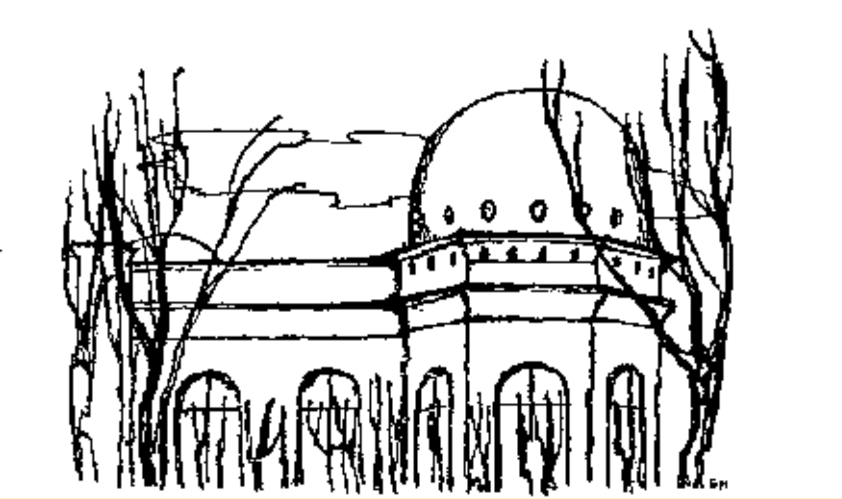

Ecole Internationale Daniel Chalonge

17th Paris Cosmology Colloquium 2013

"The new standard model of the Universe: Lambda Warm Dark Matter (LWDM) Theory vs. Observations"

Neutrino Masses, Mixings and Phases: Theory vs. Experiment

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Outline

1. The 3ν Mass-Mixing Framework

2. Oscillation searches sensitive to Δm^2

3. Oscillation searches sensitive to δm^2

4. Global 3ν analysis of all oscillation data

+ 6 exercises as homework (since this is an "Ecole" …)

Based on work with E. Lisi, A. Marrone, D. Montanino, A. Palazzo, A.M. Rotunno, ...

1. The 3ν Mass-Mixing Framework

1. Neutrino hysto(ry)gram

The discovery of flavor oscillations has raised the level of interest in neutrino physics, at the level of ~ 1.4×10^3 papers/year titled "…neutrino(s)…" on SPIRES

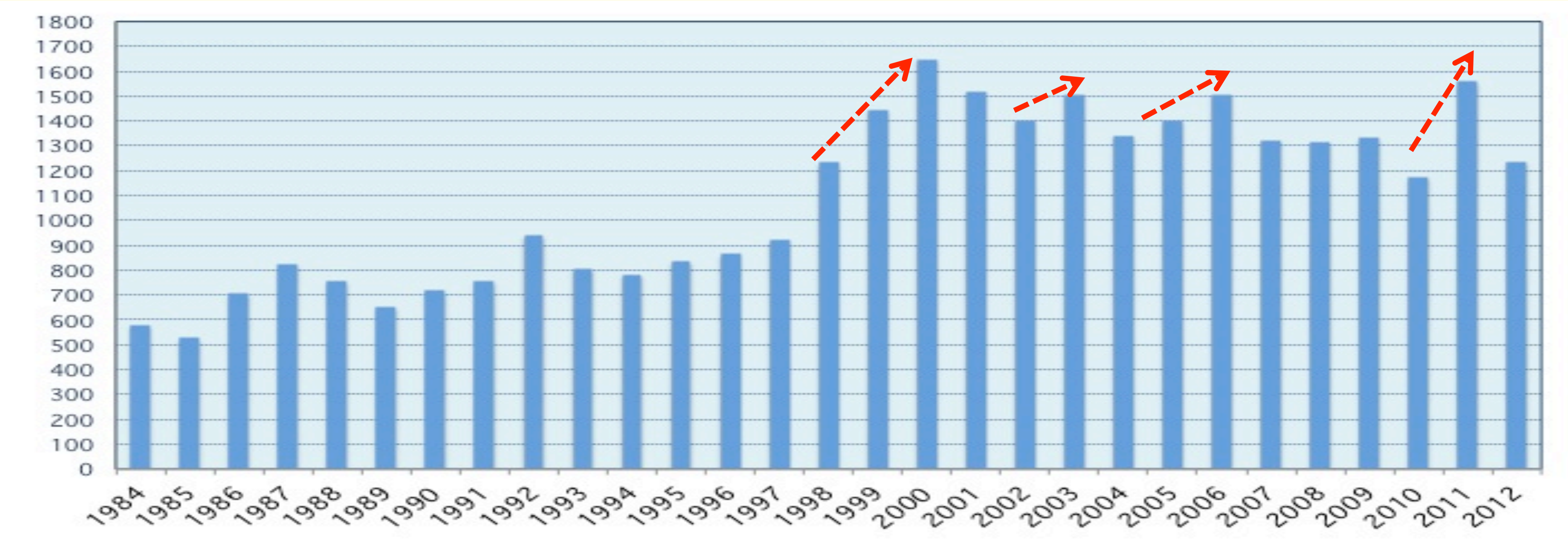
Several peaks of interest:

Atmospheric ν oscillations, Limit from CHOOZ

Solar and react. ν oscillations, Nobel 2002 to Davis & Koshiba

Accelerator ν oscillations, Cosmo limits on abs. masses

1-3 ν oscill. at 2nd gen. react. + neutrino anomalies



- **The fundamental ν parameters:** $(\Delta m^2, \theta_{23})$ $(\delta m^2, \theta_{12})$ **[Osc.patterns]** (θ_{13})
- **Basis of 3ν mixing framework essentially established in 1998-2012**

2. Notation for neutrino masses

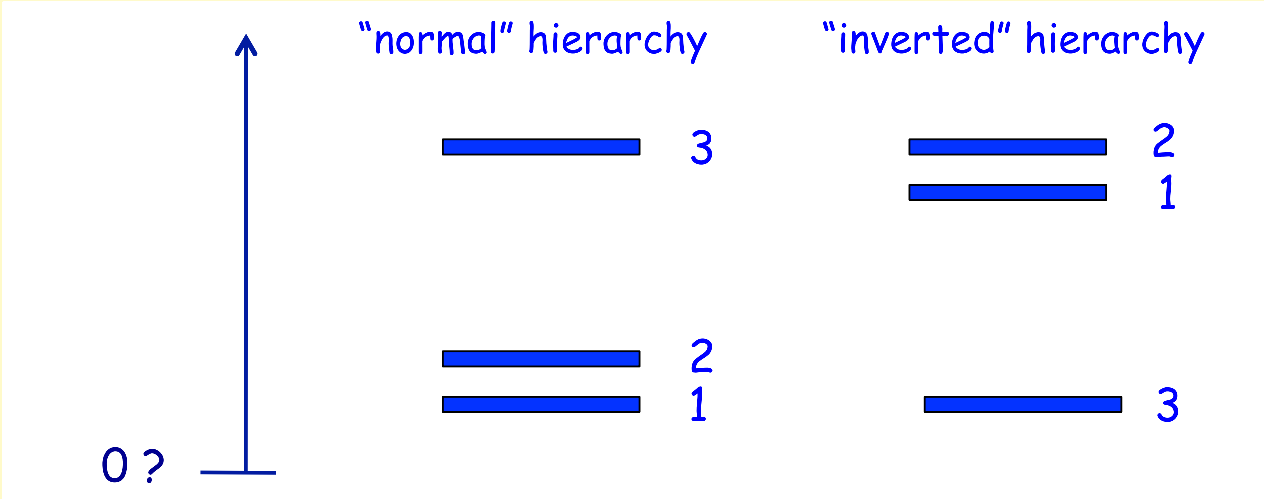
- Three mass eigenstates $\nu_1\,\nu_2\,\nu_3$ with masses $m_1\,m_2\,m_3$
- For ultrarelativistic ν in vacuum: $E = \sqrt{m_i^2 + p^2} \cong p + \dfrac{m_i^2}{2p}$
- Neutrino oscillations probe $\Delta E \approx \Delta m_{ij}^2$
- 3 neutrinos $\rightarrow$ 2 independent mass differences, say, δm^2 and Δm^2
- Experimentally very different values: $\delta m^2/\Delta m^2 \sim 1/30$

$\boldsymbol{\delta m^2 = 7.5 \times 10^{-5}\ eV^2}$ small or "**solar**" splitting

$\boldsymbol{\Delta m^2 = 2.5 \times 10^{-3}\ eV^2}$ large or "**atmospheric**" splitting

- Very difficult to probe both splittings in the same experiment!
- Absolute ν mass scale unknown: lightest $\boldsymbol{m_i}$ could be zero
- However, upper limits exist: $m_i \lesssim \mathcal{O}(\mathrm{eV})$

- Two possible arrangements, called "**hierarchies**", for the splittings



- In both hierarchies, there is "doublet" of close mass states and a "lone" mass state. Universal convention: ν_3 is the lone state, (ν_1, ν_2) is the doublet, with ν_1 being the lightest state: $m_1 < m_2$.

- Splittings: $\delta m^2 = m_2^2 - m_1^2 > 0$ (> 0 by definition)

  $\Delta m^2 = m_3^2 - m_{1,2}^2 > \text{or} < 0$ (± an important physical sign)

- We use

  $\Delta m^2 = \frac{1}{2}\left(m_{3,1}^2 - m_{3,2}^2\right)$ (our convention)

3. Notation for neutrino mixing

- Three flavor states $\nu_e\, \nu_\mu\, \nu_\tau$ coming from mixing of the mass eigenstates $\nu_1\, \nu_2\, \nu_3$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \qquad \text{i.e.} \qquad \nu_\alpha = U_{\alpha i} \nu_i$$

- If these are the only ν states in nature, then the matrix U is unitary

$$UU^\dagger = I$$

- For antineutrinos $U \to U^*$

- As for quarks, the unitary mixing matrix U can be expressed in terms of four independent physical parameters:

3 mixing angles + 1 ~~CP~~ phase

- The Particle Data Group notation is universally adopted:

$$U = O_{23}\,\Gamma_\delta\,O_{13}\,\Gamma_\delta^\dagger\,O_{12} =$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

where

$$\Gamma_\delta = \begin{pmatrix} 1 & & \\ & 1 & \\ & & e^{i\delta} \end{pmatrix} \qquad \text{and} \qquad \begin{cases} c_{ij} = \cos\theta_{ij} \\ s_{ij} = \sin\theta_{ij} \end{cases}$$

- The matrix U is often called "Pontecorvo-Maki-Nakagawa-Sakata" (PMNS) matrix.

- Experimentally we know that

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\Downarrow$	$\Downarrow$	$\Downarrow$
$\sin^2\theta_{23} \sim 0.5$	$\sin^2\theta_{13} \sim 0.02$	$\sin^2\theta_{12} \sim 0.3$
~ maximal	small	large
$(\theta_{23} \sim \pi/4)$	$(\delta = ?)$	

- The presence of two small parameters, $\sin^2\theta_{13} \sim 0.02$ and $\delta m^2/\Delta m^2 \sim 1/30$, makes 3ν mixing approximatively reducible to an "effective 2ν mixing" in several cases of phenomenological interest.

- Goal of many currents and future experiments is to find evidence of "genuine 3ν effects" beyond the 2ν approximation.

4. Neutrino flavor evolution

Since $m_i \ll E$ in almost all cases of phenomenological interest, then

- We can often set $\beta = v/c \approx 1$.

- Chirality flips (LH $\leftrightarrow$ RH) of $\mathcal{O}(m_i/E)$ can be ignored, i.e. the spinorial properties are not relevant in flavor evolution.

- One can then adopt a simple description in terms of "scalar" states $|\nu\rangle$ governed by a Hamiltonian $\mathcal{H}$

$$i \frac{d}{dx} |\nu\rangle = \mathcal{H} |\nu\rangle$$

with formal solution

$$|\nu(x)\rangle = \mathcal{S}(x,0)\, |\nu(0)\rangle$$

where $\mathcal{S}(x,0)$ is the evolution operator from 0 to x.

Let us start from the evolution in vacuum

- For a ν beam of momentum p traveling in vacuum, in the mass eigenstates basis the $\mathcal{H}$ matrix reads:

$$\mathcal{H}_{\text{mass}} = \begin{pmatrix} E_1 & & \\ & E_2 & \\ & & E_3 \end{pmatrix} \simeq p \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} + \frac{1}{2E} \begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix}$$

diagonal

- However, in the flavor basis:

$$\mathcal{H}_{\text{flavor}} = U\, \mathcal{H}_{\text{mass}}\, U^\dagger$$

non diagonal: flavor not conserved

- We shall work out several consequences of this simple Hamiltonian, and then add corrections for propagation in matter.

  Main output: flavor oscillation probabilities

$$P(\nu_\alpha \to \nu_\beta) = |S_{\beta\alpha}|^2$$

$\alpha = \beta$: "survival" (or "disappearance") probability

$\alpha \neq \beta$: "transition" (or "appearance") probability

Exercise # 1: 3ν oscillation in vacuum

It can be proved that the general form of the "transition" probability is

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i<j} \mathrm{Re}\, J^{ij}_{\alpha\beta} \sin^2\left(\frac{\Delta m^2_{ij}\, x}{4E}\right) - 2\sum_{i<j} \mathrm{Im}\, J^{ij}_{\alpha\beta} \sin\left(\frac{\Delta m^2_{ij}\, x}{2E}\right)$$

where

$$\Delta m^2_{ij} = m^2_i - m^2_j \qquad J^{ij}_{\alpha\beta} = U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j} \qquad i, j = 1, 2, 3$$

Numerically

$$\frac{\Delta m^2_{ij}\, x}{4E} = 1.267 \left(\frac{\Delta m^2_{ij}}{\mathrm{eV}^2}\right)\left(\frac{x}{\mathrm{m}}\right)\left(\frac{\mathrm{MeV}}{E}\right)$$

Exercise # 2: $3\nu \to 2\nu$ reduction for SBL reactor experiments

Short baseline reactor experiments look for $\bar{\nu}_e$ oscillations at $\begin{cases} x = L \sim \mathcal{O}(1\ \text{km}) \\ E \sim \text{few MeV} \end{cases}$

At these energies, CC reactions in the final state can produce e^+ but not μ^+ or τ^+.

Therefore, only "disappearance" $P(\bar{\nu}_e \to \bar{\nu}_e)$ is observable

but not "appearance" $\begin{cases} P(\bar{\nu}_e \to \bar{\nu}_\mu) \\ P(\bar{\nu}_e \to \bar{\nu}_\tau) \end{cases}$

Moreover, it is $\delta m^2 L/4E \ll 1$, while $\Delta m^2 L/4E \sim \mathcal{O}(1)$.

➡ It can be proved that, in the limit $\delta m^2 \sim 0$, effective 2ν oscillations occur:

$$P(\bar{\nu}_e \to \bar{\nu}_e) \simeq 1 - \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

oscillation factor (distance)

oscillation amplitude (mixing)

dependent only on θ_{13}.

We can get an intuitive understanding of the dependence on θ_{13} only.

Indeed, two of the three mixing rotations have ~ no effect

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 2\;3 \end{pmatrix}\begin{pmatrix} 1\;3 \end{pmatrix}\begin{pmatrix} 1\;2 \end{pmatrix}\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

mixes unobservable flavors (μ and τ)

mixes ~ degenerate states (ν_1 and ν_2)

It follows

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \cancel{\begin{pmatrix} 2\;3 \end{pmatrix}}\begin{pmatrix} 1\;3 \end{pmatrix}\cancel{\begin{pmatrix} 1\;2 \end{pmatrix}}\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

then only θ_{13} contributes to the mixing.

Note that in this approximation:

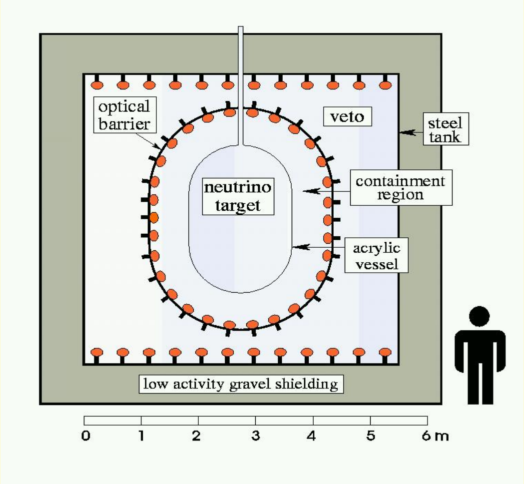
- δ is unobservable
- sign($\pm\Delta m^2$) is unobservable
- $P(\bar{\nu}_e \to \bar{\nu}_e) = P(\nu_e \to \nu_e)$

2. Oscillation searches sensitive to Δm^2

1. Oscillations searches at short baseline (SBL) reactors

The short-baseline reactor experiment CHOOZ (1998)

~ 1 km →



Probably (one of) the most cited **negative** results !

First data: Phys. Lett. B 466, 415 (1999) > 1550 citations

Final data: Eur. Phys. J. C 27, 331 (2003) > 950 citations

Production

Reactors: intense sources of $\bar{\nu}_e$ (~ $6\text{x}10^{20}$/s/reactor)

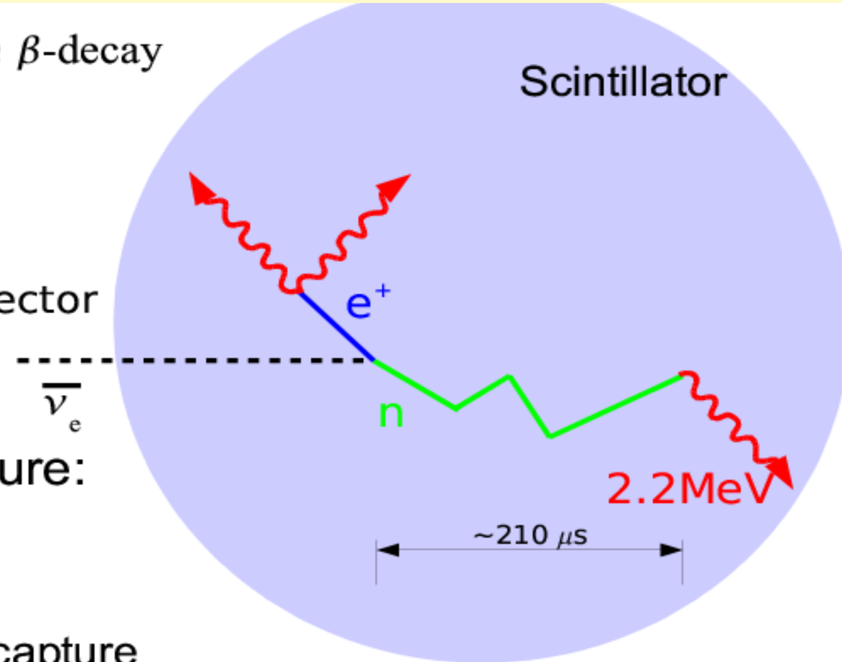
Typical available neutrino energy E ~ few MeV

Detection

Reaction Process: inverse β-decay

$$\overline{\nu}_e + p \rightarrow e^+ + n$$
$$\downarrow$$
$$n + p \rightarrow d + \gamma$$

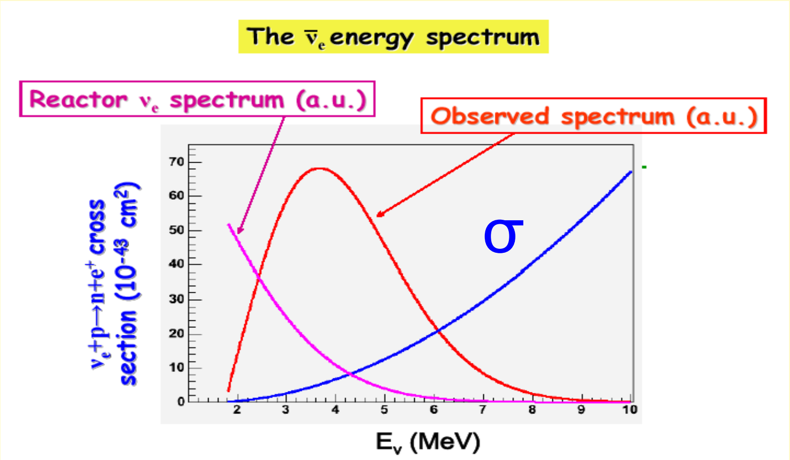
Scintillator is target and detector



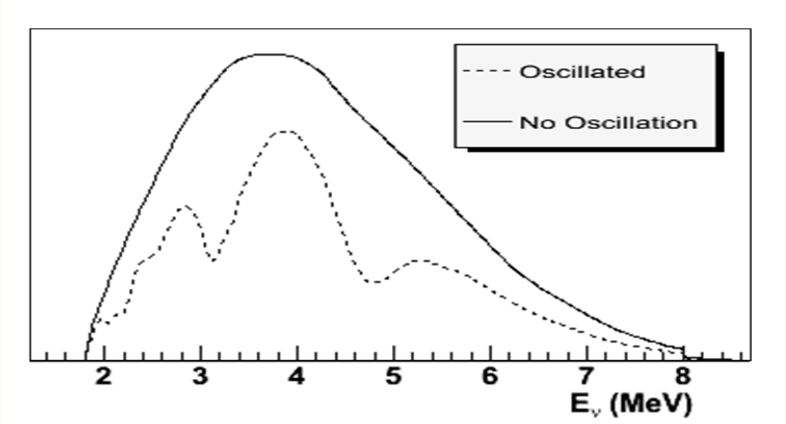
- Distinct two-step signature:
  - prompt event: positron
    $E_\nu \approx E_{e^+} + 0.8\ MeV$
  - delayed event: neutron capture after ~210μs
    - 2.2 MeV gamma

Results

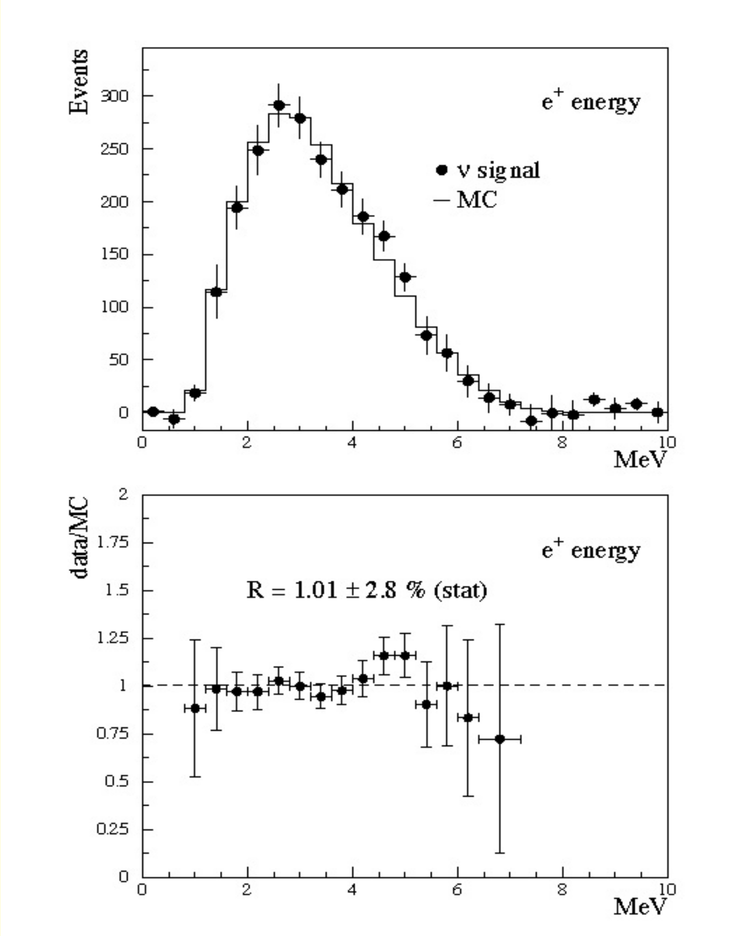
Expected spectrum (no oscill.):



Distorted with oscillations (qualitative):



CHOOZ: no oscillations
within few % error



Interpretation

We have seen:

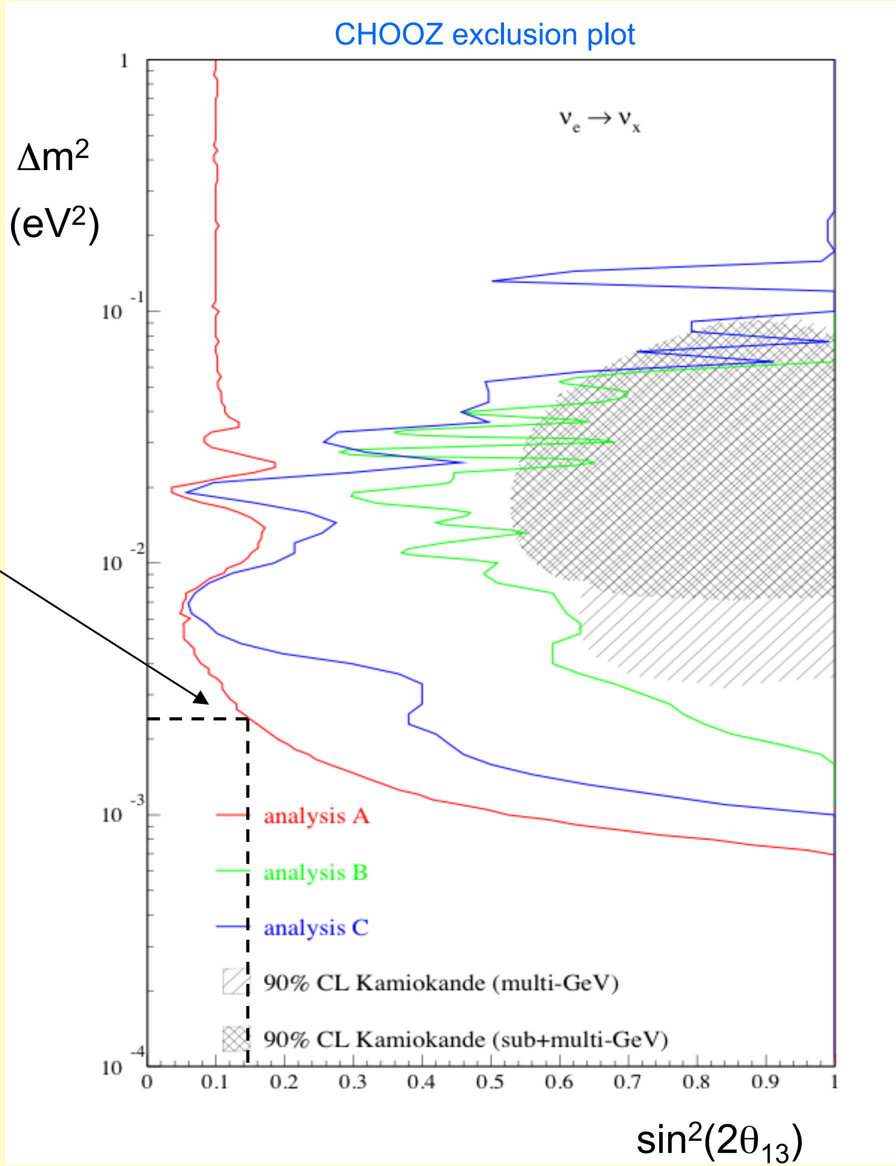
$P_{ee} = 1 - \sin^2(2\theta_{13}) \sin^2(\Delta m^2 L/4E_\nu)$

For any value of Δm^2 in the range allowed by atmospheric data (see next), we get stringent upper bound on θ_{13}

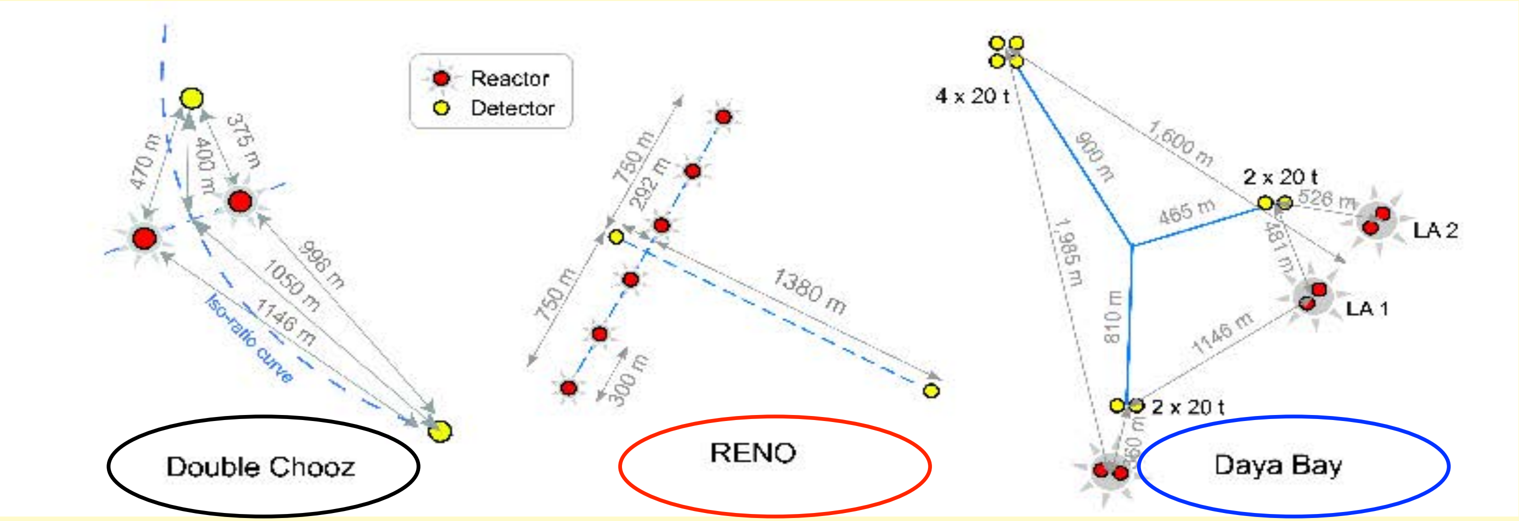
$\sin^2\theta_{13}$ < few %
(depending on Δm^2)

... Nobody could know at that time, but θ_{13} was just behind the corner (less than a factor of two in sensitivity!)

In any case, it was clear that, to reach higher θ_{13} sensitivity, it needs to use a second (close) detector to reduce systematics through **far/near** comparison



But new reactor experiments have been projected and are working at present with near & far detectors (ND & FD)

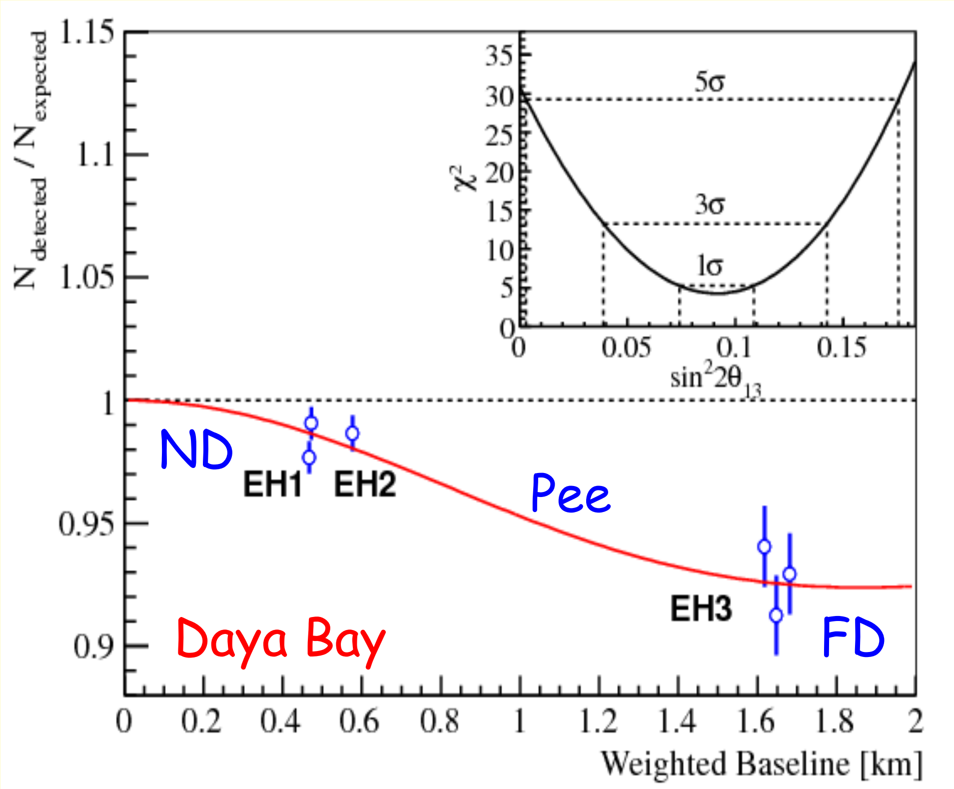


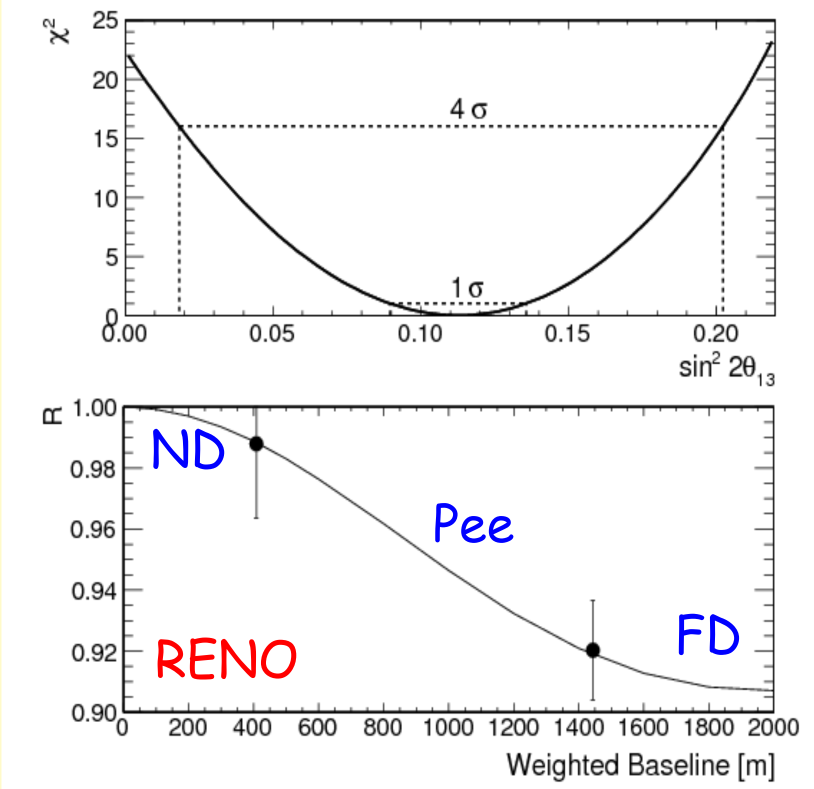
Running with FD;
ND in construction

Running with
ND & FD

Running with
ND & FD

2012: discovery of $\theta_{13}>0$! (value obtained at ~ fixed Δm^2)





Results: disappearance at FD with respect to ~ unoscillated signal at ND.

Double Chooz results (FD only) also consistent with Daya Bay & RENO.

Further data and spectral analyses expected in the near future.

The 2012 Reactor results are already included in the PDG Review

$\sin^2(2\theta_{13})$

At present time direct measurements of $\sin^2(2\theta_{13})$ are derived from the reactor $\overline{\nu}_e$ disappearance at distances corresponding to the Δm^2_{32} value, i.e. L $\sim$ 1km. Alternatively, limits can also be obtained from the analysis of the solar neutrino data and accelerator-based $\nu_\mu \to \nu_e$ experiments.

VALUE	*CL%*	*DOCUMENT ID*		*TECN*	*COMMENT*	
0.098±0.013 OUR AVERAGE						
0.086±0.041±0.030		[1] ABE	12	DCHZ	Chooz reactors	← Double CHOOZ
0.113±0.013±0.019		[2] AHN	12	RENO	Yonggwang reactors	← RENO
0.092±0.016±0.005		[3] AN	12	DAYA	Daya Bay, Ling Ao, Ling Ao-II reactors	← Daya Bay

Interestingly: value of θ_{13} was previously hinted.
Weaker signals were also coming from (see later):

• • • We do not use the following data for averages, fits, limits, etc. • • •

0.05 - 0.21	68	[6] ABE	11A	T2K	Normal mass hierarchy	← T2K
0.06 - 0.25	68	[7] ABE	11A	T2K	Inverted mass hierarchy	
0.01 - 0.09	68	[8] ADAMSON	11D	MINS	Normal mass hierarchy	← MINOS
0.03 - 0.15	68	[9] ADAMSON	11D	MINS	Inverted mass hierarchy	
0.08 ±0.03	68	[10] FOGLI	11	FIT	Global neutrino data	← Global fit (from 2008)

But now, let us proceed with other expt's mainly sensitive to Δm^2 →

Exercise # 3: One-dominant-mass-scale approximation (vacuum)

It can be proved that, in experiments mainly sensitive to Δm^2, i.e. with

$$\frac{\Delta m^2 x}{4E} \sim \mathcal{O}(1) \qquad \text{and} \qquad \frac{\delta m^2 x}{4E} << 1$$

the oscillation probabilities depend only on $|\Delta m^2|$ and on the mixing with ν_3 (elements $|U_{\alpha 3}|$, governed by θ_{23} and θ_{13}):

$$P_{\alpha\alpha} = P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha) \simeq 1 - 4\,|U_{\alpha 3}|^2(1 - |U_{\alpha 3}|^2)\sin^2\left(\frac{\Delta m^2 x}{4E}\right)$$

$$P_{\alpha\beta} = P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta) \simeq 4\,|U_{\alpha 3}|^2|U_{\beta 3}|^2\sin^2\left(\frac{\Delta m^2 x}{4E}\right) \qquad \alpha \neq \beta$$

where $|U_{e3}|^2 = s^2_{13}$, $|U_{\mu 3}|^2 = c^2_{13}s^2_{23}$, $|U_{\tau 3}|^2 = c^2_{13}c^2_{23}$.

Typically

- no sensitivity to $(\delta m^2, \theta_{12})$ of course, but also:
- no sensitivity to hierarchy or CP violating phase δ
- no difference $\nu/\bar{\nu}$.

Phenomenological note

The one-dominant-mass-scale approximation can be applied in several cases:

- atmospheric neutrino expts. (ATM) SuperKamiokande, …
- long-baseline accelerator expts. (LBL) K2K, MINOS, T2K, OPERA, …
- short-baseline reactor expts. (SBR) CHOOZ, D. CHOOZ, Daya Bay, RENO, …

OPERA (LBL) : $$P(\nu_\mu \to \nu_\tau) \simeq \cos^4\theta_{13} \sin^2 2\theta_{23} \sin^2\left(\frac{\Delta m^2 x}{4E}\right) \quad (*)$$

ATM + LBL : $$P(\nu_\mu \to \nu_\mu) \simeq 1 - 4\, c_{13}^2\, s_{23}^2\, (1 - c_{13}^2\, s_{23}^2) \sin^2\left(\frac{\Delta m^2 x}{4E}\right) \quad (*)$$

ATM + LBL : $$P(\nu_\mu \to \nu_e) \simeq \sin^2\theta_{23} \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2 x}{4E}\right) \quad (**)$$

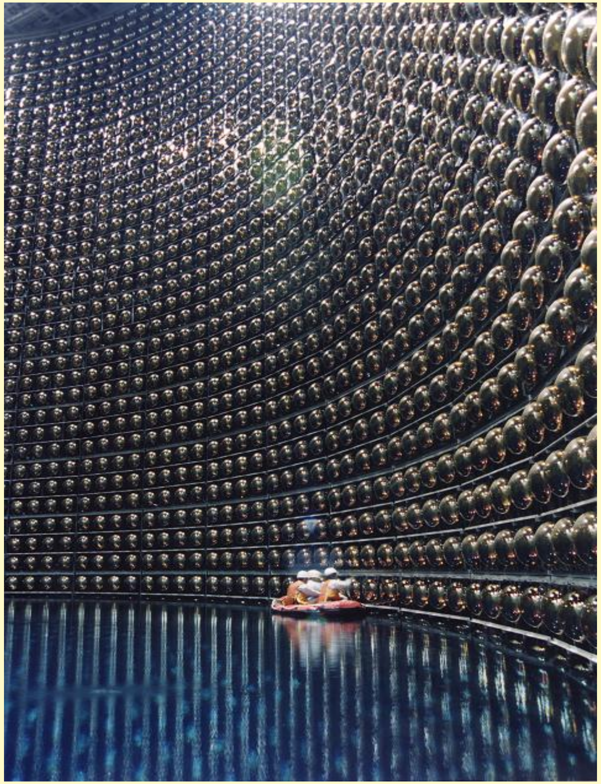
SBR : $$P(\nu_e \to \nu_e) \simeq 1 - \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2 x}{4E}\right) \quad (**)$$

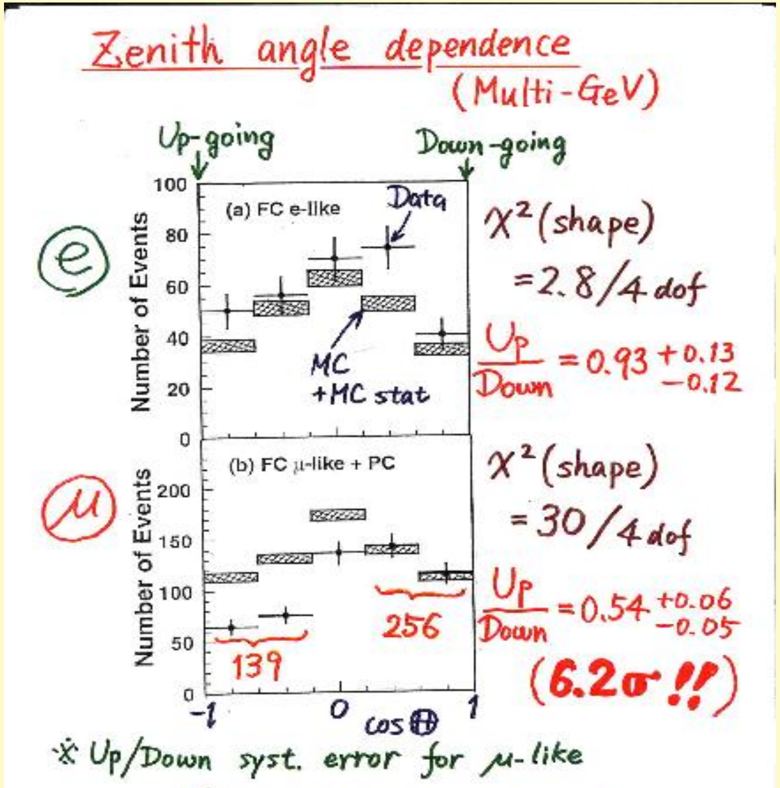
(*) reduces to the 2ν form for $\theta_{13} \to 0$ (pure $\nu_\mu \to \nu_\tau$ oscillations)

(**) vanishes for $\theta_{13} \to 0$

2. Oscillations searches with atmospheric neutrinos

The 1998 Super-Kamiokande breakthrough





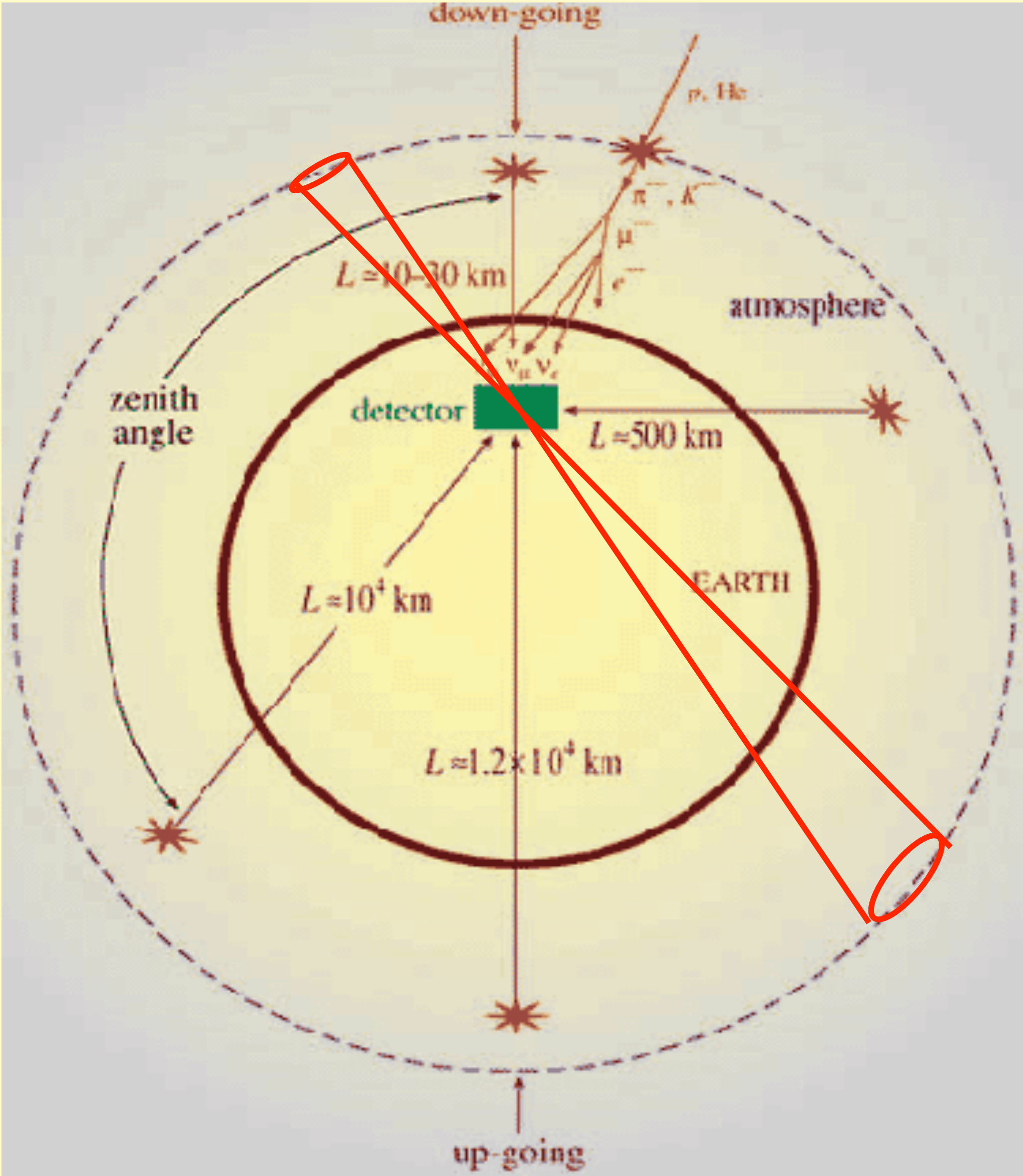
(T. Kajita at Neutrino'98, Takayama)

The atmospheric ν flux

Same ν flux from
opposite solid angles
(up-down symmetry)

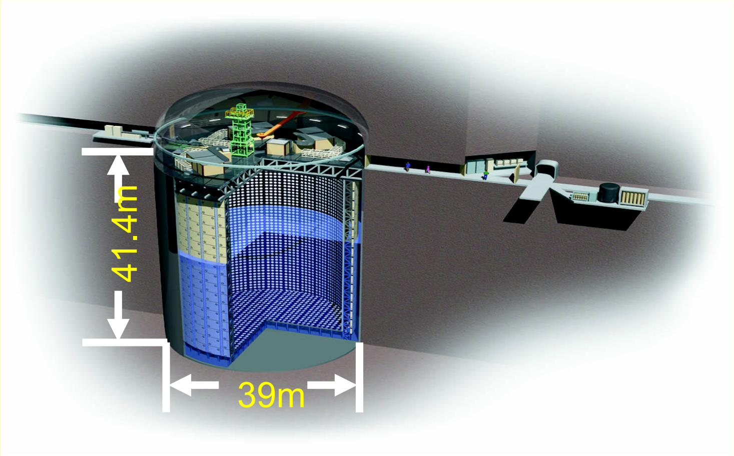
[Flux dilution (~ $1/r^2$) is compensated by a larger production surface (~ r^2)]

Should be reflected in symmetry of event zenith spectra, if energy & angle can be reconstructed well enough

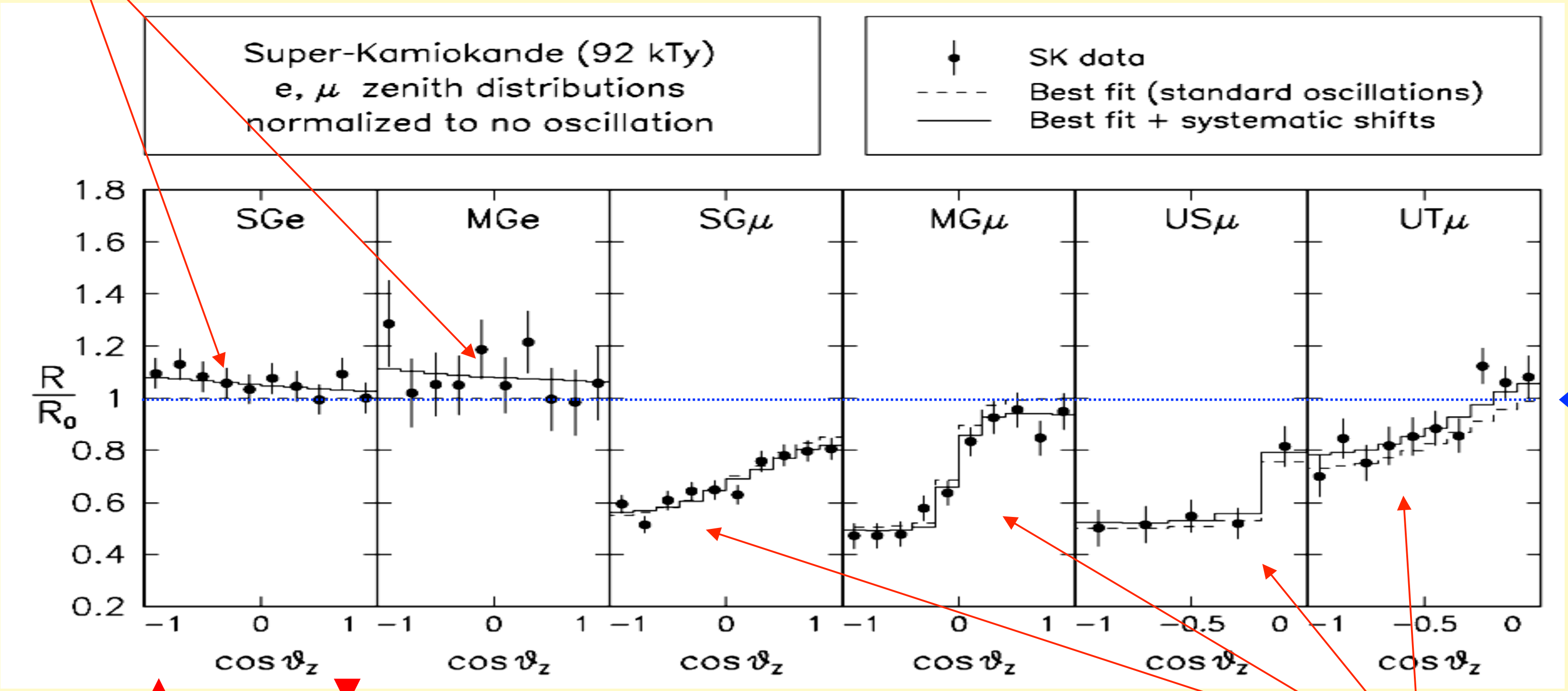


RESULTS SK zenith distributions

SGe	Sub-GeV electrons
MGe	Multi-GeV electrons
SGμ	Sub-GeV muons
MGμ	Multi-GeV muons
USμ	Upward Stopping muons
UTμ	Upward Through-going muons



electrons ~OK



no osc.

up down

muon deficit from below

Observations over several decades in L/E:

- ν_e induced events: ~ as expected
- ν_μ induced events: clear "disappearance" from below

Interpretation in terms of oscillations:

- Channel $\nu_\mu \rightarrow \nu_e$? No (or subdominant) ← CHOOZ OK!
- Channel $\nu_\mu \rightarrow \nu_\tau$? Yes (dominant)

One-mass-scale approximation (for $\theta_{13} = 0$):

$$P_{\mu\tau} = \sin^2(2\theta_{23}) \sin^2(\Delta m^2 L/4E_\nu)$$

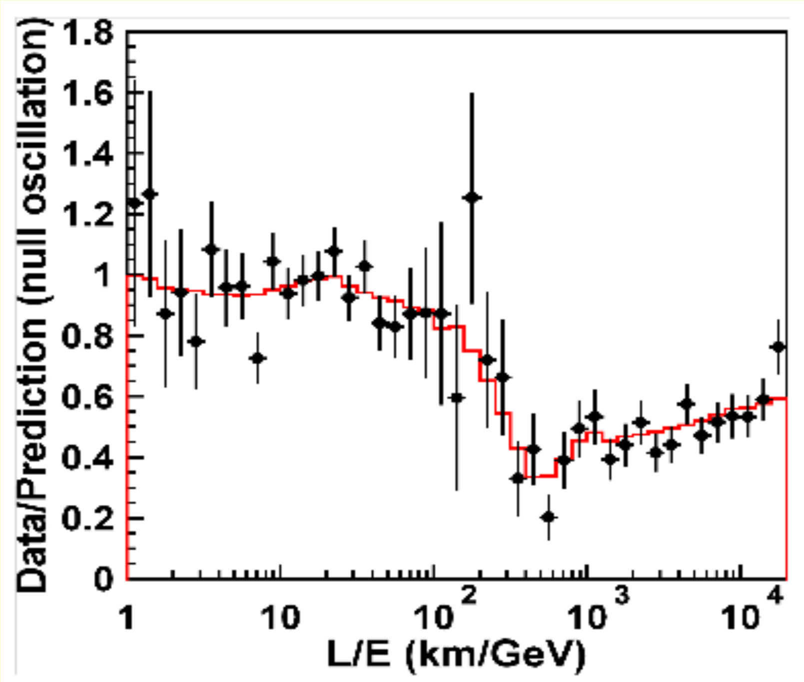
[In this channel, oscillations are ~ vacuum-like, despite the presence of Earth matter]

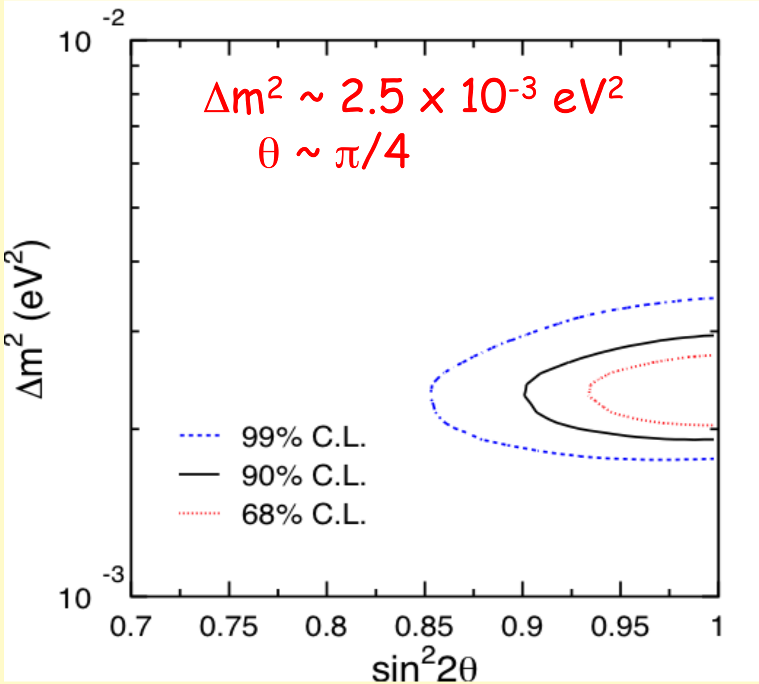
Results consistent with other atmospheric experiments using different techniques (MACRO, Soudan2) but with lower statistics.

Dedicated L/E analysis in SK "sees" half-period of oscillations

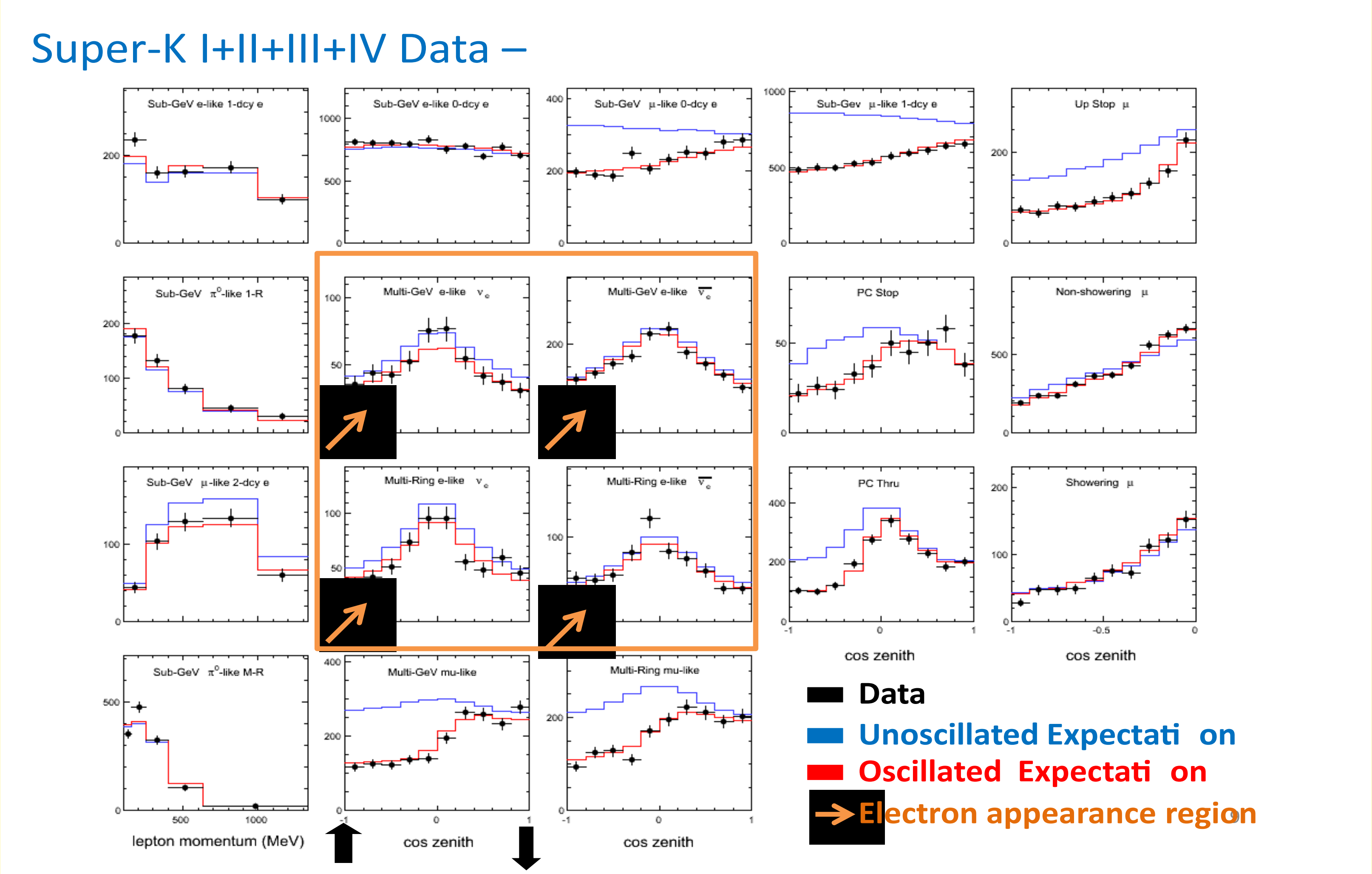
1st oscillation dip still visible despite large L & E smearing

Strong constraints on the parameters (Δm^2, θ_{23})





Latest SK data include hundreds of bins

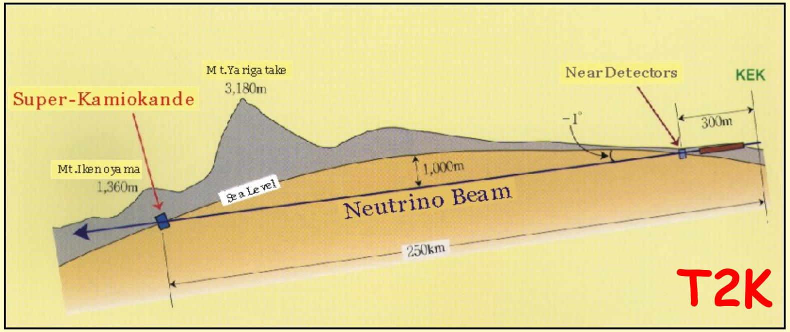


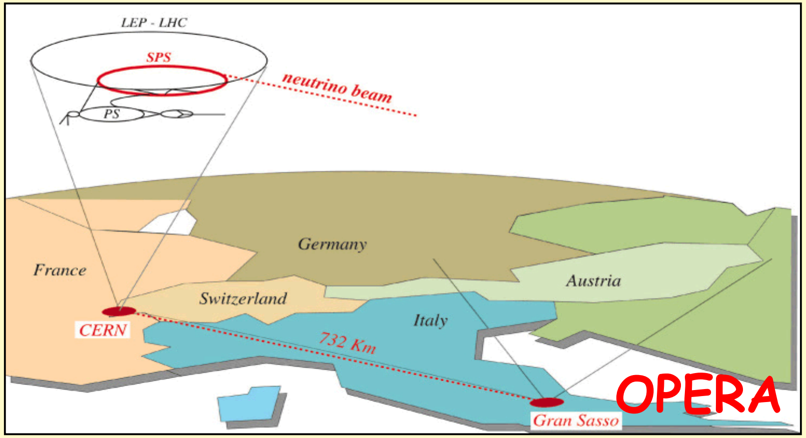
Now more attention to e-like events, to "squeeze" subleading effects

3. Oscillations searches at long baseline (LBL) accelerators

(K2K, MINOS, OPERA, T2K)

"Reproducing atmospheric ν_μ physics" in controlled conditions

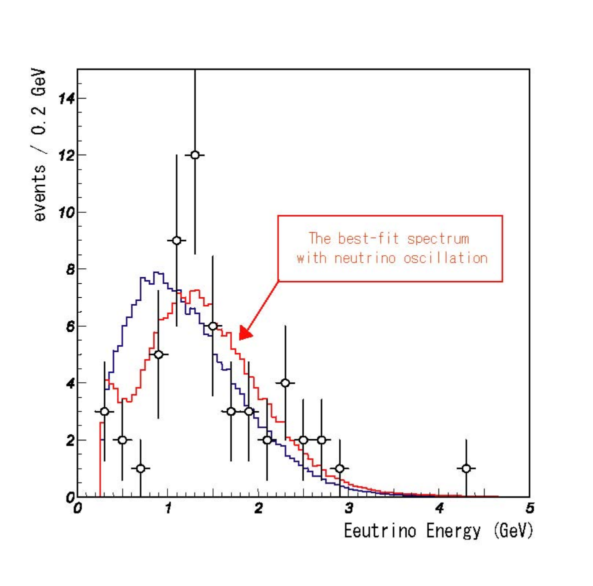




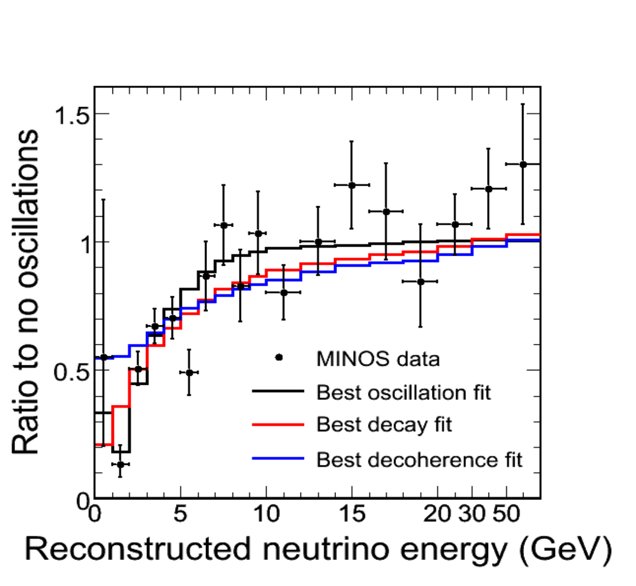
K2K, MINOS, T2K supplemented by near detectors to measure $P_{\mu\mu}$ (disappearance).

Results in muon neutrino disappearance mode, $P_{\mu\mu}$

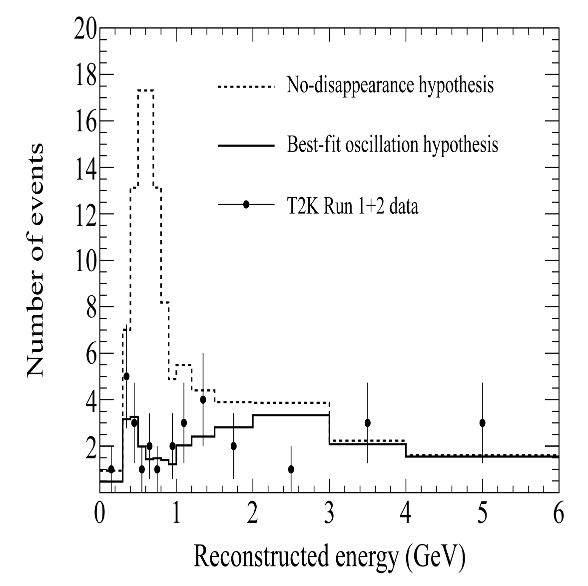
K2K



MINOS



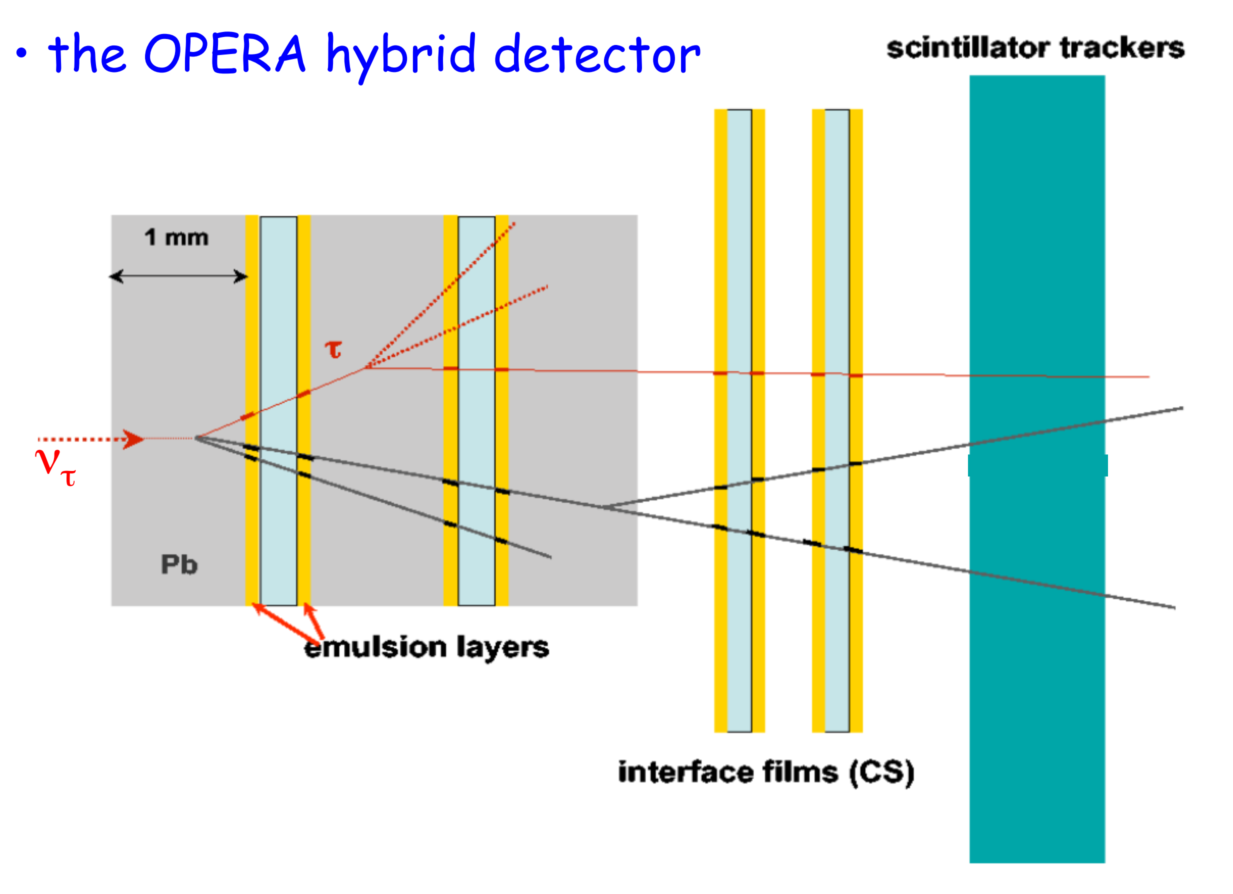
T2K



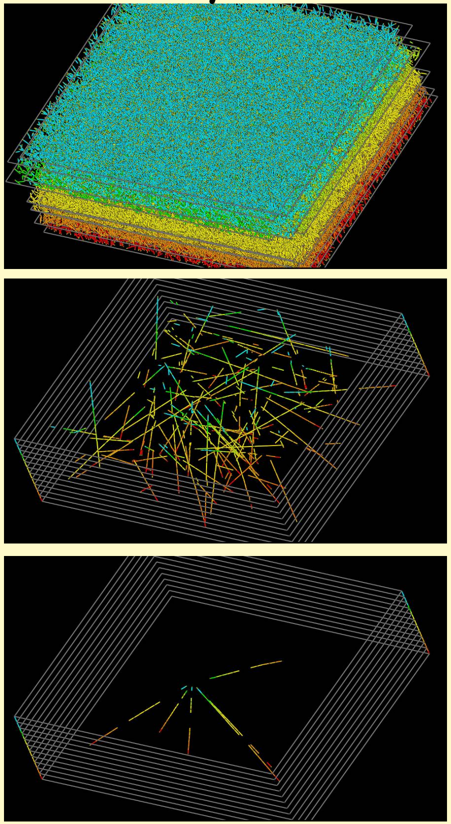
1st oscillation dip observed

[Exotic explanations without dip (decay, decoherence) excluded]

Testing dominant oscillations via τ appearance: **OPERA**



Finding needles
in a haystack…

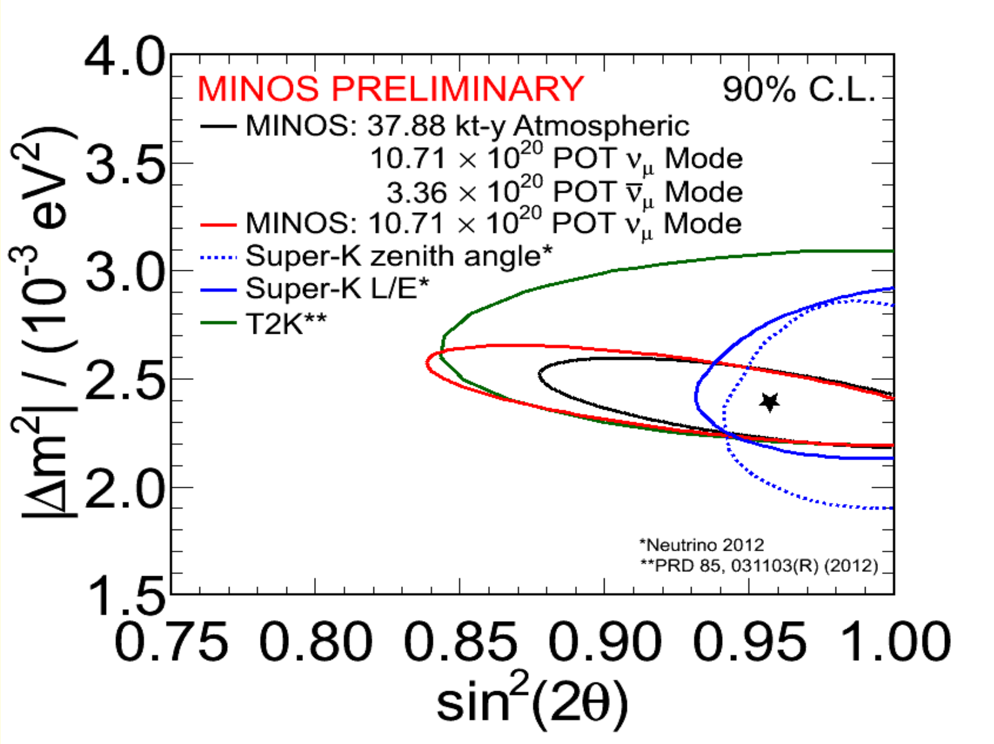


Three "τ needles" found ! (consistently with expected signal)

Interpretation

Once more … dominant $P_{\mu\tau} = \sin^2(2\theta_{23})\,\sin^2(\Delta m^2 L/4E_\nu)$

Oscillation parameters consistent among atm/LBL experiments…
… with recent, possible hints of non-maximal mixing



The format of such a “2ν” plot is, however, obsolete…

4. A note about the parameters Δm^2 and θ_{23}

They are mainly determined by ATM + LBL experiments via $P(\nu_\mu \to \nu_\mu)$ (disappearance).

- $P_{\mu\mu}$ is octant symmetric (i.e. invariant for $\theta_{23} \to \frac{\pi}{2} - \theta_{23}$) only in the limit

  $\delta m^2 \to 0$ <u>and</u> $\theta_{13} \to 0$ ⟹ $P_{\mu\mu} \cong 1 - \sin^2 2\theta_{23} \sin^2\left(\frac{\Delta m^2 x}{4E}\right)$

- For $\theta_{13} \neq 0$ it is no longer octant-symmetric:

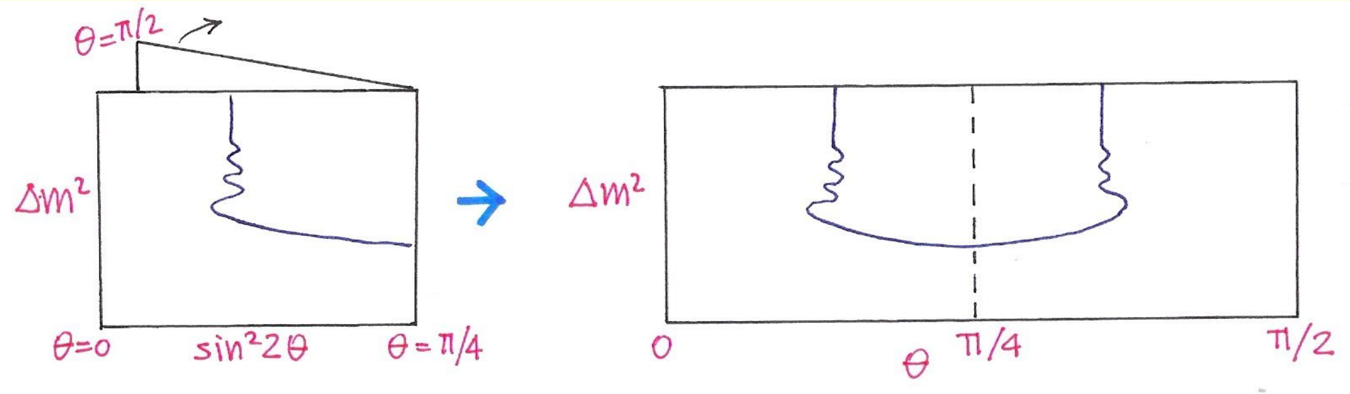
$$P_{\mu\mu} \cong 1 - 4\, c_{13}^2 s_{23}^2 \left(1 - c_{13}^2 s_{23}^2\right) \sin^2\left(\frac{\Delta m^2 x}{4E}\right)$$

- Further effects ($\delta m^2 \neq 0$, matter) also contribute to the asymmetry

So

⬇

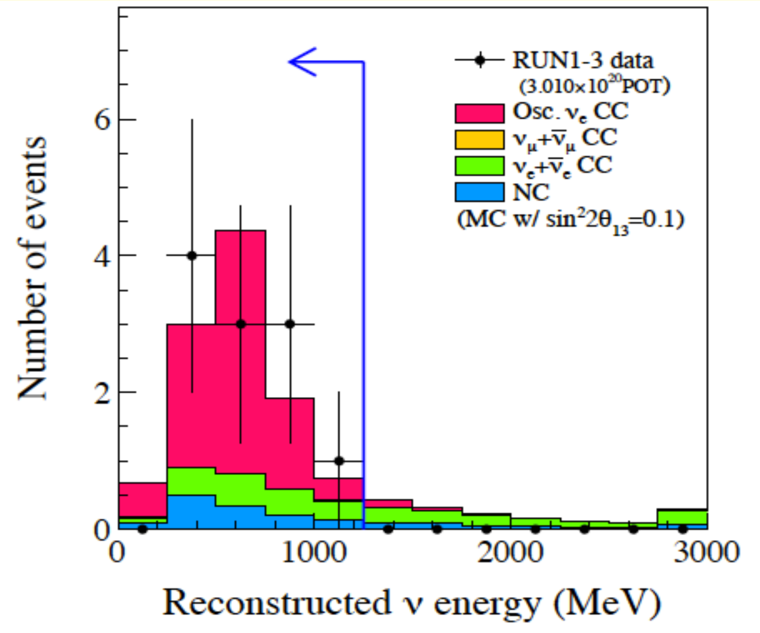
- Because of the asymmetry, it needs to unfold the 2nd octant, in order to see what is the octant to which θ_{23} belongs.



- Typical abscissa: either $\sin^2\theta$ (linear scale) or $\tan^2\theta$ (log scale)

This is even more important in the light of the

ν_e appearance in T2K (2012)

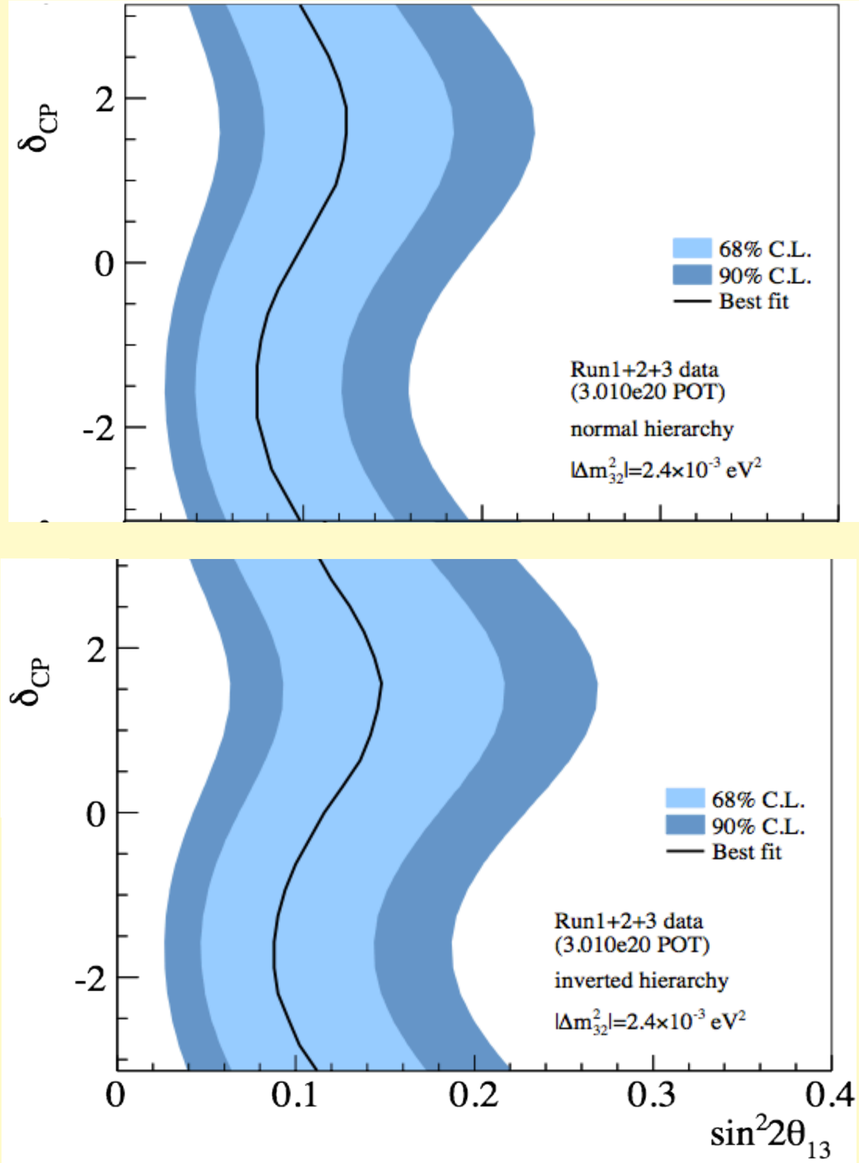


The appearance has been confirmed a few days ago (it will be important even in MINOS), and is consistent with the same θ_{13} measured at reactors (up to subleading oscillation terms).

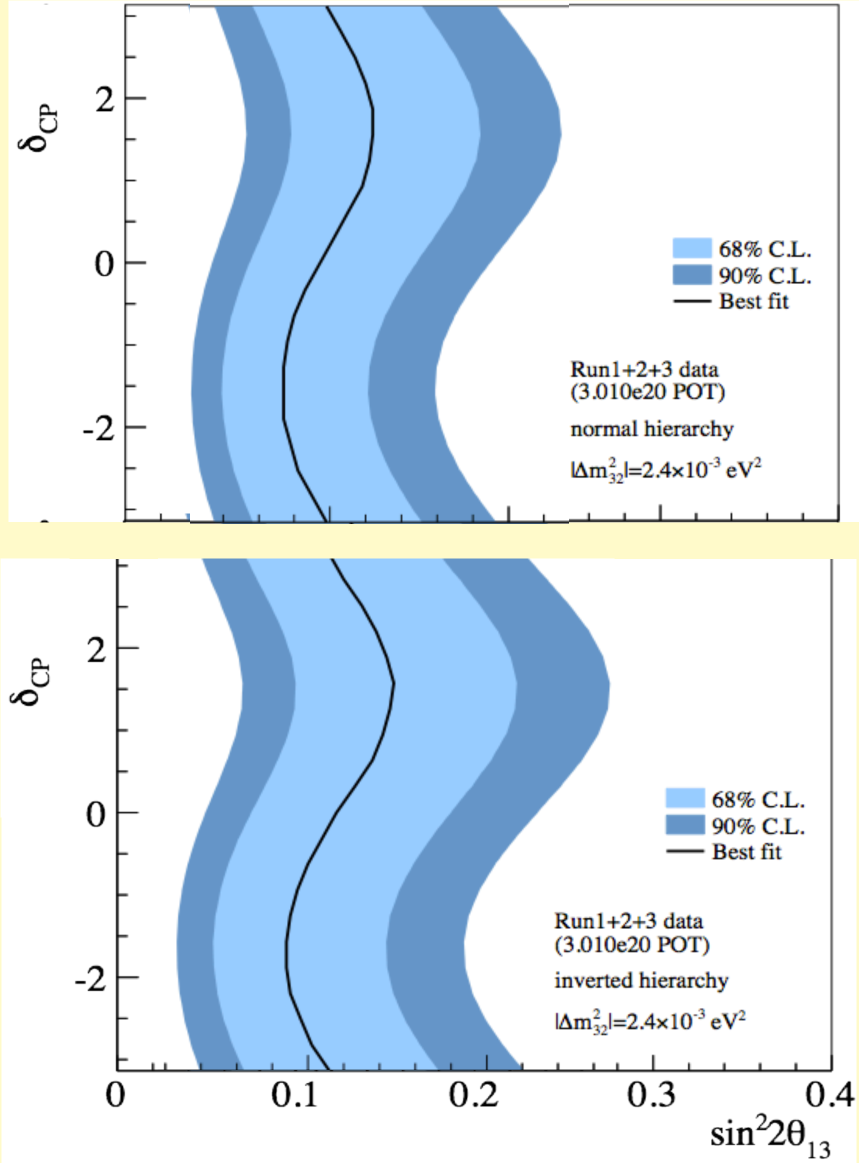
Indeed

LBL appearance: $P_{\mu e} = \sin^2\theta_{23}\sin^2(2\theta_{13})\sin^2(\Delta m^2 L/4E_\nu)$ + corrections

since it is NOT octant symmetric, anticorrelates θ_{23} and θ_{13}: the lower θ_{23}, the higher θ_{13}.



The corresponding LBL contours shown by T2K may be shifted to the left (right) for higher (lower) θ_{23}, due to the anti-correlation effect seen before …



The corresponding LBL contours shown by T2K may be shifted to the left (right) for higher (lower) θ_{23}, due to the anti-correlation effect seen before …

… this introduces obvious consequences for the comparison with θ_{23}-independent SBL reactor data

We will see the relevance of the point later, in the presentation of the global analysis.

4. A note on perspectives

- The previous experiments (LBL + ATM + SBR) allow set constraints on **$|\Delta m^2|$** and on the third-column elements of the mixing matrix (in absolute value)

$$|U| = \begin{pmatrix} \bullet & \bullet & |U_{e3}| \\ \bullet & \bullet & |U_{\mu 3}| \\ \bullet & \bullet & |U_{\tau 3}| \end{pmatrix} \quad \leftarrow \text{ functions of } \theta_{23}, \theta_{13}$$

- Next frontier: subleading effects related to sign(Δm^2), δ, θ_{12}, δm^2, matter ….

- E.g., in **atmospheric neutrinos**, all these effects are present and must be accounted for in state-of-the-art analyses.

- Unfortunately, it is difficult to observe (and then disentangle) them within the current uncertainties.

3. Oscillation searches sensitive to δm^2

Exercise # 4: experiments sensitive to δm^2 in the limit $\Delta m^2 \to \infty$

Previously we have considered expts. with sensitivity to Δm^2 in the limit $\delta m^2 \to 0$. Conversely, there are expts. with leading sensitivity to **δm^2**, for which one can take the limit **$\Delta m^2 \to \infty$**:

$$\frac{\delta m^2 x}{4E} \sim \mathcal{O}(1) \qquad\qquad \frac{\Delta m^2 x}{4E} >> 1$$

This is the case, for instance, of long-baseline reactor experiments (KamLAND) with large x and relatively low E. At low E (E ~ few MeV), the main observable is the **disappearance probability P_{ee}**. It can be proved that

$$P_{ee} \simeq \cos^4\theta_{13}\left[1 - \sin^2 2\theta_{12} \sin^2\left(\frac{\delta m^2 x}{4E}\right)\right] + \sin^4\theta_{13}$$

namely, the 3ν probability (for $\theta_{13} \neq 0$) is related to the 2ν probability (at $\theta_{13} = 0$) by the relation:

$$P_{ee}^{3\nu} = \cos^4\theta_{13}\, P_{ee}^{2\nu} + \sin^4\theta_{13}$$

independently of hierarchy, $\nu/\bar{\nu}$, $\cancel{CP}$.

It is important to note that the previous relation for $P^{3\nu}_{ee}$ in its general form

$$P^{3\nu}_{ee} = c^4_{13}\, P^{2\nu}_{ee}(\delta m^2, \theta_{12}) + s^4_{13}$$

holds not only for KamLAND, but also for solar neutrinos, where, however, $P^{2\nu}_{ee}$ takes a very different form due to matter effects in the Sun.

Therefore, via $P^{3\nu}_{ee}$, solar + KamLAND experiments allow to set constraints on δm^2 and on the 1st row elements of the mixing matrix (in absolute value)

$$|U| = \begin{pmatrix} |U_{e1}| & |U_{e2}| & |U_{e3}| \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{pmatrix} \qquad \leftarrow \text{ functions of } \theta_{12},\, \theta_{13}$$

Summary of leading sensitivity:

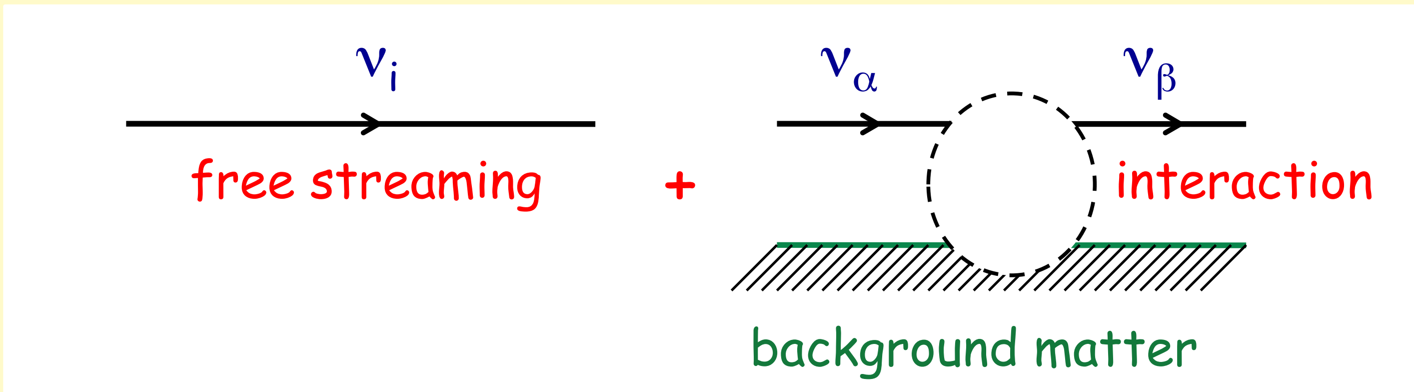
SBL reactors	$\rightarrow$		θ_{13}	$\lvert\Delta m^2\rvert$
ATM + LBL accel.	$\rightarrow$	θ_{23}	θ_{13}	$\lvert\Delta m^2\rvert$
Solar + KamLAND	$\rightarrow$	θ_{12}	θ_{13}	δm^2

1. Hamiltonian for ν oscillations in matter (the MSW effect)

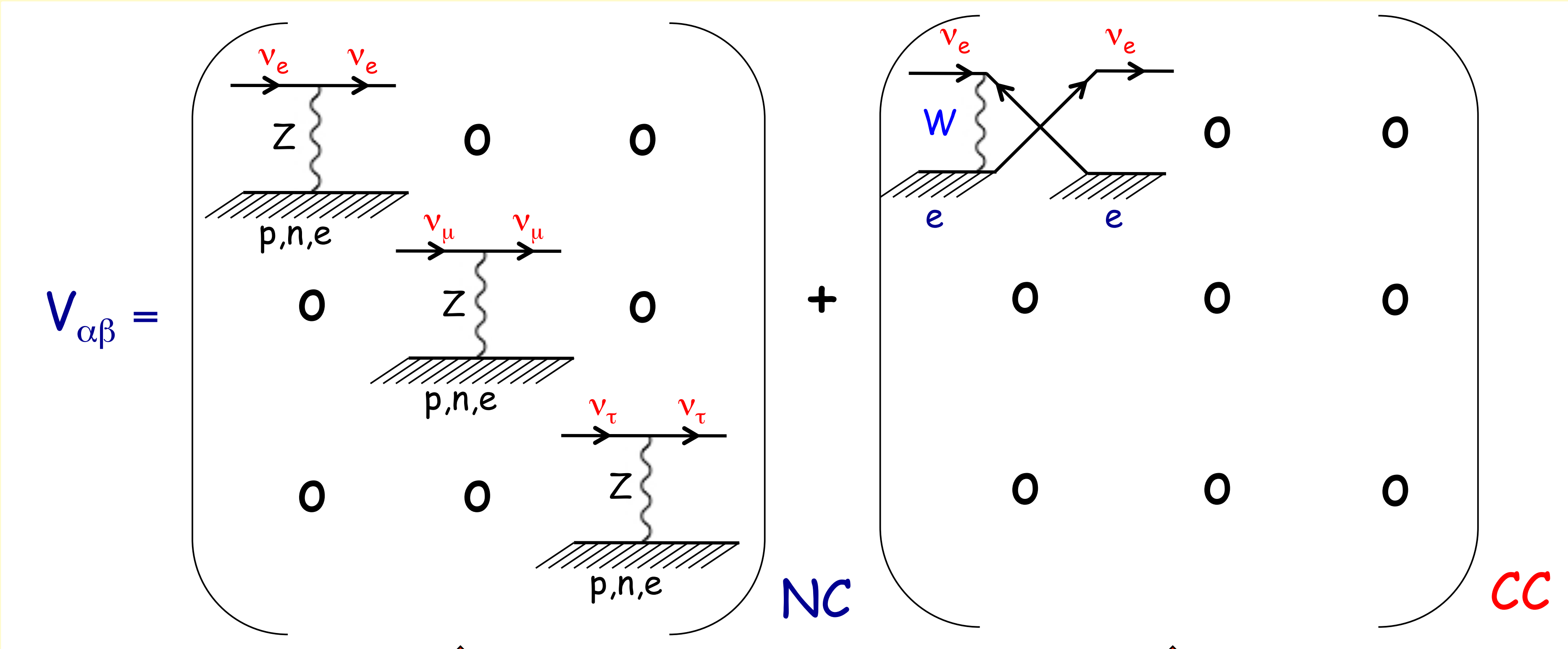
It was first realized by Wolfenstein, and later elaborated by Mikheyev and Smirnov, that neutrinos traveling in matter receive a contribution to coherent forward scattering, in the form of a tiny interaction energy $V_{\alpha\beta}$:

The Hamiltonian in the flavor basis reads

$$H_{flavor} = \underbrace{U \begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix} U^\dagger}_{\text{vacuum (kinematics)}} + \underbrace{\begin{pmatrix} V_{ee} & V_{e\mu} & V_{e\tau} \\ V_{\mu e} & V_{\mu\mu} & V_{\mu\tau} \\ V_{\tau e} & V_{\tau\mu} & V_{\tau\tau} \end{pmatrix}}_{\text{matter (dynamics)}}$$



Within the Standard Model and within ordinary matter



proportional to unity
and then unobservable

observable in
ν_e oscillations

So the relevant term is the *CC* interaction $\nu_e e^- \rightarrow \nu_e e^-$. No analogous for μ and τ, absent in the ordinary matter.

- It turns out that the V_{CC} interaction energy is

$$V = \sqrt{2}\, G_F N_e$$

where G_F is the Fermi constant, N_e the electron number density and $V \to -V$ for $\nu \to \bar{\nu}$.

- Then, the Hamiltonian of ν propagation in matter reads:

$$H_{flavor} = \frac{1}{2E} U \begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix} U^\dagger + \frac{1}{2E} \begin{pmatrix} A & & \\ & 0 & \\ & & 0 \end{pmatrix}$$

with $A = 2\sqrt{2}\, G_F N_e E$

- The relative size of matter/vacuum terms is given by $A/\Delta m_{ij}^2$. Roughly speaking, one may expect sizable effects for $A/\Delta m_{ij}^2 \sim \mathcal{O}(1)$.

- The dependence $A = A(x)$ makes the evolution non-trivial in many cases.

Exercise # 5: 2ν oscillation in matter at constant density

It can be proved that in the 2ν limit ($\theta_{13} = 0$), the ν_e survival probability reads:

$$P^{2n}_{ee}(\text{matter}) = 1 - \sin^2 2\tilde{\theta}_{12} \sin^2\left(\frac{\delta\tilde{m}^2 x}{2E}\right)$$

for N_e = const.

i.e., it has the same vacuum-like structure, but with the replacements:

$$\sin 2\tilde{\theta}_{12} = \frac{\sin 2\theta_{12}}{\sqrt{\left(\cos 2\theta_{12} - \frac{A}{\delta m^2}\right)^2 + \sin^2 2\theta_{12}}}$$

$$\delta\tilde{m}^2 = \delta m^2 \frac{\sin 2\theta_{12}}{\sin 2\tilde{\theta}_{12}}$$

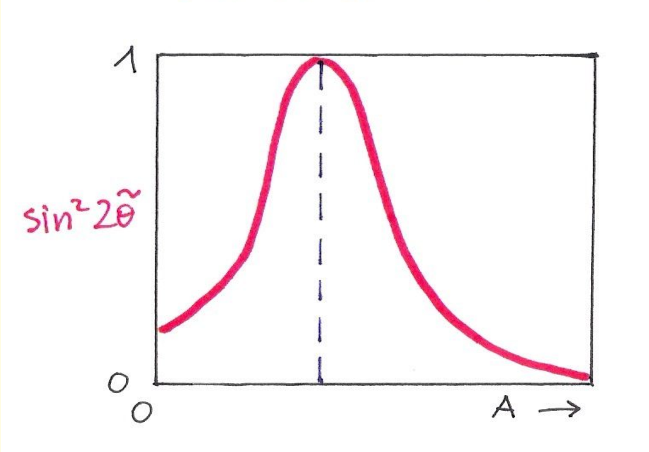
with $A = \pm 2\sqrt{2}\, G_F N_e E$ with $\begin{cases} + \text{ for } \nu \\ - \text{ for } \bar{\nu} \end{cases}$

2. The MSW resonant effect

For $A/\delta m^2 > 0$ the effective parameters have a resonant behaviour around

$$\frac{A}{\delta m^2} \simeq \cos 2\theta$$

MSW resonance



Note: only for ν, no resonance for $\bar{\nu}$ (it is $A < 0$).

Limiting cases:

$A/\delta m^2 << 1$: $(\widetilde{\delta m}^2, \tilde{\theta}) \sim (\delta m^2, \theta)$ ← vacuum-like behaviour

$A/\delta m^2 \sim \cos\theta$: $(\widetilde{\delta m}^2, \tilde{\theta}) \sim (\delta m^2 \sin 2\theta, \pi/4)$ ← resonant behaviour

$A/\delta m^2 >> 1$: $(\widetilde{\delta m}^2, \tilde{\theta}) \sim (A, \pi/2)$ ← matter dominance

Exercise # 6: 2ν oscillation in matter with slowly varying density

If $N_e(x)$ changes slowly from $x = x_i$ (with $\tilde{\theta} = \tilde{\theta}_i$) to $x = x_f$ (with $\tilde{\theta} = \tilde{\theta}_f$) while oscillations are fast, then the averaged P_{ee} probability takes the form:

$$P_{ee}^{2\nu} \sim \cos^2\tilde{\theta}_i \cos^2\tilde{\theta}_f + \sin^2\tilde{\theta}_i \sin^2\tilde{\theta}_f \qquad \leftarrow \text{ adiabatic approximation}$$

… and its application to solar neutrinos

Indeed, it turns out that, for the $(\delta m^2, \theta_{12})$ values chosen by nature, the adiabatic approximation can be applied to solar ν_e.

In this case, $\tilde{\theta}_{12}(x_f) = \theta_{12}$ (vacuum value at the exit from the Sun), while $\tilde{\theta}_{12}(x_i)$ must be evaluated at the production point x_i.

Limiting cases:

$E \lesssim$ few MeV (vacuum dominance): $A/\delta m^2 \lesssim 1$ and $\tilde{\theta}_{12}(x_i) \lesssim \theta_{12}$

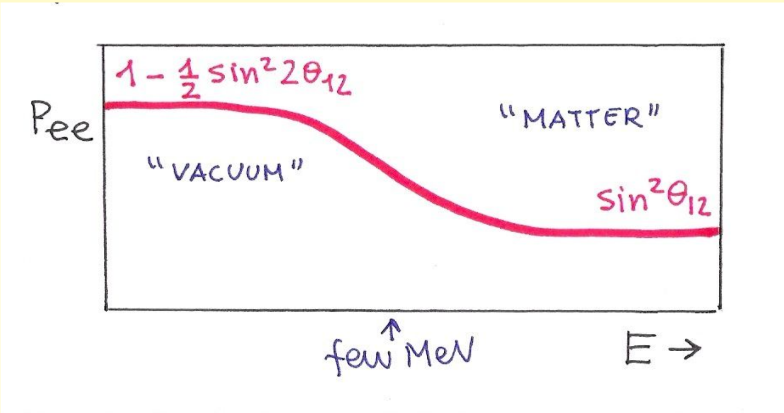
$$P_{ee} \simeq c_{12}^4 + s_{12}^4 = 1 - \frac{1}{2}\sin^2 2\theta_{12}$$

This is the averaged vacuum probability, octant symmetric.

$E \gtrsim$ few MeV (matter dominance): $A/\delta m^2 \gtrsim 1$ and $\tilde{\theta}_{12}(x_i) \sim \frac{\pi}{2}$

$$P_{ee} \simeq \sin^2\theta_{12}$$

This is the matter-dominated probability, octant-asymmetric.



The P_{ee} transition from "low" to "high" E is a signature of matter effects in the Sun.

Thanks to matter effects we can determine the octant of the mixing angle θ_{12}.

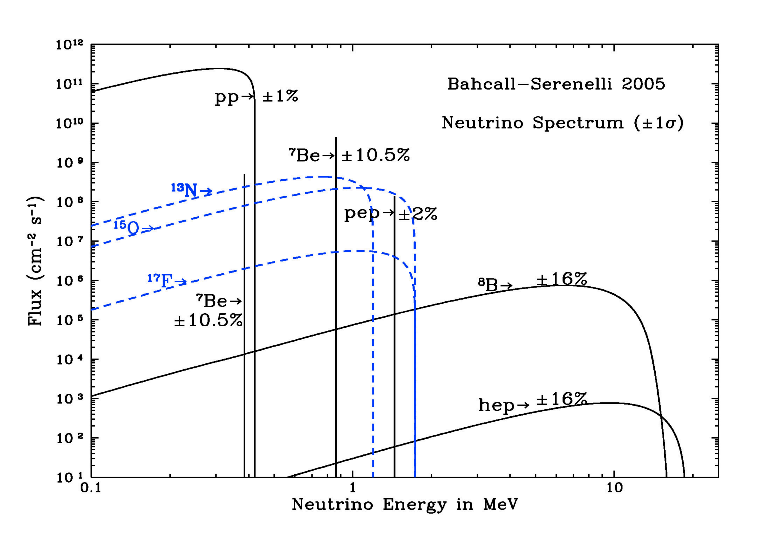
3. Solar neutrinos

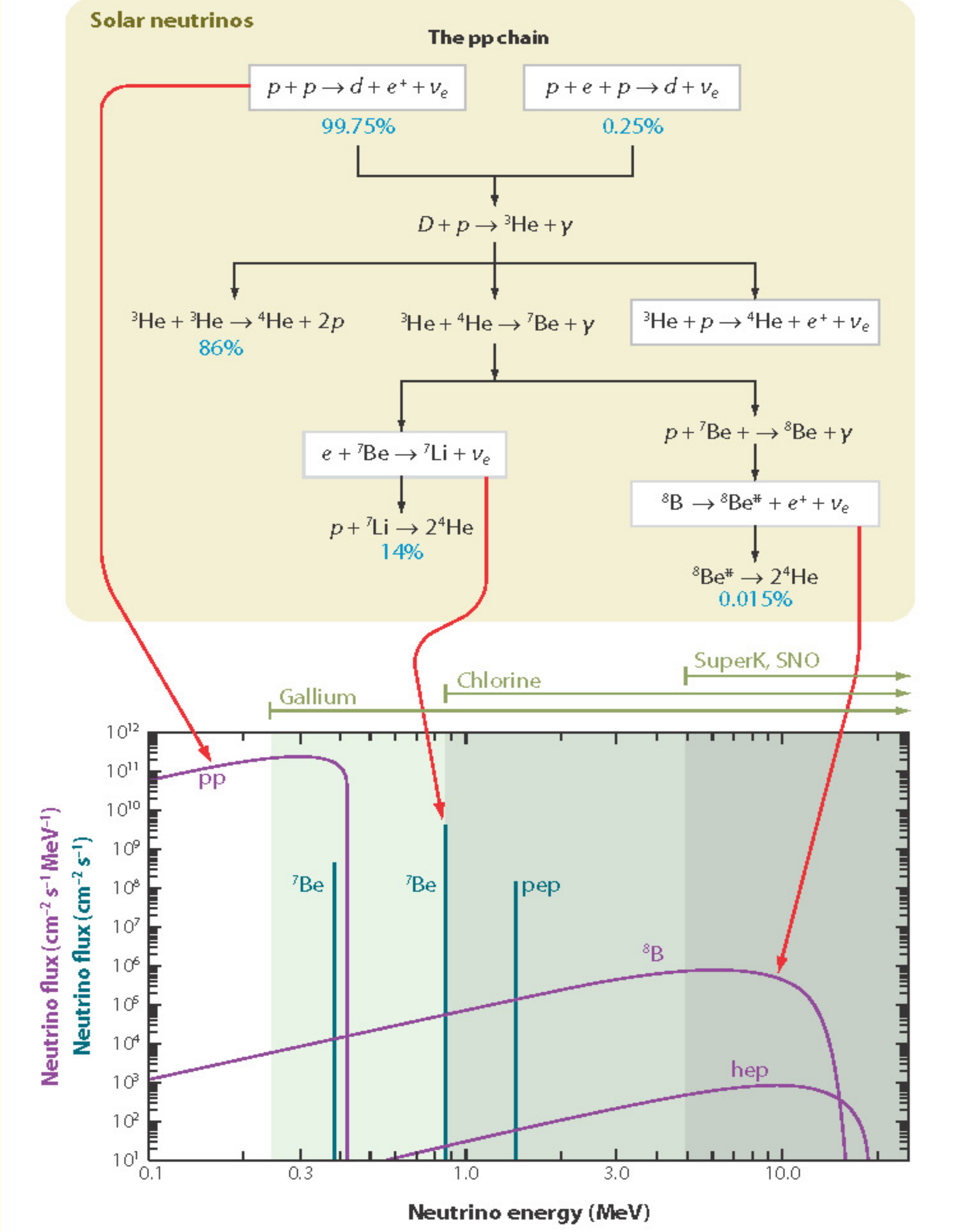
Chlorine (Homestake)

Gallium (GALLEX/GNO, SAGE)

Water (SK, SNO, Borexino)

Deuterium (SNO)





Detection

- **Radiochemical**: count the decays of unstable final-state nuclei. (low energy threshold, but energy and time info lost/integrated)

$$^{37}\mathrm{Cl} + \nu_e \rightarrow {}^{37}\mathrm{Ar} + e \quad (\mathrm{CC})$$ Homestake

$$^{71}\mathrm{Ga} + \nu_e \rightarrow {}^{71}\mathrm{Ge} + e^- \quad (\mathrm{CC})$$ GALLEX/GNO, SAGE

- **Elastic scattering**: events detected in real time with either "high" threshold (Č, directional) or "low" threshold (scintillators)

$$\nu_x + e^- \rightarrow \nu_x + e^- \quad (\mathrm{NC,CC})$$ SK, SNO, Borexino

- **Interactions on Deuterium**: *CC* events detected in real time; NC events separated statistically + using neutron counters.

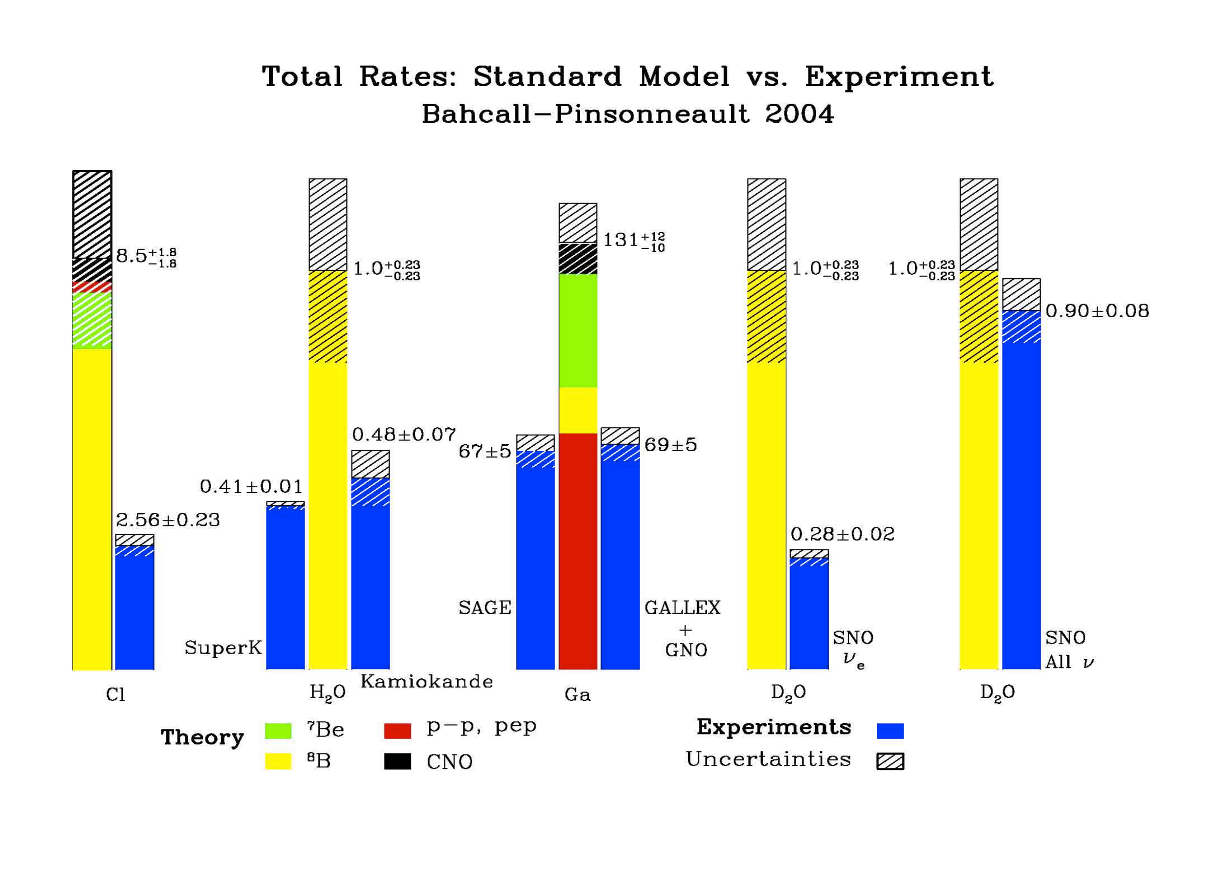
$$\nu_e + d \rightarrow p + p + e^- \quad (\mathrm{CC})$$

$$\nu_x + d \rightarrow p + n + \nu_x \quad (\mathrm{NC})$$

SNO (Sudbury Neutrino Observatory)

Results

All CC-sensitive results indicated a ν_e deficit…



…as compared to solar model expectations

Interpretation

In the "past millennium": Oscillations? Maybe, but…

- large uncertainties in the parameter space or solar model
- no clear evidence for flavor transitions ("smoking gun")

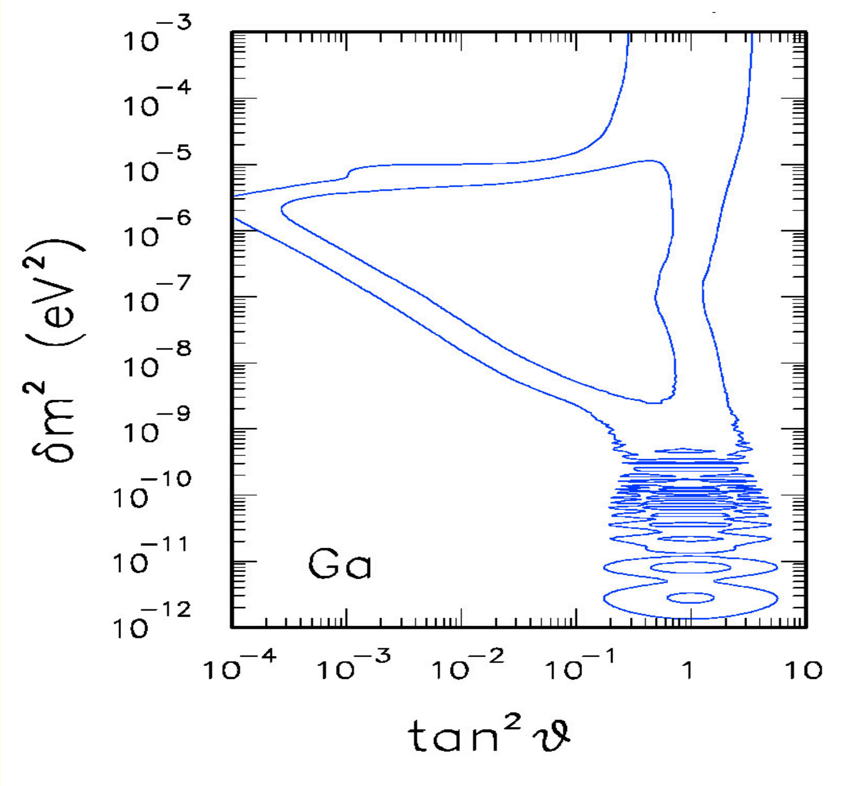
E.g., in Gallium expts:

"matter" (MSW) solutions

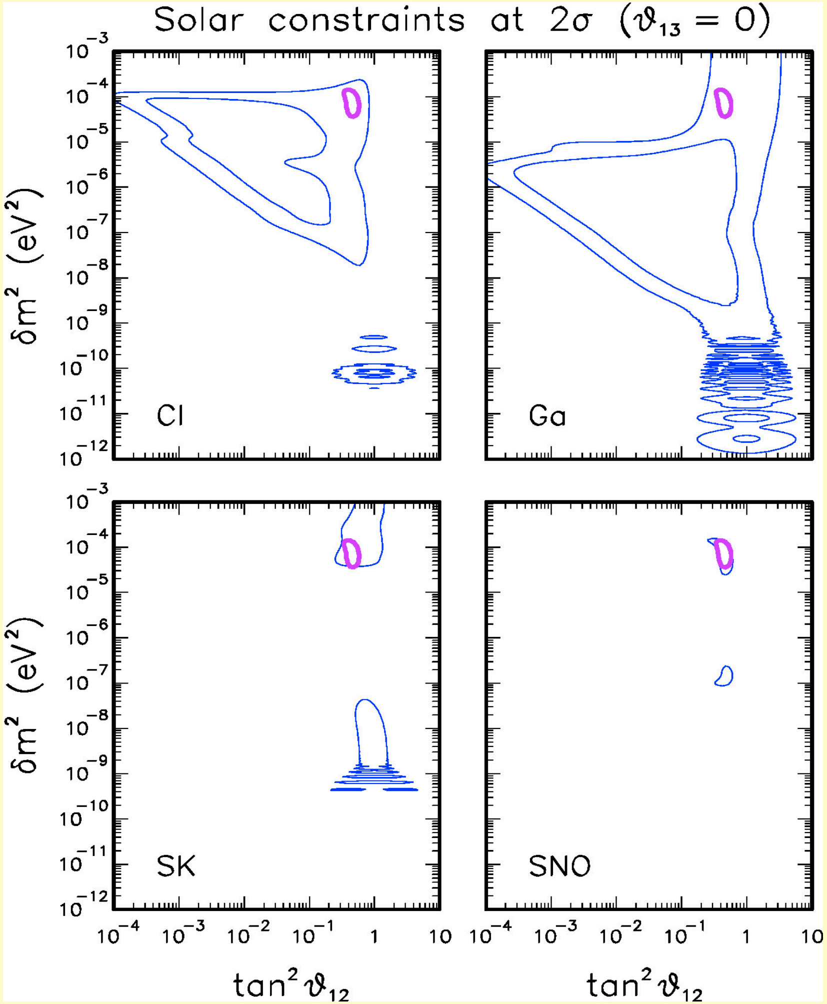
"vacuum" solutions

+ many "exotic" non-oscillatory solutions …

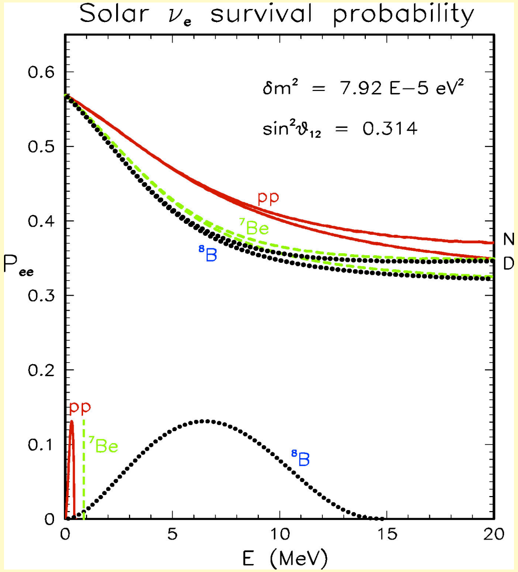
"small" mixing "large" mixing

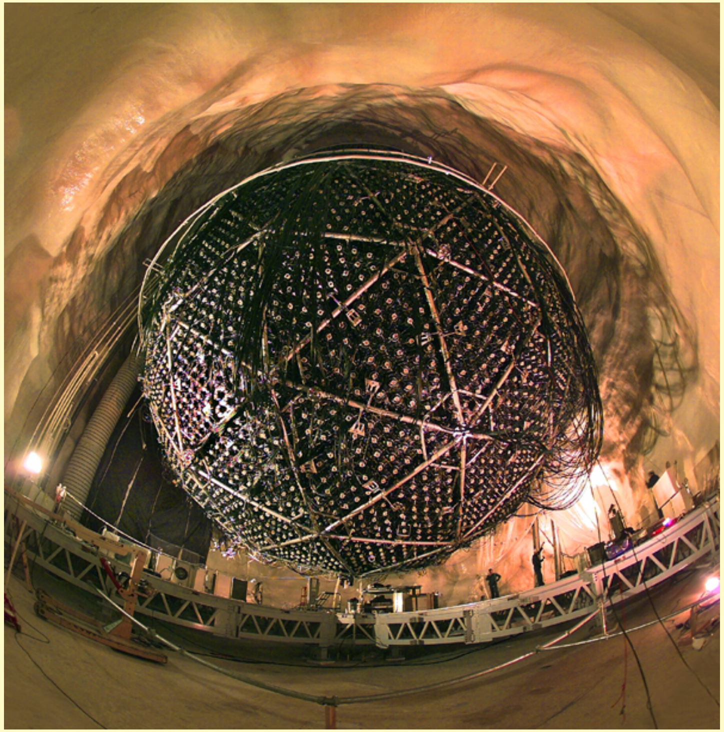


But, in 2002 ("annus mirabilis"), one global solution was finally singled out by combination of data ("large mixing angle" or LMA).



For LMA parameters, evolution is adiabatic in solar matter





Crucial role played by
Sudbury Neutrino Observatory

The breakthrough:
in deuterium one can separate CC events (induced by ν_e only) from NC events (induced by ν_e, ν_μ, ν_τ), and double check via Elastic Scattering events (due to both NC and CC).

$$
\begin{aligned}
\mathrm{CC}: &\quad \nu_e + d \rightarrow p + p + e \\
\mathrm{NC}: &\quad \nu_{e,\mu,\tau} + d \rightarrow p + n + \nu_{e,\mu,\tau} \\
\mathrm{ES}: &\quad \nu_{e,\mu,\tau} + e \rightarrow e + \nu_{e,\mu,\tau}
\end{aligned}
$$

$$\frac{\mathrm{CC}}{\mathrm{NC}} \sim \frac{\phi(\nu_e)}{\phi(\nu_e) + \phi(\nu_{\mu,\tau})}$$

thus:

$$\frac{\mathrm{CC}}{\mathrm{NC}} < 1 \;\Rightarrow\; \phi(\nu_{\mu,\tau}) > 0 \;\Rightarrow\; \nu_e \rightarrow \nu_{\mu,\tau}$$

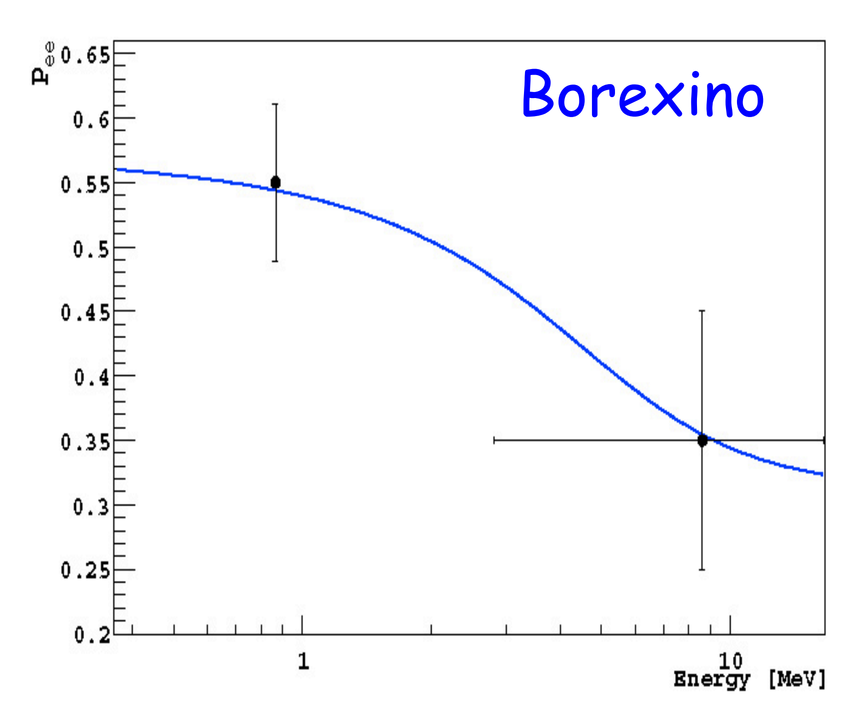
- CC/NC ~ 1/3 < 1
  "Smoking gun" proof of flavor change. Solar model OK! Also:

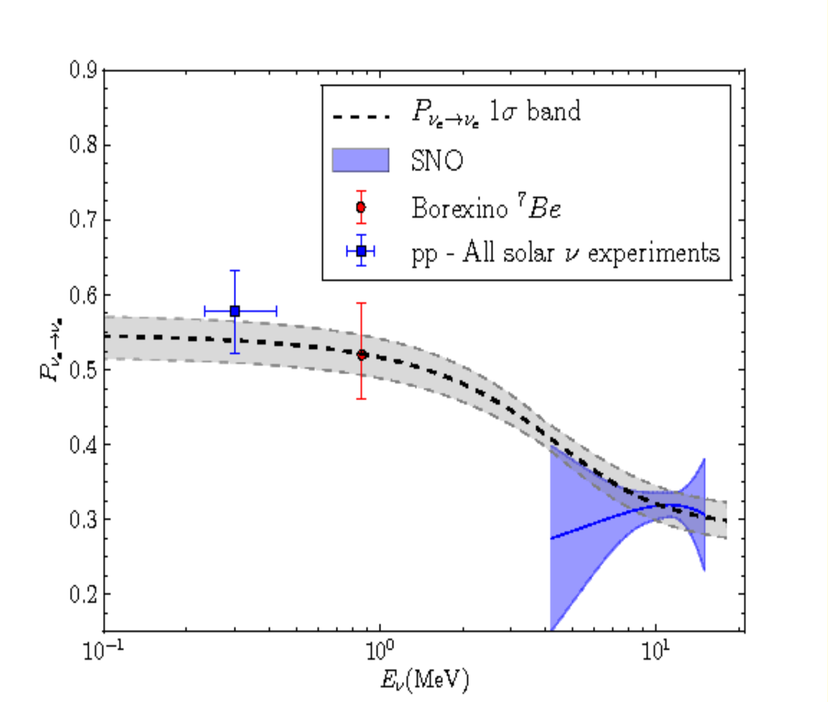
- CC/NC ~ Pee ~ $\sin^2\theta_{12}$ (LMA) ~1/3 < $\frac{1}{2}$
  Evidence of mixing in first octant + matter effects

P_{ee} pattern

Recent, direct confirmation of adiabatic Pee pattern at LMA in a single solar ν experiment: BOREXINO at Gran Sasso

Overall picture including final SNO data [Spectral rise of SNO data at low energy not yet directly observed – anomaly?]





4. KamLAND neutrinos

Also in 2002… : 1000 ton mineral oil detector, "surrounded" by nuclear reactors producing anti-ν_e. Characteristics:

$A/\delta m^2 \ll 1$ in Earth crust
(vacuum approxim. OK)
$L \sim$ 100-200 km
$E_\nu \sim$ few MeV

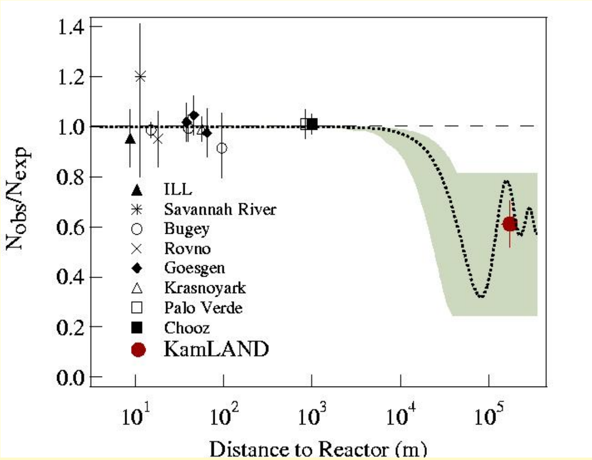
⇨

With previous (δm^2, θ) parameters it is ($\delta m^2 L/4E$) ~ $O(1)$ and reactor neutrinos should oscillate with large amplitude (large θ)

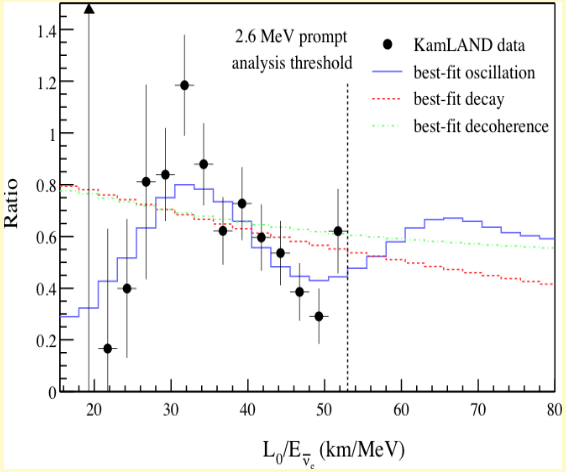
A long-baseline reactor experiment

KamLAND results

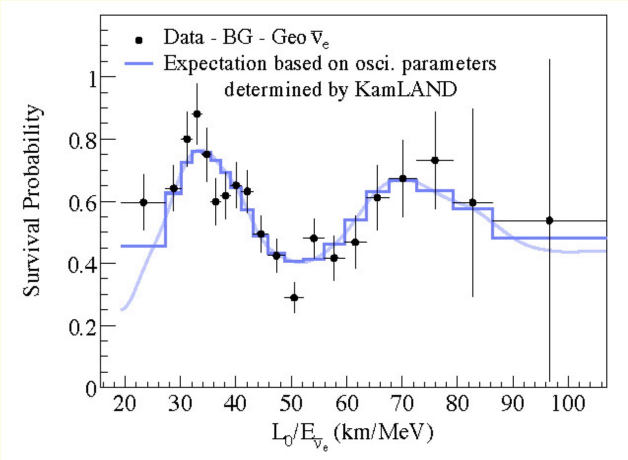
2002: electron flavor disappearance observed



2004: half-period of oscillation observed



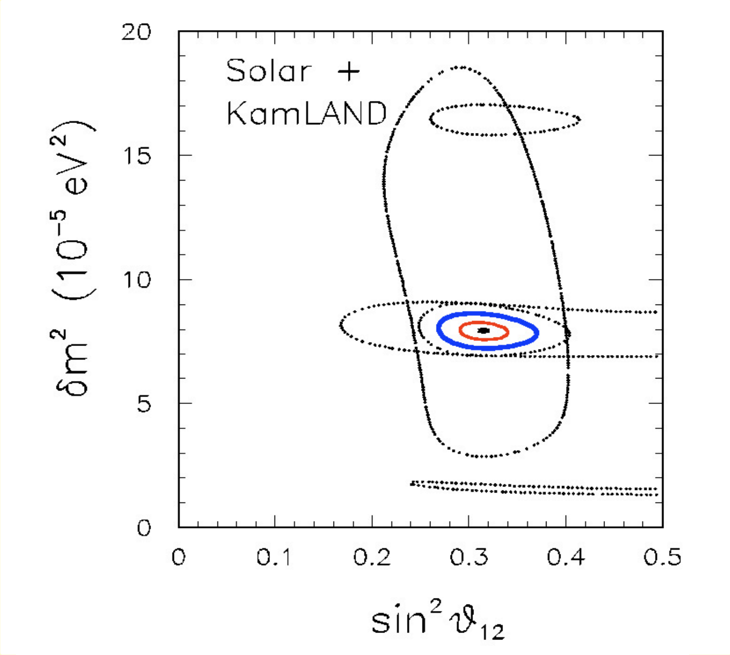
2007: one period of oscillation observed



Direct observation of δm^2 oscillations!

Interpretation in terms of 2ν oscillations

(δm^2, θ_{12}) - complementarity of solar/reactor neutrinos



KamLAND

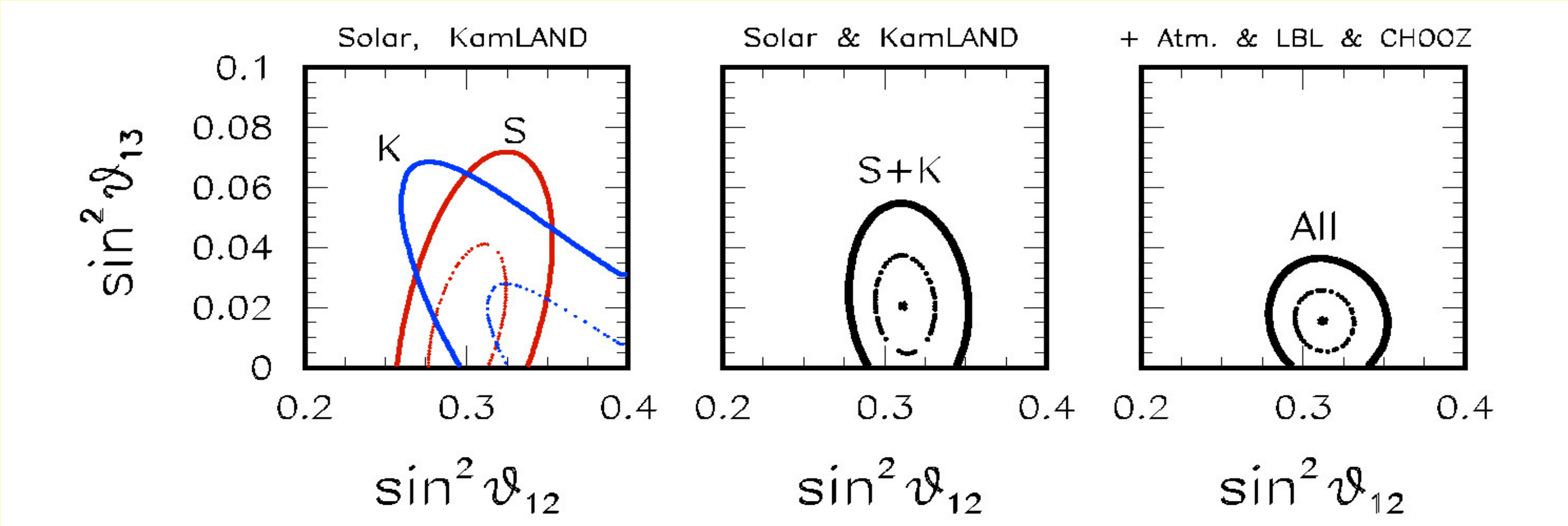
Solar

More refined (3ν) interpretation

Going beyond dominant 2ν oscillations: include subleading effects due to θ_{13} and averaged Δm^2 oscillations in vacuum/matter.

Interesting (small) effects emerge:

"Hints of $\theta_{13} > 0$ from global neutrino data analysis"
[GLF, Lisi, Marrone, Palazzo, Rotunno, PRL 101, 141801 (2008), hep-ph/0806.2649]



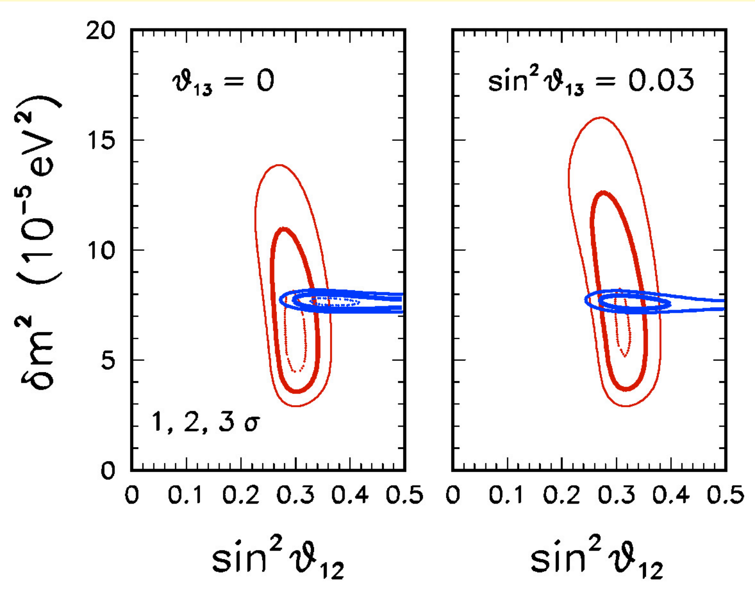
2008: A hint of $\theta_{13} > 0$, caming from the slight tension on θ_{12} (solar vs KamLAND) and from different correlation between mixing angles, related to different relative signs in P_{ee} (survival probability) of solar vs KamLAND:

Solar, high energy (LMA MSW):

$$P_{ee} \simeq (1 \underset{-}{-} 2s_{13}^2)(\underset{+}{+}s_{12}^2)$$

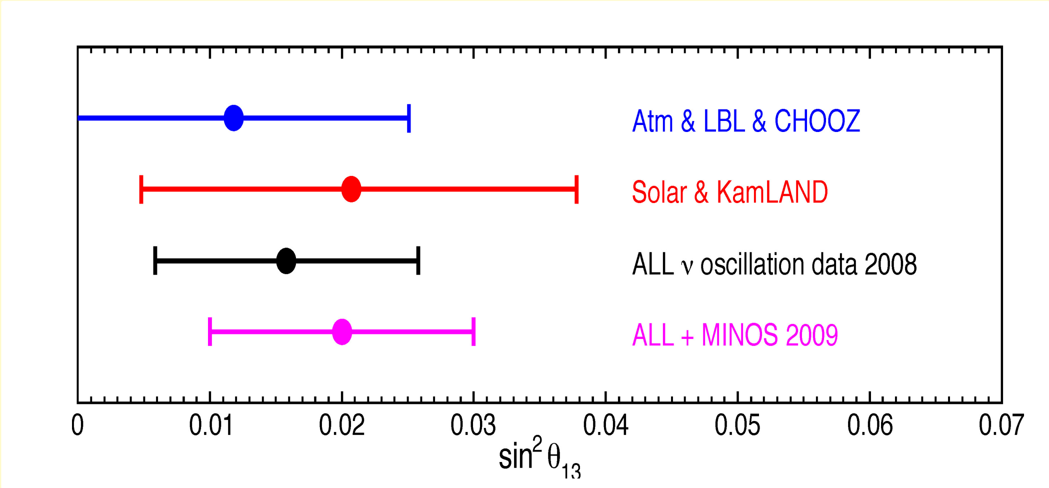
Reactor (~vacuum): KamLAND

$$P_{ee} \simeq (1 \underset{-}{-} 2s_{13}^2)(1 \underset{-}{-} 4s_{12}^2 c_{12}^2 \sin^2(\delta m^2 L/4E))$$



Slight "tension" on θ_{12} could be reduced for $\theta_{13} > 0$

2009: there were already a few independent hints of $\theta_{13} > 0$:



The grand total was:

$\sin^2\theta_{13} \approx 0.02 \pm 0.01$ (all 2009 data) arXiv:0905.3549

which represented an encouraging - and experimentally testable - 2σ indication. Actually, as already discussed, T2K (appearance) found similar θ_{13} values in 2011, and a definitive measurements emerged in 2012 from reactors (disappearance).

PDG 2012: $\sin^2\theta_{13} \approx 0.024 \pm 0.003$

This is an important test of the overall consistency of 3ν oscillations.

4. Global 3ν analysis of all oscillation data

(within the 3ν framework)

In the following:

- Oscillation parameters are extracted with their correlations from solar, atmospheric, accelerator and reactor neutrino data, as of summer 2012 (Neutrino Conference in Kyoto).

- Full 3ν probabilities included, no approximation.

- **Note about methodology**

  We combine first **LBL accelerator** data with **solar+KamLAND** data, since the latter provide the "solar parameters" needed to calculate the full 3ν LBL probabilities in matter. So, the sequence of contraints will be shown as:

  (LBL + Solar + KamLAND) + (SBL reactor) + (SK atm)

- Extracted from

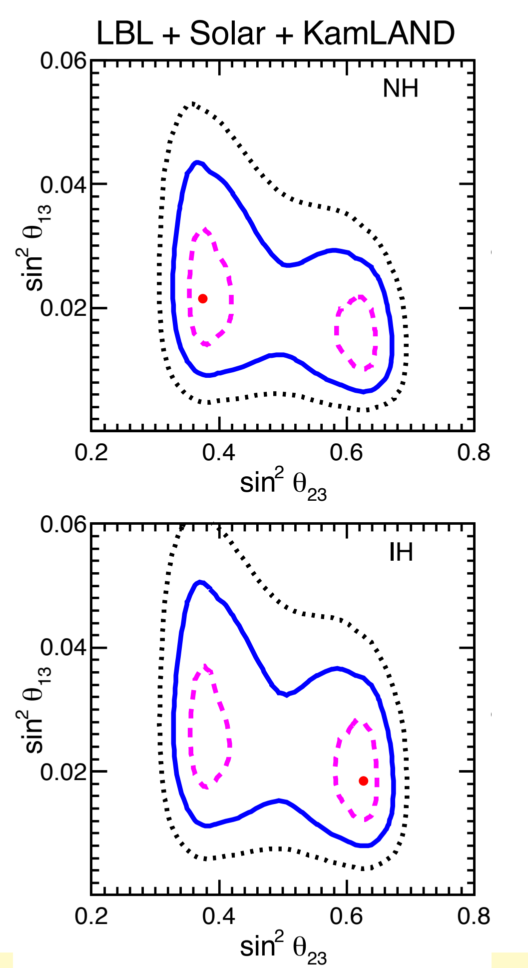
  G.L.F., E. Lisi, A. Marrone, D. Montanino, A. Rotunno, A. Palazzo,
  "Global analysis of neutrino masses, mixings and phases: entering the era of leptonic CP violation searches"
  Phys. Rev. D 86, 013012 (2012) , arXiv:1205.5254v3]

4.1 $(\theta_{13}, \theta_{23})$ correlations

From 2012 LBL appearance + disappearance data plus solar + KamLAND data:

NH

IH



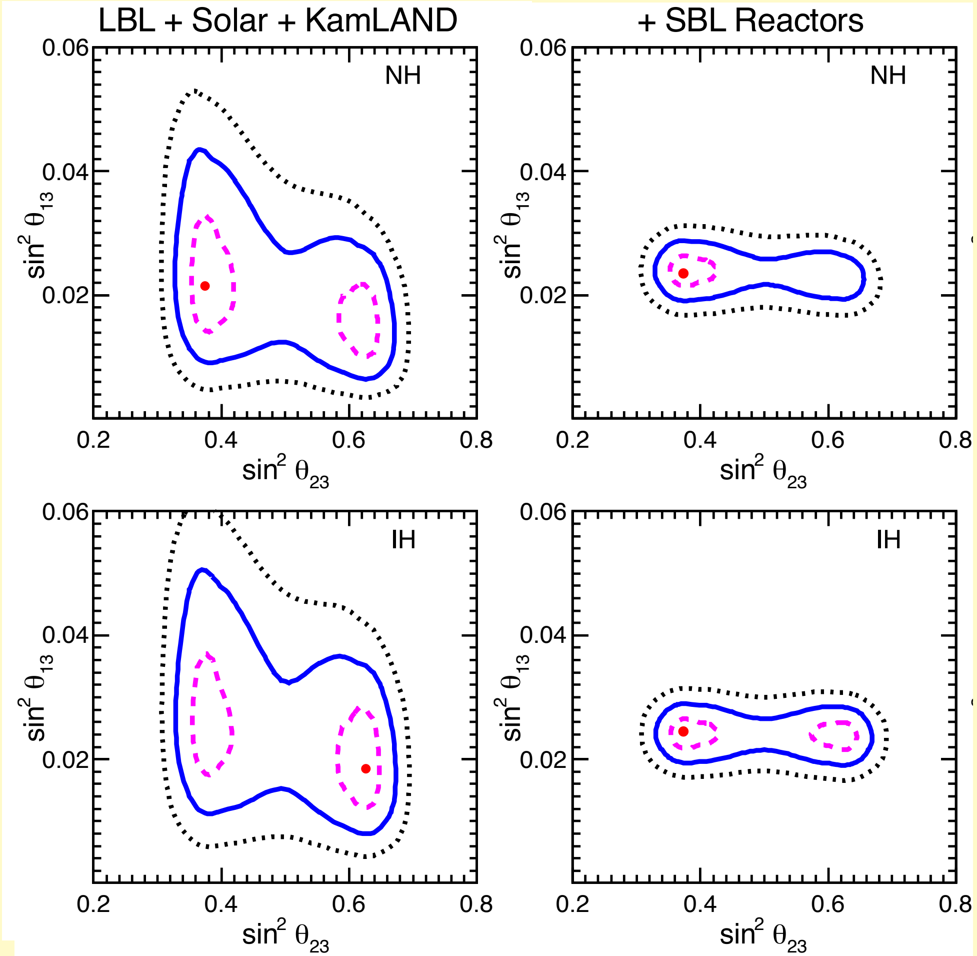
For both hierarchies, NH & IH:

- Latest LBL disappearance data from T2K and MINOS favor nonmaximal θ_{23}
- Two quasi-degenerate θ_{23} solutions are seen to emerge, in some anticorrelation with θ_{13}. The two solutions merge above ~**1**σ.
- Solar + KamLAND data happen to prefer just $\sin^2\theta_{13} \sim 0.02$, and are unable to solve the octant degeneracy.

Adding SBL reactor data (Chooz, Double Chooz, Daya Bay, RENO):

NH

IH



for both NH & IH $\sin^2\theta_{13} \sim 0.02$ preferred!

NH

further preference for the solution with

θ_{23} in the 1st octant

IH

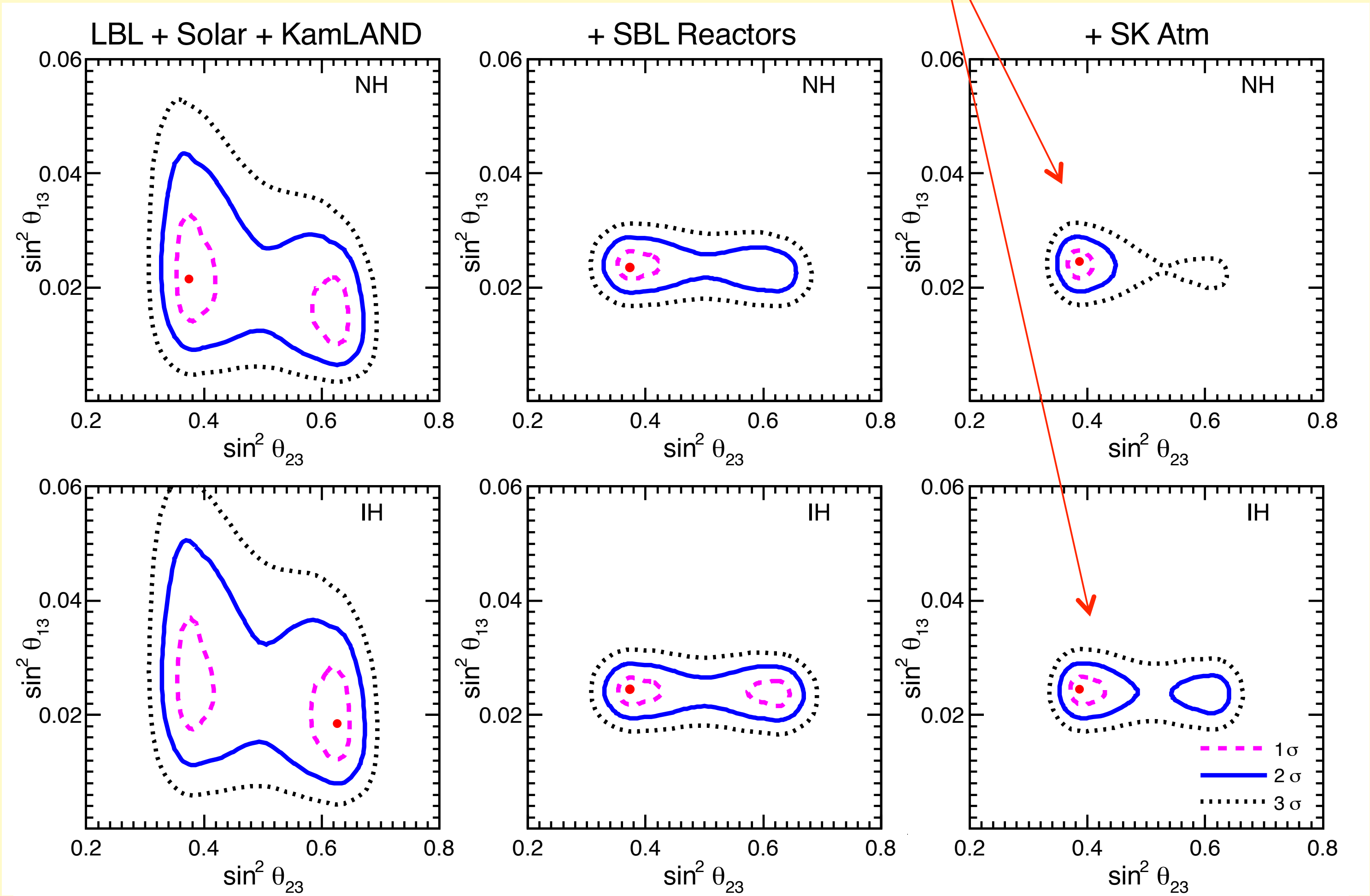
only a marginal preference for θ_{23} in the 1st octant

Adding SK atm data: the preference for θ_{23} in the 1st octant is more evident

No hint about hierarchy yet…

NH

IH

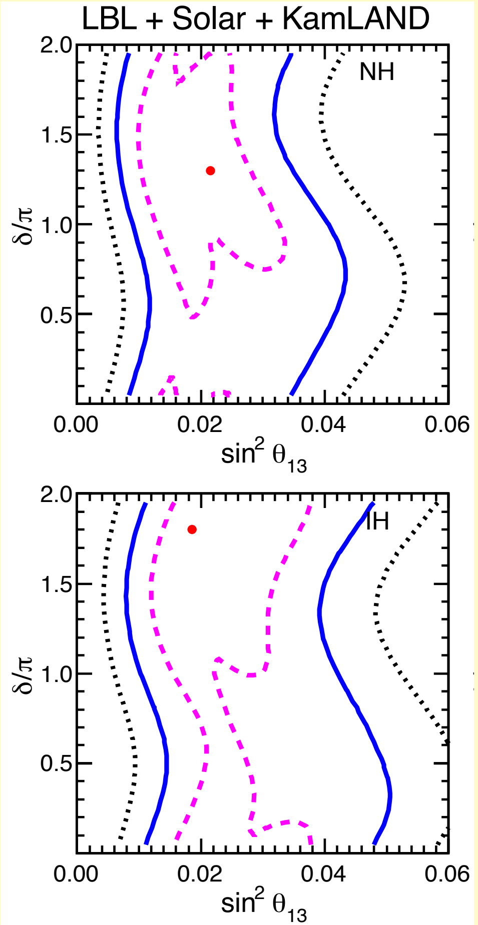


4.2 (θ_{13}, δ_{CP}) correlations

With only LBL appearance + disappearance data plus solar + KamLAND data:

NH

IH



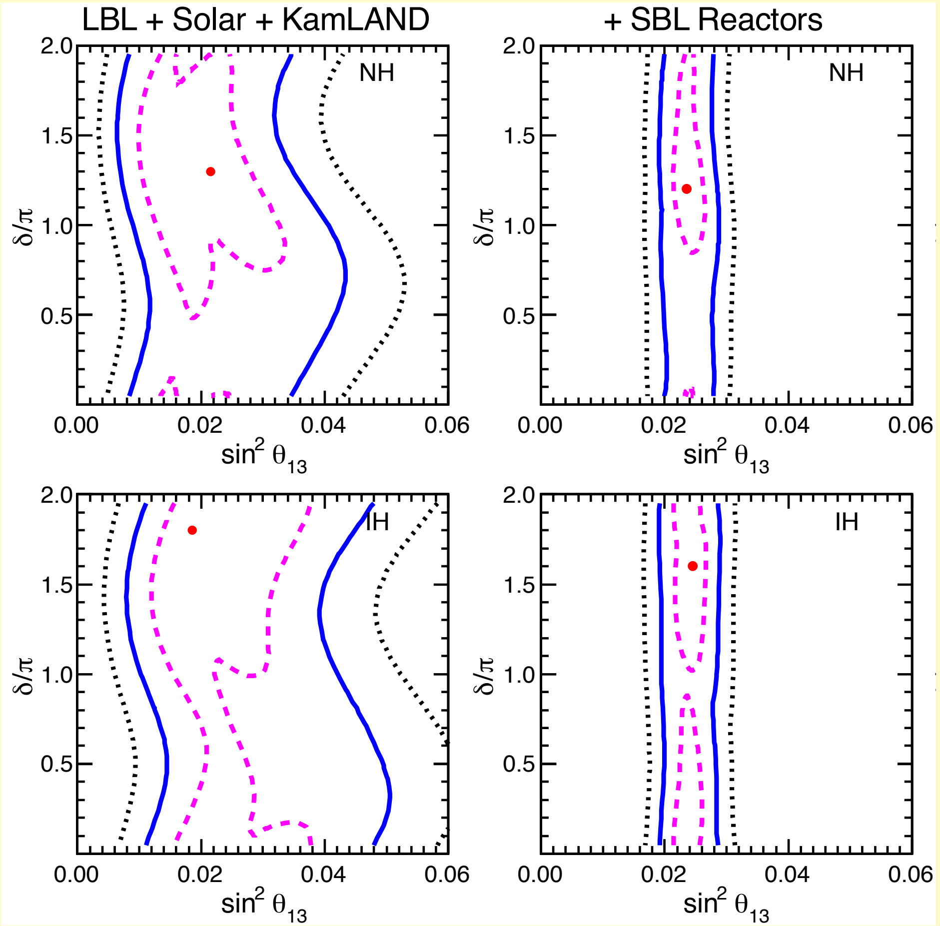
- no significant sensitivity to δ_{CP} yet: δ is basically unconstrained at $\sim 1\sigma$.
- Fuzzy 1σ contours are a side effect of θ_{23} degeneracy

Adding SBL reactor data (Daya Bay, RENO, Double Chooz):

SBL reactor data restrict θ_{13} and reduce degeneracy effects on the nσ contours.

NH

IH



NH

at most ~ 1σ sensitivity

IH

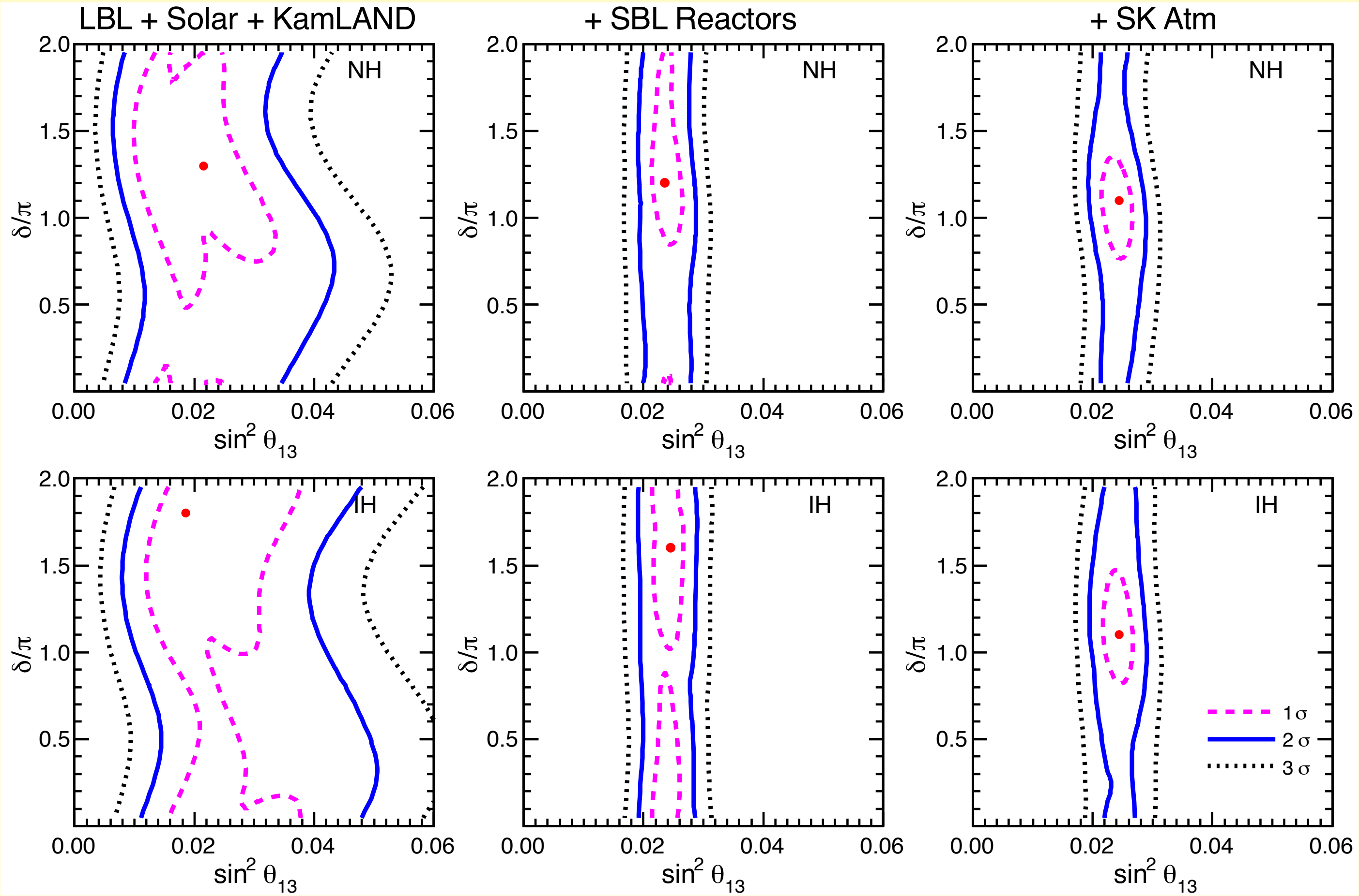
not yet sensitivity at ~ 1σ

Adding SK atmospheric data:

We find a ~ 1σ preference for θ ~ π (helps fitting sub-GeV e-like excess in SK).

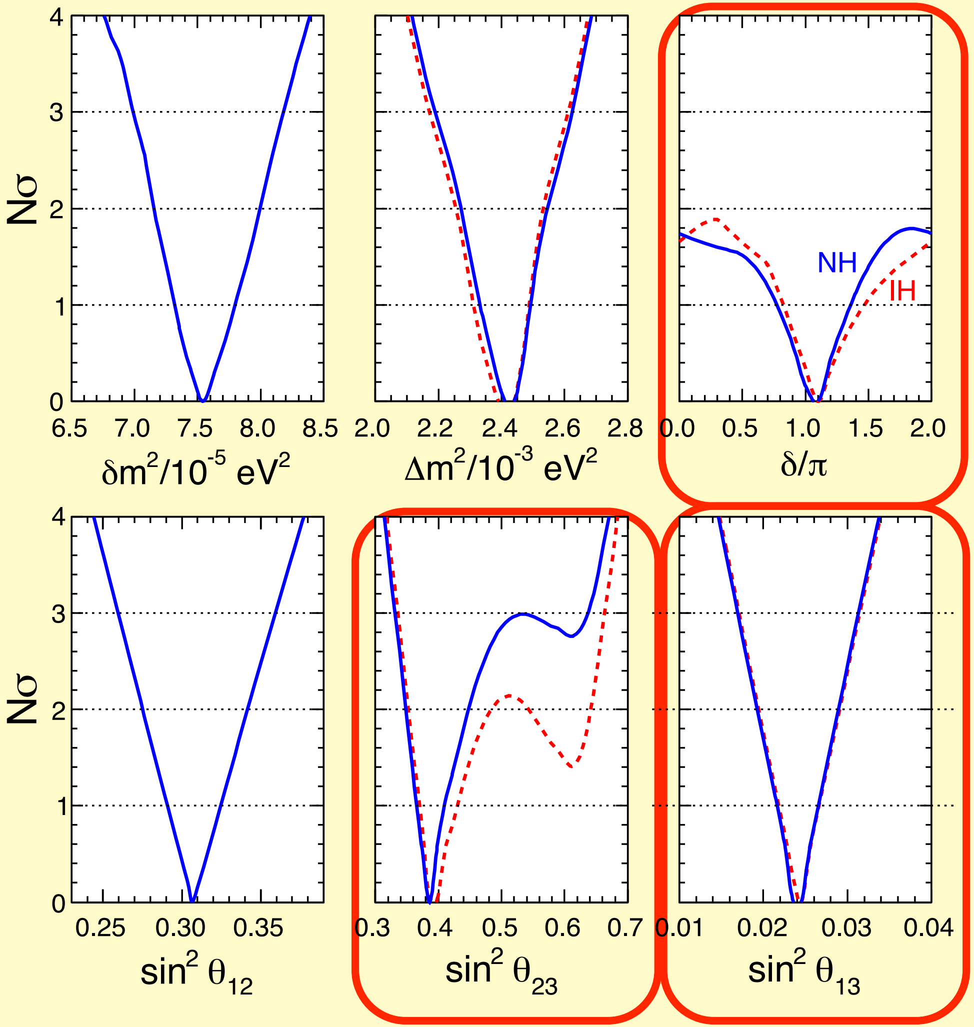
NH

IH



4.3 Conclusions

Synopsis of global 3ν oscillation analysis



- Previous hints of $\theta_{13} > 0$ are now measurements! (and basically independent of old/new reactor fluxes)
- Some hints of θ_{23} in the 1st octant are emerging at $\sim 2\sigma$, worth exploring by means of atm. and LBL+reac. data
- A possible hint of $\delta_{CP} \sim \pi$ is emerging from atm. data [Is the PMNS matrix real?]
- So far, no hints for NH ⟷ IH

Numerical 1σ, 2σ, 3σ ranges:

Parameter	Best fit	1σ range	2σ range	3σ range
$\delta m^2/10^{-5}$ eV^2 (NH or IH)	7.54	7.32 – 7.80	7.15 – 8.00	6.99 – 8.18
$\sin^2\theta_{12}/10^{-1}$ (NH or IH)	3.07	2.91 – 3.25	2.75 – 3.42	2.59 – 3.59
$\Delta m^2/10^{-3}$ eV^2 (NH)	2.43	2.33 – 2.49	2.27 – 2.55	2.19 – 2.62
$\Delta m^2/10^{-3}$ eV^2 (IH)	2.42	2.31 – 2.49	2.26 – 2.53	2.17 – 2.61
$\sin^2\theta_{13}/10^{-2}$ (NH)	2.41	2.16 – 2.66	1.93 – 2.90	1.69 – 3.13
$\sin^2\theta_{13}/10^{-2}$ (IH)	2.44	2.19 – 2.67	1.94 – 2.91	1.71 – 3.15
$\sin^2\theta_{23}/10^{-1}$ (NH)	3.86	3.65 – 4.10	3.48 – 4.48	3.31 – 6.37
$\sin^2\theta_{23}/10^{-1}$ (IH)	3.92	3.70 – 4.31	3.53 – 4.84 $\oplus$ 5.43 – 6.41	3.35 – 6.63
δ/π (NH)	1.08	0.77 – 1.36	—	—
δ/π (IH)	1.09	0.83 – 1.47	—	—

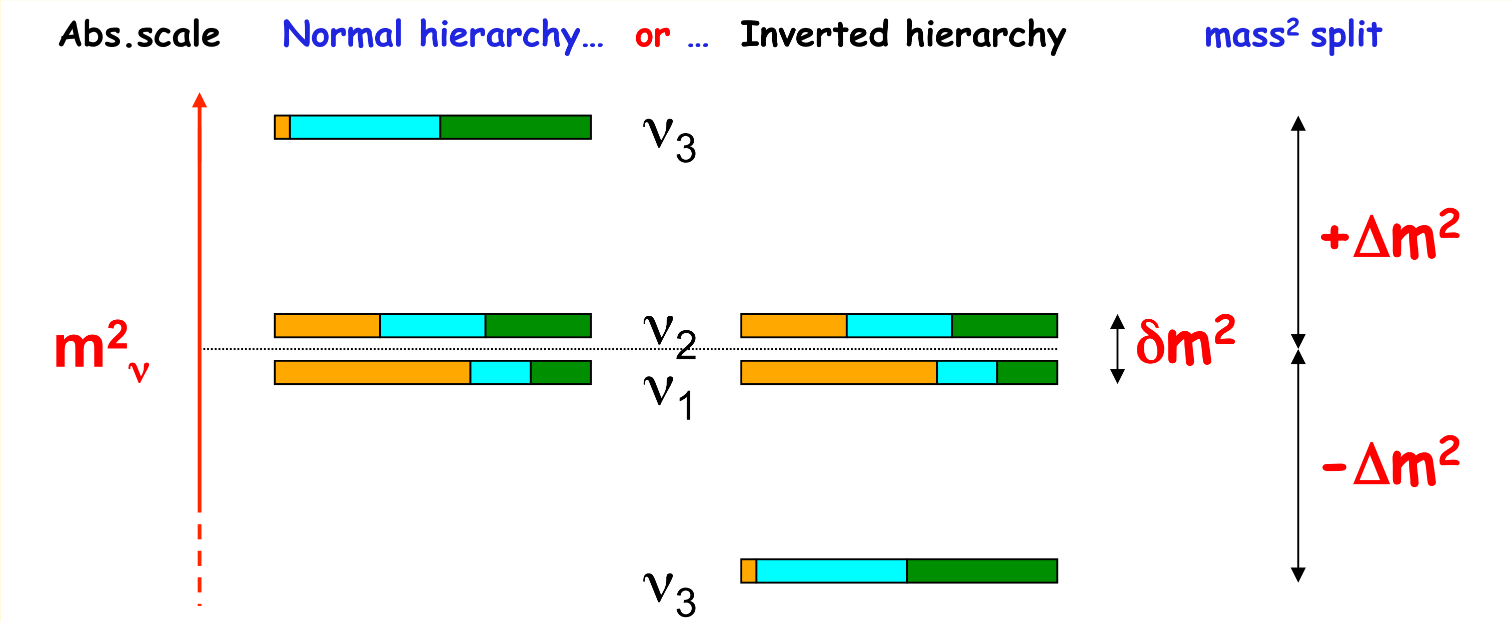
Fractional 1σ accuracy [defined as 1/6 of $\pm 3\sigma$ range]

δm^2	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	Δm^2
2.6%	5.4%	10%	14%	3.0%

Hierarchy differences well below 1σ for various data combinations

With 1 digit accuracy: 3ν framework in just one slide!

Flavors = e μ τ

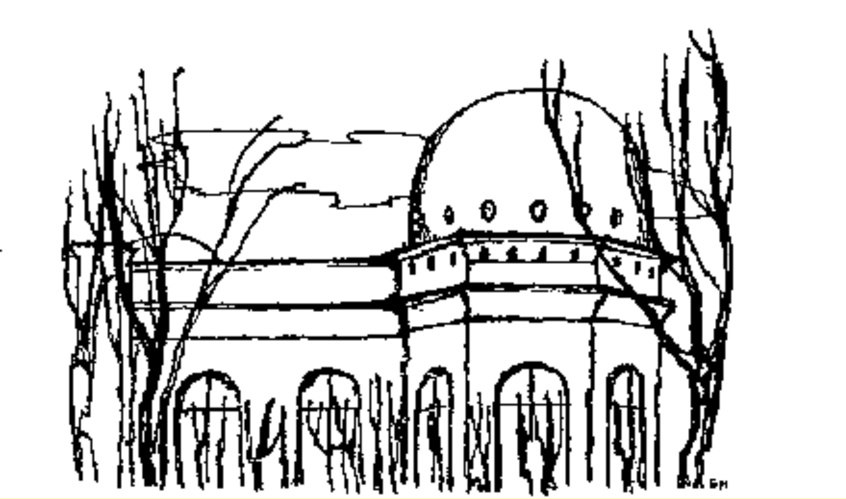


Knowns:

$\delta m^2 \sim 8 \times 10^{-5}$ eV^2
$\Delta m^2 \sim 2 \times 10^{-3}$ eV^2
$\sin^2\theta_{12} \sim 0.3$
$\sin^2\theta_{23} \sim 0.5$
$\sin^2\theta_{13} \sim 0.02$

Unkowns:

δ (CP)
sign(Δm^2)
octant($\sin^2\theta_{23}$)
absolute mass scale
Dirac/Majorana nature



Ecole Internationale Daniel Chalonge
17th Paris Cosmology Colloquium 2013

"The new standard model of the Universe: Lambda Warm Dark Matter (LWDM) Theory vs. Observations"

Thanks for your attention!