where one sterile neutrino is keV-ish, the others heavy**
  - **➔ tiny active – sterile mixings $O(m_\nu/M_R)$**

  ➔ observational hints from astronomy

  - **hints that a keV sterile particle may exist ➔ right-handed neutrino?**

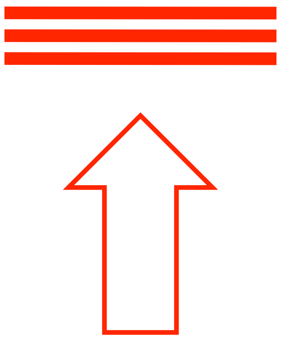
  Biermann, Kusenko & Segre, Fuller et al., Biermann & Kusenko, Stasielak et al., Loewenstein et al., Dodelson, Widrow, Dolgov, …

Non-standard Sterile Neutrino Scenarios

The standard picture:

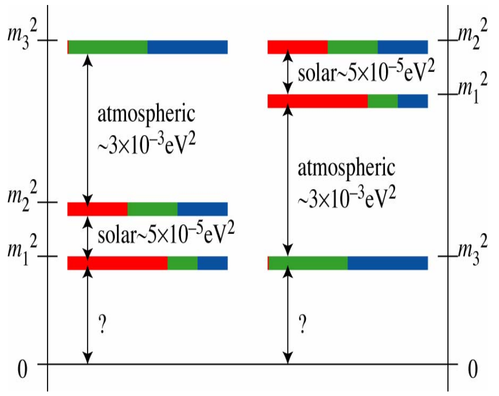
3 heavy sterile neutrinos typ. $\geq 10^{13}$ GeV

➔ leptogenesis, role in GUTs, …



Theory mechanism which makes
1 or 2 heavy sterile neutrinos light?
➔ keV sterile neutrino, tiny H-L mixing

3 light active neutrinos



➔ this cold easily be wrong

- more than 3 N_R states, …

- M_R may have special eigenvalues, …

➔ light sterile neutrinos ?!

Evidences for Light Sterile Neutrinos

Particle Physics:

Reactor anomaly, LSND, MiniBooNE, MINOS, Gallex…

➔ evidences for light sterile ν's?

➔ see talk by A. Smirnov

➔ New and better data / experiments are needed to clarify the situation

➔ maybe something exciting around the corner?

➔ but eV scale and sizable mixings

Astrophysics:

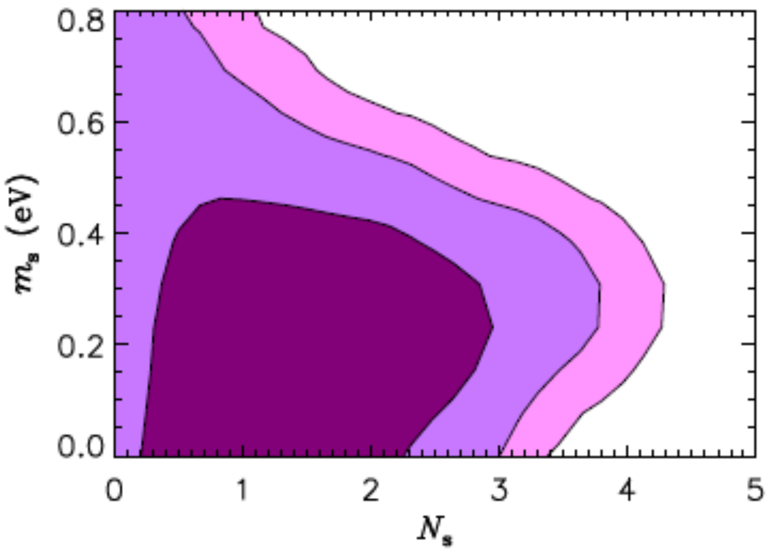
- e.g. effects of keV-ish sterile neutrinos on pulsar kicks

Kusenko, Segre, Fuller, Mocioiu, Pascoli

- …

➔ see talk by P. Biermann

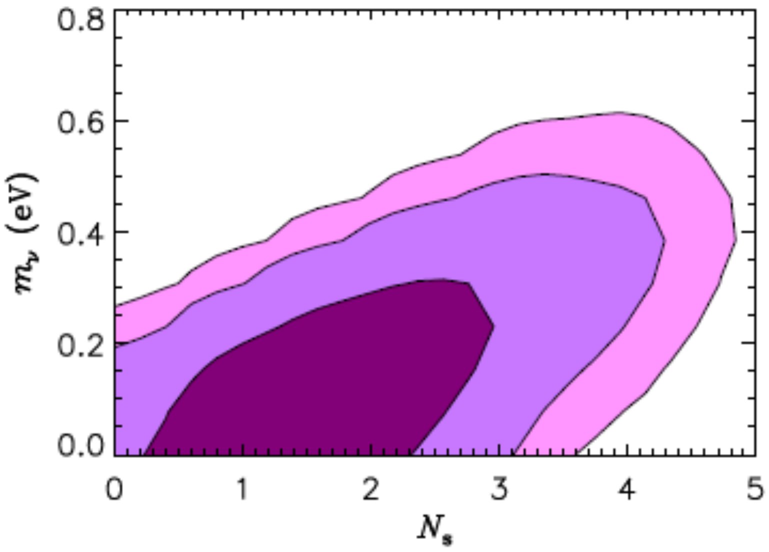
Extra Sterile Neutrinos & CMB



3 active massless neutrinos

+

N_s massive neutrinos



3 active massive neutrinos

+

N_s massless neutrinos

J. Hamann et al

➔ eV scale masses

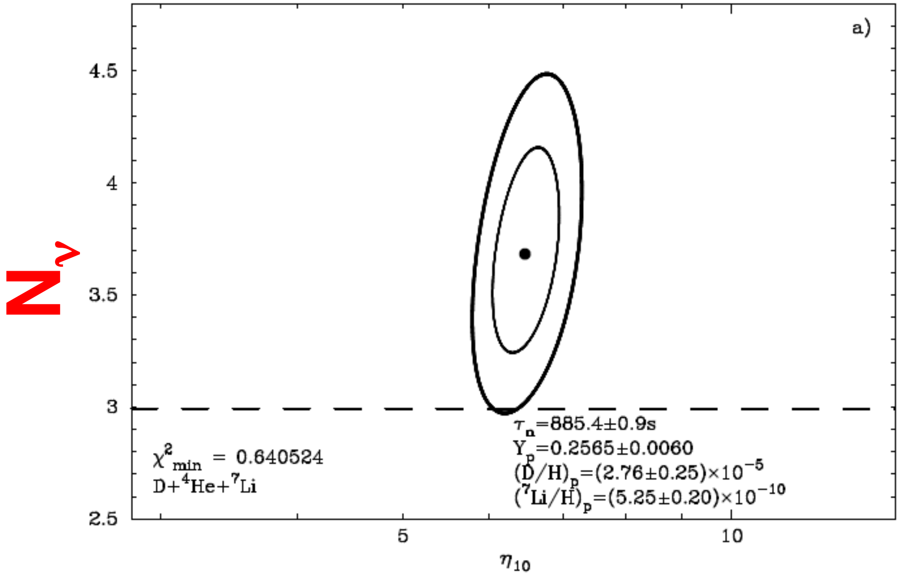
Cosmological Indications for Sterile Neutrinos

Cosmology and structure formation:

WDM works very well

➔ see talks at this conference

BBN – ‘feels’ extra neutrino-like particles:



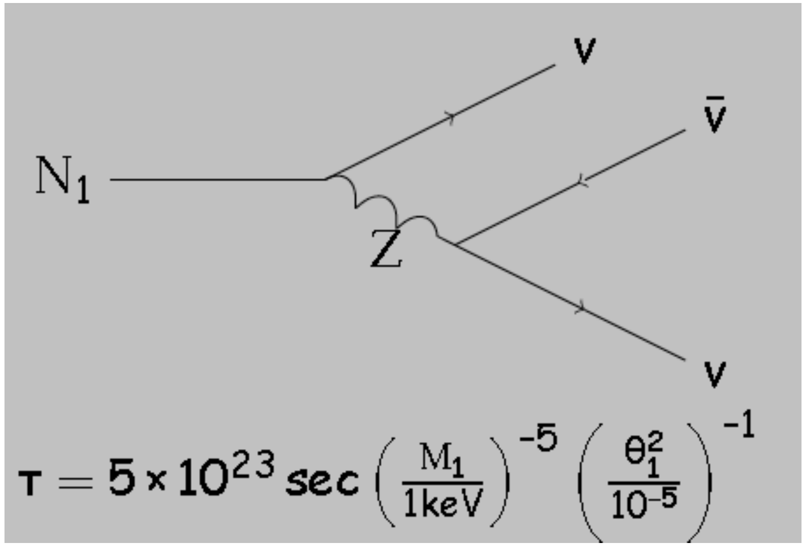
$N_\nu \simeq 3.7 \pm 1$

E. Aver, K. Olive, E. Skillman (2010)
Y. Izotov, T. Thuan(2010)

Observing keV-ish Neutrino DM

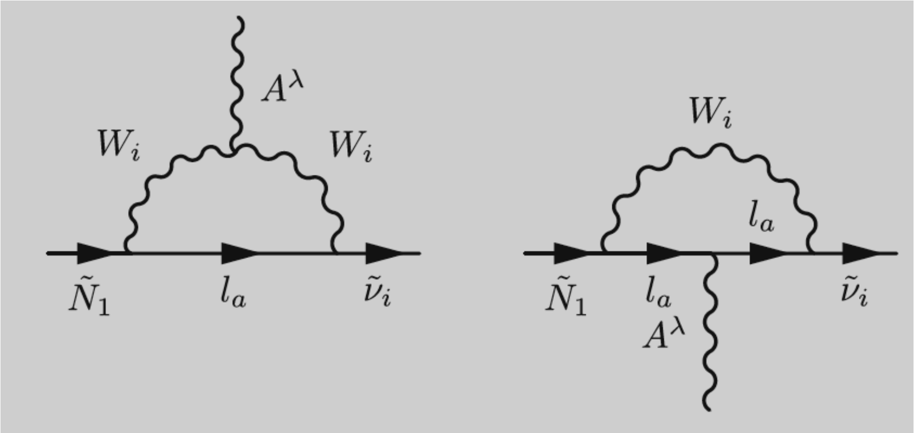
- LHC
  - sterile neutrino DM is not observable
  - WIMP-like particles still possible – but not DM
- direct searches ➔ see talk by Ch. Weinheimer
  - sterile neutrino DM very difficult; maybe in β-decay (MARE)
- astrophysics/cosmology ➔ at some level: keV X-rays

  ➔ sterile neutrino DM is decaying into active neutrinos
  - decay $N_1 \rightarrow \bar{\nu}\nu\nu$, $N_1 \rightarrow \bar{\nu}\bar{\nu}\nu$
  - not very constraining since $\tau \gg \tau_{\text{Universe}}$



$$\tau = 5 \times 10^{23}\,\text{sec} \left(\frac{M_1}{1\text{keV}}\right)^{-5} \left(\frac{\theta_1^2}{10^{-5}}\right)^{-1}$$

- - radiative decays $N_1 \rightarrow \nu\gamma$



- so far: observational limit on active-sterile mixing angle

$$\Gamma_{N_1\rightarrow\nu\gamma} \simeq 5.5\times 10^{-22}\theta_1^2\left(\frac{M_1}{1\ \mathrm{keV}}\right)^5 \mathrm{s}^{-1}$$

$$\theta_1^2 \lesssim 1.8\times 10^{-5}\left(\frac{1\ \mathrm{keV}}{M_1}\right)^5$$

- mixing tiny, but naturally expected to be tiny: O(scale ratio)

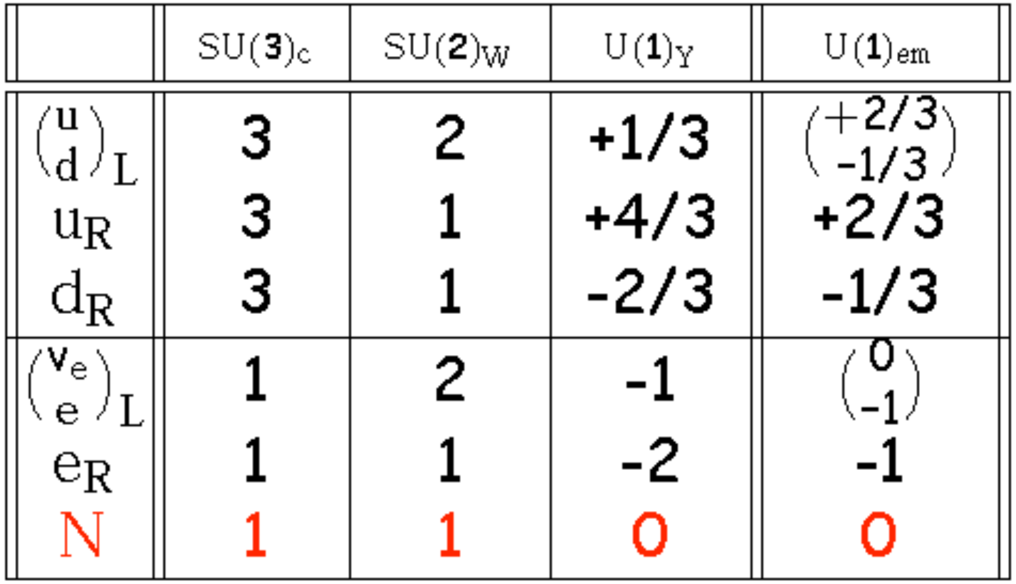
keV Neutrinos as WDM

The νMSM

Asaka, Blanchet, Shaposhnikov, 2005 Asaka, Shaposhnikov, 2005

Particle content:

- Gauge fields of $SU(3)_c \times SU(2)_W \times U(1)_Y$: **γ, $W_\pm$, Z, g**
- Higgs doublet: **$\Phi=(1,2,1)$**
- Matter



	$SU(3)_c$	$SU(2)_W$	$U(1)_Y$	$U(1)_{em}$
$\begin{pmatrix} u \\ d \end{pmatrix}_L$	3	2	+1/3	$\begin{pmatrix} +2/3 \\ -1/3 \end{pmatrix}$
u_R	3	1	+4/3	+2/3
d_R	3	1	-2/3	-1/3
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	1	2	-1	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
e_R	1	1	-2	-1
N	1	1	0	0

x3 generations

➔ lepton sector more symmetric to the quark sector

➔ Majorana masses for N

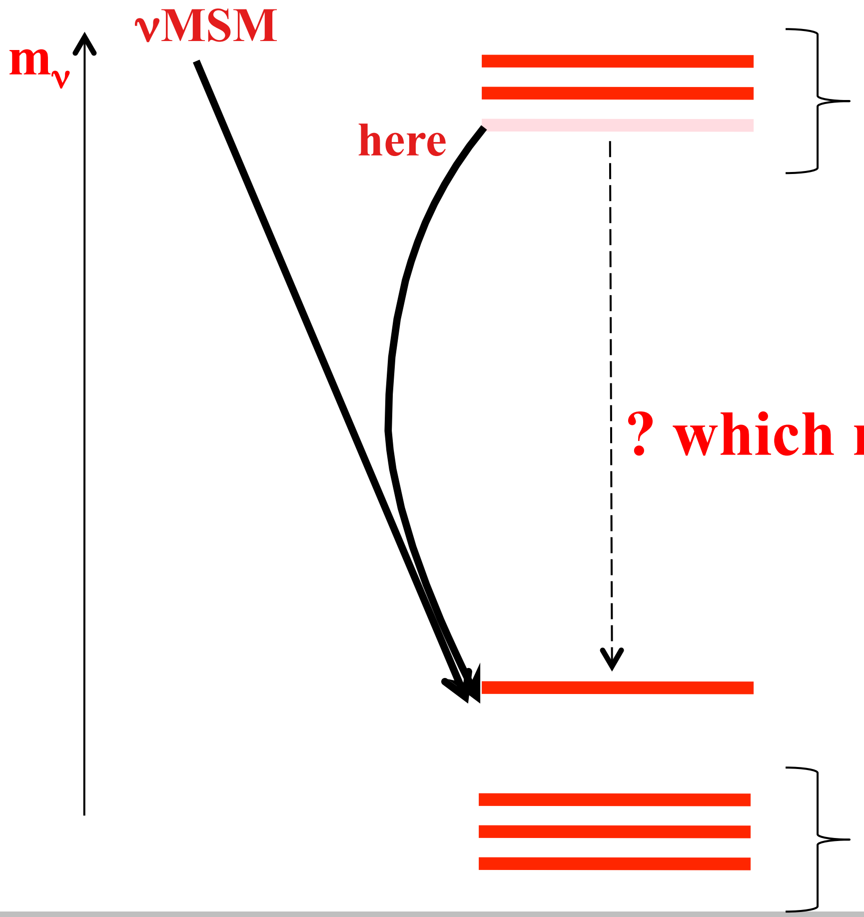
➔ choose for one sterile ν ~keV mass ➔ exceeds lifetime of Universe

Virtue and Problem of the νMSM

νMSM: Scenario with sterile ν and tiny mixing ➔ never enters thermal equilibrium

➔ requires non-thermal production from other particles (avoid over-closure)

➔ new physics before the beginning of the thermal evolution sets abundance



heavy sterile neutrinos typ. $\geq 10^{13}$ GeV

? which mechanism ?

one light sterile neutrino $\simeq$ keV = DM

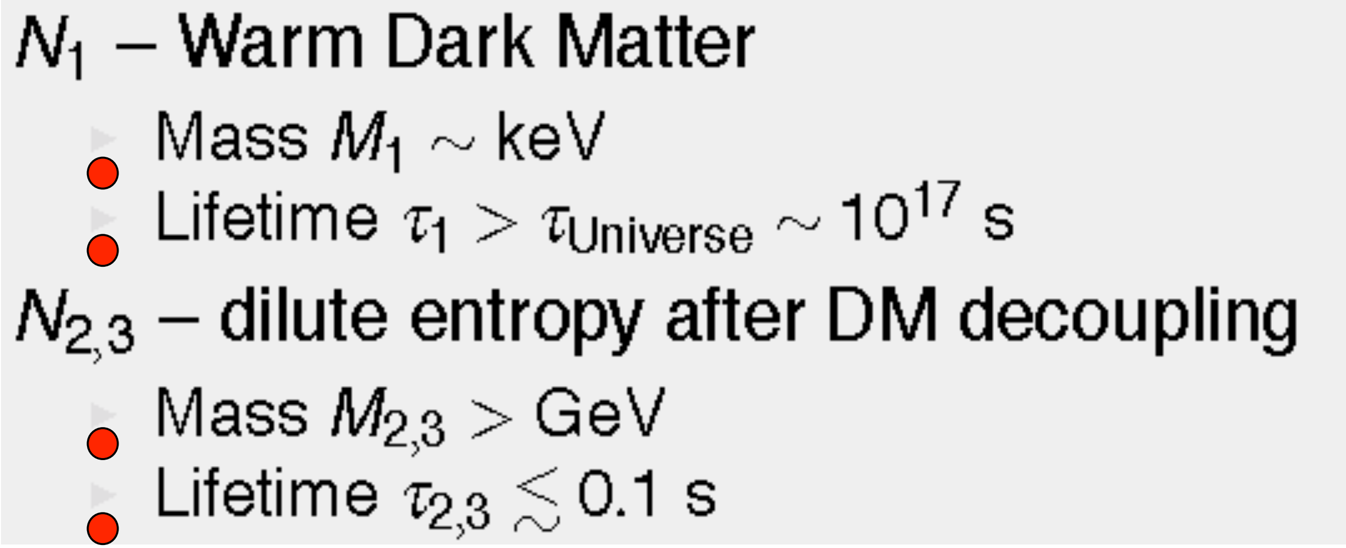
light active neutrinos < eV

Alternative Scenario with Thermal Abundance

An alternative scenario: **Bezrukov, Hettmannsperger, ML**

- **Three right-handed neutrinos N_1 , N_2 , N_3**
- **Dirac and Majorana mass terms**
- **N Charged under some (BSM) gauge group ➔ scale M (~sterile)**
- **Specific example: LR-symmetry** $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

Roles played by the sterile (~right-handed) neutrinos:



N_1 – Warm Dark Matter

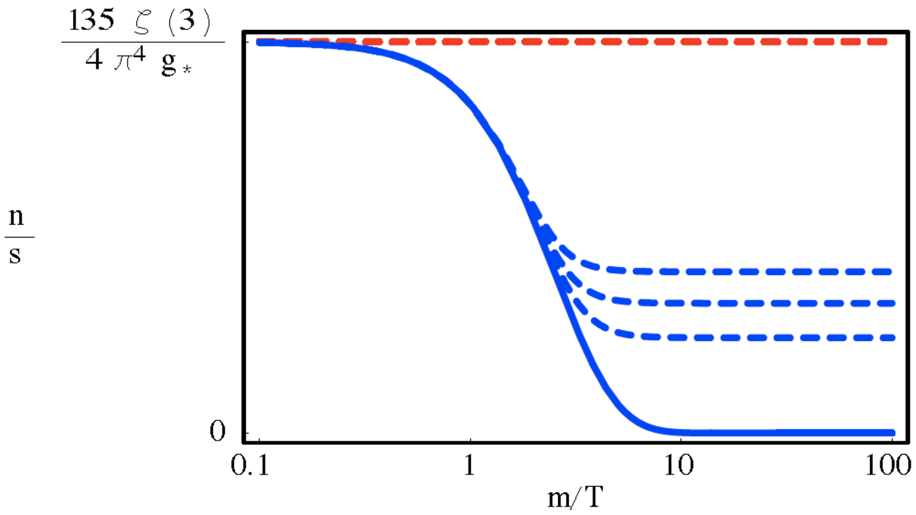
- Mass $M_1 \sim$ keV
- Lifetime $\tau_1 > \tau_{\text{Universe}} \sim 10^{17}$ s

$N_{2,3}$ – dilute entropy after DM decoupling

- Mass $M_{2,3} >$ GeV
- Lifetime $\tau_{2,3} \lesssim 0.1$ s

Obtaining the right Abundance

Usual thermal case:



HDM:

$$\frac{\Omega}{\Omega_{DM}} \simeq \left(\frac{10}{g_{*f}}\right)\left(\frac{M}{10\,\mathrm{eV}}\right)$$

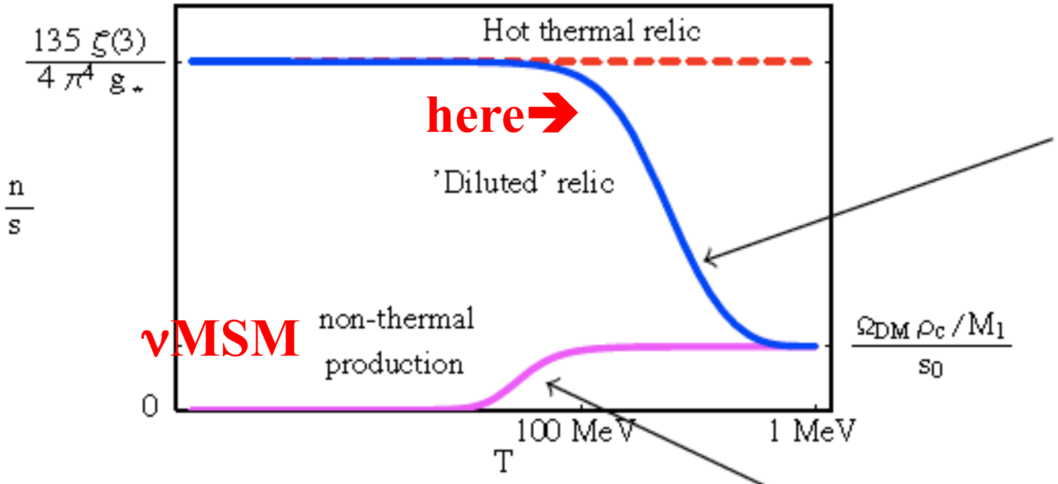
Decoupled relativistic

CDM: (M>>MeV)

$$\Omega \sim \Omega_{DM}$$

Decoupled nonrelativistic

keV sterile neutrinos:



Diluted after decoupling (entropy generated by other particle decay)

$$\Omega \sim \Omega_{DM}$$

Never entered thermal equilibrium

Sterile Neutrino DM Freeze-Out & Abundance

Decoupling of N_1 in early Universe: sterile neutrino DM is light
→ freezout while relativistic → calculation like for active neutrinos
+ suppression of annihilation x-section by M

Freeze-out temperature:

$$T_f \sim g_{*f}^{1/6} \left(\frac{M}{M_W} \right)^{4/3} (1 \div 2)\ \mathrm{MeV}$$

Abundance of N_1 today:

$$\frac{\Omega_N}{\Omega_{DM}} \simeq \frac{1}{S} \left(\frac{10.75}{g_{*f}} \right) \left(\frac{M_1}{1\,\mathrm{keV}} \right) \times 100$$

Required entropy generation factor:

$$S \simeq 100 \left(\frac{10.75}{g_{*f}} \right) \left(\frac{M_1}{1\,\mathrm{keV}} \right)$$

Entropy Generation by out-of Equilibrium Decay

Heavy particle (here: N_3) dropping out of thermal equilibrium while relativistic $T_f > M_2$: → bounds gauge scale from below

$$M > \frac{1}{g_{*f}^{1/8}} \left(\frac{M_2}{\mathrm{GeV}} \right)^{3/4} (10 \div 16)\ \mathrm{TeV}$$

→ sufficiently long lived → become non-relativistic

→ dominates expansion of Universe during its decay

→ entropy generation factor →

$$\frac{S_{\mathrm{after}}}{S_{\mathrm{before}}} = S \frac{a^3_{\mathrm{before}}}{a^3_{\mathrm{after}}}$$

$$S \simeq 0.76 \frac{\bar{g}_*^{1/4} M_2}{g_* \sqrt{\Gamma_2 M_{\mathrm{Pl}}}}$$

→ fixes decay width Γ_2

Summary of Constraints

X/γ-ray

$$\theta_1^2 \lesssim | 1.8\times 10^{-5}\left(\frac{1\,\mathrm{keV}}{M_1}\right)^5$$

$$\zeta^2 \lesssim 10^{-18}\ldots(\mathrm{keV}/M_1)^3$$

Ly-α bound

$$M_1 > 1.6\mathrm{keV}$$

BBN $\tau_2 > 0.1 \div 2\,\mathrm{sec}$

$$M_2 > \left(\frac{M_1}{1\mathrm{keV}}\right)(1.7\div 10)\ \mathrm{GeV}$$

The right abundance of the sterile neutrino N_1 is achieved if

$$\Gamma_2 \simeq 0.50\times 10^{-6}\,\bar{g}_*^{1/2}\frac{M_2^2}{M_{\mathrm{Pl}}}\left(\frac{1\mathrm{keV}}{M_1}\right)^2$$

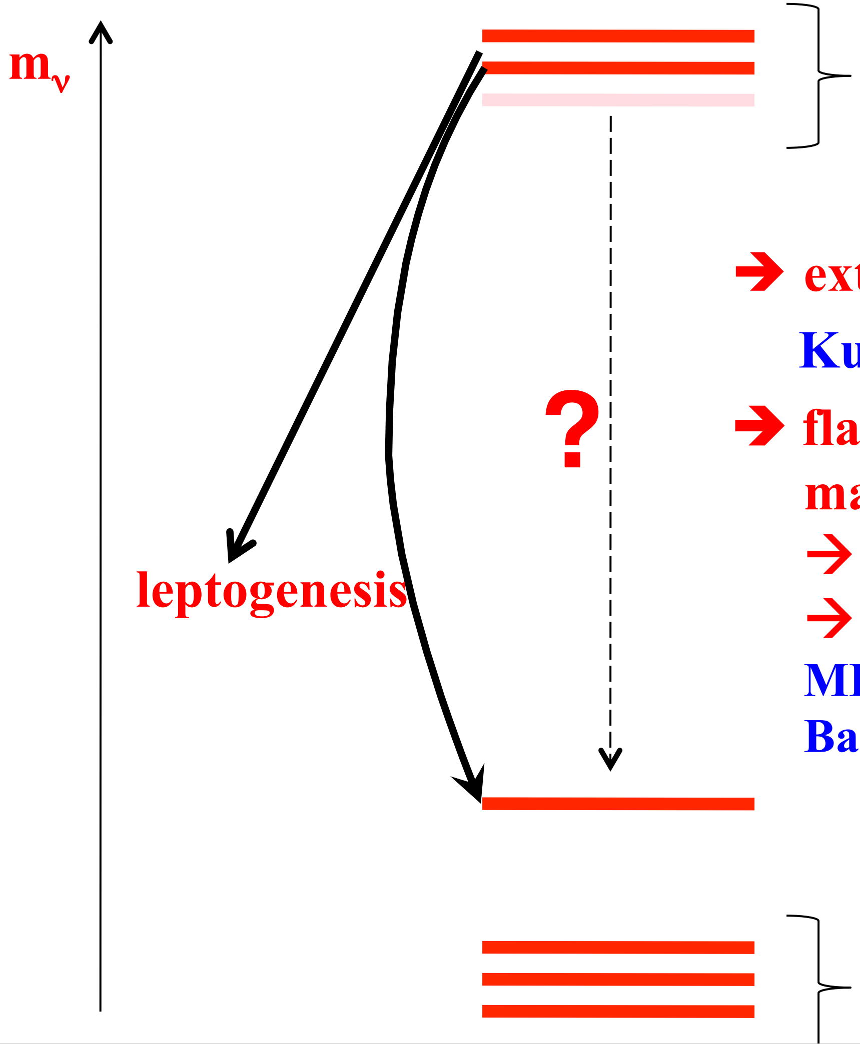
The entropy is effectively generated if the right-handed gauge scale is

$$M > g_{*\mathrm{f}}^{-1/8}\left(\frac{M_2}{1\ \mathrm{GeV}}\right)^{3/4}(10\div 16)\ \mathrm{TeV}$$

Justifying keV sterile Neutrinos

Generating keV-ish Sterile Neutrinos

Possible scenario: See-saw + a reason why 1 sterile ν is light



heavy sterile neutrinos typ. $\geq 10^{13}$ GeV

➔ extra dimensional physics ➔ 'split see-saw'

Kusenko, Takahashi, Yanagida,

➔ flavour symmetries explaining active neutrino masses + charged leptons + quarks

→ consequences for heavy mass matrix

→ L_e-L_μ-L_τ DM←→ flavour problem

ML, Merle, Niro ; Merle, Niro,
Barry, Rodejohann, Zhang

one light sterile neutrino $\simeq$ keV = DM

light active neutrinos < eV

Light Sterile Neutrinos from L_e-L_μ-L_τ

- Flavour symmetries have been studied to explain apparent regularities of masses and mixing: A4, S3, D5, …
  ➔ implications for sterile sector?
  ➔ could the same symmetries explain a keV-ish sterile ν ?

Model by Lavoura & Grimus ➔ **modified ML, Merle, Niro**

SM + ν_{iR} + softly broken U(1) ←→ $\mathcal{F} \equiv L_e - L_\mu - L_\tau$

type II see-saw ➔ +Higgs triplet $\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}$

	L_{eL}	$L_{\mu L}$	$L_{\tau L}$	e_R	μ_R	τ_R	N_{1R}	N_{2R}	N_{3R}	ϕ	Δ
$\mathcal{F}$	1	-1	-1	1	-1	-1	1	-1	-1	0	0

Neutrino Mass Terms

- Mass matric for right-handed neutrinos:

$$\mathcal{L}_{\rm mass} = -M_R^{12}\,\overline{(N_{1R})^C}\,N_{2R} - M_R^{13}\,\overline{(N_{1R})^C}\,N_{3R} + h.c.$$

- Dirac masses

$$\begin{aligned}\mathcal{L}_{\rm mass} \quad = \quad & -Y_D^{e1}\,\overline{L_{eL}}\,\tilde{\phi}\,N_{1R} - Y_D^{\mu 2}\,\overline{L_{\mu L}}\,\tilde{\phi}\,N_{2R} - Y_D^{\mu 3}\,\overline{L_{\mu L}}\,\tilde{\phi}\,N_{3R} - \\ & -Y_D^{\tau 2}\,\overline{L_{\tau L}}\,\tilde{\phi}\,N_{2R} - Y_D^{\tau 3}\,\overline{L_{\tau L}}\,\tilde{\phi}\,N_{3R} + h.c.,\end{aligned}$$

- In addition: Triplet masses

$$\mathcal{L}_{\rm mass} = -Y_L^{e\mu}\,\overline{(L_{eL})^C}\,(i\sigma_2\Delta)\,L_{\mu L} - Y_L^{e\tau}\,\overline{(L_{eL})^C}\,(i\sigma_2\Delta)\,L_{\tau L} + h.c.$$

- **Mass matrix in the basis**

$$\Psi \equiv \left((\nu_{eL})^C, (\nu_{\mu L})^C, (\nu_{\tau L})^C, N_{1R}, N_{2R}, N_{3R}\right)^T$$

➔ $$\mathcal{M}_\nu = \left(\begin{array}{ccc|ccc} 0 & m_L^{e\mu} & m_L^{e\tau} & m_D^{e1} & 0 & 0 \\ m_L^{e\mu} & 0 & 0 & 0 & m_D^{\mu 2} & m_D^{\mu 3} \\ m_L^{e\tau} & 0 & 0 & 0 & m_D^{\tau 2} & m_D^{\tau 3} \\ \hline m_D^{e1} & 0 & 0 & 0 & M_R^{12} & M_R^{13} \\ 0 & m_D^{\mu 2} & m_D^{\tau 2} & M_R^{12} & 0 & 0 \\ 0 & m_D^{\mu 3} & m_D^{\tau 3} & M_R^{13} & 0 & 0 \end{array}\right)$$

➔ **three scenarios**

- $m_D^{\alpha i} \ll m_L^{\alpha\beta} \ll M_R^{ij}$ (separation scenario),
- $m_L^{\alpha\beta} \ll m_D^{\alpha i} \ll M_R^{ij}$ (type II see-saw scenario),
- $m_L^{\alpha\beta} \sim m_D^{\alpha i} \ll M_R^{ij}$ (hybrid scenario).

$\det(M_{ij}) = 0$
➔ $M_1 = 0$
➔ massless sterile state + soft breaking
➔ light sterile ν

Implications for See-Saw

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2}(\overline{\tilde{\nu}^c_{aL}}, \overline{\tilde{N}_{aR}}) \begin{pmatrix} M_L & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \tilde{\nu}_{aL} \\ \tilde{N}^c_{aR} \end{pmatrix} + \text{H.c.}$$

- **Usual flavour (=tilde) to mass basis rotation**

$$\begin{pmatrix} \tilde{\nu}_{aL} \\ \tilde{N}^c_{aR} \end{pmatrix} \simeq \begin{pmatrix} 1 & (M_R^{-1} m_D^T)^\dagger \\ -M_R^{-1} m_D^T & 1 \end{pmatrix} \begin{pmatrix} U & 0 \\ 0 & V_R \end{pmatrix} \begin{pmatrix} \nu_{iL} \\ N^c_{IR} \end{pmatrix}$$

- **U = PMNS matrix, V_R = mixing in right-handed sector**

$$M_L - m_D M_R^{-1} m_D^T = U^\star \cdot \text{diag}(m_1, m_2, m_3) \cdot U^\dagger$$

➔**M_L = 0: Type-I**

$$M_R = V_R^\star \cdot \text{diag}(M_1, M_2, M_3) \cdot V_R^\dagger$$

- **Mixing angles between mass states, sterile neutrinos and flavour states:**

$$\theta_{aI} \equiv \frac{(m_D V_R)_{aI}}{M_I} \quad \textbf{and} \quad \theta_I^2 \equiv \sum_{a=e,\mu,\tau} |\theta_{aI}|^2$$

←→ strength of interaction (decay) of sterile neutrinos

- **Current best fit values:**

$$\Delta m^2_{\rm sol} = (7.65^{+0.69}_{-0.6}) \times 10^{-5}\ {\rm eV}^2$$
$$\Delta m^2_{\rm atm} = (2.4^{+0.35}_{-0.33}) \times 10^{-3}\ {\rm eV}^2.$$

- **Casas-Ibarra parametrization for type-I and II** (Akhmedov, Rodejohann)

$$\theta_I^2 = \frac{[\sqrt{M_R} R^T m_\nu^{\rm diag} R^\star \sqrt{M_R}]_{II}}{M_I^2}\ ,\quad m_\nu^{\rm diag} = {\rm diag}(m_1, m_2, m_3)$$

- **assume (convention) $m_1 < m_2 < m_3$ ➔ we get for the first two sterile ν's**

$$\begin{aligned} M_1\theta_1^2 &= m_3|\sin\omega_{13}|^2 + m_2|\cos\omega_{13}|^2|\sin\omega_{12}|^2 \\ &\quad + m_1|\cos\omega_{13}|^2|\cos\omega_{12}|^2, \\ M_2\theta_2^2 &= m_3|\cos\omega_{13}|^2|\sin\omega_{23}|^2 + m_2|\cos\omega_{23}\cos\omega_{12} \\ &\quad - \sin\omega_{23}\sin\omega_{13}\sin\omega_{12}|^2 + m_1|\cos\omega_{23}\sin\omega_{12} \\ &\quad + \sin\omega_{23}\sin\omega_{13}\cos\omega_{12}|^2. \end{aligned}$$

- The relation |z-w| ≥ ||z| - |w|| leads then to the following inequalities:

$$\begin{aligned} M_1\theta_1^2 &\geq m_2\{\sin^2\omega_{13} + \cos^2\omega_{13}\sin^2\omega_{12}\}, \\ M_2\theta_2^2 &\geq m_2\{\cos^2\omega_{13}\sin^2\omega_{23} + (|\cos\omega_{23}||\cos\omega_{12}| \\ &\qquad - |\sin\omega_{23}||\sin\omega_{13}||\sin\omega_{12}|)^2\}. \end{aligned}$$

- The minimum of the sum on the *rhs* is m_2 ➔

$$M_1\theta_1^2 + M_2\theta_2^2 \geq m_2 \geq \Delta m_{\text{sol}} \quad (*)$$

In words: One cannot generate active ν masses with type-I see-saw without sufficient mixings between active and sterile neutrinos

➔ conflict with bounds:

Entropy generation: $M_2\theta_2^2 \lesssim 1.8\times 10^{-3}\bar{g}_*^{1/2}\left(\frac{\text{GeV}}{M_2}\right)^2\left(\frac{\text{keV}}{M_1}\right)^2$

X-ray bound: $M_1\theta_1^2 \lesssim 2.7\times 10^{-3}\left(\frac{1.6\text{ keV}}{M_1}\right)^4$

➔ violates bound (*)

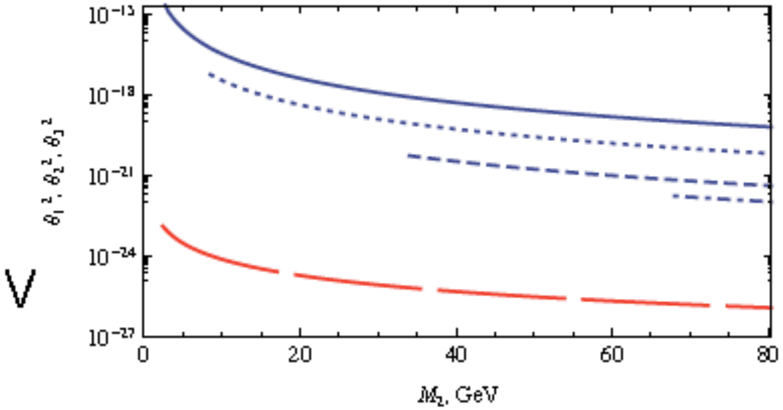
➔ type-I see-saw impossible ➔ type II

Working example with type II see-saw

Exactly LR-symmetric model:

$$\mathscr{L}_{\text{mass}} = -\frac{1}{2}\left(\overline{\nu^c_{aL}}, \overline{N_{aR}} \right) \begin{pmatrix} f\,v_L & y\,v \\ y\,v & f\,v_R \end{pmatrix} \begin{pmatrix} \nu_{aL} \\ N^c_{aR} \end{pmatrix}$$

$$m_\nu = v_L f - \frac{v^2}{v_R} y f^{-1} y\,, \qquad M_I = f_I v_R$$



$m_1 = 5.2 \times 10^{-9}$ eV

$m_2 = 8.7 \times 10^{-3}$ eV $\quad m_3 = 4.9 \times 10^{-2}$ eV

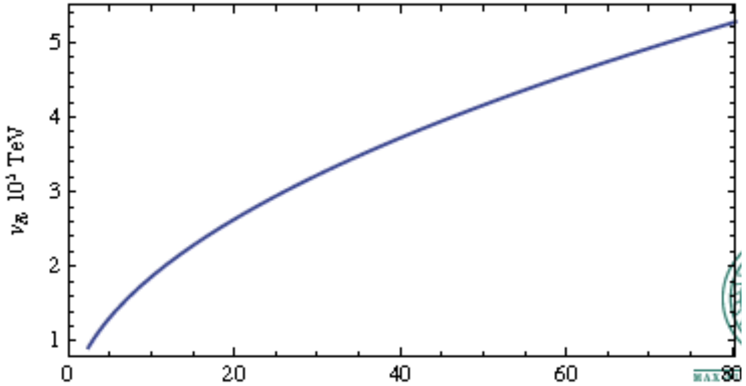
$M_1 = 1.6$keV

$M_2 = 2.7$ GeV $\qquad M_3 = 15.1$ GeV

$\theta_1^2 = \theta_2^2 = \theta_3^2 = 2.3 \times 10^{-15}$

$v_R = 9.67 \times 10^4$ TeV $\quad v_L = 313$ keV

$y = 0.027 f$



Conclusions

- **A keV-ish sterile neutrino is a very well motivated and good working Warm Dark Matter candidate ⇔ finite ν-masses**

- **Simplest realization: νMSM → requires non-thermal production**
- **Alternative: Sterile ν's which are charged under some extended gauge group → abundance from thermal production**
  → interesting constrains
  - small mixings from X-ray constraints and entropy generation (DM abundance)
  - masses bound by BBN

→ Implications for neutrino mass generation:

- type-I see-saw not possible
- type-II works ⇔ very natural in gauge extensions
- requires one sterile neutrino to be light

→ More general scenarios require just some mechanism which 'naturally' explains light sterile neutrinos