The Standard Model of the Universe: Warm Dark Matter and galaxy structure

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16th Paris Cosmology Colloquium 2012

Observatoire de Paris, Paris campus

25, 26 and 27 July 2012

tandard Cosmological Model: DM + Λ + Baryons + Radiati

- Begins by the inflationary era. Slow-Roll inflation explains horizon and flatness.
- Gravity is described by Einstein's General Relativity.
- Particle Physics described by the Standard Model of Particle Physics: $SU(3) \otimes SU(2) \otimes U(1) =$ qcd+electroweak model.
- Dark matter is non-relativistic during the matter dominated era where structure formation happens. DM is outside the SM of particle physics.
- Dark energy described by the cosmological constant Λ.

Standard Cosmological Model: ΛCDM $\Rightarrow$ ΛWDM

Dark Matter + Λ + Baryons + Radiation
begins by the Inflationary Era. Explains the Observations:

- Seven years WMAP data and further CMB data
- Light Elements Abundances
- Large Scale Structures (LSS) Observations. BAO.
- Acceleration of the Universe expansion: Supernova Luminosity/Distance and Radio Galaxies.
- Gravitational Lensing Observations
- Lyman α Forest Observations and X ray data
- Hubble Constant and Age of the Universe Measurements
- Properties of Clusters of Galaxies
- Galaxy structure explained by WDM

Dark Matter Particles

DM particles can decouple being ultrarelativistic (UR) or non-relativistic.
They can decouple at or out of thermal equilibrium.

The DM distribution function freezes out at decoupling.

The characteristic length scale is the free streaming scale (or Jeans' scale). For DM particles decoupling UR:

$$r_{lin} = 57.2 \, \mathrm{kpc} \, \frac{\mathrm{keV}}{m} \, \left(\frac{100}{g_d}\right)^{\frac{1}{3}}$$

DM particles can freely propagate over distances of the order of the free streaming scale.

Therefore, structures at scales smaller or of the order r_{lin} are erased.

For $m \sim$ keV WDM particles $r_{lin} \sim 60$ kpc, is the size of the DM cores.

CDM free streaming scale

For CDM particles with $m \sim 100$ GeV: $r_{lin} \sim 0.1$ pc

Hence CDM structures keep forming till scales as small as the solar system.

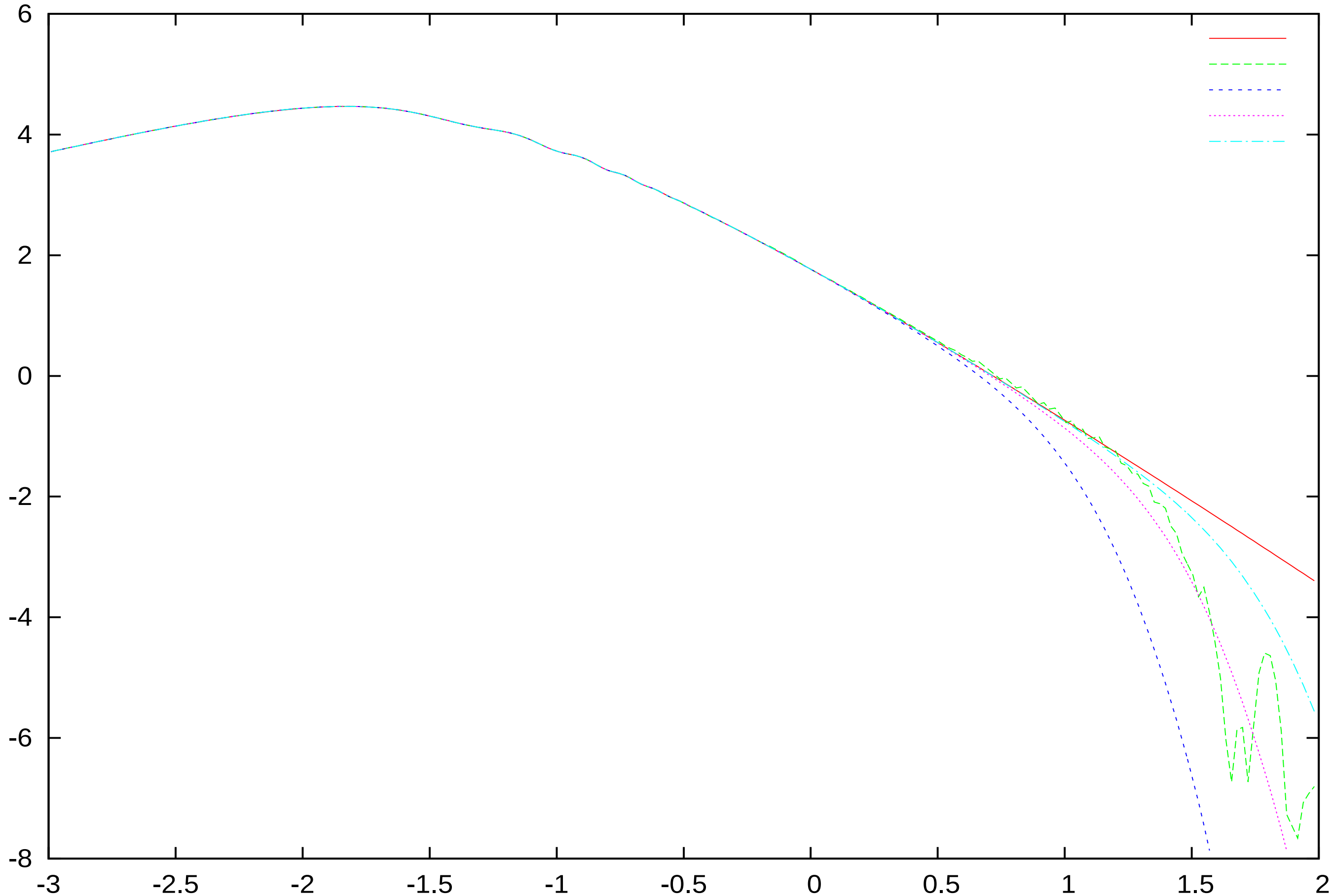
This has been explicitly verified by all CDM simulations but never observed in the sky.

There is over abundance of small structures in CDM (also called the satellite problem).

CDM has many serious conflicts with observations:

A number of pure-disk galaxies (bulgeless) are observed whose formation through CDM hierarchical clustering is unexplained (Kormendy & Freeman).

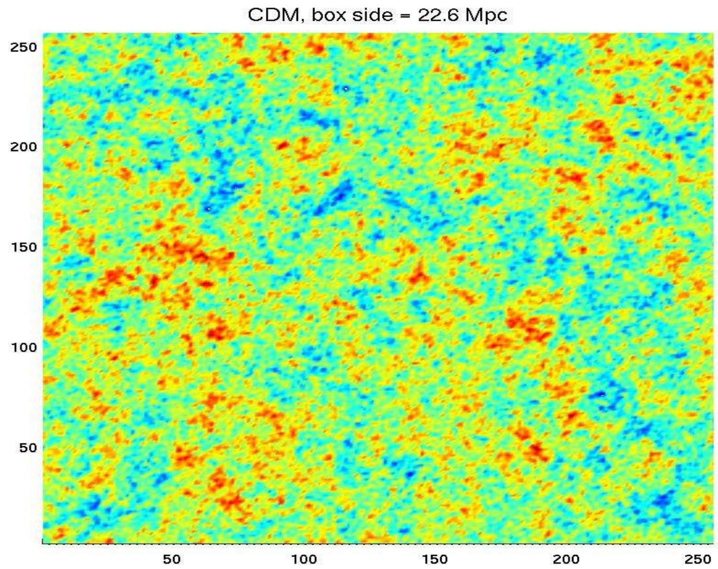
Linear primordial power today $P(k)$ vs. k Mpc h



$\log_{10} P(k)$ vs. $\log_{10}[k \text{ Mpc } h]$ for CDM, 1 keV, 2 keV, light-blue 4 keV DM particles decoupling in equil, and 1 keV sterile neutrinos. WDM cuts $P(k)$ on small scales

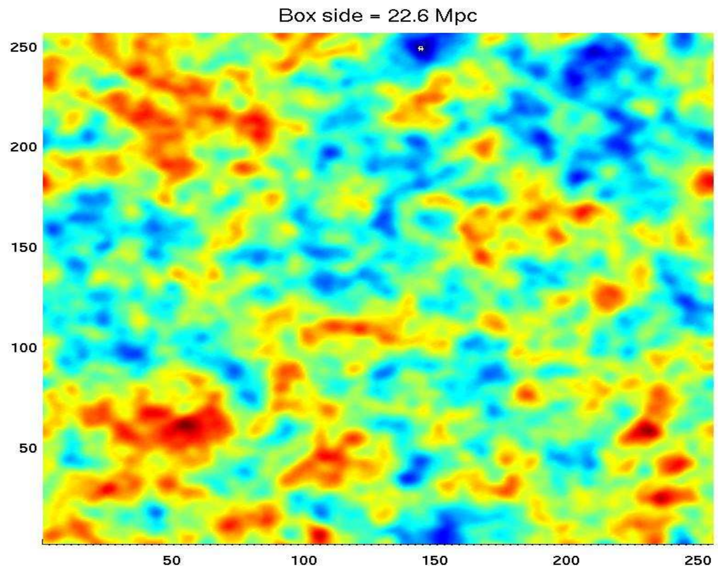
$r \lesssim 100 \ (\text{keV}/m)^{4/3}$ kpc

WDM vs. CDM linear fluctuations today

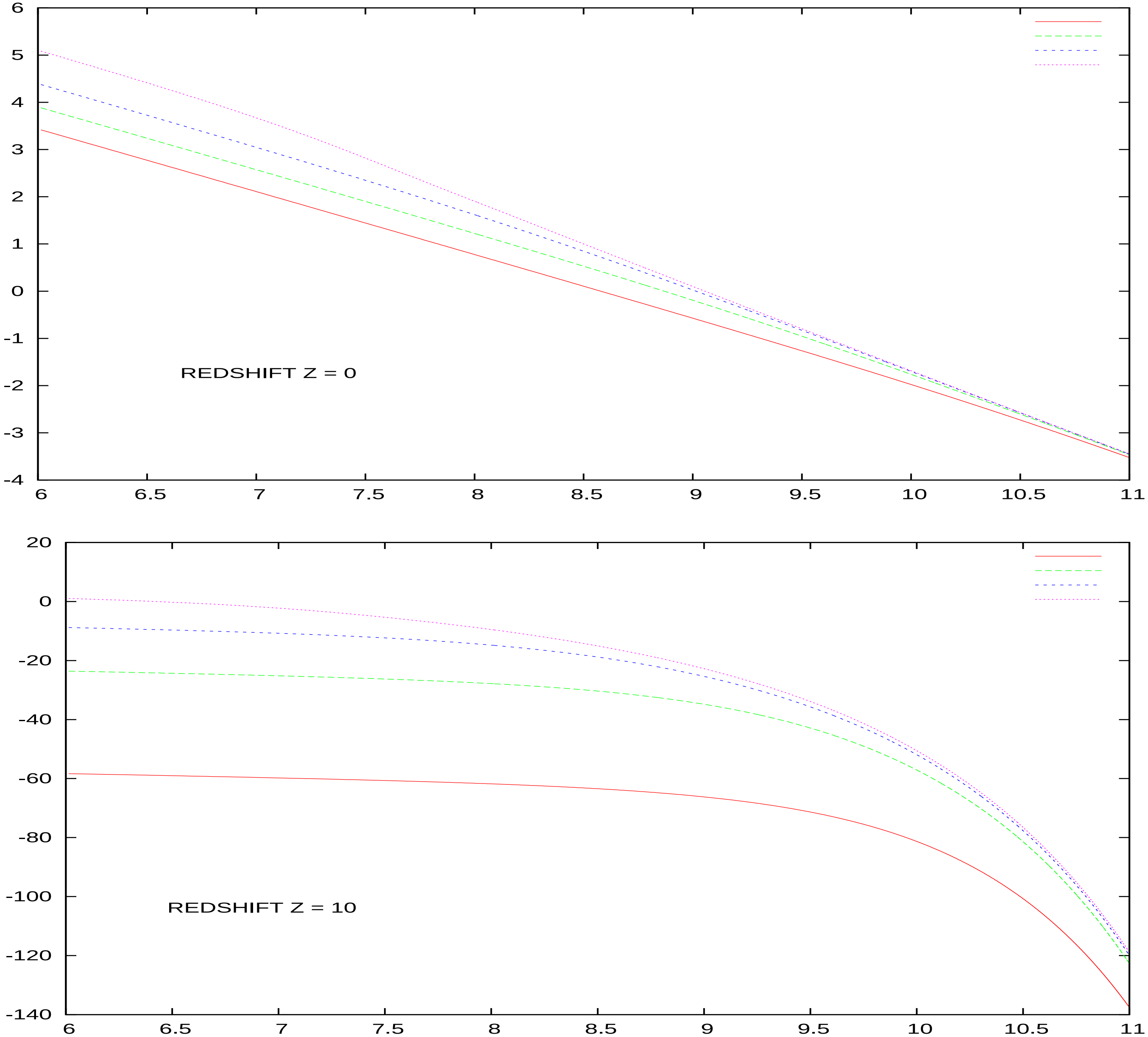


Box side = 22.6 Mpc. [C. Destri, private communication].

WDM:



$\log_{10}[(z+1)^3 dN/dM \ Mpc^3 \ 10^9 \ M_\odot]$ **vs.** $\log_{10}[h^3 \ M/M_\odot]$



CDM, 4 keV , 2 keV, 1 keV. WDM decoupling in equilibrium.

Galaxy Density Profiles: Cores vs. Cusps

Astronomical observations always find cored profiles.
Selected references:
J. van Eymeren et al. A&A (2009), M. G. Walker, J.Peñarrubia,ApJ(2012).N.Amorisco,N.Evans,MNRAS(2012).

Galaxy profiles in the linear regime: core size $\sim$ free streaming length (de Vega, Salucci, Sanchez, 2010)=

halo radius $r_0 = \begin{cases} \sim 0.05 \text{ pc cusps for CDM } (\text{m} > \text{GeV}). \\ \sim 50 \text{ kpc cores for WDM } (\text{m} \sim \text{keV}). \end{cases}$

N-body simulations for CDM give cusps (NFW profile).

N-body simulations for WDM : quantum physics needed for fermionic DM !!! (Destri, de Vega, Sanchez, 2012)

CDM simulations give a precise value
for the concentration $\equiv R_{virial}/r_0$.

CDM concentrations disagree with observed values.

The Phase-space density $Q(r)$

The phase-space density $Q \equiv \rho_{DM}/\sigma_{DM}^3$ is invariant under the cosmological expansion and can only decrease under self-gravity interactions (gravitational clustering).

Early universe value: $Q_{prim} = \frac{\rho_{prim}}{\sigma_{prim}^3} = \frac{3\sqrt{3}}{2\,\pi^2}\, g\, \frac{I_2^{\frac{5}{2}}}{I_4^{\frac{3}{2}}}\, m^4$

I_2 and I_4 are momenta of the DM distribution function.

g : $\#$ of internal degrees of freedom of the DM particle, $1 \leq g \leq 4$.

During structure formation Q decreases by a factor that we call Z, $(Z > 1)$:

$$Q_{today} = \frac{1}{Z}\, Q_{prim} \quad , \quad Z > 1$$ implies the

Tremaine-Gunn bound (1979): $m > 0.3$ keV.

The Phase-space density $Q = \rho/\sigma^3$ and its decrease factor Z

The phase-space density today Q_{today} follows observing dwarf spheroidal satellite galaxies of the Milky Way (dSphs) as well as spiral galaxies. Its value is galaxy dependent.

For dSphs $Q_{today} \sim \ (0.18 \text{ keV})^4$ Gilmore et al. 07/08.

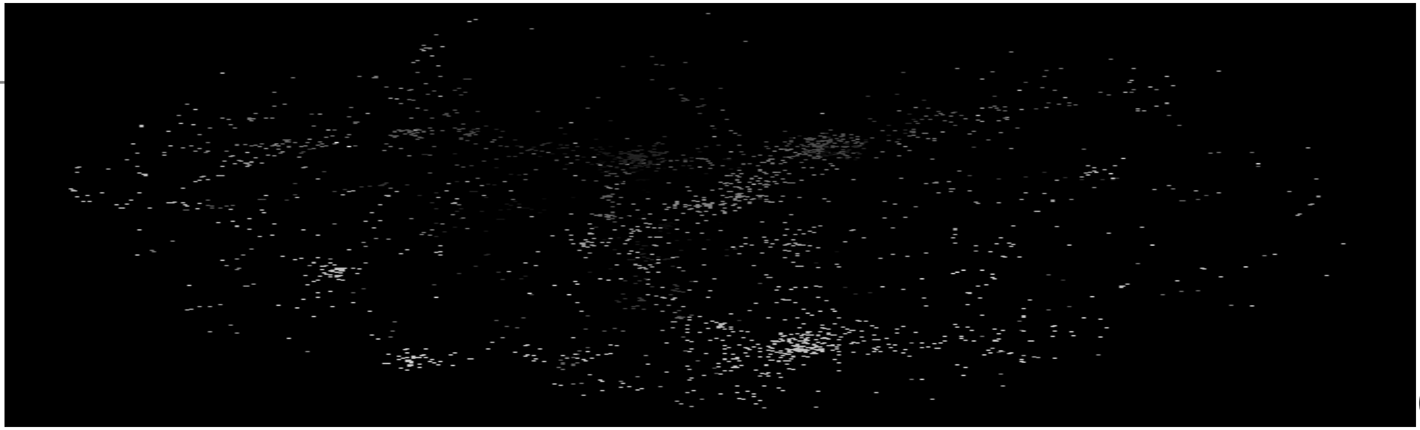
The spherical model gives $Z \simeq 41000$ and N-body simulations indicate: $10000 \gtrsim Z > \gtrsim 10$. Z is galaxy dependent.

As a consequence m is in the keV scale:1keV$\lesssim m \lesssim 10$keV. H. J. de Vega, N. G. Sanchez, MNRAS (2010)

This is true both for DM decoupling in or out of equilibrium, bosons or fermions.

It is independent of the particle physics model.

CDM and ΛWDM simulations vs. astronomical observatio

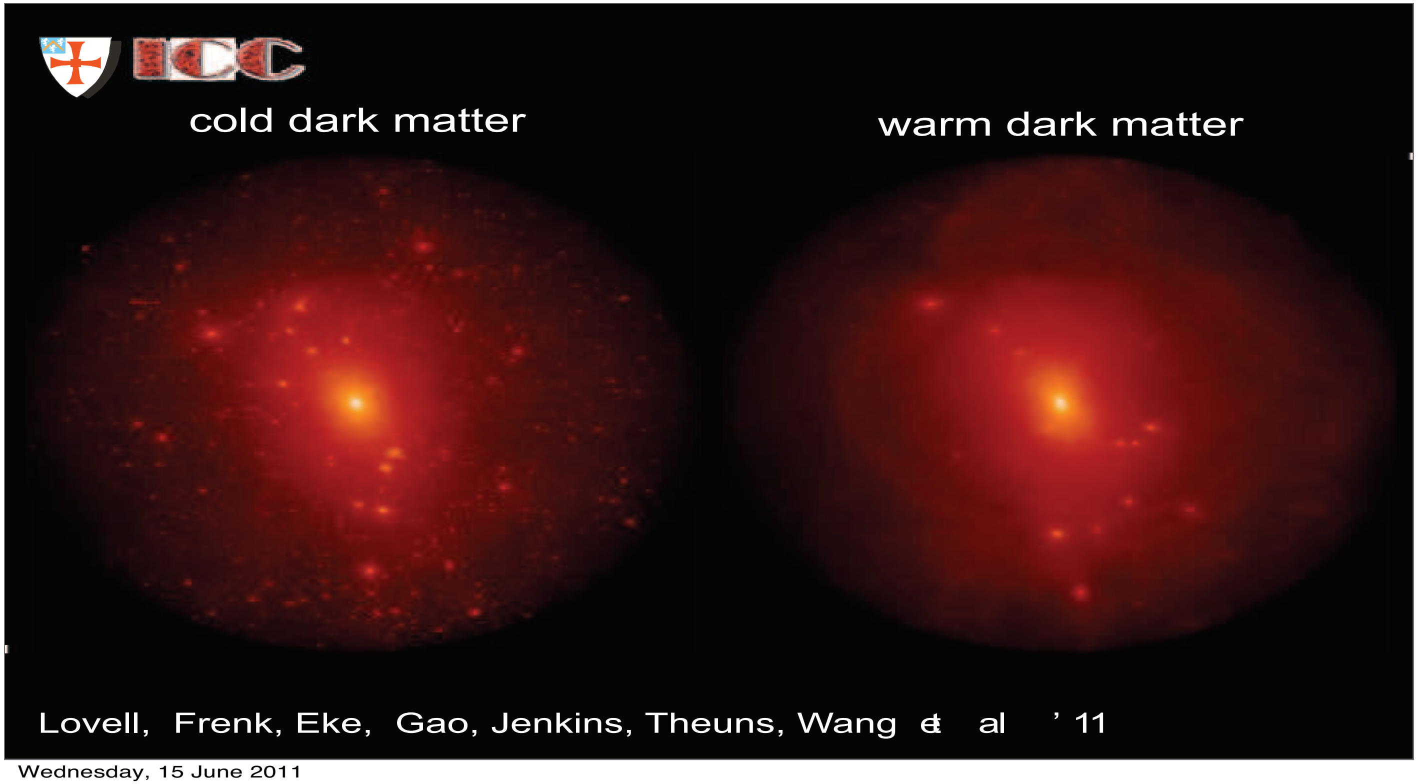


CDM

observations

1keV WDM

N-body WDM Simulations

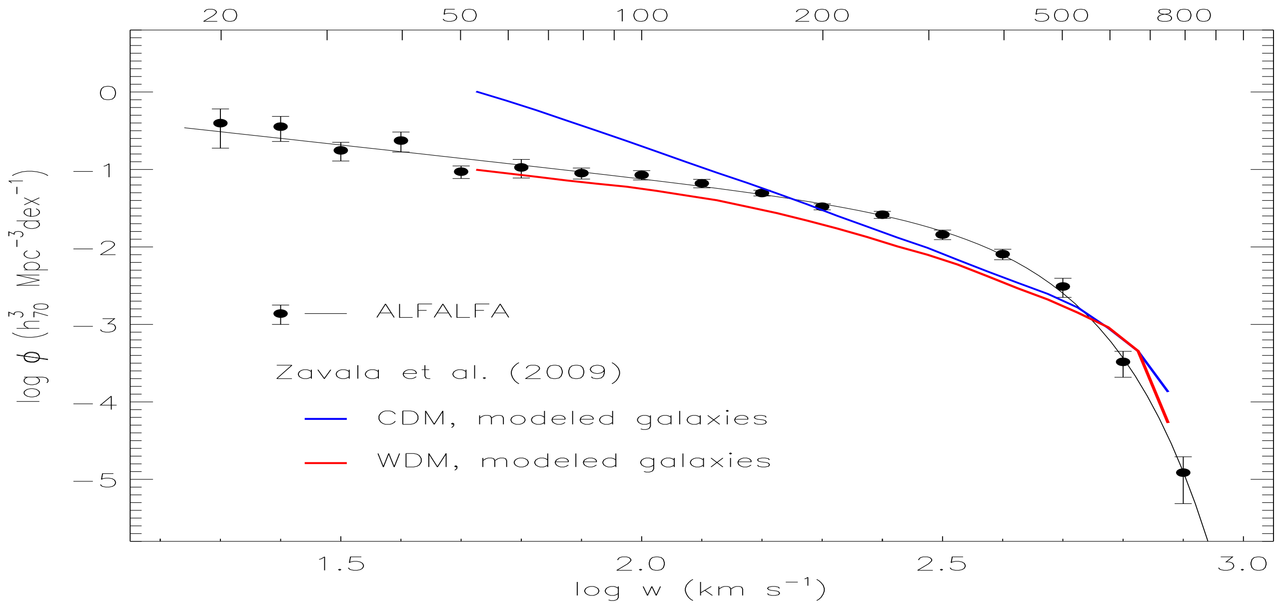


WDM subhalos are less concentrated than CDM subhalos.

WDM subhalos have the right concentration to host the bright Milky Way satellites.

Lovell et al. MNRAS (2012).

Velocity widths in galaxies



Velocity widths in galaxies from 21cm HI surveys. ALFALFA survey clearly favours WDM over CDM. (Papastergis et al. ApJ, 2011, Zavala et al. ApJ, 2009).

Notice that the WDM red curve is for $m = 1$ keV WDM particle decoupling at thermal equilibrium.

The 1 keV WDM curve falls somehow below the data suggesting a slightly larger WDM particle mass.

Quantum Bounds on Fermionic Dark Matter

The Pauli principle gives the upper bound to the phase space distribution function of spin-$\frac{1}{2}$ particles of mass m:

$$f(\vec{r}, \vec{p}) \leq 2$$

The DM mass density is given by:

$$\rho(\vec{r}) = m \int d^3p \, \frac{f(\vec{r},\vec{p})}{(2\,\pi\,\hbar)^3} = \frac{m^4}{2\,\hbar^3} \, \sigma^3(\vec{r}) \, \bar{f}(\vec{r}) \, K \ ,$$

where:

$\bar{f}(\vec{r})$ is the $\vec{p}-$average of $f(\vec{r}, \vec{p})$ over a volume $m^3 \, \sigma^3(\vec{r})$,

$\sigma(\vec{r})$ is the DM velocity dispersion, $\sigma^2(\vec{r}) \equiv < v^2(\vec{r}) > /3$

$K \sim 1$ a pure number.

The Pauli bound $\bar{f}(\vec{r}) \leq 2$ yields: $Q(\vec{r}) \equiv \frac{\rho(\vec{r})}{\sigma^3(\vec{r})} \leq K \, \frac{m^4}{\hbar^3}$

This is an absolute quantum upper bound on $Q(\vec{r})$ due to quantum physics, namely the Pauli principle.

$Q(\vec{r})$ can **never** take values larger than $K \, m^4/\hbar^3$.

In the classical limit $\hbar \to 0$ and the bound disappears.

Classical physics breaks down near the galaxy center

N-body simulations point to cuspy phase-space densities

$Q(r) = Q_s \left(\frac{r}{r_s}\right)^{-\beta}, \quad \beta \simeq 1.9 - 2, \;\; r_s =$ halo radius,

$Q(r)$ derived within classical physics tends to infinity for $r \to 0$ violating the Pauli principle bound.

Classical physics breaks down near the galaxy center.

The quantum upper bound on $Q(r)$ is valid for

$$r \geq r_q \equiv \frac{\hbar^{\frac{3}{2}}}{m^2} \sqrt{\frac{Q_s}{K}}\, r_s \quad , \quad (\text{setting } \beta = 2).$$

Classical physics breaks down for $r < r_q$ where r_q is in the parsec range for WDM $m \sim$ keV.

r_q corresponds to maximally packed fermions around the center of the galaxy. The core radii must be much larger than r_q for diluted objects as galaxies.

In atoms the electrons phase-space density turns to be significantly below the Pauli quantum bound.

Dwarf galaxies as quantum objects

de Broglie wavelength of DM particles $\lambda_{dB} = \frac{\hbar}{m \, \sigma}$

$d =$ Average distance between particles

$d = \left(\frac{m}{\rho}\right)^{\frac13} \quad , \quad \rho = \sigma^3 \, Q \quad , \quad Q =$ phase space density.

ratio: $\mathcal{R} = \frac{\lambda_{dB}}{d} = \hbar \, \left(\frac{Q}{m^4}\right)^{\frac13}$

Observed values: $0.74 \times 10^{-3} < \mathcal{R} \, \left(\frac{m}{\mathrm{keV}}\right)^{\frac13} < 0.70$

The larger $\mathcal{R}$ is for ultracompact dwarfs.
The smaller $\mathcal{R}$ is for big spirals.

The ratio $\mathcal{R}$ near unity (or above) means a QUANTUM OBJECT.

Observations alone show that compact dwarf galaxies are quantum objects (for WDM).

Summary on quantum bounds on cores

If DM were formed by bosons the quantum bound on Q does not apply and the formation of cusps would be allowed.

Astronomical observations show that DM galaxy density profiles are cored.

Thus, bosonic DM turns to be strongly disfavoured.

In all cases, cusps of fermionic DM in the galaxy density profile are artifacts produced by classical physics computations irrespective of the nature of the fermionic dark matter (HDM, WDM, CDM).

Quantum physics, namely the Pauli principle, rule out galaxy cusps for fermionic dark matter.

C. Destri, H. J. de Vega, N. G. Sanchez, ‘Fermionic warm dark matter produces galaxy cores in the observed scales’, arXiv:1204.3090.

WDM properties

WDM is characterized by

- its initial power spectrum cutted off for scales below ~ 50 kpc. Thus, structures are not formed in WDM for scales below ~ 50 kpc.
- its initial velocity dispersion. However, this is negligible for $z < 100$ where the non-linear regime starts (see Valageas, 2012).
- Classical N-body simulations break down at small distances ($\sim$ pc). Need of quantum calculations to find WDM cores.

Structure formation is hierarchical in CDM.
WDM simulations show in addition top-hat structure formation at large scales and low densities but hierarchical structure formation remains dominant.

So far, not a single objection arised against WDM.

Quantum pressure vs. gravitational pressure

quantum pressure: $P_q =$ flux of momentum $= n\ v\ p$,

$v =$ mean velocity, momentum $= p \sim \hbar/\Delta x \sim \hbar\, n^{\frac{1}{3}}$,

particle number density $= n = \frac{M_q}{\frac{4}{3}\,\pi\, R_q^3\, m}$

galaxy mass $= M_q$, galaxy halo radius $= R_q$

gravitational pressure: $P_G = \frac{G\, M_q^2}{R_q^2} \times \frac{1}{4\,\pi\, R_q^2}$

Equilibrium: $P_q = P_G \Longrightarrow$

$$R_q = \frac{3^{\frac{5}{3}}}{(4\,\pi)^{\frac{2}{3}}}\ \frac{\hbar^2}{G\, m^{\frac{8}{3}}\ M_q^{\frac{1}{3}}} = 10.6\ldots\text{pc}\left(\frac{10^6\ M_\odot}{M_q}\right)^{\frac{1}{3}}\ \left(\frac{\text{keV}}{m}\right)^{\frac{8}{3}}$$

$$v = \left(\frac{4\,\pi}{81}\right)^{\frac{1}{3}}\ \frac{G}{\hbar}\ m^{\frac{4}{3}}\ M_q^{\frac{2}{3}} = 22.9\,\frac{\text{km}}{\text{s}}\ \left(\frac{\text{keV}}{m}\right)^{\frac{4}{3}}\ \left(\frac{M_q}{10^6\ M_\odot}\right)^{\frac{2}{3}}$$

for WDM the values of $M_q,\ R_q$ and v are consistent with dwarf galaxies !! .

Dwarf spheroidal galaxies can be supported by the fermionic quantum pressure of WDM.

Self-gravitating Fermions in the Thomas-Fermi approach

WDM is non-relativistic in the MD era.

Chemical potential: $\mu(r) = \mu_0 - m\,\phi(r)$, $\phi(r) =$ grav. pot.

Poisson's equation: $\frac{d^2\mu}{dr^2} + \frac{2}{r}\,\frac{d\mu}{dr} = -4\,\pi\,G\,m\,\rho(r)$

$\rho(0) =$ finite for fermions $\Longrightarrow \frac{d\mu}{dr}(0) = 0$.

Density $\rho(r)$ in terms of the distribution function $f(E)$:

$$\rho(r) = \frac{m}{\pi^2\,\hbar^3}\int_0^\infty p^2\,dp\,f[\frac{p^2}{2\,m} - \mu(r)]$$

Thomas-Fermi eqs. determine the chemical potential $\mu(r)$

Boundary condition at
$r = R = R_{200} \sim R_{vir}$, $\rho(R_{200}) \simeq 200\,\bar{\rho}_{DM}$

The chemical potential at $r = 0$ is fixed by the value of $Q(0)$.

Self-gravitating Fermions in the Thomas-Fermi approach

Using observed values of $Q(0)$, we obtain halo radius $r_s \sim 0.1 - 10$ kpc, galaxy masses $10^5 - 10^7 \; M_\odot$ and velocity dispersions ~ 10 km/s, all consistent with the observations of dwarf galaxies.

The Thomas-Fermi approach gives realistic halo radii, larger than the quantum lower bound r_q, as expected.

In the Thomas-Fermi approach, for growing halo radius the total mass grows and the central phase-space density decreases as it happens in real galaxies.

Fermionic WDM treated quantum mechanically is able to reproduce the observed DM cores of galaxies.

Sterile Neutrinos in the SM of particle physics

SM symmetry group: $SU(3)_{color} \otimes SU(2)_{weak} \otimes U(1)^{weak}_{hypercharge}$

Leptons are color singlets and doublets under weak SU(2).

Sterile neutrinos ν_R do not participate to weak interactions. Hence, they must be singlets of color, weak SU(2) and weak hypercharge.

The nonzero Higgs vacuum expectation value induces a mixing between left ν_L and right ν_R handed neutrinos with eigenstates:

- active neutrino: $\nu_{active} \simeq \nu_L - \frac{m_D}{M}\,\nu_R$.
- sterile neutrino: $\nu_{sterile} \simeq \nu_R + \frac{m_D}{M}\,\nu_L, \quad M \gg m_D^2/M$.

and respective mass eigenvalues: $\frac{m_D^2}{M}$ and M.

Sterile Neutrinos

$M \sim 1$ keV and $m_D \sim 0.1$ eV is consistent with observations. Mixing angle: $\theta \sim \frac{m_D}{M} \sim 10^{-4}$ is appropriate to produce enough sterile neutrinos accounting for the observed DM.

Smallness of θ makes sterile neutrinos difficult to detect.

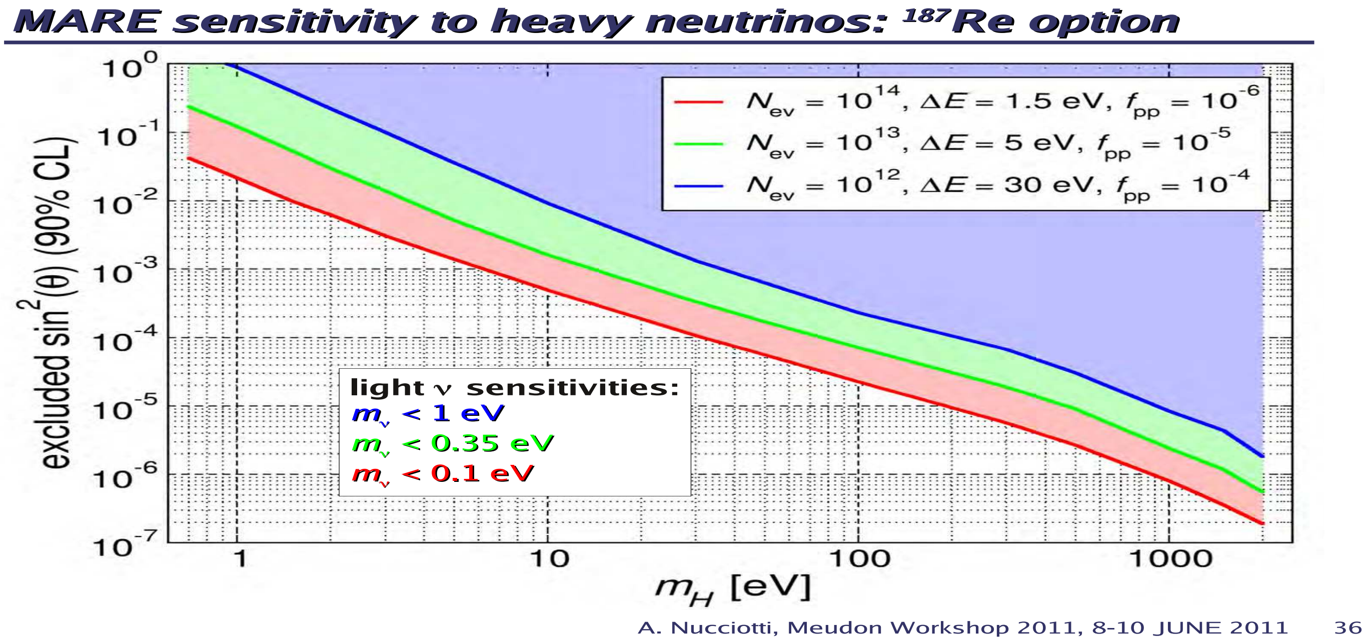
Precise measure of nucleus recoil in tritium beta decay: ${}^3H_1 \Longrightarrow {}^3He_2 + e^- + \bar{\nu}$ can show the presence of a sterile instead of the active $\bar{\nu}$ in the decay products.

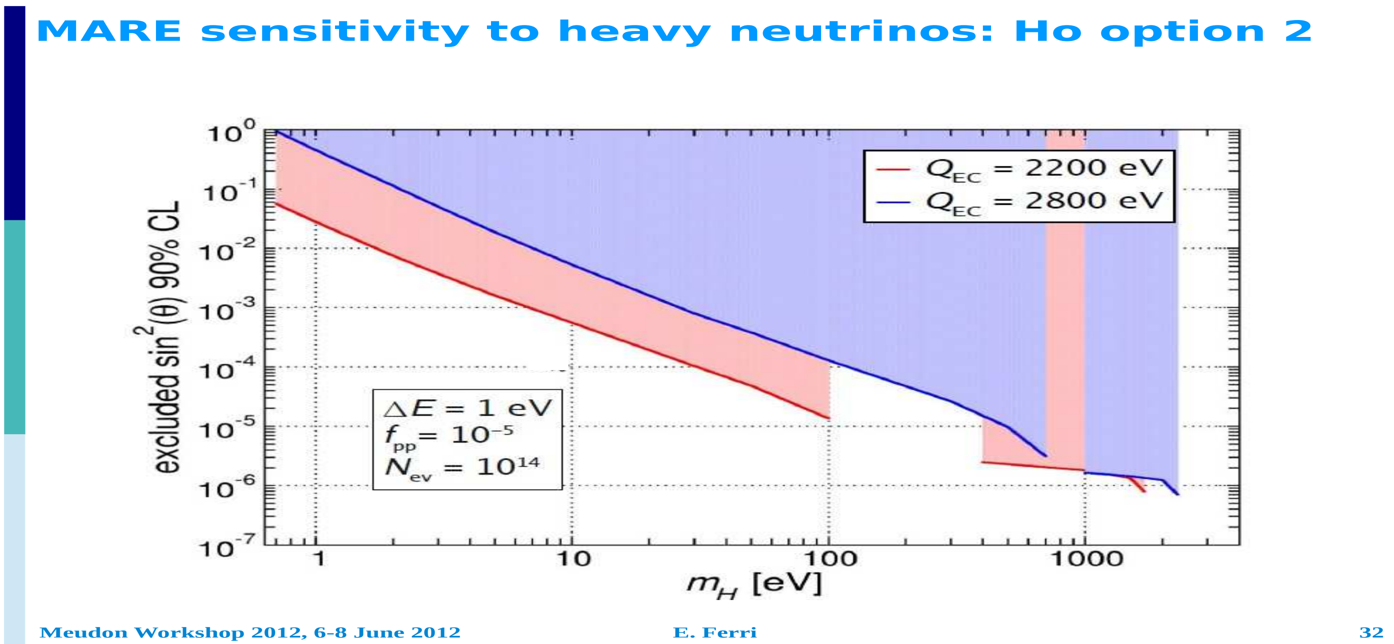
Rhenium 187 beta decay gives $\theta < 0.095$ for 1 keV steriles [Galeazzi et al. PRL, 86, 1978 (2001)].
Available energy: $Q({}^{187}Re) = 2.47$ keV, $Q({}^3H_1) = 18.6$ keV.

Future measurements: KATRIN(Karlsruhe), MARE(Milano).

Conclusion: the empty slot of right-handed neutrinos in the Standard Model of particle physics can be filled by keV-scale sterile neutrinos describing the DM.

MARE searchs in Re187 β decay and Ho163 electron capture





Sterile neutrino models

Sterile neutrinos: named by Bruno Pontecorvo (1968).

- DW: Dodelson-Widrow model (1994) sterile neutrinos produced by non-resonant mixing from active neutrinos.
- Shi-Fuller model (1998) sterile neutrinos produced by resonant mixing from active neutrinos.
- ν-MSM model (1981)-(2006) sterile neutrinos produced by a Yukawa coupling from a real scalar χ.
- DM models must reproduce $\bar{\rho}_{DM}$, galaxy and structure formation and be consistent with particle experiments.

WDM particles in different models behave just as if their masses were different (FD = thermal fermions):

$$\frac{m_{DW}}{\text{keV}} \simeq 2.85 \left(\frac{m_{FD}}{\text{keV}}\right)^{4/3}, \; m_{SF} \simeq 2.55 \, m_{FD}, \; m_{\nu\text{MSM}} \simeq 1.9 \, m_{FD}.$$

H J de Vega, N Sanchez, Phys. Rev. D85, 043517 (2012).

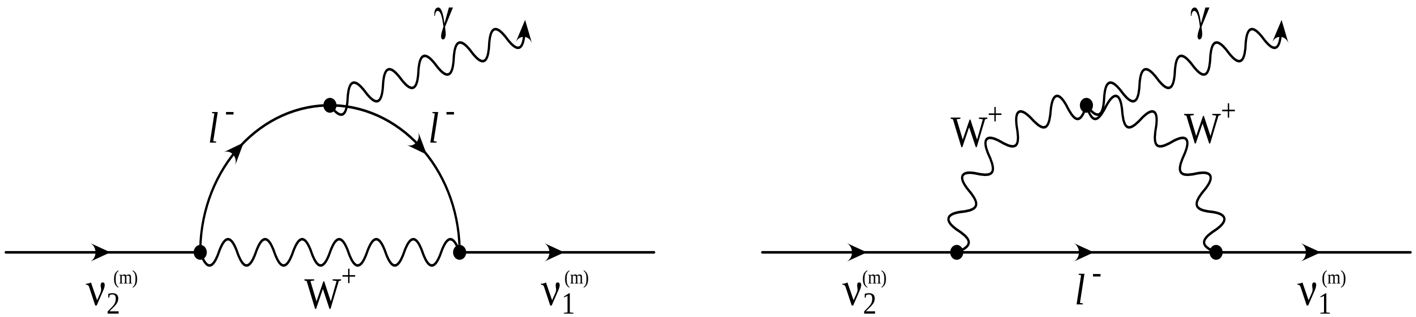
X-ray detection of DM sterile neutrinos

Sterile neutrinos ν_s decay into active neutrinos ν_a plus photons (γ) with a width:

$$\Gamma_{\nu_s \to \nu_a} = \frac{9}{256\,\pi^4}\,\alpha\,G_F^2\;\sin^2\theta\;m_{\nu_s}^5 = \frac{\sin^2\theta}{1.8\;10^{21}\;\text{sec}}\;\left(\frac{m_{\nu_s}}{\text{keV}}\right)^5$$

$$\Gamma_{\nu_s \to \nu_a} \ll \frac{1}{\text{Age of the universe} = 4.32\;10^{17}\text{sec}}$$

Pal & Wolfenstein (1982). $E_\gamma = m_{\nu_s}/2 \Rightarrow$ X-rays.

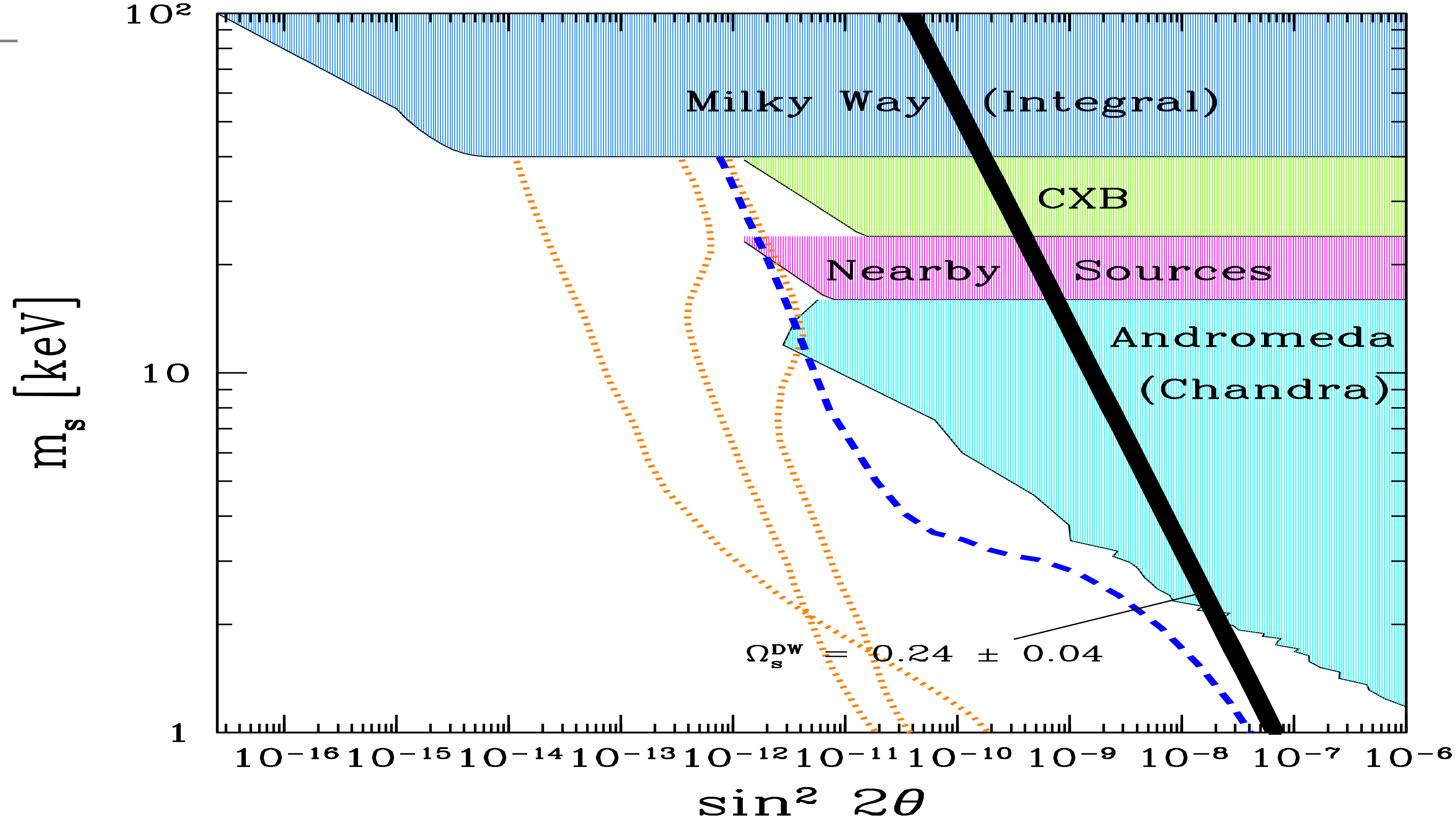


These X-rays may be seen in the sky looking to galaxies !

recent review: C. R. Watson et al. JCAP, (2012).

Future observations: DM bridge between M81 and M82 ~ 50 kpc. Overlap of DM halos.
satellite projects: IXO (killed) and Xenia (NASA).

Constraints on the sterile neutrino mass and mixing angle



Dashed = Shi-Fuller model. Dotted = Dodelson-Widrow for fermion asymmetry $L = 0.1,\ 0.01$ and 0.003.
Allowed sterile neutrino region in the right lower corner.
Main difficulty: to distinguish the sterile neutrino decay X-ray from narrow X-lines emitted by hot ions.

Further Experiments to detect Sterile Neutrinos

Lyα forest observations give limits on the sterile ν mass. However, it is the most sensitive method to the difficult-to-characterize non-linear growth of baryonic and DM structures. As a result, there are significant discrepancies between the reported mass lower limits.

Supernovae: θ unconstrained, $1 < m < 10$ keV (G. Raffelt & S. Zhou, PRD 2011).

CMB: WDM decay distorts the blackbody CMB spectrum. The projected PIXIE satellite mission (A. Kogut et al.) can measure WDM sterile neutrino mass.

Rhenium and Tritium beta decay (MARE, KATRIN). Theoretical analysis: H J de V, O. Moreno, E. Moya de Guerra, M. Ramón Medrano, N. Sánchez, arXiv:1109.3452.

Sterile Neutrinos may be observed in electron capture (EC) as in ^{163}Ho $\rightarrow$ ^{163}Dy, MARE experiment (Nuccioti et al.)

Summary: keV scale DM particles

- Reproduce the phase-space density observed in dwarf spheroidal and spiral galaxies (dV S, MNRAS 2010).
- Fermionic WDM provide cored galaxy profiles through quantum effects in agreement with observations (Destri, de Vega, Sanchez, 2012).
- The galaxy surface density $\mu_0 \equiv \rho_0 \, r_0$ is universal up to $\pm 10\%$ according to the observations. Its value $\mu_0 \simeq (18 \text{ MeV})^3$ is reproduced by WDM (dV S S, New Astronomy, 2012). CDM simulations give 1000 times the observed value of μ_0 (Hoffman et al. ApJ 2007). Density profile scales and decreases for intermediate scales with the spectral index n_s : $\rho(r) \sim r^{-1-n_s/2}$
- Alleviate the CDM satellite problem (Avila-Reese et al. 2000, Götz & Sommer-Larsen 2002, Markovic et al. JCAP 2011) and the CDM voids problem (Tikhonov et al. MNRAS 2009).

Summary: keV scale DM particles

- All direct searches of DM particles look for $m \gtrsim 1$ GeV. DM mass in the keV scale explains why nothing has been found ... e^+ and $\bar{p}$ excess in cosmic rays may be explained by astrophysics: P. L. Biermann et al. PRL (2009), P. Blasi, P. D. Serpico PRL (2009).
- Velocity widths in galaxies from 21cm HI surveys. ALFALFA survey clearly favours WDM over CDM. Papastergis et al. ApJ 2011, Zavala et al. ApJ 2009
- Combining theoretical evolution of fluctuations through the Boltzmann-Vlasov equation with galaxy data points to a DM particle mass 3 - 10 keV. The keV mass scale holds independently of the DM particle physics model.
- Highlights and conclusions of the Chalonge Meudon Workshop 2011: Warm dark matter in the galaxies, arXiv:1109.3187 and the 16th Paris Cosmology Colloquium 2011 arXiv:1203.3562, H. J. de V., N. G. S.

Future Perspectives

DM properties from galaxy observations.
keV scale DM particles are strongly favoured.
Determination of DM properties (mass, T_d) from galaxy data confronted with theory (Boltzmann-Vlasov and simulations).

Extensive WDM N-body simulations showing substructures, galaxy formation and evolution.
Quantum dynamical evolution for the WDM cores.

Sterile neutrinos ? Other particle in the keV mass scale?
Chandra, Suzaku, XMM, X-ray data: keV mass DM decay?

Neutrinos mass hierachy: active $\sim$ meV, light sterile $\sim$ eV, WDM sterile $\sim$ keV, unstable sterile $\sim$ MeV....

Sun models well reproduce the sun's chemical composition but not the heliosismology (Asplund et al. 2009).
Can DM inside the Sun help to explain the discrepancy?

Bounds from MARE on sterile neutrino mass and θ.
Could KATRIN join the search of keV sterile neutrinos?

Future research: Confront WDM theoretical predictions with observations.

Mainly at small scales, from galaxies.

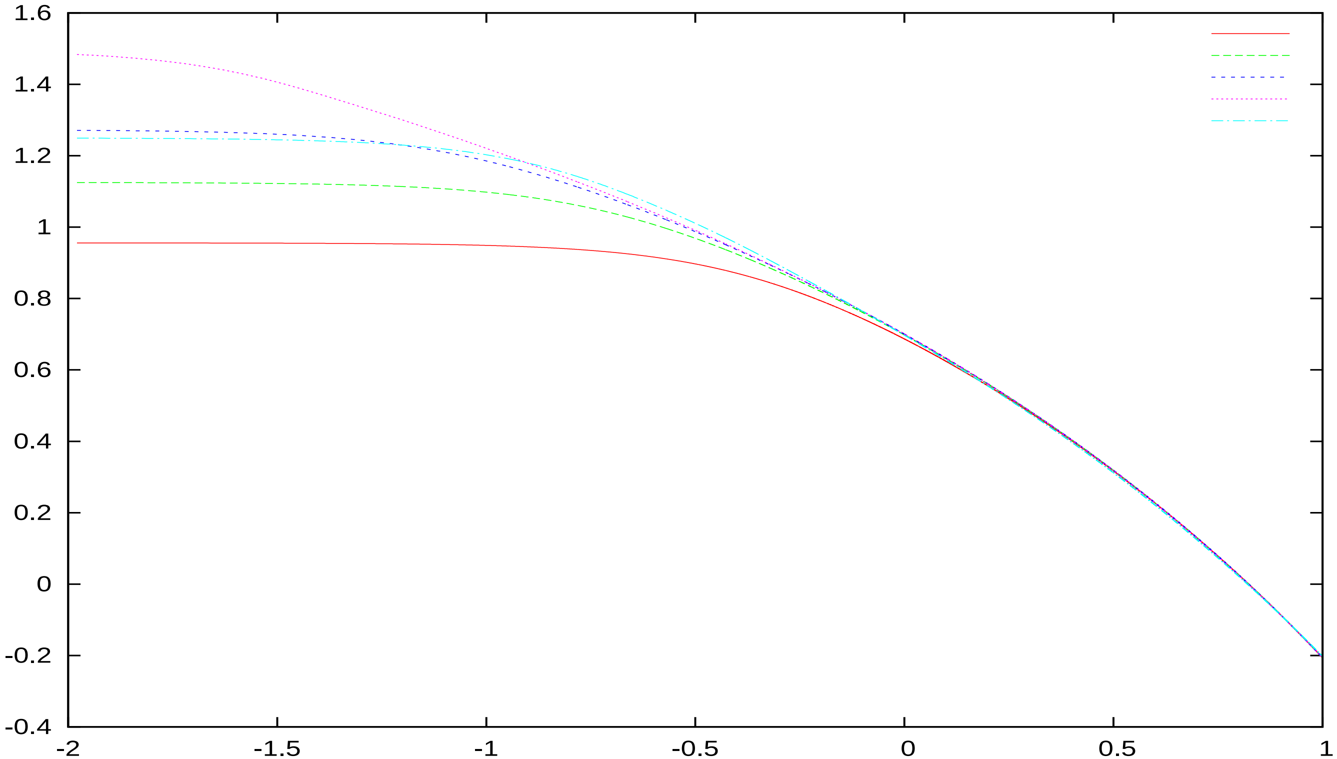
THANK YOU VERY MUCH

FOR YOUR ATTENTION!!

The expected overdensity

The expected overdensity within a radius R in the linear regime

$\sigma^2(R) = \int_0^\infty \frac{dk}{k} \, \Delta^2(k) \, W^2(kR) \quad , \quad W(kR)$: window function.



$\log \sigma(R)$ vs. $\log(R/h \text{ Mpc})$ for CDM, 1 keV, 2 keV, 4 keV DM particles decoupling in equil, and 1 keV (light-blue) sterile neutrinos. WDM flattens and reduces $\sigma(R)$ for small scales.

The Mass Function

The differential mass function gives the number of isolated bounded structures with mass between M and $M + dM$: (Press-Schechter)

$$\frac{dN}{dM} = -\frac{2\,\delta_c}{\sqrt{2\,\pi}\;\sigma^2(M,z)}\;\frac{\rho_M(z)}{M^2}\;\frac{d\sigma(M,z)}{d\ln M}\;e^{-\delta_c^2/[2\,\sigma^2(M,z)]},$$

$\delta_c = 1.686\ldots$: linear estimate for collapse from the spherical model.

$\sigma(M,z)$ is constant for WDM for small scales: small objects (galaxies) formation is suppresed in WDM in comparison with CDM.

$$\sigma(M,z) = \frac{g(z)}{z+1}\,\sigma(M,0) \quad \text{during the MD/}\Lambda\text{ dominated era.}$$

$g(z)$: takes into account the effect of the cosmological constant, $g(0) = 0.76\ ,\ g(\infty) = 1$

The Free Streaming Scale

The characteristic length scale is the free streaming scale (or Jeans' scale)

$$r_{lin} = 2\,\sqrt{1+z_{eq}}\,\left(\frac{3\,M_{Pl}^2}{H_0\,\sqrt{\Omega_{DM}}\;Q_{prim}}\right)^{\frac{1}{3}} = 21.1\,q_p^{-\frac{1}{3}}\ \text{kpc}$$

$q_p \equiv Q_{prim}/(\text{keV})^4$. DM particles can freely propagate over distances of the order of the free streaming scale.

$$r_{lin} = 57.2\,\text{kpc}\;\frac{\text{keV}}{m}\;\left(\frac{100}{g_d}\right)^{\frac{1}{3}}$$

Therefore, structures at scales smaller or of the order r_{lin} are erased.

It is useful to introduce the dimensionless wavenumber:

$$\gamma \equiv k\; r_{lin}/\sqrt{3} \quad \text{and} \quad \alpha \equiv \sqrt{3}\;\gamma/\sqrt{I_4}$$

where I_4 is the second momentum of the DM zeroth order distribution.

Relics decoupling non-relativistic

$$F_d^{NR}(p_c) = \frac{2^{\frac{5}{2}} \, \pi^{\frac{7}{2}}}{45} \; g_d \, Y_\infty \; \left(\frac{T_d}{m}\right)^{\frac{3}{2}} \; e^{-\frac{p_c^2}{2\, m \, T_d}} = \frac{2^{\frac{5}{2}} \, \pi^{\frac{7}{2}}}{45} \; \frac{g_d \, Y_\infty}{x^{\frac{3}{2}}} \; e^{-\frac{y^2}{2\,x}}$$

$Y(t) = n(t)/s(t), \; n(t)$ number of DM particles per unit volume, $s(t)$ entropy per unit volume, $x \equiv m/T_d, \; T_d < m$.

$Y_\infty = \frac{1}{\pi} \sqrt{\frac{45}{8}} \, \frac{1}{\sqrt{g_d} \, T_d \, \sigma_0 \, M_{Pl}}$ late time limit of Boltzmann.

σ_0: thermally averaged total annihilation cross-section times the velocity.

From our previous general equations for m and g_d:

$m = \frac{45}{4\,\pi^2} \, \frac{\Omega_{DM} \, \rho_c}{g \, T_\gamma^3 \, Y_\infty} = \frac{0.748}{g \, Y_\infty}$ eV and $m^{\frac{5}{2}} \, T_d^{\frac{3}{2}} = \frac{45}{2\,\pi^2} \, \frac{1}{g \, g_d \, Y_\infty} \, Z \, \frac{\rho_s}{\sigma_s^3}$

Finally: $\sqrt{m \, T_d} = 1.47 \, \left(\frac{Z}{g_d}\right)^{\frac{1}{3}} \mathrm{keV}$

We used ρ_{DM} today and the decrease of the phase space density by a factor Z.

Relics decoupling non-relativistic 2

Allowed ranges for m and T_d.

$m > T_d > b$ eV where $b > 1$ or $b \gg 1$ for DM decoupling in the RD era

$$\left(\frac{Z}{g_d}\right)^{\frac{1}{3}} \; 1.47 \text{ keV} < m < \frac{2.16}{b} \text{ MeV} \; \left(\frac{Z}{g_d}\right)^{\frac{2}{3}}$$

$g_d \simeq 3 \quad \text{for} \quad 1 \text{ eV} < T_d < 100 \text{ keV}$ and $1 < Z < 10^3$

$1.02 \text{ keV} < m < \frac{104}{b} \text{ MeV} \quad , \quad T_d < 10.2$ keV.

Only using ρ_{DM} today (ignoring the phase space density information) gives one equation with three unknowns: $m, \; T_d$ and σ_0,

$\sigma_0 = 0.16 \text{ pbarn} \, \dfrac{g}{\sqrt{g_d}} \, \dfrac{m}{T_d}$ http://pdg.lbl.gov

WIMPS with $m = 100$ GeV and $T_d = 5$ GeV require $Z \sim 10^{23}$.

Universe Inventory

The universe is spatially flat: $ds^2 = dt^2 - a^2(t)\, d\vec{x}^2$ plus small primordial fluctuations.

Dark Energy (Λ): 74 % , Dark Matter: 21 %

Baryons $+$ electrons: 4.4 % , Radiation ($\gamma + \nu$): 0.0085%

83 % of the matter in the Universe is DARK.

$$\rho(\text{today}) = 0.974\ 10^{-29}\ \frac{\text{g}}{\text{cm}^3} = 5.46\ \frac{\text{GeV}}{\text{m}^3} = (2.36\ 10^{-3}\ \text{eV})^4$$

DM dominates in the halos of galaxies (external part).

Baryons dominate around the center of galaxies.

Galaxies form out of matter collapse. Since angular momentum is conserved, when matter collapses its velocity increases. If matter can loose energy radiating, it can fall deeper than if it cannot radiate.

The quantum radius r_q for different kinds of DM

DM type	DM particle mass	r_q	
CDM	$1 - 100$ GeV	$1 - 10^4$ meters	in practice zero
WDM	$1 - 10$ keV	$0.1 - 1$ pc	compatible with observed cores
HDM	$1 - 10$ eV	kpc - Mpc	too big !

MARE searchs in Ho163 electron capture

MARE sensitivity to heavy neutrinos: Ho option 1

heavy neutrino emission in ^{163}Ho EC decay